Bonds vs. Equities: Information for Investment*

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Abstract

We provide robust empirical evidence that uncovers the reason for the observed closer relationship between the bond market versus the equity market and the macroeoconomy. Our results indicate that the tight bond market-macroeconomy link is not due to differences in the investor base, but instead to the unique transformations of asset volatility and leverage that credit spreads and equity volatility represent. We focus on the investment channel. Using firm-level data, we find that the sensitivity of investment to equity volatility is highly significant, but changes sign in the cross section of firms depending on their distance to default. This sign change confounds aggregate inference. We rationalize these findings using a simple structural model of credit risk and investment with debt overhang.

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1 Introduction

Economists and practicioners alike have long argued that there is a tight connection between bond markets and the macroeconomy. Friedman and Kuttner (1992) show that the spread between commercial paper and Treasury bills forecasts recessions. Gilchrist and Zakrajšek (2012) use firm-level data to construct a credit spread measure with substantial predictive power for consumption, inventories, and output. Philippon (2009) constructs Tobin's q from bond market data and shows that it outperforms equity-market q in predicting firm-level investment.[1]

An open question, however, is why bond market data appears to have better forecasting power for real outcomes (in particular during recessions) than equity market data. Is it because bond markets have more "smart money" and better reflect changes in financial market distortions? Or because bonds capture downside risk better, while equity prices are more affected by growth options? Our empirical evidence and model of investment with debt overhang support the latter explanation.

We focus on an investment channel. Bloom (2009) shows that shocks to uncertainty measured using implied equity volatility forecast lower investment. However, recent work by Gilchrist, Sim, and Zakrajšek (2014) shows that controlling for credit spreads substantially reduces the impact of equity volatility on investment. The structural connection between credit spreads and equity volatility has been underappreciated

[1] See also the important contributions by Friedman and Kuttner (1998), Bernanke (1990), Gertler and Lown (1999), and Gilchrist, Yankov, and Zakrajšek (2009), Giesecke, Longstaff, Schaefer, and Strebulaev (2014), Krishnamurthy and Muir (2017).

in the literature using financial market data to forecast economic activity.[2] As emphasized in the seminal works of Merton (1974) and Leland (1994), bond spreads and equity volatility are tightly related due to the fact that they both reflect a combination of firms' asset volatilities and leverage. Relatedly, Atkeson, Eisfeldt, and Weill (2017) show that (the inverse of) equity volatility and credit spreads contain very similar information about firms' financial soundness.

We establish four main facts. First, credit spreads drive out equity volatility in an empirical model of the sensitivity of firm-level investment to equity volatility and credit spreads. Second, this result is due to the heterogeneity of the sensitivity of investment to equity volatility in the cross section of firms. The sensitivity of investment to equity volatility is positive for firms far enough away from default, and otherwise it is negative. These different signs in the cross section drive the overall effect to be less significant than credit spreads. Third, the levels of both equity volatility and credit spreads are in large part driven by asset volatility and leverage, as predicted by structural models of credit risk.[3] However, credit spreads have higher loadings on leverage, while equity volatility loads more on asset volatility. This is intuitive given the priority of debt versus equity in firms' capital structures. Fourth, the sensitivity of investment to *asset* volatility is positive for all firms. Our results indicate that the closer relation between bond markets and the macroeconomy

[2]See Stock and Watson (2003) for a comprehensive survey on research using financial markets data to forecast macroeconomic outcomes.

[3]Collin-Dufresn, Goldstein, and Martin (2001) show that *changes* in credit spreads also have a common component that appears unrelated to structural determinants, however we show the majority of variation in credit spread levels, and about one third of credit spread changes, can be explained by asset volatility and financial leverage.

is not due to differences in the investor base or the presence of financial frictions, but instead is due to the precise transformation of asset volatility and leverage that equity volatility and credit spreads represent. We confirm this finding using credit spreads constructed from equity data alone.

[Figures 4, 6 and 5 about here.]

We present VAR evidence which aggregates these findings to the economy-wide level. The aggregate evidence confirms our micro-level results and documents the importance of our findings for understanding the role of uncertainty and credit spreads on aggregate activity. The aggregate investment response to a positive shock to asset volatility is positive while the response to a positive shock to credit spreads (a market proxy for distance-to-default or leverage) is negative. Figure 4 also provides an intuitive visual representation of what is driving our results. This figure plots the time series and cross section of firms' elasticity of investment with respect to equity volatility. As shown in the picture, firms with lower credit spreads which are further away from default display a positive elasticity of investment, while firms with higher credit spreads display a negative elasticity. Aggregate effects are driven by the movement of the entire cross section of firms away from and closer to their respective default boundaries. In contrast, Figure 6 shows that the elasticity of investment to credit spreads is negative for all firm quarters.

The starting point for our empirical work is a simple replication of the finding in Gilchrist, Sim, and Zakrajšek (2014) showing that (i) individually, the sensitivity of investment to both equity volatility and credit spreads is negative, but that (ii) if both are included the sensitivity of investment to credit spreads is essentially unchanged but the sensitivity of investment to equity volatility is reduced by 70%. We then show that this is because the sensitivity of investment to equity volatility varies systematically in the cross section of firms with high and low credit spreads. Firms in the lowest tercile of credit spreads have a statistically and economically significant *positive* elasticity of investment to equity volatility, while firms in the highest tercile have a statistically and economically significant *negative* elasticity. By contrast, the elasticity of investment to credit spreads is always negative. As a result, the elasticity of investment to equity volatility controlling for both credit spreads and the interaction between credit spreads and equity volatility is highly significant and positive. That is, controlling for high-credit-spread firms' negative elasticity of investment with respect to equity volatility, the relation between equity volatility and investment becomes robustly positive.

Using "fair value spreads" constructed using equity market data alone, we repeat the above analysis and show that the results are virtually identical. These fair value spreads are constructed using the results from structural models of credit risk which derive credit spreads from asset volatility and leverage.[4] Thus, the different information in equity and bond data for investment is not due to a difference in investor base or market segmentation.

According to structural models of credit risk, both credit spreads and equity volatility are driven by asset volatility and leverage. We document this using our panel data. We extract asset volatility by deleveraging equity volatility. The frac-

[4]See Arora, Bohn, and Zhu (2005) and Nazeran and Dwyer (2015).

tions of variation in equity volatility and credit spreads explained by asset volatility and leverage (including firm and time fixed effects) are 87% and 57%, respectively. In changes, the fractions of variation are 79% (equity volatility) and 35% (credit spreads). In terms of loadings, these regressions show that equity volatility is driven by both asset volatility and leverage. By contrast, the loading for credit spreads on leverage is three times as large as the loading on asset volatility in levels and fifteen times larger in changes. Thus, an increase in equity volatility could either signal an increase in asset volatility or in leverage. As credit spreads are mostly driven by leverage, an increase in credit spread always signals an increase in leverage.

We use this decomposition to show that the reason that equity and bond market data fare differently in explaining investment is due to the fact that asset volatility and leverage have opposite effects on investment. The elasticity of investment to asset volatility is significantly positive for all firms, indicating that the marginal revenue product of capital is a convex function of asset volatility shocks (see Leahy and Whited, 1996).[5] A one standard deviation increase in asset volatility is associated with a 9% standard deviation increase in firms' investment rate. Interestingly, the positive impact of asset volatility on investment gets stronger for firms with lower credit spreads. Thus, when credit spreads are low, the asset volatility effect dominates, and investment increases following a positive shock to equity volatility, while the reverse is true when credit spreads are high. As credit spreads are mostly driven by leverage, the leverage effect always dominates and the sensitivity of investment

[5]See Abel, Eberly, et al. (1994) and Dixit and Pindyck (2012) for models with convex and non-convex costs of adjustment.

to credit spreads is always negative.

We provide a model to illustrate the structural decomposition and the heterogeneity in the cross section of the relationship between equity volatility and investment. In our model, two forces drive the investment decision: debt overhang and the option value of equity. We find that an increase in asset volatility has a positive effect on investment unless the distance-to-default decreases sufficiently for the debt overhang problem to dominate. Importantly, equity volatility is a poor signal of uncertainty. Indeed, if the option value of equity increases sufficiently following a positive shock to asset volatility, equity volatility might decrease. As a result, the best signals of debt overhang and option value of equity are credit spreads and asset volatility.[6]

2 Empirical Evidence

2.1 Data and Definition

Data Collection We use S&P's Compustat quarterly database from 1984:Q1 to 2018:Q4. We exclude firms in the financial sector (SIC code 6000 to 6999) and utility sector (SIC code 4900 to 4949) and observations with negative sales. We use daily returns from the Center for Research in Security Prices (CRSP) database. Bond prices come from the Lehman/Warga (1984-2005) and ICE databases (1997-2018).

[6] We note that even in real-option models with fixed costs and inaction regions, the ultimate impact of an increase in asset volatility is to increase the frequency of non-zero investment rates (see Bloom, 2009). See also DeMarzo, Fishman, He, and Wang (2012) and Sundaresan, Wang, and Yang (2015)

We require non-missing data for variables we use to construct investment rate, equity volatility, market leverage, and credit spreads and impose a restriction that a firm need to be in the panel for at least 3 years. This selection criterion yields 1,161 unique firms with 54,033 firm-quarter observations. To ensure that our results are not driven by extreme values, we trim the sample by replacing the top and bottom 0.5% of regression variables as missing values. Below, we describe how we construct our key variables.

Investment and Equity Volatility We define investment rate as capital expenditures in quarter t scaled by net property, plant, and equipment in quarter $t-1$. Idiosyncratic equity volatility is constructed in two steps. For each firm-fiscal quarter, we extract daily excess returns using the Carhart four-factor model. Then for each regression we calculate the standard deviation of residuals over one quarter, and obtain quarterly firm-specific idiosyncratic equity volatility. We only keep observations for quarters with more than 30 trading days.

Credit Spreads We follow Gilchrist and Zakrajšek (2012) to compute bond-level credit spreads. First, we construct a theoretical risk-free bond that replicates exactly the promised cash flows. The price of this risk-free bond is calculated by discounting the promised cash flows using continuously-compounded zero-coupon Treasury yields from Gürkaynak, Sack, and Wright (2007). The credit spread of an individual bond is the difference between the yield of the actual bond and the yield of the corresponding risk-free bond. We then define the credit spread of a firm as the quarterly average of the month-end credit spreads of all bonds issued by that firm.

Asset Volatility We use data on realized equity volatility, debt, and market capitalization to derive a measure of firm-level idiosyncratic asset volatility implied by the Merton model. Realized equity volatility is computed as the standard deviation of historical daily stock returns over a quarter. The firm's debt is assumed to be equal to the sum of its current liabilities and one-half of its long-term liabilities. We implement the iterative procedure proposed by Bharath and Shumway (2008), and then use the resulting asset values to generate times series of daily asset returns. With time series of daily asset returns, we calculate the idiosyncratic asset volatility using the same methodology used for idiosyncratic equity volatility. In addition to this *realized* asset volatility measure, we also use an *implied* asset volatility measure based on the option-implied equity volatility from OptionMetrics. Our implied equity volatility data corresponds to at-the-money 30-day forward put options. The asset volatility is constructed as the unlevered equity volatility, that is, implied equity volatility times market value of equity divided by market value of assets.

Market Leverage Market leverage is defined as the ratio of market value of assets to market value of equity. The market value of assets is built as the book value of assets plus the market value of equity minus the book value of equity. Following Davies, Fama, and French (2000), the book value of equity is defined as the book value of stockholders' equity, plus balance sheet deferred taxes and investment tax credit, minus the book value of preferred stock. Depending on availability, we use the redemption, liquidation, or par value (in that order) for the book value of preferred stock. If this procedure generates missing values, we measure stockholders' equity as the book value of common equity plus the par value of preferred stock, or the book value of assets minus total liabilities.

Fair Value Spreads We also use a proprietary data set from Moody's on its Public Firm Expected Default Frequency (EDF) Metric, which is an equity-based measure of firm's probability of default. The core model used to generate the EDF metric belongs to the class of option-pricing based, structural credit risk models pioneered by Black and Scholes (1973) and Merton (1974). The Vasicek-Kealhofer (VK) model summarizes information on asset volatility, market value of assets, and the default point into one metric, distance-to-default (DD), and then maps the DD to obtain the EDF metric. The DD-to-EDF mapping step utilizes the empirical distribution of DD and frequency of realized defaults. Nazeran and Dwyer (2015) provide a detailed description of their methodology. Most importantly for our purpose, the EDF credit risk measure relies only on equity market inputs and does not contain bond market information.

Using the EDF credit risk measure, we construct a cumulative EDF (CEDF) over T years by assuming a flat term structure, that is, $CEDF_T = 1-(1-EDF)^T$. Then, we convert our physical measure of default probabilities (CEDF) to risk-neutral default probabilities (CQDF) using the following equation:

$$CQDF_T = N\left[N^{-1}\left(CEDF_T\right) + \lambda\rho\sqrt{T}\right],$$

where N is the cumulative distribution function for the standard normal distribution, λ is the market Sharpe ratio and ρ is the correlation between the underlying asset returns and market returns. Given this risk-neutral default probability measure, the spread of a zero-coupon bond with duration T can be computed as:

$$\hat{s} = -\frac{1}{T}\log(1 - CQDF_T \cdot LGD),$$

where LGD stands for the risk-neutral expected loss given default. We follow Moody's convention and set $T = 5$, $LGD = 60\%$, $\lambda = 0.546$, and $\rho = \sqrt{0.3}$ to build our "fair value spread" measure $\hat{s}$. We successfully match $39,925$ fair value spreads with our firm-quarter observations.

Covenant Tightness To measure the strength of creditor control rights, which is useful for providing empirical support for the debt overhang channel in our model, we use a covenant tightness measure based on a firm's outstanding loans in a firm-year panel.[7] Data on covenant specifications and thresholds for loans is from DealScan. There are 18 types of covenants in the data. We first compute the distance between the actual financial ratio and the covenant threshold for each type of covenant, normalized by the year-specific standard deviation of the actual financial ratios at loan issuance. We then use the minimum of the normalized distances to measure the overall covenant tightness for the firm.

[7]We thank Yueran Ma and Amir Kermani for sharing their data with us. See Kermani and Ma (2020) for more details on the covenant tightness measure.

2.2 Firm-level Panel Regressions

In this section, we present a set of firm-level panel regressions of investment rate on volatility and spreads:

$$\log[I/K]_{i,t} = \beta_1 \log X^{\sigma}_{i,t} + \beta_2 \log X^{s}_{i,t} + \eta_i + \lambda_t + \epsilon_{i,t}, \tag{1}$$

where $\log[I/K]_{i,t}$ is the log of investment rate of firm i in period t, $X^{\sigma}_{i,t}$ denotes either the idiosyncratic equity volatility $\sigma^{e}_{i,t}$ or the idiosyncratic asset volaility $\sigma_{i,t}$, and $X^{s}_{i,t}$ denotes the credit spread $s_{i,t}$, the fair value spread $\hat{s}_{i,t}$, or market leverage $[MA/ME]_{i,t}$. We control for the firm fixed effects and time fixed effects by including η_i and λ_t. We run these regressions using both the full sample and subsamples based on firms' credit spread. We also consider specifications that add an interaction term $\log X^{\sigma}_{i,t} \times \log X^{s}_{i,t}$ to the right-hand side of Equation (1).

Table 1 and Table 2 use equity volatility and credit spread/fair value spread as right-hand-side variables and establish that credit spreads drive out equity volatility in predicting investment is not due to different investor base or market segmentation, but due to the heterogeneity of sensitivity of investment to equity volatility in the cross-section. Tables 3-5 use asset volatility and credit spread/market leverage on the right-hand-side of eq. (1) and show that asset volatility and credit spread are jointly unambiguous signals for investment, which we rationalize in the model section.

Equity Volatility and Credit Spread We first replicate the results in Gilchrist, Sim, and Zakrajšek (2014) that the adverse effect of idiosyncratic equity volatility on investment rate is dampened when controlling for credit spreads. Table 1 presents estimation results of Equation (1) using equity volatility and credit spread on the right-hand side.

[Table 1 about here.]

As shown in columns 1-3 of Table 1, the coefficient on idiosyncratic equity volatility and credit spread are statistically significant and economically important on their own (columns 1-2). However, when both measures are included in the regression, the coefficient on equity volatility is substantially reduced both in terms of magnitude and statistical significance while the coefficient on credit spread is unaffected (column 3).

To see why bond spreads can drive out equity volatility, we sort firms into tercile groups based on credit spread each quarter and run the same regression for these subsamples (columns 4-6).[8] We find that the coefficient on equity volatility changes sign in the cross section: it is significantly positive among firms with low credit spread and significantly negative among firms with high credit spread. The last column shows results from the regression with an interaction term, and confirms our findings from columns 4-6. A simple back-of-the-envelope calculation suggests that the sign flip happens at a credit spread level of 194 basis points. In Appendix C, we show that regressions using lags of the independent variables generate similar results, which highlights the predictive power of these measures on investment.

[8] This method of splitting uses quarter-specific cutoffs. Using fixed cutoffs to sort all firm-quarter observations leads to similar results.

[Table 2 about here.]

In Table 2, we replace credit spreads with fair value spreads. The results are qualitatively identical to Table 1. The coefficient on equity volatility goes from significantly positive to significantly negative as firm's credit spread goes up, while the coefficient on the fair value spread remains significantly negative across the subgroups. In Appendix C, we run additional robustness checks to address concerns over sample selection bias. As the fair value spreads are constructed with only equity market information and does not contain bond market information, the results from Table 1 cannot be driven by differences in the investor base or information about financial frictions only reflected in credit spreads.

Asset Volatility and Credit Spread Equity volatility can be decomposed into asset volatility (derived from Merton's model) and market leverage. In Table 3, we run the same regression but we replace idiosyncratic equity volatility $\sigma^e_{i,t}$ with idiosyncratic asset volatility $\sigma_{i,t}$. The coefficient on asset volatility is always positive and statistically significant in the full sample and in all subgroups.[9] Interestingly, the positive impact of asset volatility on investment is statistically stronger for firms with lower credit spreads, while the reverse is true for credit spreads.

[Table 3 and Table 4 about here.]

[9]Column 6 of Table 3 suggests that the coefficient on asset volatility might flip sign when credit spread are higher than 592 basis points. However, this is due to the limitation of the linear interaction model. Estimating the coefficients on a sample of high credit spread firms or using a quadratic model implies a positive coefficient for all firms.

In the model, equity holders make investment decisions given uncertainty about future returns. In Table 4, we replicate the same exercise but with implied asset volatility from equity options (and with lags in Appendix C). The results are as strong as with volatility derived from past equity return observations, lending support to the idea that it is the expectation of future asset volatility that drives changes in investment, not past uncertainty.

[Table 5 about here.]

Given the decomposition of equity volatility, a natural question is whether the coefficient on asset volatility is also positive when asset volatility and market leverage are used together as explanatory variables for investment. We present the results in table 5. The results are inconclusive as the coefficient on asset volatility is not statistically different from zero. In the model section, we rationalize this finding by showing that, together, leverage and asset volatility are not unambiguous signals of debt overhang and option value.

Loadings Asset volatility and leverage are also important drivers for bond spreads. To understand why there is no such sign flip for credit spreads, we consider the loadings of credit spreads and equity volatility on asset volatility and leverage and estimate the following equation:

$$\log y_{i,t} = \beta_1 \log \sigma_{i,t} + \beta_2 \log[MA/ME]_{i,t} + \eta_i + \lambda_t + \epsilon_{i,t},$$

where $y_{i,t}$ is either the equity volatility ($\sigma^e_{i,t}$) or the credit spread ($s_{i,t}$), and $MA/ME_{i,t}$ is market leverage. We estimate the equation both in levels and in first differences.

[Table 6 about here.]

Table 6 summarizes the results. Columns 1-2 show how the levels of equity volatility and credit spread load on levels of asset volatility and leverage, and columns 3-4 show how the changes load on corresponding changes. Both specifications imply that changes in bond spreads are mainly driven by leverage, while changes in equity volatility are driven by both asset volatility and leverage. Since shocks to asset volatility and leverage impact investment differently, bond spreads and equity volatility contain different information for investment. In Appendix C, instead of using the firm's asset volatility and market leverage directly, we use the industry-level regressors, constructed as a simple average of all firms in the same industry excluding the firm itself. This exercise shows similar patterns that equity volatility loads more on asset volatility while credit spread loads more on leverage.

Thus, an increase in equity volatility could either signal an increase in asset volatility (positive for investment) or in leverage (negative for investment). Whether one force dominates the other changes in the cross-section. As seen in Table 3, the asset volatility effect weakens as firms' credit spreads increase, while the leverage effect strengthens.[10] Thus, the sensitivity of investment to equity volatility becomes negative when the leverage effect dominates for firms with higher credit spreads.

[10]The statistical significance of the credit spreads coefficient in Table 3 but also the leverage coefficient in Table 5 strengthens when credit spreads are higher.

Although credit spreads are also a combination of asset volatility and leverage, the loading of credit spreads on asset volatility is not large enough to ever drive a positive relation between credit spreads and investment.

Covenant As we are attributing the main force driving our results to debt overhang, we expect this coefficient to be stronger when debt holders have tighter control over cash flows resulting from investment. To test this hypothesis, we split the sample into two groups according to its covenant tightness. The observations with the overall measure of distance between actual financial ratios and covenant thresholds below median are placed in the "Tight Covenant" group, and the remaining are assigned to the "Slack Covenant" group. We estimate Equation (1) with the interaction term using the two subsamples. The results are summarized in Table 7. For the subsample with tight covenant, all the coefficients are larger in absolute value, and are statistically more significant. In particular, the positive coefficient for equity volatility is twice as large for firms with tight covenant. This exercise provides empirical support for our model with debt overhang as a key distorting force.

[Table 7 about here.]

2.3 Aggregates

Time Series To understand the implications of our findings for time series, we plot the elasticity of investment rate with respect to equity volatility, asset volatility, and credit spread across time and across firms using the estimates from the regressions with interaction terms. In Figure 4, we compute the overall coefficient on equity volatility at each credit spread level using estimates on equity volatility ($\log \sigma^e_{i,t}$) and the interaction term ($\log \sigma^e_{i,t} \times \log s_{i,t}$) reported in the last column of table 1. We repeat the procedure for asset volatility and credit spreads in Figure 5 and Figure 6. Figure 4 shows that the cross section of elasticities of investment with respect to equity volatility varies a lot over time. In particular, this coefficient is negative for the whole cross-section of firms during the Great Recession, while it is mainly positive in the late 1980s. By contrast, in Figures 5 and 6, the elasticity of investment to asset volatility remains positive and the elasticity to credit spread remains negative, both in the cross-section and over time.

VAR Analysis Using an identified vector autoregression (VAR) framework, we confirm that our micro-level results—asset volatility has positive impact on investment—still holds at the macro-level. We aggregate the variables in our sample and estimate a simple VAR consisting of the three endogeneous variables: the log of idiosyncratic asset volatility ($\log \sigma_t$), the log of credit spread ($\log s_t$), and the log of investment rate ($\log[I/K]_t$).[11] We employ a standard recursive ordering technique and consider two identification schemes, one in which credit spread has an immediate impact on asset volatility and the other where asset volatility has immediate impact on credit spread.

[Figure 7 about here.]

[11]We use the value-weighted average of $\sigma_{i,t}$, $s_{i,t}$ and $[I/K]_{i,t}$ to generate the corresponding aggregate time series. We seasonally adjust the investment rate time series by using its four-quarter moving average. All variables are detrended using the HP filter with weight 1600.

Figure 7 reports the impulse responses of investment rate to credit spread and asset volatility using the two specifications. Credit spread has a negative impact on investment while asset volatility has a positive impact. As shown in panel (a) and (c) of Figure 7, the positive impact of asset volatility on investment is economically and statistically larger in the first specification, highlighting the importance of controlling for credit spread for asset volatility to be a strong positive signal for investment in the aggregate.

Market Volatility Eisdorfer (2008) also explores the different impacts of uncertainty on firm investment in the cross-section and finds that uncertainty has a positive effect on distressed firms' investment, seemingly opposite to our results. He uses expected market volatility which is generated by applying a GARCH (1,1) model to monthly returns of the NYSE market index. This poses the question whether our results would be different using aggregate volatility instead of firm-level idiosyncratic volatility.

[Table 8 and Table 9 about here.]

To address this question, we first replicate Eisdorfer's (2008) results in Table 8. Columns 1-2 presents the results under Eisdorfer's (2008) specification, where we split the sample into financially healthy and distressed firms[12] and regress investment

[12] We select the 20th percentile of distance-to-default as the cutoff for distressed firms. Eisdorfer (2008) classifies firms with Z-score below 1.81 at the beginning of each year as distressed, which generates a subsample of distressed firms including 18.6% of total observations.

rate on aggregate equity volatility, along with the same control variables.[13] The coefficient on aggregate equity volatility is significantly negative for healthy firms and is positive but insignificant for firms closer to default. Column 3-5 presents the result under our specification, where we sort firms into tercile groups based on credit spread each quarter and regress investment rate on aggregate equity volatility in addition to idiosyncratic equity volatility and credit spread. The coefficient on idiosyncratic equity volatility still goes from significantly positive to significantly negative as firm's credit spread goes up, while the coefficient on aggregate equity volatility goes in the opposite direction. We emphasize that equity volatility is an ambiguous signal for investment, driven by asset volatility and leverage, so we run the regressions using asset volatility. As shown in Table 9, both the coefficients on aggregate asset volatility and firm-level idiosyncratic asset volatility are positive across all subgroups when controlling for firm-level credit spread, or aggregate credit spread, or both.

3 Investment Decisions with Debt Overhang

In this section, we develop a simple but general credit risk model to analyze the investment choices of a firm with outstanding debt already in place. Two forces drive the investment decision: debt overhang and the option value of equity. We

[13]The control variables are: firm size, estimated by the log the market value of the firm's total assets; market-to-book ratio; leverage, estimated by the ratio of the book value of total debt to the book value of total assets; cash flow, estimated by the ratio of operating cash flow to PP&E at the beginning of the year; the NBER recession dummy variable; the BAA-AAA yield spread; the interest rate, estimated by the nominal return on 1-month Treasury bills.

demonstrate that credit spreads and asset volatility are jointly unambiguous signals of these two forces. However, the signals provided by leverage and asset volatility or credit spreads and equity volatility are ambiguous and can change in the cross-section. All proofs are relegated to Appendix D. For ease of notation, we sometimes write $f_x(x) \equiv \frac{\partial f(x)}{\partial x}$.

Consider a firm that has risky assets in place and has funded itself partly with debt. In the first period, shareholders choose how much to invest. At the beginning of the second period, a random productivity shock is realized, and, after observing the payoff of their investment, shareholders decide whether to file for bankruptcy or not. For our basic argument, we make the following assumptions regarding the firm and its investments.

Assumption 1 (Assets in Place)**.** *The firm has existing real assets in place with a final value of $Y(\iota, z)$, which is a function of investment ι and a random productivity shock z realized in the future. The assets are normalized to have an initial value of one. That is, $\mathbb{E}[Y(0,z)] = 1$.*

Assumption 2 (Firm Liabilities)**.** *The firm is funded by equity, together with a debt claim with total face value b that is due in the second period when the asset returns are realized. In the second period, shareholders decide whether to default. Upon bankruptcy, the entirety of the firm's value is lost. Furthermore, shareholders cannot liquidate the firm ($\iota \geq 0$).*

We show that our results are robust to a relaxation of Assumption 2 featuring partial recovery in Section D.

Assumption 3 (Pricing)**.** *All securities are traded in perfect Walrasian markets. We normalize the risk-free interest rate to zero and set prices of securities equal to their expected payoff with respect to a risk-neutral distribution* $F(z)$ *of firm's asset productivity* z*, and* $Y(\iota, z)$ *with full support on* $[0, \infty)$*.*

Given our assumptions about payouts and pricing, it follows that the value of equity E and debt D are given by:

$$E(b, \iota, \underline{z}, \sigma) = \int_{\underline{z}}^{\infty} (Y(\iota, z) - b) dF(z; \sigma) - \iota,$$

$$D(b, \underline{z}, \sigma) = (1 - F(\underline{z}; \sigma))b.$$

The first order conditions for investment ι and the default threshold $\underline{z}$ imply that, at an optimum, ι and $\underline{z}$ satisfy:

$$\int_{\underline{z}}^{\infty} Y_{\iota}(\iota, z) dF(z; \sigma) = 1,$$

$$Y(\iota, \underline{z}) = b.$$

Credit spreads are given by: $cs(\underline{z}, \sigma) = F(\underline{z}; \sigma)$. To streamline our analysis, we also make assumptions on the risk distribution.

Assumption 4 (Investment Returns)**.** *The investment return function* $Y(\iota, z)$ *is continuous in both* ι *and* z*, positive, homogeneous of degree* 1 *in* z*, strictly increasing* *in ι, strictly concave in ι, and normalized to z when $\iota = 0$. That is,*

$$Y(\iota, z) = k(\iota)z \geq 0, \qquad k(0) = 1, \qquad k_\iota(\iota) > 0, \qquad k_{\iota\iota}(\iota) < 0.$$

Furthermore, the standard deviation σ of z is a finite moment of the distribution F.

Assumption 4 imposes restrictions common in models with investment. The homogeneity of degree 1 in z allows us to disentangle the effect of investment i and risk z on the investment returns $Y(\iota, z)$ and simplifies the analytics. Assumption 5 provides the only assumptions we make on the distribution of productivity shocks, $F(z)$. These assumptions are always satisfied with the Black–Scholes–Merton model and most risk distributions usually considered in finance.

Assumption 5 (Vega)**.** *The distribution of the productivity shock $F(z; \sigma)$ is such that vega is always positive:*

$$\nu(\underline{z}, \sigma) = \frac{\partial}{\partial \sigma} \mathbb{E}\left[(z - \underline{z})^+\right] > 0.$$

The model has two free parameters, leverage and asset volatility. Given the normalization $\mathbb{E}[Y(0, z)] = 1$, at the beginning of the first period leverage is simply the face value of debt b with zero investment. The model has two endogenous decision variables, investment ι and the default threshold $\underline{z}$. We use this simple model to study the behavior of investment following changes in the key observable variables from our empirical section: asset volatility σ, leverage b, credit spreads cs, and equity volatility σ^e.

Proposition 1 (Credit Spread and Asset Volatility). *Holding asset volatility constant, the partial derivative of investment with respect to credit spread is given by:*

$$\frac{\partial \iota}{\partial cs} = \frac{k_{\iota}(\iota)}{k_{\iota\iota}(\iota)} \frac{\underline{z}}{\mu(\underline{z}, \sigma)} < 0, \tag{2}$$

where $\mu(\underline{z}, \sigma) = \mathbb{E}[z|z \geq \underline{z}]\,\mathbb{P}[z \geq \underline{z}]$*. Holding credit spread constant, the partial derivative of investment with respect to volatility is given by:*

$$\frac{\partial \iota}{\partial \sigma} = -\frac{k_{\iota}(\iota)}{k_{\iota\iota}(\iota)} \frac{\nu(\underline{z}, \sigma)}{\mu(\underline{z}, \sigma)} > 0. \tag{3}$$

In Proposition 1, we provide the elasticities of investment when observing asset volatility and credit spread. Given Assumptions 1-5, the sign of these partial derivatives match our empirical results. The first term on the right-hand side of equation (2) is negative due to the concavity of $k(\iota)$ in the denominator. All other terms are positive and thus the sign of the elasticity of investment to credit spreads is always negative.

In terms of the magnitude of the negative effect of higher credit spreads on investment, consider the denominator of the second term on the right-hand side of equation (2), $\mu(\underline{z}, \sigma)$. This term represents the expected return on one unit of investment given the option to default. As the default boundary $\underline{z}$ (which is also the marginal product lost from increasing the default threshold) increases, expected returns $\mu(\underline{z}, \sigma)$ decrease and shareholders have less incentives to invest. Thus, the debt-overhang problem intensifies when the firm gets closer to default. We also note the role of the concavity of the investment return function. If effective capital is more concave in investment, the first term will be smaller because firms won't have to adjust investment as much since $k_\iota(\iota)$ increases faster for a given reduction in investment.

By contrast, investment reacts positively to an increase in volatility as the payout to shareholders is non-linear with limited downside and unlimited upside, that is, vega $\nu(\underline{z}, \sigma)$ is positive. In equation (3), the first term is the same, except for the negative sign in front of it, while the numerator of the second term reflects the option value of higher investment as volatility increases. How strong the option-value effect is depends on the distribution of productivity shocks, $F(z; \sigma)$. Again, the ratio of the marginal investment return $k_\iota(\iota)$ to investment return concavity $k_{\iota\iota}(\iota)$ determines the strength of the investment response. If the marginal investment return $k_\iota(\iota)$ is large or the marginal productivity does not fall too fast (low $|k_{\iota\iota}(\iota)|$), then the investment response to a change in volatility is stronger.

Thus, in this simple model with fairly general as well as fairly standard assumptions, the signs of the effects of credit spreads and asset volatility on investment are unambiguous. Changes in credit spreads cs signal changes in the debt-overhang burden and changes in asset volatility σ signal changes in the option value of equity. In Figure 1, we illustrate the optimal investment function with a log-normal distribution of risk.

We now compare the straightforward roles of credit spreads and asset volatility in determining investment with the more intricate relation between *leverage* and asset

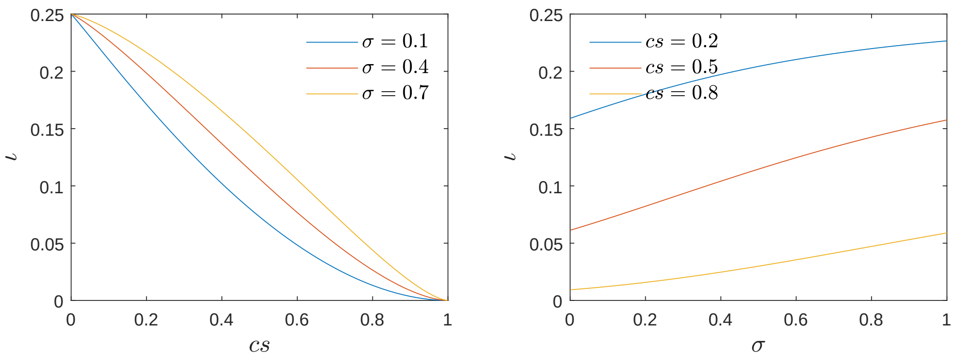

Figure 1: Optimal investment with log-normal distribution. The left picture shows the level of investment ι as a function of credit spreads cs for different levels of asset volatility σ, while the right figure shows the level of investment ι as a function of asset volatility σ for different levels of credit spreads cs. The production function is given by: $Y(\iota, z) = z(1 + \iota^{\alpha})$ where $\alpha = 0.5$.

volatility in investment decisions. This analysis exemplifies why credit spreads and asset volatility are clean empirical measures of the effects of financial soundness and option value on investment decisions.

Proposition 2 (Leverage and Asset Volatility)**.** *Holding asset volatility constant, the partial derivative of investment with respect to leverage is given by:*

$$\frac{\partial \iota}{\partial b} = \frac{k_{\iota}(\iota)}{k_{\iota\iota}(\iota)} \frac{\underline{z}}{\mu(\underline{z}, \sigma)} \xi_{b|\sigma}(\iota, \underline{z}, \sigma) < 0, \tag{4}$$

where

$$\xi_{b|\sigma}(\iota, \underline{z}, \sigma) \equiv \frac{f(\underline{z}; \sigma)}{k(\iota)} \varphi(\underline{z}, \sigma) > 0, \qquad \varphi(\iota, \underline{z}, \sigma) \equiv \left(1 + \frac{k_{\iota}(\iota)^2 \underline{z}^2 f(\underline{z}; \sigma)}{k(\iota) k_{\iota\iota}(\iota) \mu(\underline{z}; \sigma)}\right)^{-1} > 0.$$

Holding leverage constant, the partial derivative of investment with respect to volatil-

ity is given by:

$$\frac{\partial \iota}{\partial \sigma} = -\frac{k_\iota(\iota)}{k_{\iota\iota}(\iota)} \frac{\nu(\underline{z}, \sigma)}{\mu(\underline{z}; \sigma)} \xi_{\sigma|b}(\iota, \underline{z}, \sigma), \tag{5}$$

where

$$\xi_{\sigma|b}(\iota, \underline{z}, \sigma) \equiv \left(1 - \frac{\underline{z} F_\sigma(\underline{z}; \sigma)}{\nu(\underline{z}, \sigma)}\right) \varphi(\iota, \underline{z}, \sigma).$$

Proposition 2 shows that if, instead of controlling for credit spreads cs, we observe leverage b, the elasticities of investment become more intricate. Starting with equation (4), note that the first two terms on the right-hand side are equivalent to those in Proposition 1. The wedge $\xi_{b|\sigma}$ captures the additional effects of changing leverage. Note that φ is always positive at a maxima as imposed by the second-order conditions. Thus, this wedge is always positive, and the sign of the effect of leverage on investment holding asset volatility constant is always negative. The φ term captures the feedback loop between investment and default decisions. Following a decrease in investment, shareholders default more often as output decreases and thus incentives to pay back the debt also decrease. That additional force was not present in Proposition 1, since changing credit spreads $F(\underline{z}; \sigma)$ controls for the default decision $\underline{z}$ directly. Holding leverage constant instead controls for $b = Y(\iota, \underline{z})$, which is a function of both ι and $\underline{z}$.

Turning to the effect of asset volatility on investment holding leverage constant, the sign now becomes ambiguous. Relative to the effect holding credit spreads con-

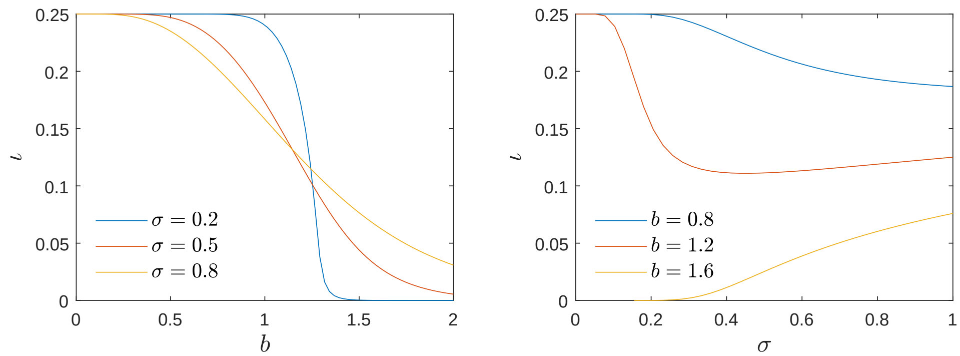

Figure 2: Optimal investment with log-normal distribution. The left picture shows the level of investment ι as a function of leverage b for different levels of asset volatility σ, while the right figure shows the level of investment ι as a function of asset volatility σ for different levels of leverage b. The production function is given by: $Y(\iota, z) = z(1 + \iota^{\alpha})$ where $\alpha = 0.5$.

stant characterized in Proposition 1, there is again a wedge, which we denote $\xi_{\sigma|b}$. Intuitively, there are two effects to increasing asset volatility holding leverage constant. The first is that the option value of investment increases. The second is that the debt overhang problem also increases. To hold leverage $b = Y(\iota, \underline{z})$ constant as asset volatility increases, the default threshold $\underline{z}$ must change and the distance to default could shrink faster than the increase in the option value. The wedge $\xi_{\sigma|b}$ captures this horserace between option value and what is lost in default as asset volatility increases. If the option value effect is strong, this term will be positive. However, if the increase in asset volatility moves a large probability mass into the default region (thus $\underline{z}F_{\sigma}(\underline{z};\sigma)$ is positive), the term can be negative. In other words, when the marginal increase in investment returns lost to default $zF_{\sigma}(\underline{z};\sigma)$ dominates the marginal increase in the option value $\nu(\underline{z},\sigma)$, shareholders reduce investment following an increase in volatility.

Which effect dominates is highly dependent on the shape of the distribution

$F(z;\sigma)$. In Figure 2, we plot the optimal investment decision as a function of asset volatility σ when holding leverage b constant assuming a log-normal distributions for z. The monotonic relation between leverage and investment holding asset volatility constant is clear. However, the relation between investment and asset volatility holding leverage constant is non-monotonic. When leverage is high, the option-value effect dominates while the debt-overhang effect dominates when leverage is low.

Next, we consider the changes in investment when observing credit spreads and equity volatility, and illustrate the intuition our model suggests for the empirical finding that the sign of the elasticity of investment with respect to equity volatility changes sign in the cross section of more and less distressed firms. First, we define equity volatility as measured in the data as:[14]

$$\sigma^e(\underline{z},\sigma) = \frac{\sigma}{\bar{\mu}(\underline{z},\sigma)},$$

where $\bar{\mu}(\underline{z},\sigma) = \mathbb{E}\left[(z-\underline{z})^{+}\right]$ is the unconditional left-truncated expectation of the payoff above the default threshold. Thus, equity is levered asset volatility, where the denominator $\bar{\mu}(\underline{z},\sigma)$ represents the impact of leverage on equity volatility. If the debt burden from leverage b increases, then the default threshold $\underline{z}$ increases as well and

[14]Defining equity volatility as:

$$\sigma^e(\underline{z},\sigma) = \frac{\sqrt{\mathbb{V}ar\left[(Y(\iota,z)-b)\,\mathbb{1}\{z \geq \underline{z}\} - \iota\right]}}{E(b,\iota,\underline{z},\sigma)} = \frac{\bar{\sigma}(\underline{z},\sigma)}{\bar{\mu}(\underline{z},\sigma) - \iota/k(\iota)},$$

where $\bar{\sigma}(\underline{z},\sigma) = \sqrt{\mathbb{V}ar\left[(z-\underline{z})^{+}\right]}$ makes the analysis untractable and is further away from how equity volatility is measured as (i) the truncation of the volatility is not reflected in the measurement of equity volatility unless default occurred and (ii) investment, as measured in Compustat, does not varies over a quarter.

equity's expected payoff $\bar{\mu}(\underline{z},\sigma)$ decreases. Conversely, if the firm is funded entirely by equity ($b=0$), then $\underline{z}$ is equal to zero—the lower bound of the support. In that case, equity volatility is equal to asset volatility ($\sigma^e(\underline{z},\sigma)=\sigma$) since $\bar{\mu}(0,\sigma)=1$.

Proposition 3 (Credit Spread and Equity Volatility)**.** *Holding equity volatility constant, the partial derivative of investment with respect to credit spreads is given by:*

$$\frac{\partial\iota}{\partial cs}=\frac{k_\iota(\iota)}{k_{\iota\iota}(\iota)}\frac{\underline{z}}{\mu(\underline{z},\sigma)}\xi_{cs|\sigma^e}(\underline{z},\sigma), \tag{6}$$

where

$$\xi_{cs|\sigma^e}(\underline{z},\sigma)\equiv 1+\sigma^e_{\underline{z}}(\underline{z},\sigma)\frac{\nu(\underline{z},\sigma)}{\underline{z}f(\underline{z})}\xi_{\sigma^e|cs}(\underline{z},\sigma).$$

Holding credit spreads constant, the partial derivative of investment with respect to equity volatility is given by:

$$\frac{\partial\iota}{\partial\sigma^e}=-\frac{k_\iota(\iota)}{k_{\iota\iota}(\iota)}\frac{\nu(\underline{z})}{\mu(\underline{z},\sigma)}\xi_{\sigma^e|cs}(\underline{z},\sigma), \tag{7}$$

where

$$\xi_{\sigma^e|cs}(\underline{z},\sigma)\equiv\left(\sigma^e_\sigma(\underline{z},\sigma)-\frac{F_\sigma(\underline{z};\sigma)}{f(\underline{z};\sigma)}\sigma^e_{\underline{z}}(\underline{z},\sigma)\right)^{-1}.$$

We again use wedges to make the distinction between Propositions 3 and 1 clear. It is easiest to start with the relation between investment and equity volatility holding credit spreads constant. To understand the additional complication when using equity volatility as a signal of uncertainty, it is useful to look at the partial derivative of equity volatility with respect to asset volatility σ and the default threshold $\underline{z}$:

$$\sigma^e_\sigma(\underline{z},\sigma) = \frac{1}{\overline{\mu}(\underline{z},\sigma)} - \frac{\sigma\nu(\underline{z},\sigma)}{\overline{\mu}(\underline{z},\sigma)^2} \qquad \text{and} \qquad \sigma^e_{\underline{z}}(\underline{z},\sigma) = \frac{\sigma\left(1 - F(\underline{z};\sigma)\right)}{\overline{\mu}(\underline{z},\sigma)^2} \geq 0.$$

Thus, when the option value impact of asset volatility $\nu(\underline{z},\sigma)$ is large, equity volatility decreases following a positive shock to asset volatility. Indeed, the increase in the payoff to equity holders (denominator of σ^e) gets larger than the relative increase in asset volatility (numerator of σ^e). Add to that effect that to keep the credit spread cs constant, the default threshold $\underline{z}$ needs to decrease, and it not surprising anymore that following a positive asset volatility shock, equity volatility might decrease. Corollary 1 makes that argument explicit.

Corollary 1 (Equity Volatility and Asset Volatility)**.** *If the total derivative of the default threshold with respect to asset volatility is such that:*

$$\frac{d\underline{z}}{d\sigma} < \frac{\sigma\nu(\underline{z},\sigma) - \overline{\mu}(\underline{z},\sigma)}{\sigma(1 - F(\underline{z};\sigma))},$$

then the total derivative of equity volatility with respect to asset volatility is negative:

$$\frac{d\sigma^e(\underline{z},\sigma)}{d\sigma} < 0.$$

These additional forces are captured by the wedges $\xi_{\sigma^e|cs}(\underline{z},\sigma)$ and $\xi_{\sigma^e|cs}(\underline{z},\sigma)$ such that the signs of the elasticities of Proposition 3 are highly dependent on the shape of the risk distribution $F(z;\sigma)$ and the level of leverage and volatility of the firm, contrarily to the robust signs of the elasticities of Proposition 1.

Lemma 2 (Existence of Credit Spread and Equity Volatility Pair)**.** *Given* $(cs, \sigma^e) \in [0,1] \times \mathbb{R}^+$*, there does not always exist a solution* $(\underline{z}, \sigma) \in \mathbb{R}^+ \times \mathbb{R}^+$ *to the following system of two equations:*

$$cs = F(\underline{z}; \sigma),$$
$$\sigma^e = \frac{\sigma}{\overline{\mu}(\underline{z}, \sigma)}.$$

Furthermore, the solution might not be unique.

Following Lemma 2, these non-monotonicities also complicate the mapping of investment decisions in the (cs, σ^e)-space. Thus, in Figure 3, we show the sign of the wedges in the (cs, σ)-space for two distributions: a log-normal distribution and a log-normal mixture distribution. In the case of the log-normal distribution, the wedges are either both positive (white area), such that the signs of the elasticities are identical to Proposition 1, or both negative (light gray area), such that the signs of the elasticities are opposite to Proposition 1.

The mixture distribution is a mixture of two log-normal distributions (see caption of Figure 3) and therefore bimodal. This risk distribution could correspond to a technology where the productivity shock is drawn from either a bad (low mean) or a good (high mean) distribution. In this case, an increase in uncertainty could have a large effect on the option value without substantially impacting default risk. Thus, a third area (dark grey) appears, where the elasticities with respect to credit spread and

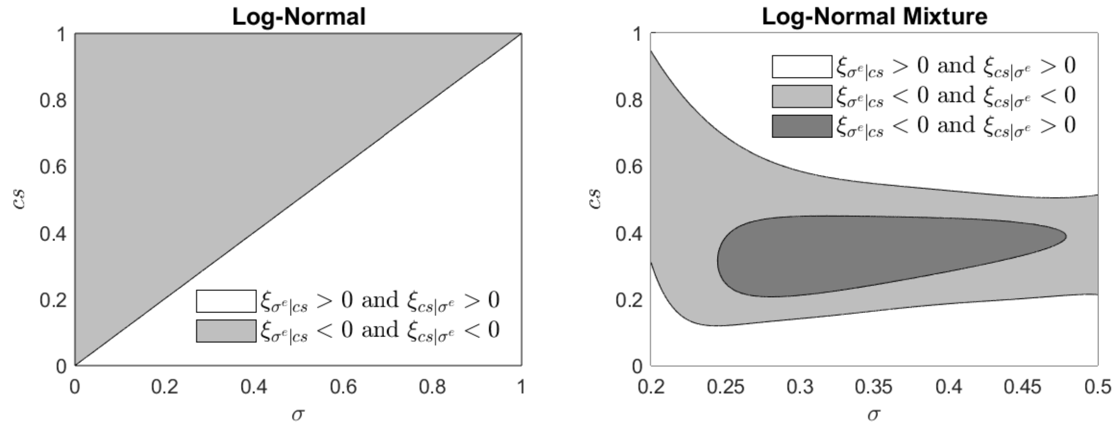

Figure 3: Sign of Wedges for Log-Normal and Log-Normal Mixture. These pictures shows the sign of the wedges of Proposition 3 in the (cs,σ)-space for the log-normal distribution (left) and a log-normal mixture distribution (right). The mixture distribution is a mixture of two log-normal distributions drawn with 50% probability with parameters $(\mu_1, \hat{\sigma})$ and $(\mu_2, \hat{\sigma})$ such that the unconditional mean of z is 1 and the standard deviation of z is σ. We set $\hat{\sigma} = 0.2$ in this example.

equity volatility are both negative. In the example of Figure 3, fixing asset volatility to 0.3, the elasticity with respect to equity volatility is positive for low credit spread level ($cs \leq 0.15$) and gets negative for high level of leverage ($0.15 \leq cs \leq 0.6$). At the same time, the elasticity with respect to credit spread is mostly negative (for $cs \leq 0.14$ and $0.21 \leq cs \leq 0.45$). Thus, in that example, we observe the same change of sign in the cross-section as in our empirical results.

In Appendix E, we show that our results hold in a setting with endogenous leverage dynamics. We extend the framework of DeMarzo and He (2020) to include an investment function and show that Proposition 1 still holds.

4 Conclusion

In our empirical analysis and model, we establish that equity volatility is an ambiguous signal of uncertainty for firm-level investment decisions. Intuitively, if a positive uncertainty shock causes a large increase in the option value of equity, equity volatility might go down. Using asset volatility instead results in an unambiguous relationship with investment: an increase in asset volatility generates an increase in the investment rate.

Overall, our model and evidence provide support for the idea that the close connection between bond markets and the macroeconomy is due to the unique non-linear transformation of asset volatility and leverage that credit spreads represent.

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Appendices

A Figures

Figure 4: This figure presents the elasticity of investment with respect to equity volatility across time and across firms using the estimates from the regressions with interaction terms. In each quarter we generate five cutoffs in the cross-section of log credit spread: $\{p_{10}, p_{30}, p_{50}, p_{70}, p_{90}\}$. Using the estimates in column 7 of Table 1 on

$$\log[I/K]_{i,t} = \beta_1 \log \sigma^e_{i,t} + \beta_2 \log s_{i,t} + \gamma \log \sigma^e_{i,t} \times \log s_{i,t} + \eta_i + \lambda_t + \epsilon_{i,t},$$

the elasticity at each cutoff point is computed as $\beta_1 + \gamma p_n$, $n = 10, 30, 50, 70, 90$.

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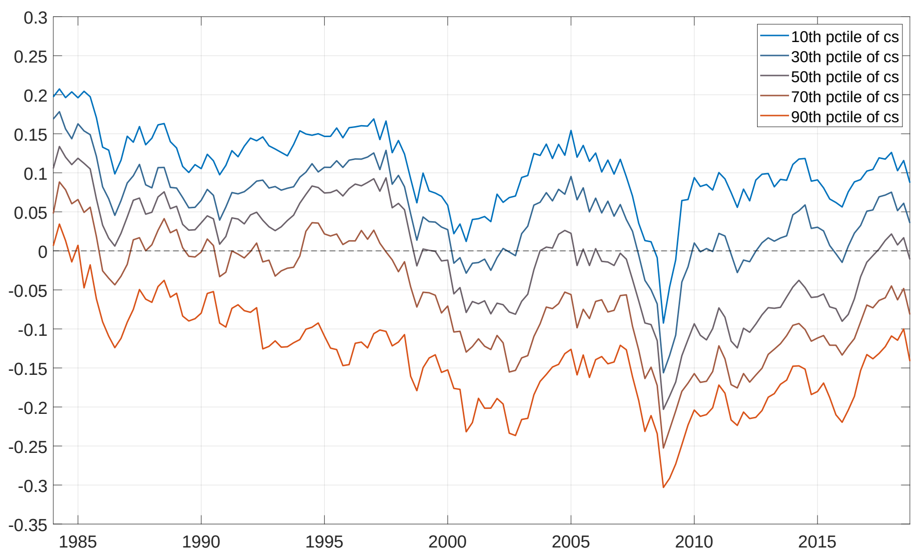


Figure 5: This figure presents the elasticity of investment with respect to asset volatility across time and across firms using the estimates from the regressions with interaction terms. In each quarter we generate five cutoffs in the cross-section of log credit spread: $\{p_{10}, p_{30}, p_{50}, p_{70}, p_{90}\}$. Using the estimates in column 5 of Table 3 on

$$\log[I/K]_{i,t} = \beta_1 \log \sigma_{i,t} + \beta_2 \log s_{i,t} + \gamma \log \sigma_{i,t} \times \log s_{i,t} + \eta_i + \lambda_t + \epsilon_{i,t},$$

the elasticity at each cutoff point is computed as $\beta_1 + \gamma p_n$, $n = 10, 30, 50, 70, 90$.

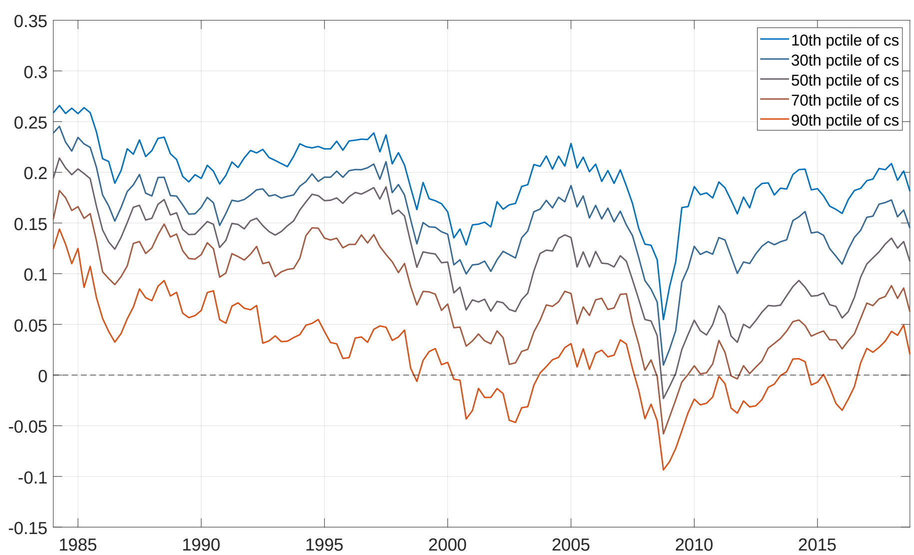


Figure 6: This figure presents the elasticity of investment with respect to credit spread across time and across firms using the estimates from the regressions with interaction terms. In each quarter we generate five cutoffs in the cross-section of log equity volatility: $\{p_{10}, p_{30}, p_{50}, p_{70}, p_{90}\}$. Using the estimates in column 7 of Table 1 on

$$\log[I/K]_{i,t} = \beta_1 \log \sigma^e_{i,t} + \beta_2 \log s_{i,t} + \gamma \log \sigma^e_{i,t} \times \log s_{i,t} + \eta_i + \lambda_t + \epsilon_{i,t},$$

the elasticity at each cutoff point is computed as $\beta_2 + \gamma p_n$, $n = 10, 30, 50, 70, 90$.

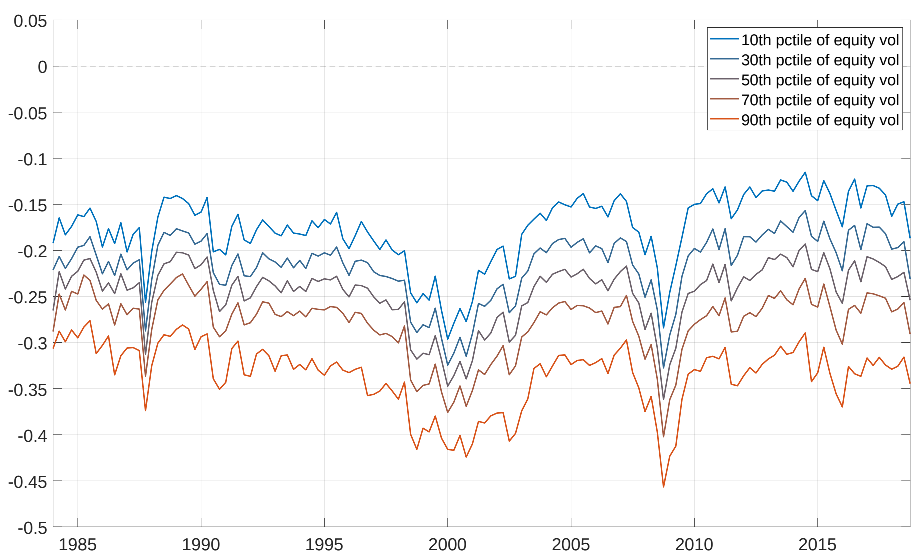
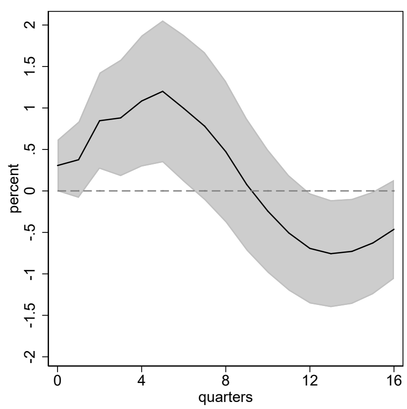
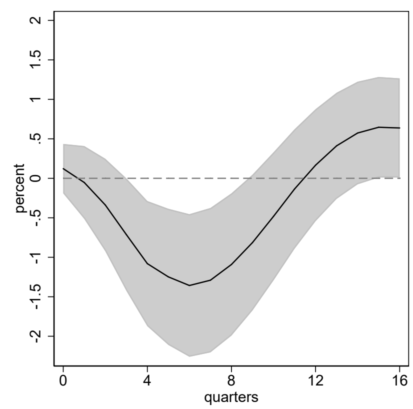


Figure 7: This figure plots the impulse responses of investment to an orthogonalized 1 standard deviation shock to asset volatility and credit spread. The VAR is estimated using four lags of each endogenous variable. Subfigures (a) and (b) correspond to the recursive ordering: (s, σ, I/K). Subfigures (c) and (d) correspond to the recursive ordering: (σ, s, I/K). The shaded bands represent the 95% confidence interval.

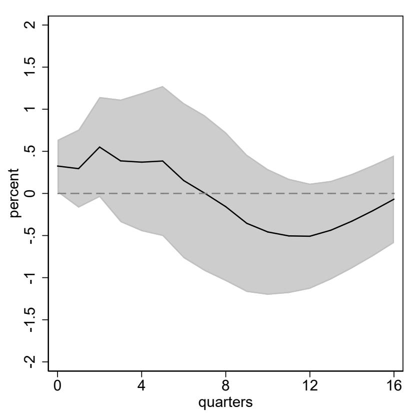
(a) Asset volatility



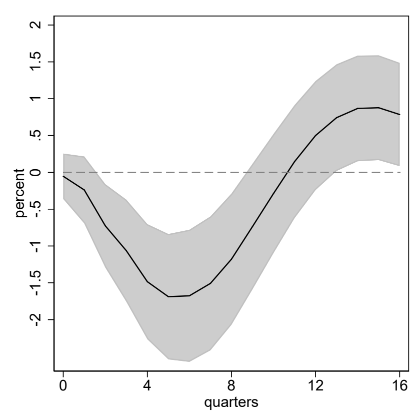
(b) Credit spread



(c) Asset volatility



(d) Credit spread



B Tables

Table 1: This table documents the relationship between equity volatility, credit spread and investment. Investment rate is regressed on all the regressors on the right-hand side in column 7, on equity volatility and credit spread spread in columns 3-6, on credit spread in column 2 and on equity volatility in column 1. Columns 4-6 use subsamples sorted by terciles on credit spread. Each observation is a firm-quarter. Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels and standard errors are clustered at the firm level.

$$\log[I/K]_{i,t} = \beta_1 \log \sigma^e_{i,t} + \beta_2 \log s_{i,t} + \gamma \log \sigma^e_{i,t} \times \log s_{i,t} + \eta_i + \lambda_t + \epsilon_{i,t}$$

	(1) all	(2) all	(3) all	(4) low cs	(5) mid cs	(6) high cs	(7) all
$\log \sigma^e_{i,t}$	-0.167***		-0.051***	0.070***	-0.015	-0.111***	0.801***
	(-8.99)		(-3.37)	(3.86)	(-0.73)	(-4.55)	(9.79)
$\log s_{i,t}$		-0.285***	-0.268***	-0.104***	-0.251***	-0.431***	-0.462***
		(-13.33)	(-12.84)	(-3.03)	(-5.68)	(-11.67)	(-16.61)
$\log \sigma^e_{i,t} \times \log s_{i,t}$							-0.152***
							(-10.21)
Firm FE	✓	✓	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓	✓	✓
Observations	52897	52900	52414	17614	17595	17205	52414
R-squared	0.101	0.125	0.126	0.144	0.122	0.116	0.134

Table 2: This table documents the relationship between equity volatility, fair value spread and investment. Investment rate is regressed on all the regressors on the right-hand side in column 7, on equity volatility and fair value spread in columns 3-6, on fair value spread in column 2 and on equity volatility in column 1. Columns 4-6 use subsamples sorted by terciles on credit spread. Each observation is a firm-quarter. Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels and standard errors are clustered at the firm level.

$$\log[I/K]_{i,t} = \beta_1 \log \sigma^e_{i,t} + \beta_2 \log \hat{s}_{i,t} + \gamma \log \sigma^e_{i,t} \times \log \hat{s}_{i,t} + \eta_i + \lambda_t + \epsilon_{i,t}$$

	(1) all	(2) all	(3) all	(4) low cs	(5) mid cs	(6) high cs	(7) all
$\log \sigma^e_{i,t}$	-0.154***		-0.008	0.079***	0.029	-0.079***	0.298***
	(-7.91)		(-0.54)	(4.31)	(1.34)	(-2.92)	(7.01)
$\log \hat{s}_{i,t}$		-0.152***	-0.148***	-0.072***	-0.119***	-0.152***	-0.232***
		(-13.96)	(-13.88)	(-4.33)	(-8.37)	(-9.15)	(-14.43)
$\log \sigma^e_{i,t} \times \log \hat{s}_{i,t}$							-0.069***
							(-7.19)
Firm FE	✓	✓	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓	✓	✓
Observations	39925	40331	39925	13272	13296	13027	39925
R-squared	0.108	0.140	0.139	0.169	0.145	0.126	0.144

Notes: Each observation is a firm-quarter. Coefficients are reported with t-statistics in parenthese. ***, **, and * indicate significance at 1%, 5% and 10% levels and standard errors are clustered at the firm level.

Table 3: This table documents the relationship between asset volatility and investment, controlling for credit spread. The regression in column 5 includes all the regressors in the estimation equation, and the regressions in columns 1-4 drop the interaction term. Columns 2-4 use subsamples sorted by terciles on credit spread. Each observation is a firm-quarter. Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels and standard errors are clustered at the firm level.

$$\log[I/K]_{i,t} = \beta_1 \log \sigma_{i,t} + \beta_2 \log s_{i,t} + \gamma \log \sigma_{i,t} \times \log s_{i,t} + \eta_i + \lambda_t + \epsilon_{i,t}$$

	(1) all	(2) all	(3) low cs	(4) mid cs	(5) high cs	(6) all
$\log \sigma_{i,t}$	0.050***	0.090***	0.113***	0.061***	0.066***	0.683***
	(3.30)	(6.34)	(6.60)	(3.11)	(3.00)	(7.45)
$\log s_{i,t}$		-0.279***	-0.101***	-0.276***	-0.485***	-0.457***
		(-12.60)	(-2.75)	(-5.98)	(-11.87)	(-13.43)
$\log \sigma_{i,t} \times \log s_{i,t}$						-0.107***
						(-6.54)
Firm FE	✓	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓	✓
Observations	47752	47384	16237	15947	15200	47384
R-squared	0.093	0.125	0.149	0.122	0.114	0.128

Table 4: This table documents the relationship between implied asset volatility and investment, controlling for credit spreads. The regression in column 5 includes all the regressors in the estimation equation, and the regressions in columns 1-4 drop the interaction term. Columns 2-4 use subsamples sorted by terciles on market leverage. Each observation is a firm-quarter. Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels and standard errors are clustered at the firm level.

$$\log[I/K]_{i,t} = \beta_1 \log \hat{\sigma}_{i,t} + \beta_2 \log s_{i,t} + \gamma \log \hat{\sigma}_{i,t} \times \log s_{i,t-1} + \eta_i + \lambda_t + \epsilon_{i,t}$$

	(1) all	(2) low cs	(3) mid cs	(4) high cs	(5) all
$\log \hat{\sigma}_{i,t}$	0.225***	0.223***	0.199***	0.185***	0.775***
	(7.61)	(4.52)	(3.94)	(4.02)	(4.45)
$\log s_{i,t}$	-0.334***	-0.152***	-0.371***	-0.560***	-0.513***
	(-10.66)	(-3.68)	(-5.67)	(-8.22)	(-8.16)
$\log \hat{\sigma}_{i,t} \times \log s_{i,t}$					-0.098***
					(-3.16)
Firm FE	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓
Observations	23162	7737	7748	7677	23162
R-squared	0.150	0.146	0.143	0.157	0.152

Table 5: This table documents the relationship between asset volatility and investment, controlling for market leverage. The regression in column 6 includes all the regressors in the estimation equation, and the regressions in columns 1-5 drop the interaction term. Columns 3-5 use subsamples sorted by terciles on market leverage. Each observation is a firm-quarter. Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels and standard errors are clustered at the firm level.

$$\log[I/K]_{i,t} = \beta_1 \log \sigma_{i,t} + \beta_2 \log[MA/ME]_{i,t} + \gamma \log \sigma_{i,t} \times \log[MA/ME]_{i,t} + \eta_i + \lambda_t + \epsilon_{i,t}$$

	(1) all	(2) low lev	(3) mid lev	(4) high lev	(5) all
$\log \sigma_{i,t}$	0.018	0.033	0.028	0.005	0.063***
	(1.23)	(1.54)	(1.39)	(0.24)	(2.98)
$\log[MA/ME]_{i,t}$	-0.460***	-0.990***	-0.525***	-0.351***	-0.543***
	(-19.38)	(-7.79)	(-7.71)	(-11.36)	(-14.65)
$\log \sigma_{i,t} \times \log[MA/ME]_{i,t}$					-0.053**
					(-2.57)
Firm FE	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓
Observations	47400	16061	15953	15386	47400
R-squared	0.149	0.141	0.114	0.126	0.149

Table 6: This table presents the loadings of equity volatility and credit spread on asset volatility and market leverage. The dependent variable $\log y_{i,t}$ denotes either equity volatility $\log \sigma^e_{i,t}$ or credit spread $\log s_{i,t}$. We report results for estimations in levels in Panel A and results for estimations in first differences in Panel B. Each observation is a firm-quarter. Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels and standard errors are clustered at the firm level.

$$\log y_{i,t} = \beta_1 \log \sigma_{i,t} + \beta_2 \log[MA/ME]_{i,t} + \eta_i + \lambda_t + \epsilon_{i,t}$$

Panel A: Levels	(1) $\log \sigma^e_{i,t}$	(2) $\log s_{i,t}$	Panel B: Changes	(3) $\Delta \log \sigma^e_{i,t}$	(4) $\Delta \log s_{i,t}$
$\log \sigma_{i,t}$	0.783***	0.180***	$\Delta \log \sigma_{i,t}$	0.780***	0.015***
	(90.77)	(15.91)		(83.77)	(5.00)
$\log[MA/ME]_{i,t}$	0.447***	0.613***	$\Delta \log[MA/ME]_{i,t}$	0.244***	0.248***
	(56.20)	(32.17)		(25.66)	(24.37)
Firm FE	✓	✓	Firm FE	✓	✓
Time FE	✓	✓	Time FE	✓	✓
Observations	47327	47250	Observations	44706	44640
R-squared	0.865	0.571	R-squared	0.794	0.345

Table 7: This table documents the relationship between equity volatility, credit spread and investment for firms with different covenant tightness. Column 1 reports estimation results for the subsample with tight covenant (distance to threshold below median). Column 2 reports estimation results for the subsample with slack covenant (distance to threshold above median). Each observation is a firm-year. Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels and standard errors are clustered at the firm level.

$$\log[I/K]_{i,t} = \beta_1 \log \sigma^e_{i,t} + \beta_2 \log s_{i,t} + \gamma \log \sigma^e_{i,t} \times \log s_{i,t} + \eta_i + \lambda_t + \epsilon_{i,t}$$

	(1) Slack Covenant	(2) Tight Covenant
$\log \sigma^e_{i,t}$	0.868***	1.599***
	(3.54)	(6.58)
$\log s_{i,t}$	-0.589***	-0.731***
	(-7.53)	(-12.35)
$\log \sigma^e_{i,t} \times \log s_{i,t}$	-0.155***	-0.282***
	(-3.40)	(-6.71)
Firm FE	✓	✓
Time FE	✓	✓
Observations	4516	4516
R-squared	0.116	0.179

Table 8: This table documents the relationship between aggregate equity volatility and firm-level investment rate. Column 1 and column 2 report estimation of the regression model with aggregate equity volatility $\log \Sigma_t^e$ and control variables $X_{i,t}$ on the right-hand side for financially healthy and distressed firms. The control variables are: firm size, estimated by the log the market value of the firm's total assets; market-to-book ratio; leverage, estimated by the ratio of the book value of total debt to the book value of total assets; cash flow, estimated by the ratio of operating cash flow to PP&E at the beginning of the year; the NBER recession dummy variable; the BAA-AAA yield spread; the interest rate, estimated by the nominal return on 1-month Treasury bills. The financially healthy (distressed) firms are observations above (below) the 20th percentile of distance-to-default. Columns 3-5 report estimation of the regression model with aggregate equity volatility $\log \Sigma_t^e$, idiosyncratic firm-level equity volatility $\log \sigma_{i,t}^e$, and credit spread $\log s_{i,t}$ on the right-hand side use subsamples sorted by terciles based on credit spread. Each observation is a firm-quarter. Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels and standard errors are clustered at the firm level.

$$\log[I/K]_{i,t} = \beta_1 \log \Sigma_{i,t}^e + \beta_2 \log \sigma_{i,t}^e + \beta_3 \log s_{i,t} + \gamma' \mathbf{X}_{i,t} + \eta_i + \epsilon_{i,t}$$

	(1) Healthy	(2) Distressed	(3) low cs	(4) mid cs	(5) high cs
$\log \Sigma_t^e$	-0.047***	0.033	-0.033***	0.045***	0.103***
	(-4.91)	(1.45)	(-3.12)	(3.47)	(6.27)
$\log \sigma_{i,t}^e$			0.145***	0.033	-0.096***
			(7.18)	(1.60)	(-4.14)
$\log s_{i,t}$			-0.084***	-0.239***	-0.364***
			(-3.38)	(-8.64)	(-12.70)
Controls	✓	✓			
Firm FE	✓	✓	✓	✓	✓
Observations	37831	7774	17614	17595	17205
R-squared	0.042	0.058	0.016	0.023	0.054

Table 9: This table documents the relationship between aggregate asset volatility, idiosyncratic asset volatility, and investment, controlling for firm-level or aggregate credit spread. Columns 1-3 control for firm-level credit spread. Columns 4-6 control for aggregate credit spread. Column 7-9 control for both, where we use $\log[s_{i,t}/S_t]$ for firm-level credit spread. Each observation is a firm-quarter. Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels and standard errors are clustered at the firm level.

$$\log[I/K]_{i,t} = \beta_1 \log \Sigma_{i,t} + \beta_2 \log \sigma_{i,t} + \beta_3 \log[s_{i,t}/S_t] + \beta_4 \log S_t + \eta_i + \epsilon_{i,t}$$

	(1) low cs	(2) mid cs	(3) high cs	(4) low cs	(5) mid cs	(6) high cs	(7) low cs	(8) mid cs	(9) high cs
$\log \Sigma_t$	-0.020	0.083***	0.126***	0.057***	0.096***	0.080***	0.054***	0.092***	0.041
	(-1.23)	(4.14)	(4.86)	(3.23)	(4.60)	(2.73)	(3.03)	(4.39)	(1.42)
$\log \sigma_{i,t}$	0.162***	0.065***	0.051**	0.129***	0.058***	0.033	0.125***	0.064***	0.050**
	(7.95)	(3.31)	(2.35)	(6.89)	(2.94)	(1.50)	(6.52)	(3.26)	(2.33)
$\log s_{i,t}$	-0.086***	-0.263***	-0.405***						
	(-3.25)	(-8.69)	(-13.34)						
$\log S_t$				-0.211***	-0.316***	-0.272***	-0.205***	-0.283***	-0.229***
				(-5.91)	(-7.79)	(-5.13)	(-5.52)	(-7.02)	(-4.52)
$\log[s_{i,t}/S_t]$							0.026	-0.237***	-0.501***
							(0.88)	(-5.18)	(-12.25)
Firm FE	✓	✓	✓	✓	✓	✓	✓	✓	✓
Observations	16236	15947	15198	16236	15947	15198	16236	15947	15198
R-squared	0.021	0.026	0.049	0.030	0.019	0.008	0.030	0.026	0.053

C Robustness Checks

In this appendix, we provide several robustness checks for the results discussed above. In particular, in Table A1 - Table A4, we show that using lagged values of equity volatility, asset volatility and credit spread generates similar results, which highlights the predictive power of these measures on investment.

In table A5, we examine the loadings of credit spreads and equity volatility on asset volatility and leverage. Instead of using the firm's asset volatility and market leverage directly as in the main text, we use the average of all firms in the same industry (excluding itself) and generate similar results.

In Table A6 through Table A9, we present the regression results replicating those in Table 1 to Table 6 in the main text. The coefficients can be interpreted as the move in the dependent variable scaled by its standard deviation associated with one standard deviation increase in the explanatory variable. These results help us interpret the economic significance of the coefficients on equity volatility and credit spread. Also, the split-sample results confirm that our cross-sectional findings are not sensitive to different dispersion of the variables in different subgroups. In Table A10, we report results for regressions that estimate the same specification as in Table 1 while using the same sample as used to generate Table 2. Comparing Table A10 with Table 2 indicates that fair value spread behaves similarly to credit spread in our investment regressions, and there are no concerns over the sample selection since we are using exactly the same sample. In Figure A1, we show the elasticity of investment with respect to credit spread using asset volatility as the moderator variable and find very similar results those in Figure 6 in the main text.

Table A1: This table replicates Table 1 using lagged values. Investment rate is regressed on all the regressors on the right-hand side in column 7, on equity volatility and credit spread spread in columns 3-6, on credit spread in column 2 and on equity volatility in column 1. Columns 4-6 use subsamples sorted by terciles on lagged values of credit spreads $\log s_{i,t-1}$. Each observation is a firm-quarter. Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels and standard errors are clustered at the firm level.

$$\log[I/K]_{i,t} = \beta_1 \log \sigma^e_{i,t-1} + \beta_2 \log s_{i,t-1} + \gamma \log \sigma^e_{i,t-1} \times \log s_{i,t-1} + \eta_i + \lambda_t + \epsilon_{i,t}$$

	(1) all	(2) all	(3) all	(4) low cs	(5) mid cs	(6) high cs	(7) all
$\log \sigma^e_{i,t-1}$	-0.182***		-0.065***	0.038**	-0.052***	-0.105***	0.830***
	(-9.89)		(-4.44)	(2.02)	(-2.60)	(-4.54)	(9.85)
$\log s_{i,t-1}$		-0.293***	-0.272***	-0.115***	-0.286***	-0.513***	-0.477***
		(-13.44)	(-12.80)	(-3.70)	(-8.04)	(-12.46)	(-16.56)
$\log \sigma^e_{i,t-1} \times \log s_{i,t-1}$							-0.160***
							(-10.51)
Firm FE	✓	✓	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓	✓	✓
Observations	51228	51220	50820	16991	17007	16822	50820
R-squared	0.101	0.126	0.126	0.136	0.100	0.119	0.135

Table A2: This table replicate Table 2 using lagged values. Investment rate is regressed on all the regressors on the right-hand side in column 7, on equity volatility and fair value spread in columns 3-6, on fair value spread in column 2 and on equity volatility in column 1. Columns 4-6 use subsamples sorted by terciles on lagged values of credit spreads $\log s_{i,t-1}$. Each observation is a firm-quarter. Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels and standard errors are clustered at the firm level.

$$\log[I/K]_{i,t} = \beta_1 \log \sigma^e_{i,t-1} + \beta_2 \log \hat{s}_{i,t-1} + \gamma \log \sigma^e_{i,t-1} \times \log \hat{s}_{i,t-1} + \eta_i + \lambda_t + \epsilon_{i,t}$$

	(1) all	(2) all	(3) all	(4) low cs	(5) mid cs	(6) high cs	(7) all
$\log \sigma^e_{i,t-1}$	-0.160***		-0.003	0.071***	0.024	-0.071***	0.334***
	(-8.29)		(-0.20)	(4.04)	(1.15)	(-2.72)	(7.52)
$\log \hat{s}_{i,t-1}$		-0.161***	-0.159***	-0.086***	-0.130***	-0.165***	-0.254***
		(-14.29)	(-14.34)	(-4.81)	(-8.93)	(-9.59)	(-14.82)
$\log \sigma^e_{i,t-1} \times \log \hat{s}_{i,t-1}$							-0.077***
							(-7.74)
Firm FE	✓	✓	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓	✓	✓
Observations	38740	37935	37600	12495	12534	12285	37600
R-squared	0.107	0.143	0.141	0.164	0.152	0.125	0.148

Table A3: This table replicates Table 3 using lagged values. The regression in column 5 includes all the regressors in the estimation equation, and the regressions in columns 1-4 drop the interaction term. Columns 2-4 use subsamples sorted by terciles on lagged values of credit spreads $\log s_{i,t-1}$. Each observation is a firm-quarter. Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels and standard errors are clustered at the firm level.

$$\log[I/K]_{i,t} = \beta_1 \log \sigma_{i,t-1} + \beta_2 \log s_{i,t-1} + \gamma \log \sigma_{i,t-1} \times \log s_{i,t-1} + \eta_i + \lambda_t + \epsilon_{i,t}$$

	(1) all	(2) low cs	(3) mid cs	(4) high cs	(5) all
$\log \sigma_{i,t-1}$	0.084***	0.086***	0.053***	0.062***	0.691***
	(5.79)	(5.14)	(2.76)	(2.78)	(7.20)
$\log s_{i,t-1}$	-0.284***	-0.104***	-0.305***	-0.512***	-0.466***
	(-12.61)	(-2.83)	(-6.56)	(-12.25)	(-13.14)
$\log \sigma_{i,t-1} \times \log s_{i,t-1}$					-0.110***
					(-6.37)
Firm FE	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓
Observations	46513	15869	15642	15002	46513
R-squared	0.126	0.147	0.124	0.117	0.130

Table A4: This table replicates Table 4 using lagged values. The regression in column 5 includes all the regressors in the estimation equation, and the regressions in columns 1-4 drop the interaction term. Columns 2-4 use subsamples sorted by terciles on lagged values of credit spreads $\log s_{i,t-1}$. Each observation is a firm-quarter. Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels and standard errors are clustered at the firm level.

$$\log[I/K]_{i,t} = \beta_1 \log \hat{\sigma}_{i,t-1} + \beta_2 \log s_{i,t-1} + \gamma \log \hat{\sigma}_{i,t-1} \times \log s_{i,t-1} + \eta_i + \lambda_t + \epsilon_{i,t}$$

	(1) all	(2) low cs	(3) mid cs	(4) high cs	(5) all
$\log \hat{\sigma}_{i,t-1}$	0.272***	0.251***	0.251***	0.243***	0.825***
	(8.58)	(4.87)	(4.90)	(4.78)	(4.60)
$\log s_{i,t-1}$	-0.334***	-0.141***	-0.377***	-0.597***	-0.515***
	(-10.40)	(-3.23)	(-5.57)	(-8.03)	(-7.89)
$\log \hat{\sigma}_{i,t-1} \times \log s_{i,t-1}$					-0.098***
					(-3.09)
Firm FE	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓
Observations	22367	7526	7473	7316	22367
R-squared	0.154	0.149	0.139	0.166	0.156

Table A5: This table presents the loadings of credit spread on the industry average of asset volatility and market leverage. The dependent variable $\log y_{i,t}$ denotes either equity volatility $\log \sigma^e_{i,t}$ or credit spread $\log s_{i,t}$. For a firm i in industry k at time t, we compute the industry average of log asset volatility excluding itself as $\frac{1}{N_k-1}\sum_{j\neq i}\log \sigma_{j,t}$. We compute the industry average of market leverage similarly. We report results for estimations in levels in Panel A and results for estimations in first differences in Panel B. Each observation is a firm-quarter. Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels and standard errors are clustered at the firm level.

$$\log y_{i,t} = \beta_1 \frac{1}{N_k-1}\sum_{j\neq i}\log \sigma_{j,t} + \beta_2 \frac{1}{N_k-1}\sum_{j\neq i}\log[MA/ME]_{j,t} + \eta_i + \lambda_t + \epsilon_{i,t}$$

Panel A: Levels	(1) $\log \sigma^e_{i,t}$	(2) $\log s_{i,t}$	Panel B: Changes	(3) $\Delta \log \sigma^e_{i,t}$	(4) $\Delta \log s_{i,t}$
$\frac{1}{N_k-1}\sum_{j\neq i}\log \sigma_{j,t}$	0.415***	0.055	$\Delta \frac{1}{N_k-1}\sum_{j\neq i}\log \sigma_{j,t}$	0.292***	0.074***
	(11.21)	(0.87)		(11.71)	(4.58)
$\frac{1}{N_k-1}\sum_{j\neq i}\log[MA/ME]_{j,t}$	0.216***	0.353***	$\Delta \frac{1}{N_k-1}\sum_{j\neq i}\log[MA/ME]_{j,t}$	0.135***	0.074***
	(5.38)	(5.30)		(3.70)	(3.15)
Firm FE	✓	✓	Firm FE	✓	✓
Time FE	✓	✓	Time FE	✓	✓
Observations	47323	47246	Observations	44702	44636
R-squared	0.372	0.432	R-squared	0.163	0.329

Table A6: This table replicate Table 1 using standardized variables. All variables are standardized with mean 0 and standard deviation 1. We use the original value of $\log s_{i,t}$ in the interaction term so it has consistent interpretation as in our main results. Each observation is a firm-quarter. Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels and standard errors are clustered at the firm level.

$$\log[I/K]_{i,t} = \beta_1 \log \sigma^e_{i,t} + \beta_2 \log s_{i,t} + \gamma \log \sigma^e_{i,t} \times \log s_{i,t} + \eta_i + \lambda_t + \epsilon_{i,t}$$

	(1) all	(2) all	(3) all	(4) low cs	(5) mid cs	(6) high cs	(7) all
$\log \sigma^e_{i,t}$	-0.117***		-0.036***	0.049***	-0.011	-0.078***	0.563***
	(-8.99)		(-3.37)	(3.86)	(-0.73)	(-4.55)	(9.79)
$\log s_{i,t}$		-0.305***	-0.287***	-0.111***	-0.269***	-0.461***	-0.270***
		(-13.33)	(-12.84)	(-3.03)	(-5.68)	(-11.67)	(-12.40)
$\log \sigma^e_{i,t} \times \log s_{i,t}$							-0.107***
							(-10.21)
Firm FE	✓	✓	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓	✓	✓
Observations	52897	52900	52414	17614	17595	17205	52414
R-squared	0.101	0.125	0.126	0.144	0.122	0.116	0.134

Table A7: This table replicates Table 2 using standardized variables. All variables are standardized with mean 0 and standard deviation 1. We use the original value of $\log \hat{s}_{i,t}$ in the interaction term so it has consistent interpretation as in our main results. Each observation is a firm-quarter. Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels and standard errors are clustered at the firm level.

$$\log[I/K]_{i,t} = \beta_1 \log \sigma^e_{i,t} + \beta_2 \log \hat{s}_{i,t} + \gamma \log \sigma^e_{i,t} \times \log \hat{s}_{i,t} + \eta_i + \lambda_t + \epsilon_{i,t}$$

	(1) all	(2) all	(3) all	(4) low cs	(5) mid cs	(6) high cs	(7) all
$\log \sigma^e_{i,t}$	-0.108***		-0.006	0.056***	0.020	-0.055***	0.209***
	(-7.91)		(-0.54)	(4.31)	(1.34)	(-2.92)	(7.01)
$\log \hat{s}_{i,t}$		-0.282***	-0.275***	-0.135***	-0.222***	-0.283***	-0.255***
		(-13.96)	(-13.88)	(-4.33)	(-8.37)	(-9.15)	(-12.94)
$\log \sigma^e_{i,t} \times \log \hat{s}_{i,t}$							-0.048***
							(-7.19)
Firm FE	✓	✓	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓	✓	✓
Observations	39925	40331	39925	13272	13296	13027	39925
R-squared	0.108	0.140	0.139	0.169	0.145	0.126	0.144

Table A8: This table replicates Table 3 using standardized variables. All variables are standardized with mean 0 and standard deviation 1. We use the original value of $\log s_{i,t}$ in the interaction term so it has consistent interpretation as in our main results. Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels and standard errors are clustered at the firm level.

$$\log[I/K]_{i,t} = \beta_1 \log \sigma_{i,t} + \beta_2 \log s_{i,t} + \gamma \log \sigma_{i,t} \times \log s_{i,t} + \eta_i + \lambda_t + \epsilon_{i,t}$$

	(1) all	(2) low cs	(3) mid cs	(4) high cs	(5) all
$\log \sigma_{i,t}$	0.058***	0.073***	0.039***	0.043***	0.444***
	(6.34)	(6.60)	(3.11)	(3.00)	(7.45)
$\log s_{i,t}$	-0.299***	-0.108***	-0.295***	-0.519***	-0.299***
	(-12.60)	(-2.75)	(-5.98)	(-11.87)	(-12.73)
$\log \sigma_{i,t} \times \log s_{i,t}$					-0.070***
					(-6.54)
Firm FE	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓
Observations	47384	16237	15947	15200	47384
R-squared	0.125	0.149	0.122	0.114	0.128

Table A9: This table replicates Table 6 using standardized variables. All variables are standardized with mean 0 and standard deviation 1. The dependent variable $\log y_{i,t}$ denotes either equity volatility $\log \sigma^e_{i,t}$ or credit spread $\log s_{i,t}$. We report results for estimations in levels in Panel A and results for estimations in first differences in Panel B.

$$\log y_{i,t} = \beta_1 \log \sigma_{i,t} + \beta_2 \log[MA/ME]_{i,t} + \eta_i + \lambda_t + \epsilon_{i,t}$$

Panel A: Levels	(1) $\log \sigma^e_{i,t}$	(2) $\log s_{i,t}$	Panel B: Changes	(3) $\Delta \log \sigma^e_{i,t}$	(4) $\Delta \log s_{i,t}$
$\log \sigma_{i,t}$	0.724***	0.109***	$\log \sigma_{i,t}$	0.721***	0.009***
	(90.77)	(15.91)		(83.77)	(5.00)
$\log[MA/ME]_{i,t}$	0.452***	0.407***	$\log[MA/ME]_{i,t}$	0.247***	0.164***
	(56.20)	(32.17)		(25.66)	(24.37)
Firm FE	✓	✓	Firm FE	✓	✓
Time FE	✓	✓	Time FE	✓	✓
Observations	47327	47250	Observations	44706	44640
R-squared	0.865	0.571	R-squared	0.794	0.345

Table A10: This table replicates regressions in Table 1 with the same sample used for generating Table 2. Each observation is a firm-quarter. Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels and standard errors are clustered at the firm level.

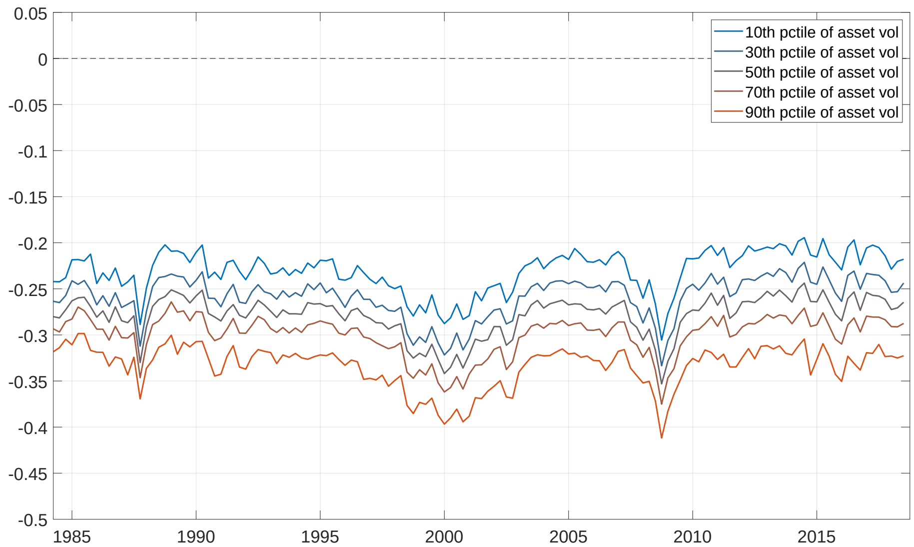
$$\log[I/K]_{i,t} = \beta_1 \log \sigma^e_{i,t} + \beta_2 \log s_{i,t} + \gamma \log \sigma^e_{i,t} \times \log s_{i,t} + \eta_i + \lambda_t + \epsilon_{i,t}$$

	(1) all	(2) all	(3) all	(4) low cs	(5) mid cs	(6) high cs	(7) all
$\log \sigma^e_{i,t}$	-0.154***		-0.048***	0.059***	-0.014	-0.118***	0.796***
	(-7.91)		(-3.00)	(3.03)	(-0.64)	(-4.35)	(8.98)
$\log s_{i,t}$		-0.278***	-0.262***	-0.111***	-0.290***	-0.392***	-0.465***
		(-11.30)	(-10.81)	(-2.92)	(-5.63)	(-8.68)	(-14.57)
$\log \sigma^e_{i,t} \times \log s_{i,t}$							-0.151***
							(-9.35)
Firm FE	✓	✓	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓	✓	✓
Observations	39925	39970	39595	13272	13296	13027	39595
R-squared	0.108	0.132	0.132	0.165	0.136	0.123	0.140

Figure A1: This figure presents the elasticity of investment with respect to credit spread across time and across firms using the estimates from the regressions with interaction terms. In each quarter we generate five cutoffs in the cross-section of log asset volatility: $\{p_{10}, p_{30}, p_{50}, p_{70}, p_{90}\}$. Using the estimates in column 7 of Table 3 on

$$\log[I/K]_{i,t} = \beta_1 \log \sigma_{i,t} + \beta_2 \log s_{i,t} + \gamma \log \sigma_{i,t} \times \log s_{i,t} + \eta_i + \lambda_t + \epsilon_{i,t},$$

the elasticity at each cutoff point is computed as $\beta_2 + \gamma p_n$, $n = 10, 30, 50, 70, 90$.



Notes: This figure is generated using the estimates on $\log s_{i,t}$ and $\log \sigma_{i,t} \times \log s_{i,t}$ in Column 5 of Table 3.

D Proofs

Shareholders maximize their expected cash flow and device when to default. Thus, the value of equity is given by:

$$E = \max_{\iota,\underline{z}} \left\{ \mathbb{E}\Big[(Y(\iota,z) - b)\mathbb{1}\{z \geq \underline{z}\}\Big] - \iota \right\}.$$

We can write the first-order conditions for investment ι and the default boundary $\underline{z}$ as:

$$\int_{\underline{z}}^{\infty} Y_\iota(\iota,z)dF(z;\sigma) - 1 = 0,$$

$$-f(\underline{z};\sigma)(Y(\iota,\underline{z}) - b) = 0,$$

and the second-order conditions for investment ι and the default boundary $\underline{z}$ as:

$$\int_{\underline{z}}^{\infty} Y_{\iota\iota}(\iota,z)dF(z;\sigma) < 0,$$

$$-f(\underline{z};\sigma)k(\iota) < 0,$$

$$-\int_{\underline{z}}^{\infty} Y_{\iota\iota}(\iota,z)dF(z;\sigma)f(\underline{z};\sigma)k(\iota) - f(\underline{z};\sigma)^2 k_\iota(\iota)^2 \underline{z}^2 > 0. \tag{8}$$

Thus, $Y_\iota(\iota,\underline{z})^2 f(\underline{z};\sigma) + k(\iota)k_{\iota\iota}(\iota)\mu(\underline{z},\sigma) < 0$.

In the following sections, we derive the partial derivatives of equity with respect to (i) credit spreads and asset volatility, (ii) leverage and asset volatility, and (iii) credit spreads and equity volatility to rationalize our empirical results.

Investment ι, Credit Spreads cs, and Asset Volatility σ Assume we observe $\boldsymbol{\theta}$ and we want to derive the partial derivatives of $\mathbf{x}$ with respect to $\boldsymbol{\theta}$. Since $\mathbf{x}$ is the solution to a system of nonlinear equations $\boldsymbol{\mathcal{D}}(\mathbf{x}, \boldsymbol{\theta})$, we need to use the multivariate implicit function theorem:

$$\frac{\partial \mathbf{x}(\boldsymbol{\theta})}{\partial \theta_i} = -\left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}), \boldsymbol{\theta})}{\partial x_j}\right]^{-1} \left[\frac{\partial \boldsymbol{\mathcal{D}}(\mathbf{x}(\boldsymbol{\theta}), \boldsymbol{\theta})}{\partial \theta_i}\right],$$

where

$$\boldsymbol{\mathcal{D}}(\mathbf{x}, \boldsymbol{\theta}) = \begin{bmatrix} \int_{\underline{z}}^{\infty} Y_\iota(\iota, z) dF(z; \sigma) - 1 \\ F(\underline{z}; \sigma) - cs \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} \iota \\ \underline{z} \end{bmatrix}, \quad \boldsymbol{\theta} = \begin{bmatrix} cs & \sigma \end{bmatrix}.$$

We can derive the Jacobian matrix of $\boldsymbol{\mathcal{D}}(\mathbf{x}, \boldsymbol{\theta})$ as:

$$\left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}), \boldsymbol{\theta})}{\partial x_j}\right] = \begin{bmatrix} \int_{\underline{z}}^{\infty} Y_{\iota\iota}(\iota, z) dF(z; \sigma) & -Y_\iota(\iota, \underline{z}) f(\underline{z}; \sigma) \\ 0 & f(\underline{z}; \sigma) \end{bmatrix}$$

and the partial derivatives as:

$$\left[\frac{\partial \boldsymbol{\mathcal{D}}(\mathbf{x}(\boldsymbol{\theta}), \boldsymbol{\theta})}{\partial cs}\right] = \begin{bmatrix} 0 \\ -1 \end{bmatrix}, \qquad \left[\frac{\partial \boldsymbol{\mathcal{D}}(\mathbf{x}(\boldsymbol{\theta}), \boldsymbol{\theta})}{\partial \sigma}\right] = \begin{bmatrix} \int_{\underline{z}}^{\infty} Y_\iota(\iota, z) f_\sigma(z) dz \\ F_\sigma(\underline{z}) \end{bmatrix}.$$

To derive the comparative statics of interest, we only need few elements of $\left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}), \boldsymbol{\theta})}{\partial x_j}\right]^{-1}$.

Thus, we get:

$$\frac{\partial \iota}{\partial cs} = \left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}), \boldsymbol{\theta})}{\partial x_j}\right]^{-1}_{12} = \frac{Y_\iota(\iota, \underline{z})}{\int_{\underline{z}}^{\infty} Y_{\iota\iota}(\iota, z) dF(z; \sigma)} = \frac{k_\iota(\iota)}{k_{\iota\iota}(\iota)} \frac{\underline{z}}{\mu(\underline{z}, \sigma)} < 0,$$

and

$$\begin{aligned} \frac{\partial \iota}{\partial \sigma} &= -\left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}), \boldsymbol{\theta})}{\partial x_j}\right]^{-1}_{11} \int_{\underline{z}}^{\infty} Y_\iota(\iota, z) f_\sigma(z) dz - \left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}), \boldsymbol{\theta})}{\partial x_j}\right]^{-1}_{12} F_\sigma(\underline{z}) \\ &= -\frac{\int_{\underline{z}}^{\infty} Y_\iota(\iota, z) f_\sigma(z) dz + Y_\iota(\iota, \underline{z}) F_\sigma(\underline{z})}{\int_{\underline{z}}^{\infty} Y_{\iota\iota}(\iota, z) dF(z; \sigma)} \\ &= -\frac{k_\iota(\iota)}{k_{\iota\iota}(\iota)} \frac{\nu(\underline{z}, \sigma)}{\mu(\underline{z}, \sigma)} > 0. \end{aligned}$$

The sign of both these partial derivatives comes directly from Assumptions 4 and 5.

Investment ι, Leverage b, and Asset Volatility σ Instead of observing credit spreads cs and asset volatility, we observe leverage b and asset volatility σ. Thus, we can write:

$$\frac{\partial \mathbf{x}(\boldsymbol{\theta})}{\partial \theta_i} = -\left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}), \boldsymbol{\theta})}{\partial x_j}\right]^{-1} \left[\frac{\partial \boldsymbol{\mathcal{D}}(\mathbf{x}(\boldsymbol{\theta}), \boldsymbol{\theta})}{\partial \theta_i}\right],$$

where

$$\boldsymbol{\mathcal{D}}(\mathbf{x}, \boldsymbol{\theta}) = \begin{bmatrix} \int_{\underline{z}}^{\infty} Y_\iota(\iota, z) dF(z; \sigma) - 1 \\ Y(\iota, \underline{z}) - b \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} \iota \\ \underline{z} \end{bmatrix}, \quad \boldsymbol{\theta} = \begin{bmatrix} b & \sigma \end{bmatrix}.$$

We can derive the Jacobian matrix of $\boldsymbol{\mathcal{D}}(\mathbf{x}, \boldsymbol{\theta})$ as:

$$\left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}), \boldsymbol{\theta})}{\partial x_j}\right] = \begin{bmatrix} \int_{\underline{z}}^{\infty} Y_{\iota\iota}(\iota, z) dF(z; \sigma) & -Y_{\iota}(\iota, \underline{z}) f(\underline{z}; \sigma) \\ Y_{\iota}(\iota, \underline{z}) & k(\iota) \end{bmatrix}$$

and the partial derivatives as:

$$\left[\frac{\partial \boldsymbol{\mathcal{D}}(\mathbf{x}(\boldsymbol{\theta}), \boldsymbol{\theta})}{\partial b}\right] = \begin{bmatrix} 0 \\ -1 \end{bmatrix}, \qquad \left[\frac{\partial \boldsymbol{\mathcal{D}}(\mathbf{x}(\boldsymbol{\theta}), \boldsymbol{\theta})}{\partial \sigma}\right] = \begin{bmatrix} \int_{\underline{z}}^{\infty} Y_{\iota}(\iota, z) f_{\sigma}(z) dz \\ 0 \end{bmatrix}.$$

To derive the comparative statics of interest, we only need few elements of $\left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}), \boldsymbol{\theta})}{\partial x_j}\right]^{-1}$. Thus, we can directly derive:

$$\begin{aligned} \frac{\partial \iota}{\partial b} &= \left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}), \boldsymbol{\theta})}{\partial x_j}\right]^{-1}_{12} \\ &= \frac{Y_{\iota}(\iota, \underline{z}) f(\underline{z}; \sigma)}{Y_{\iota}(\iota, \underline{z})^2 f(\underline{z}; \sigma) + \int_{\underline{z}}^{\infty} Y_{\iota\iota}(\iota, z) dF(z; \sigma) k(\iota)} < 0, \end{aligned}$$

$$\begin{aligned} \frac{\partial \iota}{\partial \sigma} &= -\left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}), \boldsymbol{\theta})}{\partial x_j}\right]^{-1}_{11} \int_{\underline{z}}^{\infty} Y_{\iota}(\iota, z) f_{\sigma}(z) dz \\ &= -\frac{k(\iota) \int_{\underline{z}}^{\infty} Y_{\iota}(\iota, z) f_{\sigma}(z) dz}{Y_{\iota}(\iota, \underline{z})^2 f(\underline{z}; \sigma) + \int_{\underline{z}}^{\infty} Y_{\iota\iota}(\iota, z) dF(z; \sigma) k(\iota)}, \end{aligned}$$

$$\frac{\partial \underline{z}}{\partial b} = \left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}), \boldsymbol{\theta})}{\partial x_j}\right]^{-1}_{22}$$

$$= -\frac{\int_{\underline{z}}^{\infty} Y_{\iota\iota}(\iota, z) dF(z;\sigma)}{Y_{\iota}(\iota, \underline{z})^2 f(\underline{z};\sigma) + \int_{\underline{z}}^{\infty} Y_{\iota\iota}(\iota, z) dF(z;\sigma) k(\iota)} > 0,$$

$$\frac{\partial \underline{z}}{\partial \sigma} = -\left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}), \boldsymbol{\theta})}{\partial x_j}\right]^{-1}_{21} \int_{\underline{z}}^{\infty} Y_{\iota}(\iota, z) f_{\sigma}(z) dz$$

$$= \frac{Y_{\iota}(\iota, z) \int_{\underline{z}}^{\infty} Y_{\iota}(\iota, z) f_{\sigma}(z) dz}{Y_{\iota}(\iota, \underline{z})^2 f(\underline{z};\sigma) + \int_{\underline{z}}^{\infty} Y_{\iota\iota}(\iota, z) dF(z;\sigma) k(\iota)}.$$

The sign of these partial derivatives comes directly from Assumptions 4 and 5 and the second-order condition in equation (8). Furthermore, we can derive:

$$\frac{\partial cs}{\partial \sigma} = f(\underline{z};\sigma)\frac{\partial \underline{z}}{\partial \sigma} + F_{\sigma}(\underline{z};\sigma).$$

Investment ι, Credit Spreads cs, and Equity Volatility σ^e Instead of observing credit spreads cs and asset volatility, we observe credit spreads cs and equity volatility σ^e. Thus, we can write:

$$\frac{\partial \mathbf{x}(\boldsymbol{\theta})}{\partial \theta_i} = -\left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}), \boldsymbol{\theta})}{\partial x_j}\right]^{-1} \left[\frac{\partial \boldsymbol{\mathcal{D}}(\mathbf{x}(\boldsymbol{\theta}), \boldsymbol{\theta})}{\partial \theta_i}\right],$$

where

$$\boldsymbol{\mathcal{D}}(\mathbf{x},\boldsymbol{\theta}) = \begin{bmatrix} \int_{\underline{z}}^{\infty} Y_\iota(\iota,z)dF(z;\sigma) - 1 \\ F(\underline{z};\sigma) - cs \\ \frac{\sigma}{\mu(\underline{z},\sigma)} - \sigma^e \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} \iota \\ \underline{z} \\ \sigma \end{bmatrix}, \quad \boldsymbol{\theta} = \begin{bmatrix} cs & \sigma^e \end{bmatrix}.$$

We can derive the Jacobian matrix of $\boldsymbol{\mathcal{D}}(\mathbf{x},\boldsymbol{\theta})$ as:

$$\left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}),\boldsymbol{\theta})}{\partial x_j}\right] = \begin{bmatrix} \int_{\underline{z}}^{\infty} Y_{\iota\iota}(\iota,z)dF(z;\sigma) & -Y_\iota(\iota,\underline{z})f(\underline{z};\sigma) & \int_{\underline{z}}^{\infty} Y_\iota(\iota,z)f_\sigma(z)dz \\ 0 & f(\underline{z};\sigma) & F_\sigma(\underline{z}) \\ 0 & \sigma^e_{\underline{z}} & \sigma^e_\sigma \end{bmatrix},$$

where

$$\begin{aligned} \sigma^e_{\underline{z}} &= -\frac{\sigma\bar{\mu}_{\underline{z}}(\underline{z},\sigma)}{\bar{\mu}(\underline{z},\sigma)^2} = \frac{\sigma\left(1 - F(\underline{z};\sigma)\right)}{\bar{\mu}(\underline{z},\sigma)^2}, \\ \sigma^e_\sigma &= \frac{\bar{\mu}(\underline{z},\sigma) - \sigma\nu(\underline{z},\sigma)}{\bar{\mu}(\underline{z},\sigma)^2}, \end{aligned}$$

and the partial derivatives as:

$$\left[\frac{\partial \boldsymbol{\mathcal{D}}(\mathbf{x}(\boldsymbol{\theta}),\boldsymbol{\theta})}{\partial cs}\right] = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}, \qquad \left[\frac{\partial \boldsymbol{\mathcal{D}}(\mathbf{x}(\boldsymbol{\theta}),\boldsymbol{\theta})}{\partial \sigma^e}\right] = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}.$$

To derive the comparative statics of interest, we only need two elements of $\left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}),\boldsymbol{\theta})}{\partial x_j}\right]^{-1}$.

Thus, we can directly derive:

$$\frac{\partial \iota}{\partial cs} = \left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}), \boldsymbol{\theta})}{\partial x_j}\right]^{-1}_{12} = \frac{k_\iota(\iota)}{k_{\iota\iota}(\iota)\mu(\underline{z},\sigma)} \frac{\sigma^e_{\underline{z}}(\underline{z},\sigma)\int_{\underline{z}}^{\infty} z f_\sigma(z;\sigma)dz + \sigma^e_\sigma(\underline{z},\sigma)\underline{z} f(\underline{z};\sigma)}{f(\underline{z};\sigma)\sigma^e_\sigma(\underline{z},\sigma) - F_\sigma(\underline{z};\sigma)\sigma^e_{\underline{z}}(\underline{z},\sigma)}$$

$$= \frac{k_\iota(\iota)}{k_{\iota\iota}(\iota)} \frac{\underline{z}}{\mu(\underline{z},\sigma)} \frac{\sigma^e_{\underline{z}}(\underline{z},\sigma)\int_{\underline{z}}^{\infty} \frac{z}{\underline{z}}\frac{f_\sigma(z;\sigma)}{f(\underline{z};\sigma)}dz + \sigma^e_\sigma(\underline{z},\sigma)}{\sigma^e_\sigma(\underline{z},\sigma) - \frac{F_\sigma(\underline{z})}{f(\underline{z};\sigma)}\sigma^e_{\underline{z}}(\underline{z},\sigma)}$$

$$= \frac{k_\iota(\iota)}{k_{\iota\iota}(\iota)} \frac{\underline{z}}{\mu(\underline{z},\sigma)} \left(\sigma^e_{\underline{z}}(\underline{z},\sigma)\int_{\underline{z}}^{\infty} \frac{z}{\underline{z}}\frac{f_\sigma(z;\sigma)}{f(\underline{z};\sigma)}dz + \sigma^e_\sigma(\underline{z},\sigma)\right)$$

$$\times \left(\sigma^e_\sigma(\underline{z},\sigma) - \frac{F_\sigma(\underline{z};\sigma)}{f(\underline{z};\sigma)}\sigma^e_{\underline{z}}(\underline{z},\sigma)\right)^{-1}$$

and

$$\frac{\partial \iota}{\partial \sigma^e} = \left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}), \boldsymbol{\theta})}{\partial x_j}\right]^{-1}_{13} = -\frac{k_\iota(\iota)}{k_{\iota\iota}(\iota)\mu(\underline{z},\sigma)} \frac{f(\underline{z};\sigma)\underline{z}F_\sigma(\underline{z};\sigma) + f(\underline{z};\sigma)\int_{\underline{z}}^{\infty} z f_\sigma(z;\sigma)dz}{f(\underline{z};\sigma)\sigma^e_\sigma(\underline{z},\sigma) - F_\sigma(\underline{z};\sigma)\sigma^e_{\underline{z}}(\underline{z},\sigma)}$$

$$= -\frac{k_\iota(\iota)}{k_{\iota\iota}(\iota)} \frac{\nu(\underline{z})}{\mu(\underline{z},\sigma)} \left(\sigma^e_\sigma(\underline{z},\sigma) - \frac{F_\sigma(\underline{z};\sigma)}{f(\underline{z};\sigma)}\sigma^e_{\underline{z}}(\underline{z},\sigma)\right)^{-1}.$$

We define the the credit spread wedge as:

$$\xi_{cs|\sigma^e}(\underline{z},\sigma) = \left(\sigma^e_{\underline{z}}(\underline{z},\sigma)\left(\frac{\nu(\underline{z},\sigma)}{\underline{z}f(\underline{z};\sigma)} - \frac{F_\sigma(\underline{z};\sigma)}{f(\underline{z};\sigma)}\right) + \sigma^e_\sigma(\underline{z},\sigma)\right)\zeta^e(\underline{z},\sigma)$$

$$= 1 + \sigma^e_{\underline{z}}(\underline{z},\sigma)\frac{\nu(\underline{z},\sigma)}{\underline{z}f(\underline{z};\sigma)}\zeta^e(\underline{z},\sigma),$$

and the equity volatility wedge as:

$$\xi_{\sigma^e|cs}(\underline{z},\sigma) = \left(\sigma^e_\sigma(\underline{z},\sigma) - \frac{F_\sigma(\underline{z};\sigma)}{f(\underline{z};\sigma)}\sigma^e_{\underline{z}}(\underline{z},\sigma)\right)^{-1}.$$

Investment ι, Credit Spreads with Positive Liquidation Value $\widetilde{cs}$, and Asset Volatility σ Instead of observing credit spreads cs and asset volatility σ, we observe credit spreads cs with positive liquidation value $\widetilde{cs}$ and asset volatility σ.

We define the credit spreads with positive liquidation value as:

$$\widetilde{cs} = F(\underline{z};\sigma) - \frac{\alpha}{b}\mathbb{E}\Big[Y(\iota,z)\mathbb{1}\{z \le \underline{z}\}\Big].$$

where $1-\alpha$ represents bankruptcy costs. Thus, we can write:

$$\frac{\partial \mathbf{x}(\boldsymbol{\theta})}{\partial \theta_i} = -\left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}),\boldsymbol{\theta})}{\partial x_j}\right]^{-1}\left[\frac{\partial \boldsymbol{\mathcal{D}}(\mathbf{x}(\boldsymbol{\theta}),\boldsymbol{\theta})}{\partial \theta_i}\right],$$

where

$$\boldsymbol{\mathcal{D}}(\mathbf{x},\boldsymbol{\theta}) = \begin{bmatrix} \int_{\underline{z}}^{\infty} Y_\iota(\iota,z)dF(z;\sigma) - 1 \\ F(\underline{z};\sigma) - \frac{\alpha}{b}\int_0^{\underline{z}} Y(\iota,z)dF(z;\sigma) - \widetilde{cs} \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} \iota \\ \underline{z} \end{bmatrix}, \quad \boldsymbol{\theta} = \begin{bmatrix} \widetilde{cs} & \sigma \end{bmatrix}.$$

We can derive the Jacobian matrix of $\boldsymbol{\mathcal{D}}(\mathbf{x},\boldsymbol{\theta})$ as:

$$\left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}),\boldsymbol{\theta})}{\partial x_j}\right] = \begin{bmatrix} \int_{\underline{z}}^{\infty} Y_{\iota\iota}(\iota,z)dF(z;\sigma) & -Y_\iota(\iota,\underline{z})f(\underline{z};\sigma) \\ -\frac{\alpha}{b}\int_0^{\underline{z}} Y_\iota(\iota,z)dF(z;\sigma) & f(\underline{z};\sigma)(1-\alpha) \end{bmatrix}$$

since $Y(\iota,\underline{z}) = b$, and the partial derivatives as:

$$\left[\frac{\partial \boldsymbol{\mathcal{D}}(\mathbf{x}(\boldsymbol{\theta}),\boldsymbol{\theta})}{\partial \widetilde{cs}}\right] = \begin{bmatrix} 0 \\ -1 \end{bmatrix}, \qquad \left[\frac{\partial \boldsymbol{\mathcal{D}}(\mathbf{x}(\boldsymbol{\theta}),\boldsymbol{\theta})}{\partial \sigma}\right] = \begin{bmatrix} \int_{\underline{z}}^{\infty} Y_\iota(\iota,z)f_\sigma(z)dz \\ F_\sigma(\underline{z}) - \frac{\alpha}{b}\int_0^{\underline{z}} Y(\iota,z)f_\sigma(z)dz \end{bmatrix}.$$

To derive the comparative statics of interest, we only need few elements of $\left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}),\boldsymbol{\theta})}{\partial x_j}\right]^{-1}$. Thus, we get:

$$\begin{aligned}
\frac{\partial \iota}{\partial cs} &= \left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}),\boldsymbol{\theta})}{\partial x_j}\right]^{-1}_{12} = \frac{Y_\iota(\iota,\underline{z})}{\int_{\underline{z}}^{\infty} Y_{\iota\iota}(\iota,z)dF(z;\sigma)\,(1-\alpha) - \frac{\alpha}{b}Y_\iota(\iota,\underline{z})\int_0^{\underline{z}} Y_\iota(\iota,z)dF(z;\sigma)} \\
&= \frac{k_\iota(\iota)}{k_{\iota\iota}(\iota)}\frac{\underline{z}}{\mu(\underline{z},\sigma)\,(1-\alpha) - \alpha\frac{k_\iota(\iota)}{k(\iota)}\frac{k_\iota(\iota)}{k_{\iota\iota}(\iota)}(1-\mu(\underline{z},\sigma))} \\
&= \frac{k_\iota(\iota)}{k_{\iota\iota}(\iota)}\frac{\underline{z}}{\mu(\underline{z},\sigma)}\frac{1}{(1-\alpha) - \alpha\frac{k_\iota(\iota)}{k(\iota)}\frac{k_\iota(\iota)}{k_{\iota\iota}(\iota)}\frac{1-\mu(\underline{z},\sigma)}{\mu(\underline{z},\sigma)}} < 0.
\end{aligned}$$

Indeed, since $\alpha \leq 0$, we get that $1-\alpha \geq 0$. Furthermore,

$$\begin{aligned}
\frac{\partial \iota}{\partial \sigma} &= -\left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}),\boldsymbol{\theta})}{\partial x_j}\right]^{-1}_{11}\int_{\underline{z}}^{\infty} Y_\iota(\iota,z)f_\sigma(z)dz - \left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}),\boldsymbol{\theta})}{\partial x_j}\right]^{-1}_{12}\left(F_\sigma(\underline{z}) - \frac{\alpha}{b}\int_0^{\underline{z}} Y(\iota,z)f_\sigma(z)dz\right) \\
&= -\frac{\int_{\underline{z}}^{\infty} Y_\iota(\iota,z)f_\sigma(z)dz\,(1-\alpha) + Y_\iota(\iota,\underline{z})\left(F_\sigma(\underline{z}) - \frac{\alpha}{b}\int_0^{\underline{z}} Y(\iota,z)f_\sigma(z)dz\right)}{\int_{\underline{z}}^{\infty} Y_{\iota\iota}(\iota,z)dF(z;\sigma)\,(1-\alpha) - \frac{\alpha}{b}Y_\iota(\iota,\underline{z})\int_0^{\underline{z}} Y_\iota(\iota,z)dF(z;\sigma)} \\
&= -\frac{k_\iota(\iota)\int_{\underline{z}}^{\infty} zf_\sigma(z)dz\,(1-\alpha) + k_\iota(\iota)\left(\underline{z}F_\sigma(\underline{z}) - \alpha\int_0^{\underline{z}} zf_\sigma(z)dz\right)}{\int_{\underline{z}}^{\infty} Y_{\iota\iota}(\iota,z)dF(z;\sigma)\,(1-\alpha) - \alpha\frac{k_\iota(\iota)^2}{k(\iota)}(1-\mu(\underline{z},\sigma))} \\
&= -\frac{k_\iota(\iota)(\nu(\underline{z},\sigma) - \alpha\int_0^{\infty} zf_\sigma(z)dz)}{\int_{\underline{z}}^{\infty} Y_{\iota\iota}(\iota,z)dF(z;\sigma)\,(1-\alpha) - \alpha\frac{k_\iota(\iota)^2}{k(\iota)}(1-\mu(\underline{z},\sigma))} \\
&= -\frac{k_\iota(\iota)}{k_{\iota\iota}(\iota)}\frac{\nu(\underline{z},\sigma)}{\mu(\underline{z},\sigma)}\frac{1}{(1-\alpha) - \alpha\frac{k_\iota(\iota)}{k(\iota)}\frac{k_\iota(\iota)}{k_{\iota\iota}(\iota)}\frac{1-\mu(\underline{z},\sigma)}{\mu(\underline{z},\sigma)}} > 0.
\end{aligned}$$

Indeed, $1-\mu(\underline{z},\sigma) \geq 0$.

E Endogenous Leverage Dynamics

In this appendix, we extend the framework of DeMarzo and He (2020) to include an investment function. We solve numerically the Markov perfect equilibrium and confirm that our results hold in Figure 2. We refer to DeMarzo and He (2020) for the proofs of the existence and uniqueness of the Markov perfect equilibrium.

We assume that agents are risk neutral with an exogenous discount rate of $r > 0$. The firm's assets-in-place generate operating cash flow at the rate of Y_t, which evolves according to a geometric Brownian motion:

$$dY_t/Y_t = \mu_t dt + \sigma dZ_t$$

where Z_t is a standard Brownian motion. A firm has at its disposal an investment technology with adjustment costs, such that $\iota_t Y_t$ spent allows the firm to grow its capital stock by $\mu(\iota_t)Y_t dt$, where $\mu(\cdot)$ is increasing and concave. Denote by B the aggregate face value of outstanding debt that pays a constant coupon rate of $c > 0$. The firm pays corporate taxes equal to $\pi(Y_t - cF_t)$. We assume that debt takes the form of exponentially maturing coupon bonds with a constant amortization rate ξ. Equity holders control the outstanding debt B_t through an endogenous issuance/repurchase policy $d\Gamma_t$ but cannot commit on a policy. Thus, the evolution of the outstanding face value of debt follows

$$dB_t = d\Gamma_t - \xi B_t dt.$$

In the unique Markov equilibrium, given the debt price $p(Y, B)$, the firm's issuance policy $d\Gamma_t = G_t dt$ and default time τ maximize the market value of equity:

$$E(Y, B) = \max_{\tau, \iota_t, G_t} \mathbb{E}_t \left[\int_t^\tau e^{-r(s-t)} \big[(1 - \iota_s) Y_s - \pi(Y_s - cB_s) - (c + \xi) B_s + G_s p_s \big] ds \,\middle|\, Y_t = Y, B_t = B \right].$$

Similarly, the equilibrium market price of debt must satisfy

$$p(Y, B) = \mathbb{E}_t \left[\int_t^\tau e^{-(r+\xi)} (c + \xi) ds \,\middle|\, Y_t = Y, B_t = B \right].$$

The Hamilton-Jacobi-Bellman (HJB) equation for equity holders is

$$rE(Y, B) = \max_{\iota, G} \bigg[(1 - \iota) Y - \pi(Y - cB) - (c + \xi) B_s + Gp(Y, B) + (G - \xi B) E_B(Y, F) + \mu(\iota) Y E_Y(Y, B) + \frac{1}{2} \sigma^2 Y^2 E_{YY}(Y, B) \bigg]. \tag{9}$$

Thus, in equilibrium it must be that

$$p(Y, B) = -E_B(Y, B).$$

The first-order condition for the investment rate is given by

$$1 = \mu_\iota(\iota) E_Y(Y, B).$$

In the following, we define $\{\iota(Y,B), G(Y,B)\}$ as

$$\{\iota(Y,B), G(Y,B)\} = \arg\max_{\iota,G}\Big[(1-\iota)Y - \pi(Y-cB) - (c+\xi)B_s + Gp(Y,B) + (G-\xi B)E_B(Y,F) + \mu(\iota)YE_Y(Y,B) + \frac{1}{2}\sigma^2Y^2E_{YY}(Y,B)\Big].$$

In this setting with scale-invariance, the relevant measure of leverage is given by

$$y_t \equiv Y_t/B_t,$$

and the equity value function $E(Y,B)$ and debt price $p(Y,B)$ satisfy

$$E(Y,B) = E\left(Y/B,1\right) \equiv e(y)B \quad \text{and} \quad p(Y,B) = p(Y/B,1) \equiv p(y).$$

We also define the following:

$$\iota(Y,B) \equiv \iota(y) \quad \text{and} \quad G(Y,B) \equiv g(y)B.$$

Thus, we can rewrite (9) as follows

$$(r+\xi)e(y) = \max_{\iota}\Big[(1-\iota)y - \pi(y-c) - (c+\xi) + (\mu(\iota)+\xi)ye'(y) + \frac{1}{2}\sigma^2y^2e''(y)\Big]. \tag{10}$$

The optimal default boundary is such that

$$e'(y_b) = 0.$$

The higher bound is such that

$$e'(y) = \phi y - \rho,$$

which corresponds to the value of equity without a default option. We can solve for ϕ and ρ with

$$(r+\xi)(\phi y - \rho) = \max_{\iota}\Big[(1-\iota)y - \pi(y-c) - (c+\xi) + (\mu(\iota)+\xi)\phi y\Big].$$

Thus,

$$\begin{aligned}\rho &= \frac{(1-\tau)c+\xi}{r+\xi},\\ \phi &= \frac{1-\iota^\star-\pi}{r-\mu(\iota^\star)},\\ 1 &= \mu'(\iota^\star)\phi.\end{aligned}$$

The HJB for $p(Y,B)$ is given by

$$rp(Y,B) = c + \xi(1-p(Y,B)) + (G-\xi B)p_B(Y,B) + \mu(Y,B)Yp_Y(Y,B) + \frac{1}{2}\sigma^2 Y^2 p_{YY}(Y,B).$$

where we define $\mu(Y, B) \equiv \mu(\iota(Y, B)) \equiv \mu(y)$.

Thus, we can write the HJB for $p(y)$ as

$$rp(y) = c + \xi(1 - p(y)) - (g(y) - \xi)p'(y)y + \mu(y)yp'(y) + \frac{1}{2}\sigma^2 y^2 p''(y). \tag{11}$$

where $g(y) = G(Y, B)/B$. We need $g(y)$ to be such that $p(y) = e'(y)y - e(y)$. From (10), we get

$$\begin{aligned}(r+\xi)e'(y)y = (1-\iota(y))y - \pi y - \iota'(y)y^2 + (\mu(y)+\xi)y^2 e''(y) + (\mu(y)+\xi)ye'(y) + \mu'(y)y^2 e'(y) \\ + \frac{1}{2}\sigma^2 y^3 e'''(y)) + \sigma^2 y^2 e''(y).\end{aligned}$$

Thus,

$$\begin{aligned}(r+\xi)(e'(y)y - e(y)) = (1-\pi)c + \xi - \iota'(y)y^2 + (\mu(y)+\xi)ye''(y) + \mu'(y)y^2 e'(y) \\ + \frac{1}{2}\sigma^2 y^2 e'''(y) + \frac{1}{2}\sigma^2 y^2 e''(y).\end{aligned}$$

Thus, $g(y)$ is such that

$$\begin{aligned}&c + \xi - (g(y) - \xi)p'(y)y + \mu(y)yp'(y) + \frac{1}{2}\sigma^2 y^2 p''(y) \\ &= (1-\pi)c + \xi - \iota'(y)y^2 + (\mu(y)+\xi)y^2 e''(y) + \mu'(y)y^2 e'(y) \\ &+ \frac{1}{2}\sigma^2 y^3 e'''(y)) + \frac{1}{2}\sigma^2 y^2 e''(y).\end{aligned}$$

With further algebra, we get

$$-gp'(y)y = -\pi c - \iota'(y)y^2 + \mu'(y)y^2e'(y).$$

Since $\mu'(\iota)e'(y) = 1$ and $\mu'(y) = \mu'(\iota)\iota'(y)$, we get

$$g(y) = \frac{\pi c}{p'(y)y}.$$

Plugging the solution for $g(y)$ in (11) yields

$$(r+\xi)p(y) = (1-\pi)c + \xi + (\mu(y)+\xi)yp'(y) + \frac{1}{2}\sigma^2y^2p''(y).$$

We solve numerically for the solution using ODE45 in Matlab. We use the following pseudo-algorithm.

1. Start with $y_L = 0$ and $y_H = H$, where H is a sufficiently large number.
2. Given $y_b = 1/2(y_L + y_H)$, $e(y_b) = 0$, and $e'(y_b) = 0$, we solve for $e(y)$ on $[y_b, y_B]$ where Y_B is a large number.
3. Check if $|e(Y_B) - (\phi y_B - \rho)| \leq \varepsilon$, where $\varepsilon > 0$ is a small number. If $e(Y_B) - (\phi y_B - \rho) > \varepsilon$, set $y_L = y_b$ and repeat 2-3. If $e(y_B) - (\phi y_B - \rho) < -\varepsilon$, set $y_H = y_b$ and repeat 2-3. Otherwise move to 4.
4. Start with $pp_L = 0$ and $pp_H = H$, where H is a sufficiently large number.

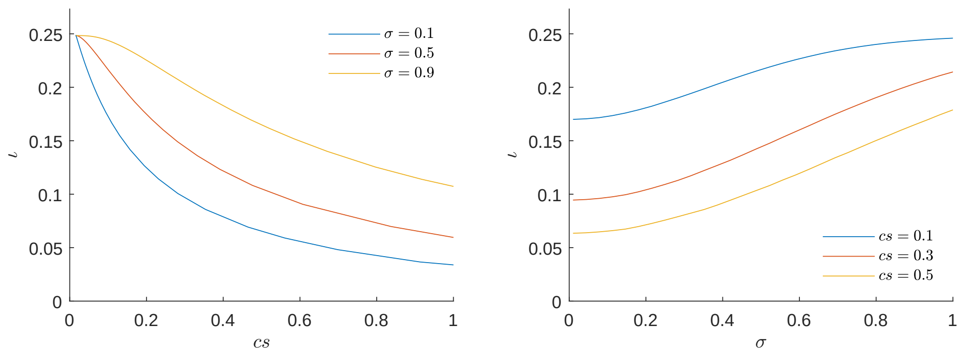


Figure 2: Optimal investment in dynamic setting with $\mu(\iota) = \frac{\log(1+\kappa\iota)}{\kappa}$, $\kappa = 100$, $r = 0.05$, $\xi = 1/8$, $c = 0.05$, $\pi = 0.3$.

5. Given $pp_b = 1/2(pp_L + pp_H)$, $p(y_b) = 0$, $p'(y_b) = pp_b$ we solve for $p(y)$ on $[y_b, y_B]$.

6. Check if $|p(y_B) - \rho| \leq \varepsilon$ If $p(Y_B) - \rho > \varepsilon$, set $p_H = p_b$ and repeat 2-3. If $p(y_B) - \rho < -\varepsilon$, set $pp_L = pp_b$ and repeat 4-5. Otherwise move to 7.

7. Check if $|p'(y_b) - e''(y_b)y_b| \leq \varepsilon$. If not, increase the precision of the ODE45 solver and restart from 1.