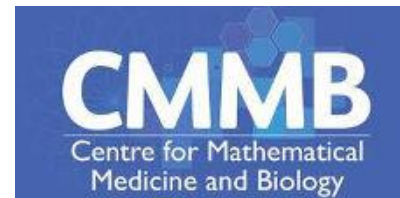




**University of
Nottingham**

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Threshold Networks 2019

Programme

July 21-24 2019

University of Nottingham
Exchange building
Jubilee campus

EPSRC

Engineering and Physical Sciences
Research Council

QJMAM
THE QUARTERLY JOURNAL OF
MECHANICS &
APPLIED
MATHEMATICS

Schedule of events

21st July 2019		
13:00 – 13:45	Stephen Coombes	Tutorial 1: Weakly coupled oscillators
13:45 – 14:00	Break	
14:00 – 14:45	Stephen Coombes	Tutorial 2: The Master Stability Function
14:45 – 15:30	Coffee break	
15:30 – 16:30	Desmond Higham	Tutorial 3: Matrix computations for discovering network structures
16:30 – 16:45	Break	
16:45 – 17:45	Renaud Lambiotte	Tutorial 4: TBA
17:45 – onwards	Welcome event	

22nd July 2019		
9:00 – 9:45		Registration
9:45 – 10:00		Opening remarks
10:00 – 11:00	Peter Mucha	Coevolving voter models
11:00 – 11:30	Coffee break	
11:30 – 12:30	Renaud Lambiotte	TBA
12:30 – 14:00	Lunch	
14:00 – 15:00	Naoki Masuda	Networks from correlation matrices: an alternative to thresholding
15:00 – 16:00	Desmond Higham	Stochastic Network Models
16:00 – onwards	Poster session	

23^d July 2019

9:00 – 9:30		Registration
9:30 – 10:30	Christian Bick	Intermittent interactions: dynamics of oscillator networks with dead zones
10:30 – 11:00	Coffee break	
11:00 – 12:00	Mario di Bernardo	Convergence and Synchronization in Networks of Piecewise Smooth Dynamical Systems
12:00 – 13:30	Lunch	
13:30 – 14:30	Yamir Moreno	Disease spreading processes through the lens of multilayer networks
14:30 – 15:30	Ginestra Bianconi	Avalanches of failures in single and multilayer networks
15:30 – 16:00	Coffee break	
16:00 – 17:00	Yi Ming Lai	Networks of nonsmooth dynamical systems: advances and applications
18:00 – onwards	Conference dinner	

24th July 2019

9:00 – 9:30		Registration
9:30 – 10:30	Jonathan Crofts	Structure-function clustering in multiplex brain networks
10:30 – 11:00	Coffee break	
11:00 – 11:50	Contributed talks	David Sirl A network epidemic model with preventive rewiring Viktor Sip Inferring the seizure propagation patterns using a data-driven model of a threshold network
11:50 – 12:50	Sarah Muldoon	Detecting state changes in evolving functional networks
12:50 – 13:00	Closing remarks	
13:00 – onwards	Refreshments and breakout	

Welcome

Welcome to our Threshold Networks Conference!

A warm welcome to Nottingham, home to both the heroic outlaw Robin Hood and the Mathematical Physicist George Green. We hope to add to these Nottingham legends with this first conference on Threshold Networks!

Many real-world problems are now being tackled with the new tools of Network Science, and it is exciting to see the advances that are being made in understanding structural-functional connectivity relationships, the organisational role of synchrony, strongly coupled network instabilities, and network control strategies. On top of this, there is a growing realisation that the use of switch-like models with 'thresholds' is a natural way to describe a wide range of dynamical processes. It is thus timely to explore 'Threshold Networks', combining Network Science and discontinuous dynamics, with an eye to both mathematical tractability and applications to biology, engineering, physics, and the social sciences.

We hope you enjoy the conference!

Yi Ming Lai
Stephen Coombes
Rüdiger Thul

Mason Porter

School of Mathematics, University of Nottingham

University of California Los Angeles

Thanks

The conference organisers would like to thank:

Volunteers
Mustafa Şayli
Abigail Cocks
Charline Tessereau
Hayley Mills

Research support staff, without whose support and assistance this conference would have been impossible

Raj Birring – Research Support Officer
Megan Hill – Research Support Administrator
James Hawkes – School Operations Officer

Sponsors

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Contents

Monday 22nd July 2019:	6
Peter Mucha	6
Renaud Lambiotte	6
Naoki Masuda	6
Desmond Higham	6
Tuesday 23rd July 2019:	7
Christian Bick.....	7
Mario di Bernardo	7
Yamir Moreno.....	7
Ginestra Bianconi	8
Yi Ming Lai	8
Wednesday 23 rd July 2019:.....	9
Jonathan Crofts.....	9
Contributed talks	9
Sarah Muldoon.....	10
Posters:	11

Monday 22nd July 2019:

Peter Mucha

Coevolving Voter Models

We consider idealized network dynamics of individuals who change their opinions and modify their connections in a coevolving process driving towards consensus. Discordant edges connecting disagreeing nodes are resolved either through one individual changing opinion to match the other or through a rewiring process, the details of which can lead to dramatically different results. We compare and contrast results under various rewiring recipes and also consider modifications that reinforce local clustering. Throughout, we identify and test approximate systems differential equations for describing the system states and the transitions between qualitatively different behaviors.

This talk represents work across multiple collaborations, including contributions from Phil Chodrow, Feng Shi, Nishant Malik, Hsuan-Wei Lee, Rick Durrett, James Gleeson, Alun Lloyd, David Sivakoff, Josh Socolar and Chris Varghese.

Renaud Lambiotte

TBA

Naoki Masuda

Networks from correlation matrices: An alternative to thresholding

Many network analyses have been exercised on correlation matrix data. Thresholding on the value of the pairwise correlation is probably the simplest and most common way to create networks from correlation matrices. However, there have been multiple criticisms on this thresholding approach, which have led to proposals of alternative methods to overcome some of the problems. We propose a method to create networks from correlation matrices based on optimisation with regularization, where we lay an edge between each pair of nodes if and only if the edge is unexpected from a null model. This is a known setting, but we will focus on the use of different null models and report applications to empirical data.

Desmond Higham

Stochastic Network Models

I will discuss a class of random graph models where a fixed set of nodes have connections that come and go over time. This type of model is relevant, for example, to on-line social interaction. For a model where new interactions are driven by the "triadic closure" principle (we are more likely to befriend people with whom we have friends in common) I will use a mean-field argument to show that bistable behaviour can arise. This is joint work with Peter Grindrod (Oxford). I will also discuss the relevance of the Friendship Paradox in this setting.

Tuesday 23rd July 2019:

Christian Bick

Intermittent interactions: dynamics of oscillator networks with dead zones

Whether two nodes in a network of dynamical units interact may depend on their states. For example, after a spike, neurons typically have a refractory period where they are desensitized to further input before they can produce another action potential. We investigate the dynamics of coupled oscillator networks where the coupling functions have "dead zones" (regions without interaction). These induce an effective coupling structure that depends on the state of the network. We analyze the interplay between dynamics and the evolving coupling structure and find solutions where units decouple and recouple as time evolves. These state-dependent dynamical systems relate to "asynchronous networks," a framework to describe dynamical systems with time-varying connectivity and typically nonsmooth dynamics. (This is joint work with P. Ashwin, M. Field, C. Poignard.)

Mario di Bernardo

Convergence and Synchronization in Networks of Piecewise Smooth Dynamical Systems

The need for coordination and cooperation among a large number of subsystems in order to achieve a common desired function is common to many applications in a wide range of different areas: from physics and technology to life and social sciences. Examples include flocking of autonomous mobile agents, fish schooling, phase synchronization of power generators, regulation of proteins and enzymes production, neuronal activity in the brain. Often, these examples can be modeled as networks of dynamical agents exchanging information over a web of complex interconnections. Typically, it is assumed that the network model is sufficiently smooth and differentiable. In many applications instead networks can be piecewise-smooth at different levels. Discontinuities can indeed affect the agent dynamics, the coupling function used to interconnect different agents and/or the evolution of the network structure itself. In this talk I will discuss recent results by my group on studying convergence and synchronization in networks of piecewise-smooth dynamical systems, including the case of Filippov systems exhibiting sliding mode solutions. After giving an overview of the problem, I will discuss different strategies based on Lyapunov stability theory and contraction theory to prove local or global convergence of all the agents towards a common asymptotic solution. Examples from applications will be used to illustrate and validate the theoretical derivations.

Yamir Moreno

Disease spreading processes through the lens of multilayer networks

Abstract: Most epidemic models assume that the spreading process takes place on a single level (be it a single population, a meta-population system or a network of contacts). The latter results from our current limited knowledge about the interplay among the various scales involved in the transmission of infectious diseases at the global scale. Therefore, pressing problems rooted at the interdependency of multi-scales call for the development of a whole new set of theoretical and simulation approaches. In this talk, we show that the recently developed framework of multilayer networks allows to tackle many of the existing challenges in the study of multi-scale diseases, ranging from interacting diseases to new phenomena like disease localization. Specifically, we (1) characterize analytically the epidemic thresholds of two interacting diseases for different scenarios and numerically compute the temporal evolution characterizing the unfolding dynamics; and (2) we present a continuous formulation of epidemic spreading on multilayer networks using a tensorial representation, showing the existence of

disease localization and the emergence of two or more transitions, which are characterized analytically and numerically through the inverse participation ratio. Our findings show the importance of considering the multilayer nature of many real systems, as this interdependency usually gives rise to new phenomenology.

[Ginestra Bianconi](#)

[Avalanches of failures in single and multilayer networks](#)

Abstract TBA

[Yi Ming Lai](#)

[Networks of nonsmooth dynamical systems: advances and applications](#)

Networks of nonsmooth dynamical systems: advances and applications

The study of coupled oscillator networks in biology, physics and engineering is now commonplace. One powerful method for studying synchrony in such networks is the master stability function (MSF), which reduces much of the problem to analysis of a low-dimensional variational equation around a periodic orbit. This can be augmented by using a piecewise linear (PWL) modelling approach, which makes the orbits accessible in closed form. However, care must be taken when the nodes in the network have jump interactions, such as pulsatile synapses or impact in physical systems. When these occur at the same time as discontinuities in the underlying orbit, they may lead to an ordering problem which has no counterpart in smoothly coupled limit cycle systems. Moreover, the smoothed versions of jump interactions do not capture the behaviour of the nonsmooth model.

We demonstrate some of these techniques, and their pitfalls, in a variety of settings ranging from neurons to pendula.

Wednesday 23rd July 2019:

Jonathan Crofts

Structure-function clustering in multiplex brain networks

Neural systems consist of large numbers of interdependent components (i.e. neurons, cortical columns, etc.) that interact in qualitatively different ways across a range of different scales. They are therefore naturally described using a multilayer or multiplex network approach, in favour of the more standard single-layer network analyses that have historically been widely applied to such systems. A key question in neuroscience is to understand how a rich functional repertoire arises within relatively static networks of structurally-connected neural populations, as this underlies higher brain (dis)function: understanding the subtle interactions between evoked 'functional connectivity' and the underlying 'structural connectivity' has the potential to address this. In this talk we shall address such issues by exploring important structure-function relations in the Macaque cortical network by modelling it as a duplex network consisting of two layers: an anatomical layer, describing the known (macro-scale) network topology of the Macaque monkey, and a functional layer derived by modelling neural activity on each node (cortical region) as a neural mass. More specifically, we shall investigate and characterize correlations between structural and functional layers, as system parameters controlling simulated neural activity are varied, by employing recently described multiplex network measures; moreover, we introduce a novel measure of multiplex structure-function clustering which allows us to investigate the emergence of functional connections that are distinct from the cortical structure on which network dynamics occurs.

Contributed talks

David Sirl

A network epidemic model with preventive rewiring

Network epidemic models have developed enormously in the last 20 years or so in response to some of the unrealistic assumptions of homogeneity in most simple epidemic models. A significant feature of most epidemic-on-a-network models is that the epidemic evolves on a static network. We consider an SIR (Susceptible - Infectious - Removed) epidemic spreading on a configuration-model network (a random network with specified degree distribution), with the addition of some simple network dynamics. The addition is to allow susceptible individuals to "drop" connections to infectious neighbours. A further extension permits such susceptible individuals to then "rewire" to connect instead with someone else in the population. Stochastic (and deterministic) epidemic models very often exhibit threshold behaviour where epidemics starting with few infective individuals either die out rapidly or take off and infect a large fraction of the population, and these models are no exception! The focus here is mainly on further analysis of the supercritical case where the epidemic can/does "take off".

For the model with dropping only (i.e. with no rewiring), we present some limit theorems (in the limit of large population size) for the temporal evolution and for the final size of the epidemic (the number of initially susceptible individuals that are ultimately recovered). For the model with rewiring included too, we show that whilst the preventive behaviour of rewiring is always rational at the individual level, it may have negative consequences at the population level.

This work is joint with Frank Ball (Nottingham), Tom Britton (Stockholm) and KaYin Leung (Stockholm).

Viktor Sip

Inferring the seizure propagation patterns using a data-driven model of a threshold network

Surgical interventions in epileptic patients aimed at the removal of the epileptogenic zone have success rates at only 60-70%. This failure can be partly attributed to the insufficient spatial sampling by the implanted intracranial electrodes during the clinical evaluation, leading to the incomplete picture of the seizure propagation patterns in the regions that are not directly observed. Utilizing the partial observations of the seizure evolution in the brain network complemented by the assumption that the epileptic seizure propagates along the structural connections that can be estimated from the diffusion-weighted MRI, we aim to infer if and when are the hidden regions recruited in the seizure. To this end we use a data-driven model of seizure recruitment and propagation across a weighted network. The simple dynamical model is enriched by strong nonlinearity caused by the discontinuous changes of the node states from normal to seizing state. For the inversion of this model we adopt the Bayesian inference framework, and we infer the parameters of the model by the Hamiltonian Monte Carlo method using the Stan software. Results of the computational experiments with the synthetic data generated by the same model show that the quality of the inference results depend on the number of observed nodes and on the connection strength. The precision of the inferred onset times increases with the number of observed nodes and with the strength of the network effects, while the capacity of the model to infer the excitabilities of the hidden nodes is limited in all studied cases. If our assumptions are fulfilled in the real epileptic seizures, these results indicate that the state of the hidden nodes during the seizure might be inferred from the incomplete observations.

This work is joint with Viktor Jirsa.

[Sarah Muldoon](#)

Detecting state changes in evolving functional networks

In functional temporal networks, where evolving communities reflect changing functional relationships between network nodes, it is important that the detected communities reflect any state changes of the system. This problem is especially relevant when analyzing neural data where the brain undergoes known changes such as from awake to anesthetized or normal to seizure states. However, many methods for performing dynamic community detection assume a temporal smoothness of the data, which does not reflect the presence of any state changes. In this talk, I will present a method for increasing the sensitivity of multilayer community detection to state changes in nodal dynamics by modeling interlayer links based on the self-similarity of network nodes between layers. This method is more appropriate for functional temporal networks from both a modeling and mathematical perspective, as it incorporates the dynamic nature of network nodes. Finally, I will show examples from both simulated and real-world neural data depicting how the use of interlayer self-similarity combined with community detection via modularity maximization can increase our ability to accurately track changes in the evolution of functional communities in dynamic networks.

Posters:

Collective irregular dynamics in random networks of excitatory and inhibitory spiking neurons
*Afifurrahman**, *Ekkehard Ullner*, *Antonio Politi*

Disease Dynamics and Control in the presence of a Reservoir
Scott Bee

The Segregation and Integration of Time-varying Functional Brain Networks of Encoding Novel Semantic Information: A Simultaneous EEG and fMRI Study
*Isil Poyraz Bilgin**, *James Douglas Saddy*, *Slawomir Jaroslaw Nasuto*

Network structure of wild and laboratory mouse immune systems
*Elohim Fonseca dos Reis**, *Mark Viney*, and *Naoki Masuda*

A mean field approach to the Deffuant model of opinion dynamics
Susan Fennell

Iterative control of partially-defined dynamical networks via pinning.
Roberto Galizia

Constructing networks from correlation matrices: An application to economical data
*Sadamori Kojaku** and *Naoki Masuda*

A Threshold Propagation Model and Application of Machine Learning for Minimization of Contagion Susceptibility in Financial Systems
Krzysztof Michalak

Maxmin- ω : A New Threshold Model on Networks
Ebrahim Patel

The Numerical Solution of Neural Field Models Posed on Realistic Cortical Domains
*Sammy Petros**, *Stephen Coombes*, *Stamatios Sotiropoulos*, *Paul Houston*, *Daniele Avitabile*

Inference on multilayer networks with latent layers
Alice Tapper

Connectivity in 1-dimensional soft random geometric graphs
Michael Wilsher

Towards understanding one-shot place learning in spatial navigation: a reinforcement learning approach
*Charline Tessereau**, *Reuben O'Dea*, *Tobias Bast*, *Mark van Rossum*, *Stephen Coombes*

Understanding sensory induced hallucinations: from neural fields to amplitude equations
*Abigail Cocks**, *Stephen Coombes*, *Alan Johnston*, *Daniele Avitabile*

Compartmentalised signaling of intracellular calcium in plants
Hayley Mills

Synchrony: from brain dynamics to Franklin Bells
*Mustafa Şayli**, *Yi Ming Lai*, *Rüdiger Thul*, *Stephen Coombes*