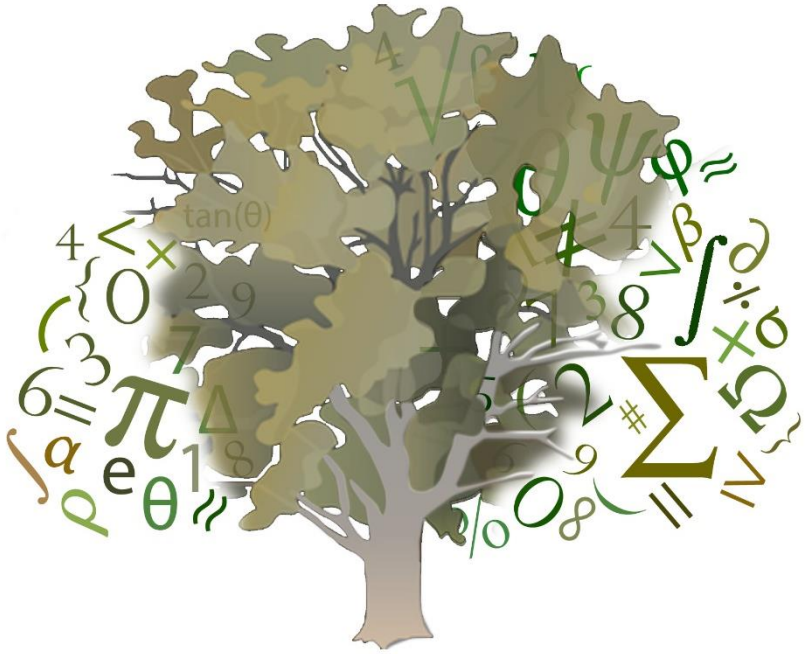

ICTMA-17
NOTTINGHAM 2015

July 19th – 24th

Modelling perspectives: looking in and across boundaries

Conference Contributions

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Conference Contributions

ICTMA-17
NOTTINGHAM 2015

1. Introduction

The Centre for Research in Mathematics Education at the University of Nottingham in the UK welcomes you to the 17th conference of the ICTMA.

We are privileged to have had the task of scheduling six plenary activities, some xxx research presentations and a small number of posters to engage almost 140 scholars from some 24 countries from across the world. Outlines of this substantial range of work are brought together in this document to allow you to devise your own programme of activity over the five days of the conference.

It is important that you make sure that you are ready to present your work at the scheduled time and also chair a session where allocated.

Making a presentation
Please note that your session has been scheduled to last 45 minutes. You are expected to present for up to 25 minutes allowing 20 minutes for questions and discussion. The chair of your session has been asked to keep time strictly within these limits to ensure smooth running of the conference. Please ensure that you keep to these times, particularly as there is only a short changeover time between sessions.

Chairing sessions
The role of the session chair is very important. The conference relies on the chair handling the session that when in this role you are asked to chair with sensitivity and firmness. If you are making a presentation yourself you will have been asked to chair a session on at least one occasion. Please arrive promptly and ensure that the presenter has arrived and is ready to make his/her presentation. Someone from the local organising group will be on hand to assist if there are any problems with the presentation equipment. Please allow a full 25 minutes for the presentation and the remaining time (up to 20 minutes) for questions and discussion. Cards will be available in the room for you to indicate that 5 and 2 minutes remain for the presentation and an additional card telling the presenter to stop. Please show these clearly from the audience so that the presenter is aware of the need to complete their presentation to time. During the ensuing questions and discussion make sure that everyone who wants to contribute is able to do so making sure that no one person dominates the discussion.

We hope that you have a stimulating and productive conference and we look forward to working with you as part of the friendly community that is ICTMA!

The local organising committee, Nottingham, July 2015.

2. Plenary Activities

ICTMA-17 has scheduled six plenary activities: four plenary presentations, a plenary session that considers the publication of a Springer volume as an outcome of the conference and a plenary panel that considers modelling and applications across the world. Five of these plenary activities are at the start of each day of the conference and the other is scheduled for the last session on Monday 20th July.

Monday 20 July 2015

9:30 – 11:00	*Japan's Ongoing Reform in Maths: Will 'Modeling' Help?*	**Keiichi Nishimura** **(Associate Professor, Gakugei University, Tokyo)** ***Response: Morten Blomhøj***

Japan's maths education is at a crossroads. Since the 1980s, Japan's performance in maths education has been well-respected globally. It has a well established curriculum in maths, which has been revised every 10 years, with good sets of textbooks, and a good tradition of teaching, supported powerfully by the practice of Lesson Study, which has been in use for over 100 years. Notwithstanding all that, there is a sense of crisis today.

The objective of the talk is to explain the background of the ongoing change and the special role that 'modelling' could potentially play within it. Dr. Nishimura will start with a brief overview of Japanese maths education and its accomplishments, particularly the special and systemic role of Lesson Study, which is not well understood outside Japan despite its popularity; he will also refer to some contemporary issues. He will then highlight 'modelling' as a relatively neglected area in Japanese maths education. By describing the last 20 years of exploring ways to introduce 'modelling' by the speaker and a small group of researchers and teachers in Japan (using lesson study as a key mechanism), the talk will clarify why and how they have arrived at 'modelling for decision making' as a critical concept - and one which has been missing in Japanese maths education to date. The talk will describe how, if introduced properly, it could change the meaning of maths for many students in Japan. Today, Japan is launching a reform to revamp the transition from high school to university with a view to changing not only the high school curriculum but also the university entrance examinations and selection systems, which have been a hidden and powerful driver of maths education to date. Dr. Nishimura will describe the motivation and proposed nature of the reform based on his continuing experience of actively shaping reforms.

Keiichi Nishimura started his career as a maths teacher, first in a Tokyo senior high school, and then in an international high school affiliated to Tokyo Gakugei University. In both places, he was active in lesson study, particularly in modelling and real context maths, and in writing textbooks. He then moved to be responsible for maths education in the Curriculum Research Centre within the Institute of Education Research of the Japanese Ministry of Education and Science and Technology (MEXT), where he became involved with national policy related research and implementation, particularly in curriculum change and assessment. He is currently an associate professor in Tokyo Gakugei University where he continues to work on national initiatives to improve maths education and is a key member of its International Math-teacher Professionalisation Using Lesson Study (IMPULS) project, funded by the Japanese government.

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Tuesday 21 July 2015

9:00 – 10:30	*Mathematical Modelling: Its role to enhance mathematical pedagogy?*	**David Burghes** **(Professor of Mathematics Education, Plymouth University, UK)** ***Response: Gloria Stillman***

At the start of the series of ICTMA conferences (Exeter, 1983) the role of mathematical modelling in Higher Education was a major theme of the presentations but a number of delegates were also keen to establish mathematical modelling in the school curriculum.

This presentation will review the progress made in the past 3 decades and reflect on the successes and problems associated with the role of mathematical (and statistical) modelling in the school curriculum, particularly in post-16 education.

Governments in the developed and developing world regard mathematics as a key discipline for economic growth and well-being in a highly technological world, with countries working hard to enhance the mathematical skills of their future workforce.

Is enhancing mathematical progress a good measure of what is needed for the future? Is it important for young people not just to enhance their mathematical progress but also to become more proficient in using and applying their skills and knowledge through mathematical and statistical modelling?

These themes and questions will be reviewed in this presentation.

David Burghes is Professor of Mathematics Education at Plymouth University and Director
of CIMT (Centre for Innovation in Mathematics Teaching). He founded the ICTMA series of conferences at Exeter University in 1983 and 1985 and has since directed three international comparative projects in mathematics, in Primary, Secondary and Teacher Training. He initiated the CIMT website (http://www.cimt.plymouth.ac.uk) which is dedicated to helping teachers of mathematics through international research and development. He is currently a consultant to the Core Maths Support Programme in the UK, focusing on the teaching and learning of mathematics through Problem Solving.Study (IMPULS) project, funded by the Japanese government.

Wednesday 22 July 2015

9:00 – 10:30	*Mathematical Modelling as a Professional Activity – Lessons for the Classroom*	**Dr. Peter Frejd** **(Senior Lecturer at Linköping University, Sweden)** ***Response: Gabriele Kaiser***

Mathematical modelling and models are used for various purposes in school and in society. Descriptions from mathematics syllabuses across the world indicate that the use of modelling activities in the mathematics classroom may contribute to developing students' understanding of how and why mathematics is used in the everyday and in the workplace. While mathematical modelling has been described as an interface between mathematics and a workplace (Damlamian Rodrigues & Sträßer, 2013), there are indications, however, that it is not emphasised in current teaching practices at upper secondary school (e.g. the preface in Kaiser et al., 2011). To strengthen the connection between industry and school as well as the teaching of mathematical modelling, it has been suggested to 'mirror' parts of expert modellers' working practice into teaching practices. This would involve, for example, spending a large proportion of learning/teaching time on formulating the problem and validating the solution, activities currently not given much space in teaching practice.
Drawing on empirical and theoretical research studies, this presentation will discuss similarities and differences between working with mathematical modelling in 'school' and mathematical modelling as a 'professional task' in the workplace . Some of the critical issues to be discussed concern goals; technology; division of labour, communication and collaboration; model construction, including the application and adaption of pre-defined models; projects; and risks involved in using the models.
Based on this discussion and examples from innovative teaching practices, approaches to simulate modelling as a 'professional activity' in educational settings will be explored. Finally, some proposals of directions for future research will be presented.

Damlamian, A., Rodrigues, J. F., & Sträßer, R. (2013). Conclusion on Educational Interfaces Between Mathematics and Industry. In A. Damlamian, J. F. Rodrigues, & R. Sträßer (Eds.), *Educational interfaces between mathematics and industry* (pp. 447-452). Cham, Switzerland: Springer.
Kaiser, G., Blum, W., Borromeo Ferri, R., & Stillman, G. (Eds.). (2011). *Trends in teaching and learning of mathematical modelling*. Springer: New York.

Peter Frejd is senior lecturer at Linköping University, Sweden, where he teaches undergraduate courses in Mathematics and Mathematics Education. His research has its main focus on mathematical modelling, including different actors' interpretation of, and work with, modelling in and out of school settings, holistic assessment and theoretical approaches to research on modelling. He has a background as a mathematics and physics teacher at upper secondary school in Sweden where he taught for more than a decade. In 2014 he was the scientific leader of a Swedish government funded development project for all in-service mathematics teachers in vocational education.

Thursday 23 July 2015

9:00 – 10:30	*Systemic change and teaching modelling*	**Prof. Malcolm Swan & Prof. Hugh Burkhardt** **(University of Nottingham, UK)** ***Response: Katja Maass***

Hugh Burkhardt and **Malcolm Swan** have recently been awarded the new ICMI Emma Castelnuovo Medal for Excellence in the Practice of Mathematics Education "*in recognition of their more than 35 years of development and implementation of innovative, influential work in the practice of mathematics education, including the development of curriculum and assessment materials, instructional design concepts, teacher preparation programs, and educational system changes.*"
In light of the recent announcement of this joint award the ICTMA local organising committee are delighted to invite Hugh and Malcolm to provide a plenary that highlights their design work that supports the teaching and learning of mathematical modelling.

Abstract
Over the last sixty years, teaching modelling has moved from initial explorations to becoming an established field, supported by research and the development well-engineered tools of various kinds. Yet, in typical classrooms around the world, autonomous student modelling remains rare. Recently, in Britain and the US at least, modelling has become an explicit element in the "intended curriculum". But the usual difficulties of embedding innovation across an education system remain. This talk will analyse these barriers and, drawing on past initiatives around the world, suggest what may be needed to overcome them.

Hugh Burkhardt led the pioneering work of the Shell Centre team in raising the quality of research-based development of materials and processes for improving school mathematics. His concern that, too often, excellent practice fails to spread through an education systems, led him to articulate the concept of "strategic design" with this focus – a key goal of MathNIC. In 2013 ISDDE awarded him its Prize for Excellence in educational design for lifetime achievement.

Malcolm Swan has led the Shell Centre design team in its research, including notably the development of materials for teaching, learning and professional development including, most recently, the 100 Classroom Challenges of the Mathematics Assessment Project. Many of these materials are renowned around the world. In 2008 he received the ISDDE Prize for "The Language of Functions and Graphs" (The Red Box).

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Friday 24 July 2015

9:30 – 11:00	***Modelling in curricula across the world***	**Plenary panel:** **Prof. Helen Doerr (USA)** **Dr. Angeles Dominguez (Mexico)** **A/Prof. Vince Geiger (Australia)** **Prof. Pauline Vos (Norway)** **Dr. Helena Wessels (South Africa)** **Panel Chair: Prof. Werner Blum (Germany)**

The plenary panel gives an opportunity to explore mathematical modelling in curricula from a range of countries that is geographically spread across the world. The plenary discussion will be stimulated by short statements from each of the panelists who have been asked to address the following questions:

- What is the role of modelling in the curriculum of your country (perhaps including other, e.g. neighbouring, countries if appropriate), including assessment?
- Are there big discrepancies between curriculum specification and actual classroom practice?
- What role - if any - has the international modelling community played in the developments in your country, and what role could it play?

		
Prof. Helen Doerr	Dr. Angeles Dominguez	Dr. Vince Geiger
		
Prof. Pauline Vos	Dr. Helena Wessels	Prof. Werner Blum

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3. Research presentations

DESIGN AND IMPLEMENTATION OF A TOOL FOR ANALYZING STUDENT PRODUCTIONS WHEN THEY SOLVE FERMI PROBLEMS

César Gallart[1], Irene Ferrando[2], Lluís M. García-Raffi[3], Lluís Albarracín[4] and Núria Gorgorió[4]

[1]Universidad Cardenal Herrera-CEU, Valencia; [2]Universitat de València; [3]Universitat Politècnica de València; [4]Universitat Autònoma de Barcelona

The implementation of modelling tasks at secondary classroom poses a challenge to the teaching staff due to this activity is not straightforward to secondary school students. With respect the classical tasks they get used to do, the modelling activity involves several competencies and putting into play abilities as could be the estimation or the capability to generate a mathematical image of the sensible world. Some of these capabilities are put in play in the context of Fermi problems (Ärlebäck, 2009) and we propose to use those that involve the estimation of large quantities as a way to introduce modelling in secondary school levels.

In this paper we present a comparative study between students with previous modelling experience and students doing these activities for the first time. We have collected data based on the productions of the students when they solve a sequence of Fermi problems focused on counting people or objects on surfaces. These problems are a sort of modelling eliciting activities. Some of the problems deal with the estimation of the number of persons that fit in the school yard, the number of persons accommodated in a sports hall, in a concert or the number of trees in Central Park (New York). The models produced by students are analyzed using a tool based on the definition of mathematical model provided by Lesh and Harel (2003). This tool considers models as conceptual systems and highlights the ways in which they are expressed, the concepts and the procedures used during the resolution.

The data analysis allows us to observe different models used to solve Fermi problems (population density, iteration of a unit or distribution of elements in a grid) and the analysis tool allows us also to distinguish some differentiating aspects in the resolution between students with previous experience and those who face modelling for their first time. Students with experience in modelling tend to create models based on more complex concepts, using more rigorous measurement procedures and a more sophisticated mathematical language (such as algebraic representations that allow the generalization of the models).

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THE MATHEMATICAL MODELING COMPETENCY REQUIRED FOR SOLVING ENGINEERING STATICS ASSIGNMENTS

Burkhard Alpers
Aalen University, Germany

Since engineers set up and work with mathematical models describing certain situations of interest, it seems to be quite obvious that the mathematical modeling competency should play a major role in engineering education. In textbooks and lecture notes important modeling quantities are introduced (like force, torque, and stress in engineering mechanics) and ways to set up models and to compute interesting quantities in these models are presented. Students are assumed to gain a better understanding of the modeling concepts and solution procedures by working on assignments. This way, students should both acquire competence and also demonstrate it in later examinations.

The mathematical modeling competency has been detailed by conceptualizing the modeling process in the so-called modeling cycle e.g. in the contribution by (Blum & Leiß, 2007) and by specifying additional components in the work by (Maaß, 2006). In this contribution we investigate which aspects of the modeling competency are required for solving tasks in two wide-spread textbooks on engineering statics based on the sample solutions given there ((Gross et al., 2013); German translation of a US textbook: (Hibbeler, 2012)). In this 'a-priori' analysis we identify those sub-processes of the modelling cycle which are needed in the solution of the problems. In order to check empirically whether the solutions in the books match with 'good' student solutions, we had two excellent students work on selected tasks and explain their thinking processes in detail. Finally, we discuss potential consequences for the split of responsibility between mathematics and engineering mechanics education regarding the mathematical modelling competency.

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MOTIVES TO VOLUNTARILY PARTICIPATE IN A MATHEMATICAL MODELLING PROJECT

Jussara Araújo, Ilaine Campos and Edmilson Torisu
Universidade Federal de Minas Gerais (UFMG), Brazil

Mathematical modelling projects have been one of the tasks proposed to the students in the context of mathematics classroom. Sometimes, it is taken for granted that there is some relationship between student's involvement in the task and his/her interest in the real problem to be solved in the modelling project. Other times, since this task is used as part of evaluation of the discipline Mathematics, student may be involved just because he/she wants to be successful in the evaluation of the discipline. In this study, our objective is to unveil some (other) possible motives that one undergraduate student of the Physics program at the university has to participate in the development of a mathematical modelling project that was not an obligatory task in the course he was student. The theoretical approach to the development of the project was the socio-critical perspective of mathematical modelling (Barbosa, 2006). We understand students' motive(s) in a Cultural Historical Activity Theory (CHAT) perspective (Leont'ev, 1978). Motives are something that move an activity and, in particular, are appropriations that occur in the activities that the individual participates in, showing the dialectical character of human relationships. Methodologically, we have adopted a critical paradigm in qualitative research (Skovsmose & Borba, 2004), and used three different procedures: 1) observation and videotaping of the modelling project; 2) report made by students, using online text editors; 3) collective interview. To this study, we will focus on data from one of the students: Rafael. As a conclusion, we point out that, although Rafael was completely involved in the modelling project, his motives were not related to the real problem to be solved in the project. Anyway, his motives were related to the academic life, but also to his personal life.

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CONFRONTATION OF ARGUMENTS IN MATHEMATICAL MODEL CONSTRUCTION

Jussara Araújo and Ilaine Campos
Universidade Federal de Minas Gerais (UFMG), Brazil

In this study, we deal with aspects of the students' practices in a learning milieu of modelling according to critical mathematics education. Our aim is to analyse the influence of different arguments in the mathematical model construction in an interaction space (Barbosa, 2007) performed by the students. In this space, two different and contradictory points of view were highlighted, one from the student Carlos and the other from the student Fernanda, on the theme of mathematical modelling project, which was the public-private partnership in the prison system in the State of Minas Gerais, Brazil. Carlos defended that the State's decision to adopt such a partnership involved financial interests and Fernanda argued that the State's interest was related to the inmates' resocialization. The interaction space was characterized by the argument construction for each of these students to defend different points of view and to select variables for the mathematical model construction. The group that has developed this learning milieu of modelling consisted of seven students in the first period of the Public Management course at the Federal University of Minas Gerais, in the second half of 2011. The study is qualitative and empirical data were constructed through unstructured and participant observation. Data analysis enables us to state that the negotiation space (Barbosa, 2007), which was constituted by the confrontation between these different arguments, originated a model that emphasized financial interests, but that expressed less certainty, through mathematics, if compared to possible models sustained in arguments supported, at first, by Carlos. Therefore, we believe that the confrontation between the different arguments weakened what Borba and Skovsmose (1997) call as an ideology of certainty, present in the first arguments about the State's financial interests.

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TEACHERS' TOOLS FOR ANTICIPATING STUDENT WORK WHEN TEACHING MODELLING

Jonas B. Ärlebäck[1] and Helen M. Doerr[2]

[1]Linköping University; [2]Syracuse University

Research on modelling draws a wide range of theoretical perspectives and curricular goals for students' learning. The teaching of mathematical modelling likewise encompasses an equally wide range of pedagogies and instructional practices. One commonly made distinction that cuts across the various theoretical perspectives on modelling is whether teaching of modelling is a goal in itself or if modelling is a vehicle for teaching and learning mathematics (Julie, 2002; Stillman, 2012). Barbosa (2006) and others argue for a third possibility for the role of modelling in schools, namely, a critical perspective where a central goal is for students to understand the role of mathematical models in society and political and economic decision making. One common goal for students' learning across all of these perspectives is for students to develop persistence and independent thinking.

While much research has attended to various aspects of students' learning within each of these perspectives, considerably less research has paid attention to the pedagogical issues within and across these perspectives. Yet, the teachers' role in supporting the development of students' abilities to persist with and work independently on modelling tasks is of central importance in all of these perspectives. The notion of student independent work places a more central role in the classroom on students' own work and on their emerging ideas. This places the teacher in the situation of needing to respond to student ideas and to manage the emergence of student interactions with the task, with each other, and in class discussion. The challenge for the teacher is to both anticipate students' work and to respond to that which cannot be fully anticipated. In this study, we examine how one teacher developed "tools" for anticipating students' work with modelling tasks.

Our research is situated in the models and modelling perspective on teaching and learning mathematics (Lesh & Doerr, 2003). Within this framework, students learn mathematics through developing their emerging models. The ability to develop, apply, and adapt a generalized model to be used in a range of contexts is the essence of what it means for students to be independent learners and problem solvers. This study took place over three years in a six week summer mathematics course designed around a model development sequence centered on the concept of average rate of change. We illustrate our findings with four "tools" that the teacher developed that helped her anticipate student work and support students' independence: (1) a data management sheet so that student could independently master needed technologies; (2) the use of a student response system to elicit student ideas, focus attention on arguments, and clarify reasoning in the moment; (3) moving from a more rigid to a more flexible use of lesson plans, allowing her to make students' emerging thinking more central to the lesson; and (4) the design of tasks to elicit alternative student approaches.

Day:	*Monday*	**Session:**	*E (16:20 – 17:05)*	**Room:**	*A42*
Age:	*Teacher Education*	**Focus:**	*Teaching*	**Type:**	*Research*

MODELLING WITH STATISTICAL DATA: CHARACTERISATION OF MODELS PRODUCED BY STUDENTS

Àngels Aymerich, Núria Gorgorió and Lluís Albarracín

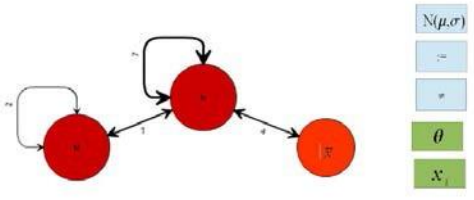
Universitat Autònoma de Barcelona

In this publication we present a modelling-eliciting activity based on the interpretation of statistical data. We carried out the task of interviewing students of 15 to 16 years of age (without any previous experience in modelling activities). We asked them to evaluate some salary figures we provided to them, with figures ranging from 14 to 50, and give us an reasoned view of what type of companies these salaries may correspond to. To achieve this they constructed mathematical models that try to explain the situation put forward to them.

For this study we considered the definition of mathematical model provided by Lesh and Harel (2003) that regards models as conceptual systems and highlights the ways in which the latter are expressed, the concepts integrated and the processes involved in these systems. The main elements of the previous definition allow us to elaborate a qualitative analysis tool of the models produced by the students involved in our research.

Our results show a wide range of concepts and mathematical procedures used to describe the companies studied, from simple elements such as considering the minimum and maximum salaries, or other more common ones such as considering centrality measures or creations elaborated as estimates of the concept of standard deviation. Given the abundance and variety of the models which the students generated, we propose a way of representing the concepts and processes involved, as well as the relationships established between them. Therefore, for each model studied we elaborated a graph characterising the model proposed. An example of this can be seen in the following graph, which illustrates three of the concepts detected (amount of data, maximum and average) and the relationships between the latter elements, as well as ideas or processes used when creating the model, such as the assumption of normality or data ordering from lowest to highest, as shown on the right side of the graph.

The wide variety of models obtained allow us to conclude that this activity is of great value from a mathematical modelling point of view. In addition, the type of analysis used opens a line of possibilities to characterise mathematical models produced by the students in order to observe the differences between them.



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USING THE SIX INSTRUCTIONAL DESIGN PRINCIPLES FOR ASSESSING MODELLING CAPABILITIES OF STUDENTS IN MATHEMATICS CLASSROOMS

Piera Biccard[1] and Dirk Wessels[2]

[1]Research Unit for Mathematics Education (RUMEUS).

[2]Stellenbosch University, South Africa

This paper presents partial findings of a study (Biccard, 2010) on the development of modelling competencies of students working in groups. Six principles for instructional design used extensively in task design to enable one to transform tasks (or create tasks) into model-eliciting activities were used as a framework for assessing modelling capabilities. This paper presents the findings of using these design principles to assess group modelling processes. This study examines how mathematical modelling competencies develop in Grade 7 students working in 3 sample groups. The study is qualitative in nature and uses a mixed approach of design research and some aspects of grounded theory. Students' progress through a modelling program is documented while the modelling competencies of students stereotyped as weak and strong are also investigated. They worked through three MEAs over a period of 4 months.

These six principles proved to be robust enough not only for assessing modelling tasks, but to assess student competencies too. They facilitated the documentation of student progress in:

- making sense of real situations,
- constructing models,
- judging their own responses and
- being able to generalise.

The six task design principles support teacher vocabulary for explaining and assessing important aspects of modelling. The framework makes advances in modelling assessment; it links up with Van den Heuvel-Panhuizen's (1996) six principles that RME held regarding assessment. The paper presents this modelling specific framework within larger assessment principles.

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MODELLING IN SCIENCES AND MATH: LEVEL OF MOTIVATION, INTEREST AND NEED TO LEARN

Maria Salett Biembengut

Pontifical Catholic University of Rio Grande do Sul - Brazil

This applied study (Biembengut, 2013) aimed at analysing the level of motivation, interest and need to learn teachers of Math and Natural Sciences (Physics, Chemistry, Biology) and their students in the Secondary School in Brazil have. The Brazilian Law of Education (LDB - Lei de Diretrizes e Bases da Educação) establishes the division of the school curriculum into large three areas: Languages and Codes; Math and Natural Sciences; and Human Sciences. Despite the division into three large areas, each area has subdivisions. The law proposes that the teachers of these three areas should meet to prepare a set of diverse joint activities to better sustain the students' knowledge. It postulates that the contents should be taught in an interdisciplinary and contextualized way, by making use of the communication and information technologies. As most of the teachers do not possess such knowledge, we have developed, for two years, the project entitled Modelling in Sciences and Math. First, we elaborated some digital didactic material composed of models in the sciences, in a way that the students would be able to work on them, going through the modelling steps. The material also proposed some scientific questions to be worked with the students in an interdisciplinary way. Subsequently, four teachers (Biology, Physics, Chemistry and Math) and their 18 volunteer students, from the same school, participated in the project. The project consisted of a course in which the teachers, by using digital media, learned how to improve their classes with the models. During the project, feelings like motivation and interest in learning 'oscillated' in teachers and students. It is possible to conclude that learning depends on a set of factors inherent to each person, according to his/her own predispositions and needs. In addition, in the still current educational structure in Brazil, the teachers' need to 'learn to teach' is obfuscated by the value attributed to being a teacher, a value that did not motivate the participant teachers in the project.

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DEVELOPING MODELING BASED MATHEMATICS TEACHING BY MEANS OF THEORIES ON CONCEPTUAL LEARNING

Morten Blomhøj and Tinne Hoff Kjeldsen
IMFUFA, NSM, Roskilde University

The context for the presentation is an in-service course for Danish teachers on the use of mathematical modelling and project work in upper secondary mathematics teaching (Blomhøj & Kjeldsen, 2006). One of the objectives of the course is to develop the teachers' abilities to deliberately use didactic theories on conceptual learning to develop their students' concept formation through the design and teaching of mathematical modeling projects.

In teaching and evaluating this in-service course, we have experienced a need for supporting more concretely the teachers' use of the presented didactic theories (e.g. Sfard, 1991; Steinbring, 1987; Vinner & Dreyfus, 1989) in their design and analyses of the students modelling activities. In this paper we present and discuss a schema for spanning the use of different representations of both process and object aspects of selected mathematical concepts involved in a given modelling context.

The schema is considered as a method that can bridge educational research and the development of teaching practice in mathematical modelling. The use of the schema is illustrated and discussed in relation to a modelling project on alcohol and THC (the active drug in cannabis) that was developed and tested in the course. The schema can function as a mediating link between theory and teaching practices and hereby support research-based development of practice.

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ANALYSIS OF PRESERVICE TEACHERS' LEVELS OF REFLECTIVITY AFTER MATHEMATICAL MODELLING ACTIVITIES WITH STUDENTS

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Mathematical Modelling became a compulsory part in curricula in several countries in the last years. The necessity to develop programmes for pre- and in-service teacher education for the learning and teaching of mathematical modelling is strongly recommended and still needed. Also the demand came up to train teachers more in their reflection competency, because mostly they do not get feedback on their teaching in school later on. Best practice examples of modelling courses which however have different foci (for an overview see Cai et al. 2014) always show that teachers need time to understand the complexity of mathematical modelling for themselves. The concept of a modelling course which offers a balance between theory and practice can be very successful regarding to the knowledge needed for teaching modelling (Borromeo Ferri & Blum, 2009).

In particular within the seminar "Modelling Days" at Kassel University, the focus on reflection was promoted explicitly. The explorative study presented in the presentation had the goal to investigate preservice teachers' levels of reflectivity after modelling activities with high school students. The analysis of 12 written reflections shows different levels of reflectivity whereas reaching high levels were rare. The development of a new model on levels of reflectivity, oriented on Hatton & Smith' (1995) model is a further result of the empirical study.

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EUROPEAN SOCCER CHAMPIONSHIP – CROSS-LINKING OF KNOWLEDGE WITHIN MATHEMATICAL MODELLING

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Mathematical Modelling within the scope of interdisciplinary project courses raises a whole bunch of research questions as well as learning opportunities. Models and modelling play a central role for gaining knowledge in the process of research-orienting learning in mathematics and natural sciences.

Without a deeper understanding of the model character of mathematical and scientific theories it is almost impossible to master these disciplines. In mathematics, there is an inexhaustible reservoir of (mathematical) models that can be used to improve our understanding of the "real world" around us. If we require a cross-linking of knowledge as well as an integration of new contents into the current knowledge network we are prepared for learning in a modern, project-oriented form. By inclusion of mathematical modelling we promote cross-fertilisation between mathematics and reality (cf. Pollak, 1979). Using mathematical modelling students learn to build bridges between the mathematics they use to explain the real world and the powerful science of abstract formulae. But in order achieve this they need to have "rich learning experiences" (cf. English, 2003) that are being made by inclusion of project-oriented modelling activities. Studies (cf. Mikelkis-Seifert & Leisner, 2003) show that quite often the understanding of modelling is poor even after long-term instruction. Anyway modelling has a general educating social relevance beyond mathematics and mathematics instruction, is suited as a structuring measure in and for math lessons, respectively, and can be implemented particularly well in project-oriented lessons.

To achieve this goal we look at the European soccer championship and show learning possibilities for mathematics education focussing on modelling activities. Two important aspects we are going to focus on are repetition and correction of mathematical deficits. On the one hand teacher students are going to derive their own model and develop concepts to implement the project with students of different ages. Later, high school students are going to work based on these studies.

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CONTEXT AND UNDERSTANDING: THE CASE OF LINEAR MODELS

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The perceived relationship between context and the teaching and learning of mathematics is varied and often contradictory. Within the mathematical modelling and application community of mathematics educators and mathematics education researchers, dealing with context (Alsina, 2007) is such as an essential element of being able to do mathematics. Outside this community, context is often portrayed as a hindrance to learning (Pfannkuch, 2011) or framed as a problematic recontextualisation (Jablonka, 2007). Within ICTMA, for example, it is well understood, there are differing definitions of modelling and certainly differing emphases in our research – this is generally seen as productive (e.g., Blum, Galbraith, Henn, & Niss, 2007); but outside the community this is presented as problematic (e.g., Williams & Goos, 2013).

This presentation focuses on the illustration, and development, of understanding related to linear functions used in modelling real situations. An extended task involving the application of linear functions was undertaken by one class of Year 9 students. Students worked on the task in pairs for three lessons. Students had access to graphing calculators and laptop computers during task implementations. Data collection included individual and audio recordings. These data has previously been analysed through a framework of affordances with a focus on understanding of (linear) functions in a technology-rich teaching and learning environment now it is being reconsidered with the focus being on identifying the impact of context on understanding. The intention is that in crossing boundaries and revisiting the data from a different perspective more can be learned. The research question to be addressed is: Does task context highlight student lack of understanding of mathematical concepts demonstrated only in an abstract fashion? The aim of data analysis is to ascertain if there were any differences in student understanding based on representations presented in the task when engagement with the context was essential or not. The analysis is intended to determine if the use of context allows one to more clearly ascertain - the level or depth of student understanding. In addition, the collaborative nature of task solving as an enabler of deepening their current understanding during task solution will be investigated.

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USING MATHTRAILS AS A DIAGNOSTIC TOOL

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Mathtrails represent a form of non-formal learning outside the classroom in mathematics education. They can be used to support exploratory and collaborative learning of students, because the students have to communicate, to argue mathematically and solve mathematical problems while doing a mathtrail (cf. Shoaf, Pollak & Schneider, 2003). The range of mathematical content in mathtrails reaches from primary to secondary mathematics and so varies the level of difficulty of selected tasks. Mathtrails can even be supported by technical devices (Jesberg & Ludwig, 2012). A fundamental idea is that students carry out mathematical tasks specifically at objects in the city or the surrounding area and solve problems by estimating or measuring realistic sizes. In this special form of mathematical modelling in particular processes of simplifying and mathematising play a crucial role that often cause problems for many students.

The presentation reports on a project in the area of instructional research, in which student teachers of the University of Hamburg have developed reality-based modelling tasks for secondary level that treat different curricular content and are based on didactic principles. On the one hand, gradations in the difficulty level of the tasks, openness and a realistic problem orientation played a special role, on the other hand, the tasks should be oriented to basic ideas and enable the students to deal with misconceptions, so that the tasks can be used by teachers as a diagnostic tool. Here, tasks are presented as examples. All tasks were based on real objects of the city center of Hamburg and were compiled to mathtrails. The mathtrails were tested with students from secondary school level and afterwards evaluated.

The students' solutions of the tasks can be analysed with a diagnostic view on basic ideas and misconceptions. The solutions show specific misconceptions and difficulties in measuring sizes adequately or translating these sizes into a mathematical model. The presentation shows exemplarily, how diagnosing the learning difficulties in the tasks of the mathtrail can identify adequate individual means for support and for the reduction of misconceptions of the students.

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"A FOUNDATION FOR UNDERSTANDING THE WORLD" UNLESS BUILT ON SHIFTING SANDS: MATHEMATICAL MODELLING, VALIDATION AND EVALUATION

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The national curriculum for mathematics in England states that studying mathematics provides, "…a foundation for understanding the world" (DfE, 2013, p. 2). This is suggestive of a mathematical modelling approach insofar that mathematical models give a description of non-mathematical practices. This is in line with a shift in international curricula to include applications and modelling, reinforced by the OECD Programme for International Student Assessment (PISA) (Wake et al, 2014). The mathematics part of the programme purports to measure the extent to which students are "able to reason quantitatively, to represent relationships or dependencies, and to connect the context and structure of a problem with mathematics when it comes to deploying mathematical skills in real world problems." (OECD, 2013, p. 15). The process of mathematical modelling (including the one in the PISA framework) is often said to include re-describing elements of a practice from the point of view of mathematical discourse ('mathematisation'), and evaluating the emergent mathematical structure and the outcomes of its analysis from the point of view of the original practice ('validation'). In our presentation, we will build on our previous analysis (Burke, et al, 2014) and explore what might constitute the validation of a mathematical model in different approaches. We wish to look at published PISA tasks as our empirical material, and investigate which mathematisation/validation strategies these examples privilege, if any. We would like to suggest that our differentiation of approaches offers a tool for developing and evaluating teaching material and classroom practice that pertain to 'mathematical modelling'.

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COURSE ON MODELLING FOR FRENCH TRAINEE TEACHERS: RELATION BETWEEN MATHEMATICAL AND PROFESSIONAL COMPETENCES

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For their fifth university education year, French trainee teachers are half time teachers in a school. Some of them have to teach two subjects, mathematics and sciences, in vocational secondary school. In 2014, at Strasbourg university, they have attended a one semester course on modelling for the grade of Master. For this course they have to conceive a modelling activity and to implant it in a class. They are assessed on the report and feedback about this implantation.

First we present the context of this course. From the anthropologic theory of didactic (Bosch, Gascon 2006) we analyze the role of different levels of didactical determination: mathematics, extra-mathematical subject, pedagogy, school, training center, educational system, society). We point the relations with research in didactic of mathematics. We show how the official curriculum enables to articulate mathematical and professional competences (Ball & al.2008 ; Maass 2006) through modelling activities.

Then we study two trainee teachers' cases. One develops a modelling activity related to geometry, the other one related to statistics, in 10th grade classes. We expose a priori and a posteriori analysis of the modelling activity in the class, pointing how competences are involved. From the double approach theory – mathematical and psychological (Robert, Rogalski, 2005), we show that mathematical competences are dominant to prepare the activity in the class. In the feedback on these activities, professionnal competences become dominant. We point two didactical problems: one about the verification of hypotheses, the other one about the role of simulation in proving. They show the importance of didactic vigilance for trainee teachers.

Various difficulties are referred by teachers. Time management is the main one with the conflict between didactical time and institutional one. We will conclude about propositions on ressources and training to take in account these difficulties and to improve the articulation between mathematical and professional competences.

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DIFFICULTIES TO TEACH MODELLING: A FRENCH-SPANISH EXPLORATION

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We present an exploratory research based on an online questionnaire in order to better know the difficulties encountered in the teaching of modelling. The questionnaire was answered by a population of 231 French or Spanish pre-service teachers, teachers, researchers, inspectors, from primary to tertiary education. We present here a preliminary analysis of the answers to this questionnaire.

The dominant difficulties (more than 50 % of agreement) are related to time and resources. The dominant positive points on modelling (more than 50 % of agreement) are related to students' development of autonomy and competence. The cluster analysis explains how other difficulties split the population in four clusters. In first cluster (36 %), much more that in the whole population there are positive about modelling and neutral about difficulties in modelling. Men, people who have studied mathematics, and secondary school teachers are strongly present. In the 2nd cluster (32%), much more than the whole population they feel negative about modelling, they find difficulties in modelling and in mathematics teaching and they don't practise modelling: tertiary education teachers or trainers are strongly represented. In 3rd cluster (20 %), much more than in the whole population they don't feel difficulties in mathematics and modelling teaching, they look positive about modelling. Spanish people are strongly present. In the 4th cluster (11%) much more than in the whole population, there are neutral on modelling and its difficulties, and pre-service or primary school teachers are strongly present.

We conclude by interpreting these results and formulating hypotheses of research to be tested in confirmatory research. We will illustrate these hypotheses with interviews of clusters paragons.

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MATHEMATICS IN A CHEMISTRY UNDERGRADUATE COURSE: A NECESSARY "EVIL"?

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The chemistry undergraduate course was founded in 2009 in Duque de Caxias, a town in the state of Rio de Janeiro, Brazil. The major problem that the course has faced since its opening is the great evasion and failure of students, especially in disciplines involving mathematics. A survey among registered and 'escaped' students found some factors that contributed to the occurrence of this phenomenon, namely: enormous difficulties in disciplines that use mathematics, physics or chemistry; irregular calendar of activities because of teacher's strikes in the last year and the campus location.

Based on this scenario, we defined as the aim of this work the diagnosis of the phenomenon of evasion and failure of students, directing such an analysis for the difficulties encountered by students in math. Through a multidisciplinary proposal we intended to incorporate significant mathematics in the course for those students, preparing to be teachers of chemistry. This incorporation of maths in a chemistry undergraduate course was realised using mathematical modelling of chemistry problems, specially in Differential and Integral Calculus.

Our project was realized in three steps:

Step 1: Survey data of students enrolled in the course through questionnaires answered by them and considering data obtained from the undergraduate secretariat;

Step 2: Discussion of issues raised in Step 1 and preparation of proposals to contribute in improving the use of mathematics by the students through modelling in chemistry;

Step 3: Application of the proposals discussed in step 2 and results obtained.

After the execution of these steps, we analysed the obtained results and we conclude that the use of mathematical modelling as a methodology to bring together the chemistry and mathematics is an effective tool in the fight against evasion and failure of students in the undergraduate course of chemistry.

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L'HOSPITAL'S WEIGHT PROBLEM: CROSSING THE BOUNDARIES BETWEEN MATH AND PHYSICS, BETWEEN APPLICATION AND MODELLING

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In this talk we will present a learning activity that has been experienced by several groups of students in the transitional mathematics course at École de technologie supérieure (ÉTS), a technical engineering school in Montreal.

The activity is based on a problem presented in 1691 by Johann Bernoulli to the Marquis de L'Hospital as an introduction to differential calculus. The potential of this problem for use in a contemporary calculus class was explored by Maanen (1991) and, later with the use of a symbolic calculator, by Drijvers (1996; 1999) who noted that some difficulties experienced by students in using the calculator had to do with their understanding of the concepts of variable and parameter.

In our activity, each team of students is provided with a physical apparatus they can manipulate in order to build and validate both their model and the predictions it allows. The frequent trips between the physical apparatus and the mathematical model shed light on the equations, parameters, and variables as well as on the students' understanding of the system.

The activity was developed in the collaborative context of *Projets mathématiques en technologie et sciences* (see website www.projetsmathematiquests.com). This partnership was born out of the need to inject hands-on activities and interdisciplinary projects into the Quebec math secondary school curriculum. It brought together teachers of mathematics and other sciences from secondary schools, cegeps, and universities to design learning activities that would motivate students for math and sciences and facilitate the transition between school levels. Building on the didactical and practical qualities of the resulting activities, their design model has been extended to postsecondary mathematics.

At the border of secondary and postsecondary instruction, L'Hospital Weight activity also sits at the boundary of math and physics. The contribution of a physicist to the design brought forward the value of crossing into physics, and the vector analysis of forces, and of reconciling this approach with that of calculus.

In our presentation of the activity, we will share our findings on its experimentation at ÉTS. Although the problem itself may look like an *application* (Stillman, 2008), we will show that the activity, as it has been designed, allows students to experience, within the practical boundaries of the course, aspects of the *modelling* process (Blum et al., 2002). We will also look into some of the conditions that must be met in order to fully exploit the interdisciplinary potential of such a modelling activity.

TEACHING MODELLING USING CALCULUS – INSIDE AND ACROSS "NATURE"

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Many notions of calculus originate from the intention to describe phenomena in natural sciences, especially in physics. But, according to Fischer & Malle (1985, p. 107), the absence of a law-of-nature-character is necessary to allow learners the free use of mathematics describing "reality". In this way the modelling perspective demonstrates a human distance to reality. Is this desirable?

Nowadays methods of calculus are commonly used in the context of social sciences, particularly in economics applications. This is sometimes a result of a modelling process, but in many cases standardized methods are taught and applied without a critical reflection. In several cases the underlying concepts are founded on skin-deep analogies rather than appropriate modelling processes relative to the original problem. It must be considered in how far this transfer is reasonable.

Even if modelling using functional relationships is regarded as reasonable, there is often no need to apply methods of calculus. Under what conditions and for what motives does this happen, anyway? In order to answer this question one must distinguish between

- a use of a calculus not reflected upon, which offers the possibility of using methods taught in school to discover certain properties of functions (local extrema, inflexion points, monotony etc.);
- a deliberately chosen approach, which allows an analysis of the model with methods of infinitesimal calculus in the first place with the aim of gaining knowledge.

Covering these topics can give prospective teachers an idea of science in general and replace their frequently observed naïve perception of science by a more enlightened attitude. So using calculus in social sciences is a means to an end, too. In the light of the coevolution of calculus and physics it is possible to discuss some epistemological and ontological issues within the meaning of a general education. The evaluation of students' short mathematical essays gives insight in students' conceptions of mathematical modelling and science in general and illustrates the didactical potential of modelling subjects inside and across "nature".

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MATHEMATICAL MODELING AND THE DIFFERENTIAL AND INTEGRAL CALCULUS TEACHING CHALLENGES.

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The reflections presented in this article are the result of a research that aimed to identify and understand the challenges of learning Differential and Integral Calculus in a Science and Tecnology Course that proposes scientific integration between Mathematical, Physical, Biological and Social knowledges. In this multidisciplinary course, a research group discuss modelling aspects of the use of digital technologies in classroom practices. The students develop mathematical skills using GeoGebra software, which supports the discussions related to the concepts of Calculus such as Functions, Limits, Derivatives and Integrals.

With a mathematical modelling point of view, the course offers different ways to re-interpretate some Calculus concepts through the use of the GeoGebra software. The course is considerate a space of mathematical discussion involving the use of digital technologies and some concepts of Differential and Integral Calculus.

The obtained results show the application of mathematical modeling providing a greater involvement by the students during the proposed activities and generated discussions. Some of the discussed models have applications in Physics and Biology, fulfilling the student academical needs in other areas as well. The use of the software as a learning tool is efficient not only in the construction of mathematical models, but also to better understand some scientific concepts. Mathematical modeling enable a closer examination of the concepts discussed in class, reaching and expanding the understanding of Differential and Integral Calculus concepts. It is also important to emphasize that there has been an increase in the student's interest in the Differential and Integral Calculus concepts and also a consequently decrease in the retention rate.

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CHANGING STUDENT PERSPECTIVES OF WORK-RELATED APPLICATIONS OF MATHEMATICS

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Applications of mathematics in work-related contexts may appear to students with limited experience of the workplace as simply mathematical processes within a superficial context and this perceived lack of authenticity affects their solutions (Palm, 2008). In contrast, applications of mathematics in the actual workplace can become deeply embedded and almost invisible to employees within routine work processes (Williams & Wake, 2007). In this paper, evidence from a recent multi-method study of young people, aged 16-18 years, on vocational training courses in England will be examined to show how, between these two positions, students' perspectives of mathematical applications in work-related contexts change as they develop a broader understanding of the workplace.

The findings are based on data from a wider study of students' experiences of learning functional mathematics in three Further Education colleges, within the areas of Construction, Public Services and Hair and Beauty. Using a comparative analysis of qualitative data from teacher interviews and student focus groups, differences are identified in the views of mathematical applications taken by trainees and professionals at different stages in their personal vocational development. Students' interpretations of mathematical applications are variable, reflecting a critical approach to the authenticity of vocational contexts, alongside a strong responsiveness to their developing understanding of the workplace. Vocational teachers, however, consistently make reference to a wider range of authentic work-related applications and state the value of a broad set of mathematical skills. By considering differences in the situation and identity of the students during their transition from school-leaver to professional, the effects of changing personal values on their judgments about mathematical applications are highlighted. Similarly, a consideration of the values of vocational teachers, in their dual roles of educator and vocational professional, suggests how perspectives continue to change with experience. This leads to the conclusion that the perspective of vocational students on work-related applications of mathematics during training constitutes an important transitional phase, in which changes in their situation, identity and experience are significant influences within a learning process that continues into employment.

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HOW STUDENTS CONNECT MATHEMATICAL MODELS TO DESCRIPTIONS OF REAL-WORLD SITUATIONS?

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Problem posing, i.e. the generation of new problems or of variants of given problems, receives ample attention in the mathematics education literature. One of the main reasons for this attention are the potentialities of problem posing to positively affect students' problem solving skills (Silver, 1994). So, asking students to generate problems themselves can lead to a better understanding of the mathematical concepts and skills that are needed for successful problem solving. The question arises whether an activity, similar to problem posing, can enhance students' modelling skills. A key step in the process of mathematical modelling involves the translation of a real-world situation into a mathematical model. The inverse activity would involve students' generation or selection of situations that fit with a given mathematical model. We will refer to this activity by the term "inverse modelling". We report an empirical study in which the potential positive effect of "inverse modelling" on modelling was investigated. Eighty 11th grade students were confronted with a multiple-choice test consisting of a modelling and an inverse modelling part. The modelling part consisted of eight items in which a real-world situation was described and participants had to connect them with an appropriate model that could be either proportional (i.e. of the form $y = ax$), affine with positive slope ($y = ax + b$ with $a > 0$), affine with negative slope ($y = ax + b$ with $a < 0$), or inverse proportional ($y = a/x$). Models were given in a formula representation and each model was appropriate for two of the given situations. The inverse modelling part consisted of eight items in which the participants had to do the opposite: connecting the same models to one of four given verbal descriptions of real-world situations. Students were randomly divided in two equal subgroups, one first receiving the modelling part and the other first receiving the inverse modelling part. Accuracy scores were analysed by a logistic regression analysis. The analysis revealed that inverse modelling not always had a positive effect on modelling: Accuracy scores on the modelling part did not differ significantly in the two subgroups. Moreover, an interaction between subgroup and model did not point in a clear direction: while the accuracy scores for modelling with a negative affine function significantly improved after inverse modelling ($p = .006$), scores for an inverse proportional function worsened ($p = .031$). For the two other models, differences between subgroups did not differ significantly. Follow-up research can further clarify these results, but clearly, just confronting students with inverse modelling in a testing context will not suffice to obtain convincing positive results.

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ASSESSING MATHEMATIZING COMPETENCES: USING STUDENTS' RESPONSE PROCESSES TO INVESTIGATE ITEM VALIDITY

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As mathematical modelling is becoming a cornerstone of many curricula, different modes of assessment are developed to investigate students' modelling competences. Haines, Crouch, and Davis (2000) developed a test for assessing the first steps that a modeller must make in modelling. This test was designed to assess students at the tertiary level and consisted of multiple-choice items. We aim to develop a test similar to the one from Haines et al. (2000), but then adapted to match the cognitive abilities of grade 9 students.

In order to develop our items we first collected qualitative data. Grade 9 students were asked to work on PISA problems. The data were analysed to explore students' mathematizing activities (e.g. assumption making, asking clarifying questions), see Djepaxhija, Vos, and Fuglestad (in press). Based on students' answers we developed multiple-choice items.

The aim of the present study is to investigate the validity of the multiple-choice items for assessing grade 9 students' mathematizing competences. Response process validity (Krathwohl, 1998) is used to analyse whether these newly developed items address the mathematizing competences we aim to assess. Four pairs of grade 9 students participated in our study.

The findings show that most multiple-choice items validly assessed grade 9 students' mathematizing competences. As intended, they made students work on a specific activity (e.g. assumption making). The students understood that they were asked to select the best alternative, which would lead to an appropriate mathematical model for the problem situation. They analysed each alternative in the context of the problem situation. Students' discussion show that they used knowledge and skills, which the items intended to assess.

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MODELING THAT MATTERS - HOW COMPUTATIONAL MEDIA INFLUENCES THE ROLE OF MODELING IN MATHEMATICS EDUCATION

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Mathematical models have a huge impact at all levels of society. Models are used for controlling processes, for enhancing human agency, to design products, to monitor and influence economic systems, and to understand the natural world. Mathematical modeling thus stands as an important topic in mathematics education and has a particular pedagogical/didactical discourse as modeling gains increased attention in research conferences and journals. Diagrammatic representations of mathematical modeling processes are increasingly being used in curriculum documents on national and transnational levels. Prominent examples are the PISA 2012 framework and the recently adopted Common Core State Standards in the United States.

An equally important and parallel trend in mathematics education research is the inclusion of digital media as a way to enhance the agency of students and provide them with new learning experiences. In this presentation, we will describe the various ways digital media are being used in modeling processes. We will critically discuss one of the most frequently used representations of modeling processes in the literature, namely that of the *modeling cycle*. We will highlight and discuss aspects of the modeling cycle that from our perspective do not fully take into account the role, function and consequences of computational media. In this context, we will specifically address these aspects: (1) that the modeling cycle presupposes and implies a linear working process; (2) that the modeling cycle falls short when it comes to accounting for the role of multiple models and/or pre-existing models used within a modeling task; (3) the modeling cycle's not highlighting or even encompassing the role of digital technology and mathematical work in its own right; and (4) the conspicuous absence of the modeling cycle in acknowledging the social and critical aspects of modeling and the use of models, that is, models not are innocent tools. We will conclude our presentation with an alternative way to think about modeling processes in order to more fully address the role of mathematical modeling in a digital era.

MODELS AND REPRESENTATIONS IN AN INTEGRATED PHYSICS AND MATHEMATICS COURSE

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This is an on-going study of a first-year integrated physics and mathematics course for engineering students. The main teaching strategies are models and modelling instruction. The course is taught in an innovative setting that promotes collaboration, transforms the classroom into a lab, and facilitates the use of technology. The research questions that guide this study are: How do students' model change over the semester in an integrated physics and mathematics course? How do students' representations vary over time in an integrated physics and mathematics course?

This qualitative investigation occurs at a large private university in northern Mexico. Forty one students enrolled in a first-semester physics course and first-semester calculus course for engineering majors, and worked in groups of three or four during the whole semester. One goal of this course is that students view mathematics as a useful tool for solving problems and applied them in physical contexts. Therefore, both instructors (one from the physics and the other from the mathematics department) are present for all classes, and the material is designed to foster the integration of both disciplines.

Most of the group collaboration activities are worked in portable whiteboards. This fosters interaction and participation within the small groups and allows at the end a whole class discussion based on the whiteboards work. Pictures of all whiteboards are shared and organized in a common virtual space. The whiteboard pictures serve as notes for students to go over the class content, material for formative assessment for the instructors, and material for research.

This study analyses the models and representations used by students during one semester of this integrated course. To that end, whiteboards from three stages (at the beginning, in the middle, and at the end) of the course are analysed focusing on the robustness of models and the variety of representations that students used. Analyses of the whiteboards revealed: a) integration of the physical and mathematical models, b) consistency and completeness of models, c) variety of representations, and d) effective written communication.

DIFFICULTIES IN DESIGNING AN MEA: EXPERIENCE IN A TEACHING STRATEGIES COURSE

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This qualitative study focuses on the models that teachers develop in designing a model-eliciting-activity (MEA). All participants were teachers enrolled in an on-line master course on teaching strategies for science and mathematics. That course emphasizes teaching strategies that have been developed by drawing on educational research in science or mathematics, such as, peer instruction, tutorials, interactive lecture demonstrations, model-eliciting activities, and generative activities. As a final project, students select one strategy to plan and implement an activity in their own classroom. Students worked collaboratively in groups of three and at least one of them implemented the activity. Planning and designing an MEA is a challenge itself, working at a distance, in an asynchronous mode, it is even harder.

Students enrolled in the master program selected this course if their discipline was teaching science or mathematics. Out of the three times that this on-line course has been taught (2011, 2012 and 2013), ten groups of three students chose to design and implement a model-eliciting activity. Collaborative groups were formed at the beginning of the semester based on discipline and grade level currently taught to cluster students with similar teaching areas. Eight of these groups implemented an activity for secondary school (grades 7 to 11), and two for tertiary education. Five of the activities were in mathematics topics, four in sciences and one in ethics.

Analysis of collaborative written reports for the planning stage shows that all groups planned their activity to facilitate the modelling cycles and most of the groups explicitly designed their activity based on the six principles for developing model-eliciting activities (Lesh, Hoover, Hole, Kelly & Post, 2000). However, in the implementation stage many teachers ended up breaking the self-assessment principle. The problem posed may have asked for a model construction, but the teacher tendency to guide their students turned the activity into a problem solving that did not ask for a generalization (reusability principle). Most of the MEA designed satisfy the reality principle and the construct documentation principle. Also, all activities allowed time for groups to share and socialized their solution. In the conclusions we will address how to help teachers to successfully plan and implement a MEA in an on-line postgraduate course.

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MATHEMATICAL MODELLING STRATEGIES OF THIRD YEAR MATHEMATICS STUDENT TEACHERS

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This paper reports on the thinking and planning strategies of third year Mathematics student teachers, who are involved in a mathematical modelling activity for the very first time. The overarching goal of the research is to deduce curricular guidelines aimed at the effective teaching and learning of mathematical modelling.

South Africa's 'new' Curriculum and Assessment Policy Statement (CAPS, 2011, p. 8) for Mathematics in Grades 10 to 12 for the first time states the following: "*Mathematical modeling is an important focal point of the curriculum. Real life problems should be incorporated into all sections whenever appropriate. Examples used should be realistic and not contrived. Contextual problems should include issues relating to health, social, economic, cultural, scientific, political and environmental issues whenever possible*". One of the modelling competencies that Mathematics learners ideally need to develop is to identify, investigate and solve real-world problems.

Mathematics student teachers should therefore be exposed to modelling tasks during their formal education. Not only should these future teachers be able to model modelling, but they should also cultivate a climate conducive towards mathematical modelling in their classrooms. A traditional Mathematics textbook problem was converted into a modelling challenge and a group of 40 third year student teachers (in purposive groups) was exposed to it. Their strategies, the nature of their engagement and their responses were carefully monitored. Proposed group solutions were collected and interpreted according to the criteria of Ng (2013), namely validity, representation and applicability. Student attitudes towards mathematical modelling and their opinions on its implementation in a classroom were also collected.

Group dynamics and their suggested 'solutions', student experiences of and their attitudes towards modelling are all aspects that richly contributed to the design of a set of curricular guidelines aimed at the effective integration of modelling into the Mathematics teacher education curriculum.

INITIAL RESULT FROM AN INTERVENTION: AFFORDANCES AND CONSTRAINTS OF AN APP FOR SMARTPHONES AND TABLETS

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Mathematical modelling and models have a real influence on our daily lives both on an individual and a societal level. Recently for example, mathematical models of how infectious diseases spread was used during the outbreak of the Ebola virus in West Africa to run simulations to forecast the behaviour of the epidemic and plan countermeasures. The gradually growing awareness of the important roles and benefits the use of models and modelling have in the society, has led to the incorporations of modelling in the mathematics curricula all over the world (Blum, et al., 2007).

Professional modellers often use different types of technology (programming, computing with computers and other ICT tools) in their work to run simulations of their evolving models to investigate their behaviour, limitations and flaws. Also in the school setting, a more frequent use of simulations in connection with different types of modelling activities has been advocated (Neunzert, 2009). The technological advances made in recent years resulting in faster computers and easy-to-work-with interfaces now offer real possibilities to also integrate simulations as integral component of modelling activities in schools.

In this paper we investigate an upper secondary mathematics classroom setting where a game app for smartphones and tables was used to simulate a pandemic outbreak. We adopted an Educational Design Research methodology with the overall goal to investigate and identify principles that can guide and facilitate the design and implementation of modelling activities using game apps' simulations to explore the mathematical models hidden in the simulations. Students worked in small groups to design and control how an infectious disease evolved, mutated, and examined what affect this had on the world's population. The theoretical ideas we use to frame the present study are based on Gibson's notions of affordances and constraints (1977).

The aim of this paper is to get an initial indication of students' experiences of using the game app intervention in learning mathematics and compare those experiences with an epistemological analysis of the intervention to develop a more mature design and implementation. In our presentation we will describe the explanatory classroom intervention where students used technology, a free app for smartphones/ tablets, to simulate a pandemic outbreak (Infection Bio War Free / Plague Inc.); provide examples of students' work; and discuss the affordances and constrain revealed by the analysis of the first implantation of the interventions in terms of students' attitudes and work, and the implications of these for the next implementation of the intervention and in formulating implementation- and design principles.

EXPLORING THE NOTION OF MATHEMATICAL LITERACY IN CURRICULA DOCUMENTS

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Members of the International Community of Mathematics and its Applications have been responsible for a significant corpus of literature related to mathematical modelling (Geiger & Frejd, in press). However, less attention has been paid by this group to research related to mathematical literacy – another theme within the field of mathematics education that focuses on the use of mathematics in the real world. The notion of mathematical literacy (known as numeracy in some international contexts) has been promoted through increasing international attention to the results of the Program for International Student Assessment (PISA), sponsored by the OECD. Within the PISA framework (OECD, 2015, p. 5), mathematical literacy is defined as:

> *…an individual's capacity to formulate, employ, and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts and tools to describe, explain and predict phenomena. It assists individuals to recognise the role that mathematics plays in the world and to make the well-founded judgments and decisions needed by constructive, engaged and reflective citizens.*

This definition emphasises the critical aspect of applying mathematics to the real world when making decisions and judgements. Further, this statement positions the application of mathematics as an essential capability for participatory and productive citizenship. While this is one definition of mathematical literacy others are embedded in national curricula documents across the world and implemented for different purposes in a variety of ways. Such differences appear to be related to cultural influences and economic and socio-political priorities within nations. The aim of this presentation is to characterise and describe how understandings of mathematical literacy and the expectations for its implementation within school curricula varies between countries. In order to compare and contrast the instantiation of mathematical literacy across nations we conduct a content analysis of a sample of national curricula documents in relation to mathematical literacy. This analysis will be used to discuss differences and similarities between the teaching and learning of mathematical literacy in different countries.

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THE PRIMACY OF 'NOTICING': A KEY TO SUCCESSFUL MODELLING

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Expertise in real world problem solving continues to be reinforced internationally as a pre-eminent educational goal. A succinct and representative statement is provided within the USA: (Common Core State Standards Initiative, 2012): which defines mathematically proficient students as those who can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.

The associated challenge contains both a task and a person dimension. There is a difference between helping students to solve individual modelling problems, and nurturing an effective problem solver over time.

The skill of 'noticing' in education (particularly teacher education) while approached with different emphases has a common theme - how teachers identify, interpret, and act upon classroom events with intent to enhance the quality of learning. But 'noticing' has been identified more widely as an essential competence to be developed within any profession.

In this paper, concerned with developing skills in modelling, we move the emphasis in 'noticing' from teacher to student. How does 'noticing' feature as an enabler of modelling ability?

The context for our study is a two-day Modelling Challenge which has been sponsored for several years by A B Paterson College, Queensland, Australia. Without previous experience in modelling students (years 10-11) have been introduced to the cyclic modelling process including an interactive example. The students (in teams of four) then use the remaining time to identify, address, and report on a modelling problem. The task of identifying the context and the specific problem(s) on which to work is the students' own. The detail of a poster summarising the modelling outcome for each team, together with a brief team presentation to the whole group, provided structural data about the substantive modelling. Additionally the students completed open questionnaire items about their approach to aspects of the task – progressively as they reached different stages of the activity. The content of written products, and observation of student activity, provided evidence of the approach, structure, and detail of the 'noticing' displayed and recorded by the students during respective phases of the modelling process.

We will discuss and illustrate features of 'noticing' that characterized the student work in different modelling phases. Some of these challenge existing stereotypes. For example, the facility with which students handle situations of data lack or redundancy.

TEACHERS AS DESIGNERS OF EFFECTIVE NUMERACY TASKS

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Numeracy, also known as mathematical literacy in some international contexts, has been a national education priority within the Australian for more than a decade. In Australia, being numerate is seen as having the capacity to apply mathematics to situations in the real world in order to make decisions and judgements or to provide solutions to problems within personal, civic and work circumstances. More recently, numeracy has been seen as a cross-curricular responsibility within schools and not simply an outcome of mathematics instruction. The view that students' development in numeracy requires a cross-curricular commitment by schools and systems was reinforced by a national numeracy review. In this review, it was recommended that:

> *…all systems and schools recognise that, while mathematics can be taught in the context of mathematics lessons, the development of numeracy requires experience in the use of mathematics beyond the mathematics classroom, and hence requires an across the curriculum commitment.* (COAG 2008, p. 7)

Additionally, the Australian Curriculum requires that numeracy be addressed across the curriculum, in every subject through the compulsory years of schooling. Thus, there are both policy and curriculum demands which create the expectation that teachers provide tasks that make use of mathematics to promote student learning within all school subjects, not just mathematics. Consequently, the effectiveness of the across the curriculum approach to numeracy is dependent on the quality of tasks developed by teachers for their students. While there is evidence that well constructed tasks are effective in improving teaching practice (Lappan & Phillips 2009), the principles behind task creation and adaptation are rarely made explicit.

This presentation will outline and describe the background to and initial progress of a nationally funded project in Australia which aims to investigate how teachers can enhance their capacity to design effective numeracy tasks by (1) determining principles of cross-curricular numeracy task design, and (2) refining teaching practices that lead to improved student performance on realistic, contextualised tasks.

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HOW TO BUILD-UP AN HYDROGEN REFUELLING STATION INFRASTRUCTURE IN GERMANY? – A PROJECT APPROACH FOR MATHEMATICS CLASSROOMS

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How and how far to reduce carbon emissions is currently a very important topic in European energy politics. We use alternative energies in our study as a matter for (mathematical) discussion and modelling with students. We consider the example of hydrogen, which is an energy carrier with zero carbon content. It can be produced from all energy resources, such as wind, solar, nuclear energy, biomass, and clean fossil fuels. Moreover, hydrogen can be converted to power and heat with high efficiency and zero emissions. Introducing new energy carriers, such as hydrogen, we have to think about fitting them in our current energy system and infrastructure. In fact, building a hydrogen refuelling station infrastructure is more complex than other fuel infrastructures due to the uncertainty of the upcoming demand, future energy costs, and number of technological options. In this project we want to focus on modelling a road map for hydrogen refuelling stations. The complexity of the model can differ in conditions (costs, energy demand etc.), precision of the model and variety of (mathematical) tools students choose. This allows to explore mathematic modelling competences (cf. Blomhoj & Jensen, 2003) related to different grades (students at high school and university), and which impact these problems have on the motivation for learning mathematics.

This research-approach in mathematical modelling (cf. Blomhoj & Jensen, 2007) is based upon modelling project carried out with high school and university students. Therefor students had to work for about one week autonomously on the above diagrammed situation. In our contribution we want to give an insight how to measure the outcome of such an approach (cf. Blum & Leiß, 2005; Borromeo-Ferri, 2011).

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MODELLING USING COMPUTER TOOLS IN MATH CLASSES

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This paper surveys modelling processes using computer tools in mathematics education theoretically, empirically and by using concrete examples.

First different modelling cycles are analyzed from a theoretical point of view in which the use of computers and other digital tools will be integrated into the modelling cycle. There are different proposals (e.g. Galbraith et.al. 2003, Savelsbergh et al. 2008, Geiger 2011, Siller & Greefrath 2010). Each focuses other aspects of the use of computer tools. Which tool is used in practice can also play a role. Here is picked up the view from the computer science education by a subject-oriented approach.

At some meaningful examples the different added value of computers in modeling and simulation is illustrated and integrated into the theoretical discussion.

In the context of a small qualitative study is also shown what activities students actually perform when using the special computer tool GeoGebra when working with modeling tasks and where these activities are related to the modelling cycle.

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MODELLING TASK DESIGN

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It is well known that mathematical modelling has helped humankind to solve a variety of problems. Thus, we now have a wide range of models specific for each discipline, models that have gone across from one discipline to other, and models that relate to different disciplines. On interdisciplinary areas, for example, on environment, models are used in attempts to address the interactions of various theoretical results, some coming from physics (meteorology) and other from chemical processes, as in the case of air pollution that has biological effects in human and other living beings. The importance of modeling in different disciplines justifies the need for research that provide insight into modelling conceptions that experts of different areas have, and also into knowing in what dimensions they value and use mathematics and mathematical modelling.

We discuss here the results of research developed in the context of activities in a seminar whose participants were teachers who debated about the organization of mathematics courses for science, engineering, mathematics and mathematics teaching students The role of modelling in the teaching of mathematics was the focus of the discussion. We observed a physicist, a chemist and a mathematician arguing about the design of tasks devised to teach mathematics and based on the modelling of various situations. We analysed the nature of their proposals in order to improve mathematics learning and the understanding of its relationships with modelling. We also focused our attention on the relationship of their proposals with the beliefs and modelling skills exhibited by the participants in the seminar exhibit.

Although this is a case study, we can say that we consider two major kind of researchers, that is, theorists and experimentalists. Experimentalists, once having real data focus their attention on data management, on creating baseline models and on graphics built with base on the statistics used in the area (realistic modelling, Kaiser et. al., 2011). Theoreticians, on the other hand, give more importance to the qualitative understanding of previously constructed models (*modelling exploring*, Doerr & Lesh, 2003), for example, variation of parameters. Their beliefs induce in the classroom the perception of mathematical models in the students and determine what the teacher thinks that their students should do in order to learn mathematics.

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SUSTAINABLE ASSESSMENT IN MATHEMATICAL MODELLING

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We discuss the role of sustainable assessment (Boud & Soler, 2015) in mathematical modelling education. Sustainable assessment is an idea that focuses on the contribution of assessment to learning beyond the timescale of a given course; it is assessment that meets the needs of the present in terms of the demands of formative and summative assessment but which also prepares students to meet their own future learning needs. An important question from this point of view is: do educational activities (including assessment) equip learners for the multiplicity of challenges they will face after graduation?

As part of a teaching innovation project in Higher Education, a mathematical modelling approach was introduced in a second year mathematics for Engineering course. This course focused significantly on the development of employability skills (e.g. discussing, explaining, working in group, presenting, etc.) as a way to make the course more relevant and engage students in meaningful mathematical activity. Part of the assessment of this course was a group presentation, in which groups of four students had to produce a mathematical model of a physical phenomena and prepare and deliver a presentation of their model and the process by which they obtained it.

In this paper, we discuss the results of the analysis of one of the group's presentation from the point of view of sustainable assessment. We video recorded some of this group's preparation meetings outside the classroom and their actual presentation, and then conducted in-depth interviews with each of the group members individually to elicit their views on this assessment as well as other parts of the course in relation to their perceived role in their learning.

Our results show that students found the group presentation challenging in various ways and that working as part of a group helped them to understand different proposed models and to succeed in giving a good presentation. Furthermore, the data shows that some of the students felt that this type of assessment helped them identify their strengths and weaknesses, and that the skills developed were therefore relevant to their future careers. However, we found that the students which were less certain about their future career were also less certain about the relevance of mathematical modelling for their degree.

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"WHY DO I HAVE TO LEARN THIS?" AN EXPLORATION OF 'RELEVANCE' IN MATHEMATICAL MODELLING EDUCATION

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In this position paper we explore the role of 'relevance' in students' motivation to engage in mathematical modelling activities.

We will give a review of literature that mentions relevance in the context of education, and relate this to mathematical modelling activities. Relevance is an important aspect within the socio-cultural context of learning, and in mathematical modelling it could be seen as mediating or connecting different parts of an activity. We will define relevance as a property of an activity in which students participate and which adds a value to them ("why should I do this task?), usually in relation to an imagined or expected future. We will also contrast 'relevance' with other concepts that support motivation, such as interest and authenticity.

We will ground our definition in three case studies of mathematical modelling education. The first case will demonstrate mathematical modelling activities that were perceived as relevant, but not interesting. The second case will demonstrate a mathematical modelling activity that was perceived as interesting, but not relevant. The third case will demonstrate mathematical modelling activities that were perceived as both interesting and relevant.

We will argue that for some students 'relevance' is more concrete than for others depending on their identities (seen as dynamic and in dialectic relation with the context in which students participate). We conclude that relevance is mediated by user value (a student can see it being useful in the real world), by authenticity (a testimony of authenticity might add relevance), and/or by a connection to an imagined future ("I could be doing this after I graduate" or "I will need this to graduate"). In particular, we claim that 'relevance' in mathematical modelling can be hidden, added or spelled out, which has important practical consequences for pedagogy, some of which will be discussed.

INVESTIGATING STUDENTS' DIFFICULTIES WITH DIFFERENTIAL EQUATIONS IN PHYSICS.

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There are numerous cases in physics where the value of a quantity and changes in that quantity are related. For example, the speed of an object depends on its acceleration; the radioactivity of a sample depends on the amount of the sample present. Except in highly idealized settings, the analysis of these cases requires students to recognize, set up, and solve a *differential equation* (DE).

In many universities, including DCU, DEs are studied in mathematics before they are applied in physics. However, the aims of mathematicians and physicists can be very different. Mathematics modules emphasize the classification of DEs and their theoretical aspects (questions of existence and uniqueness of solutions). Techniques for solving DEs are also studied. In physics modules, modelling is emphasized: students must apply mathematical knowledge to interpret a setting, recognize the need for and set up a DE, solve it, and interpret the solution.

Other potential reasons why DEs present a problem for physics students include gaps in students' mathematical knowledge, conceptual issues with DEs, and educational transfer issues. The aim of this project is to identify and understand students' difficulties with differential equations in physics, and to develop a pilot curriculum that seeks to resolve them.

The presentation will describe the pilot curriculum (a set of tutorials) that will be implemented, their contents and the justification for this methodology. They will run in conjunction with a module introducing differential equations and have a strong emphasis on transfer and modelling in physics. The tutorials have been designed from information gathered by surveying students who have completed the module as well as information from the literature. Students completed a detailed survey assessing their prior mathematical knowledge, modelling and transfer abilities, and understanding of conceptual issues. Exam papers from other mathematics exams taken by these students were also examined to inform the tutorials layout and make up.

Although this project concentrates on modelling in physics, it has the potential to expand into many other disciplines given the wide range of applications of differential equations. This is very much in line with the conference theme of working at boundaries. It aims to understand teaching and learning of mathematics at the boundaries of different forms of knowledge and to give undergraduate students the tools to succeed both in their studies and in their future careers.

MODELLING AS INTERACTIVE TRANSLATIONS AMONG PLURAL WORLDS: EXPERIMENTAL TEACHING USING THE NIGHT TIME PROBLEM

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This study examines the advantages in interpreting students' modelling from the perspective of interactive translations among plural worlds. Its position needs to be distinguished from a simplified perspective which presents mathematical modeling as involving transitions only between two fixed worlds – a real world and a mathematical world.

The specific focus of the study was the "Night time" problem in which students consider variations in the hours of daylight in different parts of the Earth as latitude North (N) increases. Twenty eight students in 10th grade in a Japanese high school participated in the study. Experimental lessons were held over a period of 100 minutes, using a structured investigation. Phase 1 introduced the phenomenon of the "midnight sun" and asked students to estimate day/night hours starting at latitude 33.3 degrees N and increasing to 66.6 degrees N. Phase 2 invited students to verify their simple estimates by drawing/measuring a geometrical model or by checking the real night time at a given latitude N. Phase 3 assisted students to build a more precise geometrical model by reflecting on their results from the second phase.

From a theoretical perspective, five different "worlds" can be applied to this problem. The first is a real world; the second is a concrete operational world where students manipulate objects; the third is a geometric world which allows them to draw/measure a model; the fourth is a symbolic operational world which represents the situation algebraically, and the fifth is a graphic representational world. Within the time available, only the first three were considered, but affordances for moving beyond these are discussed.

In the second phase, students were asked to consider the reasons for any mismatches between their initial predictions and actual findings. They were invited to draw and use a simple geometrical model of the globe and sun to ground their predictions. At the end of the teaching period, students were invited to propose problems and issues that they thought should be considered next. Students' actual suggestions were classified in terms of the interactions that took place among the three worlds presented, and for the affordances these offered for possible translations to a symbolic operational world and a graphic representational world. These interactions are important for guiding teachers to identify how to effectively promote mathematical modelling.

DESIGNING A PROBLEM BY MATHEMATICAL MODELLING AND GAMIFICATION FOR THE SOLIDS OF REVOLUTION WITH DIFFERENTIALS

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Many research studies show that mathematical modelling is an invaluable approach to learn calculus. Recent studies reveal that the gamification process is a good pedagogical approach to learn calculus too, because it motivates and keeps the students interested. In this research, a combination of these two approaches in the comprehension of the solids of revolution was established in order to improve the student learning of this concept. The objective of this research is to present the designing activities that were developed with mathematical modelling and gamification in order to construct several problems from the solids of revolutions with differentials. The work was done with two classes of engineering students.

In this study gamification was used to work with the activities of solids of revolution in the class. In the design of each of these activities the Niss and Borromeo cycle of mathematical modelling was developed taking into account each cycle step. The mathematical modelling was applied to design, develop and evaluate the activities as well. The gamification applies game mechanics and rules to motivate and create a healthy competition environment.

The research was developed in two sessions. The first session, had only problems with x-axis rotation to obtain the volume of the solid of revolution. In the second session, the problems used y-axis rotation. For each session some homework and an exam were used in order to establish an evidence of the student learning about this topic. The work involved 2 classes of 40 students divided into groups of 3 or 4 students.

In order to obtain robust results, an evaluation of each activity, that took into account the 7 steps of the Niss and Borromeo cycle, was carried out. Also the homework and the exams were evaluated in this context. For the gamification, a qualitative video analysis was carried out, so we can show how Mathematical Modelling and gamification is a good combination to learn about solids of revolution with differentials. These results will be presented during the presentation. It was shown how mathematical modelling can be combined with gamification in the comprehension of solids of revolution with differentials. It was found that the Niss and Borromeo mathematical modelling cycle can be combined with gamification. A design methodology was established using this combination: this can be extended to other concepts studied in a calculus course, even in other areas of the knowledge.

A PARALLEL MATHEMATICAL MODELLING IN ENGINEERING CALCULUS ASSISTED BY TECHNOLOGY

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The Mathematical Modeling studies place Blum and Niss (1991) as initiators of the research generated around the teaching of modeling, solving problems in the 90's. For this research study, a model is considered an abstract, simplified and idealized interpretation of a real world object, or system of relationships, an evolutionary process or a description of reality according to Henry's definition (2001).

In this study two mathematical models were used: one as a modeling process proposed by Rodriguez (2007, 2010). And the other for evaluation in the teaching process that belongs to Borromeo (2006). As Borromeo and Niss (2009) said: It is important to provide valuable mathematics education as an integral character in the students.

On the other hand, it is no longer considered that learning calculus for Engineering with technology is just an option, there have been many studies about technology for Engineering Calculus course in the past as Papert (1980), Churchhouse (1986), Cornu and Ralston (1992), Hoyles, Lagrange, Le Hung Son and Sinclair (2006) and Hoyles and Lagrange (2010) established. Hoyles and Lagrange (2010) also stated that in many countries, it is hard to conceive the world without high-speed interactivity and connectivity. Also Cornu and Ralston (1992) said that converting the technologies into a well-developed and tested curriculum for the normal teacher and the normal student is still a major challenge.

With these two frameworks, the objective of this paper is to report the research that was made about the Mathematical Modelling and technologies that were designed for teaching and learning five different mathematical models from the Engineering Calculus course: linear, quadratic, cubic, harmonic and exponential. In this research we set a parallel work for mathematical modelling, an investigation of the learning of the five different Mathematical Models was done, using Rodriguez cycle for the set up of the activities and the Borromeo and Niss (2009) cycle to evaluate these activities.

The research was made with different kinds of technology depending on which of the five models was being learned. Calculators, sensors, simulations and video analysis technologies were used. The study was done on three groups of Engineering Calculus with between 30 and 40 students each group. Qualitative and quantitative results will be reported. To finish some conclusions from the research and ideas for future work will be reported.

AN ANALYSIS OF CONSEQUENCES OF DIDACTIC THEORY FOR PRACTICE – THE CASE OF MODELLING CYCLE

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The teaching and learning of mathematical modelling is by now an established research area in mathematics education with theoretical developments, various approaches and empirical studies of various modelling activities in classrooms and their effects on students' learning (e.g. Blum & Ferri, 2009). The modelling cycle and the notion of modelling competency are central constructs in several of these theoretical works on modelling in mathematics education.

In upper secondary schools in Denmark mathematical modelling has entered into the curriculum and as a consequence also into textbooks where there are chapters on modelling featuring e.g. the modelling cycle from didactic theory. However, the modelling cycle is an analytic model of a mathematical modelling process. It is not a description of the practice of 'real world' modelling (e.g. see Frejd (2013)). We are interested in consequences of didactic theory for practice when the theory becomes part of the subject matter, so to speak. That is questions such as: What are the constraints of theory on practice? What does theory facilitate? What kind of practices has become common? Is there a more or less normative didactic setup, and if so, to what extent are common practices a consequence of such a setup?

To be more specific, our research question for this presentation is the following:

> *How does the didactic theory of modelling cycle affect the didactic practice, which constraints and conditions does it put on the teaching of mathematics?*

This will be investigated from the perspective of the Anthropological Theory of the Didactics (ATD) using the notion of didactic praxeology and didactic transposition (Bosch & Gascón, 2014). We will examine Danish curricula, exam exercises and different textbooks written by in-service teachers.

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MATHEMATICAL MODELLING WITH PROGRESSIVE LEARNING AIDS – A VIDEO STUDY

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The competency of mathematical modelling is one of the six central competencies in German mathematical educational standards already implemented more than 10 years ago. Despite the high curricular relevance of mathematical modelling, students demonstrate few modelling strategies in school practice. In addition, there is an imbalance between the amount of the teacher's support and the students' ability to work independently (Blum 2007). This qualitative empirical study addresses this imbalance and aims at supporting the competency of mathematical modelling by using progressive learning aids in a self-regulated learning environment. It also intends to evaluate the efficiency of the approach. Until now, there is only little known about the concept of progressive learning aids in mathematics education and its efficiency in self-regulated learning environments. This study uses an approach which has been developed within natural science education by the research group Kassel (Germany) and adapts it to the specific requirements of mathematical modelling. The approach comprises a set of metacognitive and cognitive support cards with successively increasing support including teacher's support as final aid. The support cards are strongly related to the specific modelling examples dealt with in class and offer more general support related to modelling strategies such as usage of the modelling cycle as metacognitive orientation, affective support and at the end example-bound aids. The approach is currently being evaluated in two seventh-grade classes of a German higher track school (so-called Gymnasium) in Hamburg. The study focuses on an external as well as an internal perspective. The external perspective is mainly surveyed by videography and audiographical means. To evaluate the internal perspective of the students, interviews are conducted.

In the first part of the presentation the motivation for conducting this research study as well as the relevant theoretical framework of progressive learning aids will be described, followed by a description of the design and the concept of progressive learning aids applied in this study. One of the self-designed modelling problems used in the study will be presented. Preliminary results of the use of the progressive learning aids when working on the presented modelling problem are finally described. They show that progressive learning aids are applicable for mathematical modelling problems. The support cards and the teacher's support are helpful in the different phases of the modelling process at varying degrees. Additionally, the use of progressive learning aids generally improves the competency of mathematical modelling. Students perceive the progressive learning aids as an enhancing and reasonable support while working on complex mathematical modelling problems. But it also becomes apparent that students do not always use support cards and prefer to ask the teacher for advice when it seems appropriate.

TRIGGERING THE DEVELOPMENT OF MATHEMATICAL MODELLING COMPETENCIES THROUGH MATHEMATICAL MODELLING AS CONTENT

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Twelve practising Mathematics teachers were introduced to mathematical modelling through mathematical modelling as content. The enculturated experience of modelling of the teachers was the use of mathematical modelling as a vehicle to introduce mathematical concepts and procedures. Another experience they were conversant with was the applications of mathematics to contextual situations after a relevant mathematical topic has been taught. At the onset of the 16-hour introductory course the teachers were presented with a photograph with some trees with one of the trees very prominent. The task was formulated as "What space is occupied by the tree?" This formulation was done in line with Davis's (1991) assertion that

> *Many problems are posed to industrial mathematicians by colleagues in other disciplines, who may not yet understand the real problems they face. Good problems need not be elegant, new, or well posed, just necessary to corporate welfare.*

The teachers worked as a group and all the sessions when they were working in the "classroom" space were video-recorded. These recordings were analysed to ascertain the affordances and constraints the adopted approach reveals for the development of mathematical modelling competencies. Results render that the teachers' mathematics pedagogical histories both limited and advanced their quest to construct a model for the given situation and that during the sense-making phase of the problem various competencies are used in an implicit manner. Implications for introducing practising teachers in similar situations to mathematical modelling are reflected on.

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SCAFFOLDING IN COMPLEX MODELLING SITUATIONS

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The support of students while solving authentic and complex modelling problems is a complex process, especially if students shall work as autonomously as possible (cf. Kaiser et al., 2013 and Kaiser & Stender, 2013). The theoretical approach of scaffolding including adaptive interventions (Wood et al., 1976) provides a theoretical framework that gives an answer to the question, how teachers should act in these situations so that students can solve the problem as independently as possible and reach an adequate answer. Research carried out amongst others within the project DISUM points out many difficulties with adaptive teacher intervention in modelling situations. In the research project described in the paper, the tutors were prepared in advance for these adaptive scaffolding activities within the frame of a university master seminar. The action of selected student groups and their tutor were videotaped while supporting students (age 14-15) who are working on complex, authentic modelling problems such as the optimal position of a bus line within a three days project session. The videotaped activities were analyzed afterwards, identifying examples of successful teacher interventions. Amongst others scaffolding measures based on the principle of minimal help could be reconstructed, referring amongst others to the idea of strategic assistance (Zech, 1996). Finally examples of successful and unsuccessful teacher activity are described. Examples of appropriate strategic interventions will be presented, based on observed interventions and referring on the theoretical framework of problem solving, a strongly related didactical approach.

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COMBINING MODELS RELATED TO DATA DISTRIBUTION THROUGH CONJECTURING AND VALIDATION: PAPER HELICOPTER EXPERIMENTATION WITH YEAR 5 STUDENTS

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Distribution is at the heart of statistics, hence it is an important objective to develop students' notions of distribution in school statistics curricula. Modelling is highlighted as a powerful vehicle for developing students' statistical ideas (e.g., Lesh, 2010). Statistical investigation requires developing ideas and models with statistical and real contextual elements (Wild & Pfannkuch, 1999). If the nature of modelling is iterative connection and overlap among models from a cognitive perspective (Ärlebäck & Doerr, 2015), it is crucial that when teaching statistics through modelling students are encouraged to combine distribution-related models so that statistical and contextual elements are coordinated in them and oriented to integrate the models. This informs the research question: through what activities, and how specifically, can students be supported to combine models? This study adopts experimental activities in modelling process (e.g., Halverscheid, 2008), that include conjecturing and validation. This paper illustrates how students combined distribution-related models using a case study involving 31 Year 5 students (aged 10-11 years) performing a *paper helicopter experimentation.* The experimentation of measuring dropped helicopters' flight times provided the students with opportunities to (1) form and validate conjectures about flight-time distribution and (2) explore changing conditions of the experiment and carry it out. As a successful case, it was shown that isolated statistical and contextual elements in the students' distribution-related models formed by initial conjectures coordinated with their models formed by re-conjecturing after trial and validation. Through the experimental activities the students grasped statistical and contextual elements from other models generated by the collection and arrangement of real data, and contrasted and coordinated these elements with ones in their own models. Triggers for revising and combining models were: (a) the awareness of data variability and its source through experimentation, (b) the recognition of empirical distribution as *a model* of experimental phenomena and (c) the exploration of features of the model. These findings will inform educators on how to facilitate student development of distribution-related models in the teaching of statistics through modelling.

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TRIGGERS FOR AND BARRIERS TO SUCCESSFUL STUDENT SWITCHING BETWEEN MODELLING CYCLES: THE CASE OF DUAL MODELLING TEACHING IN JAPAN

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The *dual modelling cycle framework* (DMCF) (Saeki & Matsuzaki, 2013) proposed as an approach for teaching modelling provides students with opportunities to attempt a real-world task, where they can refer to previously generated models from an analogous and simpler real-world task. It aims to foster mathematical understanding and modelling skills of students by switching between two modelling cycles: the initial task, and the analogous and simpler task. We have distinguished two kinds of switching: (1) *teacher's intentional switching* to facilitate student understanding of the task situation and the mathematical structure of models and (2) *students' intentional switching* to solve the initial task (Saeki et al., 2015). It was reported that although the teacher intended the students to transfer to the alternative cycle and then return to the original, some students switched further between two cycles back and forth (Kawakami et al., in press, 2015). In this paper, we attempt to clarify what the triggers for and barriers to accomplishing student switching between two cycles are, from a case study involving 33 Year 5 students (aged 10-11 years) in a Japanese school. The analysis illustrated the aspect of the student switching between two cycles, where the students connected and overlapped their generated models in each cycle by referring back and forth among the models. In the process some of the students created models dependent on the real-world/mathematical contexts and situations of two tasks, and other students developed mathematically generalized models independent of the real-world contexts and situations. We indicated two possible triggers for accomplishing the student switching: (a) awareness and exploration of alternative, similar and operable models to interpret existing models and (b) overlap between the real-world/mathematical contexts and situations of the two tasks. Furthermore, we identified three possible barriers to accomplishing the student switching: (a) insufficient understanding of the analogous relationship between the two tasks, (b) misunderstanding of the mathematical structure of models, and (c) implicit switching as a result of overlap between models preventing clear sight of the final goal.

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REALISTIC MATHEMATICS EDUCATION DESIGN THEORY AS A TOOL FOR STRUCTURING POTENTIAL MATHEMATICAL MODELLING LEARNING ACTIVITIES

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The Realistic Mathematics Education (RME) instructional design theory offers a methodological approach for designing learning sequences in which students are provided opportunities to reinvent mathematics by 'mathematizing' either real world situations or mathematical relationships and processes. However, this is very difficult to achieve in distance learning settings where students are separated from a physical tutor.

This study is a reflection on the learning activity design process that used an RME-oriented approach to develop tasks introducing basic calculus concepts in a distance learning setting. Fifteen pre-college and university students participated in a series of learning tasks loosely built around the RME construct. From an analysis of student responses to the learning tasks, it was clear that the lack of a mechanism for supporting instructional dialogue between the teacher and student made the adoption of an RME-oriented approach to distance teaching very difficult. There were also shortcomings in the actual design of the learning tasks.

This paper explores the possibility of re-designing the learning activities routed in mathematical modelling theory, using RME as a conduit for structuring modelling eliciting activities. The reflection on the attempt to adopt RME instructional design heuristics provides a unique context in which to try and infuse didactical theoretical principles for mathematical modelling while outlining the challenges that surface in the process.

The outcome is a set of re-configured learning tasks for introducing the teaching of Calculus concepts in distance or online learning settings which will still require testing in actual learning environments.

ANALYSING UNIVERSITY STUDENTS' ABILITIES IN MAKING ASSUMPTIONS IN A BALLISTICS MODEL: A CASE STUDY

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This paper investigates abilities of two groups of university students in making assumptions in a ballistics model. The first group consisted of postgraduate students majoring in applied mathematics from a New Zealand university and the second group consisted of first-year science students majoring in applied mathematics from an Australian university. The students were asked to make reasonable assumptions in a ballistics model from mechanics. We started talking about stones thrown by catapults in ancient times and proceeded to discussing firing balls from cannons in medieval times and launching projectiles and missiles in recent history. In each of the four cases – a stone, ball, projectile and missile – the distance from the starting point to the landing point was given. In addition, the maximum height for a projectile and missile was also given. The students were challenged to think in each case about the appropriateness of the following four assumptions:

- The Earth is flat;
- The Earth is an inertial system;
- Air resistance can be ignored;
- Acceleration due to gravity is constant.

It was agreed that a relative error of less than 3% was not significant. Without doing any calculations the students were asked to indicate which of the above assumptions were reasonable and which were not in each of the four cases: a stone, ball, projectile and missile. After completing the modelling exercise the correct solution was presented and discussed with the students followed by a questionnaire on their experiences in making the assumptions. Students' responses to the questionnaire on assumptions are presented and analysed in the paper following the approach suggested in (Galbraith & Stillman, 2001).

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THE PROCESS OF MATHEMATISATION IN MODELLING OF NUMBER PATTERNS IN SECONDARY SCHOOL MATHEMATICS

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This paper as part of a bigger study (Knott, 2014) that investigates the development of horizontal and vertical mathematisation and the competencies that are revealed and developed when learners work through number pattern problems in a mathematical modelling classroom. When learners mathematise, they have the opportunity to reinvent mathematics in a way similar to when it was invented (Gravemeijer, 2004). The mathematising competencies were developed for number patterns and placed on a continuum. These were: internalising, interpreting, structuring, symbolising, adjusting, organising and generalising (Van den Heuvel-Panhuizen, 2003).

A focus group consisted of seventeen Gr 10 learners in a multicultural English-medium school. The learners worked through nine learning activities in 90-minute modelling sessions. The researcher planned and described the hypothetical learning trajectory with conjectured goals and outcomes. The analysis provides a comparison of the predicted learning trajectory and the actual learning trajectory, and a detailed description of the observed horizontal and vertical mathematising competencies connected to a Local Instructional Theory. The RME theory was used as theoretical framework in a design-based research study.

The horizontal and vertical mathematising competencies for number patterns were revealed during the learning activities. The learning activities encouraged the development of number patterns competencies (Knott, 2014).

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MATHEMATICAL MODELLING FOR ENGINEERING STUDENTS: PERSPECTIVES ON VISUALISATION

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This paper offers perspectives on the visualisation of engineering students while solving differential equations (DE) with a computer algebra system (CAS). The overarching aim is to determine the influence of mathematical modelling on engineering students' visualisation. According to Arcavi (2003:217), visualisation is "*the ability, the process and the product of creation, interpretation, use of and reflection upon pictures, images, diagrams, in our minds, on paper or with technological tools, with the purpose of depicting and communicating information, thinking about and developing previously unknown ideas and advancing understandings*". In a CAS environment, students find questions relating to the interpretation of numerical tables and computer graphs problematic and often leave these unanswered.

The sample comprises a cohort of second year students registered for a three year National Diploma in Engineering at the University of Johannesburg. Most students come from poor socio-economic backgrounds and their graphing skills are often underdeveloped. These factors were carefully considered in the design of two DE problems (one relatively easy, one more advanced) inspired by scenarios reported in local daily newspapers. To determine the visualisation competencies of students, the cohort of approximately 100 students will be divided into ten groups, purposively constituted, where students will design paper-and-pen documents and programme codes on worksheets. As this is a maiden exposure to mathematical modelling, each problem is delineated into sub questions to elicit, guide and trace the process of visualisation through the modelling cycle. A relaxed setting will be created with ample time to perform the modelling tasks.

Feedback from students will be collected at the end of April and their opinions and attitudes towards mathematical modelling will be categorised via content analysis.

A complaint amongst engineering academia is the inability of students to transfer their visualisation skills across the borders of Mathematics classrooms. Results from this pilot study may shed new light on the use of mathematical modelling in striving to enhance engineering students' visualisation abilities.

IMPLEMENTING MODELLING INTO CLASSROOMS: RESULTS OF AN EMPIRICAL RESEARCH STUDY

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Mathematical modelling is one of the important mathematical competences that students should gain and develop in school. To many teachers it is unclear how to put this into practice and they feel insecure about teaching modelling. It is, therefore, essential to find appropriate didactical concepts and teaching units to effectively facilitate the modelling competencies of our students in school.

A didactical concept to foster modelling competencies in regular school lessons was developed and tested in an empirical study with 332 students of 10th grade in German secondary schools. The goal of the study was to develop a teaching unit which helps to increase the global modelling competence independently of influencing factors as the chosen topic of the teaching unit and individual factors as gender and report grades of the participating students. Based on the definition of modelling competence by Blomhøj and Jensen (2003) the focus of the study was laid on the *global modelling competence*, which was defined as the ability to undertake a full modelling process and to possess the metaknowledge of the procedure. A four-hour teaching unit was developed with a holistic and self-dependent approach based on results of the projects DISUM (Blum) and ERMO (Brand), amongst others.

Concerning the increase in the global modelling competence and the motivation of the students during the four-hour teaching unit, three hypotheses were formulated and analysed in a pre-post-designed research study. A follow-up test was also undertaken three months after the teaching unit to test the sustainability of the acquired modelling competence. The global modelling competence of the students was rated by an evaluation scheme of Siller et al. (2014). The results of the study show a significant increase in the global modelling competence independent of the influencing factors grade and topic. Concerning the gender of the students, minor differences could be identified.

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THE DUAL MODELLING CYCLE FRAMEWORK: BENEFITS FOR AUSTRALIAN STUDENTS

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Embedded within the Australian Curriculum Mathematics are the proficiency strands of understanding, fluency, problem solving and mathematical reasoning. The teaching of mathematical modelling provides students with opportunities to develop these proficiencies. In the Dual Modelling Cycle Framework (DMCF) (Saeki & Matsuzaki, 2013), students are introduced to an initial task in the first modelling cycle, then switched by their teacher to the second cycle, where a similar and simpler task is introduced. The transition between cycles is designed to encourage deeper understanding of task content, increased mathematical fluency in transitioning between task contexts, enhanced problem solving skills and improved mathematical reasoning (Saeki, Kawakami, Matsuzaki, & Lamb, accepted 2015). Use of the DMCF has resulted in successful outcomes for Japanese students (Kawakami, Saeki, & Matsuzaki, in press, 2015) and Australian students (Lamb, Kawakami, Saeki, & Matsuzaki, 2014).

In this paper we analyse responses to two tasks set using the DMCF given to 23, Year 6 students in Australia. The results indicate that the Australian students relied on their prior experiences of orientation and visual perception study when working with 2D and 3D shapes resulting in immature models during the first cycle. However, after switching to the second modelling cycle, the students were able to develop the rectangle and parallelogram models that had been anticipated for use in solving the problem tasks. The structure of the DMCF enabled the Australian students to substantially improve their understanding of the models and thus promote the key proficiencies of understanding, fluency, problem solving and reasoning.

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ALTERNATIVE CAUSES AND DISABLING CONDITIONS AS TOOLS TO STRUCTUE A REAL MODEL FROM A MESSY PROBLEM

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Statistical enquiry is by definition a modeling process, commencing with a problem in a real world setting, progressing to planning to gather data, gathering and analysing data and interpreting the findings in the light of the problem context. (Wild and Pfannkuch, 1999). Research about understanding various statistical processes and concepts at the initial stages of the modeling cycle tend to describe everyday reasoning emanating from personal situation models as idiosyncratic and at the lowest level of models of statistical reasoning. However, if statistics education wants to approach statistics as a modeling cycle, more information is needed about valid everyday reasoning processes in data rich contexts.

In this paper I report the results of an extended discussion among high school mathematics teachers who attended an introductory statistics course, and had to restructure the experiential context of prices of used cars into a real model that could be operationalised for statistical investigation. I used Sfard's (2008) theory of commognition to do discourse analysis of the discussion. I will show that the participants' everyday reasoning at the start of the modeling cycle was patterned, rather than ideosycratic and can be explained as shifts from evaluative routines, where reasoning was aimed at making sound contextual judgments, to investigative routines as relationships between concomitant variables became structured. The key impetus for the shift in discourse was allowing the participants to increase the contextual complexity in their discussion, rather than guiding them to narrow their focus. Of interest for this paper is the reasoning processes that allowed the participants to co-structure a workable real model, namely an active search for alternative causes and disabling conditions to interrogate abductive claims.

This study is of interest to teachers of statistics who want to maintain the interaction between context and mathematics during statistical investigation. The findings provide insight into reasoning at the cusp of experience and investigation of a data rich context. Such information will help to legitimise the task of structuring a real model from messy situation models as a learning goal.

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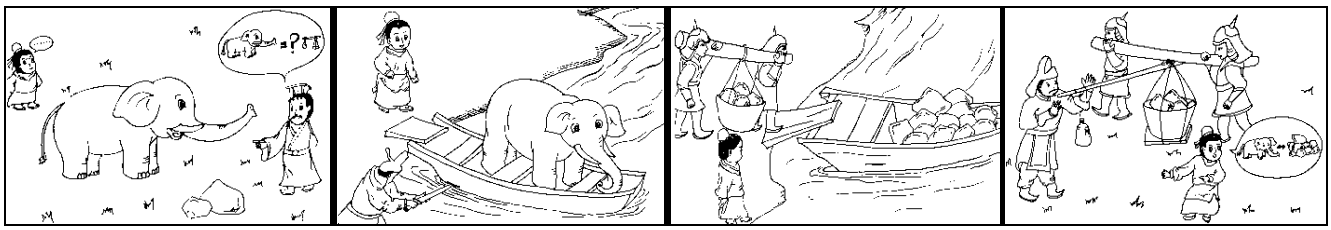
PRE SERVICE TEACHERS' KNOWLEDGE IN APPLYING STORY METAPHOR IN TEACHING MODELING: AN EXAMPLE OF WEIGHTING AN ELEPHANT

Issic K C Leung[1] and Regina M F Wong[2]

[1]The Hong Kong Institute of Education; [2]Logos Academy, Hong Kong

When teaching problem solving skill that goes beyond its abstraction to the situation that skill of mathematical modeling is required, teachers might have a good chance to explain to students why we must learn mathematical functions, and how to apply them in the context of modeling. Adopting the Gravemeijer's (1994) four levels of mathematical learning processes, namely, situation, referential, general and formal levels, our framework will extend beyond a more abstract level - the analytic level at where model will be developed. However, reaching this analytic level is not straightforward. In a relevant context, from concrete to abstract, metaphor will then be a crucial role to lead students to learn the essential contents of mathematics.

An ancient Chinese folk story tells about a young boy who used a trick to measure the weight of an elephant. It may be used as a story metaphor to teach the modeling skill and simultaneously the concept of function and its inverse.



In this investigation, through Q and A's on watching the demo video of a lesson in which teacher used this story metaphor to teach, pre service mathematics teachers were testified on how ready they use this metaphor in a lesson of mathematical modeling.

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DIDACTICAL IMPLICATION ON AN ELEMENTARY METHOD TO DIAGNOSIS THE EXISTENCE OF CHAOTIC BEHAVIOR OF A SINGLE NEURON MODEL

Issic K. C. Leung[1] and H. Y. Zhang[2]

[1]The Hong Kong Institute of Education; [2]The University of Hong Kong

Mathematics teachers may feel that advanced knowledge is not compulsory to teach elementary mathematics in schools. Being mathematics educators, we believe that advanced knowledge helps teachers to effectively deliver mathematics ingredients in their lessons. In studying advanced mathematics, we are not only learning the essential contents such as definitions and theorems, but also cultivating our attitude to strike for the knowledge of understanding. Take an example of studying the dynamics of neuron models, mathematicians have been actively involving in investigation the cognitive structure of humane thinking activities such as pattern recognition, memory, and mental calculation, which are exhibited in the form of dynamical characteristics like stability, bifurcation, and chaos. Using the finite difference method, we can discretize a given differential equation of a single neuron model $xx'(tt) = -aaxx(tt) + bb\tanh(xx(tt)), tt \geq 0$, where a and b are constants, and $xx(tt)$ is the energy possessed by the neuron at time tt to give the difference equation with step length h, $xx_{nn+1} = (1 - aah)xx_{nn} + bbh\tanh(xx_{nn})$. By using Taylor expansion of $\tanh(xx_{nn})$, xx_{nn+1} can be approximated by a cubic polynomial, i.e., $xx_{nn+1} = (1 + (bb - aa)h)xx_{nn} - (bbh/3)xx_{nn}^3$.

It can be verified that there exists a periodic point of period three in the map xx_{nn+1}. By the well-known assertion by Li & Yorke (1975): "periodic point of period three implies chaos", we ensure that chaotic behaviour exhibits in the model of single neuron stated above. This example shows an elementary method to diagnosis the existence of chaos of a neuron model represented by a nonlinear differential equation. Through a straight simplification, we may enable college students to understand advanced chaotic phenomenon of a dynamical system without a rigorous justification on its existence.

When suitable tool or approach is applied, advanced knowledge likes chaos of dynamical systems could be made understandable to non-mathematics major students. Perhaps, our example gives some hint to answer the question: *To what extent of mathematics knowledge one should possess before becoming a professional mathematics school teacher*?

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STUDENT SOLUTIONS OF AN AUTHENTIC INTER-YEAR MODELLING TASK

Matthias Ludwig
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Each day professionals use mathematical knowledge to solve problems from daily life. These mathematical problems are often unknown or hidden to teachers and students. Instead, in mathematics classes students are often exposed to problems, which lack any relation to their daily life. In this paper we present a cross-sectional study based on an authentic mathematical problem about an old skyscraper in Frankfurt, which had to be demolished. After the detonation a lot of concrete rubble and steel remained. The question was: "How long does it take to clean the area? Please estimate the number of days or give a date when all the concrete rubble is carried away. Reason mathematically!" This task was used in a regional school competition for students from grade 5 to grade 12. The students had to send in their solution before the detonation took place.

The task is obviously authentic (cf. Vos, 2011), since the context originates from reality, and the blasting could be witnessed by the students, either live or on national TV. The task question was in reality posed by inhabitants living near the skyscraper.

There were sent in 31 large-sized (DIN A1) posters from 13 schools. The posters were analysed based on criteria for the quality of the mathematical model (Jablonka, 1996). An analysis of student solutions shows, that all students use the same model (rectangular solid) to estimate the volume of the concrete rubble. In contrast to earlier cross section studies (Ludwig & Reit, 2013), the older students did not use more sophisticated mathematical tools than 5th graders, but their solutions were, in general, more detailed and took many other factors into account, which affect the answer on the removal time. To what extent these aspects lead to a better solution and how modelling tasks like this can be evaluated will be presented on the ICMTA conference. Student posters will be shown and further results (e.g. accuracy of the removal time) will be discussed in detail.

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Day:	*Monday*	**Session:**	*C (14:15 – 15:00)*	**Room:**	*A40*
Age:	*Secondary*	**Focus:**	*Learning*	**Type:**	*Practical; & Research*

PROFESSIONAL DEVELOPMENT AND MODELLING: INTERACTIONS BETWEEN BELIEFS, PRACTICES AND CLASSROOM CONTEXTS

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[1]University of Education Freiburg, Germany;
[2]University of Nottingham, England

The EU-funded research and implementation project PRIMAS (FP 7, 2010 – 2013) developed classroom materials and carried out sustained professional development (PD) courses in each of twelve countries that were designed to promote inquiry-based learning (IBL) with a focus on the modelling cycle (as defined in PISA). These courses were based on a common theoretical framework for PD and drew on a common set of web-based multimedia resources.

Using a qualitative case study research design, we characterise three different ways that teachers came to interpret IBL/modelling over the course of the PD and examine how these different interpretations interacted with other contextual factors to affect classroom practice. In total, we obtained 19 case studies, each comprising data from two interviews with the teacher, two observations of lessons and one observation of the professional development course.

Three characterisations became apparent: IBL/modelling as peripheral activities for developing student motivation; IBL/modelling as pedagogical practices for developing thinking and reasoning; IBL/modelling as a combination of activities and pedagogies for developing 'scientific processes'. These, increasingly sophisticated, viewpoints appear related to the different ways in which teachers responded to contextual factors: time constraints; syllabus coverage; assessment demand; collaboration with colleagues; relationships with school administration.

In conclusion, we suggest that these different characterisations have implications for the planning of professional development courses and the ways in which they are interpreted and acted upon by teachers.

MATHEMATICAL MODELING: STABLIZING LEARNERS' SKILLS

Azita Manouchehri
The Ohio State University, USA

In this research report, we will draw from written work collected from nearly 700 middle and high school students and follow-up interviews with a selected sample of them, to explore the relationship between personal experiences of learners and the common incorrect responses that they had provided to some application problems they were assigned. We will offer an analysis of the tasks and perspectives on potential sources for students' incorrect answers. Most immediately, the mismatch between the task designers' intentions for the type of knowledge they wished to elicit from children and children's treatment of the same tasks as modeling contexts. Interpreting the tasks using their personal life experiences motivated children's choice of variables and assumptions they had made when solving the problems.

We will elaborate on the type of capacities that teachers may need to develop in order to effectively build on the children's personal experiences to advance their mathematical modeling skills.

INCREASING MATHEMATICAL MODELING PEDAGOGICAL CAPACITY (MMPC) AMONG TEACHERS

Azita Manouchehri, Stephen Lewis, Monelle Gomez and Ali Fleming
The Ohio State University

We will describe a series of mathematical modeling experiences we used in a methods course designed for a cohort of prospective secondary mathematics teachers at our institution. This course was completed a semester prior to the prospective teachers' student-teaching phase. We will elaborate on the prospective teachers' reactions to these mathematical modeling contexts and the potential advantages they associated with their use when teaching. Lastly, we will report on our observations of the same cohort of students as they implemented similar mathematical modeling contexts in their own field placements; challenges they encountered relative to implementation and assessment of student learning outcomes. Suggestions for how mathematical modeling experiences for preserve teachers might be organized and implemented toward increasing their Mathematical Modeling Pedagogical Capacity (MMPC) will be shared.

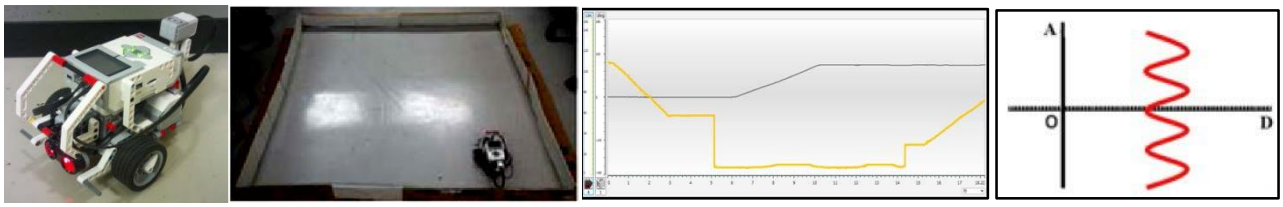
MATHEMATICAL MODELLING WORKSHOP FOR GRADUATE SCHOOL STUDENTS IN TURKEY USING LEGO®MINDSTORMS®EV3 AND GRAPHING CALCULATOR

Akio Matsuzaki[1], Kosuke Tsukahara[2], Ugur Cetinkaya[3] and Tomoko Inoue Cetinkaya[3]

[1]Saitama University; [2]Graduate school of Education, Saitama University; [3]Mathematical Power and Innovative Design Association

LEGO is located on elementary mechatronics tools (Matsuzaki, 2010), and modelling lessons were conducted using LEGO and ICT tools (Isoda & Matsuzaki, 2003; Matsuzaki & Ide, 2013). Matsuzaki have worked on modelling teaching and developing materials using LEGO with Mathematical Power and Innovative Design Association in Turkey. In this presentation we introduce mathematical modelling workshop for graduate school students in Turkey using LEGO®MINDSTORMS® EV3 and graphing calculator.

We picked up the three modelling problems in workshop: (1) the students observed the movements of a robot that was connected distance sensor and angle one, and draw two graphs (**D**istance-**T**ime graph and **A**ngle-**T**ime graph), (2) they answered the movements of the robot with referring graphs that were represented by data logging function by LEGO®MINDSTORMS®EV3, and (3) they answered the movements of robot with referring **A**ngle-**D**istance graphs.



Through the first and the second problem most of the students were able to explain the movements of the robot on graphs and read the movements of the robot from graphs. In the third problem it is not easy for them to predict and explain the movements of the robot, and they communicate opinions with their classmates.

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WHAT MAKES DIFFERENCES BETWEEN THE MODEL OF '90° SYSTEM ADVERTISEMENT' OF THE FLAG OF ENGLAND AND THE ONES BY THE STUDENTS?

Naoto Mishima[1], Akio Matsuzaki[2] and Yuki Takagi[3]

[1]Graduate School of Education, Saitama University; [2]Saitama University

[3]Saitama Prefectural Urawa High School

'90° system advertisement' is made by central projection, and looks vertical from a main camera fixed at particular point because of hallucination. There are some construction methods, for instance, using projection chart and Pythagorean theorem (Mishima, 2015). In this presentation we report a mathematics lesson including activities which were to make '90° system advertisement' of the flag of England for grade 10 students in Japan. The students made mathematical models by construction methods using projection chart or Pythagorean theorem (figure 1). The students watched the mathematical models from the main camera to check them against the flag of England, and modified them.

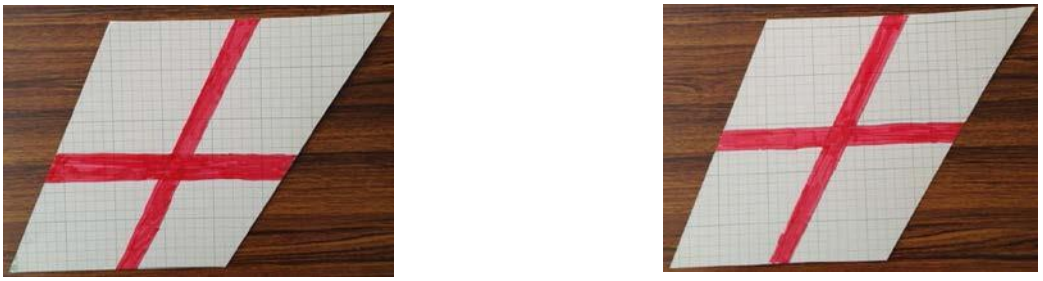

Figure 1. Mathematical models of the flag of England made by construction methods using projection chart (left) and using Pythagorean theorem (right).

The mathematical models might be characterized by construction methods, or *not*. Some of the students drew a horizontal red line on lower than middle of the flag of England by construction method using projection chart. On the other hand, some of the other students did it on the middle when using Pythagorean theorem.

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THE VELOCITY CONCEPT – HISTORY OF A MODELLING

Regina Dorothea Moeller and Hans-Stefan Siller
University of Erfurt, Koblenz Germany

Applications in physics as part of the world that we are modelling mathematically for a long time has been considered as one of the central cross-curricular topics for mathematics education. These applications often involve several mathematical fields and sometimes show a longer process of modelling activities. One of these applications is the velocity concept. Its genesis refers to a centuries-old search within the context of motion and for more than 150 years shows new technical applications.

Embedded in the concept of motion already Greek mathematicians, especially Aristotle (384-322 BC), had ideas about velocity which he combined with his observations of free fall. Before the next step by Galilei (1564-1642), Nicolas Oresme (1360) used graphical representations of changing qualities. Later Galilei used experiments to support the statement that there is a quadratic relationship between the distance travelled and the falling time (cf. Wagenschein, 1952). He succeeded in a better understanding of the concept as he did not rely on his direct perception (Einstein, 1950, p.17). Newton (1643-1726) was the first scientist who introduced the concepts of time and space and opened up the exact relationship between force and motion. Leibniz (1646-1716) developed the differential and integral computation, also considering the idea of (planetary) movements. He also introduced the way we use symbols ever since and shaped our current understanding of modelling physical phenomena.

In today's linguistic usage, we understand motion as a change in position of the (Euclidean) space over a certain period of time. We refer to lengths and time periods as conditions for the quantification of such motions. On this basis, the (average) velocity is defined as the quotient of the distance travelled over the time required.

The genesis of the velocity concept shows a potential for analysing different steps of modelling according to its historical understanding. It broadens didactical perspectives within mathematics education. Despite the scarce representation of this topic in mathematics classes (in many curricula of the German provinces the concept of velocity is only mentioned one time on the secondary level) there is a diversity of ideas which should be reflected.

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Day:	*Wednesday*	**Session:**	*N (12:40 – 13:30)*	**Room:**	*A40*
Age:	*Secondary*	**Focus:**	*Other*	**Type:**	*Practical; & Theoretical*

A MODELING PERSPECTIVE IN DESIGNING PROFESSIONAL LEARNING COMMUNITIES

Nicholas G. Mousoulides and Maria Evagorou
Department of Educational Sciences, University of Nicosia

The paper addresses how the Models and Modeling Perspective (Lesh & Doerr, 2003) can be used to understand the nature and development of teacher knowledge, and what it means for a teacher to develop mathematics content, pedagogy, and an understanding of how students develop their mathematical ideas when working with inquiry-based modeling problems. Increasingly, researchers are realising that it is crucial to work collaboratively with teachers and their students, in the reality of their own classrooms, in an attempt to better inform practice (The Design-Based Research Collective, 2003). Such collaborative paradigms include design studies, such as the multi-tiered teaching experiments, and professional development approaches that involve content-based collaborative inquiry (English, 2003; Mousoulides, 2013). In our study we adopted a three-tiered research paradigm that addresses the development of researchers, teachers, parents, and students, in an attempt to provide a learning and development environment for classroom practitioners and researchers to work together in generating meaningful change within modeling contexts (Lesh & Kelly, 2000).

In-service teachers and the parents of the students in four classes participated in a collaborative design approach to develop, and implement (the teachers) a number of model-eliciting activities. A series of workshops lasted for an academic year. During the workshops teachers and parents were introduced to the concept and examples of model-eliciting activities, and were then encouraged to collaborate with researchers to develop their own activities. 'Road Safety' and 'Health and Exercise' modeling activities were developed and implemented in the four classrooms. Researchers were present during classroom implementation, while reflective discussions between teachers, and researchers took place after classroom implementations.

Teachers and researchers both identified changes in teacher knowledge and behaviour in their classrooms. For instance, changes were evident in their pedagogical approaches in inquiry based learning, their feedback to students, and in their collaboration with other teachers. Parents also identified and appreciated a warm and collaborative environment that was developed, and welcomed the initiative as a good practice for improving their children's thinking and problem solving skills, and their communication with the teachers.

Acknowledgements
This paper is based on the work within the project MASCIL – Mathematics and Science for Life (www.mascil-project.eu). The research leading to these results/MASCIL has received funding from the European Union Seventh Framework Programme (FP7/2007-2013) under grant agreement n° 320693. This paper reflects only the author's views and the European Union is not liable for any use that may be made of the information contained herein.

INTERACTIVE DIAGRAMS USED FOR COLLABORATIVE LEARNING CONCERNING MATH MODELS OF MOTION

Elena Naftaliev
Achva Academic College & Haifa University, Israel

Our research interests are concerned with the development of knowledge about motion processes involving collaboration between students and interactive multiple representations diagrams. Interactive diagrams (IDs) are relatively small units of interactive text (in e-textbook or another material) and an important element in e-textbooks. Current technology allows for a variety of interactive tools, examples and representations as components of IDs. For example[1], IDs focused on motion may include the following components: a range of representations of motions; a repertoire of linking tools; and choices of activation of various representations. The question is, "how do a curriculum designer, a learner, and a teacher decide how and which IDs components of the text to use for different purposes in teaching-learning processes?" In an attempt to study that question, we designed three settings of IDs. The three IDs varied by the design choices of what was included in the given example and how it was represented and controlled. Regarding the what to include in the example, the animation was designed around simultaneous multi-process motions, to include motion situations known to be challenging, such as non-constant rate-of-change and 'no motion' situations, as well as surprising situations such as an 'unexpected win.' Considerations of how to design these choices were driven by the semiotic functions framework. To study specific design components, we made comparative decisions about the variety and type of representations, the control features, and the linking features. For the first step of the research, we analyzed emerging personal problem-solving processes in the context of motion by means one of the three tasks, each consisting of an ID that was designed to support different functions of inquiry teaching-learning. The findings of our research[2] show that similar tasks with different IDs should be considered as different learning settings. In the presented research, we focused on the second step, asking the students who had already been asked to address similar activities that included different IDs to share their work and participate in a group discussion. The process allowed us to analyze the social construction of knowledge in a new pedagogical setting. The results showed that the participants collaborated to generate an interactive text based on the given IDs. The text became an instrument which supported the development of shared knowledge about characteristics of kinematic phenomena (such as direction, rate of change, constant and changing speeds) and about their mathematical descriptions. Also, there is evidence that the development of shared knowledge occurred when the students defined common goals and engaged in a reflective activity about the other members' reasoning and instruments involved in the collaborative process.

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1 http://visualmath.haifa.ac.il/en/quadratic/motion_at_changing_speed

2 https://sites.google.com/site/interactivediagrams/publications

A STUDY ON UNDERGRDUATE STUDENTS' COMPETENCY TO APPLY MATHEMATICS AND MATHEMATICAL MODELING

Keiichi Nishimura[1] and Fumitaka Yanagisawa[2]

[1]Tokyo Gakugei University; [2]Thomson Reuters

Nowadays the competency to apply mathematics and mathematical modelling for workers and citizens who are expected to become leaders in society are changing. The competency that we focus on in this study is higher order skills than the mathematical literacy that PISA or PIACC assessed. The purpose of this study is to develop assessment of the undergraduate students' competency and show the results in Japan.

At first we developed 12 items with no bias in selection towards any faculty. Each item has four questions based on the mathematical modelling process. We developed three booklets that consisted of 8 items. And about 700 students from various universities and faculties had the assessment in Japan. Fig.1 shows the score distribution and distribution of a percentage of correct answers.

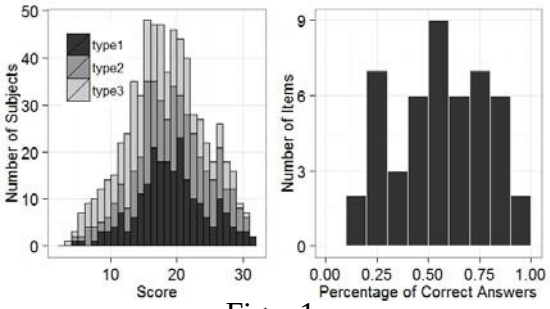

Fig. 1

Secondly, we analyzed the results by using IRT, and identified key skills to show to what extent one can perform in real world problems requiring mathematical formulation. The procedure is as follows:

1. Estimate item parameters and person parameters used in 2PL IRT model.

2. Divide assessment items into 5 level groups by the order of their item difficulty parameters.

3. Extract skills commonly required in items of the same group.

4. Make a table of these skills arranged by group and mathematical modelling process.

With the levels of difficulty, the table shows what are key skills to develop students' mathematical modelling competency. For example, while lower levels need simple use of numbers and formulations on the process of employing mathematics, upper levels need more processes such as calculating reciprocals or means to represent real world contexts, making graphs to compare concerned cases and re-examine the fairness of these comparisons.

Finally, we put subjects into 6 groups by the table of skills mentioned above. The result shows gaps between learning mathematical knowledge or techniques in a mathematical context and employing them in a real world context. For example, nearly 70% of subjects are not in levels where they can employ means, percentage of increase or reciprocals of units to compare cases in the real world, while use of these mathematical elements is taught before 7th grade in Japan.

IN PURSUIT OF A LIFE-PREPAREDNESS ORIENTATION FOR MODELLING: THE ROLE OF CONTEXTUAL REASONING AND REFLECTIVE KNOWING

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This paper provides a theoretical exposition of the notion of a 'life-preparedness orientation', the role of mathematical and contextual practices in this orientation, and, particularly, the format of modelling seen to characterise this orientation. In this regard, I argue that a life-preparedness orientation is characterised by a dominant agenda for sense-making and enhanced understanding of contextual practices. Agendas involving the transmission and acquisition of mathematical content, structures and processes, albeit important and of use in the contextual sense-making process, are ultimately subordinated to this dominant agenda. I argue further that this life-preparedness orientation is facilitated, in part, through a particular format and conception of modelling. This conception is characterised by a unique intersection of everyday knowledge and mathematical competency, and, crucially, by specific forms of reflection and reasoning. The type of reflection envisioned draws on the levels of 'reflective knowing' identified by Skovsmose (1992), and I argue that these levels facilitate reflection on and successful engagement with, primarily, the mathematical components of the modelling process. However, given the intention within a life-preparedness orientation for the prioritisation of contextual sense-making practices, I argue that this form of reflective knowing must be accompanied by a degree of contextual reasoning (c.f. Bansilal, 2013) that facilitates successful identification, interpretation and engagement with contextually relevant resources, discourse, knowledge, content, techniques and forms of participation. The combination of reflective knowing and contextual reasoning, thus, ensure a form of modelling that recognises the validity of existing structures of contextually acceptable forms of participation together with the potential and/or limitations of mathematically-structured representations and reconstructions of real-world practices. I conclude by arguing that both reflective knowing and contextual reasoning are necessary and crucial elements of modelling processes that seek to facilitate a heightened degree of preparation for life- and work-related practices.

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MATHEMATICAL MODELLING IN A LONG DISTANCE TEACHER EDUCATION COURSE IN BRAZIL: DEMOCRATISING MATHEMATICS

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Brazil has experienced an accelerated economic growth with accompanying social changes. The country is now the 7th largest economy in the world. It hosted the 2014 World Cup and it is hosting the Olympics in Rio in 2016, and is going through a tremendous amount of modernization in relation to infrastructure, including that of health and education. Nationwide a process of upgrading teacher competencies and the training of new teachers on a massive scale has beend developed by integrating the use of long distance and multimedia technologies.

To increase access to a wider audience, the *Brazilian Open University* system aims to democratize and increase access to higher education. In this regard, the study of new educational and methodological proposals such as mathematical modelling become relevant as it promotes social changes resulting from contemporary scientific and technological developments. The need to update and upgrade professional development for teachers raises new institutional solutions, methods and resources in order to meet the demand for specialized teacher education programs.

This context allows federal universities in Brazil to offer *Seminars in Mathematical Modelling* in long distance mathematics undergraduate courses, which are offered entirely in an environment mediated by new technologies and the internet. The development of the activities in these courses is conducted through the use of the Moodle platform that possesses interactional tools among teachers, tutors, and students.

Long distance education contributes and can assist students to overcome difficulties regarding the adoption of mathematical modeling courses because technological tools offered by the platforms such as Moodle are simple and funcional. Through the use of discussion forums and videoconferences, professors and tutors are able to critically analyze interactions enabled by these tools, which can contribute to the reflexive development of the elaboration of mathematical models in the Virtual Learning Environment (VLE).

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ETHNOMODELLING AS THE MATHEMATISATION OF CULTURAL PRACTICES

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Ethnomodelling is the process of elaboration of problems and questions that grow from practical contexts and the formation of an image or sense of an idealized version of the *mathema*. The focus of this perspective constitutes a critical analysis of the generation and production of knowledge (creativity) in order to critically discuss the social mechanisms of institutionalization of knowledge (academics) and its diffusion through generations (education).

By analyzing reality as a whole, this holistic context allows those engaged in the modelling process to study systems of reality in which there is an equal effort to create an understanding of all aspects and components of the system under study as well as the interrelationships among them.

These systems have revealed sophisticated mathematical ideas and practices that often include geometric principles in craft work, architecture, and the traditional practices encountered in activities and artifacts found in local and vernacular contexts, which are related to numeric relations found in measuring, classification, calculation, measuring, games, divination, navigation, astronomy, modelling, and a wide variety of other mathematical procedures and cultural artifacts.

Many Western mathematical activities are regarded as modeling by this definition and due to its cultural roots in non-Western society it can be defined as ethnomodeling of the mathematical practices found in non-Western settings. A characteristic of these new problems is that they cannot be solved using syllogistic that is a classical Aristotelian logic, but need multivalued logic, often called *fuzzy logic*, which is the logic that underlies inexact or approximate reasoning. In this direction, multivalued logic can be used in order to formalize human-like processes that are culturally bound.

This context allows the development of a definition of ethnomodelling as the translation of local mathematical ideas, procedures, and practices in which the prefix *ethno* is related to the specific mathematical knowledge possessed by the members of distinct cultural groups. Thus, ethnomodelling adds cultural perspectives to the modelling process. In this regard, it is necessary to start with the social context, reality, and interests of the students and not by enforcing a set of external values and decontextualized curricular activities without meaning for them.

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EXPLORING THE CRITICAL-REFLEXIVE DIMENSION OF MATHEMATICAL MODELLING

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According to the Brazilian National Curriculum for Mathematics (Brazil, 1998), students need to develop their ability to solve problems, make decisions, work collaboratively, and communicate effectively. This approach is based on valueing emancipatory powers, which help students face challenges posed by society by turning them into flexible, adaptive, reflexive, critical, and creative citizens.

This perspective is also related to the sociocultural dimensions of mathematics, which are closely associated with an ethnomathematics program (D'Ambrosio, 1990). This aspect emphasizes the role of mathematics in society by highlighting the necessity to analyze the role of critical and reflexive thinking about the nature of mathematical models as well as the role of the modelling process to solve everyday challenges present in the contemporary society.

Mathematical modelling provides real and concrete opportunities for students to discuss the role of mathematics as well as the nature of mathematical models (Shiraman & Kaiser, 2006). It could be understood as a language to study, understand, and comprehend problems faced daily by society. For example, mathematical modelling is used to analyze, simplify, and solve daily phenomena in order to predict results or modify the characteristics of these phenomena.

In this process, the purpose of mathematical modelling is to develop students' critical and reflexive skills that enable them to analyze and interpret data, to formulate and test hypotheses, and to develop and verify the effectiveness of the mathematical models. In so doing, the reflection on reality becomes a transformative action, which seeks to reduce the degree of complexity of reality through the choice of a system that it represents. This isolated system allows students to make representations of this reality by developing strategies that enable them to explain, understand, manage, analyze, and reflect on all parts of this system.

This process aims to optimize pedagogical conditions for teaching so that students are better able to understand a particular phenomenon in order to act effectively transform it according to the needs the community. The application of critically-reflexive dimensions of mathematical modelling makes mathematics to be seen as a dynamic and humanized subject. This process fosters abstraction, the creation of new mathematical tools, and the formulation of new concepts and theories.

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USING MODELLING AND TABLETS IN CLASSROOM TO LEARN THE QUADRATIC FUNCTION

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Many studies in mathematics education agree about pointing out the importance of introducing modelling in education to show the students the relation between mathematics and the real world. Nevertheless, the incorporation of these tools into the educational practices is still a pending issue due to a lack of resources and support material for teachers and a need of a change of the methodologies used in classroom, among others. Moreover, the information and communications technologies (ICT), which are present in the daily lives and whose use has been increasing in the last years, is another element to bring mathematics closer to the real world.

In the first part of the paper, we present the design and the results of a pilot study on the learning of the concept of the quadratic function and the meaning of the parameters through the modelling process using tablets. Our study was globally organised by the theoretical and methodological framework of the Local Theoretical Models (Filloy, Rojano & Puig, 2008) and the teaching methodology was based on problem solving. The design of the teaching situation introduces the qualitative analysis of the phenomenon and the family of functions as the main mean to manage and control the modelling process, and it takes into account the use of real data. These data were taken with classroom by the students using the app Video Physics® and they were processed in Data Analysis®. The teaching situation was implemented in a group of 11th grade students and took place in a total of three sessions. The analysis of data revealed that (a) the qualitative analysis and the students' previous knowledge are key elements to choose the function used as a model and to interpret the results in terms of the phenomenon, and (b) the students have a deeply rooted idea that the height can't take negative values and it's exactly zero on the ground.

In the second part of the paper, we present an improved design of the teaching model considering the results obtained in the preliminary one. Besides, the new design allows the students to learn the meaning of the parameters through the observation of the changes in the algebraic expression of the function and its effects on the graph. In addition, the different canonical forms used by the apps give them an opportunity to work the algebraic transformations and to learn them with sense, not as a mechanical process.

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MATHEMATICAL MODELLING AND PROOF BY RECURRENCE: AN ANALYSIS FROM A WITTGENSTEINIAN PERSPECTIVE

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The generalization of patterns from a particular case is a mathematical procedure often seen as a strategy in mathematical modelling tasks. The generalization process is often carried out by means of mathematical recurrence. Concerned about mathematics use in the context of these tasks, we investigate the question "can the recurrence used in the mathematical modelling tasks be considered as a mathematical proof?". The study is based on the writing of the Austrian philosopher Ludwig Wittgenstein, in particular, on the subject proof by recurrence (Wittgenstein, 1974; 1996; 2009). In this context, we analyzed the development of a mathematical modelling task performed by undergraduate students during a mathematical modelling course, in a Mathematics Major course. Data were collected through audio recordings, video, quizzes, and written records. The data analysis methodology is based on qualitative research and the interpretation of Wittgenstein's writings. Analysis indicate that students engaged in mathematical modelling tasks may have to develop a set of mathematical procedures associated to the proof by recurrence.

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AN EXAMINATION OF PRE-SERVICE TEACHERS' CAPACITY TO CREATE MATHEMATICAL MODELING PROBLEMS FOR CHILDREN

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This presentation will discuss the results of a study carried out with pre-service Foundation Phase teachers in South Africa. The study was designed to examine the choices that pre-service teachers make around the creation of mathematical modeling problems, based on Galbraith's (2007) Suitability Criteria for Modelling Problems. Participants were first given the opportunity to use this criteria to evaluate and discuss the suitability of selected modelling tasks. They were then asked to create their own task for Foundation Phase students that would meet the given criteria.

The presentation will discuss the analysis of these tasks and some key findings. The analysis offers insight into the pre-service teachers' ability to create suitable, age-appropriate tasks that require the creation and use of a mathematical model to find a reasonable solution to a problem. It also examines their ability to determine the mathematical content involved with a particular task. The findings highlight a strength in the pre-service teachers' ability to create real-world contexts within which to set their modelling tasks, but in many cases, an inability to construct a task and related scaffolding questions which require the use of modelling to find a solution.

The discussion of the findings will also consider two implications of the results. The first is what the pre-service teachers' choices can reveal about their mathematical knowledge for teaching and how this can inform mathematics teacher education. The second involves questions that arise from this study about the reasonability of applying such criteria in the early stages of mathematics education, particularly up to grade three.

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MATHEMATICAL MODELING WITH DATA IN A COLLABORATIVE GROUP : STUDENTS' MATHEMATICS DISPOSITIONS AND IDENTITIES

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This study examined how a collaborative mathematical modeling activity impacted students' mathematics disposition and identity development. A number of studies demonstrated that mathematical modeling, which plays a prominent role in the new US curriculum, Common Core State Standards for Mathematics (CCSSM), promoted socially situated learning environments with group collaboration, classroom discussion, initiative, and creativity, and it has the potential to develop positive disposition toward mathematics and strengthen their mathematical identity (e.g, Lesh & Doerr, 2003). As disposition reflects how one sees oneself through developing relationships with mathematics in a given context, examining mathematical identity became a pivotal means to understand one's relationship with mathematics and the context in which learning takes place.

Sixty students who enrolled in College algebra course participated in this study. A total of eighteen focal students were selected for interview based on the results of classroom observations and students' journals. The curricular tasks for the mathematical modeling activities required students to collect their own data and to use the ecological foot print as a tool for assessing human impact on the environment.

The result from analysis of interview data, students' written tasks and journals identified the aspects of mathematical modeling that impact students' mathematical dispositions and identities. The full paper will report the distinguishing features of the mathematical modeling activities that enable students to see themselves as learners of mathematics and having them view mathematics as a subject that is worthwhile through negotiating mathematical meaning and establishing the validity of mathematical models using their knowledge of real world.

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PRE-SERVICE MATHEMATICS TEACHERS' KNOWLEDGE IN MATHEMATICAL MODELING AND VIEWS ON MATHEMATICAL MODELING

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The Common Core State Standards for Mathematics (CCSSM) in the US calls for a greater emphasis on mathematical modeling. Modeling standards appear in each of the other five high school standards of mathematical content and one of the eight standards for mathematical practice. Mathematics teacher educators are challenged with preparing teachers to understand the intricacies of mathematical modeling and to define effective strategies for classroom implementation. Literature addressing secondary mathematics teachers' modeling competency or ways to increase their pedagogical knowledge regarding modeling is scarce. Stillman et al. (2007)'s framework supported the implementation of mathematical modeling and identifying modeling skills and competencies.

This study reports an exploratory study, which examined how a group of pre-service secondary mathematics teachers' knowledge and views on mathematical modeling were developed through their mathematical modeling experiences. The pre-service mathematics teachers carried out mathematical modeling activities that were included in a core mathematics content class in undergraduate STEM education program.

The data were obtained from the pre-service teachers' performance of solving and creating mathematical modeling tasks as well as the questions they were asked in an open-ended questionnaire. The pre-service teachers' performances at modeling process were noted in discussions and post-class reflections. Their written reports to the modeling tasks also served as artifacts to be examined in detail for aspects of the modeling process discussed in class. After completing the modeling tasks, the pre-service teachers responded to open-ended questions pertaining to their modeling experiences and their views on teaching and learning mathematical modeling.

The full paper will report pre-service mathematics content knowledge revealed during modeling process and their views on teaching and learning of mathematical modeling. The pre-service teachers considered modeling as content for students to learn how to model the real world beyond learning mathematics skills.

QUALITY CRITERIA FOR MATHEMATICAL MODELS IN RELATION TO THE MODELLING PURPOSE; THEIR USEFULNESS IN HIGHER ENGINEERING EDUCATION

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At the TU/e all BSc study programmes have been substantially innovated since 2013. New is a course on mathematical modelling for all BSc students. The course introduces criteria, especially developed for this case, for the quality of mathematical models in relation to the model's purpose. A modest search into the literature suggests that this list of criteria is new as well as its focus on the relation to purposes. In this study we will explore their functioning in educational practice. The criteria have been developed by the 3rd author, implemented by the 4th, and evaluated by the 1st, assisted by the 2nd.

The criteria are: *genericity* (how many different modelled systems can we handle?), *scalability* (how large can the size of the problem be?), *specialization* (how much should the intended audience know?), *audience* (how large can the intended number of stakeholders be?), *convincingness* (how plausible are the assumptions?), *distinctiveness* (how accurate, how certain, how decisive can the model outcome be?), *surprise* (to what extent can the model outcome give new insight?), *impact* (how big can the consequences of the model outcome be?).

A criterion's relevance depends on the model's purpose, i.e. explanation, prediction when, prediction what, compression, inspiration, communication, unification, analysis, abstraction, verification, exploration, decision, optimization, specification, realization, or steering & control.

Among other things, the course includes lectures and project work in groups of five. Four kinds of modelling projects were organized, corresponding to variants within the course: generic modelling, dynamic modelling, data modelling, and process modelling. In their reports, the groups had to answer reflection questions concerning the necessity and the possibilities for improvement of their modelling work, related to the criteria. Over 200 project reports will be scored on whether students actually use the criteria in their answers showing understanding.

For a one third sample pilot investigation the variable usefulness of a purpose or a criterion was defined as the number of occurrences of use showing understanding minus the number of occurrences of use showing misunderstanding. As for the purposes, only *optimization*, *analysis* and *predicting what* proved to be useful, with optimization showing the most. As for the criteria, *genericity*, *scalability*, *specialization* and *convincingness* showed most usefulness, followed by *distinctiveness* and *surprise*. At the conference, results will be shown for the whole population, followed by an explanation of the emerged pattern as well as an evaluation of the set of criteria.

DEVELOPMENT OF MODELLING COMPETENCIES IN STATISTICAL INFERENCE

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This study investigates pre-service teachers' development of modelling competencies through the observations of two teams of students training to be teachers who were engaged with modelling statistical data using linear models. Statistical models are special types of mathematical models, distinguished, in part, from other mathematical models because they are not determinist. The case studies presented in this article provide insights into situations when the study participants made inferences while they moved from the creation of an algebraic presentation of the logic underlying the simple linear regression procedure to a statistical model that best fits all the data and build this model by estimating unknown model parameters from the data.

We report on the students' model-based inferences as they build formal conceptions about variance, goodness of fit for the model that helped them to make decisions about the linear relationship between the dependent variable and independent variable as indicated by results of a flexible interview. This study also points to the potential of specific aspects of multidirectional modelling for fostering the development of students' robust knowledge of the logic of inference when modeling and investigating connections between real contexts and data, and statistical models. These insights can inform our understanding of pre-service teacher's development of modelling competencies and the ways they make inferences during their modeling process. Additionally, understanding learners' competencies/reasoning during their movement from constructing algebraic models to constructing statistical models, as well as the competencies/reasoning used during the modelling process of the model building, can help us determine what should be taught to students in both senior secondary and university settings.

BOUNDARY CROSSING BETWEEN WORKPLACE AND MATHEMATICS IN PROSPECTIVE TEACHERS' DIDACTICAL DESIGNS

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In this paper our goal is to explore prospective teachers' (PTs') didactical designs aiming to promote inquiry-based learning (IBL) and the integration of the world of work (WoW) in mathematics teaching. The study is inspired by the philosophy of the Mascil project (see: www.mascil-project.eu). In particular, the Mascil tasks provided a basis for PTs' engagement in exploring choices and transformations need to be taken in order to integrate the WoW in mathematics teaching and learning. The study took place in the context of two courses that constitute parts of a two-year master's programme in mathematics education. We encouraged PTs to use existing Mascil classroom tasks or develop their own in the same spirit as part of their teacher education activities. We (as teacher educators) initially introduced IBL and WoW in the courses (e.g., by presenting research findings from the corresponding literature) and engaged PTs' in the cycle design-implementation-analysis-reflection which was based on: (a) designing IBL tasks that connect to the WoW; (b) using the tasks in the classroom or with small volunteering groups of secondary school students; (c) analysing students' mathematizations with an emphasis on the role of workplace context and tools and (d) reflecting on their experiences and linking emergent issues with existing research. Our main focus is on the ways that PTs integrated WoW in their designs and implementations as they attempted to engage students in inquiring mathematical ideas and linking school-based and workplace-related knowledge.

Our work is informed by the emerging body of literature on boundary crossing in mathematics education and out of school situations such as work. Boundary crossing processes have been studied mainly in the workplace and in the vocational education indicating the contrast between mathematical practices in workplaces and in educational settings. Little attention, however, has been given to how this contrast can be embedded into PTs' mathematics education as a stimulus for them to engage in designing-implementing-analysing classroom activities aiming to facilitate the students' crossing from one world to the other. For instance, what are the PTs' teaching dilemmas in approaching the authenticity of workplaces in the classroom? What types of learning processes they target for the students (e.g., perspective making and perspective taking) and what is their own conceptualization in terms of boundary crossing? What types of means (e.g., choice of tasks, tools) do they use in their designs to integrate mathematical and workplace knowledge? The purpose of this study is to provide insight into these issues.

HELPING ALL STUDENTS BUILD THEIR DREAM HOMES: USING MODELLING WITH MATHEMATICS IN A LEARNING SUPPORT CLASS

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The Common Core State Standards for teaching mathematics identify mathematics modeling as a vital skill that students need to be taught. While some students in regular classes may have opportunities for modeling with mathematics, too often students who struggle are almost never given opportunities to work on modeling-based mathematics activities. McDuffie, Wohlhuter, & Breyfogle, (2011) argue that mathematics learning tasks need to be designed so that all students, even students with special learning needs, can master appropriate skills. In addition, a 2015 study by Baron showed that using authentic tasks in mathematics courses is necessary in order to teach students the ability to do effective mathematics modeling.

This presentation will describe an authentic mathematics-modeling task designed for a 7th grade pre-algebra class. The class had learning support mathematics students integrated with regular 7th grade mathematics students. The class was co-taught by a learning support teacher and a mathematics teacher. Students in the class were given a mathematics modeling task that drew on two mathematics concepts from the seventh grade mathematics curriculum: area and percent. Students were asked to calculate the cost of decorating (flooring and painting) the inside of their dream house. Using prices from a local building supply store, students had to decide what type of flooring and wall covering they could afford. Their budgets were based on research done on loans they could get from local banks.

The presentation will begin with an overview of authentic modeling task development and the role such tasks play for students who struggle with mathematics, particularly learning support students. The presentation will describe the task design, share student work of various ability levels, and discuss the evaluation method used. The presentation will conclude with a call for further efforts to use authentic modeling tasks to help all students achieve proficiency in mathematics.

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DIFFICULTY OF SOLUTION APPROACHES OF MODELLING TASKS

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Not only in Germany mathematical modelling plays an integral part in educational plans by now. However, several studies show that there is a gap between educational policies and instructional reality. For example Jordan et al. (2006) found out, that the percentage of modelling tasks in school is rather low. Focusing on the teachers' perspective as executives of curricula standards, obstacles become apparent. Schmidt (2010) found out that particularly lack of predictability, inefficient planning possibilities and assessment are reasons mentioned by teachers in this context. These statements can be attributed to the special characteristics of modelling tasks. In particular a large task space in terms of multiple solutions is at first sight hardly comprehensible. In that regard the estimation of the task difficulty and consequently the implementation of modelling tasks become a problem.

The present study focuses on the obstacles modelling tasks may involve. Hence, within the study models have been developed to determine the difficulty of solution approaches of modelling tasks. Structural considerations, as they have been applied in the field of word problems, serve as a basis. The structure of solution approaches, represented by single thought operations, related to a theory of cognitive psychology play an integral part. Simultaneity and nesting of thought operations seem to affect the difficulty of solution approaches. Thus, by analyzing the solution structure conclusions can be made to the cognitive level of the respective solution approach.

Within the study five modelling tasks have been developed to test the validity of the difficulty models. Approximately 1800 student solutions of ninth graders (15 years of age) of German grammar schools have been tested using the developed modelling tasks. The leading question was which of the models developed fits best to the student performance. Within the models emphasis was given to the question whether students perform better with theoretical simple tasks than with hard ones and vice versa. First results indicate that there are significant differences in the performance of the models in dependence of the structure of the solution approaches.

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MODELLING PRACTICES OF ENGINEERS THROUGH DIFFERENTIAL EQUATIONS AND SIMULATION

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[Presentation via Skype]

This paper aims to show the results of an innovative design of a Differential Equations (DE) course through modeling and computer simulation. In this work, we define mathematical modeling as the cyclic process based on the relation between real world and mathematical models (Niss, Blum, & Galbraith, 2007). This course has been implemented in a private university in the northeast of Mexico since 2010 (Rodriguez, 2015). In particular, we are interested in continuing the study of the development of modelling competences through the use of technology in the modelling process (Rodriguez & Quiroz, 2015). This course aims at helping students better understand the application of this mathematical object (DE) in different areas, especially those of a physical and/or social nature. The study presented is analyzed from a qualitative research paradigm. First, the qualitative approach that leads students to identify key aspects of modeling, aspects that are of great interest to future engineers, is considered. The qualitative approach includes interviews with an expert of a specific engineering field, observations in a specialised course such as Dynamics Systems, analysis of textbooks, technologies and languages used and academic discussions with an expert. The second part deals with the discussion and the collaborative design of modeling activities done by an industrial engineer with minor in Systems and a DE professor. These have led to introduce a series of activities that ask the students to model the reality around them or the reality they suppose will eventually become a part of their workplace lives. This should be done taking into account two things: the key ideas of the DE mathematical aspect, and above all, the modeling practices of the 21st century engineer. Finally, in the third part we will show the results of a developed questionnaire. This questionnaire has been administered over a period of one year to engineering students enrolled in DE courses. This seeks to validate the contribution of this approach to teaching DE courses through modeling and simulation from the engineering point of view. The results are evidence of the positive contribution of this course design with regards to the student's perception of how useful these activities are in his understanding of the basic content and above all the use of this knowledge in his future professional life.

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PRACTICAL STUDY ON GRADUATE STUDENTS' DEMONSTRATION OF PAINTINGS USING MATHEMATICS: READING BACKGROUND OF ARTS WITH MATHEMATICS

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The purpose of this study is to investigate how graduate students' practical teaching abilities have been developed through shuttling between paintings and mathematical structures. We have conducted a class on STEAM (STEM with Art) for eight graduate students who would like to become mathematics teachers. In this class, they have discovered mathematical structures in paintings, and have demonstrated them to visitors at the Otsuka Museum of Art. The aims of this class were:

- To improve their abilities to develop teaching materials which integrate mathematics and arts
- To develop teaching materials corresponding to an audience with a diversity of knowledge and skills,and
- To communicate how to appreciate paintings from a mathematical viewpoint to the audience

Four full-scale paintings reproduced on ceramic boards at the Otsuka Museum of Art that the students have demonstrated were '*School of Athens'* , '*Annunciation'* , '*Mona Lisa'* and '*the Last Supper* . The one hour guided tour of these paintings was held at two times. Specialists in the museum, a supervisor of school education, and teachers of graduate school acted as facilitators to support students' goals.

For all demonstrations, the paintings were explained using mathematical structures such as the golden triangle, golden rectangle, root rectangles and so on. The students designed their demonstrations to shuttle between mathematics and other topics such as arts, Christianity history, cultures and so on. Graduate students, specialists in the museum, a supervisor of school education, university teachiers and visitors collaborated to help students improve demonstrations.

After this class, we found the three results which accorded with its aims.

- Collaboration between graduate students and specialists in the museum acted as a catalyst that helped graduate students notice the necessary content for demonstration of paintings.
- Students who had no ideas regarding mathematical modelling realised the necessity of shuttling between mathematics and other topics through revision of demonstration contents.
- Students built their ability to improve their demonstrations by explaining the paintings to visitors at the Otsuka Museum of Art.

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INQUIRYING AND MODELLING IN A REAL ARCHAELOGICAL CONTEXT: THE IMPORTANCE OF THE SEQUENCE TASKS DESIGN

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This paper focuses on the design of a sequence based upon an archaeological context -the ruins of a Roman theatre discovered in Badalona (Catalonia, Spain)- which will be implemented in the next course 2015-16 with 12-14-years-old students. The aim of the design is to promote the inquiry and modelling students' competences and to deal with the problem integrating a multidisciplinary approach to the study of extra-mathematical questions where mathematical modelling appears as a central tool in the teaching and learning processes, in the same way as previous works like, for instance, Sala, Barquero, Font & Giménez (2015). In fact, the design is concerned about integrating, in an interdisciplinary way, the students' work with Mathematics and History.

The sequence of tasks starts with a problematic and real situation, very close to the students' quotidian context. This initial problematic situation, which is introduced to the students, is a relevant archaeological discovery in their city, Badalona: some Roman ruins that could have been a public building. They will work in teams and will have to carry out inquiry —based on real data, archaeological reports, canons of Roman architects, etc.— what kind of building could it be and with what characteristics? The students will be able to visit the place of the discovery and take photos, measures, and so on. The students have at their disposal a guide of inquiry and certain devices that allow them make progress. This guide of inquiry was designed basing on the reports of real investigation carried out by the professional archaeologists' team of the Badalona Museum (Padrós & Moranta, 2001).

Each inquiry team, following the guide, has to write a report and some documents in order to provide the teachers evidence to make the inquiry progress of their students easier to follow and to allow the assessment of the inquiry and modelling scope competences.

The design of the sequence and all the didactic devices incorporated are crucial to properly temporize the tasks in order to promote the inquiry and modelling students' competences development.

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Day: *Tuesday* **Session:** *I (14:35 – 15:20)* **Room:** *A42*
Age: *Primary/Secondary* **Focus:** *Teaching and Learning; & Other* **Type:** *Practical*

TEACHERS' PERCEPTIONS ABOUT MODELING ACTIVITIES PROPOSED IN CONTINUED EDUCATION

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This article presents the results of an applied study that aimed at identifying and comprehending the perceptions Basic Education (BE) teachers have in relation to Modeling. Research about Modeling in Brazilian Education started four decades ago, and as a consequence of time, we encounter different perceptions among researchers. As a consequence, BE teachers display distinct understandings and perceptions as regards modeling. To reach the goal of this study, we organized a 16-hour-Continued Education Course. Sixty-three BE teachers from the west region of Rio Grande do Sul state in Brazil participated. We organized the course in such a manner that we had four distinct moments: (1) we proposed two Modeling activities about the topics 'Möbius strip' and 'Vertex of a parabola' which resulted in six mathematical models; (2) we promoted reflections about the activities developed in the first encounter, followed by discussions about theories of Modeling; (3) we proposed a Modeling activity with the theme "Chocolate"; (4) we proposed the elaboration and socialization of didactic sequence projects under the same theme. The data for this study was obtained along the Course: through discussions, questions, productions and evaluations. We analyzed the data through the lenses of Cognitive Psychology and Neuroscience: perception and memory. Results revealed that experiencing the process of Modeling enabled the participant-teachers the *perception* of (a) the relation between mathematical contents and reality; (b) Modeling as a method that allows the teacher to propose activities without rigid procedures; (c) Modeling as a research method that boosts the motivation and focus required for learning math; and of (d) Modeling as not being a part of their previous training. The *expression* of the Modeling process through nets constructed from the target memories related to the theme highlighted, initially, the obtained model and in sequence, the steps related to obtaining it. The participant-teachers were able to express the large net that leads constantly to the beginning and end of the Modeling process. This process occurs, according to Luria (1991), when we build mental representations of external objects departing from sensory data obtained during the experiencing with Modeling.

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HIDDEN BENEFITS OF MODELLING FOR LEARNERS WITH SPECIAL EDUCATIONAL NEEDS (SEN)

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A study (Scott-Wilson, 2014) was conducted in the Northern Territory (Australia) to gauge the impact of mathematical modelling tasks with learners with SEN. The school was an urban public middle school for students from year 7 to year 9 from middle to low socio-economic environment. Entry requirements into a special needs class as per Northern Territory policy (Department of Education and Child Services, 2012) stipulated that learners had to show significantly below average intellectual functioning (Intelligence Quotient (IQ) of 70 or below on an individually administered IQ test), and concurrent deficits in adaptive functioning (functioning in the bottom 2% in areas such as communication, self-care, social/interpersonal skills, functional academic skills, work, health and safety) with multiple needs, and, that they require intensive support for needs and a highly individualised program to allow access to, and participation in, the curriculum. The daily mathematics lesson of the 8 learners with a range of formally diagnosed conditions was substituted with a set of modelling tasks over a period of one month. A design-based research methodology with a neo-Vygotskian design philosophy was coupled with three intensive case studies were used to monitor the effects of the modelling tasks on the learners. A dynamic assessment approach allowed for immediate modifications to be made to the design or the delivery of the design in response to the nature of the learners' interactions with the tasks. On the whole, the findings of the study were positive showing evidence of engagement and meaningful mathematical learning moments. Moreover, the study suggests several "hidden benefits" of modelling for learners with SEN. Throughout the study, the thinking of the learners was supported using principles from Feuerstein's (2013, 1990) theory of cognitive modification.

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ANALISYS OF CONCEPT OF DIFFERENTIABILITY PRESENTED IN TEXTBOOKS OF DIFFERENTIAL CALCULUS THROUGH THE THEORY OF DIDATIC TRANSPOSITION

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Currently, it is a common perception that the most abstract content can be better understood with a clearer language. In particular, in the area of the Exact Sciences, this perception between teachers and students is expanded enormously.

This work is a reflection about the didactic transposition of the concept of derivatives. For this, we analyze the main differential calculus books used in Brazilian universities, using Y. Chevallard's theory.

It is apparent that in some books, the term derivative is formally introduced without reference to the idea of rate of change. In fact, in support of the exercise of such books, there is a clear connection between derivative and limits. To reinforce this association, the exercises focus exclusively on formal calculation. Indeed, this option by way of exhaustive repetition of formulas can lead to misunderstanding of the real need for assimilation of this concept.

Finally, it should be noted that this research in initial character, aims to foster discussions about the assimilation of mathematical content from the presentation. Thus, we believe that this is a way to reduce the high retention rates seen in Calculus courses.

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AUTHENTIC MATHEMATICAL MODELLING EXPERIENCES OF UPPER SECONDARY SCHOOL STUDENTS: A CASE STUDY

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In this presentation I will present the results of my 2012 research project and my current research proposal.

The purpose of my 2012 research project was to explore authentic mathematical modelling, that is an experience that mimics that of a professional mathematical modelling team, and use that experience to develop and research an authentic mathematical modelling experience for secondary school students. In 2011 I had the opportunity to work as a member of a professional modelling team. From this experience classroom activities were developed and trialed with a group of New Zealand year 12 average ability students at a decile 10 school. The focus of the unit was the process of mathematical modelling. Data was collected on the classroom learning activities and what parts of the mathematical modelling process was remembered. The three data sources were student diaries, classroom assessment and student interviews. The results showed that an authentic modelling process is achievable within the restricted classroom environment. With the prompts provided in the classroom activity students coped well identifying the essential aspects of the situation being modelled, and there was good recall by all students for strategies to identify the essential aspects of the situation. Students did not do as well forming a model once they had identified the essential aspects. Further work is recommended in developing strategies to help students with the model formation stage. To allow for a full experience of the process of mathematical modelling more time is needed than was given to this activity.

I will present my findings from this project and their potential impact on a first year modelling course. I will discuss what aspects of the modelling experience were possible at secondary school and therefore should be easily achievable at undergraduate level; what things students enjoyed about the experience and therefore useful to include; and what things they struggled with, providing a sign that these things need good instruction and relevant time.

I will also outline, for discussion, my current research proposal on investigating best teaching practices for formulating models with a particular emphasis on tertiary teaching approaches and their effects on students' abilities to formulate models.

ENABLING ANTICIPATION IN MATHEMATISING REAL WORLD PROBLEMS IN A FLIPPED CLASSROOM

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The provision of vicarious experiences of real world situations in upper secondary school to enrich students' understanding of the world is often considered a luxury a teacher in a time poor curriculum must forego in order to cover the core elements of the curriculum (Stillman, 2007). According to the NCTM (1991, p. 89) professional standard 4, on the contrary there should be "evidence that the teacher emphasizes connections between mathematics and other disciplines and connections to daily life". In conjunction with this there is also a second purpose that can be argued for the teacher—to enable opportunities for students to develop ways of thinking commensurate with modelling situations in their social and physical worlds.

A recent innovation in upper secondary classrooms is the flipped classroom where there is a swapping of classroom and homework activities and in some cases an expansion of the curriculum through the use of electronic technologies (Bishop & Vergler, 2013). In the example which will be examined in this presentation, the teachers have used video technology to present skill work and use of mathematical techniques sometimes embedded in real world scenarios for students to view before class as assigned homework. In-class time is spent on teachers providing support as students work on problem solving activities.

The question arises: Could a flipped classroom provide the ideal situation for exacerbating the perceived restrictions of modelling and applications in the upper school through judicious use of video-clips to provide both vicarious experiences (e.g., construction of ramps for wheelchairs on a lecture hall to meet building code standards) and the intentional fostering of the critical thinking skills associated with modelling? As an example of the latter, the teacher might plan to include in the video-clip deliberate modelling of actions to assist students in mathematizing a real world problem situation such as explicitly modelling anticipating the effects of confounding variables in the situation (e.g., length of the required ramp covering the entrance to a car park) in finding a "sense of direction" (Treilibs, 1979) in mathematizing to model the situation adequately. This would require that the teacher anticipate potential pitfalls in the situation that would need to be considered and from these select those that have the most dramatic effect to capitalise on the use of the visual medium to enhance critical thinking. The only research to date on the use of flipped classrooms in secondary mathematics (Saunders, 2014) has not shown significant gain in critical thinking skills but then thinking skills associated with modelling and application were not investigated. The presentation will present the preliminary results of a new project in a local flipped classroom.

EFFECTIVENESS OF MATHEMATICAL MODELLING IN SECONDARY SCHOOL AND FUTURE RESEARCH OPPORTUNITIES

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Mathematical Modelling is gaining increased interest in countries' mathematics education standards documents (i.e. Ärlebäck, 2009; Common Core State Standards Initiative, 2010). In Sweden, mathematical modelling is one of seven mathematical abilities to develop in students. Germany includes mathematical modelling as one of six compulsory competencies (Blum & Ferri, 2009). In the United States, mathematical modelling is one of eight Standards for Mathematical Practice. Mathematical modelling activities can develop in students needed mathematics content knowledge and life skills for them to be successful in the 21st century (Stohlmann, 2013). However, there is no one agreed conceptualization of mathematical modelling (Frejd, 2013) and there are many versions of the mathematical modelling cycle (i.e. Adan, Perrenet, & Sterk, 2004; Blum & Leib, 2006; Kaiser, 1995; Lesh & Doerr, 2003a). Different interpretations of mathematical modelling could lead to decreased effectiveness. In general, learning mathematical modelling is a difficult task for students in secondary education (Galbraith & Stillman, 2006). In order to guide the research and teaching of mathematical modelling, essential elements of mathematical modelling should be detailed and an analysis of the current literature on middle and high school mathematical modelling is needed.

Kaiser (2011) noted key questions for the topic of applications and modelling: "What are research deficits? Can we identify them? What kind of empirical research is necessary?" (p. 926). This paper presentation will detail the results of an extensive literature review that describes the mathematical modelling interventions, assessments, units of analysis, and the effectiveness of mathematical modelling activites at the middle and high school level (ages 11-18). This analysis is discussed in the context of what future research is needed in mathematical modelling.

INTERNATIONAL MATHEMATICAL MODELING CHALLENGE

Henk van der Kooij

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The purpose of the IMM Challenge is to promote the teaching of mathematical modeling and applications at all educational levels for all students. It is based on the firm belief that students and teachers need to experience the power of mathematics to help better understand, analyze and solve real world problems outside of mathematics itself – and to do so in realistic contexts. The Challenge is being launched in the spirit of promoting educational change. One important way to influence secondary school culture, and teaching and learning practices, is to institute a high-level, prestigious new secondary school contest – one that will have both national and international recognition. We have therefore founded the International Mathematical Modeling Challenge (IM^2C).[1] Among the members of the organizing committee are representatives from countries that already have national modeling contests.

This will be a true team competition, held over a number of days with students able to use any inanimate resources. A major emphasis is for students to experience working with mathematics in a way that mirrors the way the world works with mathematics. When fully operational the Challenge will consist of two rounds of competition. Once the national teams have been chosen, in the first round they will work on a common problem and submit their solutions to a judging panel. Then there will be a second round hosted each year by a different country, in which the national teams present their solutions in person and engage in additional modeling experiences together with teams from other countries.

In this first pilot year, eleven countries participate only in the first national round. An international judging panel will qualify the papers of each team as Outstanding, Meritorious, and Honorable Mention. The outstanding papers will be published.

In this workshop we like to present the background and organization of the Challenge, the 2015 task and give you some taste of it and discuss papers of some teams.

[1] Information about IMMC, the organizing committee and rules for participation can be found at www.immchallenge.org

DOES STUDENTS' OVER-RELIANCE ON LINEARITY ALSO OCCUR IN ECONOMICS?

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In 2005, during a trade mission in China, Prince Filip, currently the Belgian King, expressed his optimism about the Chinese economy that, at that time, grew at a rate of 14% per year. He stated: "*It means that people's income will double in the next 7 years…*" (Huylebrouck, 2005). Likely, the Prince thought that he just had to multiply 14 by 7, which gives 98 percent. A growth of 98 percent is about one hundred, thus a doubling. It is an example of linear reasoning in a situation of "compound growth", a context in which exponential reasoning is more appropriate. In the mathematics education literature, students' overreliance on the linear model is well-known and has been investigated empirically in a variety of mathematical subdomains, at distinct educational levels and in countries having different math educational traditions (see, e.g., Van Dooren & Greer, 2010). As the introductory example illustrates, overreliance on linearity can also be a potential source of misperception of economic phenomena, but within economics, empirical research on this phenomenon is rare.

In this paper, we first provide an overview of cases from the economics and economic education literature in which linear relations are inadequately assumed in order to grasp economic situations. These cases, typically mentioned at the margin of studies of which the primary focus is on something different, were found by means of a thorough literature search. We did not restrict ourselves to the overreliance on a "*k* times *A*, thus *k* times *B*" form of reasoning (or the overuse of a $y = ax$ model), but also included situations that were inadequately modeled by means of linear functions in the more general sense ($y = ax + b$ models). Cases include, among others, supply and demand behavior, determination of prices in an inflationary economic context, as well as the concept of returns to scale.

Second, we report on the design and main results of an empirical study having students' overreliance on linearity as its research focus. In this study, business economics students were confronted with a variety of correct and incorrect linear statements about micro- and macroeconomic situations. The results showed that even these tertiary level students over-rely on linearity when analyzing such statements. It was also found that this phenomenon is affected by the economic subdomain and by the way the statements are formulated.

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SOME RELATIONS BETWEEN THE MATHEMATICAL MODELLING AND PAULO FREIRE'S CONCEPTS

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Teaching and learning are moments of the understanding process, which involves search actions, trial and error, misunderstanding, curiosity, satisfaction, pleasure (Freire, 2001). Teaching should move towards to authentic and critical thinking, in which abhors the imposition of formulas. According to Freire (1979), understanding is the result of process that requires effort, recreation and research, which are the characteristics of a democratic educational practice that aims to develop a critical awareness.

Modelling in Mathematical Education allows the development of this critical awareness. This statement is based in the assumed perspective that "Mathematical Modelling is in a set of procedures whose goal is to build a parallel to try to explain, mathematically, the phenomena present in the daily life of human, helping you to make predictions and decisions" (Burak; 1992, p. 62). Burak (2010) states that an activity of mathematical modelling has as premises: the interest of the students group, because Psychology explains that many of our actions are motivated by interest and this interest generates involvement and sustains procedures performed during the activity; the other is that the data should be collected where the interest of the group is, and it can be understood as an ethnography research and thus the teaching and the learning fit into a broader context.

In order to promote the mathematical modelling activities in classroom, Burak (2010) suggests five steps, which do not necessarily occur linearly: choice of theme; exploratory research; issue(s) identification; issue(s) resolution and study of mathematical subjects in the theme context; the critical review of the issue's solution.

To summarise, these steps to promote the mathematical modelling activities in classroom can help the students develop skills as dialoguing, thinking, arguing, reflecting, conjecturing, creating strategies, among others. The development of these skills are related to critical awareness (Freire, 1979).

In this paper, we present a theoretical study showing some relations between the actions that students do during the development of a mathematics modelling activity and the characteristics of critical awareness from Freire (1979).

CONCEPTUALISING AND MEASURING METACOGNITIVE MODELLING COMPETENCIES

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In the research and discussion about modelling competency, metacognitive competencies are considered to be an essential component of modelling competency (cf. Maaß, 2006). The importance of this sub-component of modelling competency is indicated in several studies. These studies point out that lacking meta-knowledge about the modelling process can cause substantial problems when working on a modelling task (for an overview see Stillman, 2011).

Different studies point to the relationship between mathematical performance and metacognitive competencies, however it is became clear as well that measuring metacognitive modelling competencies is challenging. Often, the use of metacognitive strategies by students cannot be observed or reconstructed, because the students do not verbalize their thinking. However, the use of self-reports contains problems as well. For example Schukajlow & Leiss (2011) in their study did neither find any correlations between self-reported general strategies and performance in modelling tasks nor correlations between self-reported task-based strategies and performance in modelling tasks

The presentation intends to report first steps of conceptualizing and measuring metacognitive modelling competencies. A detailed description of the construct metacognitive modelling competencies will be presented, followed by the description of an instrument to evaluate metacognitive modelling competencies. The instrument is based on a questionnaire, which evaluates the self-reported tackling of modelling problems. The instrument was developed in a design-based process consisting of several sub-studies. The different aims of the sub-studies, their relationship as well as the first results will be presented.

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INTERPRETING A MATHEMATICAL MODEL; COMPLEMENTARITY OF CONTEXT AND MATHEMATICS

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In many countries we see that mathematical models are used in mathematics education. For example in calculus in Dutch secondary schools, algebraic formulas are set in real-world contexts and the variables have a physical meaning. It remains a question, how students combine real-world contexts and mathematics. For example Busse (2011) found four ideal types of how students deal with contexts: reality bound, mathematics bound, ambivalent or integrating.

In a study on students' development of their understanding of the concept of derivative in real-world contexts, we studied an atomistic, yet essential aspect of the modelling process: interpreting a given mathematical model. For this, we used a task from Kaiser-Messmer (1986) in the context of cars, their petrol consumption and the distance driven. Central is a function $V(a)$, which is the volume of petrol (in liters) that depends on the travelled distance a (in km). Without an explicit function being given, the task is to interpret the formula:

$$\frac{V(a+h)-V(a)}{h}$$ (h is a value, which you can choose)

To capture students' development over time, we used a longitudinal design with task-based interviews. Ten Dutch students from the pre-university science & technology stream were studied while they moved from grade 11 to 12. We interviewed them three times, with six months intervals between interviews. We analysed students' interpretations of the above petrol formula, and how these changed over time.

We found that some students had a reality bound preference and others a mathematics bound preference. In subsequent interviews, students tended to keep their preference. Also, their interpretations became more accurate. In the final interview, half of the students were able to integrate the real-world context with mathematics. As this was independent of their initial preference, we conclude that both a reality bound and a mathematics bound preference can be basis for an integrated interpretation preference.

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DEVELOPING BOUNDARY OBJECTS TO SUPPORT TEACHERS' TEACHING AND LEARNING OF MODELLING

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In their overview of research that considers boundaries, boundary objects and boundary crossing Akkerman & Bakker (2011, p. 133) consider a boundary "as a sociocultural difference leading to a discontinuity in action or interaction" and as such a site of learning. In doing so the implication is that learning might be considered to occur when an individual or group renegotiates their relationship with an activity with which they are involved (Beach, 1999). In the study we report here we draw on such notions when considering teachers' teaching and learning about mathematical modelling.

Our research is set in the context of considering how to support teachers to introduce lessons which develop students' mathematical competencies and skills in problem solving and modelling. In doing so we have worked with teachers using a model of lesson study adapted to the English context from the Japanese model. Thus, teachers' learning might be considered as situated at the boundary between classroom and lesson-study group. This involves teachers engaging in boundary crossing with a number of boundary objects designed to facilitate this. Our concern, therefore, is to support teachers with learning at such boundaries and to help them improve their day-to-day classroom practice in lessons for problem solving whilst also developing sustainable lesson study communities that facilitate professional development.

Here we report our theorisation of how such boundary practices and learning can be effectively supported by artefacts that are specially designed for the purpose. Further, we consider such boundary objects as artefacts that are made instrumental (Trouche, 2014) by teachers for use both in their classroom to support teaching and in the lesson-study group to support their professional learning. This theoretical understanding is illustrated by extracts from a case study of our on-going research.

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WHAT ARE THE LEARNING OPPORTUNITIES OF MODELLING PROBLEMS?

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I find it interesting to understand what we actually learn as we become skilled modellers and problem solvers. I have therefore attempted to explicitly identify and categorize a number of aspects of mathematical modelling and problem solving that can be learned from solving modelling problems.

Taking a broader view and extending work that we have previously published, the investigation is based on several sources. I have analyzed what I think there is to learn in about 30 problems that I use in a university course for engineering students in modelling and problem solving, including the discussion around these problems. Other sources are my own notes from supervision in the course, written student submissions and interviews.

I have attempted to distinguish between basic knowledge and insights, where I see insights as knowledge that is usually gained through experience or observation of a recurring pattern. An example of basic modelling knowledge is that models are simplifications and approximations of reality. A modelling insight is that to benefit from a mathematical analysis you need to actively interpret aspects of a real problem so that the mathemtical problem becomes non-trivial and of some interest.

I have also tried to separate specific aspects of modelling from more general aspects of problem solving. An example of basic knowledge about problem solving is to always try something simple first. An example of a problem solving insight is the importance of really understanding the problem, and how a successively deepened understanding of a problem is a path to solving it. We here also relate to other theories of problem solving.

Of course, not all learning opportunities can be described independently of a particular problem. An applied problem contributes to a familiarity with problems that can be solved with mathematics, and how this is done. Additionally, the actual sequential experience of solving a problem is a part of the learning experience, and together this becomes a case that can be recalled at a later time.

For teaching, this analysis may be useful to increase the awareness of what one may wish to teach, to see what learning opportunities are contained in a problem, and to select a set of problems that will offer the intended learning opportunities. For learning, it may be helpful in identifying student difficulties, and what students learn from different kinds of problems.

AN INVESTIGATION OF MATHEMATICAL MODELLING AND CREATIVITY OF FUTURE TEACHERS IN A GERMANY UNIVERSITY

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This paper reports on an investigation of future teachers' solutions to a mathematical modelling problem to determine the levels of creativity evident in these models. Their views on what creativity is and how it can be developed were also explored.

Creativity plays a significant role in personal and economic success in the 21st century and can be fostered at school level through the solving of mathematical modelling problems (MMPs) (Chamberlin & Moon, 2005). Future teachers should be enabled to develop creativity through mathematical modelling, but in order to develop a better understanding of this process, they need to solve MMPs themselves (Wessels, 2014).

Twenty-six future teachers from a German university preparing for teaching mathematics in primary and/or secondary school phases solved a mathematical modelling problem in groups of 4 to 7. The models that were developed as well as the processes groups followed to solve the problem were analysed. Levels of mathematical creativity evident in these models were identified using four characteristics of creativity namely fluency, flexibility, originality and usefulness (Torrance 1974). Before solving the modelling problem, the participants also completed a questionnaire exploring their views on what creativity is, how it can be identified in students and how it might be developed in the classroom.

Preliminary results show that although these future teachers' have not had specific instruction on mathematical creativity, some of their descriptions of what it entails and how it could be developed directly or indirectly referred to the four characteristics of mathematical creativity that can be found in the literature. Data analysis is still under way.

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MISSION-DRIVEN PROGRESSIVE PRACTICE TEACHING PATTERN ON MATHEMATICAL MODELING COURSE

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One kind of progressive practice teaching pattern on Mathematical modeling course is designed in this paper. The key of this practice teaching pattern is integrating mission-driven practice into course to improve the comprehension and stimulate the creativity of students step by step. The progressive practice teaching is classified four aspects, Mission Creation, Subject Activity, Open Learning, Feedback and Evaluation. The four aspects correspond to attract the students by designing mission from life via discussion between teacher and students, integrity of basis contents and topic training in subject activity, synthesis of flexible application with its motivated self-learning, feedback and improvement. The goal is to enhance the knowledge cognitive ability and strengthen the innovation ability of students.

Mission creation is the basis of progressive mission-driven practice training. The created mission should appeal to students by selecting the interesting subject from the life via discussion among teachers and students. Personalized design would benefit to the students development. Different missions are designed to improve the students ability to imagine, learn and practice the mathematical modeling knowledge. Subject activity is the key of implementation of mission. It integrates the modelling contents into a series of subject mission, by finding the problems, trying to collect data, analyze the data, find the strategy to solve the problems. Open learning is a good way to investigate the study and research in the mission by combining students' self-learning, cooperation and teacher's guidance. Feedback and evaluation creates a spiral improvement loop of mission-driven learning pattern.

Finally, two examples, check-out time distribution analysis in supermarket and goods place style in storehouse, are demonstrated here to show the efficiency of mission-driven learning mode.

Day:	*Tuesday*	**Session:**	*G (11:50 – 12:35)*	**Room:**	*A45*
Age:	*Other*	**Focus:**	*Learning*	**Type:**	*Research; & Practical*

STUDENTS' DIFFICULTIES IN MATHEMATICAL MODELLING: OBSERVATIONS FROM TWO EXAMPLES

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It is well documented in previous research that students all around the world have problems with mathematical modelling tasks. In this presentation, we report some observations of students' difficulties revealed from two modelling examples, and their possible implications for quality teaching of mathematical modelling.

The observations are based on two modelling tasks which were used as a part of the examination requirements in a mathematical modelling course in Tsinghua University. The first task asks the students to answer how much water should be filled into a beer glass in order to make it most stable. It was used in the course of the year 2013, and 77 students submitted their answers to this modelling task. The other one asks the students to answer how far away the bricks can extend in the horizontal direction by overlapping them together. It was used in the course of the year 2014, and 161 students submitted their answers. The students' answers to both modelling tasks reveal that, although almost all of them can understand the modelling tasks correctly, many of them face difficulties in certain steps of the modelling cycle. To a certain extent, all of them have some problems in making assumptions to simplify the situation of a task. A very popular case is that, certain critical assumptions were actually made and used in a student's model but they were never mentioned explicitly anywhere. Another difficulty faced by some students is that they have problems in identifying what basic physical law should be used, and how to translate it into a mathematical formula. Besides, a prominent shortcoming in some students' answers is that their models are developed for a very specific and special case, but not for more general cases. It seems that they have either oversimplified the modelling tasks or not been qualified to extend a concrete model to a general one. These observations may provide some reflections for the teachers to improve the teaching quality in mathematical modelling classrooms.

MODELLING CHALLENGE PROGRAM AT SECONDARY LEVEL IN JAPAN

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Challenge programs on mathematical modelling and application are carried out in the world. For example, the mathematical modelling challenge program in Hamburg University, HiMCM in U.S.A., A-lympiad in Netherland, A B Paterson College Mathematical Modelling Challenge, and so forth.

The contents of mathematical modelling are generally not taught in school mathematics in Japan. Japanese students don't know the word "mathematical modelling" and its concept. Of course there were no mathematical modelling challenge programs in Japan. However, the new Japanese course of study attaches great importance to making use of practical mathematics and mathematical activities.

We think that mathematical modelling should be taught in regular mathematics classes in Japanese secondary schools, and that some challenge programs on mathematical modelling should be prepared.

Therefore, we made pilot plans to carry out modelling challenge programs for secondary school students in Kyoto every year since 2012. The last year program for secondary school students was carried out in February 2015. The detailed contents of these programs and the response of the students participating in the programs will be discussed here.

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MATHEMATICAL MODELLING OF A SOCIAL PROBLEM IN JAPAN: THE TEMPERATURE RISE AND THE DECLINING BIRTH RATE

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Two social problems were taken up as a teaching material for mathematical modelling. One is the temperature rise in Osaka caused by global warming. On August 12, 2013, the city of Shimanto in Kochi Prefecture recorded a temperature of 41.0°C, a new high for Japan. As the country heats up, temperatures are rising most rapidly in major cities; the capital Tokyo has seen a three-degree increase over the past century. The other one is the declining birth rate in Japan. Japan, like many other countries, has a declining birth rate that is likely to result in a host of socioeconomic problems. The Japanese government hopes to create a social environment that will raise the country's total fertility rate.

In the former the students made a personal choice to the linear regression model using a graphing calculator, while in the latter they did in a group. This teaching experiment focused on the students' processing the data by graphing calculators in mathematical modelling when posing their own linear regression model.

As a result, it was confirmed that, in general, the students are not able to repeat the mathematical modelling cycle in real-life problems and that the students' modelling skills in handling numerical adjustments were acquired through their discussion about their own answers. In addition, it demonstrated that it is possible to use mathematical modelling teaching materials which deal with the temperature rise and the declining birth rate for instructing junior high school students in Year 9.

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MATHEMATICAL MODELLING IN DUTCH TEXTBOOKS IS IT GENUINE MATHEMATICAL MODELLING?

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In the curriculum of Dutch upper secondary mathematics education, mathematical modelling has an explicit but not substantial role. In this study we explore and describe how modelling is implemented in the two most frequently used Dutch textbooks. After an exploration of the relevant literature we focus on the occurrence of modelling purposes which we borrow from a course on mathematical modelling in tertiary education by the last two authors: explanation, prediction 1 (when?) and 2 (what?), compression, inspiration, communication, unification, abstraction, analysis, verification, exploration, decision, optimization, specification, realization, steering & control. To answer the question how far students have to perform genuine modelling activities: conceptualizing, mathematizing, solving, interpreting, verifying, validating, iterating, communicating, and reflecting (on the modelling process), we used a study of the first two authors.

In both textbooks we analyzed about 500 tasks. The occurring purposes in the two textbooks are explanation, prediction 1 and 2, analysis, optimization and decision. Analysis and explanation have the highest occurrence, each in about 15% of the tasks. Optimization and decision occur each about 10%, Prediction 1 and 2, each about 3%. The modelling activities are mostly learning mathematical concepts and techniques and applying these on problems in non-mathematical situations while almost always the mathematical model is given or very obvious. It turns out that in more than 95% of the tasks solving is the only occurring modelling activity. In about 30% of the tasks the students have also to conceptualize and mathematize the given problem situation. Interpreting, verifying, validating, iterating, communicating and reflecting, each have a score of about 1%.

Our conclusion is that, although mathematical modelling is explicitly mentioned in the formal curriculum, in Dutch upper secondary mathematics education not mathematical modelling does not play a role, but applying, while, according to the formal curriculum students should analyze the problem situation, translate it into a mathematical model, use solving techniques within this model, give meaning to the resulting solutions in terms of the problem situation, and reflect on the modelling process. Consequently, the textbooks don't meet the requirements of the curriculum.

We see opportunities for including tasks into the textbooks where, instead of applying mathematics to non-mathematical problem situations, more genuine modelling is possible, with maybe not more modelling purposes, but with a higher occurrence of the genuine modelling activities.

4. Posters

EXPLORING TWO-VARIABLE FUNCTIONS REPRESENTATION THROUGH MODELLING OF SOUND INTENSITY WITH IPADS®

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Modelling tasks at secondary education have an interesting teaching goal. As stated by Blum and Niss (1991), the real situation has to be simplified by the problem solver. The real model has to be mathematized, that implies its data, concepts, relations, conditions and assumptions have to be translated into mathematics.

In this poster we describe a modelling experience carried out with tenth grade students. We analyze the solving strategies and the discussion generated by the students in the process of solving the modelling task. The aim of the work is to explore the mathematical concepts and procedures related to two-variable functions generated by the students. In particular we are interested in the Cartesian three dimensional representation of this kind of functions. It is important to remark that no mathematical concepts on this topic have been introduced to the pupils yet.

For this purpose, during four sessions, the students act as researchers, organized by groups. They have to answer a specific research question: *How does intensity of sound distribute throughout our classroom?* Concepts related to sound and instrumental tools are explained by the teachers along the sessions. Pupils use iPad® as sound meter in the research process. Using Decibel Ultra Pro app, the students measure real data from a sound source and try to obtain a sound map of the classroom in order to answer the research question with the teacher's guidance. Teachers provide also pupils with a physical material, Multilink® cubes, to give them the opportunity to ideate a three dimensional representation.

From the qualitative analysis of the student performances, we observe that pupils are able to elaborate an idiosyncratic representation system for this phenomenon. In this representation we find some aspects that are precursors of the Cartesian representation, especially something similar to the historical notion of *applicata* used, for instance, by Euler. This can be a first step for the construction by pupils of two variables functions. Thus, modelling situations, like this, can make fundamental contributions to the learning process in students.

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MODELLING PROJECT IN TRAINING TEACHERS OF MATHEMATICS

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The curricular bases that drive Chilean educational regulations defined modelling as an ability to develope, without clarifying what is meant by modelling. Moreover, initial training programs whose teaching quality control accreditation processes are based on the autoregulation for a proposed program training is given, not necessarily obeying agreed public policies. Recently there has been Test Inicia (Inicia, 2012) (Based on the guiding Standards for teaching careers (MINEDUC, 2014)) as a way to guide how teacher training should address as the graduates will be assessed to qualify teacher assignment with public funds.

In mathematics, there are three Standards that clearly mention the modelling. One of them refers to the concept of function, noting that "Use functions in modeling situations coming from different areas". Another standard asks "analyze mathematical models from different fields"; meaning no model building and away from the definition of Blum & Leiss (2007).

Analyzed two training programs, we realized that none of them have in an integral way for the teaching of mathematical modeling in the sense of Blum et al. (2007). The modeling is seen as the use of mathematics in which the model is given and the task is solved within mathematics. Interviews with teachers of two programs of initial teacher training in mathematics show that mathematics is used only in contexts in calculus and differential equations courses.

The poster is intended to inform the results of a pilot project, which studies the induction process to mathematical modelling in initial teacher training in mathematics. We focus in two different teacher training programs. The methodology used both for students address a modelling process, so that they achieve to create (as teachers) modelling situations based on research area and control instruments competence development will be presented.

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MAKING MODELLING TASKS USABLE IN THE CLASSROOM

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Modelling tasks in Mathematics lessons are rarely implemented in Japanese classrooms despite the growing recognition of their importance. This may be attributed to the complex nature of the modelling process, more so if it includes several possible models. Therefore, to conduct a lesson with a modelling task, it is necessary to comprehend conceivable modelling process for that task. This makes it possible to focus on the specific process of the specific model for the entire class and also individually help students who may get stuck at some point.

There are some attempts to describe the modelling process. One of them is the flowchart created by Burkhardt (1981). Notably, this flowchart suggests not only the general modelling process but also the problem states corresponding to the process, which consists of "practical situation", "mathematical model", "mathematical answers", "practical understanding" and "model's value and limitation". In this flowchart, concrete examples were given to show how the process might proceed. This was done by tabulating the problem states of each modelling process.

The modelling process is usually invisible. Hence, describing it concretely based on a task, as Burkhardt did with the table, is meaningful and useful. On the other hand, this gives little information on how each model can be made. This research focuses on how to describe the modelling process based on a task so that implementing modelling tasks in the classroom would be easier.

To improve the modelling process, some stages in making models are added and characterized. First, modellers should identify and write a more specific task to tackle from the given task in each modelling cycle. Then, they need to define the variables to focus on. Finally, they have to reflect on what is considered in the real world as they transition from one model to another.

Based on Burkhardt's flowchart and these additional features, I have described the modelling process of one task as an example. The task considered in this research is finding the range from where Mt. Fuji can be viewed. Several possible models were created for this task. Furthermore, having this variety made it worthwhile to explore connections between these models.

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ADDITIVE REASONING IN A MODELLING TASK: AN EXPERIENCE IN A BRAZILIAN ELEMENTARY SCHOOL

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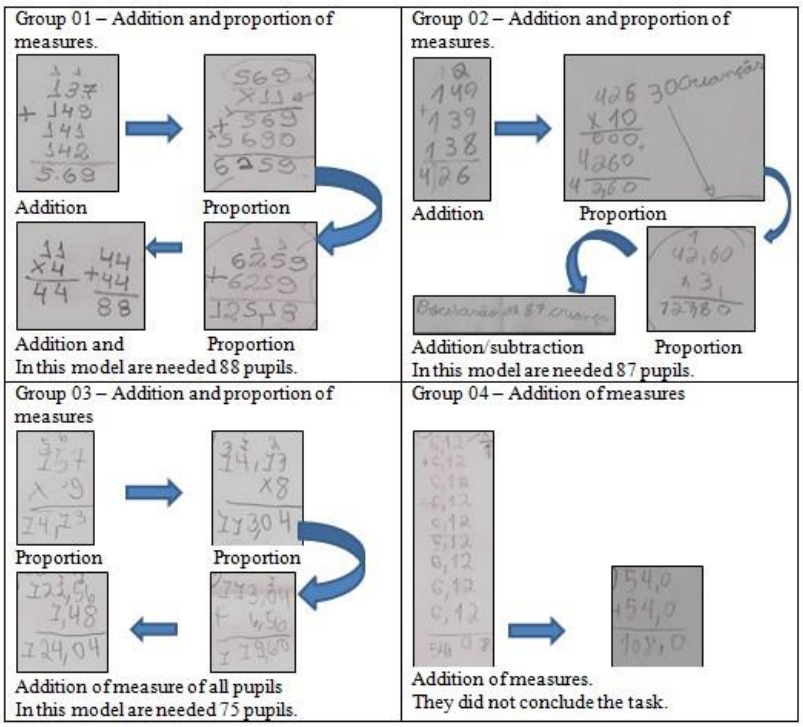
This paper presents the results obtained during a mathematical modelling activity with fifth-year students (10-year-olds) in a Brazilian elementary school. The participants (fifteen students) worked in groups. The activity "Cathedral height" was adapted from Blum and Borromeo Ferri (2009), entitled originally "Die Michaelis- Kirche". The issue involved in this modelling task was to estimate, from the height of the Cathedral (124 meters), the number of children necessary to achieve this height. For this, the groups received a form containing this information and a tape measure. We have aimed at describing the actions of the students during this activity and at identifying their strategies.

The picture besides illustrates the solution strategies of the four groups. In each case, additive reasoning was dominant. The Groups 01, 02 and 03 solved this task also with the help of multiplicative reasoning. The possibility of using division involving the height of the cathedral and the height of a student was not present in the solutions. However, the activity gave the students an environment of discussion, of exchanging information and of collectively working with the mathematical tools available to them. Among the groups that have completed the activity, also validation activities could be observed, by a comparison between how tall the students were with the height of the cathedral. Also highlighted was the importance of different models and hence different results obtained by the groups, depending on the data used by each group, favouring communication and mathematical reasoning among the students.



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5. Modelling and Applications in Nottingham and the UK – Thursday 23 July 2015

On Thursday 23rd July the organizing committee is pleased to welcome to the conference a number of teachers and educators who are involved in research and curriculum development that attempts to improve mathematics education across the UK. In a series of presentations/workshops throughout the day you will have opportunities to learn more of their work. Some of this is carried out in collaboration with researchers of the Centre for Research in Mathematics Education of the University of Nottingham, your hosts at ICTMA-17. Other work is carried out by a number of organisations that will provide insight into their efforts on the ground that carries forward the ideals of ICTMA.
We encourage you to attend some of their sessions to learn more.

O 11:00-11:45	*PRESENTATION BY UK RESEARCHER AND TEACHER: Developing Students' Capacity to Critically Assess Mathematical Ideas*	Sheila Evans; Nick Wilson	A44
P 11:50 – 12:35	*PRESENTATION BY UK ORGANISATION: Cambridge Mathematics Education Project*	Lynne McClure; Nathan Barker	A44
Q 13:45 – 14:30	*PRESENTATION BY UK ORGANISATION: Core Maths Support Programme – 'Problem Solving in Core Mathematics'*	Mick Blaylock	A42
R 14:35 – 15:20	*PRESENTATION BY UK TEACHER: Lessons for Mathematical Modelling From Lesson Study*	Jan Parry; Geoff Wake	A44
S	*PRESENTATION AND DISCUSSION BY UK TEACHER AND RESEARCHER: Teaching problem solving for concept development or problem solving – lessons from lesson study*	Dominic Hudson; Geoff Wake	A42