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Mathematical Modelling

Themes

- My background
- Why did we start the ICTMA conferences?
- Significant changes in the past 32 years
- Mathematics Modelling/Models
- The value of Mathematics for economic growth
- How can we prepare our students for a world we cannot imagine?
- The future of mathematical modelling in the teaching and learning of mathematics

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My Background in Mathematics Education

- Mathematics Degree from Sheffield University and appointed to teaching staff as I was keen to teach and Applied Maths Dept thought I might have research potential
- Loved teaching but not too keen on research but completed PhD
- Moved to Newcastle University and began to teach more in schools that at the University
- Moved to Cranfield University: employed to solve industrial and business problems and teach MSc in Applied Maths
- Developed teaching resources for School based on applications of mathematics (through the Spode Group)
- Appointed Professor of Education at Exeter University and inaugurated these International Conferences in 1983 and 1985
- Founded the Centre for Innovation in Mathematics Teaching, aimed at helping and supporting teachers at all levels to enhance their teaching of mathematics based on international evidence
- Moved to Plymouth University and working with Primary and Secondary teachers throughout England and also on the new CORE MATHS development (focused on mathematical modelling)

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Why did we start the ICTMA conferences?

- Promoting the role of mathematical modelling in the teaching of mathematics
- Providing a forum for mathematics teachers and lecturers in school and tertiary education to meet together to discuss the common issues in terms of promoting mathematical modelling
- Moving the agenda onto the international platform so that we could collaborate and learn from each other

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Significant changes in the past 30 years

- Mathematical modelling now has international recognition for its role in mathematics teaching and learning
- Computing power has increased phenomenally
- Statistical education has also increased rapidly (including statistical modelling)
- Modern applications of mathematics have increased in all disciplines including the development of efficient solutions for technological advances.

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Using and Applying Mathematical Models

- Using mathematical (or statistical) models can be thought to be less challenging than that of modelling, but someone has designed the model
- Great success in the application of Newtonian mechanics (GPS technology, New Horizons visit to Pluto, etc)
- Many other successes in the physical, biological and medical sciences (Genetic fingerprinting and DNA) but more limited success in social sciences
- Game Theory, Catastrophe Theory and Chaos Theory have limited applications in the real world

Mathematical Model: Heptathlon Scoring

Points are allocated to each of the 7 events using formulae for track events of the type:

$$P = a\,(b - T)^c$$

Here *a*, *b* and *c* are constants and T is the time taken in seconds.

For example, for the 800 m race,

$$P = 0.11193\,(254.00 - T)^{1.88}$$

- ***Is this an effective model for awarding points in the Heptathlon?***

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Mathematical Model: Credit Card Security

The last digit on a credit card is a check digit; consider the number:

7992 7398 4528 673*x*

The procedure for finding this digit is shown below:

7	9	9	2	7	3	9	8	4	5	2	8	6	7	3	*x*
14	9	18	2	14	3	18	8	8	5	4	8	12	7	6	
5	9	9	2	5	3	9	8	8	5	4	8	3	7	6	91

$91 + x$ must be divisible by 10 i.e. $x = 9$

Can this model detect single errors?

What about double errors?

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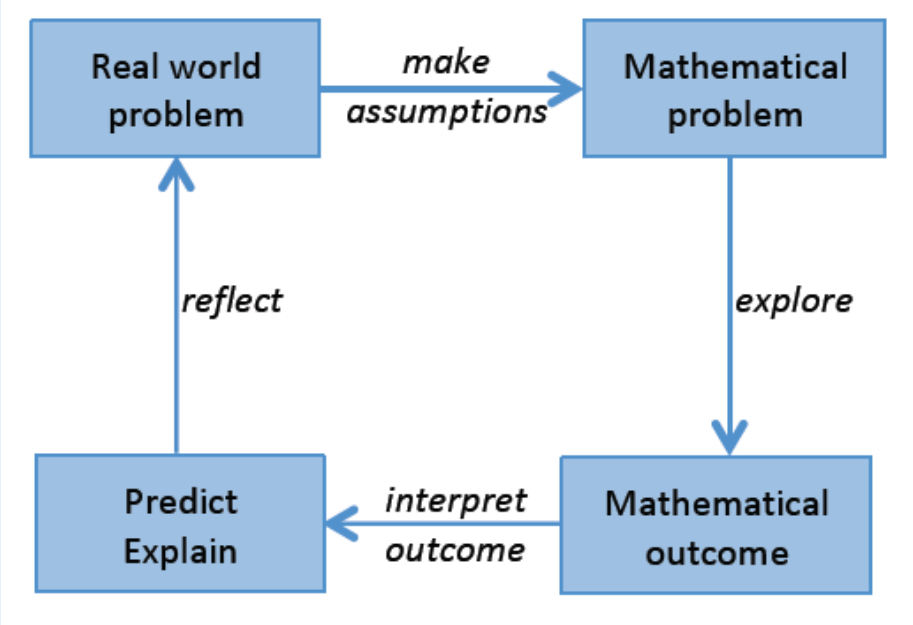
Mathematics Modelling

Modelling, where assumptions have to be made and relationships developed and results tested against known data or events, is still problematic as a teaching activity in schools and tertiary education. It can be summarised as below.



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Mathematics Model: Proportional Representation

How do you fairly allocate 5 seats in a town election where the votes cast for the 4 political parties are:

Party A: 17 920

Party B: 11 490

Party C: 11 170

Party D: 4 420

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Proportional Representation: D'Hondt Method

Divide by	A	B	C	D
1	17920*	11490*	11170*	4420
2	8960*	5745	5585	2210
3	5973*	3830	3723	1473
4	4480	2873	2793	1105

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Mathematics Modelling: Tennis ATP Rankings

- Based on points scored in competitions (but weighted)

  Grand Slams
  Barclays ATP World Tour Finals
  ATP World Tour Masters 1000
  ATP 500
  ATP 250

- Calculated every Monday based on performance in previous 52 weeks and based on:

  4 Grand Slams
  8 ATP World Tour Masters 1000
  4 Barclays ATP World Tour Finals
  6 best performances from other events

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Mathematics Modelling: Tennis ATP Rankings

	W	F	SF	QF	R16	R32	R64	R128
GS	2000	1200	720	360	180	90	45	10
Barclays	1500*							
ATP 1000	1000	600	360	180	90	45	10	
ATP 500	500	300	180	90	45	20		
ATP 200	250	150	90	45	20			

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Tennis ATP Rankings: Enhancing the model

- ***Is the model fit for purpose?***
- ***Can you suggest modifications?***

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Mathematics Modelling: Key Aspects

- Understand the real world problem
- Transform into mathematical problem
- Solve the mathematical problem
- Review the outcomes of the model and
- Use the model to validate, clarify or predict

or

- Revise the assumptions made and go through the cycle again

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The value of Mathematics for economic growth

- Governments are all giving mathematics teaching a high priority
- They believe that it is the key for economic growth; for example,

  Net present value to the UK of a 25-point increase on PISA: £4 trillion (the value of every house in the UK)

  Net present value to the UK of getting all students to 400 on PISA: £5 trillion (three times the national debt)
- About 8 years ago in England it was suggested that success in A-Level mathematics would result in an extra £10 000 per year annual salary compared with those taking A-Level examinations that did not include mathematics

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The value of Mathematics for economic growth

Programme for International Student Assessment (PISA)

	2009	2012
Sweden	496	482
United States	496	492
United Kingdom	500	502
Australia	519	512
Canada	527	518
Finland	544	525
Shanghai	579	587

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How can we prepare our students for a world we cannot imagine?

So the model that says "learn while you're at school, while you're young, the skills that you will apply during your lifetime" is no longer tenable.

The skills that you can learn when you're at school will not be applicable. They will be obsolete by the time you get into the workplace and not need them, except for one skill.

The one really competitive skill is the skill of being able to learn. It is the skill of being able not to give the right answer to questions about what you were taught in school, but to make the right response to situations that are outside the scope of what you were taught in school.

We need to produce people who know how to act when they're faced with situations for which they were not specifically prepared. (Papert, 1998)

This line of thinking is strong evidence for the teaching of mathematical modelling

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Aims and Objectives for Mathematics Education

Mathematics Department, Bishop Luffa School, Chichester

AIMS *"Our learners will become independent thinkers who enjoy working together to produce creative solutions in mathematics in unfamiliar situations."*

OBJECTIVES

Enjoy doing mathematics *– to help learners to enjoy and sense personal reward in the process of thinking, searching for patterns and solving problems*

Gain confidence and belief in abilities *- to develop learners' confidence in their ability to do mathematics*

Be willing to take risks and to persevere *– to improve learners' willingness to attempt unfamiliar problems and to develop perseverance in solving problems without being discouraged by initial setbacks*

Interact with others to develop new ideas *– to encourage students to share ideas and results, compare and evaluate strategies, challenge results, determine the validity of answers*

Although mathematical modelling is not explicitly mentioned, this is implicitly the focus for many of these objectives

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Participation in Mathematics in Post-16 Education

Students taking mathematics post-16[7.4]

	Any mathematics	Advanced mathematics
Japan	All	High
Korea	All	High
Taiwan	All	High
Estonia	All	Medium
Finland	All	Medium
Sweden	All	Medium
Russia	All	Low
Czech Republic	All	–
France	Most	Medium
USA (Mass)	Most	Medium
Germany	Most	Low
Ireland	Most	Low
Canada (BC)	Most	–
Hungary	Most	–
New Zealand	Many	High
Singapore	Many	High
Australia (NSW)	Many	Medium
Netherlands	Many	Low
Hong Kong	Some	Medium
Scotland	Some	Medium
Spain	Some	Low
England	**Few**	**Low**
Northern Ireland	Few	Low
Wales	Few	Low

Key

Any mathematics	5–19%	20–50%	51–79%	80–95%	95–100%
Advanced mathematics	0–15%		15–30%		30–100%

Data on participation in advanced mathematics were insufficient in Canada (BC), Czech Republic and Hungary.

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CORE MATHS: Post-16 Education

New initiative from the Government to target students who are not taking A-Level Mathematics but who have passed GCSE mathematics at age 16.

The Government is concerned that ENGLAND is an outlier in post-16 education provision.

Objectives

1. *Deepen competence in the selection and use of mathematical methods and techniques.*
2. *Develop confidence in representing and analysing authentic situations mathematically and in applying mathematics to address related questions and issues.*
3. *Build skills in mathematical thinking, reasoning and communication.*

Problem Solving in context is the focus for these new courses i.e. mathematical modelling

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CORE MATHS: Problem Solving Questions

In an ice skating competition, Jenna and Kim were the top two competitors.

The five judges gave them the following scores.

	Judge 1	*Judge 2*	*Judge 3*	*Judge 4*	*Judge 5*
Jenna	8	6	10	9	7
Kim	9	9	7	8	7

- ***Can you give good reasons why Jenna was declared the winner or should it have been Kim?***

CORE MATHS: Problem Solving Questions

'Reading age' is the level of *reading* ability that a person has in comparison to an average child of a particular *age.*

So that pupils' reading ages can be assessed, it is important to have an estimate of the reading ages of books written for school-age readers.

There are many ways of doing this.

- ***Design a formula or procedure to estimate the reading age of a text using a sample passage***

READIND AGE: Mathematical Models

Forecast formula

$$R = 25 - \frac{N}{10}$$

where N is the number of one-syllable words in a passage of 150 words.

FOG index

$$R = \frac{2}{5}\left(\frac{A}{n} + \frac{100L}{A}\right)$$

where A is the number of words in a passage, n the number of sentences in the passage and L the number of words containing 3 or more syllables (excluding those ending in 'ing' or 'ed').

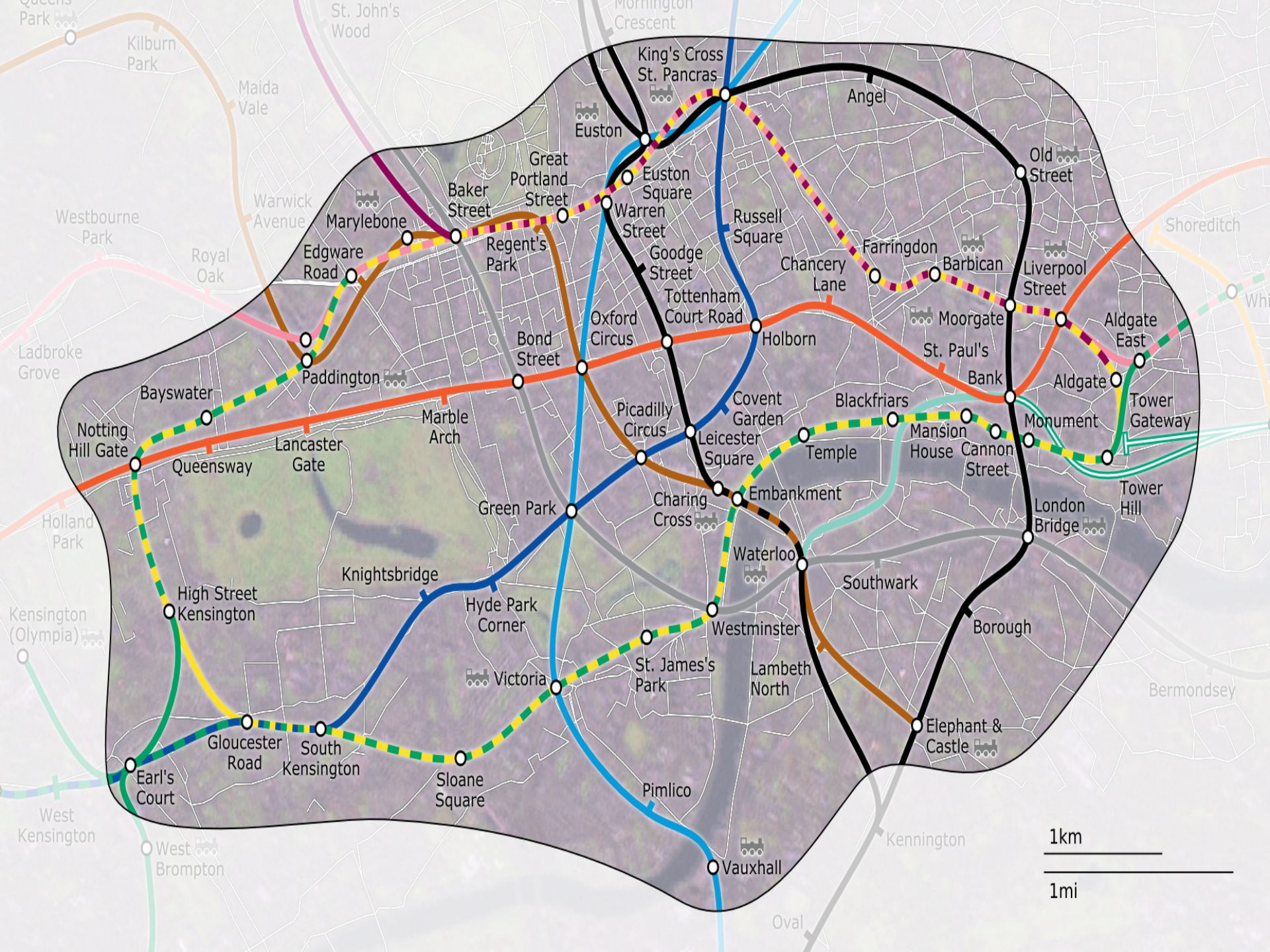
READIND AGE: Mathematical Models

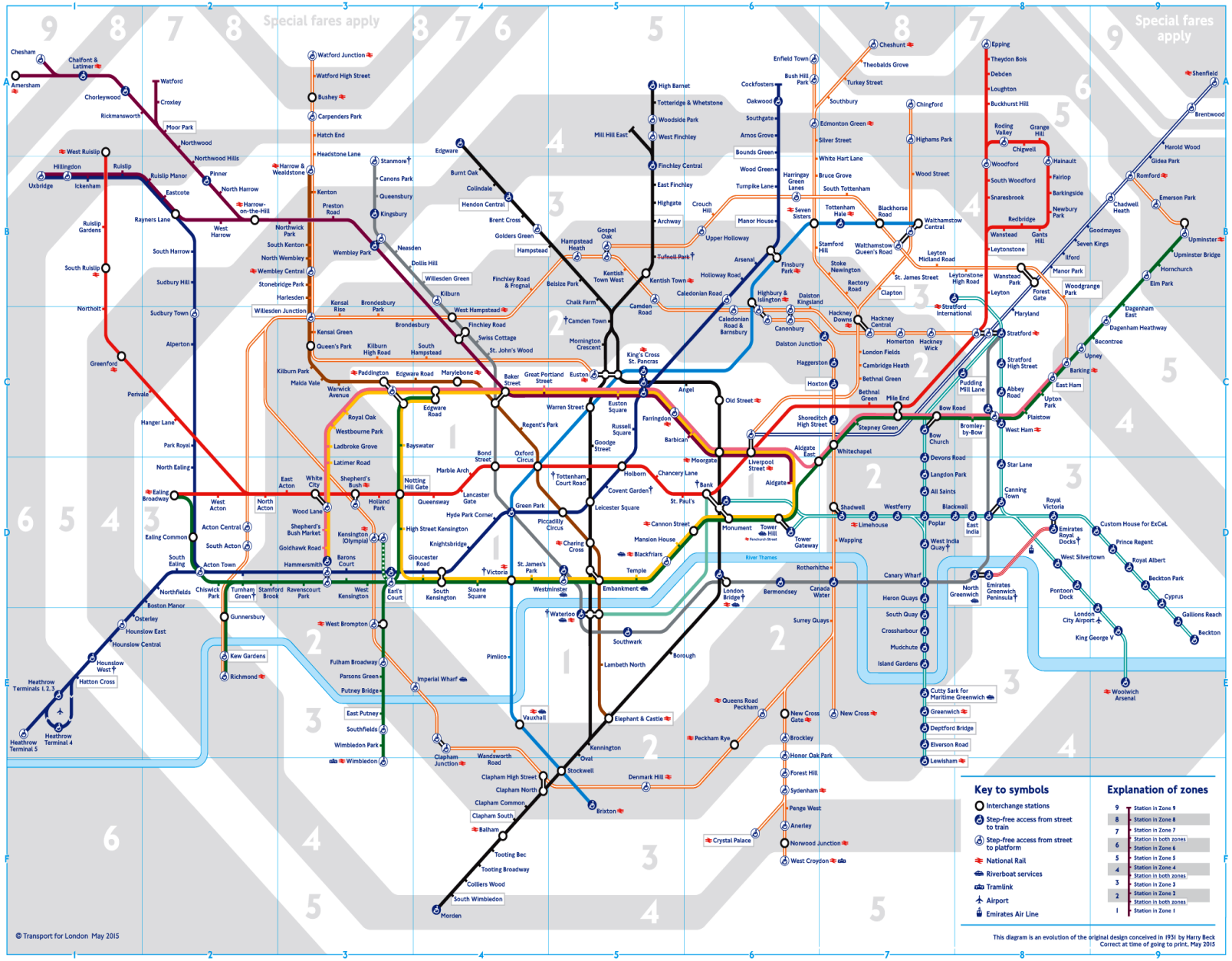
One that is well used is called the Flesch formula and this is given by:

$$\text{Reading Score} = 206.835 - 84.6S - 1.015W$$

where S is the average number of syllables per word and W is the average number of words in a sentence. This Reading Score is interpreted as in the table below:

Score	**Interpretation**
90 - 100	Easily understood an average 11 year od
60 - 70	Easily understood by 13-15 year olds
0 - 30	Best understood by University Graduates


King's Cross St. Pancras
Angel
Euston
Old Street
Great Portland Street
Euston Square
Baker Street
Warren Street
Russell Square
Marylebone
Regent's Park
Goodge Street
Farringdon
Barbican
Liverpool Street
Edgware Road
Chancery Lane
Tottenham Court Road
Moorgate
Aldgate East
Oxford Circus
Holborn
St. Paul's
Bond Street
Paddington
Bank
Aldgate
Bayswater
Covent Garden
Blackfriars
Tower Gateway
Picadilly Circus
Monument
Notting Hill Gate
Marble Arch
Mansion House
Leicester Square
Temple
Cannon Street
Queensway
Lancaster Gate
Embankment
Tower Hill
Charing Cross
Green Park
London Bridge
Waterloo
Knightsbridge
Southwark
High Street Kensington
Hyde Park Corner
Westminster
Borough
St. James's Park
Victoria
Lambeth North
Gloucester Road
South Kensington
Elephant & Castle
Earl's Court
Sloane Square
Pimlico
Vauxhall
1km
1mi




10
+3

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The future of mathematical modelling in the teaching and learning of mathematics

- The increase in Mathematics provision in Post-16 Education will focus on mathematical modelling
- Preparing our students for a world we cannot imagine will focus on helping students to gain transferable skills and this is at the heart of mathematical modelling
- The development of increasingly effective technology will require mathematical solutions to different types of problems
- ICTMA should reflect the real world problems by encouraging mathematics educators to embrace mathematical modelling to be at the heart of an effective mathematics pedagogy