ICTMA 17

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MATHEMATICAL MODELLING AS A PROFESSIONAL ACTIVITY – LESSONS FOR THE CLASSROOM

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LINKÖPING UNIVERSITY

Aim

- Some similarities and differences between working with mathematical modelling in ‘school’ and mathematical modelling as a ‘professional task’
- Approaches to simulating modelling as a ‘professional task’ in educational settings will be explored

Introduction

The relevance of using mathematics in out-of-school activities (Romberg, 1992)

Workplace mathematics is more complex and is situation dependent (e.g. Harris, 1991; Noss & Hoyles, 1996; Wedege, 2010)

Introduction

”Developing work-based learning is a crucial way to strengthen the links between the education system and the labour market, enhance youth employability and improve transitions from education to work.” (OECD, 2015, p. 27)

“Coordination between school and working life must be strengthened to ensure high quality of education and strong involvement from industry and the public sphere” (Skolverket, 2012, p. 12).

Introduction

”[A]n agenda for action is needed containing short- and long-term activities that strengthen the relation between industry and mathematics education at school” (Kaiser, van der Kooij & Wake, 2013, p. 269)

“Mathematical modelling in education is the most important educational interface between mathematics and industry” (Li, 2013, p.51)

Introduction

"For successful transfer of mathematical knowledge to client disciplines the theme of mathematical modelling is a crucial educational challenge. The lectures, books and laboratory exercises are necessary, but the actual maturing into an expert can only be achieved by '**treating real patients'** " (Heilio, 2013, p. 224)

Introduction

"The basic philosophy behind the approach . . . of the modelling workshop for higher education is that to become proficient in modelling, you must fully experience it – it is no good just watching somebody else do it, or repeat what somebody else has done – you must experience it yourself. I would liken it to the activity of swimming. You can watch others swim, you can practice exercises, but to swim, you must be in the water doing it yourself. " (Burghes,1984, p. xiii).

Introduction

Drakes (2012) argues for mirroring some parts of expert modellers' working practice into teaching practise.

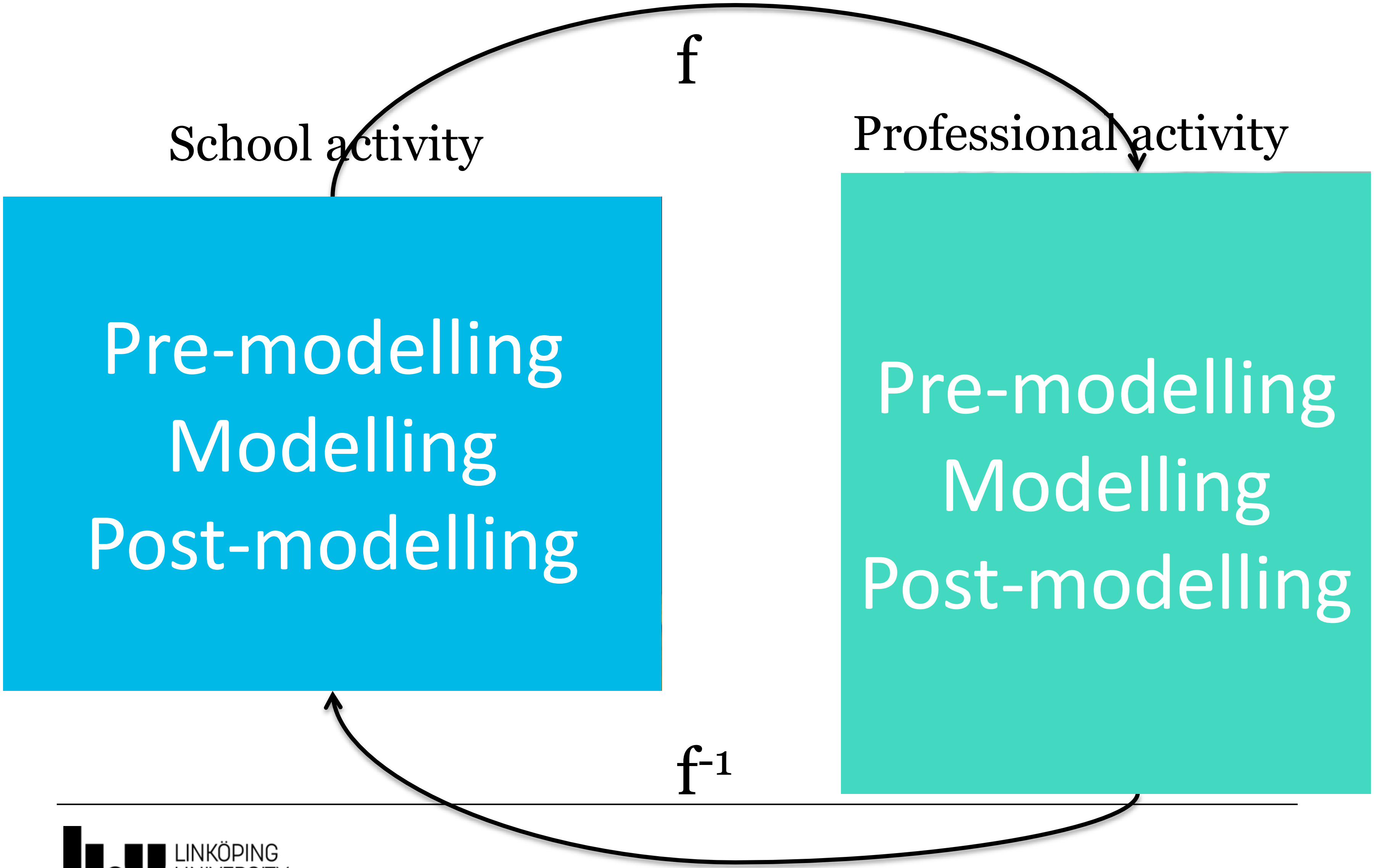
- spending a large proportion of learning/teaching time on formulating the problem and validating the solution
- "Students would ... benefit from seeing real modelling done by experts. Seeing experts deal with being stuck is informative, and helps change the belief that experts simply rely on intuition" (Drakes, 2012, p. 207)

Introduction

Summary

- Organisations, curricula documents, researchers, educators, etc. call for strengthening the relation between industry and mathematics
- Modelling
- You learn it only by doing (Burghes,1984; Heilio, 2013; Neunzert, 2013)
- Experts

Similarities and differences



School Pre-modelling

In education modelling as a mathematical classroom activity can be an **aim in itself** (to develop modelling competencies) or have an **aim to develop a broader mathematical ability** (a didactical tool to learn mathematics) (see e.g. Blum & Niss, 1991).

School Pre-modelling

Teachers or researchers usually deliver the problems to the students and there is a diversity and a richness of knowledge taught.

Some examples are related to: elementary arithmetic with base ten blocks (Speiser & Walter, 2010), sound intensity and brightness (Riede, 2010), ranking statistical data (Carmona & Greenstein, 2010; English, 2010; Mousolides et al, 2010), solving linear pattern tasks (Amit & Neria, 2010), solving problems with geometry (Stillman, Brown, & Galbraith, 2010), with technology (Confrey & Maloney, 2007), different representations of functions (Arzarello, Pezzi & Robutti, 2007), calculus (Araújo & Salvador, 2001), non linear situations (De Bock, Van Dooren & Janssens, 2007), multi-variable functions (Nisawa & Moriya, 2011), interdisciplinary projects (Ng, 2011), traffic models (Blomhøj & Hoff Kjeldsen, 2011), Determining the centre of the base area of the Tipi (Rosa & Orey, 2013), finding the length of a spiral banister (Saeki & Matsuzaki, 2013) Algal bloom problem (Geiger, 2013) etc....

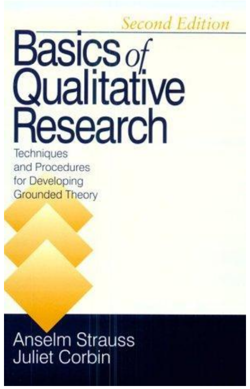
Professional Pre-modelling

Mathematical modelling as a professional task (Frejd & Bergsten, submitted)

The aim is to explore how expert modellers work with mathematical modelling in their workplace

Sector	Area of expertise (Systems to model)
Air industry	Models and simulations for aerodynamics and flight mechanics
Banking	Risk models for banking
Biology	Risk models for the spread of diseases; optimization of animal transportation
Climate	Climate models; models for aerodynamics
Insurance	Risk models used in insurance and banking
Physics	Models for new materials
Scheduling	Optimization models; scheduling
Traffic	Traffic simulations models
Various	Models for measurement instruments to identify and estimate; models for water conservation plans

Interview questions	*The main aim of the interview question is to find:*
1. What is your academic background?	Background information
2. What are you working life experiences before you got here?	Background information
3. What is your vocation and what role does mathematical modelling play in your vocation?	Background information
4. What does mathematical modelling mean to you? Has your view on modelling changed during the years? (If yes) How?	Constructors' conceptions
5. Make a general description how you work with a modelling problem (from start to end).	Pre-construction, construction, post-construction
6. Who gives you the problems to work with? What type of problems do you work with? What are the aims with the problems you get?	Pre-construction
7. What will be the use of the model? Who set the goal for the mathematical activity? Who defines the criteria?	Pre-constructing
8. How do you work with mathematical modelling in your vocation (by yourself, in groups) If it is group work how/what communication takes place? What types of artefacts are used?	Constructing
9. What kind of models do you develop (static/dynamic, deterministic/stochastic, discrete/continuous, analytic/simulations)?	Constructing
10. What are the connections between input and output?	Constructing
11. How was the necessary measurement data obtained? Is there a way to control the quality and the origin of the data? Can you give example of values and quantity of the data?	Constructing
12. What factors may have affected the investigated phenomena (measuring instrument or its use)?	Constructing
13. Is it possible to control the result? What types of assumptions have been made according to the context? Who decide what assumptions are being important? What is the accuracy of the result?	Constructing
14. How does the solution contribute to understanding and action?	Constructing
15. What is an acceptable solution? Are there other solutions? (if yes) How do you chose solution?	Constructing
16. Are there any risk to use the result? If so, how is that considered? Is ethical issues discussed?	Post-constructing
17. Is mathematical modelling something that was a part of your education in school or something you learned in your vocation?	Background information



(Jablonka, 1996)

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Professional Pre-modelling

- The modellers get orders/ problems from *clients*
- The goals are to *describe* and *simulate* a phenomenon in order to be able to *predict* (to make prognoses about the future), *design* (improving objects), or *construct* (objects)
- The models are developed to serve as basis for decision making

Professional Pre-modelling

The goals to simulate, predict…similar to the goals of *descriptive* and *prescriptive* (Niss, 2015)

In prescriptive modelling the ultimate aim is to pave the way for *taking actions based on decisions* resulting from certain kind of mathematical considerations, in other words ‘to change the world’ rather than only ’to understand the world’. (Niss, 2015)

Similarities and differences

f

School activity

Professional activity

Pre-modelling

The models are developed to serve as basis to learn modelling or mathematics. Teachers deliver the problem.

(Galbraith, 2013)

Pre-modelling

The models are developed to serve as basis for decision making. C

deliver th

Nishimura, 2015

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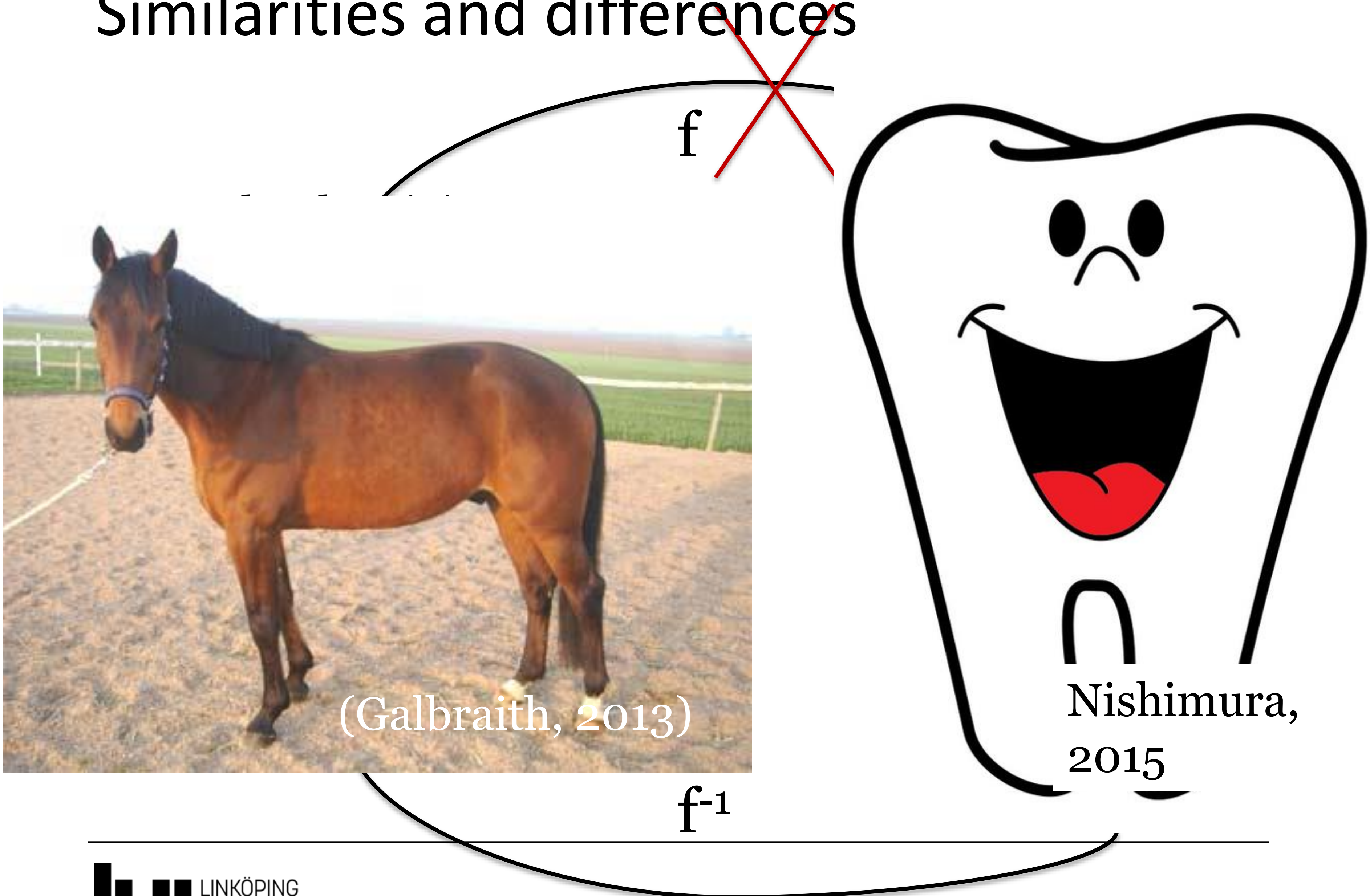
Similarities and differences



Modelling as a school activity

- There is a diversity of theoretical perspectives on mathematical modelling in mathematics education literature
- (e.g. Blum, Galbraith, Henn, & Niss, 2007; Jablonka & Gellert, 2007; Garcia, Gascón, Higueras & Bosch, 2006; Kaiser & Sriraman, 2006; Frejd, 2010; Geiger & Frejd, 2015)

ATD

Ethno-modelling

Modelling cycle

M&M

Modelling interpreted in commognition

Modelling as a school activity

Name of the Approach
Realistic or applied modelling
Contextual modelling
Educational modelling
Socio-critical and sociocultural modelling
Epistemological modelling

(Kaiser & Sriraman, 2006)

Modelling as a school activity

Realistic
or
applied
modelling

Modelling Week
Modelling Day
Modelling projects
Modelling competitions
(e.g. Hamson, 1987; Kaiser & Stender, 2013; Kaiser at al. 2013; Jiang, Xie & Ye, 2007; Haan, 2003; etc.)

Modelling as a school activity

Realistic
or
applied
modelling

Modelling as a school activity

Contextual modelling

Word problems

Modelling as a school activity

An infectious disease is spreading in a city. The number of infected increase from 200 to 400 during 24h. Develop a model and estimate the number of infected after yet another four days, if the increase is:

a) Linear
b) Exponential

Which model seems most accurate?

6277 I en stad sprids en mycket smittsam sjukdom. Antalet sjuka ökar från 200 till 300 på ett dygn. Gör en modell och beräkna hur många sjuka det finns efter ytterligare fyra dygn om ökningen är

a) linjär
b) exponentiell.
c) Vilken modell verkar mest rimlig?

a) Om ökningen är linjär ökar antalet sjuka med lika många personer, 100 st, varje dygn.
Antalet sjuka y kan beskrivas med funktionen
$y = 200 + 100x$ där x är antalet dygn från mätningens start.
$x = 5$ ger $y = 200 + 100 \cdot 5 = 700$
Antalet sjuka är 700 enligt den linjära modellen.

b) Om ökningen är exponentiell ökar antalet sjuka med lika många procent, 50 %, varje dygn. Antalet sjuka y kan beskrivas med funktionen $y = 200 \cdot 1{,}5^x$
där x är antalet dygn från mätningens start.
$x = 5$ ger $y = 200 \cdot 1{,}5^5 \approx 1\,500$
Antalet sjuka är 1 500 enligt den exponentiella modellen.

c) Om sjukdomen är mycket smittsam kommer antalet sjuka att öka snabbt.
Den exponentiella modellen är därför mest trolig.
Efter en tid avtar dock hastigheten och modellen gäller inte längre.

6278 @ För antalet besökare per år på en djurpark har man som prognos två modeller.
Modell A: $y = 40\,000 + 12\,000x$
Modell B: $y = 40\,000 \cdot 1{,}2^x$
x är antalet år efter 2011.

a) Vilken modell är exponentiell?
b) Hur många besökare har djurparken år 2011?
c) Hur många besökare har djurparken år 2013 enligt den linjära modellen?
d) Vilken modell ger störst antal besökare år 2015?

6279 Folkmängden i en stad är 2,5 miljoner och ökar med 0,1 miljoner varje år.

a) Beräkna folkmängden efter 3 år.
b) Skriv en matematisk modell som visar folkmängden y miljoner efter x år.
c) Hur många år tar det innan folkmängden överstiger 4 miljoner?

6280 I en swimmingpool finns det 1 000 bakterier per cm^3. För varje dygn ökar antalet bakterier med 80 %.

a) Beräkna antalet bakterier efter 4 dygn.
b) Skriv en matematisk modell som beskriver hur många bakterier y det finns efter x dygn.
c) Ange en ekvation som ger svar på frågan: *"Efter hur många dygn är antalet bakterier 20 000?"*

6281 En bil som köpts in för 300 000 kr sjunker i värde under första året med 30 000 kr vilket är 10 % av värdet.

a) Hur mycket är bilen värd efter 3 år om minskningen är linjär respektive exponentiell?
b) Hur många procent har bilen minskat i värde efter 3 år om minskningen är linjär respektive exponentiell?

338 6.2 FUNKTIONER

(Frejd, 2013)

Aktivitet

MODELLERA

“To ‘model’ in mathematics means to do a mathematical model, which is a simplification of a real situation. You start to do assumptions and estimations and then use computation”

3 Den spektakulära byggnaden Turning Torso i Malmö är 54 våningar hög. Konstruktionen bygger på nio kuber med fem våningsplan i varje. Varje våningsplan har omkring 400 m^2 boyta. Det finns 147 lägenheter från kub tre till kub nio.

Hela konstruktionen vrider sig 90 grader på sin väg upp.

Arkitekten Santiago Calatrava är konstnär, skulptör och ingenjör. Han är verksam i Zürich, Paris, New York och Valencia. Han inspireras av naturliga rörelser hos djur och människor.

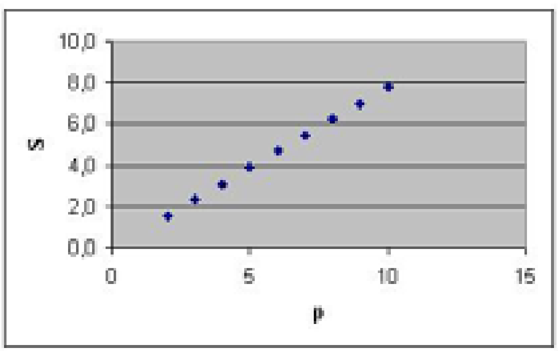
How long time does it take to walk up the stairs to the top floor?

Ärlebäck, 2009

Modelling as a school activity

Socio-critical and sociocultural Modelling

p	S
2	1,6
3	2,3
4	3,1
5	3,9
6	4,7
7	5,5
8	6,2
9	7,0
10	7,8



"The bean and corn seeds donated by the Government begin to be distributed in yesterday afternoon. There are 37.5 tons, 25 tons of bean and 12.5 tons of corn seed. About 8000 subsistence farmers will be benefited.
According to the mayor, each farmer will receive 3 kg of bean and 2 kg of corn."

(Barbosa, 2006)

Modelling as a school activity

Epistemological Modelling

(Ruiz, Bosch & Gascón, 2007)

”Workshop”

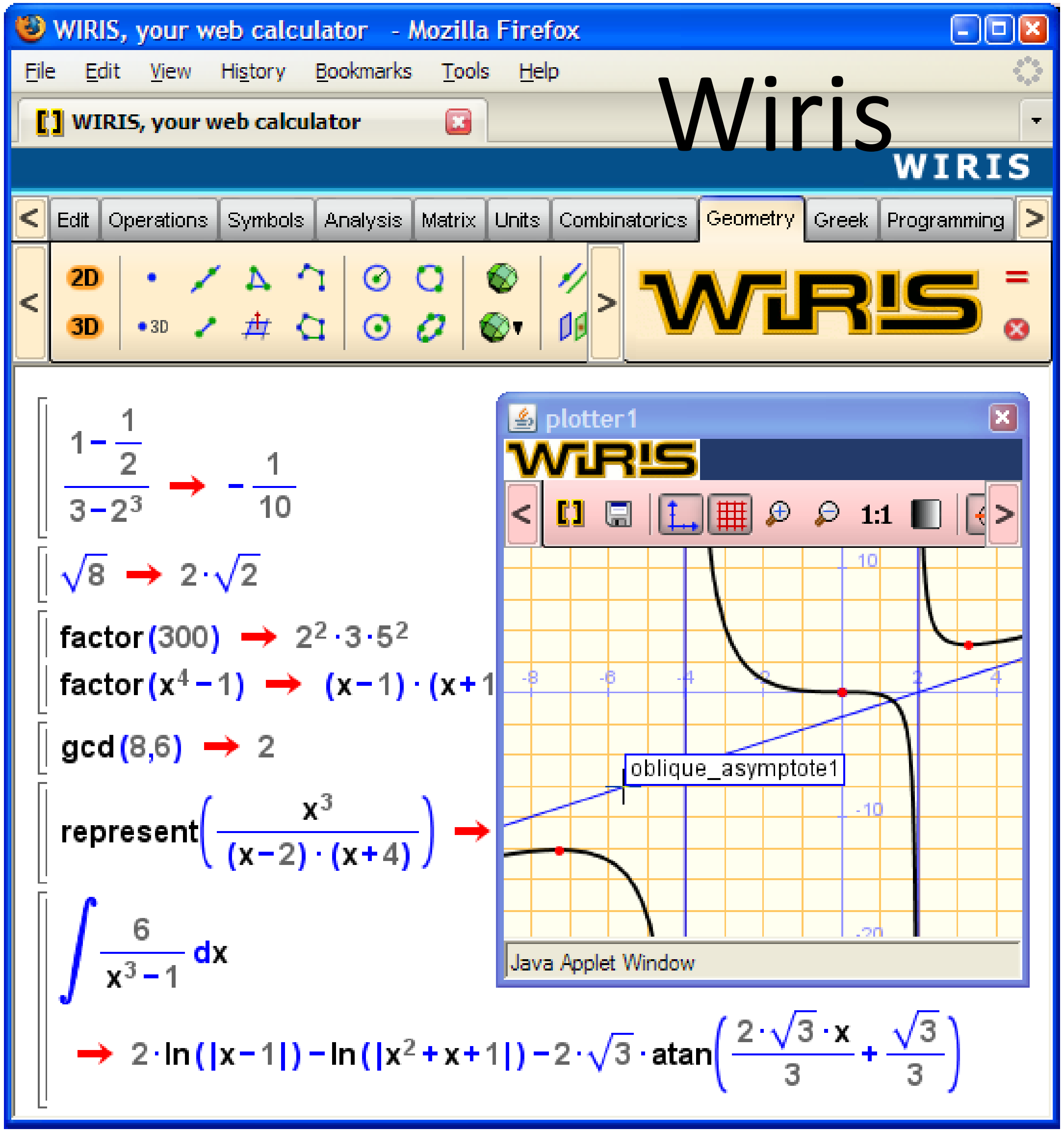
MONTH	May	June	July	August
Sold	100	329	264	
Total costs (€)	550	1122,5	960	
Total incomes (€)	520	1710,8	1372,8	
Benefits (€)	-30	588,3	412,8	

A constant unitary cost c = 2,5 €, a constant unitary price p = 5,2 € and a constant fixed cost L = 300 €.

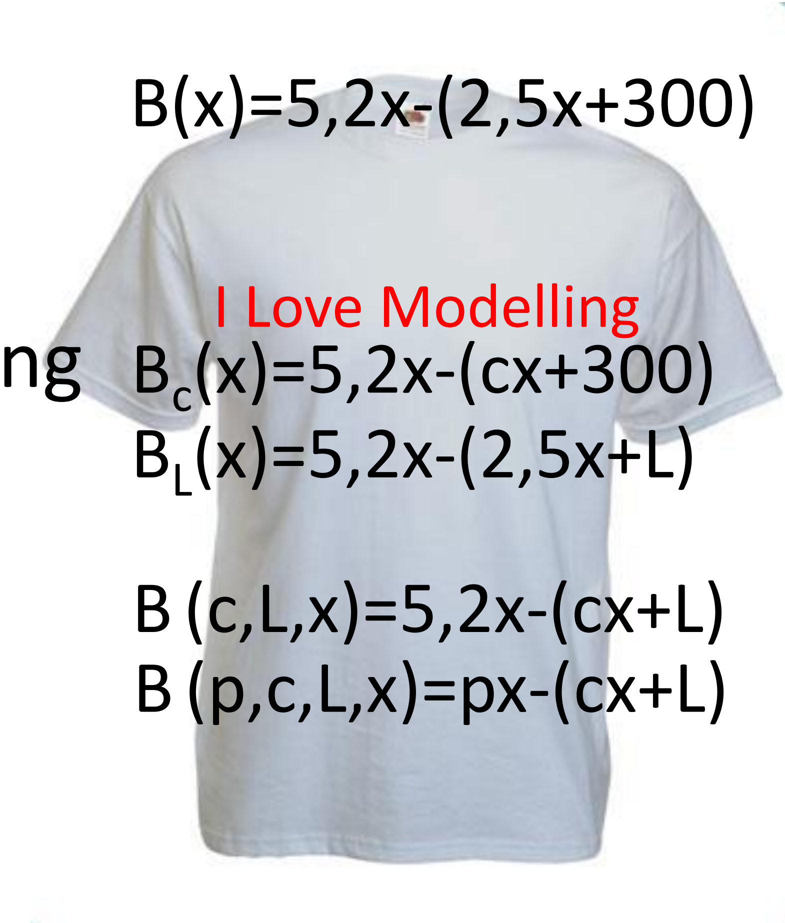
Q0: In the given initial conditions, is it possible to obtain a benefit of 3000 € in August by selling a reasonable number of T-shirts?

(Ruiz, Bosch & Gascón, 2007)

Wiris


WIRIS, your web calculator - Mozilla Firefox
File Edit View History Bookmarks Tools Help
WIRIS, your web calculator
WIRIS
Edit Operations Symbols Analysis Matrix Units Combinatorics Geometry Greek Programming
2D
3D
WIRIS
plotter1
oblique_asymptote1
Java Applet Window
factor(300) → 2²·3·5²
factor(x⁴−1) → (x−1)·(x+1
gcd(8,6) → 2
represent

1. Isolated functions of one variable
2. Family of functions of one variable and the corrsponding parametrical equation.
3. Family of functions of 2 or more variables



$B(x)=5,2x-(2,5x+300)$

$B_c(x)=5,2x-(cx+300)$

$B_L(x)=5,2x-(2,5x+L)$

$B\ (c,L,x)=5,2x-(cx+L)$

$B\ (p,c,L,x)=px-(cx+L)$

(Ruiz, Bosch & Gascón, 2007)

Modelling as a school activity

Summary of modelling as a school activity

- Collaboration
- Communication
- Adapt models
- Develop models
  - Well formulated problems
  - Use pre-defined data
  - Analyse data
  - Some technology

Modelling as a professional activity

Data-generated modelling

The work of gathering, interpreting, synthesizing, and transforming data as the underlying base for identifying variables, relationships and constraints about a phenomenon used in the development process

Theory-generated modelling

The work of setting up new equations based on already theorised and established physical equations, followed by the activation of computer resources for computational purposes to solve the new equations with aim to get information about the 'theorised' equations

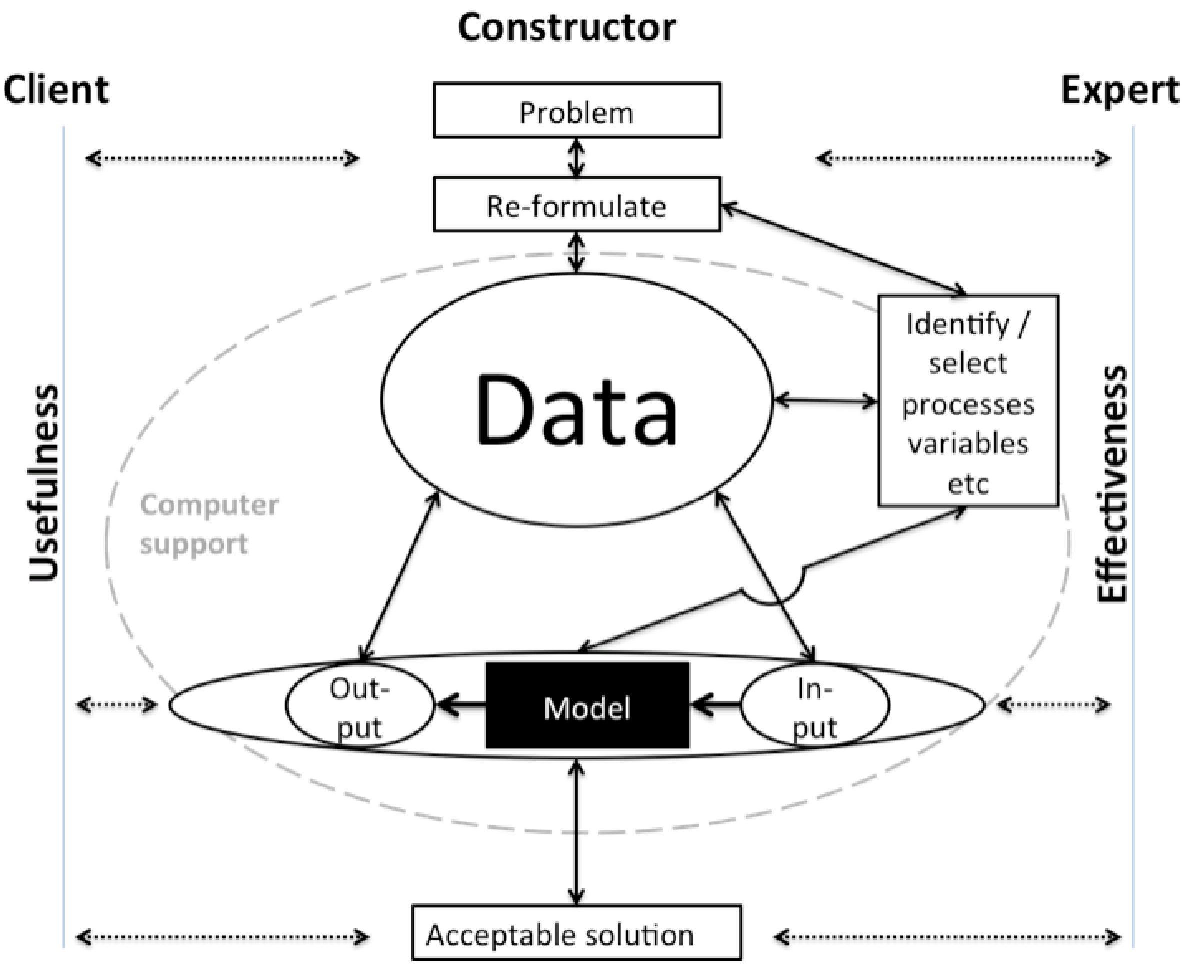
Model-generated modelling

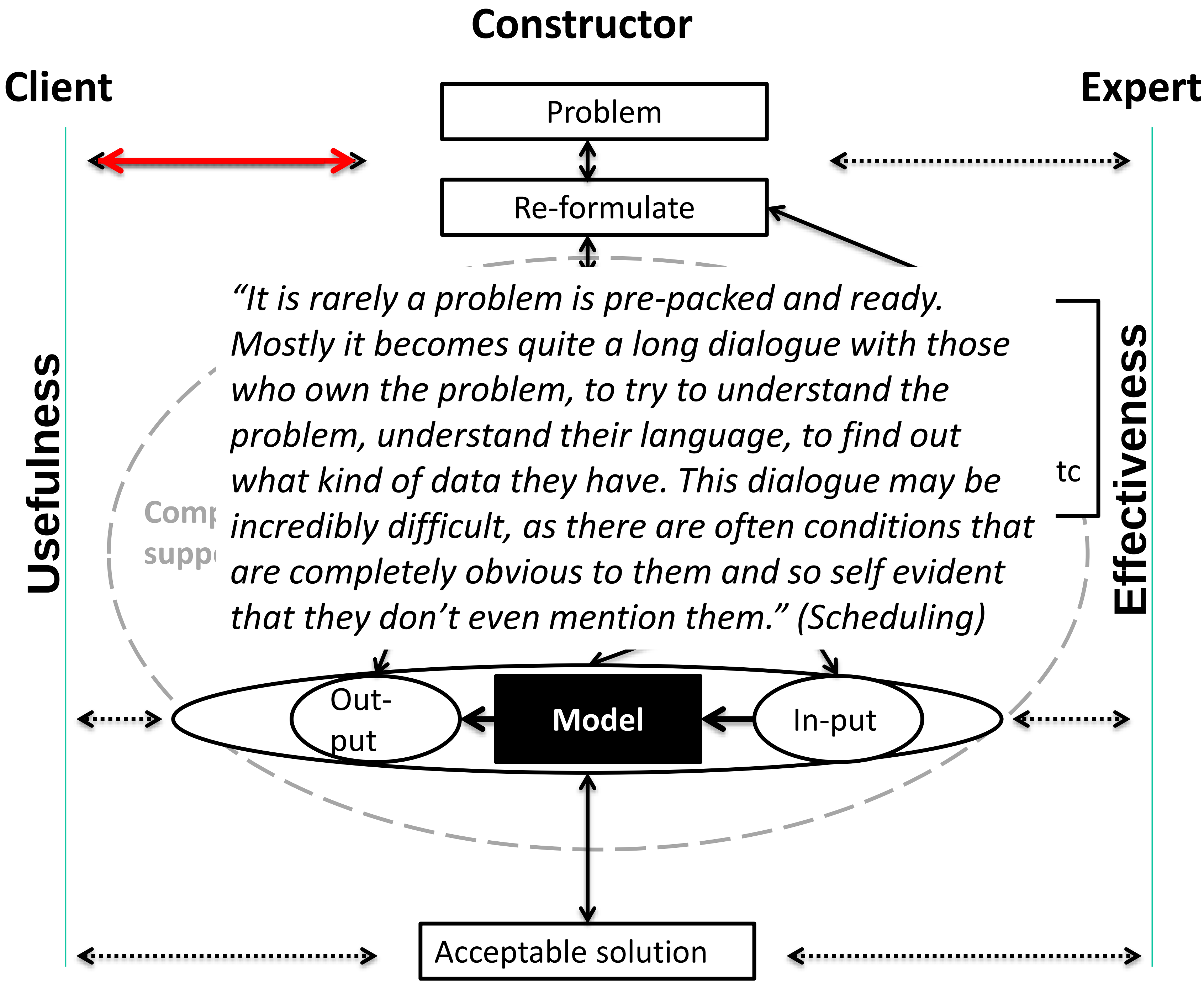
Models are constructed by identifying situations on which some mathematics or established mathematical models can be directly applied

→ *Modelling Activity Schemes*

Modelling as a professional activity

Data-generated modelling

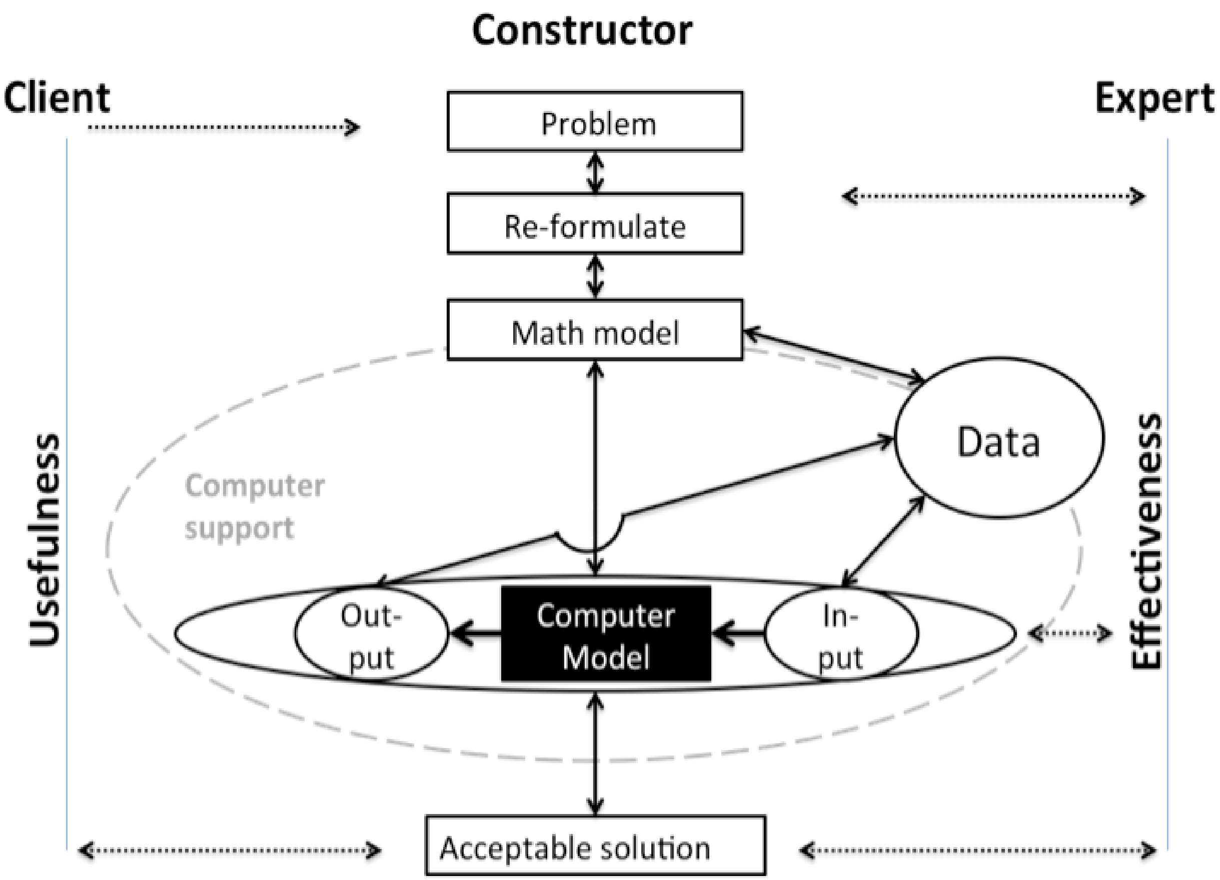


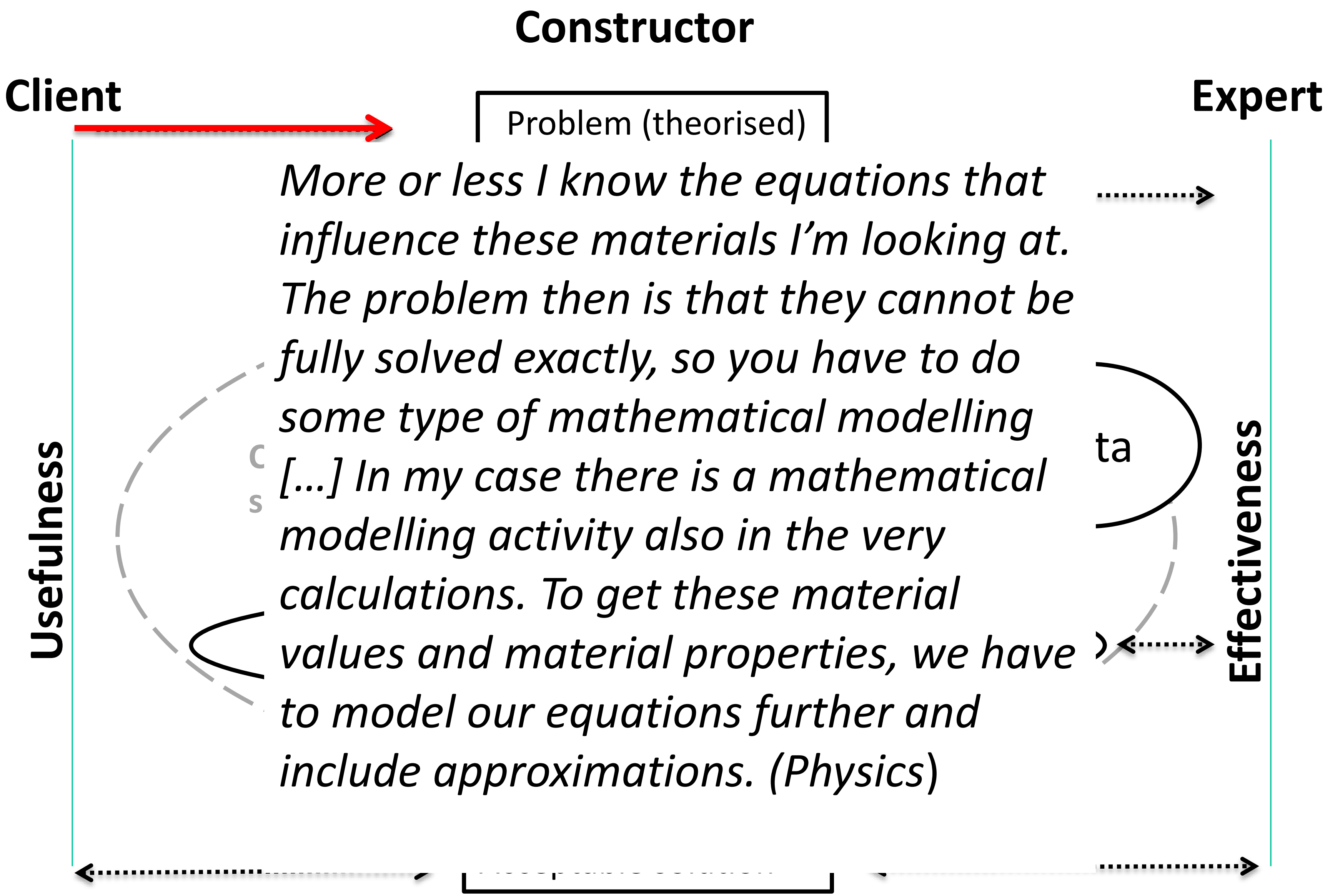


Constructor
Client
Expert
Problem
Re-formulate
Usefulness
Effectiveness
"It is rarely a problem is pre-packed and ready. Mostly it becomes quite a long dialogue with those who own the problem, to try to understand the problem, understand their language, to find out what kind of data they have. This dialogue may be incredibly difficult, as there are often conditions that are completely obvious to them and so self evident that they don't even mention them." (Scheduling)
Out-put
Model
In-put
Acceptable solution

Modelling as a professional activity

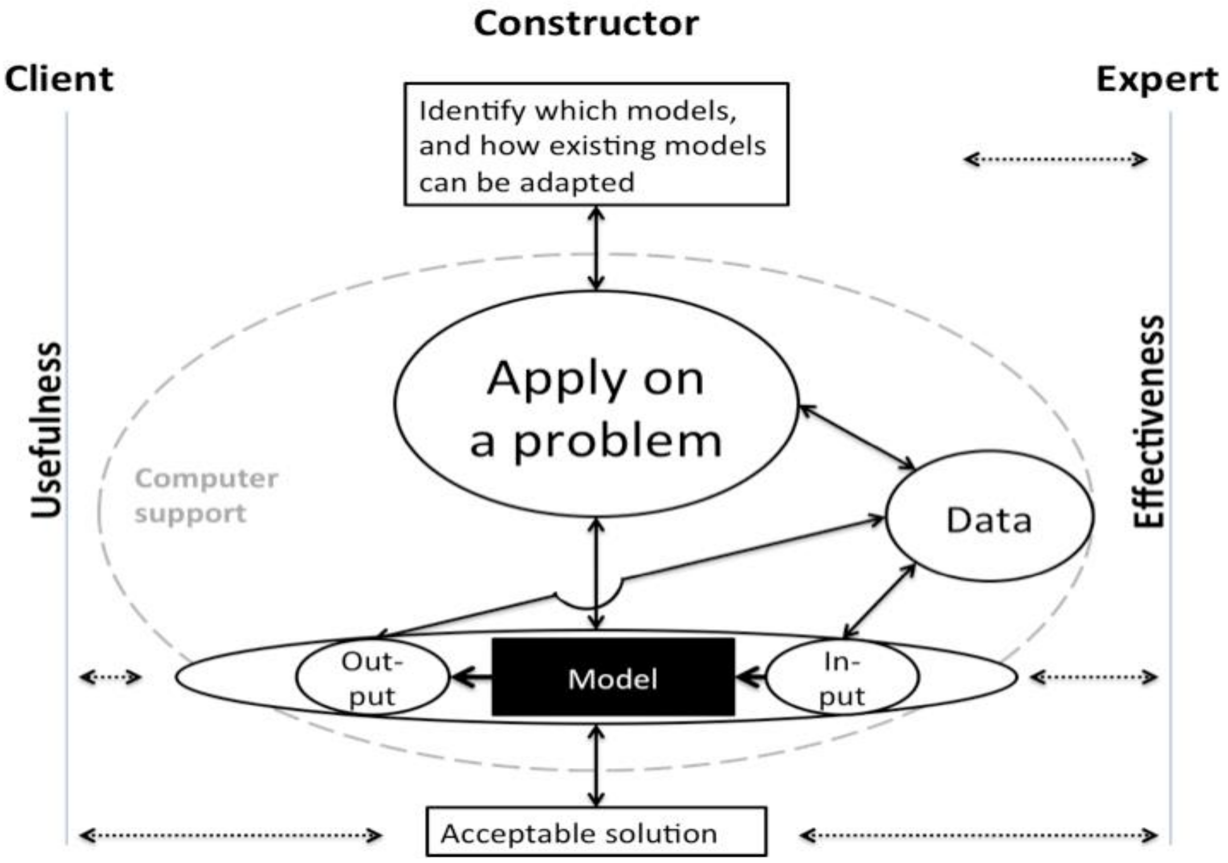
Theory-generated modelling




Constructor
Client
Expert
Problem (theorised)
Usefulness
Effectiveness
More or less I know the equations that influence these materials I’m looking at. The problem then is that they cannot be fully solved exactly, so you have to do some type of mathematical modelling […] In my case there is a mathematical modelling activity also in the very calculations. To get these material values and material properties, we have to model our equations further and include approximations. (Physics)

Modelling as a professional activity

Model-generated modelling



Constructor

Client

Expert

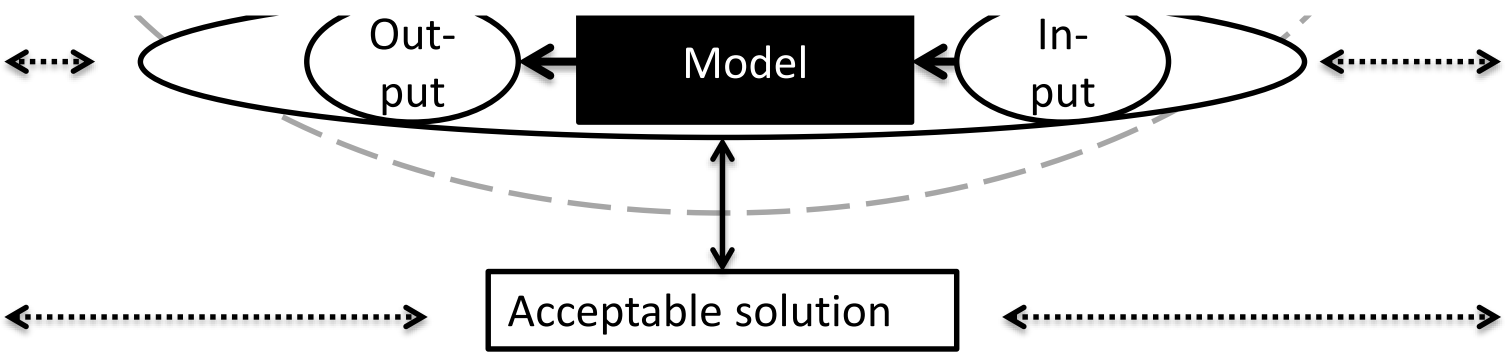
Identify which models

There are many tools constructed, so we do not build everything ourselves, instead you often start with something already existing and if you have a special product you may write some scripts that you put into the program. (Banking)

Usefu

Effecti

This part of developing models can be a large or a small part of our projects. We can use an already defined traffic simulation model. (Traffic)



Modelling as a professional activity

Modelling activity	Description
Data-generated modelling	Main focus lies within the data • frame the problem formulations • identification of parameters, variables, constants, process • serving as base for the construction and validation of the models.
Theory-generated modelling	Goal to develop a theory • development of new models based on already established (perhaps amended) theory, • re-formulating the given problem in the mathematical domain; employs computer simulations for the study and validation of the new models.
Model-generated modelling	Is a part of all constructors' work • the application of mathematics or already established (perhaps adapted) models in new contexts and situations

Modelling as a professional activity

Summary of modelling as a school activity

- Teamwork
- Computer support
- Communication between different actors (clients, operators and other experts)
- Validation
  - Expert opinions
  - Statistics and back testing on old data
  - Experiments

Similarities and differences

f

School activity

Modelling

- Collaboration
- Communication
- Adapt models
- Develop models

Professional activity

Modelling

- Collaboration
- Communication
- Adapt models
- Develop models

f^{-1}

Similarities and differences

f

School activity

Professional activity

Modelling

- Collaboration
- Communication
- Adapt models
- Develop models

Modelling

- Computer support
- Division of labour
- Knowledge and experiences
- The three schemes

f^{-1}

School Post-modelling

There are also major differences between the professional and the educational contexts in terms of objectives and **consequences of the modelling activity** (cf. Wake, 2014, p. 272). For example, in the classroom mathematical models constructed by students are **seldom** put to use in a context of practice, or in other ways involve risks.

Professional Post-modelling

Of course there are risks of using models and sometimes one talks about model risks and that is exactly that you have ***missed something****, that you use the model in a* ***context*** *where it should not be used. Or you use it even if the* ***conditions are not fulfilled****, or that the* ***assumptions maybe worked*** *when you made the model […] the customer has paid* ***money*** *today to get it back as a pension after twenty years and when you get there no money is left.* (Banking)

Professional Post-modelling

If I look at it from the perspective of the animals and an optimiser, then this farm […] should not be allowed to have cows he should have pigs […] this slaughter house you should not place here as it would create very long animal transports (Biology)

Suddenly you take away the power from people that used to have power, to be able to construct schedules for themselves, and this power is now given to outsiders. (Scheduling)

Similarities and differences

f

School activity

Professional activity

Post-Modelling

- The models are seldom put to use in a context of practice, or in other ways involve risks

Post-Modelling

- The models are put in practice
- Involve risks of using the models

f^{-1}

Conclusion

....**differences** in how modellers, teachers and students work with modelling in different practices in terms of **the goal** with the modelling activity**, the risks** involved in using the models, the **use of technology**, **division of labour** and the **construction** of mathematical models.

...**similarities** identified described as important aspects of modelling work in the different practices, such as **communication**, **collaboration**, **projects**, and the **use of applying** and adapting pre-defined models.

Discussion

These major differences between modelling work in educational and non-educational contexts seem to entail that mathematical modelling in school becomes an unrealistic utopia in terms of coherence to non-educational professional practice.

“inaccessible phenomena” (Gainsburg, 2003, p. 263)

Simulation

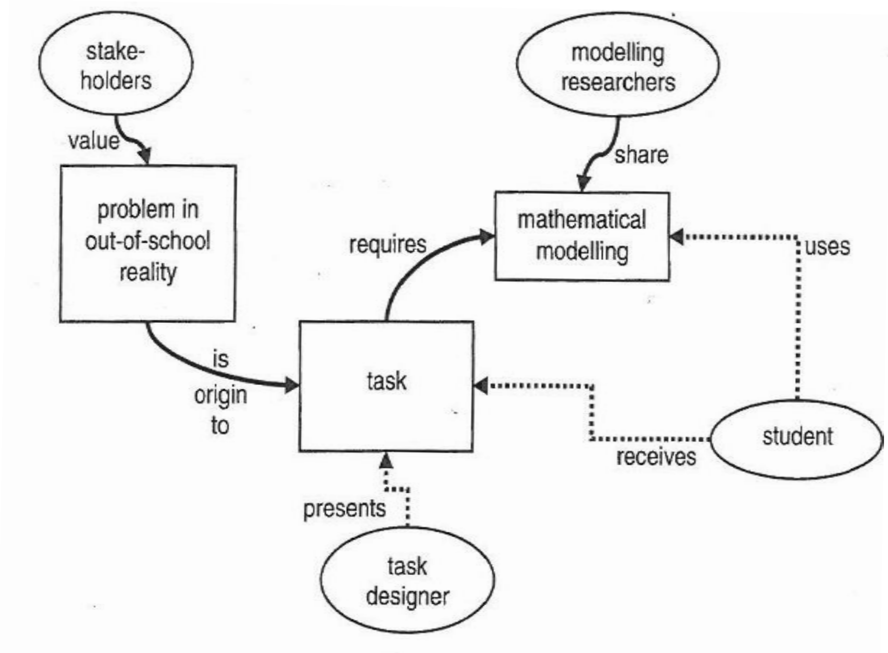


Fig. 68.3 The social construct of 'authentic' aspects in a mathematical modelling task

Clearly not created for educational purposes

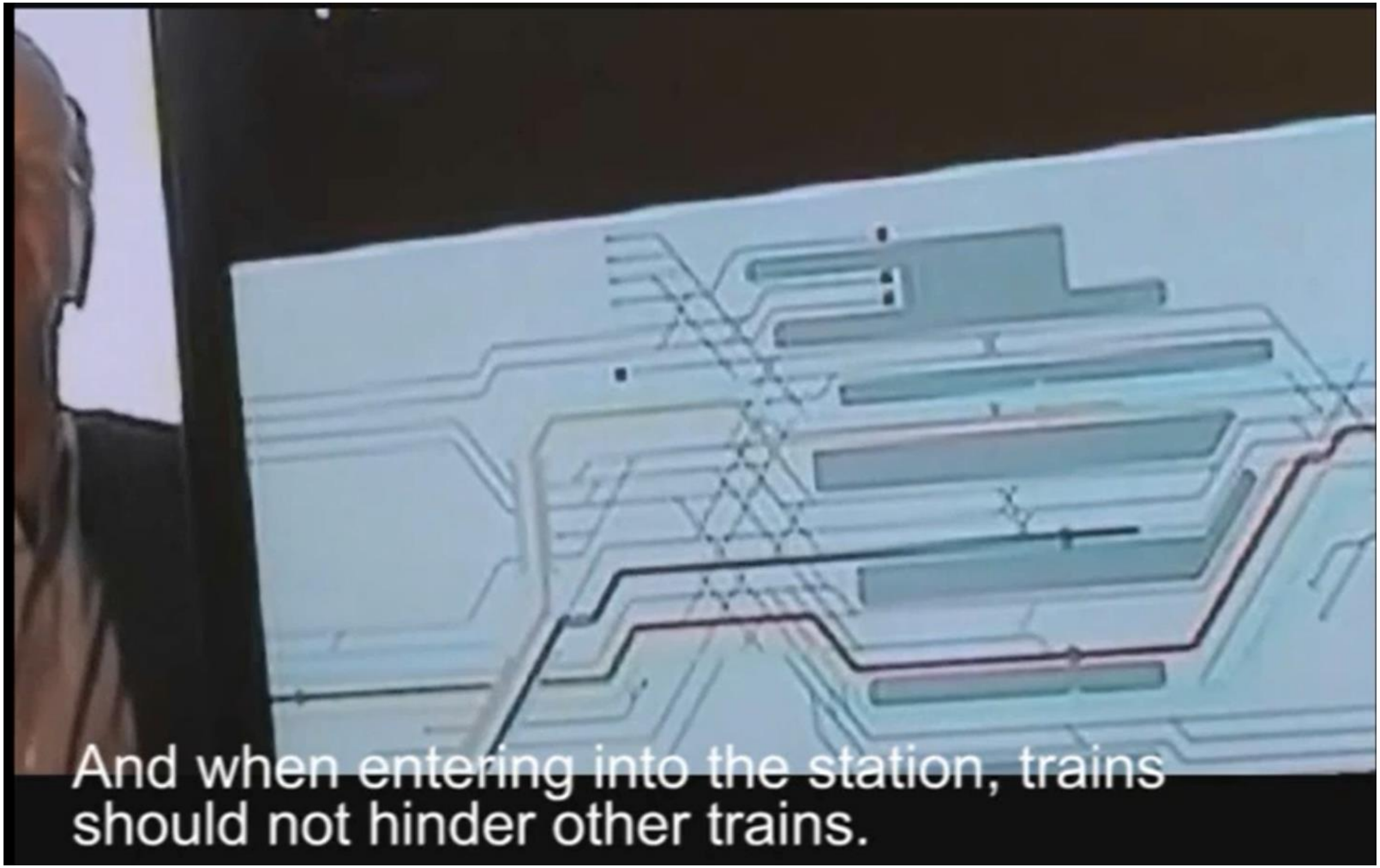
1. It connects to an out-of-school origin
2. It is binary
3. It can be applied to seperate task aspects
4. It is actor independen, which can be certified by actors (stakeholders, modelling researchers)

(Vos, 2011, p. 271)

Simulation

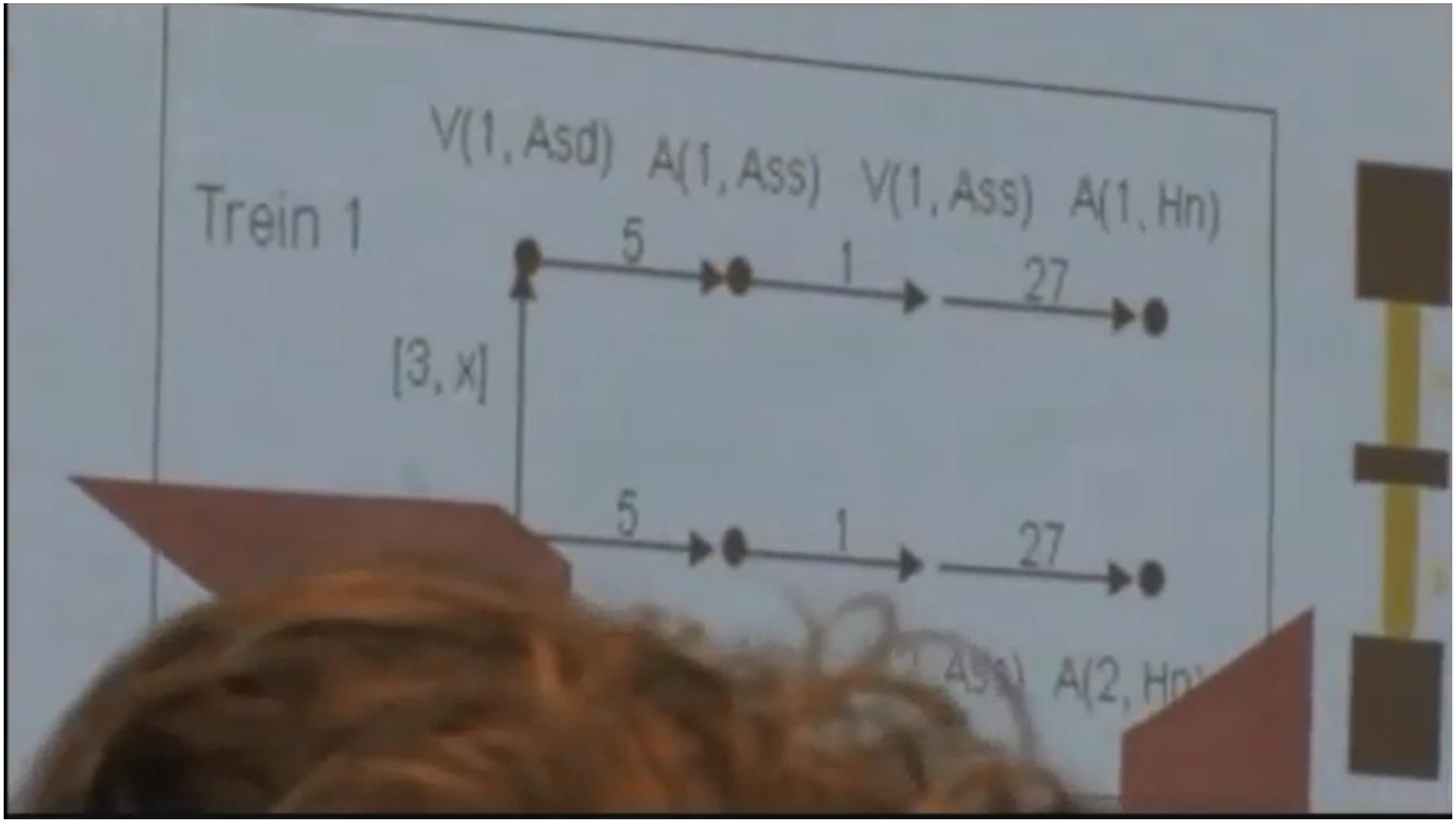
(Vos, 2015)

Simulation



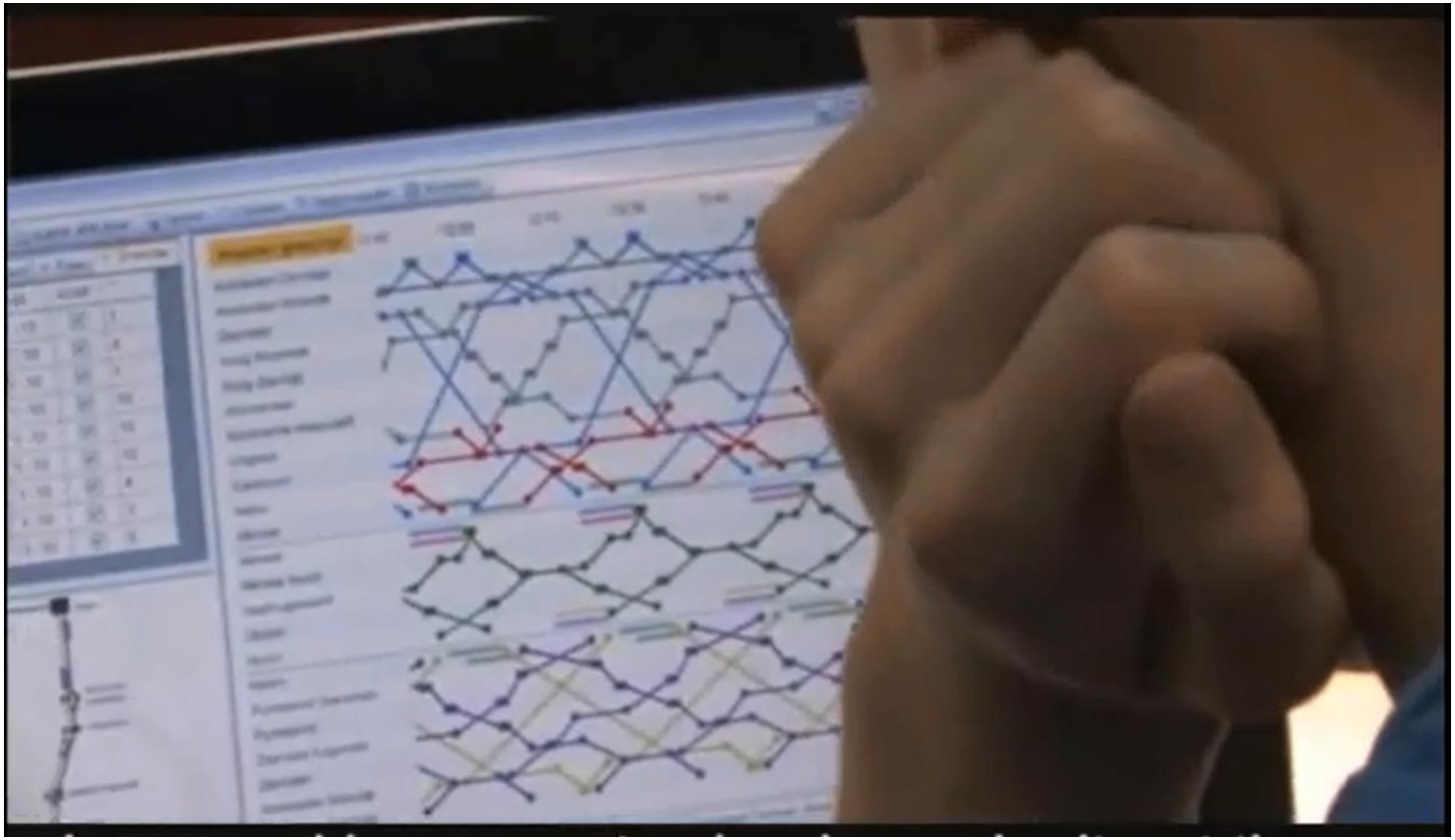
(Vos, 2015)

Simulation



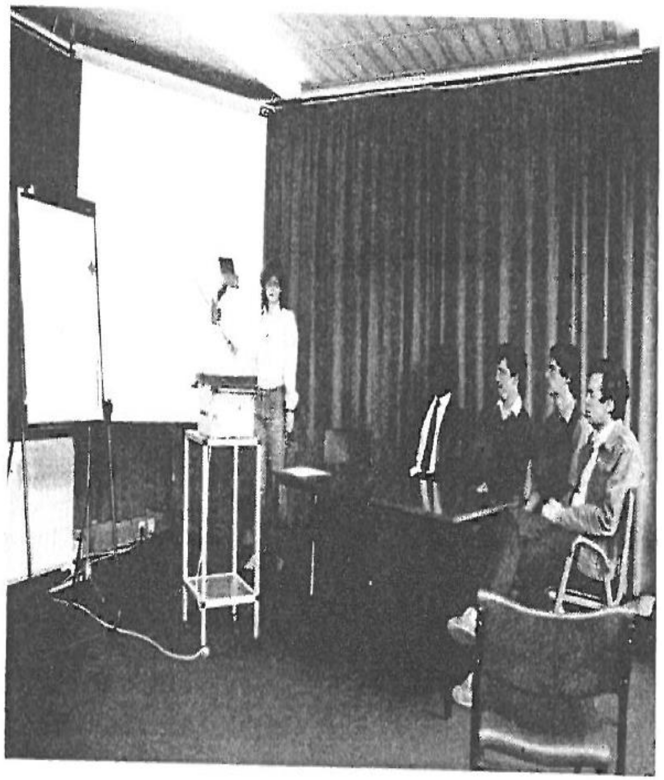
(Vos, 2015)

Simulation



(Vos, 2015)

Simulation



Management meeting (Edwards & Morton, 1987)

Terminology - simulation, gaming and role-play

- Confused in literature (Armstrong, 2003)
- Simulation is "a simplified reproduction of part of a real or imaginary world" (van Ments, 1999, p. 3)
- Gaming is "a structured system of competitive play that incorporates the material to be learned" (van Ments, 1999, p. 3)
- Role-play is "a make-believe representation of some real-life event, carried out in order to help participants [who play a role] get better at managing the event itself" (McGuire & Priestley, 1981, p. 87)

Role playing in education

- Has been known as an activity for education since 1940 (Williams, 2014) and it has gained momentum
- In school, role-playing occurs most frequently in the humanities, i.e. history, literature, politics, and sociology (van Ments, 1999), also in business (Armstrong, 2003) and healthcare education (Nestel & Tierney, 2007)
- Emphasizing decision making (e.g. Belova et al., 2013)
- ERIC - 44 hits
- Research into role play and mathematics with older students seems to be non-existent (Williams, 2014)

Role playing in education

- The idea is ”to give students the opportunity to practice interacting with others in certain roles. The situation is defined by producing a scenario and a set of role-descriptions. The scenario gives a background to the particular problem or environment and indicates the constraints which operate. The role-descriptions give profiles of the people involved” (van Ments, 1999, p. 9)

Powerful, motivating, provide meaningful contexts, increase students’ skills in collaboration and communication (Griffiths, 2005; Gifford, 2005; Ginsburg 2002, 2009; van Ments, 1999; Williams, 2014).

Role playing in education

"The findings suggest that role play is useful for mathematical learning and that it is possible to engage in complex mathematics through role play. I argue that the potency of role play is its ability to suspend disbelief and engage children as participants in a community of learners. This study also concludes there is potential for developing children's mathematical awareness and metacognition through reflecting on role play." (Williams, 2014, p.3)

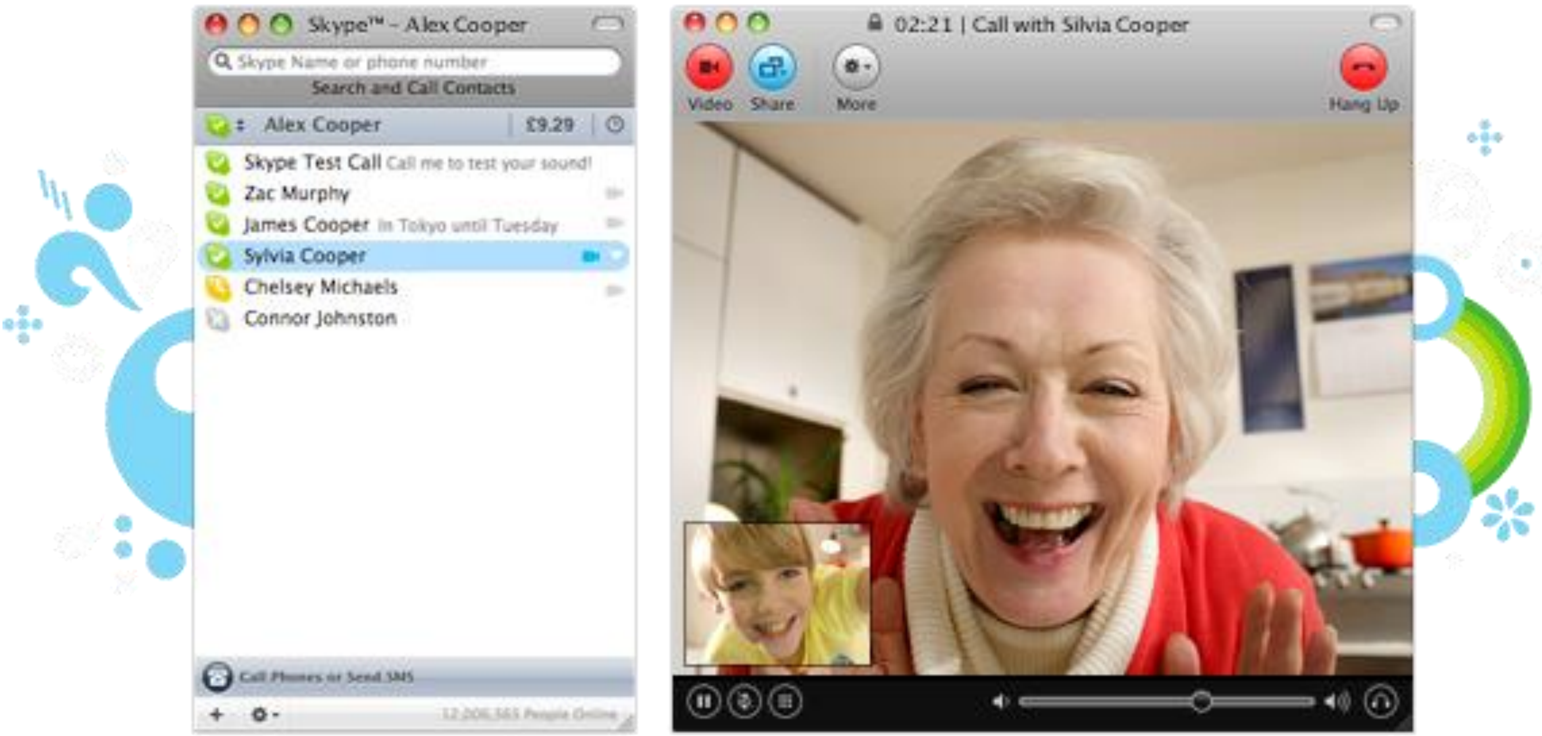
In tackling mathematical content within the role play, children were observed thinking mathematically, engaged in in gathering and ordering information, analysing information and making conjectures (Williams, 2014)

Role playing in education

- An example of a role play emphasizing aspects found in ‘modelling as a professional activity’

Role playing in education

The scenario



Role playing in education

The scenario

“The long-term goal of the predator policy in Sweden is to **achieve and conserve a healthy population** of wolf, bear, wolverine, lynx and golden eagles. A credible predator policy also requires an active management to strengthen the confidence in the predator policy and contributes to a better coexistence between humans and large predators “(p.14)... “the aim is to create a **good balance between the predator population and the impact it causes on business, public and individual interests**” (p. 16)... (SOU 2012/13:191, my translation)

How many wolves should we have in Sweden?

Role playing in education

Role-descriptions

ROLE-DESCRIPTION

Sheep farmer Liz Goat

You are the head of the sheep farmers association in Sweden and you own a farm with 500 sheep that has been attacked by Wolves several times during the years and lately the attacks have escalated.

Last weekend you protested outside house of parliament with the banner reading: "*Today famers, tomorrow unemployed.*"

You argues: "*There is nothing natural about being eaten by wolves. We are against wolves from the moment they attack our farms. The continued threat of wolf attacks on sheep is an enormous daily stress, it is omnipresent and oppressive, and farmers around me feel helpless. Those who wanted to overprotect them are going to kick themselves. The wolf reproduces and moves around very fast,*"

You are living under the motto "*We want to be able to do our job in good conditions*"

Your goal is to reduce the number of wolves. As the head of the sheep farmers association, you will be the leader of a group of experts that will explore statistics on sheep attacks by wolves and it impacts it cause on farms. Your work will consist of coordinate and assist the other participants in your group so they will develop a mathematical model that describes a health wolf's population, based on your economic interests and your working conditions.

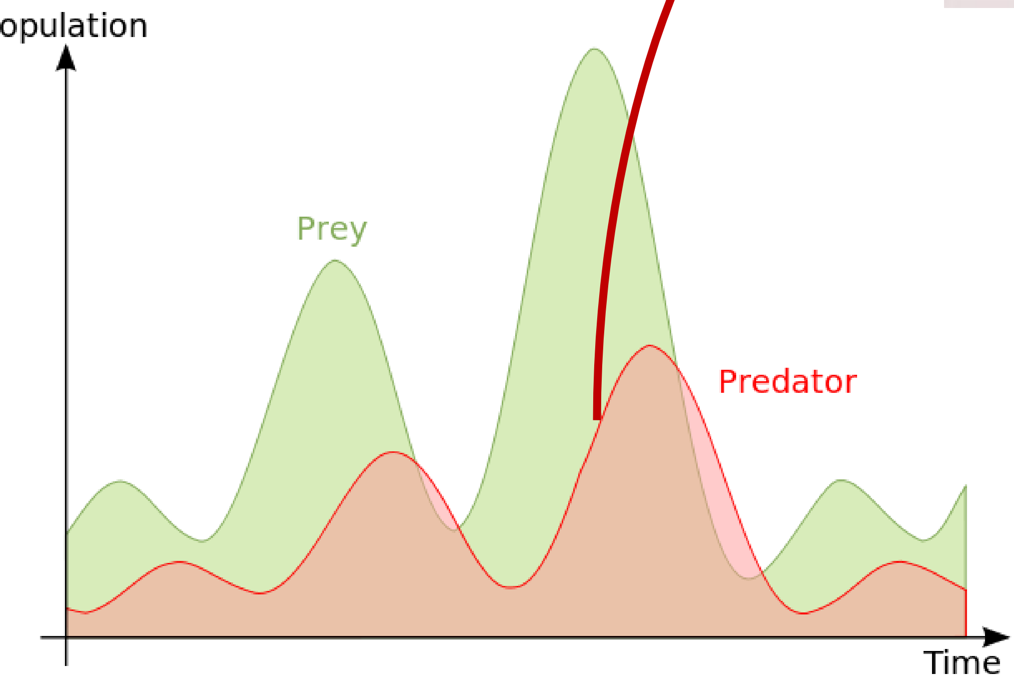
Liz Goat in action

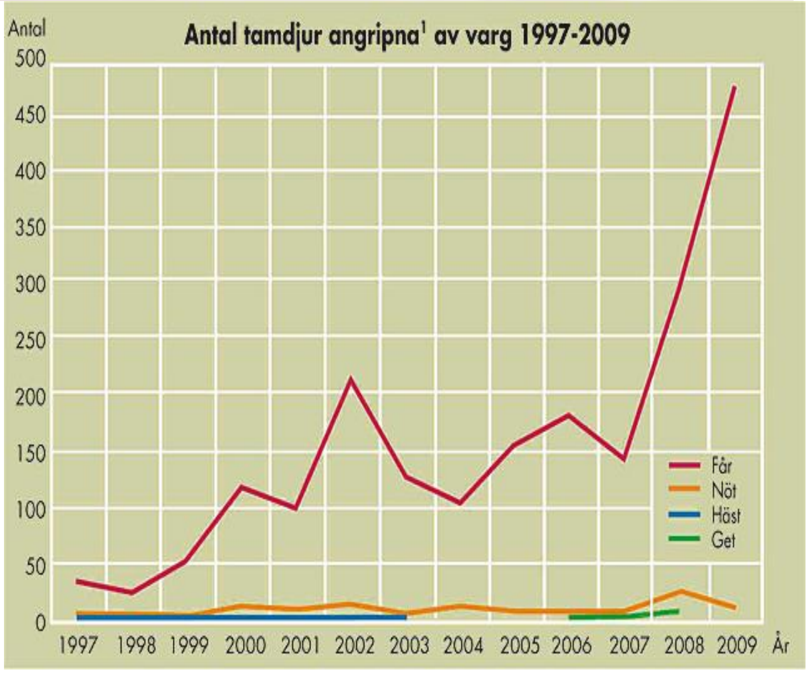
Role playing in education

Role-descriptions

Students preparation/ research

$$\frac{dx}{dt} = x(\alpha - \beta y)$$
$$\frac{dy}{dt} = -y(\gamma - \delta x)$$





Role playing in education

Expert guidance/expert opinions/ expert evaluation

Role playing in education

The presentation

(Edwards & Morton, 1987)

Role playing in education

Debriefing

The teacher may allow an opportunity to discuss and correct errors that may occur during or after the role-play. (van Ments, 1999)

Role playing in education

Implementing role-play

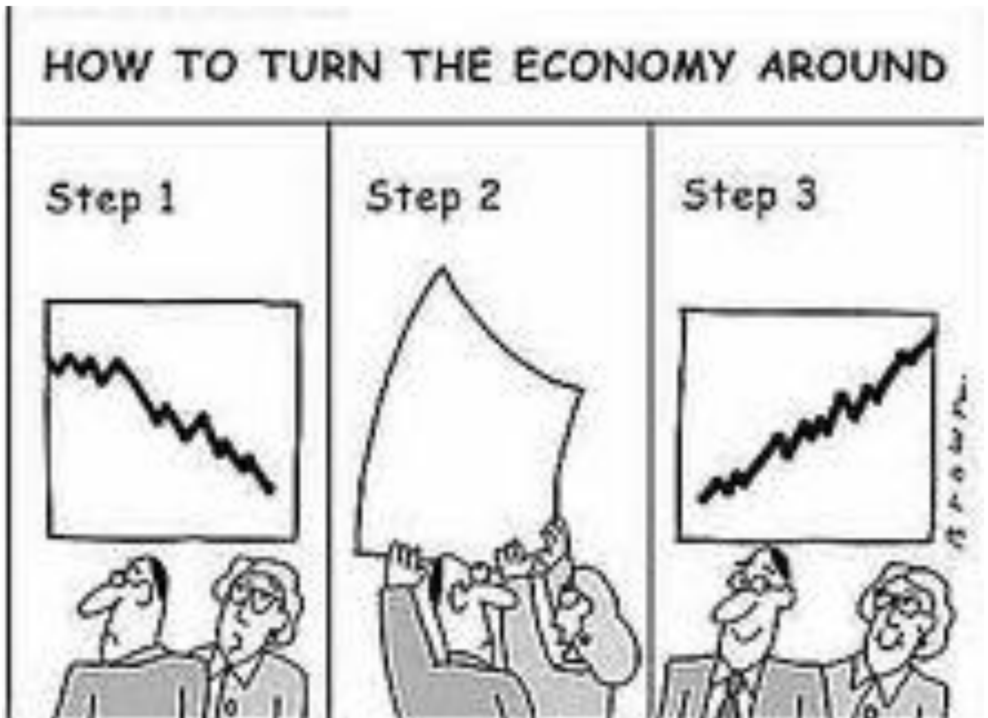
1. Objectives
2. Choose Context & Roles
3. Introduction
4. Student preparation/research
5. The Role-Play
6. Debriefing
7. Assessment

http://serc.carleton.edu/introgeo/roleplaying/howto.html

Future research (some examples)

1. What challenges and opportunities are there of using role-play for teaching and learning modelling?
2. To what extent is it possible to simulate professional modelling practice with the use of role play?
3. What are the various theories relating to the contribution of role play to learning modelling?
4. What are students attitudes in relation to role play and modelling?
5. How may assessment of modelling be organized in the activity of role play?

Thank you for your attention!



MATHEMATICAL MODELLING AS A PROFESSIONAL ACTIVITY – LESSONS FOR THE CLASSROOM

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