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Good Donors or Good Recipients? A Repeated Moral Hazard Model of Aid Allocation

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Alessia Isopi and Fabrizio Mattesini

Abstract

We propose a repeated moral hazard model with full commitment and limited punishment to study the problem of aid allocation in environments characterized by asymmetric information. The donor (*principal*) finances a three-period development program and the elite of the recipient country (*agent*), involved in the realization of the project, can affect the final output through adequate policies. The donor has the goal to help the poor of the recipient country, but she may also be conditioned by non altruistic motives. We show that when the moral hazard problem is relevant, under a wide set of parameter values, optimal aid contracts should be conditional on the previous result of the project. We distinguish between *weak conditionality*, which means that aid depends only on the previous performance of the project and *strong conditionality*, which means that aid depends on the whole history of the project. Unconditional aid may be an optimal contractual arrangement for the donor if the moral hazard issue is not very important or if the donor gives aid merely for strategic or economic reasons. An entirely altruistic donor will never provide unconditional aid. On the other hand, if she has a strong desire to help the recipient she should never deny aid to it.

JEL Classification: F35, D82

Keywords: Foreign Aid, Conditionality, Moral Hazard

Centre for Research in Economic Development and International Trade, University of Nottingham

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Outline

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Abstract

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1 Introduction

Despite the large amount of resources employed to assist developing countries in the last fifty years, almost everybody today agrees that foreign aid does not work as it should, raising serious doubts on whether the current practices in aid allocation are appropriate with respect to the main goals of development assistance [1]. One of the major factors behind the failure of foreign aid in promoting growth is that, in developing countries, the implementation of aid projects is plagued with information/incentive issues that seriously undermine their effectiveness.

Aid projects usually require the involvement of a *donor* (multilateral organization or a single country) that through an *aid agency* and a *contractor* administer the details of the aid program and a *recipient*, i.e. the bureaucratic elite of the recipient country that locally implements the aid project. Although the involvement of local elites is crucial to the success of the projects [2] the conflict of objectives that often arises between donors and recipients, and the possibility that recipients use aid devoted to projects and programs to advance their own interests rather than pursuing the aims of the donors give rise to both moral hazard and adverse selection problems.

One remedy that is often advocated to solve this type of problems is *conditionality*, i.e. the practice of linking the disbursement of aid to some specific actions undertaken by the recipient country or to some results. Conditionality may be applied *economy wide* as with IMF performance criteria or many of the provisions of the World Bank's structural adjustments programs; but it can be also applied to an individual development *project*, with requirements for actions that will affect its viability and/or results obtained in a previous stage of the project (Killick et al., 1998).

At the macro level, conditionality *on good policies* has been extremely popular since the beginning of the 90's[3], especially among international financial organizations[4], although the debate on its effectiveness is still opened[5] and some authors have emphasized

[1] For an extensive discussion on this issue see Hansen and Tarp (2001) and Rajan and Subramanian, (2005).

[2] As a matter of fact, the DAC Principles clearly state that "*Development assistance is a co-operative partnership exercise between donors and recipients. (...) Aid supports activities for which developing countries have final responsibility and ownership. Project performance depends on both donor and recipient action. Both have an interest in, and responsibility for, the best use of scarce public funds* (DAC, 2000)."

[3] The second generation conditionality we make reference to is mainly devoted to promote democratic reform, human rights and economic development.

[4] On this issue, see for example the Review of World Bank Conditionality(2005).

[5] Empirical studies on the effectiveness of conditionality include Collier (1997), Devarajan et al. (2001), Dollar and Svensson (1998), Killick (1995, 1997), Killick et al. (1998), Mosley et al. (2003) and World Bank (1998), among others.

its limits, deriving from the time inconsistency of conditional aid policies [6].

When we focus on individual projects, instead, conditionality on performance does not appear to be a widely used practice. Econometric evidence on this issue is very limited, exception made for Isopi and Mavrotas (2009) that show that the majority of donors still disburse aid without a serious assessment of previous results. Some descriptive evidence on the lack of incentive schemes in aid allocation, however, can be found in Hoebink and Stokke (2005), that study the performance of individual European donors. Not only donors seem to disburse aid without assessing previous performance, but often do not even evaluate or verify whether recipients have accomplished the required tasks. A developed culture for evaluating cooperation projects and policies, for example, seems missing in Italy and in France. In Finland, after an early evaluation came up with highly critical conclusions, no serious attempt was made for more than a decade to re-evaluate the impact of Finnish foreign aid. In Germany, planning and monitoring of projects suffered for long time from the lack of coordination among the several private organizations appointed for that, which resulted in analysis of general development themes rather than proper projects reviews. In Spain "*The absence of an evaluation of activities until very recently was especially significant because it has impeded self-correction and the learning process derived from a critical analysis of previous experiences*"(Hoebink and Stokke, 2005).

As Easterly (2006) and Easterly and Pfuntze (2007) have shown, the only requirement that recipients are frequently asked to meet is that foreign aid is spent on goods and services provided by the donor, (i.e. *tied aid*).

Two interesting questions, then, arise: i) why do donors seem so reluctant to use incentive schemes in the implementation of aid projects? Is it simply due to institutional inertia and bureaucratic inefficiency or rather to rational behavior? ii) What types of "aid contracts" should donors offer to achieve internationally shared objectives, such as lifting countries out of poverty?

To answer these questions, we propose a model that analyzes the optimal aid allocation process in environments where i) donors aim at helping the poor, but may also be conditioned by non-altruistic motives; ii) local elites or local bureaucracies must actively cooperate for the realization of aid projects; iii) significant inefficiencies arise in the implementation of aid projects, due to asymmetric information and moral hazard; iv) donors are limited in the possibility of punishing recipients for bad performance by a zero bound on the level of aid. Since aid disbursement is not usually a one shot phenomenon but is rather a multiperiod relationship, we build on the repeated moral hazard literature developed, among others, by Lambert (1983), Holmstrom (1982), Fudenberg and Tirole (1990), Laffont and Martimort (2002) and we analyze a situation where the

[6] See, among others, Svensson, (2003); Drazen, (2002); Candel-Sánchez, (2008).

government of a donor country agrees to finance a multi-period project, such as a poverty eradication program.

Applications of agency models to conditionality are usually found at a macro-level, focusing in particular on multilateral lending. Murshed and Sen (1995) use a principal-agent model to capture the stylized facts of multilateral aid negotiations on non economic conditionality such as military expenditure decrease. Svensson (2000, 2003) and Candel-Sánchez (2008) study the time inconsistency problem that, lacking a commitment technology, arises when a donor tries to make aid conditional on economic reforms. Drazen (2002) analyzes the effects of multilateral conditionality when the IMF's objectives are shared by the government in power, but also face strong domestic opposition, concluding that, in some cases, unconditional lending is better than conditionality. Calmette and Kilkenny (2001) study the effect of asymmetric information when international charity is given to developing countries and conclude that poorest recipients are those that pay the highest costs associated with asymmetric information. Azam and Laffont (2003) and Cordella and Dell'Ariccia (2002) propose adverse selection models where the governments of donor and recipient countries have different preferences over the allocation of aid. Azam and Laffont (2003) highlight how aid contracts can be designed to induce governments to reveal their preferences toward the poor. Cordella and Dell'Ariccia (2002) show that making aid conditional on some observable public expenditure may be optimal under imperfect information but may imply aid rationing.

Our model differs from these papers in several aspects. We concentrate on micro-implementation of aid project and we focus on bilateral aid. This allows us to depart from the usual assumption that donors are entirely altruistic and to explicitly consider the possibility that donors' policies are driven also by strategic/economic considerations[7]. The case of an entirely altruistic donor can be seen as a particular case of our more general framework. Moreover, we abstract from commitment issues and we assume that a donor can always carry out the contracts signed *ex-ante* by donors and recipients. Finally, we consider an environment where donors, instead of conditioning the disbursement of aid to some structural reform or macroeconomic policy indicator, can actually condition the disbursement of aid to the previous results of the project which represents an imperfect signal of the level of effort exerted by the elite that leads the recipient country. Actually, when we consider bilateral aid, conditionality on performance is often the only type of conditionality that can be imposed. Unlike a multilateral institution, a bilateral donor is often not in the position, politically, to require institutional reforms or macroeconomic policies.

[7] The empirical literature on aid allocation has highlighted that when we consider bilateral aid allocation economic/ strategic interests must be taken into account besides poverty reduction (Alesina and Dollar, 2000).

Within this set-up, we show that although we assume full commitment it may be optimal, for a donor, to offer unconditional aid contracts. The choice between conditional and unconditional aid is a crucial consequence of the motivations of the donor. When the donor has a strong interest in favoring the elite of the recipient country for strategic/economic reasons, she will want to minimize the costs that an incentive mechanism imposes on this social group and might find it optimal to accept an inefficient use of resources by the recipient.

Conditionality on results is always optimal when there are significant informational asymmetries and donors are more oriented toward recipient needs. When entirely altruistic, a rational donor will always offer conditional contracts. This allows us to obtain an important normative implication: if the objective of the donor is lifting a developing country out of poverty, she must try to impose some form of incentive mechanism that eliminates a wasteful use of resources by the local elite.

Conditionality, when optimal, may take different forms, depending on the memory of the process required by the structure of incentives. We distinguish between two types of conditionality: *strong conditionality* and *weak conditionality*. In the first case, aid is tied to the whole history of the project; in the second case, only the recent performance of the recipient country is taken into account in disbursing aid. This second possible outcome is a consequence of the limit that we impose on the maximum punishment that the recipient can get. Because of this, a recipient may always have the incentive to participate on the project and the "right" incentives can be induced by conditioning aid only on the performance of the previous period.

The preferences of the donor affect also the size of aid contracts. We consider the weighted sum of the utilities that a donor gets from giving aid to the elite of the recipient country (for strategic/economic reasons) and to the poor (for purely altruistic reasons) as a measure of the *interest* that the donor has for a specific recipient and we find that the higher is this *interest*, the higher is the level of aid that the donor will grant, in every possible state of the world. In our model, optimal conditional contracts allow for the possibility that recipient countries receive no aid after a bad performance but we find that a donor will never deny aid if she is very concerned about the welfare of the recipient country, either for altruistic reasons or for strategic/economic motives.

In Section 2, the setup of the model is described. Section 3 defines the full information case, while optimal foreign aid contracts under incomplete information are derived in Section 4. Section 5 concludes.

2 The Model

The problem we study in this paper is a typical situation that arises in foreign aid allocation. A donor wants to improve the economic conditions of a developing country, with the particular (but not exclusive) intention of enhancing the welfare of the poor. The country, however, is run by another social group (the elite) that gets a share of total output and, by managing the resources provided by the donor, has a crucial influence on the outcome of development projects.

This setting can be modeled as a *dynamic principal-agent model with limited punishment*, where three players agree on a contract in order to produce a certain level of output. The principal (the donor) is altruistic in the sense that her utility depends on the utility of the other players, that we call type I agents and type II agents. Type I agents (the elite of the recipient country) are endowed with the technology needed to carry out production and decide both on whether to accept the contract and on the level of effort. Type II agents (the poor) do not own any technology and do not choose the level of effort, but benefit from the actions of the other two players. The principal commits to transfer a nonnegative amount of resources to both agents, conditional on the actions undertaken by type I agents. Contracts last three periods. Differently from the standard principal-agent model, where the transfer is simply a reward to the agent for the effort exerted, here it is also the input required each period to carry out production in the following stage. The principal determines the transfer on a period by period basis, to finance the different stages of the project, after having observed past outcomes. In order to get the project off the ground, we assume that type I agents provide an initial level of capital exogenously given, a_0.

We assume that the level of output of the project produced in the economy is given by[8]:

$$q_t(\theta_t, a_{t-1}) = \theta_t + a_{t-1} \tag{1}$$

where θ_t is a random variable that can take only the values $\{\bar{\theta}; \underline{\theta}\}$ where $\bar{\theta} > \underline{\theta}$ and a_{t-1} is the amount of resources transferred by the principal in period $t-1$ and used by type I agents in next period's production. A type I agent can choose between a high level of effort, $i = 1$ or a low level of effort $i = 0$. The stochastic influence of a type I agent's action on production is characterized by the probabilities $Pr(\theta = \bar{\theta}|i = 1) = \pi_1$ and $Pr(\theta = \bar{\theta}|i = 0) = \pi_0$ with $\pi_1 > \pi_0$. Undertaking the high level of effort implies a disutility ψ. Effort improves production in the sense of first order stochastic dominance, i.e. $Pr(\theta \leq \theta^* \mid i)$ is decreasing in i for any given θ^*. The principal cannot observe

[8]We choose a production function where the random component enters additively in order to keep the analysis as simple as possible.

the level of effort exerted by type I agents, but only the level of output that is realized in each period. Upon observing the realization of output, the principal decides on the transfer a_t.

Type I agents are always able to capture a fraction γ of the output q_t. We assume that this fraction is exogenous. The parameter γ is meant to reflect the income distribution in the recipient country and expresses not only the contribution that type I agents give to the project due to the ownership of the technology, but also the political power they have inside the country. Type II agents are always able to capture a fraction $(1-\gamma)$ of the final output. Undertaking $i=1$, however, implies for type I agents a cost in terms of utility, ψ. The program has the following structure:

1. *time 0*: an amount a_0 is provided by type I agents to start the project. This amount is fixed and exogenously given and represents the start up cost of the project;
2. *time 1*: either an output $\bar{q}_1 = q(a_0, \bar{\theta})$ is realized or an output $\underline{q_1} = q(a_0, \underline{\theta})$. Upon observing the realization of the project, the donor determines an amount of resources $a_1(q_1)$, $q_1 \in [\bar{q}_1, \underline{q_1}]$ to devote to the continuation of the project;
3. *time 2*: either an output $\bar{q}_2 = q(a_1, \bar{\theta})$ is realized or an output $\underline{q_2} = q(a_1, \underline{\theta})$. After having observed q_2, the donor determines $a_2(q_2)$, $q_2 \in [\bar{q}_2, \underline{q_2}]$ and delivers it to the recipient;
4. *time 3*: either an output $\bar{q}_3 = q(a_2, \bar{\theta})$ is realized or an output $\underline{q_3} = q(a_2, \underline{\theta})$. The project is completed and the game is over.

The intra period utility function for type I agents is given by:

$$u^e_{0,t} = \gamma[\pi_0(\bar{\theta} + a_{t-1}) + (1-\pi_0)(\underline{\theta} + a_{t-1})] \tag{2}$$

if they undertake no effort and

$$u^e_{1,t} = \gamma[\pi_1(\bar{\theta} + a_{t-1}) + (1-\pi_1)(\underline{\theta} + a_{t-1})] - \psi \tag{3}$$

if they exert effort. The intra period utility function of type II agents is given by:

$$u^p_{0,t} = (1-\gamma)[\pi_0(\bar{\theta} + a_{t-1}) + (1-\pi_0)(\underline{\theta} + a_{t-1})] \tag{4}$$

if type I agents undertake no effort and

$$u^p_{1,t} = (1-\gamma)[\pi_1(\bar{\theta} + a_{t-1}) + (1-\pi_1)(\underline{\theta} + a_{t-1})] \tag{5}$$

if type I agents exert effort.

The donor country gives aid in order to enhance the development process in the recipient country. His main objective is to favor the poor of the recipient country and this is captured by the fact that the donor maximizes the utility of type II agents. However, the form of the donor's utility function takes into account the effects that its actions have on the utility of type I agents which, in this model, have an active role in the production process. This intends to capture the idea that donors may not be entirely oriented toward the needs of the poor but may also desire to influence the elite of the recipient country for political and strategic/economic reasons.

Hence, the instantaneous utility function of the donor is a weighted average of the utility of the two social groups in the recipient country, minus the cost of giving aid, that is:

$$V_d = E[\lambda u^e + (1-\lambda)u^p] - C(a_t) \tag{6}$$

The parameter λ represents the weight that the donor assigns to the utility function of each population group in the recipient country. The cost function $C(a_t)$ represents the opportunity cost incurred by the donor in subtracting resources that could otherwise be directed towards domestic investment. We assume a quadratic cost function with $\delta > 0$:

$$C(a_t) = \delta a_t^2. \tag{7}$$

Define, now

$$F = [\lambda\gamma + (1-\gamma)(1-\lambda)]; \quad \tilde{\theta} = \pi_0\bar{\theta} + (1-\pi_0)\underline{\theta}; \quad \hat{\theta} = \pi_1\bar{\theta} + (1-\pi_1)\underline{\theta}; \quad \Delta\pi = \pi_1 - \pi_0$$

We normalize the model by setting $\tilde{\theta} = \pi_0\bar{\theta} + (1-\pi_0)\underline{\theta} = 0$. Moreover, in order to simplify the notation, we denote the conditional contracts in the following way: $\bar{a}_1 = a_1(\bar{q_1})$ and $\underline{a_1} = a_1(\underline{q_1})$ represent the first period transfers. Similarly, $\bar{a_2}(\bar{q_1}) = a_2(q_2(\bar{\theta}, a_1(\bar{q_1})))$ and $\bar{a_2}(\underline{q_1}) = a_2(q_2(\bar{\theta}, a_1(\underline{q_1})))$ and $\underline{a_2}(\bar{q_1}) = a_2(q_2(\underline{\theta}, a_1(\bar{q_1})))$ and $\underline{a_2}(\underline{q_1}) = a_2(q_2(\underline{\theta}, a_1(\underline{q_1})))$ represent the second period transfers after the realization of output q_2.

If type I agents maximize the first best contract, denoting by $\epsilon > 1-\pi_1$ the intertemporal discount factor, then the donor's objective function is given by:

$$\begin{aligned} V_1 \quad = \quad & \pi_1\Big\{F[\hat{\theta} + \bar{a_1}] - \lambda\psi - \delta\bar{a_1}^2 + \epsilon\Big[\pi_1\Big(F[\hat{\theta} + \bar{a_2}(\bar{q_1})] - \lambda\psi - \delta\bar{a_2}^2(\bar{q_1})\Big) + \\ & (1-\pi_1)\Big(F[\hat{\theta} + \underline{a_2}(\bar{q_1})] - \lambda\psi - \delta\underline{a_2}^2(\bar{q_1})\Big)\Big]\Big\} + (1-\pi_1)\Big\{F[\hat{\theta} + \underline{a_1}] - \lambda\psi - \delta\underline{a_1}^2 + \\ & \epsilon\Big[\pi_1\Big(F[\hat{\theta} + \bar{a_2}(\underline{q_1})] - \lambda\psi - \delta\bar{a_2}^2(\underline{q_1})\Big) + (1-\pi_1)\Big(F[\hat{\theta} + \underline{a_2}(\underline{q_1})] - \lambda\psi - \delta\underline{a_2}^2(\underline{q_1})\Big)\Big]\Big\} \end{aligned} \tag{8}$$

Usually, in principal-agent models, the principal wishes that the agent undertakes the best level of effort and therefore optimal contracts are found by maximizing the principal's objective function under the assumption that the agent chooses $i = 1$. In our model, however, this is not necessarily true, since the donor maximizes a weighted sum of the utility of type I and type II agents. Since in turn, the welfare of type I agents depends on the cost of exerting the high level of effort, the donor internalizes this cost. This implies that it is non necessarily optimal for a donor to always induce good behavior by type I agents.

Define donor's utility when type I agents undertake the low level of effort as:

$$\begin{aligned} V_0 \quad = \quad & \pi_0 \Big\{ F\bar{a_1} - \delta\bar{a_1}^2 + \epsilon\Big[\pi_0\Big(F\bar{a_2}(\bar{q_1})] - \delta\bar{a_2}^2(\bar{q_1})\Big) + (1-\pi_0)\Big(F\underline{a_2}(\bar{q_1}) - \delta\underline{a_2}^2(\bar{q_1})\Big)\Big\} \\ + (1-\pi_0)\Big\{ & F\underline{a_1} - \delta\underline{a_1}^2 + \epsilon\Big[\pi_0\Big(F\bar{a_2}(\underline{q_1}) - \delta\bar{a_2}^2(\underline{q_1})\Big) + (1-\pi_0)\Big(F\underline{a_2}(\underline{q_1}) - \delta\underline{a_2}^2(\underline{q_1})\Big)\Big]\Big\} \end{aligned} \tag{9}$$

If the overall utility a donor obtains when type I agents undertake the high level of effort is greater than the utility she obtains when type I agents undertake the low level of effort, i.e. if:

$$V_1 > V_0 \tag{10}$$

then equilibrium contracts are those that maximize (8) subject to the intertemporal incentive compatibility constraint:

$$\begin{aligned} & \gamma\Delta\pi\Big(\bar{a_1} - \underline{a}_1\Big) + \epsilon\gamma(\Delta\pi)^2\Big[\bar{a_2}(\bar{q_1}) - \underline{a}_2(\bar{q_1})\Big] \geq \\ & \qquad (1+\epsilon)\Big[\psi - \gamma\hat{\theta}\Big] + \epsilon\gamma(\Delta\pi)^2\Big[\bar{a_2}(\underline{q_1}) - \underline{a}_2(\underline{q_1})\Big] \end{aligned} \tag{11}$$

the intertemporal participation constraint:

$$\begin{aligned} & \pi_1\Big\{\gamma[\hat{\theta} + \bar{a_1}] + \epsilon\Big[\pi_1\gamma[\hat{\theta} + \bar{a}_2(\bar{q_1})] + (1-\pi_1)\gamma[\hat{\theta} + \underline{a}_2(\overline{q_1})]\Big]\Big\} + \\ & \quad (1-\pi_1)\Big\{\gamma[\hat{\theta} + \underline{a_1}] + \epsilon\Big[\pi_1\gamma[\hat{\theta} + \bar{a}_2(\underline{q_1})] + (1-\pi_1)\gamma[\hat{\theta} + \underline{a}_2(\underline{q_1})]\Big]\Big\} \geq (1+\epsilon)\psi \end{aligned} \tag{12}$$

and the non negativity constraints:

$$\bar{a_1} \geq 0; \quad \underline{a_1} \geq 0; \quad \bar{a}_2(\bar{q_1}) \geq 0; \quad \bar{a}_2(\underline{q_1}) \geq 0; \quad \underline{a}_2(\bar{q_1}) \geq 0; \quad \underline{a}_2(\underline{q_1}) \geq 0. \tag{13}$$

When instead (10) is not satisfied, then the donor maximizes (9) subject to (13) and:

$$\begin{aligned} & \pi_0\Big\{\gamma\bar{a_1} + \epsilon\Big[\pi_0\gamma\bar{a}_2(\bar{q_1}) + (1-\pi_0)\gamma\underline{a}_2(\overline{q_1})\Big]\Big\} + \\ & \qquad (1-\pi_0)\Big\{\gamma\underline{a_1} + \epsilon\Big[\pi_0\gamma\bar{a}_2(\underline{q_1}) + (1-\pi_0)\gamma\underline{a}_2(\underline{q_1})\Big]\Big\} \geq 0 \end{aligned} \tag{14}$$

It is worth, at this point, to emphasize the peculiarities of this model with respect to the standard repeated moral hazard framework[9]. First, the transfer from the principal

[9]See for example Laffont and Martimort, (2000).

to the agent is not a pure reward but is also an input in next period's production. Therefore, in deciding the transfer, the principal must weight the need to provide the "right" incentives to type I agents with the need to provide for the continuation of the project. Second, by imposing the non negativity constraints on the level of aid, we limit the possible punishment that the donor can inflict to the agent. Third, the particular utility function of the donor (i.e. a weighted sum of the utility of type I and II agents), implies that a transfer is not only a cost to the donor but also a benefit[10].

The preferences of the donor are expressed by the composite parameter F, which is the sum of two terms: i) the weight a donor assigns to the elite in her utility function multiplied by the share of output the elite receives and ii) the weight a donor assigns to the poor multiplied by the share of output that the poor receive. We can then consider F as an indicator of the *interest* that the donor has for the recipient country as a whole. If the elite receives a large amount of output (γ is high), a high F, due to a high λ, implies that the donor is interested on the welfare of the recipient country because of strategic/economic reasons. On the other hand, if γ is low, i.e. the poor receive a high fraction of the output, a high F means that the donor is altruistic, i.e. is concerned in the welfare of the poor.

3 Complete Information

Assume, first, complete information. If the agent does not choose the high level of effort, his deviation can be properly detected both by the principal and the third party. The agent can be punished and the third party commits to always enforce the punishment.

The optimal contract is the one that solves the principal's maximization problem (8) subject to type I agents' participation constraint (12) and the non negativity constraints (13). Define first

$$\omega = \frac{\psi - \gamma\hat{\theta}}{\gamma\Delta\pi}$$

we can, then, state:

Proposition 1. *Equilibrium contracts imply either a constant transfer in every possible state and time, or a transfer which is independent of the realization of θ but increasing over time. In particular,*

1. *if* $\frac{F}{2\delta} \geq \Delta\pi\omega$*:*

$$\bar{a}_1 = \underline{a_1} = \bar{a}_2(\bar{q}_1) = \bar{a}_2(\underline{q_1}) = \underline{a}_2(\bar{q}_1) = \underline{a}_2(\underline{q_1}) = \frac{F}{2\delta}; \tag{15}$$

[10]Observe that is reasonable as donors do receive a benefit from being seen to give aid.

2. *if* $\frac{F}{2\delta} < \Delta\pi\omega$:

$$\bar{a}_1 = \underline{a_1} = \frac{\psi - \gamma\hat{\theta}}{\gamma} \tag{16}$$

$$\bar{a}_2(\bar{q_1}) = \bar{a}_2(\underline{q_1}) = \underline{a}_2(\bar{q_1}) = \underline{a}_2(\underline{q_1}) = 2\Big(\frac{\psi - \gamma\hat{\theta}}{\gamma}\Big). \tag{17}$$

Proof 1. *See Appendix.*

Proposition 1 implies two possible equilibria. In the first one, the principal will always provide the same unconditional transfer either in period $t = 1$ or $t = 2$. In the second equilibrium, transfers are again independent of the state of the world, but they are increasing in the cost of effort, ψ. Period 2 transfers are larger than period $t = 1$ transfers.

The first equilibrium occurs when ψ is low and/or $\frac{F}{2\delta}$ is high, i.e. when the cost of exerting effort is low relative to the interest of the donor on the welfare of the recipient country. In this case, participation by the agent is always guaranteed and, given the absence of a moral hazard problem, the principal is free to set the level of aid in every period only on the basis of her preferences.

The second type of contracts prevails if $\gamma\frac{F}{2\delta} + \gamma\hat{\theta} \leq \Delta\pi\psi$. In this case, local elites find it very costly to undertake the agreed project and the donor does not have a strong motivation for helping the recipient. The level of aid must therefore be sufficient to compensate this social group for its effort and to induce its active involvement. The equilibrium contract does not depend on the preferences of the principal but is determined only by the participation constraint and must be increasing in ψ. In order to induce type I agents to participate in the contract in both periods, the reward in period 2 must be greater than the reward in period 1.

Summing up, under complete information, the donor will offer unconditional contracts. In both cases, the elite receives full insurance from the risk-neutral donor and the transfer is independent of the realization of θ. However, in the first equilibrium, the preferences of the donor toward the recipient country determine the size of the aid transfers across periods. In the second equilibrium, the level of the transfers is simply increasing in the cost of effort and over time.

4 Incomplete Information

Assume, now, that the effort exerted by the agent is not observed by the principal. Differently from the complete information case, optimal contracts must also satisfy the incentive compatibility constraint (11). We start our analysis by assuming that condition (10) is satisfied, i.e. that it is optimal for the principal to always induce type I agents to undertake the high level of effort. We will then explore under what conditions this assumption does not hold. Therefore, solving problem (8)-(13), we can state:

Proposition 2. *Suppose* (10) *holds. We distinguish between two possible types of contracts. The first occurs when $\omega \leq 0$. In this case, an optimizing donor offers unconditional contracts in every period. The aid transfers are given by:*

$$\bar{a}_1 = \underline{a}_1 = \bar{a}_2(\bar{q_1}) = \underline{a}_2(\bar{q_1}) = \bar{a}_2(\underline{q_1}) = \underline{a}_2(\underline{q_1}) = \frac{F}{2\delta} \tag{18}$$

The second type of contract occurs when instead $\omega > 0$. In this case, long term equilibrium contracts may exhibit weak *or* strong conditionality*. Under* weak conditionality*, the optimal level of aid depends only on the outcome of the project of the previous period. Under* strong conditionality*, the optimal level of aid is tied to the outcome of the project in any previous periods.*

Weakly conditional contracts arise when $\frac{F}{2\delta} > \Delta\pi\omega$. In this case:

$$\bar{a}_1 = \bar{a}_2 = \frac{F}{2\delta} + (1 - \pi_1)\omega \tag{19}$$

$$\underline{a}_1 = \underline{a}_2 = \begin{cases} \frac{F}{2\delta} - \pi_1\omega & \text{if} \quad \frac{F}{2\delta} > \pi_1\omega & \text{(20a)} \\ 0 & \text{if} \quad \frac{F}{2\delta} \leq \pi_1\omega & \text{(20b)} \end{cases}$$

Strongly conditional contracts arise when $\frac{F}{2\delta} \leq \Delta\pi\omega$. The optimal payoffs are given by:

$$\bar{a}_1 = \frac{F}{2\delta} + (1 - \pi_1)\omega \tag{21}$$

$$\underline{a}_1 = \begin{cases} \frac{F}{2\delta} - \pi_1\omega & \text{if} \quad \frac{F}{2\delta} - \pi_1\omega \geq 0 & \text{(22a)} \\ 0 & \text{if} \quad \frac{F}{2\delta} - \pi_1\omega < 0 & \text{(22b)} \end{cases}$$

$$\bar{a}_2(\bar{q_1}) = \frac{F}{2\delta} + \omega(2 - \pi_1 - \pi_0) \tag{23}$$

$$\underline{a}_2(\bar{q_1}) = \bar{a}_2(\underline{q_1}) \begin{cases} \frac{F}{2\delta} + \omega(1 - \pi_1 - \pi_0) & \text{if} \quad \frac{F}{2\delta} + \omega(1 - \pi_1 - \pi_0) \geq 0 & \text{(24a)} \\ 0 & \text{if} \quad \frac{F}{2\delta} + \omega(1 - \pi_1 - \pi_0) < 0 & \text{(24b)} \end{cases}$$

$$\underline{a}_2(\underline{q_1}) = 0 \tag{25}$$

Proof 2. *See Appendix.*

Proposition 2 tells us that even if condition (10) is satisfied, unconditional contracts may be optimal. Unconditional contracts are offered when $\omega \leq 0$, which happens when ψ is low and/or γ is high. In this case, the moral hazard issue is not relevant and donors will offer the first best contracts. Since the cost of exerting effort for the elite is low and/or the elite is able to capture a large part of the outcome of the project, a donor does not need to impose an incentive mechanism and will set the level of aid unconditionally just on the basis of the parameters F and δ.

When, instead, $\omega > 0$ the donor will offer conditional contracts. In this model, conditionality may take different forms. An optimizing donor, in fact, must determine not only whether the recent performance is relevant in granting aid, but also whether the whole history of the project must be taken into account, i.e. whether contracts should exhibit memory. Aid can be made conditional either on the level of output obtained in all previous periods or on the results obtained only in the previous period. We label the first case as *strong conditionality* and the second as *weak conditionality*.

Strong conditionality occurs when $\frac{F}{2\delta} \leq \Delta\pi\omega$. In this case, a maximizing donor offers one contract with six conditional payoffs: two payoffs at time 1 that are conditional on the realization of the output in period 0 and four payoffs at time 2 that depend on the outcome of the project at time 0 and the outcome of the project at time 1. *Weak conditionality* instead occurs when $\frac{F}{2\delta} > \Delta\pi\omega$, in which case period 2 payoffs are made conditional on the realization of the output in period 1, but will not depend on what happened in period 0.

Strong conditionality is obtained when the participation constraint is binding in every period, while *weak conditionality* is obtained when the participation constraint in period $t = 2$ is not binding. The fact that the participation constraint is slack at $t = 2$ depends on the limit that we impose on the maximum punishment that the recipient can get. Since the recipient can, at most, receive no aid for the following period, it is possible that the donor offers contracts which will be always accepted by the recipient. When participation is always guaranteed, the need to establish a relationship between the level of aid granted in period 2 and the level of aid promised in period 1 is severed and therefore it is sufficient for a donor to make aid conditional only on the outcome achieved in the previous period.

Notice that while the choice between conditional or unconditional contracts depends only on ψ and γ, the type of conditionality depends also on the parameter F, which expresses the preferences of the donor country. The higher the utility the donor receives from granting aid, the more aid contracts will be favorable to the recipient and the more the participation of the agent is guaranteed. Therefore, *weak conditionality* will be more likely to occur when F is high, while *strong conditionality* is more likely to occur when F is low. The *interest* of the donor for the welfare of the recipient affects not only the degree of conditionality of aid contracts, but also the size of the aid transfers. In all cases, aid is larger the greater is F and the lower is δ.

Notice also that both under *strong* and *weak conditionality*, the moral hazard problem may be so strong that non negativity constraints become binding and the optimal transfers are equal to zero. The system of incentives is a system of reward and punishments: when the cost of exerting effort is very high, the principal will always apply the maximum punishment. Again, a no aid policy is more likely to be adopted the lower is $\frac{F}{2\delta}$. This highlights the role played by the preference of the principal in this model: the past performance of the recipient country is not a sufficient reason for denying foreign aid. A zero aid policy is also the signal of a lack of interest of the donor toward the recipient country.

As we anticipated at the beginning of this section, the results in Proposition 2 hold only if (10) is satisfied. We now explore under which conditions this assumption does not hold.

Proposition 3. *If*

$$\frac{[F(\lambda,\gamma)-\lambda\gamma]\hat{\theta}}{\Delta\pi\gamma}-\lambda\omega-\left\{\frac{\delta\omega^2\pi_1[(1-\pi_1)^2-\epsilon\kappa]}{(1+\epsilon)\Delta\pi\gamma}+\frac{(1-\epsilon-\pi_1)F(\lambda,\gamma)^2}{4\delta(1+\epsilon)\Delta\pi\gamma}\right\}<0 \tag{26}$$

$$\textit{where} \qquad \kappa=[1+(2-\pi_1)(1-\pi_1-\pi_0)^2+2(1-\pi_1-\pi_0)]$$

an optimizing donor offers unconditional contracts, even though incentive compatible contracts exist. In this case:

$$\bar{a_1}=\underline{a_1}=\bar{a}_2(\bar{q_1})=\bar{a}_2(\underline{q_1})=\underline{a}_2(\bar{q_1})=\underline{a}_2(\underline{q_1})=\frac{F}{2\delta} \tag{27}$$

Proof 3. *See Appendix.*

Proposition 3 states that in some cases, although incentive compatible contracts exist, the donor prefers not to offer them and accepts that type I agents exert the low level of effort, which is more likely to happen the higher is ψ and the lower is γ. This result derives from the fact that, unlike what happens in the standard principal-agent model, the utility of the agent enters the utility of the principal. The donor, therefore, internalizes the cost of effort: when this cost is high, she will prefer that type I agents opt for the low level of effort, which implies a wasteful use of resources from the recipient country.

When the donor is totally altruistic and she does not derive utility from the utility of type I agents ($\lambda = 0$), equation (26) is never satisfied and the donor will always try to impose an incentive mechanism. If donors want to sincerely help the poor they will never accept that local elites undermine the final outcome of the project by exerting low effort. Our model, therefore, has a strong normative implication: also bilateral donors should move to aid conditionality.

More generally, notice that differentiating (26) with respect to λ, we get:

$$\frac{\psi}{\Delta\pi\gamma}-\frac{(2\gamma-1)\hat{\theta}}{\Delta\pi\gamma}+\frac{(1-\epsilon-\pi_1)2F(\lambda,\gamma)(2\gamma-1))}{4\delta(1+\epsilon\Delta\pi\gamma)} \tag{28}$$

which, since $\epsilon \geq (1 - \pi_1)$ is always positive if i) $\gamma < 1/2$ and ii) $\gamma > 1/2$ and ψ is high. The more the donor is interested in supporting local elites for strategic/economic reasons, the more likely she will provide unconditional contracts.

5 Conclusions

In this paper, we analyze the problem of aid allocation from the point of view of a donor that is motivated both by a sincere desire to help the poor of the recipient country and by strategic/economic interests.

We consider an environment characterized by repeated moral-hazard, full commitment and limited punishment to study why conditionality on performance it seems not being a standard practice in bilateral aid allocation.

We find that unconditional aid, i.e. aid transfer that is not conditional on past performance, may represent an optimal strategy for donor countries when: i) the moral hazard issue is not very important and ii) motivations behind aid allocation reflect strategic/economic interests. This will never happen if the donor is entirely altruistic but may happen if strategic/economic interests are relevant. The implications of this result are quite strong: the widespread practice of giving unconditional aid could be a signal of the fact that foreign aid is not always recipient needs oriented since past performance represent recipient effort in achieving needs.

Conditional aid instead is always optimal when the moral hazard problem is relevant and donors have the unique goal of helping the poor. When it is optimal for a donor to offer conditional contracts, she can choose between *weakly conditional* contracts, where only the outcome of the previous period is taken into account or *strongly conditional* contracts, where aid is tied to the whole history of the project. The type of conditionality depends on the preferences of the donor: the more the donor is interested in the welfare of the recipient, the more likely is that aid is tied only to the most recent performance. The size of the contracts offered is a function of two factors: the preferences of the principal and the disutility of effort exerted by the agent, which in this model is represented by local elites.

Some kind of conditionality may be needed in order to guarantee that the recipient works effectively toward the success of the project, but if a donor has a strong desire to help a developing country, either for altruistic reasons or for strategic/economic motives, she should never deny aid to it.

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6 Appendix

6.1 Proof of Proposition 1

Proof 1. *In order to solve the intertemporal maximization program, the strategy is the following: we first solve the problem at $t = 2$ assuming that the participation constraint* (12) *is always binding. Then, we move to $t = 1$, solving the maximization program and deriving the optimal contracts in every period. At the end, we check the conditions under which the assumption concerning the participation constraint is satisfied.*

*Denote by $\tilde{a}_2(q_1)$ the "*promised expected level of aid*", i.e. the amount that the donor country promises to the recipient at the beginning of the project, as a function of the realization of the previous period's output.*

The maximization problem at $t = 2$ is given by:

$$\max_{\{\bar{a_2}(q_1);\underline{a_2}(q_1)\}} \quad \pi_1\Big[F[\hat{\theta} + \bar{a_2}(q_1)] - \lambda\psi - \delta\bar{a_2}^2(q_1)\Big] + (1-\pi_1)\Big[F[\hat{\theta} + \underline{a_2}(q_1)] - \lambda\psi - \delta\underline{a_2}^2(q_1)\Big] \tag{29}$$

$$s.t. \qquad \gamma\pi_1[\hat{\theta} + \bar{a_2}(q_1)] + (1-\pi_1)\gamma[\hat{\theta} + \underline{a_2}(q_1)] = \psi + \gamma\tilde{a}_2(q_1) \tag{30}$$

The FOCs are given by:

$$\pi_1 F - \pi_1 2\delta\bar{a_2}(q_1) + \mu_2\gamma\pi_1 = 0 \Rightarrow \bar{a_2}(q_1) = \frac{\mu_2\gamma + F}{2\delta} \tag{31}$$

$$(1-\pi_1)F - (1-\pi_1)2\delta\underline{a_2}(q_1) + \mu_2\gamma(1-\pi_1) = 0 \Rightarrow \underline{a_2}(q_1) = \frac{\mu_2\gamma + F}{2\delta} \tag{32}$$

where μ_2 is the Lagrange multiplier associated to the individual rationality constraint in $t = 2$, (30)*. Equations* (31) *and* (32) *together imply $\bar{a_2}(q_1) = \underline{a_2}(q_1)$. Since we assumed* (30) *to be binding:*

$$\mu_2 = \frac{-F + 2\delta\bar{a_2}(q_1)}{\gamma} > 0 \tag{33}$$

Plugging (33) *into* (30) *and solving for $\bar{a_2}(q_1) = \underline{a_2}(q_1)$, we get:*

$$\bar{a_2}(q_1) = \underline{a_2}(q_1) = \frac{\psi - \gamma\hat{\theta}}{\gamma} + \tilde{a}_2(q_1). \tag{34}$$

We now consider the maximization problem in $t = 1$, which is given by:

$$\max_{\{\bar{a_1};\underline{a_1};\tilde{a}_2(\bar{q_1});\tilde{a}_2(\underline{q_1})\}} \quad \pi_1\Big\{F[\hat{\theta} + \bar{a_1}] - \lambda\psi - \delta\bar{a_1}^2 + \epsilon\Big[\pi_1\Big(F[\hat{\theta} + \tilde{a}_2(\bar{q_1})] - \lambda\psi - \delta\tilde{a}_2^2(\bar{q_1})\Big)$$
$$+(1-\pi_1)\Big(F[\hat{\theta} + \tilde{a}_2(\underline{q_1})] - \lambda\psi - \delta\tilde{a}_2^2(\underline{q_1})\Big)\Big]\Big\} + (1-\pi_1)\Big\{F[\hat{\theta} + \underline{a_1}] - \lambda\psi - \delta\underline{a_1}^2 +$$
$$\epsilon\Big[\pi_1\Big(F[\hat{\theta} + \tilde{a}_2(\bar{q_1})] - \lambda\psi - \delta\tilde{a}_2^2(\bar{q_1})\Big) + (1-\pi_1)\Big(F[\hat{\theta} + \tilde{a}_2(\underline{q_1})] - \lambda\psi - \delta\tilde{a}_2^2(\underline{q_1})\Big)\Big]\Big\} \tag{35}$$

s.t. $$\gamma\pi_1[\bar{a_1}+\hat{\theta}]+\gamma(1-\pi_1)[\hat{\theta}+\underline{a_1}]+\epsilon\Big[\gamma\pi_1[\hat{\theta}+\tilde{a}_2(\bar{q_1})]+\gamma(1-\pi_1)[\hat{\theta}+\tilde{a}_2(\underline{q_1})]\Big] \geq (1+\epsilon)\psi \tag{36}$$

$$\bar{a_1} \geq 0; \qquad \underline{a_1} \geq 0; \quad \tilde{a}_2(\bar{q_1}) \geq 0; \quad \tilde{a}_2(\underline{q_1}) \geq 0 \tag{37}$$

where (35) *is the objective function at* $t = 1$, (36) *is the individual rationality constraint at* $t = 1$ *and* (37) *are the non negativity constraints at* $t = 1$.

Assume, now, that (36) *is binding and ignore for the moment the non negativity constraints.*

The FOCs are given by:

$$\pi_1 F - \pi_1 2\delta\bar{a_1} + \mu_1\gamma\pi_1 = 0 \tag{38}$$

$$(1-\pi_1)F - (1-\pi_1)2\delta\underline{a_1} + \mu_1\gamma(1-\pi_1) = 0 \tag{39}$$

$$\pi_1 F - \pi_1 2\delta\tilde{a}_2(\bar{q_1}) + \mu_1\gamma\pi_1 = 0 \tag{40}$$

$$(1-\pi_1)F - (1-\pi_1)2\delta\tilde{a}_2(\underline{q_1}) + \mu_1\gamma(1-\pi_1) = 0 \tag{41}$$

where μ_1 *is the Lagrange multiplier associated to the individual rationality constraint in* $t = 1$. *Equations* (38), (39), (40), (41) *together imply* $\bar{a_1} = \underline{a_1} = \tilde{a}_2(\bar{q_1}) = \tilde{a}_2(\underline{q_1})$.

Plugging this result into (36), *we get:*

$$\bar{a_1} = \underline{a_1} = \tilde{a}_2(\bar{q_1}) = \tilde{a}_2(\underline{q_1}) = \frac{\psi - \gamma\hat{\theta}}{\gamma} \tag{42}$$

In order to check if (36), *the individual rationality constraint at* $t = 1$, *is binding we can plug* (42) *into equation* (38) *and solve for* μ_1:

$$\mu_1 = \frac{2\delta}{\gamma}\Big[\frac{\psi - \gamma\hat{\theta}}{\gamma} - \frac{F}{2\delta}\Big] \tag{43}$$

If $\frac{\psi-\gamma\hat{\theta}}{\gamma} > \frac{F}{2\delta}$, *then* $\mu_1 > 0$ *and the optimal contracts in period* $t = 1$ *are given by:*

$$\bar{a_1} = \underline{a_1} = \tilde{a}_2(\bar{q_1}) = \tilde{a}_2(\underline{q_1}) = \frac{\psi - \gamma\hat{\theta}}{\gamma} \tag{44}$$

When, instead, $\frac{\psi-\gamma\hat{\theta}}{\gamma} \leq \frac{F}{2\delta}$, *then* $\mu_1 = 0$ *and the optimal payoffs at* $t = 1$ *are given by:*

$$\bar{a_1} = \underline{a_1} = \tilde{a}_2(\bar{q_1}) = \tilde{a}_2(\underline{q_1}) = \frac{F}{2\delta} \tag{45}$$

We now go back and check whether the conditions under which the assumption $\mu_2 > 0$ *is satisfied. Again, we have two possible cases: when* $\frac{\psi-\gamma\hat{\theta}}{\gamma} > \frac{F}{2\delta}$ *plugging* (44) *into* (33), *gives us* $\mu_2 > 0$. *Therefore, the optimal contracts in period* $t = 2$ *are given by:*

$$\bar{a}_2(\bar{q_1}) = \bar{a}_2(\underline{q_1}) = \underline{a}_2(\bar{q_1}) = \underline{a}_2(\underline{q_1}) = 2\Big(\frac{\psi - \gamma\hat{\theta}}{\gamma}\Big)$$

When, instead, $\frac{\psi-\gamma\hat{\theta}}{\gamma} \leq \frac{F}{2\delta}$ *and the intermediate payoffs are given by* (45)*, then* $\mu_2 = 0$ *and therefore the final payoffs at* $t = 2$ *are given by:*

$$\bar{a}_2(\bar{q_1}) = \bar{a}_2(\underline{q_1}) = \underline{a}_2(\bar{q_1}) = \underline{a}_2(\underline{q_1}) = \frac{F}{2\delta}.$$

Q.E.D.

6.2 Proof of Proposition 2

Proof 2. ***i)*** $\omega < 0$

Let us, now, first consider the case $\omega < 0$*. In order to prove Proposition 2, the strategy is the following: we first solve the maximization problem at* $t = 2$*, assuming that the participation constraint is always binding. We then move to* $t = 1$*, solving the maximization program and deriving the optimal contracts in every period. Given the full solution to the problem, we then check under what condition the assumption that the participation constraint in* $t = 2$ *holds.*

The maximization problem at $t = 2$ *is given by:*

$$\max_{\{\bar{a_2}(q_1);\quad \underline{a_2}(q_1)\}} \quad \pi_1\Big[F[\hat{\theta} + \bar{a_2}(q_1)] - \lambda\psi - \delta\bar{a_2}^2(q_1)\Big] + (1-\pi_1)\Big[F[\hat{\theta} + \underline{a_2}(q_1)] - \lambda\psi - \delta\underline{a_2}^2(q_1)\Big] \tag{46}$$

$$sub: \qquad [\bar{a_2}(q_1) - \underline{a_2}(q_1)] \geq \omega \tag{47}$$

$$\gamma\pi_1[\hat{\theta} + \bar{a_2}(q_1)] + (1-\pi_1)\gamma[\hat{\theta} + \underline{a_2}(q_1)] \geq \psi + \gamma\tilde{a}_2(q_1) \tag{48}$$

Assume that the individual rationality constraint in $t = 2$*,* (48)*, is binding and suppose, instead, the incentive compatibility constraint in* $t = 2$*,* (47)*, is not binding. The FOCS are given by:*

$$\pi_1 F - \pi_1 2\delta\bar{a_2}(q_1) + \nu_2\gamma\pi_1 = 0 \Rightarrow \bar{a_2}(q_1) = \frac{\nu_2\gamma + F}{2\delta} \tag{49}$$

$$(1-\pi_1)F - (1-\pi_1)2\delta\underline{a_2}(q_1) + \nu_2\gamma(1-\pi_1) = 0 \Rightarrow \underline{a_2}(q_1) = \frac{\nu_2\gamma + F}{2\delta} \tag{50}$$

where ν_2 *is the Lagrange multiplier associated to* (48)*. Equation* (49) *and* (50) *imply* $\bar{a_2}(q_1) = \underline{a_2}(q_1)$*. Substituting this result into* (47)*, we notice that*

$$0 > \omega$$

which implies that equation (47) *at* $t = 2$ *is always satisfied as a strong inequality. Since we assumed* (48) *to be binding, the solution in* $t = 2$ *is given by:*

$$\bar{a_2}(q_1) = \underline{a_2}(q_1) = \frac{\psi - \gamma\hat{\theta}}{\gamma} + \tilde{a}_2(q_1). \tag{51}$$

We now move to period $t = 1$ where the maximization program is given by:

$$\begin{aligned} \max \qquad & \pi_1\Big\{F[\hat{\theta}+\bar{a_1}] - \lambda\psi - \delta\bar{a_1}^2 + \epsilon\Big[\pi_1\Big(F[\hat{\theta}+\tilde{a}_2(\bar{q_1})] - \lambda\psi - \delta\tilde{a}_2^2(\bar{q_1})\Big) \\ & + (1-\pi_1)\Big(F[\hat{\theta}+\tilde{a}_2(\underline{q_1})] - \lambda\psi - \delta\tilde{a}_2^2(\underline{q_1})\Big)\Big]\Big\} + (1-\pi_1)\Big\{F[\hat{\theta}+\underline{a_1}] - \lambda\psi - \delta\underline{a_1}^2 + \\ & \epsilon\Big[\pi_1\Big(F[\hat{\theta}+\tilde{a}_2(\bar{q_1})] - \lambda\psi - \delta\tilde{a}_2^2(\bar{q_1})\Big) + (1-\pi_1)\Big(F[\hat{\theta}+\tilde{a}_2(\underline{q_1})] - \lambda\psi - \delta\tilde{a}_2^2(\underline{q_1})\Big)\Big]\Big\} \end{aligned} \tag{52}$$

$$s.t. \qquad [\bar{a_1} - \underline{a_1}] + \epsilon[\tilde{a}_2(\bar{q_1}) - \tilde{a}_2(\underline{q_1})] \geq (1+\epsilon)\omega \tag{53}$$

$$\gamma\pi_1[\bar{a_1}+\hat{\theta}] + \gamma(1-\pi_1)[\hat{\theta}+\underline{a_1}] + \epsilon\gamma[\pi_1[\hat{\theta}+\tilde{a}_2(\bar{q_1})] + (1-\pi_1)[\hat{\theta}+\tilde{a}_2(\underline{q_1})]] \geq (1+\epsilon)\psi \tag{54}$$

$$\bar{a_1} \geq 0; \qquad \underline{a_1} \geq 0; \qquad \tilde{a}_2(\bar{q_1}) \geq 0 \qquad \tilde{a}_2(\underline{q_1}) \geq 0 \tag{55}$$

Assume the incentive compatibility in $t = 1$, (53), is not binding. Denoting by IC the left hand side of equation (54) and by IR the left hand side of equation (54), we can rewrite equation (53) as:

$$IC = IR - \Big[\gamma\Big(\pi_0\bar{a}_1 + (1-\pi_0)\underline{a_1}\Big)\Big] + \epsilon\Big[\gamma\Big(\pi_0\tilde{a}_2(\bar{q_1}) + (1-\pi_0)\tilde{a}_2(\underline{q_1})\Big)\Big] \geq 0 \tag{56}$$

which implies that in $t = 1$, if $IC \geq 0$ then $IR > 0$ and we can ignore it. Since we assumed also incentive compatibility at $t = 1$ is not binding, the optimal contracts are given by:

$$\bar{a_1} = \underline{a_1} = \tilde{a}_2(\underline{q_1}) = \tilde{a}_2(\bar{q_1}) = \frac{F}{2\delta} \tag{57}$$

Let us now check whether the assumption on the incentive compatibility constraint in $t = 1$ holds. Substituting (57) into (53), we see again that:

$$0 > \omega \tag{58}$$

which implies that the incentive compatibility constraint in $t = 1$ is not binding.

In order to obtain the final payoffs in $t = 2$, we substitute now (57) into (51), still under the assumption that IR in $t = 2$ is binding and we get:

$$\bar{a_2}(\bar{q_1}) = \bar{a_2}(\underline{q_1}) = \underline{a_2}(\bar{q_1}) = \underline{a_2}(\underline{q_1}) = \begin{cases} \dfrac{\psi - \gamma\hat{\theta}}{\gamma} + \dfrac{F}{2\delta} & if \quad \frac{\psi-\gamma\hat{\theta}}{\gamma} + \frac{F}{2\delta} \geq 0 \quad (59a) \\ 0 & if \quad \frac{\psi-\gamma\hat{\theta}}{\gamma} + \frac{F}{2\delta} < 0 \quad (59b) \end{cases}$$

Let us now check whether the assumption that the individual rationality constraint is binding in $t = 2$ holds. From equation (49) or (50), we get:

$$\nu_2 = \frac{-F + 2\delta a_2(q_1)}{\gamma} \tag{60}$$

Substituting, now, equation (59a) into (60), and since we are under the assumption that $\omega < 0$, we get:

$$\nu_2 = \frac{2\delta(\psi - \gamma\hat{\theta})}{\gamma} < 0 \tag{61}$$

which implies a contradiction since by assumption $\nu_2 > 0$. The individual rationality constraint in t = 2 therefore is not binding. The optimal contracts in this case are given by:

$$\bar{a_2}(\bar{q}_1) = \bar{a_2}(\underline{q_1}) = \underline{a_2}(\bar{q}_1) = \underline{a_2}(\underline{q_1}) = \frac{F}{2\delta} \tag{62}$$

ii) $\omega \geq 0$

Consider now the case $\omega \geq 0$. We start again focusing on problem (46)-(48) and we first prove:

Claim 1. *The incentive compatibility constraint,* (47)*, in* $t = 2$ *is binding.*

Proof. Let's assume that the incentive compatibility in $t = 2$, (47), holds as a strict inequality. In this case, the FOCs are given by (49) and (50), that together imply $\bar{a_2}(q_1)$ $= \underline{a_2}(q_1)$. Substituting this result into equation (47), we immediately see that:

$$0 \geq \psi - \gamma\hat{\theta} = \gamma\Delta\pi\omega \tag{63}$$

which leads to a contradiction. Therefore, the incentive compatibility constraint in $t = 2$ must be binding. □

Claim 2. *The incentive compatibility constraint in* $t = 1$, (53)*, is binding and the individual rationality constraint in* $t = 1$, (54)*, is redundant.*

Proof. Consider the maximization program in $t = 1$ (52)- (55). At this point, we first solve the problem ignoring the non negativity constraints: we will then check the conditions under which they are satisfied. In order to solve (52)- (55), we assume first that the incentive compatibility constraint is not binding and since we previously showed that in $t = 1$ if $IC \geq 0$, then $IR > 0$, the FOCs imply:

$$\bar{a_1} = \underline{a_1} = \tilde{a}_2(\underline{q_1}) = \tilde{a}_2(\bar{q}_1) = \frac{F}{2\delta} \tag{64}$$

Substituting (64) into equation (53), we obtain:

$$0 \geq (1 + \epsilon)[\psi - \gamma\hat{\theta}] \tag{65}$$

but, since we are under the assumption that $\omega = \frac{\psi - \gamma\hat{\theta}}{\gamma\Delta\pi} > 0$, this implies a contradiction and therefore, the incentive compatibility constraint must be binding. □

Claim 3. *The optimal contracts in* $t = 1$ *are given by* (21) *and* (22).

Proof. Given Claim 2 in order to solve problem (52)-(55) we simply maximize (52) under (53). The FOCs are given by:

$$\pi_1 F - \pi_1 2\delta\bar{a_1} + \eta_1\gamma\Delta\pi = 0 \tag{66}$$

$$(1-\pi_1)F - (1-\pi_1)2\delta \underline{a_1} + \eta_1\gamma\Delta\pi = 0 \tag{67}$$

$$\pi_1 F - \pi_1 2\delta \tilde{a_2}(\bar{q_1}) + \eta_1\gamma\Delta\pi = 0 \tag{68}$$

$$(1-\pi_1)F - (1-\pi_1)2\delta \tilde{a_2}(\underline{q_1}) + \eta_1\gamma\Delta\pi = 0 \tag{69}$$

where η_1 is the Lagrange multiplier associated to the incentive compatibility constraint at $t = 1$.

Dividing (66) by (67) and (68) by (69), we get:

$$\frac{\pi_1 F - \pi_1 2\delta \bar{a_1}}{(1-\pi_1)F - (1-\pi_1)2\delta \underline{a_1}} = -1 \tag{70}$$

and

$$\frac{\pi_1 F - \pi_1 2\delta \tilde{a_2}(\bar{q_1})}{(1-\pi_1)F - (1-\pi_1)2\delta \tilde{a_2}(\underline{q_1})} = -1 \tag{71}$$

which imply that $\bar{a_1} = \tilde{a_2}(\bar{q_1})$ and $\underline{a_1} = \tilde{a_2}(\underline{q_1})$. From (70), we therefore get:

$$\bar{a_1} = \frac{F - (1-\pi_1)2\delta \underline{a_1}}{\pi_1 2\delta} \tag{72}$$

Since $\bar{a_1} = \tilde{a_2}(\bar{q_1})$ and $\underline{a_1} = \tilde{a_2}(\underline{q_1})$, the incentive compatibility constraint, (53), can be written as:

$$(\bar{a_1} - \underline{a_1}) = \omega \tag{73}$$

Solving now (72) and (73) with respect to $\underline{a_1}$ and $\bar{a_1}$, we finally prove the claim. □

Claim 4. *If $\frac{F}{2\delta} > \omega\gamma\Delta\pi$ then the individual rationality constraint in $t = 2$, (48), is not binding and the payoffs at $t = 2$ are given by (19) and (20).*

Proof. Let us go back to period $t = 2$ maximization problem. Assume now individual rationality constraint, (48), at $t = 2$ is binding. Given Claim 1 the incentive compatibility constraint becomes:

$$\bar{a_2} = \underline{a_2} + \omega$$

Substituting into (46), the objective function can be written as:

$$\pi_1[F(\hat{\theta} + \omega + \underline{a_2}) - \delta(\underline{a_2} + \omega)^2] + (1-\pi_1)[F(\hat{\theta} + \underline{a_2}) - \delta \underline{a_2}^2]. \tag{74}$$

The FOC with respect to $\underline{a_2}$ is:

$$\pi_1[F - 2\delta(\underline{a_2} + \omega)] + (1-\pi_1)[F - 2\delta \underline{a_2}] = 0 \tag{75}$$

$$\underline{a_2} = \frac{F}{2\delta} - \pi_1\omega \tag{76}$$

and substituting into (47), we get also:

$$\bar{a_2} = \frac{F}{2\delta} + (1-\pi_1)\omega. \tag{77}$$

In order to verify whether the assumption concerning the individual constraint in $t = 2$ is consistent with the overall constraints of the model, substitute (76) and (77) into the intertemporal individual rationality constraint (12). Notice that in this case the intertemporal IR constraint reduces to:

$$\frac{F}{2\delta} > \psi - \gamma\hat{\theta} \tag{78}$$

Clearly, if $\frac{F}{2\delta} \geq \psi - \gamma\hat{\theta}$ the assumption that the individual rationality constraint in $t = 2$ is binding implies a contradiction and this proves the claim. □

Claim 4, together with Claim 3 proves the second part of the Proposition 2.

Claim 5. *If $\frac{F}{2\delta} \leq \psi - \gamma\hat{\theta}$ then the individual rationality constraint is binding and the payoff at $t = 2$ are given by* (23), (24) *and* (25) .

Proof. Claim 1 already proved that the incentive compatibility constraint in $t = 2$ is binding. Claim 4, instead, tells us that if $\frac{F}{2\delta} \geq \psi - \gamma\hat{\theta}$, then the participation constraint in $t = 2$ is also binding. Consider, therefore, the maximization problem in $t = 2$ (46)-(48).

Then, the FOC are given by:

$$\pi_1 F - \pi_1 2\delta \bar{a_2}(q_1) + \eta_2\gamma\Delta\pi + \kappa_2\gamma\pi_1 = 0 \tag{79}$$

$$(1 - \pi_1)F - (1 - \pi_1)2\delta \underline{a_2}(q_1) - \eta_2\gamma\Delta\pi + \kappa_2\gamma(1 - \pi_1) = 0 \tag{80}$$

where η_2 is the Lagrange multiplier associated to the incentive compatibility constraint at $t = 2$ and κ_2 is the Lagrange multiplier associated to the individual rationality constraint at $t = 2$. Since the constraints are both binding we can solve equation (47) for $\bar{a_2}(q_1)$:

$$\bar{a_2}(q_1) = \frac{\psi - \gamma\hat{\theta}}{\gamma\Delta\pi} + \underline{a_2}(q_1) \tag{81}$$

and then plugging (81) into (48), we obtain:

$$\bar{a_2}(q_1) = \tilde{a}_2(q_1) + \frac{(1 - \pi_0)(\psi - \gamma\hat{\theta})}{\gamma\Delta\pi} \tag{82}$$

$$\underline{a_2}(q_1) = \tilde{a}_2(q_1) - \frac{\pi_0(\psi - \gamma\hat{\theta})}{\gamma\Delta\pi} \tag{83}$$

Combining now (82) and (83) with the contracts in $t = 1$ proved by Claim 3, we finally get:

$$\bar{a_2}(\bar{q_1}) = \frac{F}{2\delta} + \omega(2 - \pi_1) - \pi_0) \tag{84}$$

$$\underline{a_2}(\bar{q_1}) = \bar{a_2}(\underline{q_1}) \begin{cases} \dfrac{F}{2\delta} + \omega(1 - \pi_1 - \pi_0) & \text{if} \quad \frac{F}{2\delta} + \omega(1 - \pi_1 - \pi_0) \geq 0 \quad (85a) \\ 0 & \text{if} \quad \frac{F}{2\delta} + \omega(1 - \pi_1 - \pi_0) < 0 \quad (85b) \end{cases}$$

$$\underline{a_2}(\underline{q_1}) = \begin{cases} \frac{F}{2\delta} - (\pi_1 + \pi_0)\omega & \text{if} \quad \frac{F}{2\delta} - (\pi_1 + \pi_0)\omega \geq 0 \quad (86a) \\ 0 & \text{if} \quad \frac{F}{2\delta} - (\pi_1 + \pi_0)\omega < 0 \quad (86b) \end{cases}$$

Now, we can proceed checking the non negativity constraints. Notice since in this case we have $\frac{F}{2\delta} \leq \omega\gamma\Delta\pi$, this implies $\underline{a_2}(\underline{q_1}) = 0$. □

Claim 5, together with Claim 3 proves the last part of Proposition 2.

6.3 Proof of Proposition 3

We first prove the following:

Claim 6. *If condition* (10) *is not satisfied, the donor maximizes* (9) *subject to* (13) *and* (14)*. The optimal contracts are given by:*

$$\bar{a_1} = \underline{a_1} = \bar{a_2}(\bar{q_1}) = \bar{a_2}(\underline{q_1}) = \underline{a_2}(\bar{q_1}) = \underline{a_2}(\underline{q_1}) = \frac{F}{2\delta} \tag{87}$$

Proof. We start by solving the problem at $t = 2$, assuming that the participation constraint (12) is always binding, checking later the conditions under which this assumption holds. We then move to $t = 1$, solving the maximization program and deriving the optimal contracts in every period.

The maximization problem at $t = 2$ is given by:

$$\max_{\{\bar{a_2}(q_1);\underline{a_2}(q_1)\}} \quad \pi_0\Big[F\bar{a_2}(q_1) - \delta\bar{a_2}^2(q_1)\Big] + (1-\pi_0)\Big[F\underline{a_2}(q_1)] - \delta\underline{a_2}^2(q_1)\Big] \tag{88}$$

$$\text{s.t.} \qquad \pi_0\bar{a_2}(q_1) + (1-\pi_0)\underline{a_2}(q_1) = \tilde{a}_2(q_1) \tag{89}$$

The FOCs are given by:

$$\pi_0 F - \pi_0 2\delta\bar{a_2}(q_1) + \upsilon_2\pi_0 = 0 \Rightarrow \bar{a_2}(q_1) = \frac{\upsilon_2 + F}{2\delta} \tag{90}$$

$$(1-\pi_0)F - (1-\pi_0)2\delta\underline{a_2}(q_1) + \upsilon_2(1-\pi_0) = 0 \Rightarrow \underline{a_2}(q_1) = \frac{\upsilon_2 + F}{2\delta} \tag{91}$$

where υ_2 is the Lagrange multiplier associated to the individual rationality constraint in $t = 2$, (89). Equations (90) and (91) together imply $\bar{a_2}(q_1) = \underline{a_2}(q_1)$:

$$\upsilon_2 = 2\delta\bar{a_2}(q_1) - F > 0 \tag{92}$$

Plugging (92) into (89) and solving for $\bar{a_2}(q_1) = \underline{a_2}(q_1)$, we get:

$$\bar{a_2}(q_1) = \underline{a_2}(q_1) = \tilde{a}_2(q_1) \tag{93}$$

The maximization problem in $t = 1$ is given by:

$$\begin{aligned} \max \qquad & \pi_0\Big\{F\bar{a_1} - \delta\bar{a_1}^2 + \epsilon\Big[\pi_0\Big(F\tilde{a}_2(\bar{q_1}) - \delta\tilde{a}_2^2(\bar{q_1})\Big) \\ & +(1-\pi_0)\Big(F\tilde{a}_2(\underline{q_1}) - \delta\tilde{a}_2^2(\underline{q_1})\Big)\Big]\Big\} + (1-\pi_0)\Big\{F\underline{a_1} - \delta\underline{a_1}^2 + \\ & \epsilon\Big[\pi_0\Big(F\tilde{a}_2(\bar{q_1}) - \delta\tilde{a}_2^2(\bar{q_1})\Big) + (1-\pi_0)\Big(F\tilde{a}_2(\underline{q_1}) - \delta\tilde{a}_2^2(\underline{q_1})\Big)\Big]\Big\} \end{aligned} \tag{94}$$

$$\text{s.t.} \qquad \gamma[\pi_0\bar{a_1} + (1-\pi_0)\underline{a_1}] + \epsilon\gamma[\pi_0\tilde{a}_2(\bar{q_1}) + (1-\pi_0)\tilde{a}_2(\underline{q_1})] \geq 0 \tag{95}$$

$$\bar{a_1} \geq 0; \quad \underline{a_1} \geq 0; \quad \tilde{a}_2(\bar{q_1}) \geq 0; \quad \tilde{a}_2(\underline{q_1}) \geq 0. \tag{96}$$

where (94) is the objective function at $t = 1$, (95) is the individual rationality constraint at $t = 1$ and (96) are the non negativity constraints at $t = 1$.

Assume, now, that (95) is binding and ignore for the moment the non negativity constraints.

The FOCs are given by:

$$\pi_0 F - \pi_0 2\delta\bar{a_1} + \upsilon_1\gamma\pi_0 = 0 \tag{97}$$

$$(1-\pi_0)F - (1-\pi_0)2\delta\underline{a_1} + \upsilon_1\gamma(1-\pi_0) = 0 \tag{98}$$

$$\pi_0 F - \pi_0 2\delta\tilde{a_2}(\bar{q_1}) + \upsilon_1\gamma\pi_0 = 0 \tag{99}$$

$$(1-\pi_0)F - (1-\pi_0)2\delta\tilde{a_2}(\underline{q_1}) + \upsilon_1\gamma(1-\pi_0) = 0 \tag{100}$$

where υ_1 is the Lagrange multiplier associated to the individual rationality constraint in $t = 1$. Equations (97), (98), (99), (100) together imply $\bar{a_1} = \underline{a_1} = \tilde{a}_2(\bar{q_1}) = \tilde{a}_2(\underline{q_1})$.

Plugging this result into (95), we get:

$$\bar{a_1} = \underline{a_1} = \tilde{a}_2(\bar{q_1}) = \tilde{a}_2(\underline{q_1}) = 0 \tag{101}$$

In order to check if the individual rationality constraint at $t = 1$, (95), is binding we can plug (101) into equation (97), solving for μ_1:

$$\mu_1 = -\frac{F}{\gamma} < 0 \tag{102}$$

which is a contradiction, therefore, the participation constraint in $t = 1$, (95), cannot be binding.

The optimal contracts in $t = 1$ and the intermediate payoffs, therefore, will be given by:

$$\bar{a_1} = \underline{a_1} = \tilde{a}_2(\bar{q_1}) = \tilde{a}_2(\underline{q_1}) = \frac{F}{2\delta} \tag{103}$$

We can check now whether the participation constraint in $t = 2$, (89), is binding or not. Substituting (103) into (92), it turns out that $\upsilon_2 = 0$, which implies that also the participation constraint in $t = 2$ is not binding.

The optimal contracts in $t = 2$ are given by:

$$\bar{a}_2(\bar{q_1}) = \bar{a}_2(\underline{q_1}) = \underline{a}_2(\bar{q_1}) = \underline{a}_2(\underline{q_1}) = \frac{F}{2\delta} \tag{104}$$

□

Proof 3. *In order to verify when condition* (10) *is satisfied, we substitute on the left hand side of* (10) *the contracts given by Proposition 2 and on the right hand side the contracts given by Claim 6, that we have just proved.*

Consider first the case in which $\frac{F}{2\delta} > \Delta\pi\omega$*. In this case, we substitute* (19) *and* (20) *into left hand side of* (10) *and* (27) *on the right hand side and after few steps we get:*

$$\frac{[F(\lambda,\gamma)-\lambda\gamma]\hat{\theta}}{\Delta\pi\gamma} - \lambda\omega - \frac{\delta\omega^2\pi_1(1-\pi_1)}{\Delta\pi\gamma} < 0 \tag{105}$$

When instead $\frac{F}{2\delta} \leq \Delta\pi\omega$*, we substitute, first, on the left hand side of* (10) *equations* (21), (22a),(23), (24a), (25) *and* (27) *on the right hand side. After few steps, we get:*

$$\frac{[F(\lambda,\gamma)-\lambda\gamma]\hat{\theta}}{\Delta\pi\gamma} - \lambda\omega - \frac{\delta\omega^2\pi_1[(1-\pi_1)^2-\epsilon\kappa]}{(1+\epsilon)\Delta\pi\gamma} + \frac{(1-\epsilon-\pi_1)F(\lambda,\gamma)^2}{4\delta(1+\epsilon)\Delta\pi\gamma} < 0 \tag{106}$$

where $\kappa = [1+(2-\pi_1)(1-\pi_1-\pi_0)^2 + 2(1-\pi_1-\pi_0)]$.

We then substitute on the left hand side of (10) (21), (22a),(23), (24b), (25) *and* (27) *on the right hand side. After few steps, we get:*

$$\frac{[F(\lambda,\gamma)-\lambda\gamma]\hat{\theta}}{\Delta\pi\gamma} - \lambda\omega - \frac{\delta\omega^2\pi_1[(1-\pi_1)+\epsilon\kappa]}{(1+\epsilon)\Delta\pi\gamma} + \frac{\epsilon(1-\pi_1)F(\lambda,\gamma)^2}{4\delta(1+\epsilon)\Delta\pi\gamma} < 0 \tag{107}$$

The last case we consider occurs when we substitute on the left hand side of (10) (21) (22b), (23), (24b), (25) *and* (27) *on the right hand side. After few steps we get:*

$$\frac{[F(\lambda,\gamma)-\lambda\gamma]\hat{\theta}}{\Delta\pi\gamma} - \lambda\omega - \frac{\delta\omega^2[(1-\pi_1)^2+\epsilon(2-\pi_1-\pi_0)^2]}{(1+\pi_1\epsilon)\Delta\pi\gamma} < 0 \tag{108}$$

We now prove that (106) *implies* (105), (107) *and* (108)*. Subtracting* (105)*and* (106)*, we get:*

$$\frac{\delta\omega^2\pi_1}{(1+\epsilon)\Delta\pi\gamma}[(1-\pi_1)(1+\epsilon)-[(1-\pi_1)^2-\epsilon\kappa]] - \frac{(1-\epsilon-\pi_1)F^2(\lambda,\gamma)}{4\delta(1+\epsilon)\Delta\pi\gamma} > 0 \tag{109}$$

Since $(1-\epsilon-\pi_1) < 0$*, the difference is positive implying that* (105) > (106)*.*

Subtracting, now, (107)*and* (106)*, we get:*

$$\frac{\delta\omega^2\pi_1}{(1+\epsilon)\Delta\pi\gamma}[(1-\pi_1)+\epsilon\kappa-(1-\pi_1)^2+\epsilon\kappa] + \frac{F^2(\lambda,\gamma)[(2\epsilon-1)+\pi_1(1-\epsilon)]}{4\delta(1+\epsilon)\Delta\pi\gamma} > 0 \tag{110}$$

which implies that (107) > (106)*.*

In order to compare (106) *and* (108)*, we subtract the first to the second and we get:*

$$\frac{\delta\omega^2}{\Delta\pi\gamma}[\frac{\pi_1(1-\pi_1)^2}{(1+\epsilon)} - \frac{\pi_1\epsilon\kappa}{(1+\epsilon)} - \frac{(1-\pi_1)^2}{(1+\pi_1\epsilon)} - \frac{\epsilon(2-\pi_1-\pi_0)^2}{(1+\pi_1\epsilon)} + \frac{F^2(\lambda,\gamma)(1-\epsilon-\pi_1)}{4\delta(1+\epsilon)\Delta\pi\gamma} < 0 \tag{111}$$

which implies that (108) > (106)*. Therefore, we can write* (26) *as follows:*

$$\frac{[F(\lambda,\gamma)-\lambda\gamma]\hat{\theta}}{\Delta\pi\gamma} - \lambda\omega - \left\{\frac{\delta\omega^2\pi_1[(1-\pi_1)^2-\epsilon\kappa]}{(1+\epsilon)\Delta\pi\gamma} + \frac{(1-\epsilon-\pi_1)F(\lambda,\gamma)^2}{4\delta(1+\epsilon)\Delta\pi\gamma}\right\} < 0. \tag{112}$$