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Risk Connectivity and Risk Mitigation: An Analytical Framework

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Abstract

This paper studies the decision problem of risk averse single-output producers and suppliers under uncertainties in input prices, in a two-moment decision model with the presence of a dependent background risk. This framework is based on the utility from the expected value and the standard deviation of the uncertain random total profit of the supplier. Our theoretical framework for studying producers' responses to risks allows not only for analysing risk averse suppliers' attitude towards endogenous and background risks, but also to identify how the changes in the connectivity (i.e. correlation) between these two broad sources of risks will affect the risk averse suppliers' decision at the optimum. All comparative static effects are described in terms of the relative sensitivity of the supplier towards risks. This analytical framework has a number of potential application in development economics, such as optimal production decision under energy price uncertainty, output price uncertainty, and exchange rate uncertainty.

JEL Classification: D21; D81.

Keywords: Supply chain management; Risk management; two-moment decision model; background risk.

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Outline

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1. Introduction

Exposure to risk is inherent within all businesses. Managers devote most of their efforts to avoid and mitigate risks for meeting their targets. Thereby, *risk* is a negative consequence or loss that materializes with a certain probability (e.g., Wagner and Bode 2006, 2008, 2009; Tang and Musa 2011). The higher the loss and probability of risk, the higher is the negative impact on a firm's performance. Supply Chains (SC) of firms are inherently vulnerable to risk. While SCs have become more complex and globalized during the past decades, more severe SC incidents are reported in the news and academic world (e.g., Sheffi, 2007; Waters, 2011). The number of negative events affecting SCs exceed by far the memorable natural hazards like the Japanese Tsunami or hurricane Katrina in the US. Two main *impacts of SC risks* have been confirmed by several authors (e.g., Meena et al., 2011; Tang and Musa, 2011). On the one hand, Hendricks and Sighal (2005) observed a negative impact on *financial performance* of a firm after SC risk incidents. On the other hand, materialized risks have a negative *impact* on the *operational performance* of a SC (throughput, service level, lead times, etc.) (Wagner and Bode, 2008). Hence, SC risks must be considered by firms as seriously as other business risks (Elkins *et al.* 2005; Wagner and Bode, 2009; Heckmann et al., 2015).

In a mature stream of literature on *SC risk* (*e.g.*, Peck 2005, 2006; Wagner and Bode 2006; Pettit et al., 2013), various kinds of risks have been identified and classified. For instance, Chopra and Sodhi (2004) in their seminal work, characterized nine risk sources in SCs (disruptions, delays, systems, forecasts, intellectual property, procurement, receivables, inventory, and capacity). Whereas a simpler categorization by Jüttner (2005) comprises risks on the multiple fronts of demand, supply, and the environment. Rao and Goldsby (2009) and Sodhi et al. (2012) found that most of the articles are considered to be either qualitative or conceptual and more recently, Heckmann et al. (2015) classified them into four groups like modelling, conceptual, case study, and survey based. For quantitative descriptions of SC risks, one can find the following methodological approaches: optimization, multivariate analysis, stochastic programming, simulation, and real options (Tang and Musa, 2011). This demonstrates a certain difficulty to find a uniform or standardized method for describing SC risks and leaves great potential for developing applicable models for decision making. Since risks in practice are rather manifold, simultaneously hard to assess, and hidden along the entire SC, it is less feasible to fit them all into a specific risk model.

The unique contribution of this paper is to employ a two-moment decision model in the context of a risk averse supplier facing uncertainties in input prices, with the presence of a correlated background risk, that can be thought of an aggregate of all other sources of supply uncertainty other than input price risk. For analytical simplicity, we have considered a single output supplier with fixed contract, facing stochastic production costs, stemmed from the uncertainties in input prices. The major advantage of the mean–standard deviation model lies in its simplicity and ease of interpretation. Its effects can be illustrated in terms of risk and returns, and such models remain two-dimensional even with multidimensional risks or choice variables. This approach enables us to directly model such decision problem without any specific assumptions on the higher-order and cross derivatives of the utility function. To the best of our knowledge, this is the first study contributing to the literature of supply chain uncertainty in this context.

Although this modelling technique sometimes is misinterpreted as the special case of the standard von Neumann–Morgenstern expected utility framework, the two-moment decision making modelling approach is completely different and a novel-yet-simplest approach. The reason is: when the random variables under some choice set differ only in terms of the scale (standard deviation) and location (mean) parameters of the distribution, then an expected utility ranking of these random variables can be based on the means and standard deviations of the alternatives' risky outcomes, if uncertainty represented by a stochastic variable and the decision maker's decision variable interact in a linear way (Meyer, 1987) when uncertainty stems from the change in distribution of only one random variable (which is, in the context of this research, either only randomness in input prices or randomness in output prices). However, in the presence of above-mentioned background risk, on the top of price risks, with linear interaction between the risk averse supplier’s decision and the two sources of risks, the location – scale condition holds if these two risks are jointly elliptically distributed (see for example, Chamberlain, 1983; Owen and Rabinovitch, 1983; Eichner and Wagener, 2009).

The rest of the paper in structured the following pattern: First, a review of the relevant literature looks into SC risks and their management. At this point we identify the most important risks and mitigation measures on the supply-side of a SC. Second, methods are explained and data collected from a group of SC experts. A detailed simulation thus investigates the impact of mitigation measures on total SC risk. Finally, theoretical and practical applications are discussed in the conclusion section.

2. Supply-side Risks and Risks Management

When investigating risks and the impact of mitigation measures in SCs, there are two areas of academic literature that support the research questions: (1) Risk in general and supply-side risks in particular, and (2) Risk Management.

(1) *Risk* is a negative consequence or loss that materializes with a certain probability (Wagner and Bode, 2006; Tang and Musa, 2011). The higher the loss and probability of a risk, the higher the negative impact on firm performance. Several researchers identified such supply chain risks (e.g., Chopra and Sodhi, 2004; Wagner and Bode, 2009; Hopp et al., 2012). *Supply-side risks* are of highest importance due to the high cost share of the procurement function. Since supply-side risks are a mature field of research, we conducted a literature survey in the selected management review, operations management, and MS/OR journals using Google scholar and Scopus. The search included articles from the year 2004 to 2015. In total, we identified 33 comparatively different SC risks reported in these articles. Based on the frequency of occurrences, Table 1 lists the top ten SC risks. The given risks are still highly aggregated as more likely domains or areas of risk.

Table 1: Important supply-side risks based on literature survey

Supply-side Risks	**Selected Sources**
Contract risk	Elkins et al. (2005); Van Weele (2010)
Natural hazard risk	Manuj and Mentzer (2008); Wagner and Bode (2009); Tang and Musa (2011); Meena et al. (2011)
Technology, process, and infrastructure risk	Olson and Wu (2010); Ritchie and Brindley (2009); Ivanov and Sokolov (2010)
Supplier default risk	Ritchie and Brindley (2007); Van Weele (2010)
Supply quality risk	Chopra and Sodhi, (2004); Tuncel and Alpan, (2010)
Logistics/transportation risk	Zsidisin, Ragatz, and Melnyk (2005); Wagner and Bode (2009)
Supplier capacity risk	Peck (2005); Bode and Wagner (2009)
Price risk	Hallikas et al. (2004); Zsidisin, Ragatz, and Melnyk (2005); Wagner and Bode (2009)
Supplier lead time risk	Talluri et al. (2013)
Socio-political risk	Tang and Musa (2011); Ivanov and Sokolov (2010)

It is possible to find many sub-risks in these for further analysis. SC *risk connectivity* is the degree of interdependency of risks occurring at the same level or node of the SC. Risks, even at the same level or node of a supply chain, are interconnected: supplier default risk can be connected with supplier quality risk or supplier capacity risk; socio-political risk can be connected with supplier default or quality risk.

2. The Model

Consider a single-output competitive producer's profit function under supply uncertainty brought about input price risks.

$$\tilde{\pi} = p_x F(v) - \widetilde{p_v} v + \tilde{Z} \tag{1}$$

Where, $\widetilde{p_v}$ is distributed according to an objective cumulative distribution function over support $\left[\underline{p_v}, \overline{p_v}\right]$, denoting random per-unit price of input v to produce output x. The production function $F(v)$ is concave, with $F'(v) > 0, F''(v) < 0$. There is a aggregated background risk component, $\tilde{Z}$. Such background risk may be viewed as a weighted average of all random components affecting the supplier's decision other than the pricing risk; namely quality uncertainty, contract risk, natural hazard risk, technology, process, and infrastructure risk, logistic/transportation risks, supplier capacity risk, lead time risk, and socio-political risk. For analytical simplicity, we assume that the background risk is additive. Expectation of background risk is: $E(\tilde{Z}) = \mu_Z$. Input price risk and background risk are correlated. The preference function of the firm is $U = V(\mu, \sigma)$, with $V_\mu(\mu, \sigma) > 0$, $V_\sigma(\mu, \sigma) < 0$. In particular, we are implicitly assuming that the supplier is risk averse.

Expected profit is given by:

$$\mu = E(\tilde{\pi}) = p_x F(v) - \mu_v v + \mu_Z.$$

Profit risk is defined as follows:

$$\sigma = \sqrt{\sigma_v{}^2 v^2 + \sigma_Z{}^2 - 2v\mathrm{cov}\left(\widetilde{p_v}, \tilde{Z}\right)},$$

where σ_v, σ_Z and $\mathrm{cov}\left(\widetilde{p_v}, \tilde{Z}\right)$ are respectively the standard deviation of input price risk, standard deviation of background risk, and the correlation between both sources of risk. Without any loss of generality, we are assuming linear correlation between these two elliptically distributed risks (Embrechts et al., 2002). Therefore, while expected profits exhibit a 'trend shock', profit risk does not. Notwithstanding, the standard deviation of profits depends upon both $\widetilde{p_v}$ and $\tilde{Z}$, since optimum profits depend upon the realisation of $\widetilde{p_v}$ and $\tilde{Z}$. Hence both arguments in the preference function are affected by volatility in input prices and the background risk.

The marginal rate of substitution (MRS) between risk and return is defined by

$$S = -\frac{V_\sigma(\mu,\sigma)}{V_\mu(\mu,\sigma)}.$$

$S > 0$ is the two-parameter equivalent to Arrow–Pratt measure of absolute risk aversion (or, equivalently, risk attitude). The supplier solves the following problem

$$\max_{(v\geq 0)} V(\mu,\sigma). \qquad (2)$$

Before proceeding to the comparative static exercises, let us introduce fewer concepts that will be used in the analyses.

Definition 1. The elasticity of the marginal rate of substitution between risk and return with respect to the standard deviation of the supplier's random final profit is

$$\epsilon_\sigma(\mu,\sigma) = \frac{\partial S(\mu,\sigma)}{\partial\sigma}\frac{\sigma}{S(\mu,\sigma)}, \qquad \text{with } \sigma > 0.$$

The elasticity indicates the percentage change in risk aversion over the percentage change in final profit standard deviation, keeping the mean μ constant.

Definition 2. The elasticity of the marginal rate of substitution between risk and return with respect to the mean of final profit is defined as

$$\epsilon_\mu(\mu,\sigma) = \frac{\partial S(\mu,\sigma)}{\partial\mu}\frac{\mu}{S(\mu,\sigma)}.$$

The elasticity $\epsilon_\mu(\mu,\sigma)$ indicates the percentage change in risk aversion over the percentage change in expected final profit, keeping the standard deviation σ constant.

With these definitions in hand, let us begin with our first set of comparative static exercises, i.e. decision of optimal input usage only with respect to the changes in the distribution of the input prices.

3. Optimum input supply without background risk

To start with, we are assuming background risks, $\tilde{Z}$, is zero, i.e.

$$\tilde{\pi} = p_x F(v) - \widetilde{p_v} v \qquad (1.1)$$

The key comparative static exercise we are going to explore is how much does our supplier optimally uses input (and equivalently, optimally supplies) when facing uncertainties regarding the input prices.

When we consider interior solutions of this decision problem (since, corner solution would fetch the possibility of $v^* = 0$, which is not the focus of this paper: the risk averse supplier does always like to supply positive quantity at the optimum), the optimum is then determined by

$$\left(p_x F'(v^*) - \mu_v - \sigma_v\, S(\mu^*, \sigma^*)\right) V_\mu(\mu^*, \sigma^*) = 0 \qquad (3)$$

Or,

$$\frac{\left(p_x F'(v^*) - \mu_v\right)}{\sigma_v} = S\left(\mu^*, \sigma^*\right) \qquad (4)$$

Where $V_\mu(\mu^*, \sigma^*) > 0, V_\sigma(\mu^*, \sigma^*) < 0$, and the asterisk denotes the optimum. Hence, we are now going to demonstrate the comparative static properties of the model in relative terms, i.e., the comparative statics depend on how sensitively the supplier's risk attitude responds to changes in expected final profit and risk. The left-hand side of this marginal condition (i.e. Equation (4)) describes the slope of the opportunity line; the right-hand side denotes the slope of the indifference curve.

Corollary 1: When two single-product suppliers are competing among each other, but with different risk attitude, for example, $S_1 < S_2$ (i.e. firm 1 is more risk loving than firm 2), we would obtain $v_1^* > v_2^*$.

Proof. Given Equation (4), we have

$$(p_x F'(v_2^*) - \mu_v) > (p_x F'(v_1^*) - \mu_v)$$

Since $R(K)$ is a concave function, we obtain $v_1^* > v_2^*$. (Q.E.D.)

Now let us first trace out the change in optimum input usage and output supply owing to the increase in the input price risk (i.e. increase in σ_v).

Since, $V_\mu(\mu^*, \sigma^*) > 0$, Equation (3) can be rewritten further as:

$$p_x F'(v^*) - \mu_v - \sigma_v S(\mu^*, \sigma^*) = 0 \tag{3.1}$$

Implicit differentiation of Equation (3.1) with respect to (w.r.t. hereafter) σ_v yields

$$\text{sgn}\ (dv^*/d\sigma_v) = -\text{sgn}\ S(\mu^*, \sigma^*)[1 + (\partial S(\mu^*, \sigma^*)/\partial\sigma)(\sigma^*/S(\mu^*, \sigma^*))]$$

$$= -\text{sgn}[1 + \epsilon_\sigma(\mu^*, \sigma^*)] \tag{5}$$

Hence, a risk averse supplier may optimally supply less when input price-risk increases, however, if and only if the elasticity of risk aversion is greater than -1. This leads to the following proposition.

Proposition 1. *Higher volatility in input prices leads to a reduction in optimum supply if and only if* $\epsilon_\sigma(\mu^*, \sigma^*) > -1$.

Now we shall explore the relationship between the firm's optimum supply decision with respect to a change in the expected input price, i.e., μ_v.

Implicit differentiation of Equation (3.1) w.r.t. μ_v yields

$$\text{sgn}\ (dv^*/d\mu_v) = \text{sgn}\big(R^*\epsilon_\mu(\mu^*, \sigma^*) - 1\big) \tag{6}$$

Where $R^* = \sigma^* S(\mu^*, \sigma^*)/\mu^*$ can be shown $(0, 1]$:

$$R^* = \left(\frac{p_x F'(v^*) - \mu_v}{\sigma_v}\right)\left(\frac{\sigma_v v^*}{p_x F(v^*) - \mu_v v^*}\right)$$

$$= \frac{p_x v^* F'(v^*) - \mu_v v^*}{p_x F(v^*) - \mu_v v^*} \leq 1,$$

Since by the concavity property, $F'(v^*) < F(v^*)/v^*$. Hence, $(dv^*/d\mu_v) < 0$, if and only if $\epsilon_\mu < 1$. Hence, we can arrive at our next proposition.

Proposition 2. *An increase in the expected input prices may induce the supplier to supply less if and only if* $\epsilon_\mu(\mu^*, \sigma^*) < 1$.

Any change in the distribution of the input prices leads to an unequivocally negative "substitution effect" (lower supply due to higher price-risk) and an ambiguous "wealth effect" (similar to "income effect" in economics). Thus, the net effect (of the changes in the distribution of input price risk) on v^* depends on the relative magnitudes of these two effects.

4. Analysis with Background Risk.

With background risk, the equivalent F.O.C. is:

$$p_x F'(v^*) - \mu_v - \left(\frac{\partial \sigma}{\partial v}\right)_{v=v^*} S(\mu^*, \sigma^*) = 0 \tag{7}$$

Now, we have $\left(\frac{\partial \sigma}{\partial v}\right)_{v=v^*} = \left[\sigma_v^2 v^* - \mathrm{cov}\left(\widetilde{p_v}, \tilde{Z}\right)\right]\frac{1}{\sigma} = \left(p_x F'(v^*) - \mu_v\right)/S(\mu^*, \sigma^*) > 0$, since for optimal supply to be positive $(v^* > 0)$, expected markup (i.e. $p_x F'(v^*) - \mu_v$) must also be positive. In other words, we are dealing with the interior solution of the F.O.C. in Equation (7).

Next, let us trace out the impact on the decision to optimally supply owing to the changes in the covariance and also in the distribution of background risk (in relative terms).

Implicit differentiation of Equation (7) w.r.t. $\mathrm{cov}\left(\widetilde{p_v}, \tilde{Z}\right)$ yields

$$\mathrm{sgn}\left(\partial v^* / \partial \mathrm{cov}\left(\widetilde{p_v}, \tilde{Z}\right)\right) = \mathrm{sgn}\left[\epsilon_\sigma - \frac{1}{R(v^*)}\right]$$

where $R(v^*) = \left(\frac{\partial \sigma}{\partial v}\right)_{v=v^*}\left(\frac{v^*}{\sigma(v^*)}\right) > 0$ (8)

Therefore, for $\partial v^* / \partial \mathrm{cov}\left(\widetilde{p_v}, \tilde{Z}\right) < 0$, we must have $\epsilon_\sigma < \frac{1}{R(v^*)}$. This result confirms that our risk averse supplier will be inclined to supply lesser quantity when the input price is more highly correlated with the background risk, compared to the scenario when the correlation between the two sources of risk is low, if and only if $\epsilon_\sigma < \frac{1}{R(v^*)}$. These results can be summarized in our third proposition.

Proposition 3. *If both sources of risks become more concordant, the supplier would behave in more risk aversion fashion, if and only if* $\epsilon_\sigma < \frac{1}{R(v^*)}$.

Moving on to tracing out the implication of change in the volatility of background risk, ceteris paribus, implicit differentiation of Equation (7) w.r.t. σ_Z gives

$$\text{sgn}\ (\partial v^*/\partial \sigma_Z) = -\text{sgn}\ [(\partial\sigma/\partial v)_{v=v^*}(\sigma_Z/\sigma)S_\sigma(\mu^*,\sigma^*)]. \qquad (9)$$

Therefore, $S_\sigma(\mu^*,\sigma^*) > 0$. If we assume, following Eichner and Wagener (2009) that the risk averse supplier's indifference curves enter the μ- axis of the (μ,σ) plane with zero slope [i.e. $S(\mu,0) = V_\sigma(\mu,0) = 0$], then as Eichner and Wagener (2003) proved, we too would have $S - \sigma S_\sigma < 0$, which is equivalent to stating that $S_{\sigma\sigma} > 0$. This property is termed as "variance vulnerability" property (a' la Eichner and Wagener, 2003) of the risk averse supplier.

In other words, higher degree of background risks will prevent the risk averse supplier from supplying more if and only if the indifference curve in the (μ,σ)-plane is upward sloping and convex from the origin. Hence, we arrive at our next proposition.

Proposition 4. *The risk averse supplier will reduce optimum supply in response to the increased volatility of the background risk, if and only if* $S_\sigma(\mu^*,\sigma^*) > 0$.

Similarly, by implicit differentiation of Equation (7) w.r.t. μ_Z we obtain

$$\text{sgn}\ (\partial v^*/\partial \mu_z\ ) = \text{sgn}\left((d\sigma/dv)_{v=v^*}S_\mu(\mu^*,\sigma^*)\right)$$

This implies $S_\mu(\mu^*,\sigma^*) < 0$, and the supplier shows decreasing absolute risk aversion (DARA hereafter) for $\partial v^*/\partial \mu_z\ > 0$. Hence we can state the following proposition.

Proposition 5. *The risk averse supplier may optimally supply more with higher expected background risk if and only if* $S_\mu(\mu^*,\sigma^*) < 0$.

5. A Parametric Example.

Let us exemplify our propositions and their significance by a parametric example. We apply the following specific utility function, likewise Saha (1997); Eichner and Wagener (2009); Broll et al. (2015); Broll and Mukherjee (2017); and so on.

$$V = \mu^a - \sigma^b \tag{10}$$

The first-order condition of the supplier's decision problem suggests slope of the opportunity line must be equal to the MRS.

Our optimization exercise becomes

$$\max V(\mu, \sigma)$$

$$\text{with } \mu = p_x F(v) - \mu_v v + \mu_Z$$

and

$$\sigma = \left[\sigma_v{}^2 v^2 + \sigma_Z{}^2 - 2v\text{cov}\left(\widetilde{p_v}, \tilde{Z}\right)\right]^{\frac{1}{2}}.$$

The first order condition would become:

$$a\mu^{a-1}[p_x F'(v^*) - \mu_v] - b\left[\sigma_v{}^2 v^{*2} + \sigma_Z{}^2 - 2v\text{cov}\left(\widetilde{p_v}, \tilde{Z}\right)\right]^{b-2}\left[\sigma_v^2 v^* - \text{cov}\left(\widetilde{p_v}, \tilde{Z}\right)\right] = 0$$

Or,

$$\Phi = 0 \tag{11}$$

According to the "Definition 1" and "Definition 2", the relative changes in the degree of risk aversion with respect to the standard deviation and mean of the random final profit are respectively,

$$\epsilon_\sigma = b - 1, \epsilon_\mu = 1 - a.$$

Hence, from the F.O.C. we can derive the following results as corollaries to the propositions 1-5.

Corollary 2. Under the preferences given by (10), we have

(a) *An increase in σ_v (ceteris paribus), leads to the risk averse supplier to supply less optimally if and only if $b > 2$.*

Proof. $\frac{\partial v^*}{\partial \sigma_v} < 0 \Leftrightarrow \Phi_{\sigma_v} < 0$

$$\Leftrightarrow -b\sigma^{b-4}v^*\sigma_v\left[2\sigma^2 + (b-2)v^*\left\{\sigma_v^2 v^* - \mathrm{cov}\left(\widetilde{p_v}, \tilde{Z}\right)\right\}\right] < 0$$

Since, $\left\{\sigma_v^2 v^* - \mathrm{cov}\left(\widetilde{p_v}, \tilde{Z}\right)\right\} = (p_x F^{'}(v^*) - \mu_v)/S(\mu^*, \sigma^*)$, which is positive as we are dealing with interior solution of the decision problem where the supplier always chooses to supply positive quantity, we can state that $b > 2$ or $\epsilon_\sigma (= b-1) > 1 > -1$ is the necessary and sufficient condition for the risk averse supplier to reduce the optimal quantity to be supplied, which is equivalent to our generic result in **Proposition 1** (which states that the necessary and sufficient condition for $\frac{\partial v^*}{\partial \sigma_v} < 0$ is $\epsilon_\sigma > -1$).

(b) *An increase in μ_v (ceteris paribus) induces the supplier into more risk taking behaviour if and only if, $a > 0$.*

Proof. $\frac{\partial v^*}{\partial \mu_v} < 0 \Leftrightarrow \Phi_{\sigma_v} < 0$

$$\Leftrightarrow -a\mu^{a-2}\left[\mu_Z + av^*(p_x F'(v^*) - \mu_v) + p_x\left(F(v) - vF'(v)\right)\right] < 0.$$

Now, since $F(v)$ is concave, $\left(F(v) - vF'(v)\right) > 0$, for interior solution (i.e. positive v^*), $(p_x F'(v^*) - \mu_v) > 0$ (as argued before); $\frac{\partial v^*}{\partial \mu_v} < 0$ if and only if, $a > 0$, or $\epsilon_\mu (= 1 - a) < 1$, which is also identical to our generic result in **Proposition 2**.

(c) *Owing to an increase in $cov\left(\widetilde{p_v}, \tilde{Z}\right)$, the supplier would behave in more risk aversion fashion, if and only if $b < \left(1/\left\{\sigma_v^2 v^* - cov\left(\widetilde{p_v}, \tilde{Z}\right)\right\} + 2\right)$.*

Proof. $\partial v^*/\partial \mathrm{cov}\left(\widetilde{p_v}, \tilde{Z}\right) < 0 \Leftrightarrow \Phi_{\mathrm{cov}(\widetilde{p_v}, \tilde{Z})} < 0$, or,

$$\mathrm{sgn}\left(\partial v^*/\partial \mathrm{cov}\left(\widetilde{p_v}, \tilde{Z}\right)\right) = \mathrm{sgn}\left[(b-2) - 1/\left\{\sigma_v^2 v^* - \mathrm{cov}\left(\widetilde{p_v}, \tilde{Z}\right)\right\}\right] < 0$$

This would be true if and only if $(b-1) < \left(1/\left\{\sigma_v^2 v^* - \mathrm{cov}\left(\widetilde{p_v}, \tilde{Z}\right)\right\} + 1\right)$, which is also identical to our generic result in **Proposition 3**.

(d) *Owing to increase in σ_Z, the risk averse supplier will reduce optimum supply if and only if $b > 2$.*

Proof. $\partial v^*/\partial \sigma_Z < 0, \Leftrightarrow \Phi_{\sigma_Z} < 0$, or

$$\Leftrightarrow b(2-b)\{\sigma_v^2 v^* - \text{cov}(\widetilde{p_v}, \tilde{Z})\}\sigma_Z \sigma^{b-4} < 0.$$

This would hold if and only if $b > 2$, i.e. $\left(b/a\right)(b-1)\mu^{1-a}\sigma^{b-2} > 0$ (since, $a > 0$), or equivalently, $S_\sigma(\mu^*, \sigma^*) > 0$. One should also note that $b > 2$ implies $S_{\sigma\sigma}(\mu^*, \sigma^*) > 0$ as well (i.e. "variance vulnerability" property also holds). Therefore, this result also confirms our generic result in **Proposition 4**.

(e) *The risk averse supplier may optimally supply more with higher expected background risk if and only if*

Proof. $\frac{\partial v^*}{\partial \mu_Z} > 0, \Leftrightarrow \Phi_{\mu_Z} > 0$

$$\Leftrightarrow\ a(a-1)\mu^{a-2}[p_x F'(v^*) - \mu_v] > 0$$

Since, for interior solution (i.e. positive v^*), $(p_x F'(v^*) - \mu_v) > 0$ (as argued before), $\frac{\partial v^*}{\partial \mu_Z} > 0$ if and only if $a > 1$, or $\epsilon_\mu (= 1 - a) < 0$, which is also identical to our generic result in **Proposition 5**.

Thus all our comparative static results in Sections 3 and 4, stated in terms of propositions 1-5 are satisfied in terms of the preference parameters a and b in the context of this specific market.

6. Empirical Relevance

In this section, we show how to an empirically estimate a risk averse supplier's preferences under mean–standard deviation approach of final profit.

Using our specific utility function $V = \mu^a - \sigma^b$, we obtain from the F.O.C. in Equation (4)

$$(p_x F'(v^*) - \mu_v) = \left(b/a\right)\mu^{1-a}(v^*)^{b-1}\sigma_v^b$$

Taking logarithms in both sides of Equation (12) we obtain,

$$\ln(p_x F'(v^*) - \mu_v)_t - \ln\left({}^{b}/_{a}\right) + (a-1)\ln\mu_t + (1-b)\ln v_t^* - b\ln\sigma_{v_t} + \delta_t = 0 \qquad (13)$$

Where "t" denotes time-observation, δ_t is error term.

Let us present some numerical examples to illustrate propositions 1-2. Let the expected marginal revenue is 1.3, $a = 1$; $\mu_v = 0.3$. We consider three possible values for $b\,(= 2, 2.5, 1)$. Now we want to check for each of these three values for b, if we increase σ_v from 0.1 to 1 (with an increment of 0.1), then how the optimum v^* will be changed. We shall use the following formula for optimum v^*:

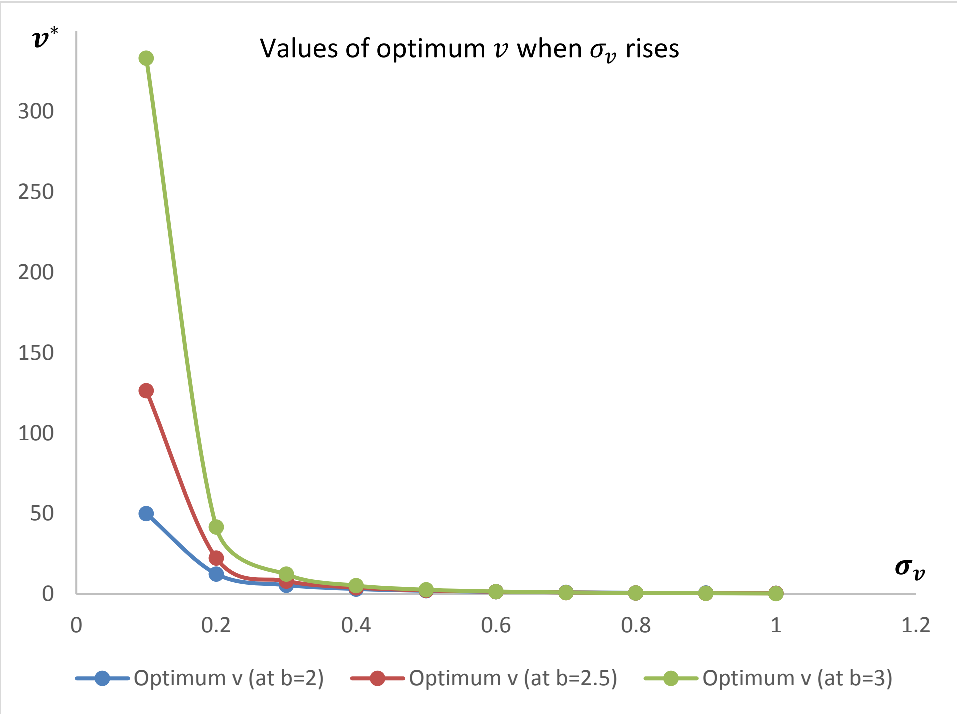
$$v^* = \left[\frac{a(p_x F(v^*) - \mu_v v^*)^{a-1}}{b\sigma_v^b}(p_x F'(v^*) - \mu_v)\right]^{\frac{1}{b-1}} \qquad (14)$$

So let us first solve Equation (14) for given σ_v and at $b = 2$. We obtain $v^* = 50$. Then for $\sigma_v = 0.2, 0.3, \ldots, 1$; the respective values of v^* will be 12.50, 5.56, 3.13, 2, 1.39, 1.02, 0.78, 0.62, 0.5. Similarly, we solve for optimum v^* at $b = 2.5$ and $b = 3$; each for $\sigma_v = 0.1(0.1)1$. Let me summarise the results in the table below:

Table 1: Optimum v for different σ_v at $a = 1$; $\mu_v = 0.3$; expected MR = 1.3.

	σ_v =0.1	σ_v= 0.2	σ_v= 0.3	σ_v= 0.4	σ_v= 0.5	σ_v= 0.6	σ_v= 0.7	σ_v= 0.8	σ_v= 0.9	σ_v= 1
$b = 2$	50.00	12.50	5.56	3.13	2.00	1.39	1.02	0.78	0.62	0.50
b =2.5	126.49	22.36	8.11	3.95	2.26	1.43	0.98	0.70	0.52	0.40
b =3	333.33	41.67	12.35	5.21	2.67	1.54	0.97	0.65	0.46	0.33

Figure 1: Values of optimum v when σ_v rises



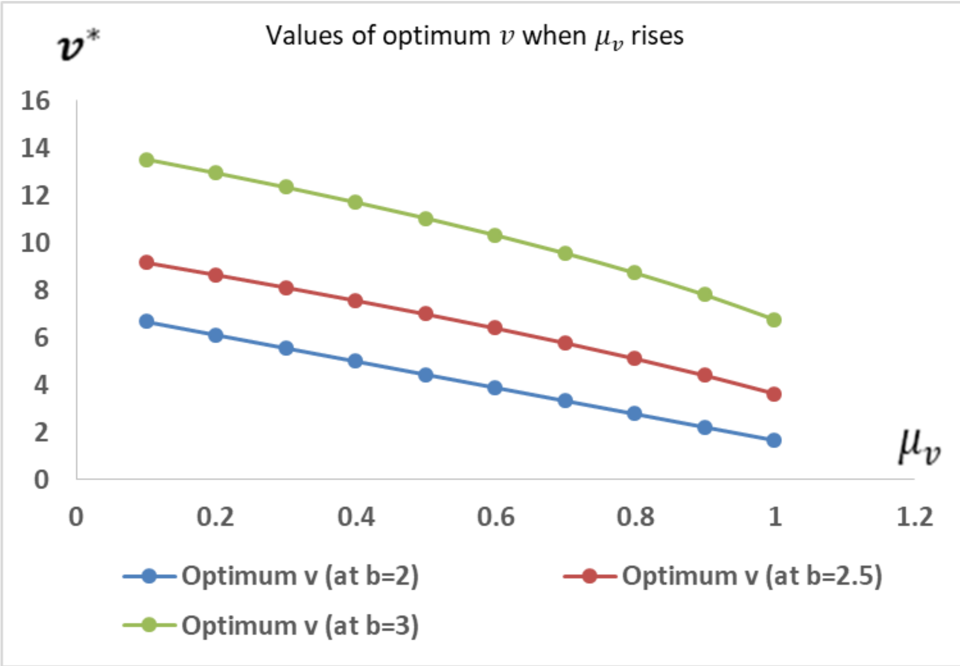
From above table and figure, we can see clearly that when risk increases, the supplier's optimum supply v^* will decrease. Also one should note that $\epsilon_\sigma (= b - 1) > 1 > -1$ for each of the cases $b = 2, 2.5, 1$. This is in line with our Proposition 1 and Corollary 2(a).

Turning to Proposition 2, taking the expected marginal revenue as 1.3, $a = 1$; $\sigma_v = 0.3$; we obtain the following results:

Table 2: Optimum v for different μ_v at $a = 1$; $\sigma_v = 0.3$; expected MR = 1.3.

	μ_v =0.1	μ_v= 0.2	μ_v= 0.3	μ_v= 0.4	μ_v= 0.5	μ_v= 0.6	μ_v= 0.7	μ_v= 0.8	μ_v= 0.9	μ_v= 1
$b = 2$	6.67	6.11	5.56	5.00	4.44	3.89	3.33	2.78	2.22	1.67
b =2.5	9.16	8.65	8.11	7.56	6.99	6.40	5.77	5.11	4.41	3.64
b =3	13.52	12.95	12.35	11.71	11.04	10.33	9.56	8.73	7.81	6.76

Figure 2: Values of optimum v when μ_v rises



Therefore, when the mean of the input prices increases, the supplier's optimum supply falls when $a > 0(= 1)$. This is in line with Proposition 2 and Corollary 2(b).

7. Concluding Remarks

The aim of this paper has been to explore the decision of a risk averse supplier on how much to be optimally supplied under input price risk, with the presence of a correlated background risk. In the first case, when the supplier only faces changes in the distribution of the input prices, the risk averse supplier's optimum supply decision is contingent upon the relative sensitivity of the risk aversion, i.e. the willingness to sell for changes in the distribution of the input prices.

In the second case, the presence of additional supply-side risks, clubbed as a collective "background risk", affects the decision making through additional riskiness and correlation with the input price risk. When the risk averse suppler confronts with higher background risk (ceteris paribus) or, most importantly, "tightly coupled" risks (i.e. interaction of the highly correlated price risk and background risk), the supplier would behave in more risk averse way. However, with only an increase in the expectation of the background risk, the supplier may supply more abroad if and only if the risk preference structure exhibits DARA (in the context of two-moment decision model).

An attractive feature of the conditions we derive for the decision problem of the risk averse supplier using the two-moment approach is their simplicity in interpretation – the sufficiency conditions are based on the supplier's relative sensitivity towards risks, which is more intuitive and appealing as empirically testable predictions; in contrast to the alternative (such as expected utility) approaches, which would depend on higher-order derivatives of utility functions and their composites.

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