CREDIT Research Paper

No. 21/05

A Subscription vs. Appropriation Framework for Natural Resource Conflicts

by

Dripto Bakshi and Indraneel Dasgupta

Abstract

We examine how cross-community cost or benefit spillovers, arising from the consumption of group-specific public goods, affect both inter-group conflicts over the appropriation of such goods and decentralized private provision for their production. Our model integrates production versus appropriation choices, vis-à-vis group-specific public goods, with their decentralized voluntary supply, against a backdrop of such cross-community consumption spillovers. Our flexible and general formulation of consumption spillovers incorporates earlier specifications as alternative special cases. We show that stronger negative (or weaker positive) consumption spillovers across communities may reduce inter-group conflict and increase aggregate income (and consumption) in society under certain conditions. Thus, stronger negative consumption spillovers may have socially beneficial consequences. We also identify conditions under which their impact will be both conflict-augmenting and income-compressing. Our general theoretical analysis offers a conceptual structure within which to organize investigation of feedback loops linking ethnic conflict and natural resource degradation in developing country contexts.

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Keywords: Production versus appropriation, Rent-seeking; Public good contest; Public bad; Natural resource conflict

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1. Introduction

Many forms of social conflict are rooted in the fact that certain kinds of collectively consumed environment-related items have the characteristics of group-specific public goods. Their consumption, use or exploitation in some particular manner generates benefits that accrue in a non-rival manner to all members of one specific group. However, those benefits spill over to members of another group only to a lesser extent, or not at all. Indeed, in extreme cases, members of another group may all suffer some cost, so that what constitutes a public good within one community may constitute a public bad for another. In these situations, all members of either community benefit if a larger proportion of that item is reserved for exclusive use by their own community. Thus, there arises scope for inter-community conflict over environmental or natural resource policy and, more generally, modes of collective consumption.

To fix ideas, consider the case of cow protection in India. Orthodox Hindus consider the cow sacred. Hindu nationalist governments in many Indian states have drastically expanded the ambit of laws against cow slaughter and increased the penalty for their violation in recent years. At the same time, vigilante groups have engaged in violence against, and even murder of, cattle traders suspected of transporting cattle to other states and neighboring countries for slaughter. Consequently, farmers, unable to sell, have increasingly taken to abandoning their economically unproductive bullocks and older cows. This has led to a dramatic increase in the number of stray cattle, which in turn poses a serious threat to standing crops, imposes large fencing costs on farmers, degrades common grazing land and increases methane emissions. Thus, the crackdown on cow slaughter, and its attendant restrictions on inter-state and inter-country cattle trade, may be seen as generating non-rival and non-excludable benefits for orthodox Hindus who are not farmers, but constituting a public bad for the farming community, with significant negative environmental consequences. Evidently, this creates scope for political conflict over the content and implementation of laws against cow slaughter between the two groups.[1]

Inter-group conflicts over sharing of group-specific public goods in general, and such conflicts with environmental implications in particular, often acquire greater salience in developing countries, due to traditional concentration of particular ethnic groups in specific economic locations. For example, resource

[1] For detailed discussions, see Alavi [2019] and Chari [2019]. Relatedly, in Sweden, while many nature lovers consider the Swedish wolf a public good, it constitutes a public bad for reindeer herders, whose livestock it preys on [Bostedt, 1999]. As noted by Buchholz *et al.* [2018], similar examples involving costly preservation of different animals can be provided from around the world.

conflicts between pastoralists and cultivators in West Africa often acquire broader ethno-linguistic and religious colors, because of strong correlations between such identities and economic interests.[2] In India, laws governing hunting and exploitation of forest produce, and conversion of forest land for commercial forestry, mining or industrial purposes, often bring traditionally forest-dwelling tribal communities into conflict with non-tribals [Aggarwal, 2020]. Thus, cross-community spillovers with environmental implications (whether positive or negative) in the group-specific use of natural resources carry important implications for both social conflict and overall economic wellbeing in many different, in particular developing country, contexts. But what exactly are those implications?

With a given stock of some natural resource, stronger negative spillovers, or greater exclusivity, across communities in its exploitation would imply stronger incentives for individuals to help appropriate that resource for exclusive use of their own community. This may be intuitively expected to increase conflict between competing communities and divert more resources overall to appropriation, rather than productive activities. But what happens then to individual incentives to contribute to public goods necessary for the maintenance or expansion of that resource stock itself?

For example, if degradation of common village grazing land leads to more destruction of standing crops by stray cattle, conflicts between farmers and 'cow protector' vigilante groups can be expected to increase. But what would happen to decentralized individual incentives to contribute to local veterinary clinics or NGOs that undertake immunization and treatment programs for all local cattle, thereby maintaining its stock? If reduced rainfall due to climatic change alters migration patterns of pastoralists and thereby causes greater damage to cultivators, conflict between the two groups over control of local waterbodies would rise. But what happens to individual incentives to contribute to the maintenance of local irrigation projects that replenish these contested waterbodies? If commercial afforestation involves planting trees that deplete groundwater more and thereby make both fruit trees and animal life less viable, conflict between hunter-gatherer communities and commercial planters, over what proportion of a given forest area should be allocated to commercial forestry, would rise. But what happens then to individual incentives to participate in forest protection associations meant to resist timber smuggling (or land grab attempts by third parties such as cultivator communities and mining companies)?

If individual incentives to contribute to its maintenance decline sufficiently overall, the stock of the resource being contested over would fall. The dampening effect of that contraction would counteract the

[2] See Shettima and Tar [2008] for a discussion of the vast literature on such conflicts. Dasgupta and Kanbur [2005a, 2005b, 2007, 2011] develop the idea that intra-group sharing of group-specific public goods generates the cohesion of interest necessary to both make and maintain stable identity *communities* out of groups of individuals.

conflict-expanding effect of stronger negative spillovers. What are the conditions, then, that determine whether the net effect would turn out to be conflict-expanding? Is it possible that stronger cross-community negative spillovers would actually increase the deployment of resources to production rather than appropriation, and thus increase aggregate social output, instead of reducing both, as one might perhaps intuitively expect? The purpose of this paper is to develop a parsimonious theoretical framework to address these issues.

We develop a model of a society populated by individuals who are partitioned into two equal-sized groups, or communities. All individuals are endowed with one unit of some resource ('money' or 'effort'). Individuals have to allocate their endowment among three alternative uses in decentralized manner. They can convert it to a privately consumed numeraire good, use it to subscribe to a society-wide common pool, or expend it on a Tullock [1980] style appropriation contest between the two communities. The common pool, or fund, generated through individual subscriptions produces, according to a strictly concave and iso-elastic production technology, the item whose inter-group division is contested over. This item has the characteristics of a group-specific public good, in that, ceteris paribus, all members of either group are strictly better off if their own group achieves a larger share. The benefit received by members of either group from a given production of the contested good falls monotonically as the share achieved by its antagonist increases, according to a convex and iso-elastic *loss function*. A more elastic loss function implies a lower loss to a community from its antagonist achieving any given share, as does a fall in its scale parameter. Thus, the elasticity and scale parameters of the loss function capture the strength of consumption spillovers and externalities, whether positive or negative, impacting on either community in consequence of its antagonist exploiting any given share of the contested good accruing to it. All individuals allocate their respective endowments simultaneously.

Examining the properties of the Nash equilibrium, we find the following. Whether stronger cross-community negative consumption spillovers, modeled as either a fall in the elasticity of the loss function or an increase in its scale factor, will serve to increase group conflict depends critically on the production technology for the contested good. The total amount of resources wasted on appropriation will fall when the output elasticity of that production technology, which may vary over the open unit interval, is sufficiently close to unity. However, it will rise otherwise. A mean-preserving decrease in the spread of the scale parameters of the loss functions has the same effect. Given any elasticity of the production technology for the contested good, stronger cross-community negative consumption spillovers must reduce aggregate social income, measured in units of the numeraire good, when the numerical size of the communities exceeds a threshold value. In this very specific sense, stronger cross-community negative consumption spillovers may affect aggregate societal well-being adversely, as might be intuitively expected. Strikingly, however, given any arbitrary community size, and any arbitrary production elasticity for the contested good, there also exist parametric configurations under which the effect would be positive. Specifically, aggregate social consumption is likely to expand when the strength of negative spillovers increases further, from an already high initial level.

A large theoretical literature has developed, stemming from the seminal contributions by Hirshleifer [1991] and Skaperdas [1992], which investigates how, when property rights are not fully secure, appropriation opportunities impact on production incentives, and vice-versa.[3] In these (so-called 'production vs. appropriation') models, the magnitude of the item open to appropriation – the size of the prize – is endogenously determined. The item open to appropriation is typically one whose consumption is fully rival across individuals – a standard private good. Our analysis belongs to this tradition, in its focus on endogenous determination of the size of the prize, and, in consequence, mutual determination of production and appropriation. However, we extend this literature by highlighting inter-group appropriation conflicts over items which exhibit public good characteristics within groups; i.e., over group-specific public goods, whose size is endogenously determined through such interplay, via a process of society-wide voluntary subscriptions.

Parallel to the production vs. appropriation literature, and originating from Katz *et al.* [1990] and Ursprung [1990], a large theoretical literature has also developed to address inter-group contests over group-specific public goods. This literature originally developed to examine inter-community conflicts over the sharing of state investment in public goods of localized or jurisdiction-specific benefit like schools, roads, hospitals, security, public art and local anti-pollution measures when the communities exhibit locational segregation, but subsequently incorporated many other applications.[4] Most recently, it has expanded to include investigations of ethno-linguistic and religious conflicts over identity goods and social norms.[5] These models however almost all belong to the 'rent-seeking', rather than 'production and appropriation', tradition, in that the size of the public good prize being contested over between groups is

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[3] See, for example, Murphy *et al.* [1993], Grossman and Kim [1996], Anderton *et al.* [1999], Noh [2002], Hausken [2005], Caruso [2010], Dal Bo and Dal Bo [2011, 2012], Mitra and Ray [2014], Cornes *et al.* [2019] and Bakshi and Dasgupta [2020].

[4] Contributions include Katz *et al.* [1990], Ursprung [1990], Gradstein [1993], Riaz *et al.* [1995], Baik [2008, 2016], Epstein and Mealem [2009], Nitzan and Ueda [2009], Esteban and Ray [2001, 2011a, 2011b], Lee [2012], Kolmar and Rommeswinkel [2013], Chowdhury *et al.* [2013], Dasgupta [2017], Bakshi and Dasgupta [2018, 2020], Cheikbossian and Fayat [2018], Dasgupta and Guha Neogi [2018], Baik and Jung [2021] and Dasgupta and Pal [2021].

[5] Esteban and Ray [2011a], Dasgupta [2017], Bakshi and Dasgupta [2018, 2020], Dasgupta and Guha Neogi [2018] and Dasgupta and Pal [2021] are examples of such recent application.

exogenously given, instead of being determined as an endogenous consequence of the interplay of production and expropriation.[6] Our model expands this literature precisely through such endogenization.[7]

Thus, like Bakshi and Dasgupta [2020], the present paper builds a bridge between the production and appropriation literature and that on inter-group contests over group-specific public goods. However, we depart fundamentally from that analysis by endogenizing the size of the public good prize being contested over between groups.[8]

The third dimension along which we expand the literature is by incorporating a quite general and flexible specification of cross-community consumption spillovers and externality effects with regard to the contested and group-specific public good. Ihori [2000] and Buchholz *et al.* [2018] formalize the idea that an item may be a public good for members of one group, but a public bad for members of another. They develop models where the benefits members of one group receive from an item generated by voluntary contributions on their part may be reduced by another item similarly generated by members of another group. We expand this idea to the domain of public good contests. In our model, the benefits received by all members of one group from its share of an item, acquired in consequence of contestation with another group (over a stock produced through voluntary contributions by members of *both* groups), may be reduced (or augmented) by the use its antagonist group makes of its own share. These inter-group spillover or externality effects may be asymmetric – the strength of the spillovers from, say, community *A* to community *B* may be different from that from *B* to *A*. This permitted asymmetry extends the formulation introduced by Dasgupta and Guha Neogi [2018], incorporating it as a special case. Indeed, we even permit the association of positive spillovers in one direction with negative spillovers in another. Furthermore, the strength of these spillovers may vary in a non-linear fashion with the level of consumption. This last feature of our model extends it beyond the linear aggregative structure adopted by both Ihori [2000] and Buchholz *et al.* [2018], which constitutes a special, limiting case of our analysis.

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[6] One partial exception is Gradstein [1993]. See footnote 7 below.

[7] Gradstein [1993] examines endogenous determination of the size of a local public good prize in the context of inter-jurisdiction contests over location of that local public good. The local public good, whose size is determined according to the preferences of members of the winning jurisdiction, is however generated through taxation of the entire society in his model, not through society-wide voluntary subscriptions (as in ours). Another related contribution is Cheikbossian [2008], who focuses on the size of (uniform lump-sum) tax-funded government spending on public good provision, when two competing groups have differing preferences over that size and can lobby the government in Tullock [1980] fashion to reflect their respective preferences more closely. Neither voluntary private supply of public goods, nor group-specificity in their consumption, appears in that analysis. These however constitute critical elements of our model.

[8] The present paper also ignores both within-group conflict over sharing of private consumption and cross-territorial conflict spillovers – key features of the model in Bakshi and Dasgupta [2020].

Lastly, our contribution has relevance for the literature on voluntary contribution to public goods. This literature typically asks the following question: how would an exogenously supplied redistribution of income/wealth affect private supply of public goods?[9] The question we address instead is: how would inter-group conflict over their distribution affect private supply of public goods?

Section 2 sets up the model. Equilibrium outcomes are characterized in section 3. Our main comparative static conclusions are presented in section 4. Section 5 discusses some possible variants of our model. Section 6 concludes. Detailed proofs of propositions are provided in an appendix.

2. The Model

Consider a society partitioned into two mutually exclusive equal-sized groups or communities, H and M. Each community has n members, $n \geq 1$. Given any community $k \in \{H, M\}$, we shall use $-k$ to denote the other. Individuals are indexed by a pair $\langle i, k \rangle$, where $i \in \{1,2, \dots, n\}$ and $k \in \{H, M\}$. Every individual in this society is endowed with one unit of a numeraire good, C (intuitively, 'money' or 'effort') which she can directly convert to fully rival and non-contestable consumption (c_{ik}), contribute to a common fund for the production of some contestable good Y (y_{ik}), or allocate to appropriation, i.e., inter-group conflict over division of that contestable good so produced (x_{ik}). Thus, each individual's budget constraint is:

$$c_{ik} + y_{ik} + x_{ik} = 1; \tag{1}$$

with $c_{ik}, y_{ik}, x_{ik} \geq 0$. The size of the common fund for production of Y, B, is given by the sum of individual contributions, so that:

$$B = \sum_{i=1}^{n} y_{iH} + \sum_{i=1}^{n} y_{iM}; \tag{2}$$

and that good is produced according to a strictly concave and iso-elastic production function:

$$\tilde{Y} = B^{\alpha}, \tag{3}$$

where $\alpha \in (0,1)$. The parameter α measures the output elasticity of Y (i.e., $\frac{Bd\tilde{Y}}{\tilde{Y}dB}$). Intuitively, the common fund B may be identified with either a pool of voluntary labor, or a sum of money raised through decentralized and voluntary individual subscriptions, that may be deployed to produce or maintain some

[9] This literature originates from Bergstrom *et al.* [1986]. See, for example, Andreoni [1990], Cornes [1993], Cornes and Sandler [1994, 1996, 2000] and Buchholz *et al.* [2018] for subsequent developments. Dasgupta and Kanbur [2005a, 2005b, 2007, 2011] analyze how private supply of public goods affects anti-poverty transfer policy, demand for income or wealth redistribution, and welfare inequality.

collective good. The assumed strict concavity of the production function ascribed to *Y* may be interpreted in standard fashion as a technological given. Evidently, the case of inter-group conflict over division of the common fund itself constitutes one limiting case of our model ($\alpha = 1$). We discuss a possible alternative formulation of the production technology for *Y* in Section 5 below.

The good *Y* is contestable at a community level - its division between competing user groups is determined by a political process involving group-specific resource investment in lobbying, bribery, and possibly violence. More formally, the proportion of any produced amount of that good unilaterally controlled, and exploited or utilized, by community *k* is given by the symmetric Tullock (1980) contest success function:

$$p_k = \frac{X_k}{X} \text{ if } X > 0, \text{ and } \frac{1}{2} \text{ otherwise;} \tag{4}$$

where the community conflict allocations are simply the sum of individual members' allocations, so that $X_k = \sum_{i=1}^{n} x_{ik}$, and total (society-wide) conflict allocation is defined as $X = X_H + X_M$.[10]

The consumption of *Y* may, in effect, be less than or more than fully rival across communities due to cross-community consumption spillovers, which may be either positive or negative. This key feature of our model is captured via a distinction between a community's *control* share of that good, p_k, arrived at through a process of Tullock contestation, and the proportion that it would need to be able to use/exploit, in order to achieve the same benefit, in case its antagonist *did not* exploit or utilize its own control share, p_{-k}. We term this the community's *effective* share:

$$g_k = \max\{1 - a_k \left(\frac{p_{-k}{}^{\rho}}{\rho}\right), 0\}; \tag{5}$$

where $a_k \in (0,2)$ and $\rho \geq 1$. We shall provide a detailed explanation of the distinction between control and effective shares, and the various properties of the latter, once we finish specifying our model.

Consumption of *Y* is at least partly non-rivalrous within each community, so that the each member of community *k* has, effectively, consumption access to $\frac{\tilde{Y} g_k}{n^{\theta}}$ amount of the good *Y*, with $\theta \in [0,1)$. *Y* is a

[10] One can generalize the model to permit inter-community differences in conflict efficiency, by replacing (4) with the following condition: $p_H = \frac{\vartheta X_H}{X_M + \vartheta X_H}$ if $[X_M + \vartheta X_H] > 0$, and $\frac{\vartheta}{1+\vartheta}$ otherwise; where $\vartheta > 0$. Then ϑ captures the relative productivity of investment in appropriation by *H*. Evidently, (4) is simply the symmetric special case of this general formulation, where $\vartheta = 1$. This generalization would considerably increase the algebraic burden, but not add anything of substance to our comparative static conclusions.

pure public good within either community when $\theta = 0$, and a pure intra-community private good in the limiting case of $\theta = 1$. For any individual i in community k, the payoff function is given by:

$$u_{ik} = c_{ik} + \frac{T\tilde{Y}g_k}{n^\theta}; \qquad (6)$$

where $T \in (0,1]$. All individuals simultaneously choose their private consumption c_{ik}, production common fund subscription y_{ik} and expropriation investment x_{ik} so as to maximize (6), subject to the constraints (1) – (5) above.

Control vs. effective share

We now proceed to lay out in detail the key features of the effective share function introduced in (5). To fix ideas, it is helpful to start with an illustrative example. Suppose pastoralists and cultivators both contribute voluntary labor for maintaining a network of drainage channels that feed rainwater into a reservoir. Through a process of political contestation, pastoralists (*H*) acquire exclusive access to the reservoir for p_H proportion of the year, and cultivators (*M*) the rest. Pastoralists use the reservoir as a water source for their herds, while cultivators use its water for irrigation. Then the control shares of the two communities are, respectively, p_H and p_M. Suppose now that pesticides and chemical fertilizers used by the cultivators seep into the water of the reservoir, reducing the survival rate of the pastoralists' livestock, even as greater irrigation by the cultivators, by increasing their crops, allows the pastoralists' animals to forage better, increasing their survival rate. The proportion of the livestock which survives the year is then determined jointly by these two effects, as well as by the proportion of the year that the pastoralists can access the reservoir (their control share). This aggregate survival rate is captured by the pastoralists' effective share g_H. Notice that, by (5), any positive effective share on their part implies:

$$g_H = p_H + \left[p_M - a_H\left(\frac{{p_M}^\rho}{\rho}\right)\right].$$

The second term on the right hand side captures the aggregate effect on livestock survival stemming from the cultivators' mode of utilization of their (control) share of access to the reservoir (p_M) – the net spillover effect of their actions on the pastoralists. Suppose, for example, that access is equally shared, so that both control shares take the value ½. Suppose further that $a_H = \frac{4}{3}$. Then, if $\rho = 2$, the pastoralists' effective share takes the value $\frac{5}{6}$: greater than their control share ½. Pastoralists can access the reservoir only half the year, which, by itself, would let only half their livestock survive. However, the actions taken by cultivators while exploiting their access to the reservoir generates net positive spillovers for the pastoralists, which increases the survival rate of their livestock to $\frac{5}{6}$. In the absence of these actions, then, the pastoralists would need access to the reservoir for an additional four months per year to attain the same survival rate - for ten months instead of the assumed six.

Now suppose, as before, that $p_H = \frac{1}{2}, a_H = \frac{4}{3}$, but $\rho = 1$. Then, $g_H = \frac{1}{3}$. In this case, negative spillovers from cultivators to pastoralists reduce the latter's effective share below their control share. Pastoralists can access the reservoir only half the year (as before), which, by itself, would let half their livestock survive. However, pesticides and chemical fertilizers applied by cultivators poison the water of the reservoir to such an extent that only a third of the pastoralists' livestock actually survive after drinking it. Clearly, this situation for the pastoralists is identical to one where cultivators do not poison the water, but permit others to access the reservoir only four months of the year, instead of the assumed six.

The general case of positive externalities and spillovers is captured by $g_k > p_k$.[11] Then, the use of its control share p_{-k} of the public good by its opponent augments the benefit k can receive from unilateral exploitation of its own control share, p_k, to the extent of $\tilde{Y}(g_k - p_k)$. If its antagonist were to stop exploiting its control share, community k's control share would need to be augmented by $(g_k - p_k)$, for it to be able to achieve the same benefit from *Y* as before. In our example, such positive externalities and spillovers may be generated by cultivators for pastoralists if their farming practices generate grass or fodder that the pastoralists' herds can feed on.

Alternatively, $g_k > p_k$ may also capture a combination of imperfect rivalry and incomplete excludability in consumption across groups. Within this class, since $\lim_{a_k \to 0} g_k = \lim_{\rho \to \infty} g_k = 1$, fully non-rival and non-excludable (but spillover-less) consumption of *Y* across communities constitutes one limiting case. In this case, the benefits that can be derived by a community, say *M*, from *Y* does not depend on how much of that good is unilaterally exploited by *H*, instead of *M*. In terms of our example, this may capture a situation where: (a) water from the reservoir seeps into and thereby replenishes aquifers which supply sufficient water for cultivators' needs throughout the year, and (b) cultivators can entirely prevent pastoralists' herds from destroying crops, say through fencing. Then what proportion of the year the pastoralists (*H*) have exclusive access to the reservoir (their control share p_H) makes no difference to the cultivators (*M*), who can extract all the water they need from the aquifers which are replenished by water from the reservoir. The pastoralists cannot prevent this seepage regardless of the extent of their control over the surface of the reservoir; nor can their herds affect the cultivators' crops in any manner. Thus,

[11] Note that the symmetric specification introduced by Dasgupta and Guha Neogi [2018], viz., $[g_k = 1 - {p_{-k}}^{\rho}]$, with $\rho > 1$, falls in this class.

regardless of their control share, the cultivators' effective share of the reservoir, g_M, is 1. Notice that non-excludability may be asymmetric – it may be possible (though not rational) for cultivators to prevent pastoralists from accessing the reservoir. Similarly, there may be spillovers from the former to the latter – say through contamination of the reservoir water due to the use of pesticides. Hence, neither the control share nor the access share of the pastoralists need equal 1 even in this case.

As $g_k = p_k$ at $a_k = \rho = 1$, fully rival, excludable and spillover-less consumption constitutes another special case of the effective share function g_k. In this case, each community can only benefit from the public good to the extent it comes to own or control it through the process of political contestation, so that control and access shares become identical. In other words, the actual unilateral use of its control share by, say, *M* does not impose any additional cost or benefit on H - only possession matters.[12] In the context of our example, this captures the case where: (a) physical access to the reservoir alone determines water availability for either community, and (b) cultivators do not contaminate the water for pastoralists, nor do pastoralists' herds affect or benefit from cultivators' crops in any manner.

When $g_k < p_k$, not only does the use of its control share p_{-k} of the public good by its opponent imply a loss of access for community k by the same proportion, such use also imposes a net additional cost on *k* via negative externalities and spill-over effects. The magnitude of this cost, measured in units of the contested good, is $\tilde{Y}(p_k - g_k)$. Thus, if its antagonist were to merely cease exploiting its own share, community *k* would receive this additional benefit, even with the same control share p_k. Intuitively, this may happen through an across-the-board decline in the productivity of *k*'s stock of the good *Y*, because of negative externalities and spillovers stemming from the actions its antagonist takes while unilaterally exploiting its own stock.[13] In our example, such negative externalities and spillovers are imposed on

[12] This particular special case of our model is the standard formulation adopted in the literature on contests over group-specific public goods.

[13] Ihori [2000] and Buchholz *et al.* [2018] use a linear aggregative structure for net benefits to capture the idea that an item may constitute a public good for one community, but a public bad for another. Since, given $\rho = 1$, $[g_k = 1 - a_k p_{-k}]$, translated to our framework, their formulation, in essence, falls out as a special case of our formulation when $a_k > 1$ and $\rho = 1$. When the contest success function is interpreted as providing the success probabilities, instead of the group *shares* as in our model, assuming risk neutrality, the expected utility of an individual must take the form: $\left[Eu_{ik} = c_{ik} + \left(\frac{T\tilde{Y}}{n^\theta}\right)\left[p_k + (1-p_k)\left(1 - \frac{a_k}{\rho}\right)\right] = c_{ik} + \left(\frac{T\tilde{Y}}{n^\theta}\right)\left[1 - a_k\left(\frac{p_{-k}}{\rho}\right)\right]\right]$. Hence, the individual pay-off function must always reduce to this special limiting form, regardless of the specification of the effective share function in (5). Thus, the linear aggregative structure considered by Ihori [2000] and Buchholz *et al.* [2018] can alternatively be derived as a special case of our formulation when: (a) $\left(\frac{a_k}{\rho}\right) > 1$, (b) the contest success function is assumed to provide success probabilities instead of group shares, and (c) agents are assumed to be risk-neutral expected utility maximizers.

pastoralists by cultivators through their farming practices, which poison the reservoir water and thereby increase the mortality rate of the former's livestock. Conversely, foraging by such livestock, by destroying standing crops, generates negative spillovers from pastoralists to cultivators.

Notice that the effective share function in (5) is monotonically increasing in own control share. Thus, the inter-community contest over control shares remains salient, despite the effective share diverging from the control share. Recalling (5), let $L_k \equiv a_k \left(\frac{p_{-k}{}^{\rho}}{\rho}\right)$. The term L_k can be thought of as a *loss* function, in that it specifies the loss (measured in terms of effective share) to k, that is generated in consequence of its antagonist exploiting any given control share. As noted above, $[L_k > p_{-k}]$ implies negative spillovers from its antagonist to community k, while $[L_k < p_{-k}]$ implies positive spillovers. Notice that spillovers must necessarily be positive if $\left[\left(\frac{a_k}{\rho}\right) < 1\right]$, and necessarily negative if $[\left(\frac{a_k}{\rho}\right) > 1$ and $\rho = 1]$. However, since:

$$a_k \left(\frac{p_{-k}{}^{\rho}}{\rho}\right) - p_{-k} = p_{-k}\left[p_{-k}{}^{\rho-1}\left(\frac{a_k}{\rho}\right) - 1\right];$$

when $\left[\left(\frac{a_k}{\rho}\right), \rho > 1\right]$, spillovers will be negative at high values of p_{-k}, but positive at low values, reflecting non-monotone effects of its antagonist's actions on community k.[14]

Changes in the parameters a_H, a_M and ρ capture changes in cross-community spillovers. The effective share g_k is monotonically increasing in ρ and monotonically decreasing in a_k. The parameter ρ measures the *elasticity* of the loss function of either community with respect to its antagonist's control share. Higher loss elasticity implies a lower loss to either community from any positive control share accruing to its antagonist. It also implies a lower loss to either community from a marginal increase in its antagonist's control share. Thus, intuitively, a more elastic loss function implies, in effect, a *symmetric* decline in inter-group consumption rivalry with respect to Y. In the context of our example, greater use by cultivators of non-contaminating farming methods, in conjunction with greater adoption by pastoralists of feeding practices which reduce foraging (and consequent crop destruction) by their herds, might be modeled through a rise in the loss elasticity. We ignore inter-group asymmetry in loss elasticity for the sake of

[14] Recall that an interpretation of the contest success function as generative of success probabilities leads to a reduced form linear aggregative structure for g_k (recall footnote 13). Under a linear aggregative structure, $\left[\left(\frac{a_k}{\rho}\right) > 1\right]$ must always imply negative spillovers, regardless of the value of ρ. In this specific sense, our interpretation of the contest success function, as generating group shares, yields a strictly richer set of possibilities than the alternative interpretation in terms of success probabilities (with risk-neutral expected utility-maximizing agents).

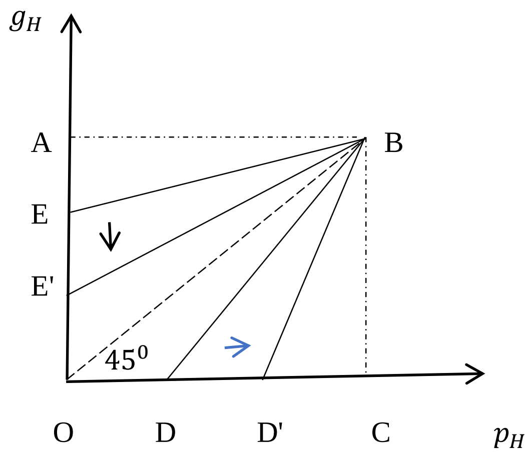
simplicity and tractability – permitting such asymmetry greatly adds to the algebra, without much compensation in the way of substantive additional insights.

Since $\lim_{\rho \to 1} \frac{dL_k}{dp_{-k}} = \lim_{p_{-k} \to 1} \frac{dL_k}{dp_{-k}} = a_k$, the scale parameter a_k provides the upper bound for the loss to k due to a marginal increase in its antagonist's control share. We shall accordingly call a_k the *marginal degradation* rate for community *k*. This is the channel through which we introduce spillover *asymmetry* in our model. The marginal degradation rates may vary across communities - a_H need not be equal to a_M. Thus, the spillover effect on *H* due to *M*'s exploitation of its control share may differ from that flowing in the reverse direction, even under an equal division of control. It follows that equal control shares do not, in general, imply equal effective shares in our model.

Lastly, note that the lower bound of *0* imposed on effective shares in (5) implies that the effect of any negative spillovers from, say, *H* to *M*, is confined to degrading the latter's stock of the contestable good. Such negative spillovers do not affect the benefits derived by members of *M* from their non-contested (and private) consumption. This assumption is made entirely for expositional clarity. As we shall establish in Section 3 below, equilibrium effective shares must be positive under our parametric restrictions. The lower bound of *0* will thus be non-binding in equilibrium. We can therefore eliminate it altogether and thereby relax (5) to $\left[g_k = 1 - a_k \left(\frac{{p_{-k}}^{\rho}}{\rho}\right)\right]$, without any consequences for our conclusions.

Figure 1 below illustrates the relationship between control and effective shares for the linear case of unit loss elasticity ($\rho = 1$), and for a given community, say *H*. The distance OA equals the distance OC, with both representing 1. The broken horizontal line AB represents the limiting case of perfectly non-rival, non-excludable and spillover-less consumption of *Y* by *H* (i.e., of $a_H = 0$). Here the benefit received by members of *H* from *Y* is entirely independent of its use by *M*, and thus of the proportion of *Y* that *H* has exclusive control over (its control share p_H). EB represents the case of net positive spillovers flowing to *H* from its antagonist's consumption (or that of imperfect rivalry and incomplete excludability) – the case where $0 < a_H < 1$. Here, the effective share of *H*, g_H, exceeds its control share whenever *H* has less than perfect control (i.e., whenever $p_H < 1$). The 45^0 line OB captures fully rival, excludable and spillover-less consumption between the two communities; here, $a_H = 1$. Control and effective shares turn identical in this case. The schedule ODB represents net negative spillovers due to *M*'s actions degrading the stock of the contested good controlled by *H* (alongside perfect rivalry and complete excludability in consumption of the contested good across communities). Due to such degradation, the effective share of *H* falls short of its control share whenever *H* has less than perfect control over *Y*. Here, $1 < a_H < 2$. Given the loss elasticity, weaker positive spillovers (or stronger negative spillovers) from *M* to *H* are captured by a rise in the latter's marginal degradation rate a_H, which swivels the effective share schedule of *H* downward. Such a change is depicted in Figure 1, for the case of net positive spillovers from *M* to *H*, by the downward shift of EB to E'B. For the case of net negative spillovers from *M* to *H*, this change is depicted in Figure 1 by the rightward shift of ODB to OD'B.

Figure 1: Control share vs. effective share of *H* under $\rho = 1$



3. Equilibrium

We proceed now to characterize the equilibria of our model. In light of (1)-(6), remembering that $[p_k = 1 - p_{-k}]$, and assuming an interior solution, the first order conditions yield:

$$\text{for every } k \in \{H, M\}\text{: } X_{-k} T\tilde{Y} \left(\frac{dg_k}{dp_k}\right) = n^{\theta} X^2. \tag{7}$$

From (7), denoting equilibrium values of all variables by the superscript *E*, the relative marginal degradation rate by A_k ($A_k \equiv \frac{a_k}{a_{-k}}$), noting (5), and that, by (4), $\frac{X_{-k}}{X_k} = \frac{p_{-k}}{p_k}$, we have the equilibrium control share ratio:

$$\left(\frac{{p_{-k}}^E}{{p_k}^E}\right)^{\rho} = A_{-k}. \tag{8}$$

Equation (8) implies:

$$p_k^E = \left({A_{-k}}^{\frac{1}{\rho}} + 1\right)^{-1}. \tag{9}$$

Together, (5) and (9) yield:

$$g_k^E = g_{-k}^E = 1 - \frac{a_H a_M}{\rho\left(a_H^{\frac{1}{\rho}} + a_M^{\frac{1}{\rho}}\right)^{\rho}}. \tag{10}$$

Thus, the equilibrium control shares differ across communities when their marginal degradation rates differ. The community with the higher marginal degradation rate stands to lose more from its antagonist achieving any given control share. It consequently allocates more resources to appropriation – to the contest over sharing of Y, and therefore receives the higher control share. In marked contrast, the equilibrium effective shares are always identical across communities. Thus, control shares provide a misleading picture of cross-community benefit disparity – any control advantage is completely compensated by a higher degradation rate – indeed, such control advantage comes about only as a compensatory response to higher negative spillovers, as captured by a higher degradation rate. Recall that $a_k \in (0,2)$ and $\rho \geq 1$ by assumption, which implies [$\frac{1}{\rho\left(a_H^{\frac{-1}{\rho}} + a_M^{\frac{-1}{\rho}}\right)^{\rho}} < 1$]. It then follows immediately from (10) that equilibrium effective shares must be positive, less than 1, increasing in the spillover (loss) elasticity and decreasing in the marginal degradation rates. These properties are stated more formally in Observation 1.

Observation 1. *For all* $k \in \{H, M\}$,

(i) $\lim_{\rho\to\infty} g_k^E = 1$ *and* $\left[1 > \lim_{\rho\to 1} g_k^E > 0\right]$;

(ii) $\frac{dg_k^E}{d\rho} > 0$;

and

(iii) $\frac{dg_k^E}{da_H}, \frac{dg_k^E}{da_M} < 0$.

Now notice that, in light of (10), the first order conditions also imply that, in an interior equilibrium:

$$T\left(\frac{\alpha}{B^{1-\alpha}}\right)\left(\frac{g_H}{n^{\theta}}\right) = T\left(\frac{\alpha}{B^{1-\alpha}}\right)\left(\frac{g_M}{n^{\theta}}\right) = 1. \tag{11}$$

Together, (10) and (11) imply:

$$B^E = \left(\left(\frac{T\alpha}{n^\theta}\right) \left(1 - \frac{a_H a_M}{\rho\left(a_H^{\frac{1}{\rho}} + a_M^{\frac{1}{\rho}}\right)^\rho} \right) \right)^{\frac{1}{1-\alpha}}. \tag{12}$$

The following conclusions may be then deduced about the equilibrium size of the subscription fund, *B*, dedicated to the production of the contested good *Y*.

Observation 2.

(i) $B^E \in (0,1)$*;*

(ii) $\frac{dB^E}{d\rho} > 0$ *and* $\frac{dB^E}{da_M}, \frac{dB^E}{da_H} < 0$*;*

(iii) *there exists* $\hat{\alpha} \in (0,1)$ *such that* $\frac{dB^E}{d\alpha} > 0$ *(resp.* < 0*) at every* $\alpha < \hat{\alpha}$ *(resp.*$> \hat{\alpha}$ *).*

Proof. See the appendix.

Total subscription for production of *Y* increases as the loss elasticity rises, reducing inter-community rivalry in its consumption and thereby increasing the effective shares of both communities. It decreases if either marginal degradation rate rises, reducing both effective shares. The effect of a marginal increase in the output elasticity of the contested good is however non-monotone. At low levels of such elasticity, total subscription rises as the elasticity increases, but it falls at output elasticities close to 1.

Since the equilibrium subscription fund expands as the loss function becomes more elastic, or as either marginal degradation rate declines, it is obvious from (3) that the amount of the contested good produced, $\tilde{Y}$, must correspondingly expand as well. More interestingly, it turns out that the latter necessarily declines as its production technology becomes more elastic, even when more input is forthcoming in response to such an increase (recall Observation 2(iii)).

Observation 3.

(i) $\tilde{Y}^E \in (0,1)$*;*

(ii) $\frac{d\tilde{Y}^E}{d\rho} > 0$ *and* $\frac{d\tilde{Y}^E}{da_M}, \frac{d\tilde{Y}^E}{da_H} < 0$*;*

(iii) $\frac{d\tilde{Y}^E}{d\alpha} < 0.$

Proof. See the appendix.

We now turn to characterizing aggregate conflict, measured in standard fashion by the total resource expended on appropriation, in equilibrium. Using (3) and (7), we get:

$$X^E = \left(\frac{T}{n^\theta}\right)(B^E)^\alpha\left(\left(\frac{dg_H(p_H^E)}{dp_H}\right)^{-1} + \left(\frac{dg_M(p_M^E)}{dp_M}\right)^{-1}\right)^{-1}; \tag{13}$$

where $\frac{dg_H(p_H^E)}{dp_H}$ and $\frac{dg_M(p_M^E)}{dp_M}$ are the derivatives of the effective share functions in (5) with respect to the corresponding own control shares, evaluated at the equilibrium values of the latter. Combining (4), (9) and (13), we get the expressions for community conflict allocations:

$$\text{for every } k \in \{H.M\}: X_k^E = \left(\frac{T}{n^\theta}\right)(B^E)^\alpha\left(\left(\frac{dg_H(p_H^E)}{dp_H}\right)^{-1} + \left(\frac{dg_M(p_M^E)}{dp_M}\right)^{-1}\right)^{-1}\left(A_{-k}{}^{\frac{1}{\rho}} + 1\right)^{-1}. \tag{14}$$

By (4) and (5), recalling that $a_k \in (0,2)$ by assumption, and that $p_H^E, p_M^E > 0$ by (9),

$$0 < \frac{dg_k(p_k^E)}{dp_k} = a_k\left(p_{-k}^E\right)^{\rho-1} < 2. \tag{15}$$

An explicit solution for X^E can be derived by combining (9), (12), (13) and (15), while those for X_H^E and X_M^E are derived by combining (9), (12), (14) and (15). Note that, since, by assumption, $T \in (0,1], \alpha \in (0,1)$ and $n^\theta \geq 1$, together, Observation 2(i), (13) and (15) imply that:

$$X^E \in (0,1). \tag{16}$$

Remark 1. Since each community must have resource endowment of at least 1 (as $n \geq 1$), Observation 2(i) and (16) imply that an interior Nash equilibrium must be feasible. Any interior equilibrium must satisfy (9) and (12)-(15), and satisfaction of these conditions guarantees the interiority of the equilibrium. It can checked that, given our assumption $a_H, a_M \in (0,2)$, every Nash equilibrium must satisfy equations (9) and (12)-(15). Hence, our model uniquely characterizes total common fund subscription and group conflict allocations in equilibrium. However, individual consumption of the private good *C*, individual common fund subscription and individual conflict contribution are all indeterminate, as are group private consumption and group common fund subscription levels. Thus, the model produces multiple Nash equilibria – essentially a consequence of the quasi-linearity of the reduced form of our preference specification.

4. Conflict, Aggregate Income and Cross-community Spillovers

We are now ready to address our central questions. What happens to aggregate conflict as cross-community negative consumption spillover effects become stronger, leading to either a decline in the loss elasticity or an increase in the marginal degradation rates? Furthermore, how do such changes affect aggregate social income (or consumption), measured in units of the numeraire good, *C*?

Noting (1)-(6), define the total consumption of community $k \in \{H, M\}$, measured in terms of units of the numeraire good – the private consumption good *C* – as follows:

$$W_k^E = \sum_{i=1}^{n}(1 - x_{ik} - y_{ik}) + nT\tilde{Y}^E(\frac{g_k^E}{n^\theta}). \tag{17}$$

Total equilibrium consumption in society, measured in units of *C*, is therefore:

$$W^E \equiv W_H^E + W_M^E = \left(2n - X^E - \left(\tilde{Y}^E\right)^{\frac{1}{\alpha}}\right) + \left(\frac{T}{n^{\theta-1}}\right)\tilde{Y}^E(g_H^E + g_M^E). \tag{18}$$

It is evident from (18) that cross-community spillovers affect aggregate social consumption in equilibrium, measured in units of the numeraire good, through three distinct, but inter-related, channels. These are: aggregate conflict investment (X^E), magnitude of the contested good ($\tilde{Y}^E$) and size of the effective shares (g_H^E and g_M^E).

Ceteris paribus, stronger negative spillovers from its antagonist increase the benefit, in terms of a gain in its effective share, to a community from a marginal increase in its control share (recall (15)). By itself, this effect will, clearly, incentivize greater conflict over control shares (note (13)). However, stronger cross-community negative spillovers, by effectively degrading more the stock of *Y* controlled by either community) will also reduce the equilibrium effective shares (note (10)). Since this reduces the benefit to either community from contributing to the common pool for production of the contestable good, the common fund will shrink, correspondingly reducing the production of *Y* (recall (12)). This fall in the size of the prize being contested over will dampen individual incentives to engage in appropriation. Thus, two contradictory effects obtain, which makes the net conflict consequence of stronger negative spillovers a priori ambiguous, and therefore non-trivial.

Is it possible, then, for stronger negative spillovers across communities to, somewhat counter-intuitively, reduce inter-community conflict? If so, noting (18), we have the intriguing possibility that greater negative consumption spillovers across communities may prove socially beneficial, in terms of their role in expanding the extent of social peace, and thus aggregate consumption. However, as already noted, they will reduce voluntary contributions to a common fund for production of a good whose consumption is at least partly non-rivalrous within a community. This effect may exacerbate the standard inefficiency associated with decentralized provision of any at least partly-public good, as well as that due to less than full marginal benefit of its subscription accruing to either community. The fall in the effective shares brought about by stronger negative spillovers will depress the numeraire good measure of aggregate consumption as well. Can one then find parametric configurations under which the combination of these three effects is positive, so that, counter-intuitively, stronger negative spillovers end up expanding both peace and aggregate consumption (or, equivalently, production or income) in society?

To fix ideas, it is helpful to begin with an illustrative example.

Example 1. Suppose $T = 1, \theta = 0, \rho = 1, a_H = a_M = b \in (0,2)$. Then, from (12), (13) and (15),

$$B^E = (\alpha)^{\frac{1}{1-\alpha}}\left(1 - \frac{b}{2}\right)^{\frac{1}{1-\alpha}}; \quad (19)$$

$$X^E = (\alpha)^{\frac{\alpha}{1-\alpha}}\left(1 - \frac{b}{2}\right)^{\frac{\alpha}{1-\alpha}}\left(\frac{b}{2}\right). \quad (20)$$

(i) We have, from (20):

$$\left(\frac{b}{X^E}\right)\left(\frac{dX^E}{db}\right) = \left[1 - \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{2}{b} - 1\right)^{-1}\right]. \quad (21)$$

By (21), any increase in the output elasticity reduces the elasticity of aggregate conflict with respect to the common degradation rate. Furthermore, $\frac{dX^E}{db} < 0$ (resp. > 0) when $\alpha >$ (resp. $<$) $\left(1 - \frac{b}{2}\right)$. Thus, starting from some initial level, say $\ddot{b}$, any increase in the common degradation rate will reduce aggregate conflict if the output elasticity is greater than $\left(1 - \frac{\ddot{b}}{2}\right)$. However, if the output elasticity falls below this threshold, a marginal increase in the common degradation rate will expand conflict. Thus, stronger negative spillovers depress conflict when the production technology for the contested good is relative elastic, but expand it otherwise. We shall generalize and expand this idea in Proposition 1 below.

(ii) Now, using (15) and (18)-(20),

$$W^E = 2n + \tilde{Y}^E \left[n(2-b) - \left(\frac{b}{2}\right)\right] - \left(\tilde{Y}^E\right)^{\frac{1}{\alpha}}. \tag{22}$$

Using (22), and recalling that, by (19), $\left[\tilde{Y}^E = (\alpha)^{\frac{\alpha}{1-\alpha}}\left(1-\frac{b}{2}\right)^{\frac{\alpha}{1-\alpha}}\right]$, we get:

$$\frac{dW^E}{db} = -\tilde{Y}^E \left[\frac{1}{2} + n\left(\frac{1}{1-\alpha}\right) - \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{1}{2-b}\right)\right]. \tag{23}$$

From (23),

$$\frac{dW^E}{db} < 0 \text{ iff } \left[n > \left(\frac{\alpha}{2-b}\right) - \frac{(1-\alpha)}{2}\right]. \tag{24}$$

By (24), a marginal increase in the common degradation rate will reduce aggregate social consumption when the common population size exceeds $\left[\left(\frac{\alpha}{2-b}\right) - \frac{(1-\alpha)}{2}\right]$, given $T = 1, \theta = 0$, and $\rho = 1$. For example, at $\left[b = 1, \alpha = \frac{1}{2}\right]$, $\frac{dW^E}{db} < 0$ for all $n \geq 1$; whereas, at $\left[b = \frac{19}{10}, \alpha = \frac{9}{10}\right]$, $\frac{dW^E}{db} < 0$ for all $n \geq 9$. These examples provide a concrete illustration of a general phenomenon that we shall identify and underline in Proposition 2 below.

(iii) Lastly, it follows from (24) that, given $T = 1, \theta = 0$, and $\rho = 1$, $\frac{dW^E}{db} > 0$ iff $\left[b > 2\left(\frac{1-2\alpha+2n}{1-\alpha+2n}\right)\right]$. Thus, any increase in the common degradation rate, b, over $\left(2\left(\frac{1-2\alpha+2n}{1-\alpha+2n}\right), 2\right)$ must increase aggregate social consumption. For example, when $n = 1, \alpha = \frac{1}{2}$, any increase in the common degradation rate over over $\left(\frac{8}{5}, 2\right)$ must expand it. Thus, our example suggests that, when negative spillovers are already at high levels (ρ is close to 1 and a_H, a_M are both close to 2), further increases in their strength might expand social consumption. Proposition 3 below formalizes and generalizes this idea. □

Proposition 1. *Let* $\eta_b \equiv \left(\frac{bdX^E}{X^E db}\right)$.

(i) $\eta_\alpha < 0$.

(ii) *For every* $k \in \{H, M\}$*: (a)* $\frac{d\left(\eta_{a_k}\right)}{d\alpha} < 0$ *and (b) there exists* $\breve{\gamma} \in (0,1)$ *such that:* $\eta_{a_k} > 0$ *(resp.* < 0*) if* $\alpha < \breve{\gamma}$ *(resp.* $> \breve{\gamma}$*).*

(iii) *Let* $a_H = a + \Delta$, *let* $a_M = a - \Delta$, *and suppose* $\Delta > 0$. *Then* $\frac{d(\eta_\Delta)}{d\alpha} > 0$*; furthermore, there exists* $\delta \in (0,1)$ *such that:* $\eta_\Delta < 0$ *(resp.* > 0*) if* $\alpha < \delta$ *(resp.* $> \delta$*).*

(iv) $\frac{d(\eta_\rho)}{d\alpha} > 0$*; furthermore, there exists* $\gamma \in (0,1)$ *such that:* $\eta_\rho < 0$ *(resp.* > 0*) if* $\alpha < \gamma$ *(resp.* $> \gamma$*).*

Proof. See the appendix.

Proposition 1 specifies the sign of the elasticity of aggregate conflict with respect to the different parameters of the model. This elasticity with respect to the production technology parameter (α) is negative, implying that conflict falls as the production technology becomes more elastic (Proposition 1(i)). This happens because an increase in the latter reduces the production of the contested good (Observation 3(iii)), but does not affect effective shares (recall (10)). The impact of changes in the spillover parameters is however more complicated. The conflict elasticity with respect to either marginal degradation rate declines monotonically as the output elasticity rises – it is positive at low output elasticities, but negative at high ones (Proposition 1(ii)). Thus, greater negative spillover from, say, *M* to *H*, due to a rise in the marginal degradation rate for *H*, will reduce aggregate conflict when the output elasticity is close to 1, but increase it when the latter is close to 0. A mean-preserving decrease in the spread of the marginal degradation rates has the same effect (Proposition 1(iii)), as does a reduction in the elasticity of the loss function (Proposition 1(iv)). Thus, in sum, stronger negative spill-overs across communities, instead of increasing conflict, may indeed dampen it when the output elasticity is close to 1, as suggested by Example 1(part (i)) earlier.

Intuitively, as already noted, stronger negative spillovers reduce subscriptions and thereby contract the size of the prize (i.e., the contested good, *Y*). Such contraction reduces the incentive to invest resources in appropriation. Stronger negative spillovers however increase the incentive to invest in appropriation for any given level of the prize. When the output elasticity is close to 1, the first effect dominates. The second effect dominates when the output elasticity is close to 0.

We now turn to aggregate social consumption. From (18), we get the following conclusions.

Proposition 2.

(i) *There must exist* $\dot{n}$ *such that, if* $n > \dot{n}$*, then: (a)* $\frac{dW^E}{da_H}, \frac{dW^E}{da_M} < 0$*, and (b) given* $(a_H + a_M)$*,* $\frac{dW^E}{d\Delta} > 0$*, whenever* $\Delta \equiv \frac{\|a_H - a_M\|}{2} > 0$*.*

(ii) *There must exist* $\widehat{\widehat{n}}$ *such that, if* $n > \widehat{\widehat{n}}$*, then* $\frac{dW^E}{d\rho} > 0$*.*

(iii) *There must exist* $\hat{n}$ *such that, if* $n > \hat{n}$, *then* $\frac{dW^E}{d\alpha} < 0$.

Proof. See the appendix.

Proposition 2 essentially implies that, given the other parameters of the model, stronger negative cross-community spillovers unambiguously reduce aggregate social consumption when the communities are sufficiently numerous (parts (i) and (ii)). More rigorously, given the other parameters, one can always find a threshold community size with the following property: stronger negative cross-community spillovers, whether in the form of higher marginal degradation rates or a lower loss elasticity, must reduce aggregate social consumption whenever the communities are larger than this threshold size. A mean-preserving contraction in the spread of the marginal degradation rates and a rise in the output elasticity will both have the same effect when the communities are above this size threshold. The exact threshold size of the communities will of course depend on the parameters being held constant, as illustrated in Example 1 (part (ii)). All these conclusions essentially arise from the fact that combined negative effect of any fall in the subscription fund and in effective shares must dominate any consumption gains from consequent lower expenditure on appropriation when the communities are sufficiently large.

Nonetheless, given any arbitrary community size, there always exist parametric configurations under which, at the margin, greater negative spillovers will increase aggregate consumption in society. This holds, in particular, under a combination of high marginal degradation rates and low loss elasticity (recall part (iii) of Example 1). This somewhat paradoxical possibility is presented formally in Proposition 3 below.

Proposition 3. *Suppose* $a_H, a_M = a$. *Then, given any* $n \geq 1$, *there exist* $\breve{a} > 0, \breve{b} > 1$ *such that, whenever* $[a \in (\breve{a}, 2)$ *and* $\rho \in [1, \breve{b})]$,

$$[\frac{dW^E}{da_H}, \frac{dW^E}{da_M}, \frac{dW^E}{d\alpha} > 0 \text{ and } \frac{dW^E}{d\rho} < 0].$$

Proof. *See the appendix.*

Proposition 3 implies the following. Given any community size and any elasticity of the production technology for the contested good, one can find a threshold level of the marginal degradation rate, say $\boldsymbol{\breve{a}}$, and a threshold level of the loss elasticity, say $\boldsymbol{\breve{b}}$, with the following property. Consider any initial situation where: (a) the two marginal degradation rates are identical and above $\breve{\boldsymbol{a}}$, while (b) the loss elasticity is below $\breve{\boldsymbol{b}}$. This initial situation can be broadly interpreted as one characterized by negative cross-community spillovers that are strong enough to exceed a certain threshold level. Any increase of either degradation rate, from this initial situation, must improve aggregate social consumption (or equivalently, production or income) measured in units of the numeraire good. The same holds for a marginal increase in the output elasticity. Any decrease in the loss elasticity from this initial situation must improve aggregate social consumption as well. Thus, in sum, greater negative spillovers across communities may be aggregate consumption augmenting, and, in this specific sense, socially beneficial, irrespective of the size of the communities. Intuitively, this is likely when the strength of negative spillovers increases further, from an already high initial level.

Notice that a linear aggregative structure for the effective share function implies $\rho = 1$. It follows from Proposition 3 that, when the effective share function is assumed to have this form, there will always exist a threshold, beyond which stronger negative spillovers will be consumption augmenting.

5. Variants

We now identify and discuss two possible variants of our model in Section 3, one of which involves an alternative production technology for the contested good, and the other a more general specification of individual preferences.

(i) We have assumed, via (2) and (3), that the subscriptions of the two communities combine in summative fashion to generate the total input allocation for the contestable good. An alternative modeling strategy might be to assume that each community separately produces the contestable good using the total subscription of its own community members as the input pool: $[B_k = \sum_{i=1}^{n} y_{ik}]$. Letting $\tilde{Y}_k$ denote the amount of the contestable good produced by community k (i.e., $\tilde{Y}_k \equiv (B_k)^{\alpha}$), the total stock of Y is then given as: $[\tilde{Y} = \tilde{Y}_H + \tilde{Y}_M]$, instead of (3). Notice that the equations derived from the first order conditions, (7) and (11), remain unchanged under this formulation (except that B is replaced, sequentially, by B_H and B_M in (11)). Since (7) remains unchanged, equations (8)-(10) remain unchanged as well, as does Observation 1. The expression for total common pool subscription in (12) now comes to stand for that within either community, $B_H^E = B_M^E$, so that we have $B^E = 2B_H^E$, with B_H^E given by (12). It follows that we now get $B_k^E, \tilde{Y}_k \in (0,1)$, so that $B^E, \tilde{Y}^E \in (0,2)$, while the comparative static properties stated in Observation 2 ((ii) and (iii)) and Observation 3 ((ii) and (iii)) remain unchanged. Equation (13) now comes to take the form:

$$X^E = 2\left(\frac{T}{n^\theta}\right)(B_H^E)^\alpha\left(\left(\frac{dg_H(p_H^E)}{dp_H}\right)^{-1} + \left(\frac{dg_M(p_M^E)}{dp_M}\right)^{-1}\right)^{-1},$$

and (14) is modified correspondingly. The claim in (15) remains unchanged, while that in (16) changes to: $X^E \in (0,2)$. The claims made in Remark 1 now come to hold under the assumption that $n \geq 3$, with the added proviso that each group's total contribution to the production of Y, i.e., B_k^E, is now uniquely defined by (12). Thus, in this alternative formulation, for each group, total private consumption, total subscription to the production of the contestable good and total expenditure on appropriation all come to be uniquely defined in equilibrium. However, individual private consumption, individual conflict allocation and individual contribution to the production of *Y* all remain indeterminate, as in our benchmark model (recall Remark 1). Since the expressions, respectively, for total spending on *Y* in (12) and total conflict allocation in (13) change only by a multiplicative factor of 2, the comparative static results presented in Propositions 1-3 below remain unchanged. Thus, in sum, very little of substance changes if we replace the specification of the contestable good's production technology adopted in our benchmark formulation by the alternative specification discussed above. A further generalization, which permits the elasticity of that production technology to vary across communities (i.e., $\tilde{Y}_k = (B_k)^{\alpha_k}$, where α_H need not equal α_M), drastically complicates the algebra, but does not yield much additional insight.

(ii) The linear specification of the utility function adopted by us via (6) evidently constitutes a limiting case of the quasi-linear form:

$$u_{ik} = c_{ik} + T\left(\frac{\tilde{Y}g_k}{n^\theta}\right)^\gamma;$$

where $\gamma \in (0,1)$. The assumed linearity of the utility function in the private consumption good, *C*, enables us to use that good as the numeraire commodity, and thereby arrive at aggregate measures of social consumption in terms of units of *C* (recall Propositions 2 and 3). We cannot consistently use the contestable group-specific public good, *Y*, as the numeraire good because its marginal utility, even in our linear case of (6), cannot be defined independently of the endogenous variable g_k. Thus, we cannot dispense with the assumed linearity of our utility function in the private good. However, our comparative static conclusions, summarized by Propositions 1-3, are consistent with diminishing marginal utility from the contested good. Propositions 1-3 are derived implicitly for the limiting case ($\gamma = 1$) of the quasi-linear form defined above. Since all relevant functions are continuous, it follows that Propositions 1-3 must continue to hold when γ is less than 1, but sufficiently close to it (i.e., within some non-empty interval $(\varepsilon, 1)$). Thus, the assumed linearity of our utility function drastically simplifies the algebra, but is not necessary for our main conclusions to hold. However, it does appear an open question whether Propositions 1-3 hold for every value of γ in the $(0,1)$ interval.

6. Concluding Remarks

This paper has developed a parsimonious theoretical framework to examine how cross-community cost or benefit spillovers, arising from the consumption or exploitation of group-specific public goods, affects both inter-group conflicts over the appropriation of such goods and decentralized private provision for their production. We have offered a model which integrates production versus appropriation choices, vis-à-vis group-specific public goods, with their decentralized voluntary supply, against a backdrop of such cross-community consumption spillovers. Our flexible and general formulation of consumption spillovers incorporates earlier specifications as alternative special cases. We have shown that, somewhat counter-intuitively, stronger negative (or weaker positive) consumption spillovers across communities may serve to reduce inter-group conflict and increase aggregate income (and consumption) in society under certain conditions, which we have identified. Thus, stronger negative consumption spillovers, under certain conditions, may have socially beneficial consequences. Of course, their impact will be both conflict-augmenting and income-compressing, as may be intuitively expected, under other conditions. We have identified these latter conditions as well.

In many different developing country contexts, climate change and environmental degradation (as well as state policy, population growth or market pressures) increase the costs imposed on one ethnic group due to another group's exploitation of some natural resource. This increases inter-group competition over natural resources and often triggers persistent and widespread social conflict. Such conflict in turn affects decentralized community-level mechanisms for the maintenance and augmentation of the contested natural resource, which feeds back into the original conflict. Our general theoretical analysis offers a broad conceptual structure within which to organize case studies of such feedback loops, linking ethnic conflict and natural resource degradation, in specific developing country contexts.

Dasgupta and Guha Neogi [2018] have examined how within-group fragmentation affects inter-group contests over group-specific public goods, while Esteban and Ray [2001], generalizing Olson [1965], have investigated how group size affects such conflicts under quite broad preference specifications. Analogous incorporation of within-group fragmentation and group size effects in suitably augmented versions of our model may generate useful insights. How income/wealth inequality and polarization affect social conflict [Esteban and Ray, 2011b] within our framework remains an open question. One may use alternatives to our perfect-substitutes summative specification for each community's aggregate group conflict effort, such as a constant elasticity of substitution aggregation [Kolmar and Rommeswinkel, 2013; Cheikbossian and Fayat, 2018], the best-shot specification [Chowdhury *et al.,* 2013] or the weakest-link formulation [Lee, 2012]. Similar alternatives have also been applied to specify the public good's production technology in voluntary subscription models [Cornes, 1993]. The consequences of cross-community spillovers with endogenous provisioning, in public good contests under such alternative specifications, whether of the conflict technology or the public good's production technology, constitute another promising avenue of future enquiry. We look forward to these and other extensions in future work.

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Appendix: Proofs

Proof of Observation 2. From Observation 1 ((i) and (ii)), for all $k \in \{H, M\}$, $g_k^E \in (0,1)$. By assumption, $T \in (0,1]$ and $\alpha \in (0,1)$, and $n \geq 1$. Then part (i) of Observation 1 follows immediately from (12) in light of (10). Now, from (12), we get:

$$B^E = \left(\left(\frac{T\alpha}{n^\theta}\right) \left(1 - \frac{1}{\rho\left(a_H^{\frac{-1}{\rho}} + a_M^{\frac{-1}{\rho}}\right)^{\rho}} \right) \right)^{\frac{1}{1-\alpha}} = \left(\left(\frac{T\alpha}{n^\theta}\right) \left(1 - \frac{a_M}{\rho\left(1 + \left(\frac{a_M}{a_H}\right)^{\frac{1}{\rho}}\right)^{\rho}} \right) \right)^{\frac{1}{1-\alpha}}. \quad \text{(A.1)}$$

Without loss of generality, suppose $\left(\frac{a_M}{a_H}\right) \leq 1$. Then part (ii) of Observation 2 follows immediately from (A.1). Now, from (A.1),

$$\ln B^E = \left(\frac{1}{1-\alpha}\right) \left[\ln \alpha + \ln \left(\left(\frac{T}{n^\theta}\right) \left(1 - \frac{a_H a_M}{\rho\left(a_H^{\frac{1}{\rho}} + a_M^{\frac{1}{\rho}}\right)^{\rho}} \right) \right) \right];$$

implying:

$$\left(\frac{(1-\alpha)^2\alpha}{B^E}\right)\frac{dB^E}{d\alpha}=\left[(1-\alpha)+\ln\left(\left(\frac{T\alpha}{n^\theta}\right)\left(1-\frac{a_Ha_M}{\rho\left(a_H^{\frac{1}{\rho}}+a_M^{\frac{1}{\rho}}\right)^\rho}\right)\right)^\alpha\right]. \tag{A.2}$$

Let $Z\equiv\left[(1-\alpha)+\ln\left(\left(\frac{T\alpha}{n^\theta}\right)\left(1-\frac{a_Ha_M}{\rho\left(a_H^{\frac{1}{\rho}}+a_M^{\frac{1}{\rho}}\right)^\rho}\right)\right)^\alpha\right]$. Then $\lim_{\alpha\to 0}Z=1$, and $\lim_{\alpha\to 1}Z=\ln\left(\left(\frac{T}{n^\theta}\right)\left(1-\frac{a_Ha_M}{\rho\left(a_H^{\frac{1}{\rho}}+a_M^{\frac{1}{\rho}}\right)^\rho}\right)\right)<0$. Now let $\tilde{Z}\equiv\alpha\ln\left(\left(\frac{T\alpha}{n^\theta}\right)\left(1-\frac{a_Ha_M}{\rho\left(a_H^{\frac{1}{\rho}}+a_M^{\frac{1}{\rho}}\right)^\rho}\right)\right)$. Then $\frac{d\tilde{Z}}{d\alpha}=\ln\left(\left(\frac{T\alpha}{n^\theta}\right)\left(1-\frac{a_Ha_M}{\rho\left(a_H^{\frac{1}{\rho}}+a_M^{\frac{1}{\rho}}\right)^\rho}\right)\right)+1$; implying $\frac{dZ}{d\alpha}=\ln\left(\left(\frac{T\alpha}{n^\theta}\right)\left(1-\frac{a_Ha_M}{\rho\left(a_H^{\frac{1}{\rho}}+a_M^{\frac{1}{\rho}}\right)^\rho}\right)\right)<0$. Part (iii) of Observation 2 follows. □

Proof of Observation 3. Parts (i) and (ii) of Observation 3 follow immediately from parts (i) and (ii), respectively, of Observation 2, in light of (3). Now, using (3) and (12),

$$\tilde{Y}^E=\left(\left(\frac{T\alpha}{n^\theta}\right)\left(1-\frac{a_Ha_M}{\rho\left(a_H^{\frac{1}{\rho}}+a_M^{\frac{1}{\rho}}\right)^\rho}\right)\right)^{\frac{\alpha}{1-\alpha}}. \tag{A.3}$$

Hence

$$\ln\tilde{Y}^E=\left(\frac{\alpha}{1-\alpha}\right)\left[\ln\alpha+\ln\left(\left(\frac{T}{n^\theta}\right)\left(1-\frac{a_Ha_M}{\rho\left(a_H^{\frac{1}{\rho}}+a_M^{\frac{1}{\rho}}\right)^\rho}\right)\right)\right];$$

implying:

$$\left(\frac{(1-\alpha)^2}{\tilde{Y}}\right)\frac{d\tilde{Y}^E}{d\alpha} = \left[(1-\alpha) + \ln\left(\left(\frac{T\alpha}{n^\theta}\right)\left(1 - \frac{a_H a_M}{\rho\left(a_H^{\frac{1}{\rho}} + a_M^{\frac{1}{\rho}}\right)^\rho}\right)\right)\right]. \tag{A.4}$$

Let $Z \equiv \left[(1-\alpha) + \ln\left(\left(\frac{T\alpha}{n^\theta}\right)\left(1 - \frac{a_H a_M}{\rho\left(a_H^{\frac{1}{\rho}} + a_M^{\frac{1}{\rho}}\right)^\rho}\right)\right)\right]$. Then $\lim_{\alpha\to 0} Z = -\infty$, and $\lim_{\alpha\to 1} Z = \ln\left(\left(\frac{T}{n^\theta}\right)\left(1 - \frac{a_H a_M}{\rho\left(a_H^{\frac{1}{\rho}} + a_M^{\frac{1}{\rho}}\right)^\rho}\right)\right) < 0$. Furthermore, $\frac{dZ}{d\alpha} = -1 + \frac{1}{\alpha} > 0$. Part (iii) of Observation 3 follows. □

Proof of Proposition 1. By (5) and (9), $\left(\left(\frac{dg_H(p_H^E)}{dp_H}\right)^{-1} + \left(\frac{dg_M(p_M^E)}{dp_M}\right)^{-1}\right)^{-1}$ is independent of α. Part (i) of Proposition 1 then follows directly from (13) in light of (3) and Observation 3(iii).

We shall prove parts (ii), (iii) and (iv) of Proposition 1 via the following lemma.

Lemma 1.

(i) *For every* $k \in \{H, M\}$, $\lim_{\alpha\to 0}\left(\frac{a_k dX^E}{X^E da_k}\right) = p_{-k}^E$, $\lim_{\alpha\to 1}\left(\frac{a_k dX^E}{X^E da_k}\right) = -\infty$, *and* $\frac{d\left(\frac{a_k dX^E}{X^E da_k}\right)}{d\alpha} < 0$.

(ii) *Let* $a_H = a + \Delta$, *let* $a_M = a - \Delta$, *and suppose* $\Delta > 0$. *Then,* $\lim_{\alpha\to 0}\left(\frac{\Delta dX^E}{X^E d\Delta}\right) = \Delta\left[\frac{p_M^E}{a_H} - \frac{p_H^E}{a_M}\right] < 0$, $\lim_{\alpha\to 1}\left(\frac{\Delta dX^E}{X^E d\Delta}\right) = \infty$, *and* $\frac{d\left(\frac{\Delta dX^E}{X^E d\Delta}\right)}{d\alpha} > 0$.

(iii) $\lim_{\alpha\to 0}\left(\frac{\rho dX^E}{X^E d\rho}\right) = K$, $\lim_{\alpha\to 1}\left(\frac{\rho dX^E}{X^E d\rho}\right) = \infty$, *and* $\frac{d\left(\frac{\rho dX^E}{X^E d\rho}\right)}{d\alpha} > 0$; *where* $K = \rho\left(\frac{\ln p_M^E + \left(\frac{p_H^E}{p_M^E}\right)\ln p_H^E}{1 + \left(\frac{p_H^E}{p_M^E}\right)}\right) < 0$.

Proof of Lemma 1.

(i) Using (13),

$$\left(\frac{n^{\theta}}{T}\right)X^{E}=\tilde{Y}^{E}\left(\left(\frac{dg_H(p_H^E)}{dp_H}\right)^{-1}+\left(\frac{dg_M(p_M^E)}{dp_M}\right)^{-1}\right)^{-1}. \tag{A.5}$$

Hence,

$$\left(\frac{n^{\theta}}{T}\right)\left(\left(\frac{dg_H(p_H^E)}{dp_H}\right)^{-1}+\left(\frac{dg_M(p_M^E)}{dp_M}\right)^{-1}\right)\left(\frac{dX^E}{da_k}\right)=\frac{d\tilde{Y}^E}{da_k}-\tilde{Y}^E\left(\left(\frac{dg_H(p_H^E)}{dp_H}\right)^{-1}+\left(\frac{dg_M(p_M^E)}{dp_M}\right)^{-1}\right)^{-1}\left[\left(\frac{\partial\left(\left(\frac{dg_H(p_H^E)}{dp_H}\right)^{-1}+\left(\frac{dg_M(p_M^E)}{dp_M}\right)^{-1}\right)}{\partial p_H}\right)\left(\frac{dp_H}{da_k}\right)+\left(\frac{\partial\left(\left(\frac{dg_H(p_H^E)}{dp_H}\right)^{-1}+\left(\frac{dg_M(p_M^E)}{dp_M}\right)^{-1}\right)}{\partial a_k}\right)\right]. \tag{A.6}$$

Define $S\equiv\left(\frac{\partial\left(\left(\frac{dg_H}{dp_H}\right)^{-1}+\left(\frac{dg_M}{dp_M}\right)^{-1}\right)}{\partial p_H}\right)$. Then, noting that $p_H=1-p_M$, we get:

$$S=-\left(\left(\frac{dg_H}{dp_H}\right)^{-2}\frac{d^2g_H}{dp_H{}^2}-\left(\frac{dg_M}{dp_M}\right)^{-2}\frac{d^2g_M}{dp_M{}^2}\right). \tag{A.7}$$

Recall that, from (5): $\frac{dg_k}{dp_k}=a_kp_{-k}{}^{\rho-1}$, $\frac{d^2g_k}{dp_k{}^2}=-\left(\frac{\rho-1}{p_{-k}}\right)\frac{dg_k}{dp_k}$. Then, from (A.7), we have:

$$S=\left(\frac{dg_M}{dp_M}\right)^{-1}(\rho-1)\left(\left(\frac{dg_M}{dp_M}\right)\left(\frac{dg_H}{dp_H}\right)^{-1}\left(\frac{1}{p_M}\right)-\left(\frac{1}{p_H}\right)\right).$$

Since $\left(\frac{dg_M}{dp_M}\right)\left(\frac{dg_H}{dp_H}\right)^{-1}=\left(\frac{a_M}{a_H}\right)\left(\frac{p_H}{p_M}\right)^{\rho-1}$, and, from (8), $\left[1=\left(\frac{a_H}{a_M}\right)\left(\frac{p_M}{p_H}\right)^{\rho}\right]$ in equilibrium, we then have:

$$S^E=\left(\frac{dg_M}{dp_M}\right)^{-1}(\rho-1)\left(\left(\frac{p_H}{p_M}\right)^{-1}\left(\frac{1}{p_M}\right)-\left(\frac{1}{p_H}\right)\right)=0. \tag{A.8}$$

In light of (A.8), (A.6) reduces to:

$$\left(\frac{n^{\theta}}{T}\right)\left(\left(\frac{dg_H(p_H^E)}{dp_H}\right)^{-1}+\left(\frac{dg_M(p_M^E)}{dp_M}\right)^{-1}\right)\left(\frac{dX^E}{da_k}\right)=\frac{d\tilde{Y}^E}{da_k}-\tilde{Y}^E\left(\left(\frac{dg_H(p_H^E)}{dp_H}\right)^{-1}+\left(\frac{dg_M(p_M^E)}{dp_M}\right)^{-1}\right)^{-1}\left(\frac{\partial\left(\left(\frac{dg_H(p_H^E)}{dp_H}\right)^{-1}+\left(\frac{dg_M(p_M^E)}{dp_M}\right)^{-1}\right)}{\partial a_k}\right). \tag{A.9}$$

Now recall that, by (5):

$$\left(\frac{dg_H}{dp_H}\right)^{-1}+\left(\frac{dg_M}{dp_M}\right)^{-1}=\left(\frac{1}{a_Hp_M{}^{\rho-1}}+\frac{1}{a_Mp_H{}^{\rho-1}}\right), \tag{A.10}$$

implying:

$$\left(\frac{\partial\left(\left(\frac{dg_H}{dp_H}\right)^{-1}+\left(\frac{dg_M}{dp_M}\right)^{-1}\right)}{\partial a_k}\right) = -{a_k}^{-1}\left(\frac{dg_k}{dp_k}\right)^{-1}. \tag{A.11}$$

Combining (A.10) and (A.11), and recalling that $A_k = \frac{a_k}{a_{-k}}$, we get:

$$\left(\left(\frac{dg_H}{dp_H}\right)^{-1} + \left(\frac{dg_M}{dp_M}\right)^{-1}\right)^{-1}\left[\left(\frac{\partial\left(\left(\frac{dg_H}{dp_H}\right)^{-1}+\left(\frac{dg_M}{dp_M}\right)^{-1}\right)}{\partial a_k}\right)\right] = -\frac{{a_k}^{-1}}{\left(1+A_k\left(\frac{p_{-k}}{p_k}\right)^{\rho-1}\right)}. \tag{A.12}$$

Hence, combining (A.9) and (A.12), we have:

$$\left(\frac{n^\theta}{T}\right)\left(\left(\frac{dg_H(p_H^E)}{dp_H}\right)^{-1} + \left(\frac{dg_M(p_M^E)}{dp_M}\right)^{-1}\right)\left(\frac{dX^E}{da_k}\right) = \frac{d\tilde{Y}^E}{da_k} + \frac{{a_k}^{-1}\tilde{Y}^E}{\left(1+A_k\left(\frac{p_{-k}}{p_k}\right)^{\rho-1}\right)}. \tag{A.13}$$

By (13),

$$\left(\frac{n^\theta}{T}\right)\left(\left(\frac{dg_H(p_H^E)}{dp_H}\right)^{-1} + \left(\frac{dg_M(p_M^E)}{dp_M}\right)^{-1}\right) = \tilde{Y}^E(X^E)^{-1}. \tag{A.14}$$

Since, by (3), $\left[\left(\frac{d\tilde{Y}^E}{da_k}\right) = \alpha\tilde{Y}^E(B^E)^{-1}\left(\frac{dB^E}{da_k}\right)\right]$, (A.13) and (A.14) together yield:

$$a_k(X^E)^{-1}\left(\frac{dX^E}{da_k}\right) = \alpha a_k(B^E)^{-1}\left(\frac{dB^E}{da_k}\right) + \frac{1}{\left(1+A_k\left(\frac{p_{-k}}{p_k}\right)^{\rho-1}\right)}. \tag{A.15}$$

Since, from (8), $\left[1 = \left(\frac{a_H}{a_M}\right)\left(\frac{p_M}{p_H}\right)^\rho\right]$ in equilibrium, (A.15) reduces to:

$$a_k(X^E)^{-1}\left(\frac{dX^E}{da_k}\right) = \alpha a_k(B^E)^{-1}\left(\frac{dB^E}{da_k}\right) + p_{-k}^E. \tag{A.16}$$

Now, from (11),

$$B^E = \left(\frac{\alpha T g_k^E}{n^\theta}\right)^{\frac{1}{1-\alpha}}, \tag{A.17}$$

so that:

$$\alpha\left(\frac{a_k}{B^E}\right)\frac{dB^E}{da_k} = a_k\left(\frac{\alpha}{1-\alpha}\right)\left(g_k^E\right)^{-1}\left(\frac{d{g^E}_k}{da_k}\right). \tag{A.18}$$

From (9) and (10),

$$\frac{dg_k^E}{da_k} = -\rho^{-1}\left(p_{-k}^E\right)^{(\rho+1)}. \tag{A.19}$$

Combining (A.16), (A.18) and (A.19), we get:

$$a_k (X^E)^{-1}\left(\frac{dX^E}{da_k}\right) = p_{-k}^E\left(1 - a_k\left(\frac{\alpha}{1-\alpha}\right)\left(g_k^E\right)^{-1}\rho^{-1}\left(p_{-k}^E\right)^{\rho}\right). \tag{A.20}$$

Noting that, by (9) and (10), both p_{-k}^E and g_k^E are independent of α, part (i) of Lemma 1 follows.

(ii) From (A.20), recalling (10),

$$\Delta(X^E)^{-1}\left[\frac{dX^E}{d\Delta}\right] = \Delta(X^E)^{-1}\left[\frac{dX^E}{da_H} - \frac{dX^E}{da_M}\right]$$

$$= \left(\frac{\alpha}{1-\alpha}\right)\Delta(g_M^E)^{-1}\rho^{-1}\left[(p_H^E)^{(\rho+1)} - (p_M^E)^{(\rho+1)}\right] + \Delta[a_H{}^{-1}p_M^E - a_M{}^{-1}p_H^E].$$

Since $\Delta > 0$, $a_H > a_M$. Hence, by (8), $p_H^E > p_M^E$, so that $[a_H{}^{-1}p_M^E - a_M{}^{-1}p_H^E] < 0$ and $\left[(p_H^E)^{(\rho+1)} - (p_M^E)^{(\rho+1)}\right] > 0$. Part (ii) of Lemma 1 follows.

(iii) From (A.5),

$$\left(\frac{n^\theta}{T}\right)\left(\left(\frac{dg_H(p_H^E)}{dp_H}\right)^{-1} + \left(\frac{dg_M(p_M^E)}{dp_M}\right)^{-1}\right)\left(\frac{dX^E}{d\rho}\right) = \frac{d\tilde{Y}^E}{d\rho} - \tilde{Y}^E\left(\left(\frac{dg_H(p_H^E)}{dp_H}\right)^{-1} + \left(\frac{dg_M(p_M^E)}{dp_M}\right)^{-1}\right)^{-1}\left[\left(\frac{\partial\left(\left(\frac{dg_H(p_H^E)}{dp_H}\right)^{-1} + \left(\frac{dg_M(p_M^E)}{dp_M}\right)^{-1}\right)}{\partial p_H}\right)\left(\frac{dp_H}{d\rho}\right) + \left(\frac{\partial\left(\left(\frac{dg_H(p_H^E)}{dp_H}\right)^{-1} + \left(\frac{dg_M(p_M^E)}{dp_M}\right)^{-1}\right)}{\partial \rho}\right)\right]; \tag{A.21}$$

which, in light of (A.7), (A.8), (A.10) and (A.14) reduces to:

$$(X^E)^{-1}\left(\frac{dX^E}{d\rho}\right) = \tilde{Y}^{E^{-1}}\left(\frac{d\tilde{Y}^E}{d\rho}\right) - \left(\frac{1}{a_H(p_M^E)^{\rho-1}} + \frac{1}{a_M(p_H^E)^{\rho-1}}\right)^{-1}\left(\frac{\partial\left(\left(\frac{dg_H}{dp_H}\right)^{-1} + \left(\frac{dg_M}{dp_M}\right)^{-1}\right)}{\partial\rho}\right). \tag{A.22}$$

From (A.10),

$$\left(\frac{\partial\left(\left(\frac{dg_H}{dp_H}\right)^{-1}+\left(\frac{dg_M}{dp_M}\right)^{-1}\right)}{\partial\rho}\right)=-\left(\left(\frac{dg_H}{dp_H}\right)^{-1}(\ln p_M)+\left(\frac{dg_M}{dp_M}\right)^{-1}(\ln p_H)\right). \tag{A.23}$$

Using (5), (A.22) and (A.23), we then get:

$$(X^E)^{-1}\left(\frac{dX^E}{d\rho}\right)=\tilde{Y}^{E^{-1}}\left(\frac{d\tilde{Y}^E}{d\rho}\right)+\left(\frac{1}{a_H(p_M^E)^{\rho-1}}+\frac{1}{a_M(p_H^E)^{\rho-1}}\right)^{-1}\left(\frac{\ln p_M^E}{a_H(p_M^E)^{\rho-1}}+\frac{\ln p_H^E}{a_M(p_H^E)^{\rho-1}}\right). \tag{A.24}$$

Since, by (3), $\left[\left(\frac{d\tilde{Y}^E}{d\rho}\right)=\alpha\tilde{Y}^E(B^E)^{-1}\left(\frac{dB^E}{d\rho}\right)\right]$, in light of (8), (A.24) reduces to:

$$\rho(X^E)^{-1}\left(\frac{dX^E}{d\rho}\right)=\rho\alpha(B^E)^{-1}\left(\frac{dB^E}{d\rho}\right)+\rho\left(\frac{\ln p_M^E+\left(\frac{p_H^E}{p_M^E}\right)\ln p_H^E}{1+\left(\frac{p_H^E}{p_M^E}\right)}\right). \tag{A.25}$$

Without loss of generality, suppose $A_M \geq 1$. From (A.17), we have:

$$\rho\alpha(B^E)^{-1}\frac{dB^E}{d\rho}=\rho\left(\frac{\alpha}{1-\alpha}\right)(g_H^E)^{-1}\left(\frac{dg_H^E}{d\rho}\right). \tag{A.26}$$

Then, (A.25) reduces to:

$$\rho(X^E)^{-1}\left(\frac{dX^E}{d\rho}\right)=\rho\left(\frac{\alpha}{1-\alpha}\right)(g_H^E)^{-1}\left(\frac{dg_H^E}{d\rho}\right)+\rho\left(\frac{\ln p_M^E+\left(\frac{p_H^E}{p_M^E}\right)\ln p_H^E}{1+\left(\frac{p_H^E}{p_M^E}\right)}\right). \tag{A.27}$$

Now, from (5),

$$\frac{dg^E{}_H}{d\rho}=-a_H\left(\frac{p_M^{E\,\rho}}{\rho}\right)\left[\ln p_M^E-\left(\frac{\rho}{p_M^E}\right)\frac{dp_H^E}{d\rho}-\frac{1}{\rho}\right]. \tag{A.28}$$

From (9), $\frac{dp_H^E}{d\rho}=\left(A_M{}^{\frac{1}{\rho}}+1\right)^{-2}\left(\frac{A_M{}^{\frac{1}{\rho}}}{\rho^2}\right)\ln(A_M)\geq 0$, since $A_M \geq 1$ by assumption. Hence, from (A.28), $\frac{dg_H^E}{d\rho}$ is positive, finite and independent of α. The terms g_H^E and p_H^E are both finite and independent of α as well. Lemma 1(iii) then follows from (A.27). □

Parts (ii), (iii) and (iv) of Proposition 1 follow immediately from parts (i), (ii) and (iii) of Lemma 1, respectively. □

Proof of Proposition 2.

(i) From (18),

$$\frac{dW^E}{da_k} = \left(-\frac{dX^E}{da_k} - \frac{dB^E}{da_k}\right) + \alpha\left(\frac{T}{n^{\theta-1}}\right)(B^E)^{\alpha-1}\left(\frac{dB^E}{da_k}\right)(g_H^E + g_M^E) + \left(\frac{T}{n^{\theta-1}}\right)\tilde{Y}^E\left(\frac{d(g_H^E+g_M^E)}{da_k}\right). \quad \text{(A.29)}$$

Together, (A.29) and (11) yield:

$$:\frac{dW^E}{da_k} = \left((2n-1)\left(\frac{dB^E}{da_k}\right) - \frac{dX^E}{da_k}\right) + \left(\frac{T}{n^{\theta-1}}\right)\tilde{Y}^E\left(\frac{d(g_H^E+g_M^E)}{da_k}\right). \quad \text{(A.30)}$$

In light of (A.16), (A.30) reduces to:

$$:\frac{dW^E}{da_k} = [(2n-1) - \alpha(B^E)^{-1}X^E]\left(\frac{dB^E}{da_k}\right) - \frac{X^E p_{-k}^E}{a_k} + \left(\frac{T}{n^{\theta-1}}\right)\tilde{Y}^E\left(\frac{d(g_H^E+g_M^E)}{da_k}\right). \quad \text{(A.31)}$$

From (11) and (13),

$$\alpha\left(\frac{X^E}{B^E}\right) = g_H{}^{-1}\left(\left(\frac{dg_H(p_H^E)}{dp_H}\right)^{-1} + \left(\frac{dg_M(p_M^E)}{dp_M}\right)^{-1}\right)^{-1}. \quad \text{(A.32)}$$

From (8) and (A.10),

$$g_H{}^{-1}\left(\left(\frac{dg_H}{dp_H}\right)^{-1} + \left(\frac{dg_M}{dp_M}\right)^{-1}\right)^{-1} = \frac{\rho a_H p_M{}^{\rho}}{\rho - a_H p_M{}^{\rho}}, \quad \text{(A.33)}$$

In light of (A.32) and (A.33), (A.31) yields:

$$\frac{dW^E}{da_k} = \left[\frac{(2n-1)\left(\rho - a_H(p_M^E)^{\rho}\right) - \rho a_H(p_M^E)^{\rho}}{\rho - a_H(p_M^E)^{\rho}}\right]\left(\frac{dB^E}{da_k}\right) - \frac{X^E p_{-k}^E}{a_k} + \left(\frac{T}{n^{\theta-1}}\right)\tilde{Y}^E\left(\frac{d(g_H^E+g_M^E)}{da_k}\right). \quad \text{(A.34)}$$

Now notice that, by (11), (A.18) and (A.19),

$$\frac{dB^E}{da_k} = -\left(\frac{\alpha}{1-\alpha}\right)(\tilde{Y}^E)\left(\frac{T}{n^{\theta}}\right)\rho^{-1}(p_{-k}^E)^{(\rho+1)}; \quad \text{(A.35)}$$

From (10), $\left(\frac{dg^E{}_H}{da_k}\right) = \left(\frac{dg^E{}_M}{da_k}\right)$. Then (A.19), (A.34) and (A.35) yield:

$$-(\tilde{Y}^E)^{-1}\frac{dW^E}{da_k} = \left(\left[\frac{(2n-1)\left(\rho - a_H(p_M^E)^{\rho}\right) - \rho a_H(p_M^E)^{\rho}}{\rho - a_H(p_M^E)^{\rho}}\right]\left(\frac{\alpha}{1-\alpha}\right) + 2n\right)\left(\frac{T}{n^{\theta}}\right)\rho^{-1}(p_{-k}^E)^{(\rho+1)} + \frac{X^E p_{-k}^E}{\tilde{Y}^E a_k}. \quad \text{(A.36)}$$

Together, (A.5) and (A.36) yield:

$$-(\tilde{Y}^E)^{-1}\left(\frac{n^{\theta}}{T}\right)\frac{dW^E}{da_k} = \left(\left[\frac{(2n-1)(\rho - a_H(p_M^E)^{\rho}) - \rho a_H(p_M^E)^{\rho}}{\rho - a_H(p_M^E)^{\rho}}\right]\left(\frac{\alpha}{1-\alpha}\right) + 2n\right)\rho^{-1}(p_{-k}^E)^{(\rho+1)}$$

$$+\left(\frac{p_{-k}^E}{a_k}\right)\left(\left(\frac{dg_H(p_H^E)}{dp_H}\right)^{-1}+\left(\frac{dg_M(p_M^E)}{dp_M}\right)^{-1}\right)^{-1}. \quad (A.37)$$

Consider the term $Z \equiv (2n-1)(\rho - a_H(p_M^E)^\rho) - \rho a_H(p_M^E)^\rho$. By Observation 1, $(\rho - a_H(p_M^E)^\rho) > 0$. Then there must exist $\dot{n}$ such that, if $n > \dot{n}$, then $Z > 0$. Part (a) of Proposition 2(i) follows. Now, using (A.37), and noting (10), we have:

$$-\left(\tilde{Y}^E\right)^{-1}\left(\frac{n^\theta}{T}\right)\left(\frac{dW^E}{da_H}-\frac{dW^E}{da_M}\right)=$$

$$\left(\left[\frac{(2n-1)\left(\rho-a_H(p_M^E)^\rho\right)-\rho a_H(p_M^E)^\rho}{\rho-a_H(p_M^E)^\rho}\right]\left(\frac{\alpha}{1-\alpha}\right)+2n\right)\rho^{-1}\left[(p_M^E)^{(\rho+1)}-(p_H^E)^{(\rho+1)}\right]+\left[\left(\frac{p_M^E}{a_H}\right)-\right.$$

$$\left.\left(\frac{p_H^E}{a_M}\right)\right]\left(\left(\frac{dg_H(p_H^E)}{dp_H}\right)^{-1}+\left(\frac{dg_M(p_M^E)}{dp_M}\right)^{-1}\right)^{-1}. \quad (A.38)$$

Without loss of generality, suppose $a_H > a_M$. Then, from (8), $p_H^E > p_M^E$. Part (b) of Proposition 2(i) then follows from (A.38).

(ii) From (18),

$$\frac{dW^E}{d\rho}=\left(-\frac{dX^E}{d\rho}-\frac{dB^E}{d\rho}\right)+\alpha\left(\frac{T}{n^{\theta-1}}\right)(B^E)^{\alpha-1}\left(\frac{dB^E}{d\rho}\right)(g_H^E+g_M^E)+\left(\frac{T}{n^{\theta-1}}\right)\tilde{Y}^E\left(\frac{d(g_H^E+g_M^E)}{d\rho}\right). \quad (A.39)$$

Together, (A.39) and (11) yield:

$$:\frac{dW^E}{d\rho}=\left((2n-1)\left(\frac{dB^E}{d\rho}\right)-\frac{dX^E}{d\rho}\right)+n\left(\frac{B^E}{\alpha}\right)(g_H^E)^{-1}\left(\frac{d(g_H^E+g_M^E)}{d\rho}\right). \quad (A.40)$$

In light of (A.25), (A.40) reduces to:

$$\frac{dW^E}{d\rho}=[(2n-1)-\alpha(B^E)^{-1}X^E]\left(\frac{dB^E}{d\rho}\right)-X^E\left(\frac{\ln p_M^E+\left(\frac{p_H^E}{p_M^E}\right)\ln p_H^E}{1+\left(\frac{p_H^E}{p_M^E}\right)}\right)+n\left(\frac{B^E}{\alpha}\right)(g_H^E)^{-1}\left(\frac{d(g_H^E+g_M^E)}{d\rho}\right). \quad (A.41)$$

Noting (A.32) and (A.33), (A.41) further reduces to:

$$\frac{dW^E}{d\rho}=\left[\frac{(2n-1)\left(\rho-a_H(p_M^E)^\rho\right)-\rho a_H(p_M^E)^\rho}{\rho-a_H(p_M^E)^\rho}\right]\left(\frac{dB^E}{d\rho}\right)$$

$$-X^E\left(\frac{\ln p_M^E+\left(\frac{p_H^E}{p_M^E}\right)\ln p_H^E}{1+\left(\frac{p_H^E}{p_M^E}\right)}\right)+n\left(\frac{B^E}{\alpha}\right)(g_H^E)^{-1}\left(\frac{d(g_H^E+g_M^E)}{d\rho}\right). \quad (A.42)$$

Together, (A.42), (A.26) and (10) yield:

$$\left(\frac{\alpha g_H^E}{B^E}\right)\left(\frac{dW^E}{d\rho}\right) =$$

$$\left(\left[\frac{(2n-1)\left(\rho-a_H(p_M^E)^\rho\right)-\rho a_H(p_M^E)^\rho}{\rho-a_H(p_M^E)^\rho}\right]\left(\frac{\alpha}{1-\alpha}\right)+2n\right)\left(\frac{dg_H^E}{d\rho}\right)-\left(\frac{\alpha g_H^E X^E}{B^E}\right)\left(\frac{\ln p_M^E+\left(\frac{p_H^E}{p_M^E}\right)\ln p_H^E}{1+\left(\frac{p_H^E}{p_M^E}\right)}\right). \tag{A.43}$$

By (A.28), $\frac{dg_H^E}{d\rho}$ is positive. By Observation 1, $(\rho - a_H(p_M^E)^\rho) > 0$. Note that, by (12) and (13), $\frac{X^E}{B^E}$ is positive and finite, as is $\lim_{\alpha\to 1}\frac{X^E}{B^E}$. Then there must exist $\hat{\tilde{n}}$ such that, if $n > \hat{\tilde{n}}$, then $[(2n-1)(\rho - a_H(p_M^E)^\rho) - \rho a_H(p_M^E)^\rho] > 0$. Proposition 2(ii) follows.

(iii) Combining (13) and (18) and using (10), we have:

$$\frac{dW^E}{d\alpha} = g_H^E\left(\frac{T}{n^\theta}\right)\left[2n - {g_H^E}^{-1}\left(\left(\frac{dg_H(p_H^E)}{dp_H}\right)^{-1}+\left(\frac{dg_M(p_M^E)}{dp_M}\right)^{-1}\right)^{-1}\right]\frac{d\tilde{Y}^E}{d\alpha}-\frac{dB^E}{d\alpha}. \tag{A.44}$$

Together, (A.33) and (A.44) yield:

$$\frac{dW^E}{d\alpha} = g_H^E\left(\frac{T}{n^\theta}\right)\left[2n - \frac{\rho a_H(p_M^E)^\rho}{\rho-a_H(p_M^E)^\rho}\right]\frac{d\tilde{Y}^E}{d\alpha}-\frac{dB^E}{d\alpha}. \tag{A.45}$$

Since: $\left[\frac{dB^E}{d\alpha} = \left(\frac{B^E}{\alpha\tilde{Y}^E}\right)\frac{d\tilde{Y}^E}{d\alpha} - \left(\frac{B^E}{\alpha}\right)\ln B^E\right]$, (A.45) yields:

$$\frac{dW^E}{d\alpha} = \left[g_H^E T\left[2n^{1-\theta} - \frac{\rho a_H(p_M^E)^\rho}{n^\theta\left(\rho-a_H(p_M^E)^\rho\right)}\right]-\left(\frac{{B^E}^{1-\alpha}}{\alpha}\right)\right]\frac{d\tilde{Y}^E}{d\alpha}+\left(\frac{B^E}{\alpha}\right)\ln B^E. \tag{A.46}$$

Noting that $B^E \in (0,1)$ by Observation 2(i), so that $[\ln B^E < 0]$, and recalling that $\frac{d\tilde{Y}^E}{d\alpha} < 0$ by Observation 3(iii), part (iii) of Proposition 2 follows from (A.46). □

Proof of Proposition 3.

Let $Z \equiv \frac{\rho a_H (p_M^E)^\rho}{\rho - a_H (p_M^E)^\rho}$. By (9), $\left[a_H (p_M^E)^\rho = \frac{1}{\left(a_H^{\frac{-1}{\rho}} + a_M^{\frac{-1}{\rho}} \right)^\rho} \right]$. Hence, when $a_H = a_M = a, \rho = 1$, we have $\left[a_H (p_M^E)^\rho = \frac{a}{2} \right]$, so that $\lim_{\rho \to 1, a \to 2} Z = \infty$. Proposition 3 then follows, by continuity, from (A.37), (A.43) and (A.46). □