CREDIT Research Paper

No. 21/06

Internal v. External Rent-seeking with In-group Inequality and Public Good Provision

by

Dripto Bakshi and Indraneel Dasgupta

Abstract

We examine how inequality in the endowment of secure wealth, mediated through voluntary public good provision, affects rent-seeking within and between groups. We model a scenario where two communities, each internally differentiated into rich, intermediate and poor segments, contest one another for the division of some rent. Any rent accruing to a community is distributed internally according to another, simultaneous, contest. Individuals first decide how much of their endowments to allocate to the two contests. They subsequently decide how to allocate their remaining wealth and rental income between private consumption and a community-specific public good. We find that greater endowment inequality among the non-rich, both within and across communities, aggravates inter-group rent-seeking. Within-group rent-seeking may rise as well. In contrast, higher such inequality between the rich and others within a community depresses between-group conflict. Within-group conflict may fall as well. The 'paradox of power' is violated for both kinds of conflict – better-endowed individuals are more successful in the internal conflict, while better-endowed groups are more successful in the external conflict.

JEL Classification No.: D72; D74

Keywords: Internal v. external rent-seeking, Ethnic conflict, Intra-group inequality, Inter-group inequality, Public good provision.

Centre for Research in Economic Development and International Trade, University of Nottingham

CREDIT Research Paper

No. 21/06

Internal v. External Rent-seeking with In-group Inequality and Public Good Provision

by

Dripto Bakshi and Indraneel Dasgupta

Outline

The Authors

Dripto Bakshi is Assistant Professor at the Indian Institute of Technology, Kharagpur; Indraneel Dasgupta is Professor in the Economic Research Unit, Indian Statistical Institute, Kolkata, Research Fellow at IZA Bonn and an External CREDIT Fellow.

Corresponding Author: indraneel@isical.ac.in

Research Papers at www.nottingham.ac.uk/economics/credit/

1. Introduction

How do stable class divisions, in the sense of inequality in the distribution of *secure* wealth endowments, within a community affect its conflict over divisible resource-sharing with another community? The purpose of this paper is to shed theoretical light on this issue. We consider a scenario where two communities, each internally differentiated into rich, intermediate and poor classes/segments, contest one another for the division of some exogenously given resource, i.e., rent. Any share of the rent accruing to a community is distributed internally between its constituent classes according to an endogenously determined, i.e., contested, sharing rule. Each member of either community has some given secure (i.e., uncontested) wealth or resource endowment ('money' or 'labor/effort' in efficiency units). Individuals first decide how much of that endowment to allocate simultaneously to the internal and external rent-seeking contests. They subsequently decide how to allocate their total income (their initial endowment net of their conflict expenditure, plus their individual share of the rent) between private consumption and contribution to a community-specific public good. We focus on how the distribution of secure initial endowments among the non-rich segments of either community affects inter-community rent-seeking conflict, as well that within either community. We find that an expansion in endowment inequality among the non-rich, both within and across communities, aggravates inter-group rent-seeking conflict. Within-group rent-seeking may rise as well. Thus, higher vertical or horizontal inequality in the distribution of initial endowments among the non-rich may imply greater wastage of social resources on rent-seeking. Higher vertical endowment inequality among the non-rich within a community may however lead to less horizontal inequality in the distribution of net rental income across communities. Conversely, higher endowment inequality between the rich and others within a community depresses between-group conflict; strikingly, within group conflict may fall as well. The so-called 'paradox of power' is violated for both kinds of conflict – better-endowed individuals may be more successful in the internal conflict, while better-endowed groups may be more successful in the external conflict.

Our results stand in sharp contrast to those found in the quite small existing literature on inter-group rent-seeking conflict with endogenous internal sharing rules, where group members are not connected to one another through common consumption of a privately produced public good. Specifically, they draw attention to the possible importance of changes in the distribution of resource endowments among different sections of the non-rich within a society, rather than changes only in that between the rich and others, in determining the intensity of (both out and in-group) rent-seeking conflicts. In so doing, they offer one possible rationalization of the positive relationship between greater inequality (both within and across communities) and inter-group conflict noted in the empirical literature, even as they provide a major caveat. This caveat pertains to the theoretical case for going beyond the currently common two-fold disaggregation of aggregate inequality into within-group and between-group components, towards a further decomposition of the former into its within-non-rich and between rich and non-rich components, in empirical studies of conflict intensity. Furthermore, our analysis rationalizes asymmetries in the impact of community-specific resource acquisition on inter-group conflict noted in some empirical studies.

Empirical research often fails to find any robust relationship between overall (i.e., country-wide) inequality and conflict (e.g., Lichbach 1989, Fearon and Laitin 2003 and Collier and Hoeffler 2004; see Østby 2013 for a survey). More disaggregated investigations, focusing on vertical (i.e., inter-personal) inequality *within* groups or horizontal inequality (i.e., inequality *between* groups) appear to fare better. Østby *et al.* (2009) found a positive and significant relation between within-region inequalities and conflict onset using data from a sample of 22 Sub-Saharan African countries. Kuhn and Weidmann (2015) introduced a global data set on within-group inequality using nightlight emissions and found that greater economic inequality within an ethnic group significantly increases the risk of conflict, especially if political or economic inequalities between groups provide a motive. Huber and Mayoral (2019), analyzing cross-country data, found a robust positive association between the level of inequality within a group and the *severity* of civil war, measured using battle deaths. Between-group disparities seem to be positively related with conflict as well (Stewart 2008, Østby 2008). The main hypothesis posited and investigated in this literature is that larger the income gap between a group and other groups in society, the greater the likelihood that the group will initiate a conflict. Cederman *et al.* (2011, 2013, 2015) offer empirical support for this hypothesis. However, there is contradictory evidence as well. Mitra and Ray (2014) find that Hindu-Muslim conflict intensifies in India when the income gap between these groups narrows. Morelli and Rohner (2013) find in cross-country analysis that when oil is discovered in the territory of a poor group, the probability of civil war increases significantly.

The empirical findings discussed above highlight the possibility of a relationship between inequalities, both within and between well-defined social groups, or communities, cleaved along 'ethnic' (i.e., non-class identity) divides such as race, language, religion or caste, and social conflict. This possibility broadly motivates our theoretical analysis. We seek to develop a general theoretical framework, integrating between and within-group rent-seeking conflicts, which allows one to examine how inequalities in the distribution of secure endowments, both within and between social groups, affect the intensity of such conflicts. Such a theoretical framework would allow one to rationalize, in a broad interpretative sense, the empirical findings mentioned above, even as it provides a priori hypotheses for organizing further empirical research. We focus on the intensity of ongoing conflicts (e.g., Esteban and Ray 2011, Mitra and Ray 2014, Huber and Mayoral 2019), rather than conflict onset. However, to the extent that more intense non-violent political contestation increases the risk of a descent into violent conflict, including civil war, our analysis carries suggestive implications for the onset of civil war as well.

The structure of the problem we pose is the following. Suppose there is a given amount of some appropriable monetary/monetizable resource (e.g., land, mineral revenue, foreign aid or policy-induced monopoly profit). Suppose further that all consumption within a group is private, and there are no pecuniary externalities connecting the consumption bundles of different group members. Then each member's benefit from investing in appropriation of such a resource depends uniquely on her own personal share of the spoils. The consumption allocations of other group members have no bearing on the wellbeing of any individual. The extent of prior inequality in the distribution of non-alienable income/wealth within the group should therefore have no bearing on the marginal individual benefit from rent-seeking. Nor should the magnitude of income/wealth inequality across groups competing to appropriate the resource make any difference, since the money value of its stock is given independently of such inequality. Greater inequality can affect rent-seeking only by affecting the marginal opportunity cost of rent-seeking – by altering the distribution of relative individual returns from productive labor.[1] However, this cost channel turns tenuous if lower individual return from productive labor is strongly correlated with lower efficiency in rent-seeking, so that lower absolute return from productive labor does not imply higher relative return from rent-seeking.[2] It loses salience when the primary driver of inequality is differential possession of non-labor sources of income such as inherited land or financial assets, rather than idiosyncratic differences in labor productivity. One, or both, these features appear likely to obtain in many real-life contexts. A priori, it seems then that inequality should not matter for appropriation conflicts over exogenously determined rents in such contexts.

The cost-benefit logic outlined above however gets more complicated when a group also happens to be a *community*, i.e., when members' welfare levels are inter-connected through common consumption of a group-specific public good, produced by voluntary subscriptions.[3] A large prior income or wealth gap between rich and non-rich members of a community may lead to the non-rich all free-riding on the rich for public good provision. A higher personal share of any income gain from appropriation then has the direct

[1] The opportunity cost channel is typically the avenue through which inequality is assumed to influence conflict in the literature. Collier and Hoeffler (2004) draw attention to economic proxies for the viability of rebellion, which essentially influence its cost, as key determinants of civil war. See also footnotes 6 and 7 below.

[2] To illustrate, skilled lawyers, experienced journalists and erudite academics are all much sought after by lobbying organizations, efficient agitational activities require managerial abilities that are often transferable to top-end business activities, and modern armies require highly skilled technical personnel to operate their war machines. Even when it comes to unskilled labor, greater physical strength may entail both greater effectiveness in violence and greater efficiency in productive market labor.

[3] Dasgupta and Kanbur (2007) argue that, intuitively, it is precisely the presence of some group-specific public good(s) that makes an identity community out of a mere group (i.e., an arbitrary collection) of individuals.

consequence of increasing a non-rich member's marginal gain from investing in such appropriation, as earlier. However, such a higher share, by correspondingly reducing the shares of rich individuals, reduces the amount of the public good produced within the group in consequence of such appropriation. This reduces the welfare of the non-rich person. The second effect counteracts the first, reducing the incentive of the non-rich to invest in intra-community appropriation, and correspondingly increasing their incentive to invest in inter-community expropriation. Suppose now that the marginal valuation of the intra-group public good, relative to that of private consumption, increases with the latter.[4] Then, as the secure endowments of the non-poor increase, their incentive to expropriate the rich within their community falls, even as their incentive to expropriate the other community rises. However, if this leads to rent-seeking gains for the community as a whole, then that second-order effect, by expanding the size of the prize, may increase the intensity of internal contestation. In any case, inequality in resource distribution now comes to influence the inter-group resource conflict, as well as the internal one.

The following questions then suggest themselves. What happens to the intensity of (i.e., aggregate resource expenditure on) rent-seeking, whether external or internal, as the secure endowments of the non-poor expand within a community? What happens to such expenditure when vertical endowment inequality contracts between the poor and the intermediate strata of a community? How is rent-seeking conflict affected by a decline in horizontal endowment inequality among the non-poor (i.e., between the non-poor of the two contending communities)? These are the questions we address.

In modeling communities, visualized in terms of racial, linguistic or religious identity groups, as collections of individuals held together by voluntary contributions to a group-specific public good, we follow the lead of Dasgupta and Kanbur (2005a, 2005b, 2007, 2011).[5] We extend this literature by

[4] This models the argument, common among historians, sociologists and political scientists, that in-group activities, practices, symbols or institutions considered constitutive of group identity often appear to be valued more by individuals higher up in the class ladder, at least within a wide range. Hobsbawm (1987, chapter 6) has highlighted the essentially middle and lower-middle class basis of ethno-linguistic nationalism in Europe in the 1875–1914 period. Hobsbawm (1987, p. 160) also provides an interesting illustration of class-specific differences in military participation from Britain. Volunteer enlistment of working-class soldiers during the South African War (1899–1902) rose and fell with unemployment. This was however not the case for volunteer recruitment from lower-middle and white-collar classes. This phenomenon may be interpreted as signaling greater susceptibility of these classes to the ideology of nationalism. Many historians have argued that working class support for the Nazi party in Germany was higher among its better off segments (see Brustein 1998 for an overview). Analyzing data from a 2019 country-wide electoral survey in India, Chibber and Verma (2019, Table 5) found their measures of religious practice, Hindu nationalism and ethno-political majoritarianism all to be the lowest among the group they defined as being of 'very low socioeconomic status'. Our assumption is also, in effect, one way of generating the outcome that higher income for the non-rich improves group cohesiveness (Sambanis and Milanovic 2011).

[5] Standard examples of such group-specific public goods include ethno-linguistic or religious institutions, practices and festivals. When communities are residentially segregated, local public goods such as law enforcement, roads, parks, museums, public libraries, art galleries, village wells, sports clubs etc., all constitute examples as well. See Dasgupta and Kanbur (2007) and Dasgupta (2017) for extended discussions.

explicitly modeling processes of both inter and intra community resource conflict within such a context. In modeling both between and within group conflict, our analysis locates itself within the literature on simultaneous between and within community rent-seeking contests.[6] We contribute to this literature by analyzing how decentralized voluntary in-group public good provision mediates the relationship between endowment inequality and individual rent-seeking incentives. We thus integrate the literature on resource allocation within an unequal community, characterized by decentralized voluntary provision of public goods, with that on inter-group resource conflict with endogenous within-group sharing rules. As already mentioned, we show that this integration produces novel and sometimes counter-intuitive conclusions. [7]

Section 2 sets up the model. Our comparative static results are presented, discussed and located in the context of earlier literature in section 3. Section 4 outlines possible extensions and generalizations. Section 5 highlights some implications of our theoretical findings for organizing empirical research. Section 6 provides concluding remarks. Proofs of propositions are relegated to an appendix.

---

[6] See, for example, Glazer (2002), Hausken (2005), Münster (2007), Dasgupta (2009), Choi et al. (2016), Bakshi and Dasgupta (2020) and Dasgupta and Pal (2021) for models of simultaneous internal and external conflict. In its integration of internal and external conflicts, our paper is also related to the contributions by Katz and Tokatlidu (1996), Wärneryd (1998), Stein and Rapoport (2004) and Cruz and Torrens (2019). However, unlike our simultaneous formulation, internal and external conflicts take place in different stages in their models. Of all these contributions, the conflict consequences of within-group inequality in resource endowments are addressed only by Dasgupta (2009), Cruz and Torrens (2019) and Dasgupta and Pal (2021). Dasgupta (2009) considers the specific case of a working class engaged in conflict with a unified employing class over enforcement of a rent-generating minimum, or 'living' wage, even as two sections of that working class, differentiated by their reservation wage, contest one another for a larger share of the rent. Relative changes in reservation wages, by affecting relative wage premia, affect both capital-labor and intra-labor conflicts. The channel through which endowment inequality affects conflict is evidently very different in our model, generating very different conclusions. Inequality affects conflict by influencing the opportunity cost of conflict investment in both Cruz and Torrens (2019) and the internal versus external conflict variant of their benchmark model discussed by Dasgupta and Pal (2021), whereas it affects conflict through the marginal benefit channel in our model, via voluntary public good provision. As we discuss in section 3 below, the comparative static conclusions generated by our model are accordingly different from theirs.

[7] Other related contributions are by Esteban and Ray (2008, 2011) and Mitra and Ray (2014). Esteban and Ray (2008, 2011) develop models of inter-group conflict that explicitly analyse the role of rich and poor within a group. However, their models do not have an internal versus external contest structure. The model in Esteban and Ray (2008), which examines the formation of cross-class coalitions within ethnic groups rather than class coalitions across ethnic divides, has a distant family resemblance with ours. However, in its focus on (a) class and ethnic conflicts as alternate (rather than joint) possibilities, and (b) interpretation of class conflict in a cross-ethnic (rather than intra-ethnic) manner, it differs greatly from our analysis. Esteban and Ray (2011) do not consider intra-group conflict at all. In both models, intra-group inequality affects inter-group conflict by affecting the opportunity cost of engaging in conflict. The theoretical model of inter-group conflict in Mitra and Ray (2014), which focuses on inter-group (rather than intra-group) inequality as a driver of group conflict, also relies heavily on inequality driving the opportunity cost of appropriation. As already noted, inequality affects conflict through a very different channel in our model - by affecting the marginal benefit from engaging in conflict, mediated through the private provision of intra-group public goods, even as the opportunity cost of doing so remains independent of the income or wealth distribution.

2. The model

Consider a society consisting of two groups or 'communities', F and H, containing three members each. Each community contains one rich (R), one middle or intermediate class (M) and one poor (P) member, demarcated according to their relative resource endowments. The set $\aleph \equiv \{F, H\} \times \{R, M, P\}$ will denote the set of all individuals in society, whereas the ordered pair $\langle c, g\rangle \in \aleph$ will denote a generic individual belonging to the community $c \in \{F, H\}$ and the resource class $g \in \{R, M, P\}$. Individual $\langle c, g\rangle \in \aleph$ has secure (i.e., non-contestable) resource endowment I_c^g. This endowment can be interpreted simply as holdings of some divisible and convertible item ('money'), or labor in efficiency units.[8] Secure resource endowments I_c^g come in multiples of some positive lower bound, normalized to unity, and there is some threshold resource endowment I separating the rich from the non-rich. Thus, non-contestable endowments can only take integer values: $I_c^g \in \{1,2,3, \dots\}$. Furthermore, $[I_c^R > I + 1 > I > I_c^M \geq I_c^P \geq 1]$, where $I \in \{3, 4, \dots\}$.[9] Note that I_H^g need not be the same as I_F^g. Thus, endowment class position is group-referent rather than absolute. The intermediate member of one group may be worse endowed than even the poor member of another group, while the rich member of a group may be poorer than the rich member of another group. Given any community $c \in \{F, H\}$, we shall use $-c$ to denote its antagonist group/community.

All agents live for two periods. In period 1, the two communities engage in a Tullock (1980) contest over the division of a monetary or monetizable resource (rent), even as the members of each community simultaneously contest one another, in the same fashion, for division of the rent accruing to their own community. To illustrate, one may think of, for example, a scenario where two internally wealth-differentiated regions within a country contest each other for the division of a fixed amount of foreign aid or mineral revenue, even as individuals within each region contest one another over sharing of the spoils.

Individuals consume, in period 2, their secure initial endowment net of their period 1 rent-seeking expenditure (or, when endowments are interpreted as labor in efficiency units, income generated from productive labor in a one-to-one manner in period 1) and period 1 rental income. Thus, all individuals simultaneously decide how much resource to invest in the two rent-seeking contests, internal and external,

[8] The first interpretation is meant to model the case where inherited ownership of non-labor resources, such as land or financial capital, is the primary driver of resource inequality, rather than inter-personal differences in labor productivity. The second interpretation captures the case of a strong positive correlation between efficiency in productive labor and that in appropriation activities. Recall the discussion in Section 1 (p. 3, especially footnote 2).

[9] If two non-rich members of a community have identical resource endowments, then one is arbitrarily defined as 'poor' and the other 'middle/intermediate' class.

occurring parallel to one another, in period 1, so as to maximize their utility in period 2. The amount of the rent being contested is set at unity for notational convenience.[10]

Let x_c^g denote the external rent-seeking expenditure by individual $\langle c, g \rangle \in \aleph$. The share of the rent going to her community, c, is then given as:

$$s_c = \left(\frac{X_c}{X}\right) \text{ if } X > 0,$$

$$= \frac{1}{2} \text{ otherwise;} \tag{1}$$

where X_c is the total expenditure on external rent-seeking by community c, $X_c \equiv x_c^R + x_c^M + x_c^P$; and $X \equiv X_H + X_F$. In line with standard practice, we shall use the total expenditure on external rent-seeking, X, as the measure of aggregate inter-community conflict in society.

Analogously, let y_c^g denote the internal rent-seeking expenditure by individual $\langle c, g \rangle$. The proportion of the rent going to her community accruing to this individual is then given as:

$$\gamma_c^g = \left(\frac{y_c^g}{Y_c}\right) \text{ if } Y_c > 0,$$

$$= \frac{1}{3} \text{ otherwise;} \tag{2}$$

where Y_c is the total expenditure on internal rent-seeking by community c, $Y_c \equiv y_c^R + y_c^M + y_c^P$. Its total expenditure on internal rent-seeking, Y_c, will serve as our measure of conflict within any community c. Define $Y \equiv Y_F + Y_H$. Then Y will measure aggregate intra-community conflict in our society.

The net resource/income available to any individual $\langle c, g \rangle \in \aleph$ in period 2 is therefore:

$$i_c^g \equiv I_c^g + s_c \gamma_c^g - x_c^g - y_c^g. \tag{3}$$

All consumption occurs in period 2. In period 2, all individuals simultaneously allocate their respective period 2 net incomes, i_c^g, between private consumption (v_c^g) and a community-specific pure public good. All prices/conversion ratios are set at unity for notational simplicity.

Let B_c^g denote the spending by individual $\langle c, g \rangle$ on her community's public good. The total amount of the public good generated within a community, B_c, is given simply by the total spending on that good by

[10] Our main comparative static conclusions will only require the size of the contested rent to be *at most* unity.

all members of the community: $B_c = B_c^P + B_c^M + B_c^R$. We shall denote by B_c^{-g} the total spending on the community's public good by all its members other than g.

The period 2 problem of individual $\langle c, g\rangle$ is:

$$\underset{v_c^g, B_c}{Max}\left[V\left(v_c^g\right) + \beta B_c\right], \quad (4)$$

where $\beta > 0$, subject to the budget constraint:

$$v_c^g + B_c = i_c^g + B_c^{-g}; \quad (5)$$

and the additional constraint:

$$B_c \geq B_c^{-g}. \quad (6)$$

The second constraint simply incorporates the assumption that individuals cannot divert other community members' public good contributions to their own private consumption.[11] We proceed to impose some necessary structure on preferences.

Assumption 1. For every $k \in \{0,1,2,3,\ldots\}$, and for all $\vartheta \in [0,1]$:

$$[V(k + \vartheta) = V(k) + A(k)\vartheta],$$

where $V(0) = 0$, $A(0) \in \Re_{++}$, $\lim_{k\to\infty} A(k) = 0$, and $A(.)$ is continuous and differentiable up to the second degree in its argument, with $\frac{1}{A(.)}$ increasing and strictly convex.

Assumption 2. $A(I + 1) < \beta < A(I)$.

Assumption 1 imposes a variant of a standard quasi-linear utility function, one which exhibits a piece-wise linear concave form for the private consumption valuation function V. The marginal utility of private consumption, bounded from above, remains constant for small changes, i.e., within any unit interval, but declines in discrete fashion across any pair of unit intervals. The assumption of piece-wise linearity is made to ignore changes in marginal utility of private consumption brought about by changes in net rental income, so as to focus on such changes brought about by exogenous changes in secure resource endowments

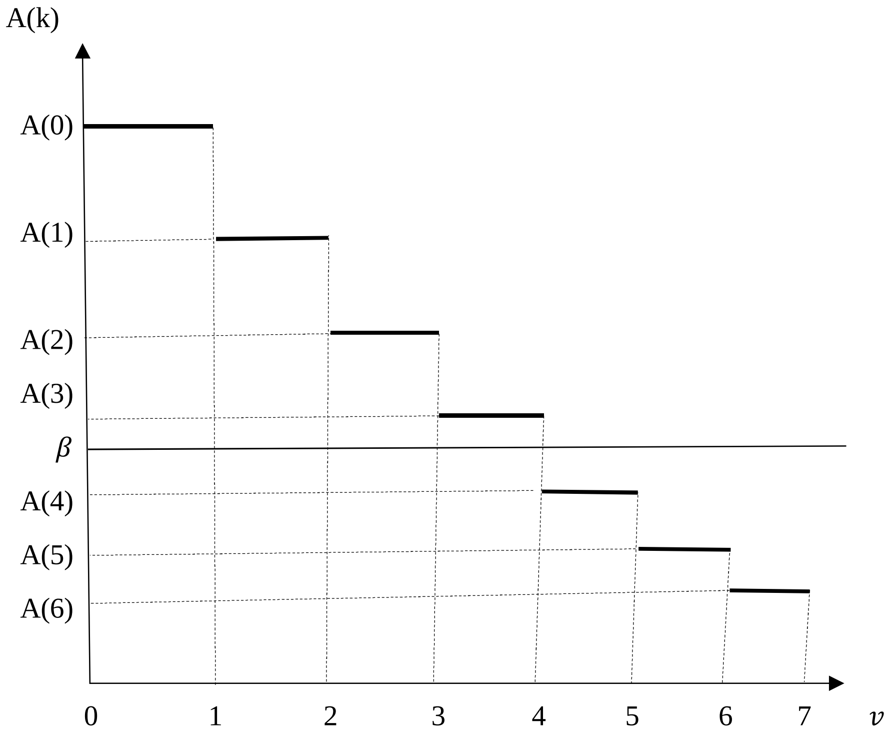
[11] The assumption of preferences being identical across groups is made for notational ease and can be relaxed, as can the assumptions that communities and endowment classes are of identical sizes, the public good is pure, and the marginal utility of the within-group public good is constant. We discuss these extensions in Section 4 below.

alone. The assumptions regarding the curvature of the marginal utility function A ensure that the marginal rate of substitution between public and private consumption ($\frac{\beta}{A(.)}$) increases, at an increasing rate, as the magnitude of the secure endowment rises – properties we shall exploit in our comparative static analysis. Now recall that, by construction, for all $c \in \{F, H\}$, we have: $[I_c^R > I + 1 > I > I_c^M \geq I_c^P \geq 1]$, and I, I_c^R, I_c^M, I_c^P are all integers. Recall further that the maximum possible rental income is 1. Hence, noting (3) and (4), in light of Assumption 1, Assumption 2 implies that the marginal utility of private consumption must always be greater than that of intra-group public consumption for the intermediate and poor members of either group. It thus ensures that poor and intermediate individuals are always too resource-poor to contribute to their group's public good. Such individuals must spend their entire period 2 income on private consumption. Assumption 2 also ensures that the marginal utility of private consumption is lower than that of public consumption for the rich if they do not contribute to their community's public good (provided they satisfy the obvious rationality condition of not spending more on rent-seeking than the total amount of the rent itself). Rich individuals must therefore contribute positive amounts to their respective group-specific public goods out of their period 2 incomes. The non-rich members of the community will free-ride on the rich for provision of their community's public good.

Example 1. A simple example helps to illustrate the structure imposed by Assumptions 1 and 2. Suppose the marginal utility function A takes the form $(1 + a)^{-\theta}$, with $\theta > 1$. Then, for every $v \in [0,1], V(v) = v$; for every $v \in [1,2], V(v) = 1 + (2)^{-\theta}(v - 1)$; for every $v \in [2,3], V(v) = 1 + (2)^{-\theta} + (3)^{-\theta}(v - 2)$, and so on. We have $\frac{d\left(\frac{1}{A(a)}\right)}{da} = -\frac{A'}{A^2}$, $\frac{d^2\left(\frac{1}{A(a)}\right)}{da^2} = \frac{1}{A^2}\left(\frac{2A'^2}{A} - A''\right)$. Since $A' = -\theta(1 + a)^{-(\theta+1)} < 0, \frac{d\left(\frac{1}{A(a)}\right)}{da} > 0$. Since $A'' = \theta(\theta + 1)(1 + a)^{-(\theta+2)}, \left(\frac{2A'^2}{A} - A''\right) = (1 + a)^{-(\theta+2)}\theta(\theta - 1) > 0$ (since $\theta > 1$). Hence $\frac{d^2\left(\frac{1}{A(a)}\right)}{da^2} > 0$. Thus, the example satisfies Assumption 1. Now suppose $I = 3$ (so that $I_c^P, I_c^M \in \{1,2\}, I_c^R \in \{5,6,7, \ldots\}$) and $\theta = 2$. Then Assumption 2 holds whenever $\beta \in \left(\frac{1}{25}, \frac{1}{16}\right)$. ■

Assumption 1 implies that the marginal utility of private consumption takes the form of a step function, while Assumption 2 structures the individual period 2 resource allocations. Figure 1 below illustrates these two restrictions for a community for the special case where $I = 3$ (so that $I_c^P = 1, I_c^M = 2$, and $I_c^R \in \{5,6,7, \ldots\}$). The vertical axis represents marginal utility of private consumption, while the horizontal axis measures private consumption itself. The thick step schedule shows how marginal utility of private consumption changes with increases in its magnitude, while the thin horizontal unbroken line between the steps corresponding to A(3) and A(4) represents the marginal utility of the public good.

Figure 1: Period 2 allocation



In light of Assumptions 1 and 2 and the preceding discussion, period 2 equilibrium consumption bundles can be characterized as follows.

Lemma 1. Given Assumptions 1 and 2, and assuming that the rich do not incur a net loss from rent-seeking in period 1, in any Nash equilibrium of the period 2 public good contributions subgame, for all $c \in \{F, H\}$; we must have: (i) $v_c^P = i_c^P$, $v_c^M = i_c^M$; (ii) $v_c^R = I + 1$; and (iii) $B_c = i_c^R - (I + 1)$.

Together, (3)-(4), Assumptions 1 and 2, and Lemma 1 yield the period 2 equilibrium utility levels:

$$u_c^R = V(I + 1) + \beta[(I_c^R - I - 1) + (s_c\gamma_c^R - x_c^R - y_c^R)]; \tag{7}$$

$$u_c^M = V(I_c^M) + A(I_c^M)(s_c\gamma_c^M - x_c^M - y_c^M) + \beta[(I_c^R - I - 1) + (s_c\gamma_c^R - x_c^R - y_c^R)]; \tag{8}$$

$$u_c^P = V(I_c^P) + A(I_c^P)(s_c\gamma_c^P - x_c^P - y_c^P) + \beta[(I_c^R - I - 1) + (s_c\gamma_c^R - x_c^R - y_c^R)]; \qquad (9)$$

whenever $[(s_c\gamma_c^g - x_c^g - y_c^g) \geq 0$ for every $g \in \{R, M, P\}]$, i.e., no individual suffers a net personal income loss from her rent-seeking activities. Clearly, this condition must be automatically satisfied in any rent-seeking equilibrium as a participation constraint, since individuals can always avoid a net loss by refusing to participate in any kind of rent-seeking activities.

We shall term a rent-seeking Nash equilibrium as *interior* if, for every individual, the first order conditions hold with equality for both internal and external rent-seeking in period 1. In the rest of the paper, we shall consider only subgame perfect interior Nash equilibria. Suppose then that (7)-(9) do indeed define the Nash equilibrium utility levels in some period 2 subgame. Using the first order conditions for internal and external rent-seeking for *R* individuals, recalling (1), (2) and (7), and assuming interiority, we have:

$$\beta\left[s_c\left(\frac{y_c^P + y_c^M}{{Y_c}^2}\right) - 1\right] = \beta\left[\left(\frac{X_{-c}}{X^2}\right)\gamma_c^R - 1\right] = 0. \qquad (10)$$

The first term denotes the net marginal utility to a rich individual from internal rent-seeking, while the second term denotes that from external rent-seeking. Analogously, assuming interior solutions, we have the equilibrium conditions for *M* and *P*, from (8) and (9) respectively:

$$\left[A(I_c^M)\left(s_c\left(\frac{y_c^P + y_c^R}{{Y_c}^2}\right) - 1\right) - s_c\beta\left(\frac{y_c^R}{{Y_c}^2}\right)\right] = \left[A(I_c^M)\left(\left(\frac{X_{-c}}{X^2}\right)\gamma_c^M - 1\right) + \beta\left(\frac{X_{-c}}{X^2}\right)\gamma_c^R\right] = 0; \qquad (11)$$

$$\left[A(I_c^P)\left(s_c\left(\frac{y_c^M + y_c^R}{{Y_c}^2}\right) - 1\right) - s_c\beta\left(\frac{y_c^R}{{Y_c}^2}\right)\right] = \left[A(I_c^P)\left(\left(\frac{X_{-c}}{X^2}\right)\gamma_c^P - 1\right) + \beta\left(\frac{X_{-c}}{X^2}\right)\gamma_c^R\right] = 0. \qquad (12)$$

As before, the first expression in square brackets in each case denotes the net marginal utility from internal rent-seeking, while the second such expression denotes that from external rent-seeking. Equations (11) and (12) capture how non-rich individuals internalize losses and gains to the rich in their community, respectively, from their own internal and external rent-seeking activities, through the mediation of consequent changes in public good supply by the rich. Using (1), (2), (10)-(12) and Lemma 1, we can characterize the conflict allocations and internal conflict allocations in any interior equilibrium as follows.

Proposition 1. Given Assumptions 1 and 2, in any sub-game perfect interior Nash equilibrium of the rent-seeking game, for all $c \in \{F, H\}$; we must have:

$$X_c = \gamma_{-c}^R\left(\frac{\gamma_c^R}{\gamma_F^R + \gamma_H^R}\right)^2, \qquad (13)$$

$$Y_c = \frac{(1-\gamma_c^R)\gamma_c^R}{(\gamma_F^R+\gamma_H^R)}, \tag{14}$$

$$\gamma_c^R = \left[3 - \beta\left(\frac{1}{A(I_c^M)} + \frac{1}{A(I_c^P)}\right)\right]^{-1}, \tag{15}$$

and

$$\text{for every } g \in \{M, P\}\text{: } \gamma_c^g = \gamma_c^R \left[1 - \frac{\beta}{A(I_c^g)}\right]. \tag{16}$$

Proof: See the appendix. ■

In light of (2), Proposition 1 connects equilibrium group expenditures on external conflict and individual expenditures on internal conflict to the parameters of the model – viz., the underlying distribution of secure endowments within the non-rich population and the marginal utility of the group-specific public good (β). In light of (1), Proposition 1 also yields the equilibrium inter-community division of the contested rent:

$$\text{for all } c \in \{F, H\}\text{: } s_c = \frac{\gamma_c^R}{\gamma_F^R+\gamma_H^R}. \tag{17}$$

By (13), (14) and (17), equilibrium aggregate conflict allocations, as well as inter-community shares, depend only on the intra-community shares of the rich. The share of the rich within any community in turn depends on the distribution of non-contested resource endowments among the non-rich population of that community (recall (15)). Changes in the distribution of non-contested resource endowments among the non-rich population of either community affect internal conflict – by changing the relative marginal valuation of public good contributions by the rich vis-à-vis their own private consumption. The equilibrium rental share of the rich of that community is consequently affected, which in turn affects external conflict. The inter-community division of the contested rent is thereby altered, which in turn feeds back into within-community conflict. The exact way in which these inter-linkages play out, and their net consequences, are the issues we shall take up in the next section.

Remark 1. In light of Assumptions 1 and 2, Proposition 1 implies $\left[1 > \gamma_c^R > \frac{1}{3} > \gamma_c^P \geq \gamma_c^M > 0\right]$, with $\gamma_c^P > \gamma_c^M$ whenever $I_c^M > I_c^P$ (recall (15) and (16)). Furthermore, it follows from Proposition 1 ((13)-(16)) that, in any sub-game perfect interior Nash equilibrium,

$$\text{for every } \langle c, g\rangle \in \aleph\text{: } \left[s_c\gamma_c^g - y_c^g = s_c\gamma_c^g\gamma_c^R > 0\right].$$

Thus, all agents make positive monetary gains in the internal conflict. The rich achieve the highest net monetary gain in the internal conflict, followed by the poor. The lowest net monetary gain in the internal conflict is made by intermediate individuals (when they are better endowed than the poor). Furthermore, in any sub-game perfect interior Nash equilibrium,

for every $c \in \{F, H\}$: $[s_c - X_c - Y_c = {s_c}^2 \gamma_c^R > 0]$.

Thus, each community derives a positive return from rent-seeking, net of its total expenditure on internal and external rent-seeking. As already noted, given any positive rent acquired from external rent-seeking, every community member generates a positive personal return from internal rent-seeking. Hence, there must exist interior subgame perfect Nash equilibria where all individuals generate positive rental gains net of their personal expenditures on internal and external rent-seeking combined.

Remark 2. Our model uniquely defines individual equilibrium allocations to internal conflict (recall (2) and (14)-(16)), as well as aggregate community contributions to external conflict (recall (13)). However, contributions by individual community members to external conflict are indeterminate. Thus, the model generates multiple equilibria.

Remark 3. Violation of Assumption 2, if it leads to the non-poor contributing to their community's public good, eliminates internal conflict. To see this, suppose $A(I) < A(I_c^P - 1) < \beta < A(I_c^P - 2)$ in some community c. Then, in period 2, all members of c contribute to its public good, so that, for every $g \in \{P, M, R\}$, $v_c^g = I_c^P - 1$; and $B_c = (I_c^P + I_c^M + I_c^R) - 3(I_c^P - 1) + (s_c - X_c - Y_c)$. Thus, individual consumption within c comes to depend only on total period 2 income of that community, and turns independent of personal incomes. Obviously, then, there is no longer any incentive for any member of c to expropriate other members – internal rent-seeking disappears.[12]

Remark 4. Proposition 1 has a bearing on the 'paradox of power' well-known in the literature (Hirshleifer 1991, Skaperdas 1992, and Skaperdas and Styropoulos 1997). In its general form, this paradox implies that agents who are worse endowed in production do better in appropriation. However, by Proposition 1 (16), the largest internal share accrues to the rich within either community in our model, as does the greatest net monetary gain in the internal conflict (recall Remark 1). Proposition 1 (16) also implies that the lower her resource endowment, the higher the internal share of her community's gross rental income accruing to a non-rich agent. Furthermore, among the non-rich, the lower the endowment, the higher the net monetary gain in the internal conflict (Remark 1). Thus, the paradox of power holds in a broad sense

[12] This disappearance follows from the well-known neutrality property of games of voluntary contribution to pure public goods (Bergstrom *et al.* 1986). It was first highlighted at an intuitive level by Dasgupta and Kanbur (2005b, 2007). More formal treatments are provided by Dasgupta and Guha Neogi (2018) and Jelnov and Klunover (2020).

in our model, so far as internal conflict is concerned, among the non-rich, but is violated between the rich and the non-rich (in contrast to the case in Cruz and Torrens 2019). Proposition 1 also implies that the lower the endowment of any non-rich agent, the lower the rental share of her community as a whole (recall (16) and (17)). This implies a violation of the paradox of power with regard to external conflict as well.

Notice that, by Proposition 1 (recall (13) and (14)), aggregate equilibrium external and internal conflict allocations must satisfy, respectively,

$$X \equiv X_F + X_H = \frac{\gamma_F^R \gamma_H^R}{\gamma_F^R + \gamma_H^R}, \tag{18}$$

$$Y \equiv Y_F + Y_H = 1 - \frac{\left({\gamma_F^R}^2 + {\gamma_H^R}^2\right)}{\left(\gamma_F^R + \gamma_H^R\right)}. \tag{19}$$

Overall equilibrium conflict allocation in society, by (18)-(19), is:

$$Z \equiv X + Y = 1 - (\gamma_F^R + \gamma_H^R) + \frac{3\gamma_F^R \gamma_H^R}{(\gamma_F^R + \gamma_H^R)}. \tag{20}$$

From (13), (14) and (17), the net equilibrium gain from rent-seeking to any community c is:

$$R_c \equiv [s_c - X_c - Y_c] = \frac{\gamma_c^R}{\left(1 + \frac{\gamma_{-c}^R}{\gamma_c^R}\right)^2}. \tag{21}$$

3. Endowment inequality and rent-seeking

How do changes in the distribution of secure resource endowments affect conflict over the division of contestable rent, whether between or within communities? We now turn to this question. Recall that secure endowments can only take integer values: $I_c^g \in \{1,2,3, ...\}$; and $[I_c^R > I + 1 > I > I_c^M \geq I_c^P \geq 1]$, where $I \in \{3, 4, ...\}$. Our comparative static conclusions presented in the section will implicitly assume that the resource distribution being compared all satisfy this structure – we shall refrain from stating so explicitly in the statements of the propositions for the sake of brevity.

First consider a rise in the endowment of some non-rich individual within a community.

Proposition 2. Let Assumptions 1 and 2 hold. Consider, for any $c \in \{F, H\}$ and any $g \in \{M, P\}$, an increase in the resource endowment from $I_c^{g^*}$ to $I_c^{g'}$. Denoting the variables in any sub-game perfect interior Nash equilibrium of the rent-seeking game under the two distributions by the corresponding superscripts * and ′, respectively, and letting $-g$ denote the member of $\{M, P\}$ other that g, we must have:

(i) $\gamma_c^{g^*} > \gamma_c^{g'}$, $\gamma_c^{R^*} < \gamma_c^{R'}$ and $\gamma_c^{-g^*} < \gamma_c^{-g'}$;

(ii) $X_c{}^* < X_c'$;

(iii) $s_c{}^* < s_c{}'$;

(iv) (*a*) if $\left(\frac{\gamma_{-c}^{R}{}'}{\gamma_c^{R'}}\right) > 1$ (resp. $\frac{\gamma_{-c}^{R}{}^*}{\gamma_c^{R^*}} < 1$), then $X_{-c}{}' >$ (resp.<) $X_{-c}{}^*$ and (*b*) $X^* < X'$;

(v) (*a*) $Y_{-c}{}' < Y_{-c}{}^*$, (*b*) if $\left(\frac{\gamma_c^{R^*}}{\gamma_{-c}^{R}{}^*}\right) > (\sqrt{2}-1)$ then $Y' < Y^*$, if $\left(\frac{\gamma_c^{R'}}{\gamma_{-c}^{R}{}'}\right) < (\sqrt{2}-1)$ then $Y' > Y^*$ and (*c*) if $\left(\frac{\gamma_c^{R'}}{\gamma_{-c}^{R}{}'}\right) < (\sqrt{2}-1)$ then $Y_c{}' > Y_c{}^*$;

(vi) if $\left(\frac{\gamma_c^{R^*}}{\gamma_{-c}^{R}{}^*}\right) > (\sqrt{3}-1)$ then $Z' < Z^*$, if $\left(\frac{\gamma_c^{R'}}{\gamma_{-c}^{R}{}'}\right) < (\sqrt{3}-1)$ then $Z' > Z^*$;

and

(vii) $R_c{}' > R_c{}^*, R_{-c}{}' < R_{-c}{}^*$.

Proof. See the appendix.

Proposition 1 and (17)-(21) above imply that any change in the distribution of non-contestable endowments among the non-rich affect the equilibrium values of the different variables of interest (viz., allocations to internal and external conflicts, inter-community rental shares and net rent-seeking incomes) *only* through their effects on the internal rental shares of the rich (i.e., on γ_F^R and γ_H^R). Consider a rise in the endowment of either the poor or intermediate individual within a community, say *F*. Such a rise would increase that individual's relative marginal valuation of the public good, and therefore, her benefit from a marginal increase in the rental income of the rich. Consequently, her marginal incentive to expropriate the rich would fall, leading to a lower rental share for her and a higher rental share for the rich (and the other non-rich individual). This is stated in Proposition 2(i). Then, since the net benefit from investing in the external conflict must be identical across all individuals within *F* in any interior equilibrium (recall (10)-(12)), all members of *F* acquire greater incentive to invest in the external conflict. *F* consequently becomes more aggressive in the external conflict - external conflict allocation by that community increases (Proposition 2(ii)), increasing its equilibrium share of the rent (Proposition 2(iii)). By Proposition 2(iv*a*), if *F*'s antagonist was initially dominant (i.e., received a larger rental share in consequence of a higher internal share for its rich), then it increases its own external conflict allocation in response to greater aggression by *F*. Otherwise, it withdraws, i.e., reduces its external conflict allocation, in response. However, in either case, resource expenditure on inter-community conflict must go up overall (Proposition 2(iv*b*)), implying that such conflict increases in intensity. Aggregate internal conflict, as well as overall social conflict, may go up as well under certain parametric conditions (Proposition 2, parts (v*b*) and (v*i*)). Intuitively, this happens if *F* is greatly dominated by *H*, in that it receives a much smaller share of the rent. A sufficient condition for this to happen is that the relative share of *F* be less than $(\sqrt{2}-1)$ in the final situation. Otherwise, aggregate internal conflict falls even as aggregate external conflict increases, so that overall conflict in society may fall as well. By Proposition 2 (parts (v*b*) and (vi)) a sufficient condition for this is that the initial relative rental share of *F* be greater than $(\sqrt{3}-1)$. Lastly, the net rental income of *F* must rise, while that of its antagonist must fall (Proposition 2(vii)).

Remark 5. By Proposition 2(ii) and Proposition 2(v*c*), both external and internal conflict allocations may increase within a community. Thus, internal and external conflict may move together, or in different directions, within a community, depending on the endowment distribution both within that community and its antagonist. This is in contrast to the models of Bakshi and Dasgupta (2020) and Dasgupta and Pal (2021), where internal and external conflict necessarily move in opposite directions. The rise in the internal share of the rich within a community, say *F*, leads a substitution of resources within *F*, from internal to external conflict. Thus, the substitution effect reduces internal conflict. However, due to greater investment in external conflict, *F* now comes to acquire a larger share of the rent. Thus, the size of the prize goes up in the internal contest. This income effect increases individual allocations to internal conflict. Whether internal conflict would rise within *F* therefore depends on whether the income effect is stronger than the substitution effect. [13]

Remark 6. By Proposition 1, a change in the endowment of the rich has no effect on conflict. It follows therefore from Proposition 2 that any decline in endowment inequality between the rich and the non-rich within a community, in the form of a reduction in the endowment of the former associated with an expansion in that of at least one member of the latter, has the same effects as a pure endowment expansion among the non-rich of that community. Thus, the comparative static results specified in Proposition 2 hold for any such mean-preserving reduction in intra-group endowment inequality between the rich and the non-rich as well. Proposition 2 therefore implies that communities exhibiting less inequality between the rich and others may exhibit greater aggression, and be more successful, in resource conflicts against other

[13] Relatedly, Dasgupta and Kanbur (2007) find that a rise in the resource endowment of the non-rich would reduce internal, i.e., class, conflict within a community. This happens in their case because the size of the prize in the internal conflict is exogenous in their model, unlike the situation in ours.

communities. In so doing, it generates one formal defense of the idea that lower inequality between the rich and the non-rich expands group cohesion and thus group aggression (Sambanis and Milanovic 2011), against the opposite conclusion by Dasgupta (2007), Esteban and Ray (2011) and Dasgupta and Pal (2021), that it makes a group less aggressive. Lower inequality between the rich and the non-rich, may, somewhat paradoxically, end up increasing within-group conflict as well (recall Remark 5).

What happens if inequality among the non-rich expands within a community, in the sense of a mean-preserving increase in the endowment spread between the poor and the intermediate classes? Proposition 2 implies there are contradictory forces at work. The net effects are not, therefore, *a priori* obvious. It turns out that such an increase within a community, say *F*, increases the internal rental share of the rich within *F*. It thus has the same effects as a unilateral increase in the endowment of a non-rich individual (Proposition 2), or a mean-preserving decline in endowment inequality between the rich and the non-rich (Remark 6). As in those cases, aggregate external conflict must rise; aggregate internal conflict and total overall conflict may do so as well. Internal conflict may go up too within the community that becomes more unequal. Rental income net of conflict investment increases for the community that becomes more unequal, while it decreases for its antagonist. Thus, interestingly, horizontal, i.e., cross-community inequality in the distribution of net rental income may decrease in consequence of an increase in vertical, i.e., within-community inequality. These results are collated formally in Proposition 3 below.

Proposition 3. Let Assumptions 1 and 2 hold. Consider a change in non-contested resource endowments among the non-rich within any community $c \in \{F, H\}$, such that I_c^M increases from $I_c^{M^*}$ to $I_c^{M'}$, and I_c^P decreases from $I_c^{P^*}$ to $I_c^{P'}$, with $\left[\left(I_c^{M^*} + I_c^{P^*}\right) = \left(I_c^{M'} + I_c^{P'}\right)\right]$ and $\left[I_c^{M^*} > I_c^{P^*}\right]$. Then, denoting the variables in any sub-game perfect interior Nash equilibrium of the rent-seeking game under the two distributions by the corresponding superscripts * and ′, respectively:

(i) $\gamma_c^{M^*} > \gamma_c^{M'}, \gamma_c^{P^*} < \gamma_c^{P'}, \gamma_c^{R^*} < \gamma_c^{R'}$;

and

(ii) the inequalities specified under parts (ii)-(vii) of Proposition 2 hold.

Proof. See the appendix.

Proposition 3 supports the conclusion of Dasgupta (2007), Esteban and Ray (2011) and Dasgupta and Pal (2021) regarding the positive correlation between internal inequality and external aggression, to the extent that it refers to inequality among the non-rich. However, as already noted, Proposition 2 contradicts their conclusion when it pertains to inequality between the rich and the non-rich (Remark 6).

Remark 7. Propositions 2 and 3 carry an interesting implication regarding the relationship between the *composition* of social conflict and endowment growth among the non-rich when the non-rich have identical endowments across communities (i.e., $I_F^M = I_H^M$ and $I_F^P = I_H^P$). In such a world, by Proposition 1 (15), $\left(\frac{\gamma_c^R}{\gamma_{-c}^R}\right) = 1 > \left(\sqrt{3} - 1\right) > \left(\sqrt{2} - 1\right)$. From that initial situation, an increase in the endowment of any member of the non-rich segment of either community must reduce both aggregate internal conflict (Proposition 2(v*b*)) and total overall conflict (Proposition 2(vi)), even as it increases aggregate external conflict (Proposition 2(b)). Hence, the relative weight of between-group conflict in aggregate social conflict must unambiguously increase, even as the society as a whole becomes less conflict-ridden. The same outcome holds for a mean-preserving increase in endowment inequality among the non-rich (Proposition 3), or a mean-preserving decrease in such inequality between the rich and the non-rich, within either community (Remark 6).

Lastly, what of a rise in cross-community inequality among the non-rich? To answer this question in an intuitively transparent manner, we shall consider a simple stylized setting that abstracts from inequality among the non-rich within a community (the focus of Proposition 3) to concentrate on inequality among them across communities. Assume therefore that all non-poor individuals have identical endowments within a community, but such endowments vary across communities. Consider a mean-preserving increase in the spread of endowments of the non-rich across communities. Proposition 4 catalogues the consequences of such a rise in inequality. As with within-community inequality, a rise in cross-community inequality will increase inter-community conflict in our model. There exist initial endowment distributions under which both aggregate internal conflict and overall conflict will increase as well. Intuitively, this will happen if the initial relative gap between the two communities in their non-rich endowments is sufficiently small. The net rental income of the better-off community (which becomes internally less unequal and more aggressive) will rise, while that of the worse-off community (which becomes more unequal) will fall. Greater cross-community inequality in initial non-poor endowments will be reflected in higher cross-community inequality in net rental income.

Proposition 4. Let Assumptions 1 and 2 hold. Suppose, for every $c \in \{F, H\}$: Let $I_c^M = I_c^P = \overline{I_c}$. Without loss of generality, suppose $\left[\overline{I_F} > \left(\frac{\overline{I_H} + \overline{I_F}}{2}\right)\right]$ in some initial situation. Consider any increase in $\overline{I_F}$ from that initial situation matched by an identical decrease in $\overline{I_H}$, so that $\left(\overline{I_H} + \overline{I_F}\right)$ remains unchanged. Then, denoting the variables in any sub-game perfect interior Nash equilibrium of the rent-seeking game under the initial and altered distributions by the corresponding superscripts * and ′, respectively:

(i) $\gamma_F^{R^*} < \gamma_F^{R'}$ and $\gamma_H^{R^*} > \gamma_H^{R'}$;

(ii) $X^* < X'$;

(iii) $Y^* < Y'$ when $\dfrac{\left[3-2\beta\left(\frac{1}{A\left(\overline{I_H}'\right)}\right)\right]}{\left[3-2\beta\left(\frac{1}{A\left(\overline{I_F}'\right)}\right)\right]} < \left(\sqrt{2}-1\right)$;

(iv) $Z^* < Z'$ when $\dfrac{\left[3-2\beta\left(\frac{1}{A\left(\overline{I_H}'\right)}\right)\right]}{\left[3-2\beta\left(\frac{1}{A\left(\overline{I_F}'\right)}\right)\right]} < \left(\sqrt{3}-1\right)$;

and

(v) ${R_F}^* < {R_F}'$ and ${R_H}^* > {R_H}'$.

Proof. See the appendix.

The assumption of identical endowments among the non-rich within a community made in the statement of Proposition 4 is only for expositional clarity. The conclusions presented in Proposition 4 actually hold for the general case with differing spreads within communities. Proposition 4 is indeed proved in the appendix for the general case where $I_F^M \geq I_H^M, I_F^P \geq I_H^P$ and $\overline{I_F} > \overline{I_H}$. Thus, we only need (i) one non-rich endowment distribution to dominate another in a first-order sense, and (ii) the endowment spread among the non-rich to remain invariant within each community, for the conclusions presented in Proposition 4 to hold.

Propositions 3 and 4 depend heavily on the idea that the marginal rate of substitution between public and private consumption ($\frac{\beta}{A(.)}$) increases at an increasing rate as the magnitude of the secure endowment rises, built into our model via Assumption 1. The results collated there get reversed if, instead, one assumes that this increases at a decreasing rate. This would happen, for example, if it is assumed that $\theta < 1$ in the illustration provided in Section 2 (recall Example 1 there).

4. Extensions

We now discuss some possible generalizations of our model.

(i) *Variable population*

The generalization to multiple members in each endowment class, and differential membership across both classes and communities, is straightforward. Such a generalization adds nothing of substance to the analysis of the impact of endowment inequality on rent-seeking expenditure – our focus in this paper. It is however easy to check that such a generalization can be made to yield a version of a property common in the literature, stemming from Olson (1965) – given the endowment distribution, communities with larger non-rich populations will be less successful in inter-group rent-seeking.

(ii) *Impure public goods*

Our assumption that members' contributions to within-community public goods are perfect substitutes can be generalized to permit private benefits from contributions, say due to a 'warm glow' from giving (e.g., Dasgupta and Kanbur 2007). One may, for example, assume that the marginal utility from additional units of the public good contributed by oneself is β, as in our benchmark model, but that from additional units contributed by other community members is some fraction, $\varepsilon \in (0,1]$, of β. Evidently, our model is a special case of this generalization, where $\varepsilon = 1$. Assumption 2 however continues to ensure that only R individuals will contribute to the public good even in the general case. Consequently, our comparative static conclusions will continue to hold.

(iii) *Differential public good valuation*

We have assumed that the marginal utility of the public good is identical across communities. This is merely for notational simplicity. We can permit the marginal utility of the public good – the parameter β – to be community-specific (β_F need not equal β_H), provided Assumption 2 holds individually for both communities. Propositions 2 and 3 continue to hold under this generalization. Proposition 4 does so as well, provided $\beta_F \geq \beta_H$ (so that the community with the better-endowed non-rich population continues to exhibit a higher internal rental share for its rich). Notice however that, by (13)-(15), the lower the value of β for the non-rich, the less sensitive conflict variables are to changes in endowments of the non-poor. Hence, a version of our model that allows the marginal utility of the public good to be community-specific can generate inter-group asymmetries in the relationship between endowment inequality and rent-seeking conflict. Similar changes in endowments may have widely diverging effects on conflict in terms of quantity (though not quality), depending on their community location.

(iv) *Diminishing marginal utility from the public good*

Instead of the linear formulation for utility from the intra-group public good, we could have adopted a strictly concave specification. For example, we could have assumed that preferences are given by:

$$u = V\left(v_c^g\right) + \beta {B_c}^{\varepsilon};$$

with $V(.)$ satisfying Assumption 1, as before, and $\varepsilon \in (0,1)$. Obviously, our benchmark specification in (4) constitutes a limiting case, where $\varepsilon = 1$. One could then suitably redefine Assumption 2 to ensure that rich agents generate their respective community's public good, whereas the non-rich continue to free-ride, as in our benchmark model. The comparative static conclusions presented in Propositions 2-4 remain unchanged. However, a rise in the endowment of the rich within any particular community, by increasing the amount of that community's public good, would now reduce its marginal utility. So long as the marginal propensity to spend on the public good does not rise more than proportionately, this effect would induce the non-poor to increase their expenditure on internal appropriation. The rental share of the rich would accordingly fall, reducing the community's external conflict allocation. Thus, this version can be made to generate, at the cost of a major increase in expositional complexity, the conclusion that a community may become externally less aggressive as the rich within that community become richer. This only validates the conclusion already yielded by our benchmark model, that lower inequality within a community between its rich and non-rich may make that community externally more aggressive (recall Remark 6). Thus, little insight is gained by assuming diminishing marginal utility from the public good, and much expositional simplicity is lost thereby.

5. Empirical implications

Our results carry important implications for organizing empirical research into the connections between inequality and social conflict. As discussed in Section 1, an expanding body of empirical literature draws attention to the importance of disaggregating inequality measures into within and between group components in the context of conflict research. Our theoretical analysis suggests that a further decomposition of within-group inequality – into its between rich and non-rich and within non-rich components – may provide important empirical insights. Furthermore, inequality between the non-rich of different groups, rather than merely that between groups as a whole, may be salient for inter-group conflict as well. Second, our theoretical framework offers some hypotheses regarding how different forms of inequality may affect within-group conflict alongside between-group conflict. Recent empirical analyses of social conflict often focus on the former, while neglecting the latter. Our model generates jointly testable hypotheses regarding the relationship between the two forms of conflict.

Lastly, our analysis provides one rationalization of why one might observe asymmetries in the impact of community-specific resource acquisition on inter-group conflict. Mitra and Ray (2014) find that a rise in average consumption by Muslims in India leads to greater Hindu-Muslim conflict, but greater average consumption by Hindus does not. Their theoretical model cannot explain this asymmetry. To the extent that greater average consumption among Muslims in India reflects resource acquisition by the non-rich segments among them, our Proposition 2 rationalizes the first part of their finding. There is indeed a body of evidence that wealth accumulation by Muslims in many parts of India has been driven in large measure by remittances from unskilled and semi-skilled workers employed in Persian Gulf countries, and has therefore significantly benefited the poor and intermediate segments of that community. It has also been argued that such wealth accumulation has led to greater political assertion by Muslims in many areas, which in turn has created exacerbated Hindu-Muslim conflicts.[14] On the other hand, the absence of any significant impact of Hindu prosperity on Hindu-Muslim conflict is rationalized by our model if (a) it is the case that wealth acquisition among Hindus in India has been largely concentrated among its rich segments, and/or (b) caste divisions and norms of ritual purity among Hindus lead to low access for the non-rich to public goods generated by the rich. The first follows directly from the fact that an endowment expansion for the rich has no effect on conflict in our benchmark model (Remark 6), and makes a community less aggressive in the modified version with diminishing marginal utility of the public good (Section 4(iv)). The second implies a low value of β for non-rich Hindus in a generalization of the model that permits inter-community differences in public good valuation (Section 4(iii)).[15]

6. Concluding remarks

This paper has examined how prior within-group inequality in secure resource endowments, mediated through the supply of in-group public goods by the rich, affects both internal and external rent-seeking conflicts. Changes in endowments impact on conflict in our model by changing the marginal benefit from rent-seeking, via changes in the marginal valuation of the intra-group public good vis-à-vis private consumption, by the non-rich. We find that changes in inequality between the rich and others within a group, and that among the non-rich within a group, affect conflict in opposite ways. A mean-preserving

[14] See Emmerich (2020) for an extensive discussion of Muslim-identitarian political assertion in India, and its connection with Gulf remittances in states such as Kerala.

[15] There is evidence that growth has been significantly pro-rich in India over the last three decades (e.g., Chancel and Piketty 2017). It is reasonable to assume that rising inequality among Hindus, who constitute about 80% of the population and have higher average income than Muslims, accounts for a large part of the rise in overall inequality. For caste divisions and caste conflicts among Hindus, see, for example, the discussions in Pai (2013) and Dasgupta and Pal (2021).

rise in endowment inequality between the rich and the non-rich within a group makes that group less aggressive, and therefore less successful, in the external conflict. Overall inter-group conflict falls in consequence. Internal conflict may fall within the group that becomes more unequal as well. A similar rise in inequality among the non-rich within a group has opposite effects. A mean-preserving rise in endowment inequality between the non-rich of two contending groups expands inter-group conflict. The 'paradox of power' is violated for both kinds of conflict – better-endowed individuals are more successful in the internal conflict, while better-endowed groups are more successful in the external conflict.

Our results offer new organizing principles for carrying out empirical conflict research. They suggest that disaggregating within-group inequality into its between rich and non-rich and within non-rich components may provide valuable insights in the context of empirical conflict analysis. Inequality measurement between the non-rich of different groups, rather than merely that between groups as a whole, may be salient for such analysis as well. Furthermore, our analysis rationalizes asymmetries in the impact of community-specific resource acquisition on inter-group conflict noted in some empirical studies. We look forward to such empirical applications of the theoretical findings of this paper in future research. At a theoretical level, one may deploy alternatives to our perfect-substitutes summative specification for each community's aggregate group conflict effort, such as a constant elasticity of substitution aggregation (Kolmar and Rommeswinkel 2013, Cheikbossian and Fayat 2018), the best-shot specification (Chowdhury *et al.* 2013) or the weakest-link formulation (Lee 2012). Extensions of our model using these alternative specifications of the contest success function may generate useful insights.

Acknowledgements

This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

References

Bakshi, D. and Dasgupta, I. (2020). Identity conflict with cross-border spillovers, *Defence and Peace Economics*, **31**(7), 786-809.

Bergstrom, T.C., Blume, L., Varian, H. (1986). On the private provision of public goods, *Journal of Public Economics*, **29**, 25–49.

Brustein, W. (1998). *The Logic of Evil – The Social Origins of the Nazi Party 1925–1933*. New Haven, Yale University Press.

Cederman, L.-E., Weidmann, N., and Gleditsch, K. (2011). Horizontal inequalities and ethnonationalist civil war: a global comparison, *American Political Science Review*, **105**(3): 478–495.

Cederman, L.-E., Gleditsch, K., and Buhaug, H. (2013). *Inequality, grievances, and civil war*. Cambridge: Cambridge University Press.

Cederman, L.-E., Weidmann, N., and Bormann, N.-C. (2015). Triangulating horizontal inequality: toward improved conflict analysis, *Journal of Peace Research*, **52**(6), 806–821.

Chancel, L. and Piketty, T. (2017). Indian income inequality, 1922-2015: From British Raj to Billionaire Raj? *WID.world Working Paper Series No. 2017/11*, World Inequality Lab, Paris School of Economics; https://wid.world/document/chancelpiketty2017widworld/

Cheikbossian, G. and Fayat, R. (2018). Group size, collective action and complementarities in efforts, *Economics Letters*, **168**, 77-81.

Chibber, P. and Verma, R. (2019). The rise of the second dominant party system in India: BJP's new social coalition in 2019, *Studies in Indian Politics*, **7**(2), 131–148.

Choi, J.P., Chowdhury, S.M., and Kim, J. (2016). Group contests with internal conflict and power asymmetry, *Scandinavian Journal of Economics*, **118**, 816–840.

Chowdhury, S. M., Lee, D. and Sheremeta, R, M. (2013). Top guns may not fire: best-shot group contests with group-specific public good prizes, *Journal of Economic Behavior and Organization*, **92**, 94–103.

Collier, P. and Hoeffler, A. (2004). Greed and grievance in civil war, *Oxford Economic Papers*, **56**, 563-595.

Cruz, I.L. and Torrens, G. (2019). The paradox of power revisited: internal and external conflict, *Economic Theory*, **68**(2), 421-460.

Dasgupta, I. and Guha Neogi, R. (2018). Between-group contests over group-specific public goods with within-group fragmentation, *Public Choice*, **174**(3-4), 315-334.

Dasgupta, I. (2017). Linguistic assimilation and ethno-religious conflict, in W. Buchholtz and D. Ruebbelke (eds.) *The Theory of Externalities and Public Goods: Essays in Memory of Richard C. Cornes*; Berlin, Springer: 219-242.

Dasgupta, I. (2009). 'Living' wage, class conflict and ethnic strife, *Journal of Economic Behavior and Organization*, **72**(2), 750-765.

Dasgupta, I. and Kanbur, R. (2005a). Community and anti-poverty targeting, *Journal of Economic Inequality*, **3**(3), 281-302.

Dasgupta, I. and Kanbur, R. (2005b). Bridging communal divides: separation, patronage, integration, in C. Barrett (ed.) *The Social Economics of Poverty: On Identities, Groups, Communities and Networks*; London, Routledge: 146-170.

Dasgupta, I. and Kanbur, R. (2007). Community and class antagonism, *Journal of Public Economics*, **91** (9), 1816-1842.

Dasgupta, I. and Kanbur, R. (2011). Does philanthropy reduce inequality? *Journal of Economic Inequality*, **9**, 1–21.

Dasgupta, I. and Pal, S. (2021). Touch thee not: group conflict, caste power and untouchability in rural India. *Journal of Comparative Economics*, **49**(2), 442-466.

Emmerich, A. (2020). *Islamic Movements in India: Moderation and its Discontents*. New York, Routledge.

Esteban, J. and Ray, D. (2008). On the salience of ethnic conflict, *American Economic Review*, **98**, 2185–2202.

Esteban, J. and Ray, D. (2011). A model of ethnic conflict, *Journal of the European Economic Association*, **9**, 496–521.

Fearon J. and Laitin, D. (2003). Ethnicity, insurgency, and civil war, *American Political Science Review*, **97**(1), 75-90.

Glazer, A. (2002). Allies as rivals: internal and external rent seeking, *Journal of Economic Behavior and Organization*, **48**, 155-162.

Hobsbawm, E. J. (1987). *The Age of Empire*. London, Abacus.

Hobsbawm, E. J. (1992). *Nations and Nationalism since 1780*, 2nd edn. New York, Cambridge University Press.

Hausken, K. (2005). Production and conflict models versus rent-seeking models, *Public Choice*, **123**, 59–93.

Hirshleifer, J. (1991). The paradox of power, *Economics & Politics*, **3**(3), 177–200.

Huber, J. and Mayoral, L. (2019). Group inequality and the severity of civil conflict, *Journal of Economic Growth*, **24**, 1–41.

Jelnov, A. and Klunover, D. (2020). When does the private provision of a public good prevent conflict? *Economic Letters*, **192**, Art. 109225.

Katz, E. and Tokatlidu, J. (1996). Group competition for rents, *European Journal of Political Economy*, **12**, 599-607.

Kolmar, M. and Rommeswinkel, H. (2013). Contests with group-specific public goods and complementarities in efforts, *Journal of Economic Behavior and Organization*, **89**, 9–22.

Kuhn, P. and Weidmann, N. (2015). Unequal we fight: between-and within-group inequality and ethnic civil war, *Political Science Research and Methods*, **3**(3), 543-568.

Lichbach, M. (1989). An evaluation of "Does economic inequality breed political conflict? *World Politics* **41**(4): 431–470.

Lee, D. (2012). Weakest-link contests with group-specific public good prizes, *European Journal of Political Economy*, **28**(2), 238–248.

Mitra, A. and Ray, D. (2014). Implications of an economic theory of conflict: Hindu-Muslim violence in India, *Journal of Political Economy*, **122**(4), 719-765.

Morelli, M. and Rohner, D. (2015). Resource concentration and civil wars. *Journal of Development Economics*, **117**(C), 32–47.

Münster, J. (2007). Simultaneous inter- and intra-group conflicts, *Economic Theory*, **32**: 333–352.

Olson, M. (1965). *The Logic of Collective Action: Public Goods and the Theory of Groups*. Cambridge, MA, Harvard University Press.

Østby G. (2008). Polarization, horizontal inequalities and violent civil conflict, *Journal of Peace Research*, **45**(2), 143-62.

Østby G, Nordås, R. and Rød, J.K. (2009). Regional inequalities and civil conflict in sub-Saharan Africa 1986–2004. *International Studies Quarterly* **53**(2), 301–324.

Pai, S. (2013). *Dalit Assertion* New Delhi: Oxford University Press.

Sambanis, N. and Milanovic, B. (2011). Explaining the demand for sovereignty, *Policy research working paper series, vol. 5888*; The World Bank.

Stewart, F. (2008). *Horizontal Inequalities and Conflict: Understanding Group Violence in Multiethnic Societies*, (ed.), Basingstoke: Palgrave Macmillan.

Skaperdas, S. (1992). Cooperation, conflict and power in the absence of property rights, *American Economic Review* **82**, 720–739.

Skaperdas, S. and Syropoulos, C. (1997). The distribution of income in the presence of appropriative activities, *Economica* **64**, 101–117.

Stein, W. and Rapoport, A. (2004). Asymmetric two-stage group rent-seeking: comparison of two contest structures, *Public Choice* **118**, 151-167.

Tullock, G. (1980). Efficient rent seeking, in J.M. Buchanan, R.D. Tollison and G. Tullock (eds.) *Toward a Theory of the Rent-seeking Society*. College Station, Texas A and M University Press: 97-112.

Wärneryd, K. (1998). Distributional conflict and jurisdictional organization, *Journal of Public Economics* **69**, 435-450.

Appendix

Proof of Proposition 1.

Let Assumptions 1 and 2 hold. Suppose that all members of some community *c* are in an interior equilibrium. Then (10)-(12) above must all hold.

First consider internal conflict. Using the first expression in (10):

$$s_c\left(\frac{y_c^M+y_c^P}{{Y_c}^2}\right)=1. \tag{N1}$$

Using the first expressions in (11) and (12), we have, respectively:

$$s_c\left(\frac{y_c^R}{{Y_c}^2}\right)\left[1-\left(\frac{\beta}{A(I_c^M)}\right)\right]=1-s_c\left(\frac{y_c^P}{{Y_c}^2}\right); \tag{N2}$$

and

$$s_c\left(\frac{y_c^R}{{Y_c}^2}\right)\left[1-\left(\frac{\beta}{A(I_c^P)}\right)\right]=1-s_c\left(\frac{y_c^M}{{Y_c}^2}\right). \tag{N3}$$

Together, (N2) and (N3) yield:

$$s_c\left(\frac{y_c^R}{{Y_c}^2}\right)\left[2-\beta\left(\frac{1}{A(I_c^M)}+\frac{1}{A(I_c^P)}\right)\right]=2-s_c\left(\frac{y_c^M+y_c^P}{{Y_c}^2}\right). \tag{N4}$$

Combining (N1) and (N4):

$$s_c\left(\frac{y_c^R}{{Y_c}^2}\right)\left[2-\beta\left(\frac{1}{A(I_c^M)}+\frac{1}{A(I_c^P)}\right)\right]=1. \tag{N5}$$

From (N1) and (N5),

$$\left(\frac{y_c^M + y_c^P}{y_c^R}\right) = \left[2 - \beta\left(\frac{1}{A(I_c^M)} + \frac{1}{A(I_c^P)}\right)\right];$$

which, in light of (2), yields:

$$1 > \gamma_c^R = \left[3 - \beta\left(\frac{1}{A(I_c^M)} + \frac{1}{A(I_c^P)}\right)\right]^{-1} > \frac{1}{3}. \tag{N6}$$

From (2), (N5) and (N6), we get:

$$Y_c = s_c\left[3 - \beta\left(\frac{1}{A(I_c^M)} + \frac{1}{A(I_c^P)}\right)\right]^{-1}\left[2 - \beta\left(\frac{1}{A(I_c^M)} + \frac{1}{A(I_c^P)}\right)\right]. \tag{N7}$$

From (2), (N2), (N6) and (N7),

$$\gamma_c^P = \gamma_c^R\left[1 - \beta\left(\frac{1}{A(I_c^P)}\right)\right] = \left[3 - \beta\left(\frac{1}{A(I_c^M)} + \frac{1}{A(I_c^P)}\right)\right]^{-1}\left[1 - \beta\left(\frac{1}{A(I_c^P)}\right)\right]. \tag{N8}$$

Analogously, from (2), (N3), (N6) and (N7),

$$\gamma_c^M = \gamma_c^R\left[1 - \beta\left(\frac{1}{A(I_c^M)}\right)\right] = \left[3 - \beta\left(\frac{1}{A(I_c^M)} + \frac{1}{A(I_c^P)}\right)\right]^{-1}\left[1 - \beta\left(\frac{1}{A(I_c^M)}\right)\right]. \tag{N9}$$

Now consider external conflict for community c. Recalling the second expression in (10)-(12), we have:

$$\text{for every } g \in \{M, P\}\text{: } \left[A(I_c^g)\left(\left(\frac{X_{-c}}{X^2}\right)\gamma_c^g - 1\right) + \beta\left(\frac{X_{-c}}{X^2}\right)\gamma_c^R\right] = \left[\left(\frac{X_{-c}}{X^2}\right)\gamma_c^R - 1\right] = 0. \tag{N10}$$

(N10) yields the community reaction functions:

$$\text{for every } c \in \{F, H\}\text{: } X_c = (X_{-c})^{\frac{1}{2}}\gamma_c^{R\frac{1}{2}} - X_{-c}. \tag{N11}$$

From (N11), recalling (1), we get the equilibrium community rental shares:

$$s_c = \frac{\gamma_c^R}{\gamma_F^R + \gamma_H^R}. \tag{N12}$$

Now notice that, from (N10), (N12) and (1),

$$X = s_{-c}\gamma_c^R = \left(\frac{\gamma_F^R\gamma_H^R}{\gamma_F^R + \gamma_H^R}\right). \tag{N13}$$

Noting that $(X_c = s_c X)$ by (1), we get the expression for equilibrium external conflict allocations specified in (13) from (N12) and (N13). Now, from (N6) and (N7),

$$Y_c = s_c(1 - \gamma_c^R). \tag{N14}$$

The equilibrium internal conflict allocations, as defined by (14), then follow from (N12) and (N14). Equation (15) follows from (N6) above, while equation (16) follows from (N6), (N8) and (N9). ■

We shall prove Proposition 2 via the following lemma.

Lemma 2. Let Assumptions 1 and 2 hold. Consider a change in the distribution of secure endowments among the non-rich within some community $c \in \{F, H\}$. Let the superscripts * and ′ denote the outcomes, in any interior sub-game perfect Nash equilibrium of the rent-seeking game under the initial distribution and the altered distribution, respectively, and suppose $\left[\gamma_c^{R^*} < \gamma_c^{R'}\right]$. Then:

(i) $s_c{}^* < s_c{}'$;

(ii) (a) $X_c{}^* < X_c'$, (b) $X^* < X'$ and (c) if $\left(\frac{\gamma_{-c}^{R}{}'}{\gamma_c^{R'}}\right) > 1$ (resp. $\frac{\gamma_{-c}^{R}{}^*}{\gamma_c^{R^*}} < 1$), then $X_{-c}{}' >$ (resp.<) $X_{-c}{}^*$;

(iii) (a) $Y_{-c}{}' < Y_{-c}{}^*$, (b) if $\left(\frac{\gamma_c^{R^*}}{\gamma_{-c}^{R}{}^*}\right) > \left(\sqrt{2}-1\right)$ then $Y' < Y^*$, if $\left(\frac{\gamma_c^{R'}}{\gamma_{-c}^{R}{}'}\right) < \left(\sqrt{2}-1\right)$ then $Y' > Y^*$ and (c) if $\left(\frac{\gamma_c^{R'}}{\gamma_{-c}^{R}{}'}\right) < \left(\sqrt{2}-1\right)$ then $Y_c{}' > Y_c{}^*$;

(iv) if $\left(\frac{\gamma_c^{R^*}}{\gamma_{-c}^{R}{}^*}\right) > \left(\sqrt{3}-1\right)$ then $Z' < Z^*$, if $\left(\frac{\gamma_c^{R'}}{\gamma_{-c}^{R}{}'}\right) < \left(\sqrt{3}-1\right)$ then $Z' > Z^*$;

and

(v) $R_c{}' > R_c{}^*, R_{-c}{}' < R_{-c}{}^*$.

Proof of Lemma 2.

(i) Part (i) follows directly from (17).

(ii) By (15), γ_{-c}^{R} is independent of γ_c^R in equilibrium. Part (iia) then follows directly from (13), and part (iib) from (18). Note now that, by (13),

$$X_{-c} = \left(\frac{\gamma_{-c}^{R}}{\frac{\gamma_{-c}^{R}}{\sqrt{\gamma_c^R}} + \sqrt{\gamma_c^R}}\right)^2. \tag{N15}$$

Consider $D \equiv \frac{\gamma_{-c}^{R}}{\sqrt{\gamma_c^R}} + \sqrt{\gamma_c^R}$. Since: $\frac{\partial D}{\partial\left(\sqrt{\gamma_c^R}\right)} = 1 - \frac{\gamma_{-c}^{R}}{\gamma_c^R}$, $\frac{\partial D}{\partial\left(\sqrt{\gamma_c^R}\right)} > 0$ (resp. < 0) if $\frac{\gamma_{-c}^{R}}{\gamma_c^R} < 1$ (resp. > 1). Since γ_{-c}^{R} is independent of γ_c^R in equilibrium, so that $\gamma_{-c}^{R}{}^* = \gamma_{-c}^{R}{}'$, part (iic) follows.

(iii) Part (iiia) follows directly from (14), since, by (15), γ_{-c}^R is independent of γ_c^R in equilibrium. Now, recalling the equilibrium expression for Y in (19), we have:

$$\frac{\partial Y}{\partial \gamma_c^R} = -\frac{(2\gamma_c^R)(\gamma_c^R+\gamma_{-c}^R)-\left({\gamma_c^R}^2+{\gamma_{-c}^R}^2\right)}{\left(\gamma_F^R+\gamma_H^R\right)^2} = \frac{2}{\left(\frac{\gamma_c^R}{\gamma_{-c}^R}+1\right)^2} - 1.$$

Thus, $\frac{\partial Y}{\partial \gamma_c^R} > 0$ (resp. < 0) if $\left(\frac{\gamma_c^R}{\gamma_{-c}^R}\right) < \left(\sqrt{2}-1\right)$ (resp. $> \left(\sqrt{2}-1\right)$. Part (iiib) of Lemma 2 follows, since, by (15), γ_{-c}^R is independent of γ_c^R in equilibrium. Part (iiic) is immediate from parts (iiia) and (iiib).

(iv) Recalling the equilibrium expression for Z in (20), we have:

$$\frac{\partial Z}{\partial \gamma_c^R} = \frac{3}{\left(\frac{\gamma_c^R}{\gamma_{-c}^R}+1\right)^2} - 1.$$

Thus, $\frac{\partial Z}{\partial \gamma_c^R} > 0$ (resp. < 0) if $\left(\frac{\gamma_c^R}{\gamma_{-c}^R}\right) < \left(\sqrt{3}-1\right)$ (resp. $> \left(\sqrt{3}-1\right)$. Recalling that, by (15), γ_{-c}^R is independent of γ_c^R in equilibrium, part (iv) follows.

(v) Part (v) follows immediately from (21). ■

Proof of Proposition 2.

Given Assumptions 1-2, part (i) of Proposition 2 follows immediately from (15) and (16). Since ${\gamma_c^R}^* < {\gamma_c^R}'$ by part (i), and since all the relevant variables depend only on γ_F^R and γ_M^R by Proposition 1 ((13)-(16)), part (ii) of Proposition 2 follows from Lemma 2. ■

Proof of Proposition 3.

Let $\overline{I_c} \equiv \frac{I_c^M+I_c^P}{2}$, and let $\Delta_c \equiv I_c^M - \overline{I_c}$. Furthermore, let:

$$a(k) \equiv \frac{1}{A(k)}. \qquad \text{(N16)}$$

By Assumption 1,

$$a'(k) > 0, \text{and } a''(k) > 0. \qquad \text{(N17)}$$

Using (N16), we have:

$$D_c \equiv \left(\frac{1}{A(I_c^M)} + \frac{1}{A(I_c^P)}\right) = \left(a\left(\overline{I_c}+\Delta_c\right) + a\left(\overline{I_c}-\Delta_c\right)\right). \qquad \text{(N18)}$$

Hence, noting (N17),

$$\frac{\partial D_c}{\partial \Delta} = \left(a'\left(\overline{I_c} + \Delta\right) - a'\left(\overline{I_c} - \Delta\right)\right) > 0. \quad \text{(N19)}$$

From (15), $\frac{\partial \gamma_c^R}{\partial D_c} > 0$. Hence, noting (N19), $\frac{\partial \gamma_c^R}{\partial \Delta} > 0$, so that ${\gamma_c^R}^* < {\gamma_c^R}'$. Recalling that $A'(.) < 0$ by Assumption 1, it follows from (16) that ${\gamma_c^P}^* < {\gamma_c^P}'$. Since ${\gamma_c^R}^* + {\gamma_c^M}^* + {\gamma_c^P}^* = {\gamma_c^R}' + {\gamma_c^M}' + {\gamma_c^P}' = 1$, we then have ${\gamma_c^M}^* > {\gamma_c^M}'$. Since ${\gamma_c^R}^* < {\gamma_c^R}'$ (as proved above), part (ii) follows from Lemma 2. ■

Proof of Proposition 4.

Let $\overline{I_c} \equiv \frac{I_c^M + I_c^P}{2}$, and let $\Delta_c \equiv I_c^M - \overline{I_c}$. Without loss of generality, suppose $I_F^M \geq I_H^M, I_F^P \geq I_H^P$ and $\overline{I_F} > \overline{I_H}$ in the initial situation.

(i) From (15), recalling (N16):

$$\gamma_c^R = \left[3 - \beta\left(a\left(\overline{I_c} + \Delta_c\right) + a\left(\overline{I_c} - \Delta_c\right)\right)\right]^{-1}. \quad \text{(N20)}$$

Part (i) of Proposition 4 is immediate from (N17) and (N20).

(ii) Now, From (N13),

$$X = \left(\frac{1}{{\gamma_F^R}^{-1} + {\gamma_H^R}^{-1}}\right) = \left(\frac{1}{6 - \beta\left(a\left(\overline{I_F} + \Delta_F\right) + a\left(\overline{I_F} - \Delta_F\right) + a\left(\overline{I_H} + \Delta_H\right) + a\left(\overline{I_H} - \Delta_H\right)\right)}\right). \quad \text{(N21)}$$

Consider

$$D \equiv a\left(\overline{I_F} + \Delta_F\right) + a\left(\overline{I_F} - \Delta_F\right) + a\left(\overline{I_H} + \Delta_H\right) + a\left(\overline{I_H} - \Delta_H\right).$$

We have, for spread-preserving increases in the mean non-rich endowment within both communities:

$$dD = \left[a'\left(\overline{I_F} + \Delta_F\right) + a'\left(\overline{I_F} - \Delta_F\right)\right]d\overline{I_F} + \left[a'\left(\overline{I_H} + \Delta_H\right) + a'\left(\overline{I_H} - \Delta_H\right)\right]d\overline{I_H}.$$

When such increases keep the aggregate non-rich endowment across the two communities invariant, $d\overline{I_H} = -d\overline{I_F}$, so that:

$$dD = \left[a'\left(\overline{I_F} + \Delta_F\right) - a'\left(\overline{I_H} + \Delta_H\right) + a'\left(\overline{I_F} - \Delta_F\right) - a'\left(\overline{I_H} - \Delta_H\right)\right]d\overline{I_F}.$$

Recall that, by assumption, $I_F^M \geq I_H^M, I_F^P \geq I_H^P$ and $\overline{I_F} > \overline{I_H}$ in the initial situation. Then, recalling (N17), we have:

$$\frac{dD}{d\overline{I_F}} > 0 \text{ when } d\overline{I_H} = -d\overline{I_F}.$$

Hence, (N21) implies:

$\frac{dX}{d\overline{I_F}} > 0$ when $d\overline{I_H} = -d\overline{I_F}$.

Part (ii) of Proposition 4 follows.

(iii) Recall that, by assumption, $I_F^M \geq I_H^M, I_F^P \geq I_H^P$ and that $\overline{I_F} > \overline{I_H}$ in the initial situation. Then, by (N17) and (N20), $\gamma_F^{R^*} > \gamma_H^{R^*}$. Furthermore, by (N17) and (N20), any increase in the non-rich mean endowment within *F*, $\overline{I_F}$ **,** matched by a corresponding decline in that within *H*, $\overline{I_H}$, with the intra-community spreads Δ_F and Δ_H held constant, must increase γ_F^R and reduce γ_H^R. Now, from (19),

for every $c \in \{F, H\}$: $\frac{\partial Y}{\partial \gamma_c^R} > 0$ (resp. < 0) if $\left(\frac{\gamma_c^R}{\gamma_{-c}^R}\right) < \left(\sqrt{2} - 1\right)$ (resp. > $\left(\sqrt{2} - 1\right)$.

Hence, any increase in the non-rich mean endowment within *F*, $\overline{I_F}$ **,** matched by a corresponding decline in that within *H*, $\overline{I_H}$, with the intra-community spreads Δ_F and Δ_H held constant, must increase *Y* if:

$$\left(\frac{\gamma_F^{R'}}{\gamma_H^{R'}}\right) < \left(\sqrt{2} - 1\right) \text{ and } \left(\frac{\gamma_H^{R'}}{\gamma_F^{R'}}\right) > \left(\sqrt{2} - 1\right);$$

i.e., if

$$\left(\frac{\gamma_F^{R'}}{\gamma_H^{R'}}\right) < \left(\sqrt{2} - 1\right) \text{ and } \left(\frac{\gamma_F^{R'}}{\gamma_H^{R'}}\right) < \frac{1}{\left(\sqrt{2}-1\right)}.$$

The first inequality implies the second. Therefore, *Y* must increase if $\left(\frac{\gamma_F^{R'}}{\gamma_H^{R'}}\right) < \left(\sqrt{2} - 1\right)$. From (15), this condition reduces to:

$$\frac{\left[3-\beta\left(\frac{1}{A\left(I_H^{M'}\right)}+\frac{1}{A\left(I_H^{P'}\right)}\right)\right]}{\left[3-\beta\left(\frac{1}{A\left(I_F^{M'}\right)}+\frac{1}{A\left(I_F^{P'}\right)}\right)\right]} < \left(\sqrt{2} - 1\right).$$

Part (iii) of Proposition 4 follows.

(iv) The proof of part (iv) of Proposition 4 is analogous to that of part (iii) and is therefore omitted.

(v) Recall that, by (N17) and (N20), any increase in the non-rich mean endowment within *F*, $\overline{I_F}$ **,** matched by a corresponding decline in that within *H*, $\overline{I_H}$, with the intra-community spreads Δ_F and Δ_H held constant, must increase γ_F^R and reduce γ_H^R. The proof of part (v) then follows immediately from (21). ■