UNIVERSITY OF NOTTINGHAM

Discussion Papers in Economics

Discussion Paper
No. 01/03

DEFINING POVERTY LINES AS A FRACTION OF CENTRAL TENDENCY

by Christophe Muller

March 2001

DP 01/03
ISSN 1360-2438

UNIVERSITY OF NOTTINGHAM

Discussion Papers in Economics

Discussion Paper
No. 01/03

DEFINING POVERTY LINES AS A FRACTION OF CENTRAL TENDENCY

by Christophe Muller

Christophe Muller is CREDIT Research Fellow, School of Economics, University of Nottingham

March 2001

DEFINING POVERTY LINES AS A FRACTION OF CENTRAL TENDENCY

Christophe MULLER

School of Economics, University of Nottingham,
University Park, Nottingham NG7 2RD,
United Kingdom.
Fax: +44 0115 951 4159
Email: christophe.muller@nottingham.AC.UK

December 20^{th}, 2000

Codes JEL: I32 (Measurement and Analysis of Poverty), O15 (Income Distribution), D31 (Personal Income Distribution).

This research was carried out during my visit to the Department of Agricultural Economics at the University of California in Berkeley. I am grateful to the British Academy for a grant. I thank participants in a seminar in Nottingham University. Remaining errors are mine.

Abstract

Using parametric formulae under lognormality for a broad family of poverty measures, we show that when inequality measured by the Gini coefficient is constant, defining the poverty line as a fraction of a central tendency of the living standard distribution restricts the evolution of the poverty measures to be stable. Moreover, when the Gini coefficient only moderately changes, most of the poverty change can be better considered as a change in inequality. We illustrate these questions by using data from the United States, for which we compare the evolution of measured poverty for poverty lines anchored on various central tendencies.

Résumé

A partir de formules paramétriques sous lognormalité pour une large famille de mesures de pauvreté, nous montrons que lorsque l'inégalité mesurée par l'indice de Gini est constante définir le seuil de pauvreté en utilisant une fraction d'une tendance centrale de la distribution des niveaux de vie restreint l'évolution des mesures de pauvreté à être stable. De plus, quand l'indice de Gini change seulement modérément, la plupart du changement de la pauvreté peut être mieux considéré en tant que changement de l'inégalité. Nous illustrons ces questions à partir de données des Etats-Unis, pour lesquelles nous comparons l'évolution de la pauvreté pour des lignes de pauvreté ancrées sur diverses tendances centrales différentes.

1 Introduction

It has long been accepted that relative notions of poverty are important, notably to account for the evolution of perceptions of basic needs in society[1]. Being poor in a population of poor people can be considered very different from being poor in a wealthy environment. This concern is often met by updating the poverty line across time. In particular, it is often considered that the poverty line should be relative to the distribution of living standards.

The literature about poverty lines is extensive[2], and very varied rules to define these lines have been proposed. Various fractions of the median or the mean of the living standard distribution have been widely used to update poverty lines[3], notably for dynamic poverty analyses. Half of the mean of the living standard distribution and half of the median are popular poverty lines used by national and international administrations.

There exist other updating procedures such as poverty lines anchored on the mean living standard of a representative group of households whose living standards are considered to be close to the desired poverty line (Ravallion (1998)) or poverty lines relying on subjective perceptions (collected by using special surveys) of poverty by individuals[4]. We do not deal with these procedures in this paper.

We focus in this paper on two major poverty measures: (1) the Watts measure[5] that is one of the most popular axiomatically sound poverty measure, (2) the head-count index which, while having mediocre axiomatic properties, is certainly the most used poverty measure. We also extend the analysis to a large class of poverty measures considered under lognormality of the living standard distribution.

The aim of this article is to show that under reasonable distribution assumptions, using a fraction of a central tendency as the poverty line restricts

[1]Sen (1983), Seidl (1988), Foster (1998).

[2]van Praag, Goedhart and Kapteyn (1978), van Praag, Spit and van de Stadt (1982), Hagenaars and van Praag (1985), Callan and Nolan (1991), Ravallion and Sen (1996), Short, Garner, Johnson and Shea (1998), Pradhan and Ravallion (1998), Ravallion (1998).

[3]Fuchs (1969), Plotnick and Skidmore (1975), Fiegehen, Lensley and Smith (1977), O'Higgins and Jenkins (1990), Central Statistical Authority (1997), Oxley (1998), Stewart (1998).

[4]van Praag, Goedhart and Kapteyn (1978), van Praag, Spit and van de Stadt (1982), Hagenaars and van Praag (1985), Pradhan and Ravallion (1998).

[5]Watts (1968), Zheng (1993).

the evolution of poverty statistics to be stable when the inequality is stable. Therefore, such updating procedure is ill-adapted to the study of dynamic poverty, which is confused for usual low levels of inequality changes with dynamic inequality described by the Gini coefficient. In section 2, we present parametric formulae of the Watts poverty measure and the head-count index under lognormality of incomes. We analyse in section 3 the properties of poverty measures when poverty lines are updated by using a fraction of central tendency. Moreover, a comparison of different central tendencies for fixing the poverty line is provided. In section 4, we present a short application using US data. Finally, section 5 concludes. The proofs are in the appendix.

2 Parametric Formulae for the Watts Poverty Measure and for the Head-Count Index

The Watts poverty measure is defined as

$$W = \int_0^z - \ln(y/z)\, d\mu(y) \tag{1}$$

where μ is the cumulative density function of living standards y and z is the poverty line. The Watts measure satisfies the focus, monotonicity, transfer and transfer sensitivity axioms. It is also continuous, subgroup consistent and decomposable (Foster, Greer, Thorbecke (1984), Zheng(1993)). Moreover, when related to social welfare functions, the Watts measure is the unique measure indicating the absolute amount of social welfare loss caused by poverty that satisfies monotonicity, continuity, decomposability and scale invariance (Zheng (1993, 1997)). Because of its axiomatic properties, it is often a better representation of poverty than other frequently used poverty indicators. The head-count index is the proportion of the poor in the whole population, $P_0 = \int_0^z \, d\mu(y)$ and is the most popular poverty indicator.

The lognormal distribution approximation has been used in applied analysis of living standards (Alaiz and Victoria-Feser (1990), Slesnick (1993)). Although it has sometimes (e.g. van Praag, Hagenaars and van Eck (1983)) been found statistically consistent with income data, other distribution models for living standards or incomes sometimes yield better goodness-of-fit.

However, what we want in this paper is not so much the goodness-of-fit of the model as to obtain simultaneously a simple parametric expression of the Watts measure, the head-count index and an inequality measure, which is generally not possible with other distributions. As a matter of fact, we use the lognormal model as a simple way of illustrating a general argument that could be extended to more flexible specifications of the income distribution. Ultimately, the usefulness of the distribution model depends on the use one wants to make of it. In this paper, a more statistically adequate distribution model would not allow us to present our point as clearly and would not allow us to exploit the availability of parameter estimates for the lognormal distribution in the US. Muller (1999) derives a parametric formula for the Watts poverty measure. The formula for the head-count index, P_0, is straightforward.

Proposition 1

If the living standard, y, follows a lognormal distribution such that ln(y) $\sim$ N(m, σ^2), then the Watts poverty measure is equal to:

$$W = (\ln z - m)\,\Phi\left(\frac{\ln z - m}{\sigma}\right) + \sigma\,\phi\left(\frac{\ln z - m}{\sigma}\right) \tag{2}$$

where ϕ and Φ are respectively the p.d.f. and c.d.f. of the standard normal distribution.

$$P_0 = \Phi\left(\frac{\ln z - m}{\sigma}\right) \tag{3}$$

The variance of the logarithms, here equal to σ^2, is a well known inequality measure, although it is not always consistent with Lorenz ordering (Foster and Ok (1999)). However, under lognormality, the Gini coefficient is $G = 2\Phi(\sigma/\sqrt{2}) - 1$ where Φ is the c.d.f. of the standard normal law and σ is on a one-to-one correspondence with the Gini coefficient. To be brief, we shall mention only the inequality described by σ, although all qualitative statements in this paper are also valid with inequality measured by the Gini coefficient.

When maximum likelihood estimators are used, standard errors of the Watts measure can be derived. These are new results.

Proposition 2 *Under lognormality,*

a) The standard error of the Watts measure estimated with its maximum likelihood estimator (i.e. by replacing the distribution parameters in its formula by the MLE) is:

$\sigma(W) = \frac{\sigma}{\sqrt{n}} \sqrt{\left(\Phi\left(\frac{\ln z - m}{\sigma}\right)\right)^2 + \frac{1}{2}\left(\phi\left(\frac{\ln z - m}{\sigma}\right)\right)^2}$, *where n is the sample size.*

Under lognormality, $\sigma(W)$ can be consistently estimated by replacing the parameters m and σ by their respective MLE.

b) The standard error of the head-count index estimated with its maximum likelihood estimator is:

$\sigma(P_0) = \frac{\phi\left(\frac{\ln z - m}{\sigma}\right)}{\sqrt{n}} \sqrt{1 + \frac{(\ln z - m)^2}{2\sigma^2}}$. *Under lognormality, $\sigma(P_0)$ can be consistently estimated by replacing the parameters m and σ by their respective MLE.*

Proof: The standard errors are derived from the standard errors of the MLE, related to the information matrix and from the application of the delta rule.

3 Restrictions for Poverty Measurement

We now show that when updating the poverty line by defining it as a fraction of the median (or the mean or the mode), the aggregate poverty levels measured by the Watts measure, the head-count index and a large class of poverty measures are conserved under lognormality when σ is constant.

Let us first remember that the median of a lognormal distribution LN(m, σ^2) is e^m, the mode is e$^{m-\sigma^2}$and the mean is e$^{m+\sigma^2/2}$. Then, for example, a poverty line defined as a fraction of the median has a formula: $z = K\, e^m$, with K a given number between 0 and 1. Let us consider the general class of poverty measures that can be written as $F\left(\frac{\ln z - m}{\sigma}, \sigma\right)$ under lognormality. We obtain the following results.

Proposition 3

a) Under lognornormality when the variance of the logarithms is constant (i.e. also when the Gini coefficient is constant), using a proportion of the *median (or the mean or the mode) of the living standard distribution to update the poverty line as the distribution varies yields a fixed estimate of poverty as measured by any poverty measure of the form* $F(\frac{\ln z - m}{\sigma}, \sigma)$.

b) The Watts poverty measure and the Head-Count Index are special cases of this class of poverty measures.

One sometimes expects that σ and other inequality measures vary much less across years than usual poverty measures. For example, the estimates in Datt and Ravallion (1992) for India and Brazil in the 1980s show a much smaller temporal relative variation for the Gini coefficient than for the head-count index. The deviation between the smallest and the largest observed levels of the Gini coefficient G across years is 5.6 % for rural India, 6.6 % for urban India and 6.0 % for Brazil. In contrast, the deviation between the smallest and the largest observed levels of the head-count index is 43.0 % for rural India, 26 % for urban India and 33 % for Brazil. This relative stability of the Gini coefficient across years is likely to hold for other common inequality measures, such as the variance of the logarithms. Then, in a first approximation and in some contexts, σ and G may change little when compared with changes in poverty measures. In these cases, the conditions of Proposition 3.1 can be considered as approximately satisfied. Then, the danger of using fractions of central tendencies as updating rules for the poverty line is plain. Indeed, it is likely that such methods restrict one to obtain only stable measures of poverty evolution. This may have damaging consequences for poverty policies if poverty lines anchored on minimal needs of representative groups, or other types of poverty lines, show very different poverty evolution.

Alternatively, when σ changes only moderately across periods, Proposition 3.1 indicates that most of the change in poverty can be better considered as a change in inequality, as measured by the variance of logarithms or by the Gini coefficient, rather than as a specific poverty phenomenon. Indeed, at the first order we have with the above relative poverty lines: $dF = \frac{\partial F}{\partial \sigma} - \frac{\partial F}{\partial Z}\left(\frac{\ln z - m}{\sigma}\right) = \frac{\partial F}{\partial \sigma}$.

When inequality changes only moderately, poverty measures that can be written as $F(\ln z - m, \sigma)$ mostly reflect this change rather than that which can be specific in poverty evolution. Other notions of poverty lines may be more appropriate and should allow the separation of analyses of poverty changes and inequality changes.

In the situation where σ varies, it is also of interest to investigate the impact of choosing a fraction of a given central tendency (median, mean or mode) for the updating rule.

Proposition 4 *For all poverty measures of the type F(Z,* σ*) where* $Z = \frac{\ln z - m}{\sigma}$*, with the above relative poverty lines:*

$\frac{dF}{d\sigma} = \frac{\partial F}{\partial \sigma} + \frac{\partial F}{\partial Z}\frac{\ln p}{\sigma^2}$ *for a poverty line equal to the* p^{th} *fraction of the median.*

$\frac{dF}{d\sigma} = \frac{\partial F}{\partial \sigma} + \frac{\partial F}{\partial Z}\left(\frac{\ln p}{\sigma^2} - \frac{1}{2}\right)$ *for a poverty line equal to the* p^{th} *fraction of the mean.*

$\frac{dF}{d\sigma} = \frac{\partial F}{\partial \sigma} + \frac{\partial F}{\partial Z}\left(\frac{\ln p}{\sigma^2} + 1\right)$ *for a poverty line equal to the* p^{th} *fraction of the mode.*

Subsequently, we can now examine the variations of the head-count index and of the Watts measure.

Proposition 5 *In the cases of the head-count index (*P_0*) and the Watts measure (W) we have:*

a) $P_0 = \Phi(Z)$, $\frac{\partial P_0}{\partial Z} = \phi(Z)$ *and* $\frac{\partial P_0}{\partial \sigma} = 0$.

b) $W = \sigma.(Z\Phi(Z) + \phi(Z))$, $\frac{\partial W}{\partial Z} = \sigma\Phi(Z)$ *and* $\frac{\partial W}{\partial \sigma} = Z\Phi(Z) + \phi(Z)$.

The previous formulae can help to understand the differences between the evolution of different poverty indicators or poverty indicators calculated by using different relative poverty lines.

Proposition 6 *a) The evolution of* P_0 *is attenuated when compared with the evolution of W and when using the above relative poverty lines.*

b) If $\frac{\partial F}{\partial Z} > 0$ *(as expected for usual poverty measures) and for small variations of* σ*, the variations in measured poverty with the considered class of poverty measures are increasing functions of p if 1/p is the fraction used in the calculation of the relative poverty line. This is true for all three considered central tendencies.*

If $\frac{\partial F}{\partial Z} < 0$*, these variations in measured poverty are decreasing functions of* p*.*

c) With the same class of poverty measures and for small variations of σ*, the evolutions of poverty with relative poverty lines based on the same fraction of the mean and the median are closer than those based on the same fraction of the mode.*

Other parametric approaches less dependent on the lognormality assumption are possible, but deliver formulae less easy to manipulate. For example, Datt and Ravallion (1992) derive parametric formulae for Foster-Greer-Thorbecke poverty indices P_0, P_1 and P_2, under assumptions of parametrised Lorenz curves of types Beta and Generalised Quadratic. However, these are only implicit formulae and the calculation of poverty measures must be made using extrapolation methods as roots of complicated equations. This generally rules out any explicit analysis for these measures under such broad distribution assumptions. Moreover, the parameters intervening in these Lorenz curves are not obvious to interpret and cannot be assimilated to inequality measures or other simple social welfare notion. Therefore, we choose not to follow this approach and we rather rely on an approximate lognormal representation. We now illustrate the analysis with data from the United States.

4 Empirical Illustration

We estimate poverty in the United States by using the Watts poverty measure and the head-count index. Minimum Khi-Square estimates[6] of parameters m and σ of the lognormal distribution have been published in McDonald and Ransom (1979). The distribution is that of US family nominal income for 1960 and 1969-75. We use these data because estimates of m and σ for the lognormal distribution are available.

Tables 1 through 4 show the parameter estimates for m and σ and the estimates of the poverty measures for all years. Figures 1 and 2 show the evolution of the two poverty indicators across years for different poverty lines. The poverty lines are defined as follows: zr1 = median/2, zr2 = median/4, zr3 = mean/2, zr4 = mean/4, zr5 = mode/2, zr6 = mode/4. Poverty lines

[6] In practice these estimates are close to the MLE. This justifies to use the above formulae for the standard errors.

zr1 to zr6 are defined relatively to the income distribution. Poverty lines za1 to za6 are absolute poverty lines, which have similar definitions to zr1-zr6, with the corresponding central tendency of the 1960 income distribution. The standard errors that are shown for 1960 and 1969 prove that the results are robust to random sample variations..

For all the considered relative poverty lines, poverty slightly fluctuates across years parallel to the fluctuations of inequality as measured by σ. The relative variations of both poverty measures with the updated poverty lines are small (a few percent), although they are often non negligible. They correspond to oscillations of poverty across time around a somewhat stable position. This is consistent with σ being not very different in 1960 and in 1975. However, there are sometimes large rises and falls in poverty between two years, even with updated poverty lines. Thus, the drop in relative poverty of 1973 is caused by a fall in inequality as measured by σ. Clearly, over the whole period, the evolution of inequality determines the evolution of relative poverty. Moreover, using half the mean or half the mode instead of half the median makes a substantial difference in the level of measured poverty, but is less important in the measured evolution pattern of poverty.

In contrast, with the absolute poverty lines based on 1960 central tendencies, the estimates show large and regularly negative relative variations (starting from 1971) of Watts poverty and poverty incidence. Omitting to update poverty yields an over-optimistic picture of steady poverty disappearance.

The evolutions of Watts poverty and the incidence of poverty are parallel, whether absolute or relative poverty lines are used. However, with relative poverty lines the changes in poverty are attenuated when using the head-count index, which does not deliver a correct representation of poverty dynamics by hiding the large changes in poverty severity. With absolute poverty lines, the attenuation associated with the head-count index is much less visible.

Stronger relative variations in poverty (much stronger when using relative poverty lines) are found with poverty lines defined as a quarter of a central tendency rather than with poverty lines defined as half of the same central tendency. With these relative poverty lines, the very poor systematically appear to be more vulnerable to poverty fluctuations across years than the moderately poor. This type of result may not be robust to poverty lines calculated differently.

On the whole, poverty changes measured with poverty lines based on the same fraction of the median or the mean are quite similar. In contrast, measured poverty variations obtained with poverty lines based on a fraction of the mode are comparatively attenuated with relative poverty lines and often accentuated with absolute poverty lines.

5 Conclusion

We investigate in this paper the consequences of updating poverty lines by using fractions of central tendencies of the living standard distribution. We show that under lognormal approximation and if the Gini coefficient of inequality does not change, the measured evolution of poverty is restricted to be stable with these updating rules. When the Gini coefficient changes moderately most of the change in poverty can be better considered as a change in inequality, rather than as a specific poverty phenomenon. An illustration based on US data confirms the theoretical results and shows the impact caused by the choice of a particular central tendency in defining the poverty line. Caution must therefore be employed when using this type of relative poverty lines. It seems also important to check the robustness of results to the use of different procedures when updating the poverty line, such as anchoring the definition on a representative group of households, or using subjective answers.

BIBLIOGRAPHY

ALAIZ, M.P. and M.-P. VICTORIA-FESER, 1996, "Modelling Income Distribution in Spain: A Robust Parametric Approach", Distributional Analysis research Programme, Working Paper no 20, LSE-STICERD, June.

BUTLER, R.J. and J.B. McDONALD, 1989, "Using Incomplete Moments to Measure Inequality", *Journal of Econometrics*, 42, 109-119.

CALLAN, T. and B. NOLAN, 1991, "Concepts of Poverty and the Poverty Line", *Journal of Economic Surveys*, Vol. 5, No. 3.

CENTRAL STATISTICAL AUTHORITY, 1997, "*Poverty Situation in Ethiopia*", Ministery of Economic Development and Cooperation, Addis Abeba.

CHIPMAN, J.S., 1974, "The Welfare Ranking of Pareto Distributions", *Journal of Economic Theory*, 9, 275-282.

DATT, G. and M. RAVALLION, 1992, "Growth and redistribution components of changes in poverty measures", *Journal of Development Economics*, 38, 275-295.

FIEGEHEN, G.C., P.S. LANSLEY and A.D. SMITH, 1977, "*Poverty and Progress in Britain 1953-73*", Cambridge University Press.

FOSTER, J.E., "Absolute versus Relative Poverty", *American Economic Association Papers and Proceedings*, 335-341, May 1998.

FOSTER, J., J. GREER, E. THORBECKE, 1984, "A Class of Decomposable Poverty Measures", *Econometrica*, Vol. 52, No. 3, May.

FOSTER, J.E. and E.A. OK, 1999, "Lorenz Dominance and the variance of Logarithms", *Econometrica*, Vol. 67, No. 4, 901-907, July.

FUCHS, V., "Comment on Measuring the Size of the Low-Income Population" in Lee Soltow, ed., "Six papers on the size distribution of wealth and income", New York, National Bureau of Economic Research, 198-202, 1969.

HAGENAARS, A.J.H. and B.M.S. VAN PRAAG, 1985, "A Synthesis of Poverty Line Definitions", *Review of Income and Wealth*, Serie 31, No. 1-4.

McDONALD, J.B., 1984, "Some Generalised Functions for the Size Distribution of Income", *Econometrica*, Vol. 52, No. 3, May.

McDONALD, J.B. and B.C. JENSEN, 1979, "An Analysis of Some Properties of Alternative Measures of Income Inequality Based on the Gamma Distribution Function", *Journal of the American Statistical Association*, vol. 78, 856-860, December.

McDONALD, J.B. and M.R. RANSOM, 1979, "Functional Forms, Estimation Techniques and the Distribution of Income", *Econometrica*, Vol. 47, No. 6, November.

MULLER, C., 1999, "The Properties of the Watts Poverty Index under Lognormality", Discussion Paper, School of Economics, Nottingham.

O'HIGGINS, M. and S. JENKINS, "Poverty in the EC: Estimates for 1975, 1980 and 1985" in R. Teekens and B. M.S. van Praag (eds.), "*Analysing poverty in the European Communauty: Policy issues, research options, and data sources*", Luxembourg, Office of Official Publications of the European Communities, 187-212, 1990.

OXLEY, H., 1998, "Income dynamics: inter-generational evidence" in "*Persistent Poverty and Lifetime Inequality: The Evidence*", Proceedings from a workshop held at H M Treasury, London, November.

PLOTNICK, G. and R. SKIDMORE, 1975, "*Progress Against Poverty. A Review of the 1964-74 Decade*", Academic Press.

PRADHAN, M. and M. RAVALLION, 1998, "Measuring Poverty Using Qualitative Perceptions of Welfare", mimeo The World Bank, August.

RAVALLION, M., 1998, "Poverty Lines in Theory and Practice", Working Paper World Bank, March.

RAVALLION, M. and B. SEN, 1996, "When Method Matters: Monitoring Poverty in Bangladesh", *Economic Development and Cultural Change*, 761-792.

SEIDL, C., "Poverty Measurement: A Survey", in Bos, D., Rose, M. and Seidl, C. (Eds.), "*Welfare and Efficiency in Public Economics*", Springer-Verlag, Berlin, 1988.

SEN, "Poor, Relatively Speaking", *Oxford Economic Papers*, 35, 153-169, 1983.

SHORT, K., T.I. GARNER, D.S. JOHNSON and M. SHEA, 1998, "Redefining Poverty Measurement in the U.S.: Examining the Impact on Inequality and Poverty", mimeo U.S. Bureau of the Census, August.

SLESNICK, D.T., 1993, "Gaining Ground: Poverty in the Postwar United States", *Journal of Political Economy*, vol. 101, No 1.

STEWART, M., 1998, "Low pay, no pay dynamics" in "*Persistent Poverty and Lifetime Inequality: The Evidence*", Proceedings from a workshop held at H M Treasury, London, November.

VAN PRAAG, B., T. GOEDHART and A. KAPTEYN, 1978, "The Poverty Line - A Pilot Survey in Europe", *The Review of Economics and Statistics*, 461-465.

VAN PRAAG, B., A. HAGENAARS and W. VAN ECK, 1983, "The Influence of Classification and Observation Errors on the Measurement of Income Inequality", *Econometrica*, Vol. 51, No. 4, July.

VAN PRAAG, B., J.S. SPIT and H. VAN DE STADT, 1982, "A Comparison between the Food ratio Poverty Line and the Leyden Poverty Line", *The Review of Economics and Statistics,* 691-694.

WATTS, H.W., 1968, "An Economic Definition of Poverty" in D.P. Moynihan (Ed..), "*On Understanding Poverty*", 316-329, Basic Book, New York.

ZHENG, B., 1993, "An Axiomatic Characterization of the Watts Poverty Index", *Economic Letters*, 42, 81-86.

ZHENG, B., 1997, "Aggregate Poverty Measures", *Journal of Economic Surveys*, Vol. 11, No. 2.

Appendix

Table 1: Estimates of Watts poverty measures

	1960	1969	1970	1971	1972	1973	1974
m	1.5853	2.1339	2.1832	2.2004	2.3031	2.38532	2.4511
σ	0.7984	0.8220	0.8461	0.8263	0.8444	0.7664	0.7747
$\Delta\sigma$	–	–	0.0241	−0.0198	0.0181	−0.078	0.0083
$zr1$	0.0849 (0.00188)	0.0915 (0.00136)	0.0983	0.0927	0.0978	0.0763	0.0785
$zr2$	0.0133 (0.000590)	0.0155 (0.000429)	0.0179	0.0159	0.0177	0.0107	0.0113
$zr3$	0.1657 (0.00278)	0.1804 (0.00205)	0.1960	0.1831	0.1949	0.1466	0.1514
$zr4$	0.0335 (0.00105)	0.0394 (0.000793)	0.0459	0.0405	0.0455	0.0265	0.0282
$zr5$	0.0158 (0.000657)	0.0163 (0.000443)	0.0168	0.0164	0.0167	0.0150	0.0152
$zr6$	0.00143 (0.000148)	0.00160 (0.0000977)	0.00177	0.00163	0.00176	0.00121	0.00127
$za1$	0.0849	0.0235	0.0233	0.0199	0.0165	0.00746	0.0063
$za2$	0.0133	0.00256	0.00271	0.00210	0.00173	0.000488	0.000409
$za3$	0.1657	0.0538	0.0526	0.0465	0.0389	0.0208	0.0179
$za4$	0.0335	0.00763	0.00781	0.00636	0.00524	0.00186	0.00157
$za5$	0.0158	0.00313	0.00329	0.00257	0.00212	0.000623	0.000523
$za6$	0.00143	0.000197	0.000227	0.000156	0.000131	0.0000211	0.0000177

$\Delta\sigma$ is $\sigma_t - \sigma_{t-1}$. The first two rows show the estimates of m and σ for all years. The following rows show the estimates of the Watts poverty measure for all years and each poverty line. Standard errors of indicators with relative poverty lines for 1960 and 1969 are in parentheses.

Table 2: Annual Relative Variations of Watts Poverty Measures

	1970	1971	1972	1973	1974	1975
m	2.1832	2.2004	2.3031	2.38532	2.4511	2.5204
σ	0.8461	0.8263	0.8444	0.7664	0.7747	0.7767
$zr1$	0.0743	-0.0569	0.0552	-0.219	0.0289	0.00681
$zr2$	0.1551	-0.111	0.114	-0.397	0.0613	0.0142
$zr3$	0.0867	-0.0657	0.0642	-0.247	0.0330	0.00777
$zr4$	0.1662	-0.118	0.122	-0.416	0.0648	0.0150
$zr5$	0.0291	-0.0230	0.0216	-0.104	0.0142	0.00331
$zr6$	0.1083	-0.0804	0.0799	-0.312	0.0468	0.0108
$za1$	-0.00603	-0.145	-0.170	-0.549	-0.147	-0.202
$za2$	0.0565	-0.224	-0.174	-0.718	-0.161	-0.246
$za3$	-0.0234	-0.114	-0.163	-0.465	-0.138	-0.182
$za4$	0.0231	-0.186	-0.174	-0.644	-0.156	-0.226
$za5$	0.0500	-0.217	-0.174	-0.706	-0.160	-0.242
$za6$	0.1510	-0.310	-0.161	-0.838	-0.163	-0.283

Table 3: Estimates of the Head-Count Index

	1960	1969	1970	1971	1972	1973	1974
m	1.5853	2.1339	2.1832	2.2004	2.3031	2.38532	2.4511
σ	0.7984	0.8220	0.8461	0.8263	0.8444	0.7664	0.7747
$zr1$	0.192 (0.00220)	0.199 (0.00161)	0.206	0.200	0.205	0.182	0.185
$zr2$	0.0412 (0.000961)	0.0458 (0.000741)	0.0506	0.0467	0.0503	0.0352	0.0367
$zr3$	0.0319 (0.00258)	0.332 (0.00188)	0.345	0.335	0.345	0.301	0.305
$zr4$	0.0905 (0.00154)	0.101 (0.00117)	0.112	0.103	0.111	0.0769	0.0804
$zr5$	0.0478 (0.00105)	0.0479 (0.000763)	0.0479	0.0479	0.0479	0.0473	0.0475
$zr6$	0.00562 (0.000226)	0.00606 (0.000173)	0.00648	0.00613	0.00645	0.00500	0.0517
$za1$	0.192	0.0654	0.0635	0.0566	0.0473	0.0256	0.0220
$za2$	0.0412	0.00928	0.00951	0.00771	0.00635	0.00216	0.00182
$za3$	0.319	0.130	0.125	0.115	0.0979	0.0627	0.0546
$za4$	0.0905	0.0246	0.0245	0.0208	0.0172	0.00740	0.00628
$za5$	0.0478	0.0111	0.0113	0.00926	0.00763	0.00271	0.00229
$za6$	0.00562	0.000875	0.000972	0.000702	0.000583	0.000114	0.0000957

Standard errors of indicators with relative poverty lines for 1960 and 1969 are in parentheses.

Table 4: Annual Relative Variations of the Head-Count Index

	1970	1971	1972	1973	1974	1975
m	2.1832	2.2004	2.3031	2.38532	2.4511	2.5204
σ	0.8461	0.8263	0.8444	0.7664	0.7747	0.7767
$zr1$	0.0339	−0.0269	0.0253	−0.111	0.0141	0.00332
$zr2$	0.104	−0.0782	0.0774	−0.299	0.0434	0.0101
$zr3$	0.0396	−0.0312	0.0295	−0.127	0.0160	0.00379
$zr4$	0.109	−0.0811	0.0806	−0.308	0.0448	0.0104
$zr5$	−0.000169	0.000352	−0.000247	−0.0114	0.00290	0.000633
$zr6$	0.0698	−0.0533	0.0516	−0.224	0.0324	0.00753
$za1$	−0.0293	−0.107	−0.164	−0.457	−0.140	−0.184
$za2$	0.0238	−0.188	−0.176	−0.658	−0.158	−0.232
$za3$	−0.0420	−0.0773	−0.152	−0.359	−0.127	−0.169
$za4$	−0.00529	−0.149	−0.173	−0.570	−0.151	−0.210
$za5$	0.0181	−0.181	−0.176	−0.644	−0.157	−0.228
$za6$	0.110	−0.277	−0.169	−0.803	−0.164	−0.273

Proof of Proposition 3.1:

Let be a poverty measure of the form $F(\frac{\ln z-m}{\sigma}, \sigma) = F(Z, \sigma)$ with $Z = \frac{\ln z-m}{\sigma}$. Then,
$dF = \frac{\partial F}{\partial \sigma}d\sigma + \frac{\partial F}{\partial Z}dZ$ and $dZ = \frac{1}{z\sigma}dz - \frac{1}{\sigma}dm - \frac{\ln z-m}{\sigma^2}d\sigma$. This gives
$dF = \frac{1}{\sigma}\frac{\partial F}{\partial Z}(\frac{dz}{z} - dm) + (\frac{\partial F}{\partial \sigma} - \frac{\partial F}{\partial Z}\frac{\ln z-m}{\sigma^2})d\sigma$.
Therefore, if σ is constant, $dF = 0$ is equivalent to $\frac{dz}{z} - dm = 0$, except in case where $\frac{\partial F}{\partial Z} = 0$, which is generically negligible. By integration one obtains: $z = K(\sigma)e^m$, where $K(\sigma)$ is a function of σ only.

If $K(\sigma) = 1/p$ with $0 < p < 1$, then $z = \frac{e^m}{p}$ is the pth fraction of the median. If $K(\sigma) = e^{\sigma^2/2}/p$, then $z = \frac{e^{m+\sigma^2/2}}{p}$ is the pth fraction of the mean. If $K(\sigma) = e^{-\sigma^2}/p$, then $z = \frac{e^{m-\sigma^2}}{p}$ is the pth fraction of the mode. QED.

Proof of Proposition 3.2: Deduced from $\frac{dF}{d\sigma} = \frac{\partial F}{\partial \sigma} - \frac{\partial F}{\partial Z}\frac{\ln z-m}{\sigma^2}$. QED.

Proof of Proposition 3.3: a) Indeed, $\frac{\partial H}{\partial \sigma} = 0$ implies that the evolution of inequality represented by σ has no effect on P_0. Then, P_0 is stable if the formula of the relative poverty line compensates for the change in parameter m.

b) This is shown by the formulae of $\frac{\partial F}{\partial \sigma}$.
c) Indeed, the formulae of $\frac{dF}{d\sigma}/\frac{dF}{dZ}$ show that it differs by 1/2 for median and mean, by 1 for median and mode, by 1.5 for mean and mode. QED.

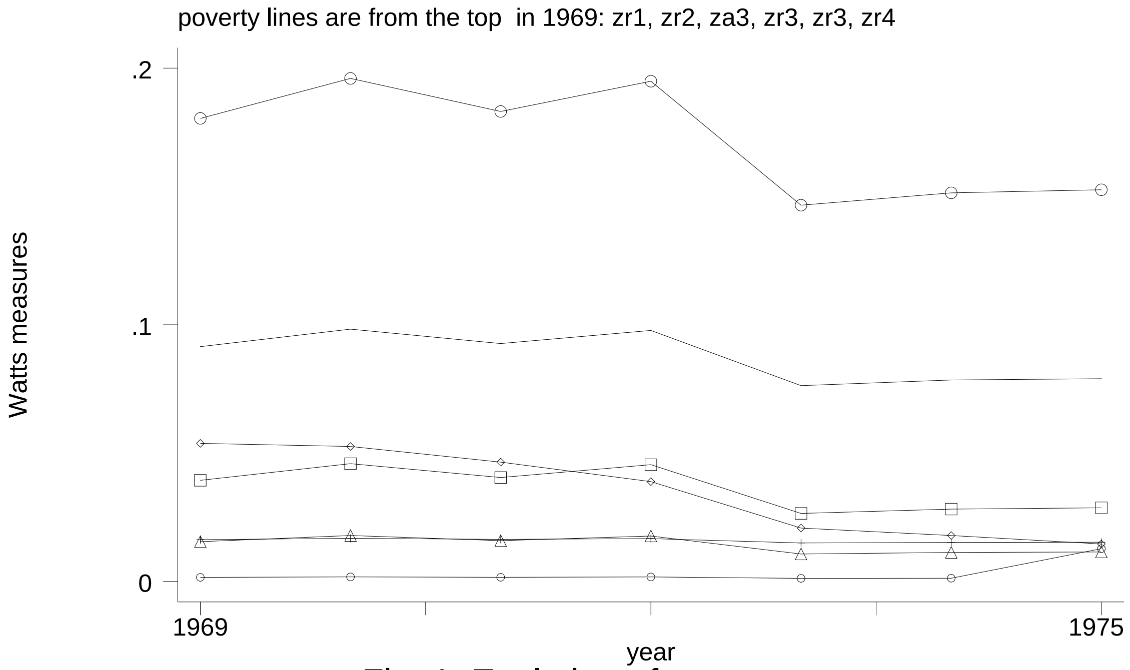
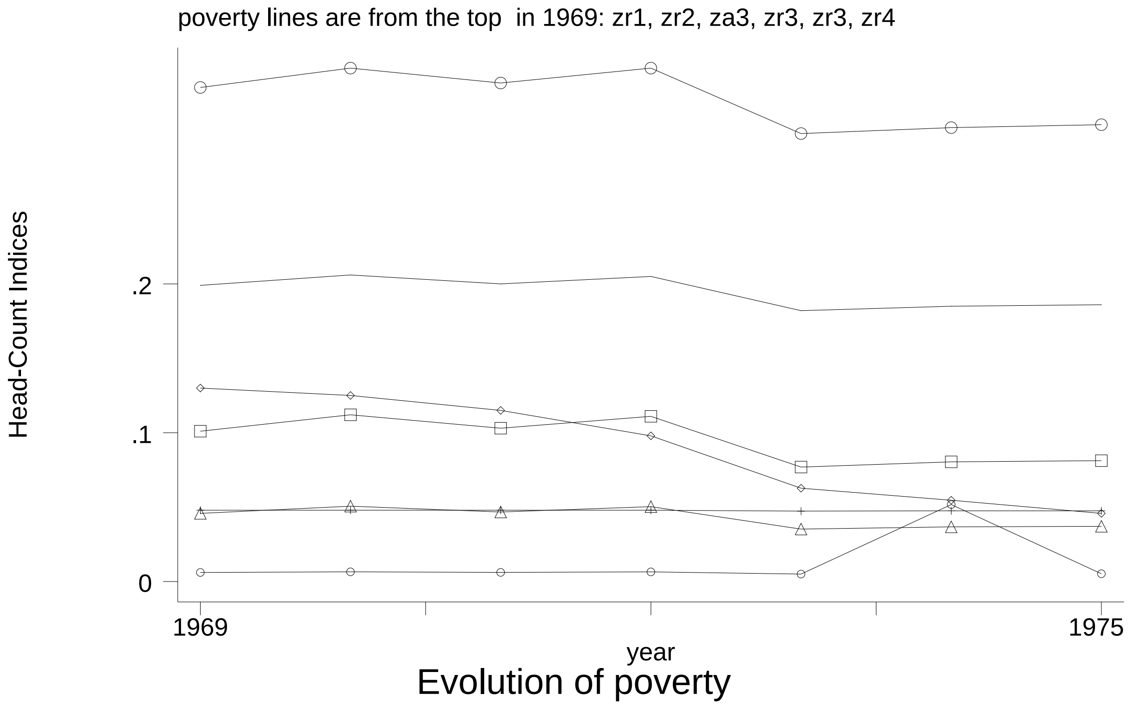


Fig. 1: Evolution of poverty


poverty lines are from the top in 1969: zr1, zr2, za3, zr3, zr3, zr4
Head-Count Indices
.2
.1
0
1969
1975
year
Evolution of poverty