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Privatisation in a Regulated Market, Open to Foreign Competition

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Abstract

The emerging literature on interaction between strategic trade theory and privatisation uses a simple example to argue that the irrelevance result is invalidated if the domestic market is open to foreign competition.

This paper uses a fairly general framework to show that anything a regulated privatized industry can achieve can always be mimicked by instructing the public firm to pursue an appropriate policy in a regulated mixed market structure. In the presence of a foreign competitor, this requires the public firm to follow the adjusted marginal cost pricing rule. But the public firm will not follow this rule if it is obliged to move with the private firms simultaneously. Hence, this study suggests that in the first best world open to foreign competition, we may concentrate on timing of the game as a source of inefficiency in a regulated mixed market structure rather than the ownership of the domestic firms.

JEL: D43, F14, L33
Keywords: Privatization, International Mixed Oligopoly, Strategic Trade policy.

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1 Introduction

In recent decades, many countries around the world have been involved in privatisation while at the same time they have used commercial policy to regulate the market. The effect of privatisation and commercial policy instruments on an imperfect market have been analyzed by the mixed oligopoly literature and strategic industrial and trade theory separately. Based on the mixed oligopoly literature, the presence of a publicly-owned welfare-maximizing enterprise as a direct regulatory device can improve welfare if the market is not competitive enough (De-Fraja and Delbono,1989). The existing literature on strategic industrial and trade theory asserts that in an imperfect market, when firms choose variables which are strategic substitutes (e.g., outputs), the government of the domestic country can improve welfare by using a subsidy to shrink the wedge between the marginal cost and price in the domestic market and shift the industry profit to the advantage of the domestic firms (Eaton and Grossman,1986). While these two regulatory devices can improve welfare separately in an imperfectly competitive market, the effect of a combination of these instruments is ambiguous.

The first attempt to explore the connection between privatisation and strategic industrial policy was made by White (1996). He used a regulated mixed oligopoly model for a closed economy with a linear inverse demand function and an identical quadratic cost function across the firms. He constructed a two stage game in which at the first stage the government uses an output subsidy. Then at the second stage, firms choose their outputs simultaneously. He found that privatisation does not change the optimal subsidy and welfare levels. The reason simply is that in equilibrium all firms adopt marginal cost pricing. Myles (2002) shows that the irrelevance result suggested by White (1996) holds for more general forms of demand and cost functions. Furthermore, it does not depend on the order of firms' moves.

While the irrelevance result for a regulated market survives several generalizations, Pal and White (1998) present an example to show how the introduction of a foreign firm to the model can lead to its violation. They extend the model of White (1996) to an international context and show that, if the domestic market is open to foreign competition and all firms move at the same time, privatisation always increases the domestic welfare and it decreases the level of optimal subsidy which is required to regulate the market. This result provides a strong argument in favor of privatisation because it claims that, even if the public firm is just as efficient as the private firms, welfare may still be enhanced by privatizing it.

In this study we extend the basic model of Pal and White (1998) from the linear-quadratic case to accommodate more general forms of demand and cost functions. In this fairly general framework, we argue that the welfare improvement of privatisation in a regulated market open to foreign competition is not a general result. Assuming that the public authority uses an output subsidy and the operation of the public sector as two alternative regulatory devices at the same time, we will show that anything a regulated privatized industry can achieve can always be mimicked by instructing the public firm to follow an appropriate policy in a regulated mixed market structure. Although an appropriate policy for the public firm in the presence of a foreign firm is not following the marginal cost pricing rule (Fjell and Pal,1996), we show that there still exists a pricing policy which can ensure the first best of the domestic economy. To allow the public firm to follow this pricing policy, the interaction of firms needs to be modelled by using a sequential game. Following this pricing policy, we may establish the irrelevance result even in the presence of a foreign competitor.

This study analyzes privatisation of a regulated market by using a mixed oligopoly model where a state-owned firm co-exists with a privately-owned firm in an imperfectly competitive market regulated by an optimal subsidy. When privatisation occurs it turns the mixed oligopoly into a standard oligopoly where only privately-owned firms compete. We consider three regimes. These regimes differ only in the way that the domestic firms are regulated. In all these regimes, the public authority uses the operation of the public sector and/or an optimal subsidy in advance of the private firms' moves. In the first regime, all domestic firms are publicly owned and a board of public managers regulates the home industry. In the second regime, private firms and a public firm are competing in the domestic market where the behaviour of the domestic private firms is regulated by an output subsidy. In the third regime, a regulated privatized industry regime, all firms in the home industry are profit-maximizing private firms and the government sets an optimal subsidy in advance of firms' moves. Our comparison shows that even in the presence of a foreign competitor we may still have an irrelevance result if the public sector considers the effect of its strategic choice on private firms.

This paper is organized as follows. First, section 2 introduces a general framework as a triopoly model. Then, in section 3 establishes the irrelevance result for a closed economy based on marginal cost pricing rule Section 4 extends the model to an international context, presents the adjusted marginal cost pricing rule which is associated with the first best of the economy and analyzes the equilibrium behaviour of firms in different regimes. Section 5 concludes.

2 The Basic Framework

Consider a country with a domestic market for a homogeneous good produced by 2 domestic firms and one foreign firm. If the home industry output is denoted by Q^h, then the total output is $Q = Q^h + q_f$ where q_f denotes the output of the foreign firm.

It is assumed throughout that:

- **Assumption 1)** *The inverse demand $p(Q)$ is a single-valued, twice continuously differentiable and log-concave function with $p'(Q) < 0$.*
- **Assumption 2)** *There exists $\bar{Q} < \infty$ where $p(\bar{Q}) = 0$ and $p(Q) > 0$ for all $Q \in [0, \bar{Q})$.*
- **Assumption 3)** *All firms have an identical, single-valued, twice continuously differentiable and strictly convex cost function with $c'(q) > 0$ and $c''(q) > 0$ for all $q \geq 0$ and $c(0) = 0$.*

The log-concavity of the inverse demand function is a weaker assumption than concavity. It asserts that $p(.)$ satisfies $p'(Q) + Qp''(Q) < 0$ for all $Q \in [0, \bar{Q})$. The strict convexity assumption imposed on the cost function is quite common in mixed oligopoly models. Otherwise, the public sector supplies the whole market alone, leaving no room for the coexistence of the public enterprise and the private firms.

We assume that there is a benevolent public authority in the domestic country which regulates the market by using the operation of the public firms and/or a subsidy s per unit of the production of the domestic private firm q_m. The subsidy is financed by a lump-sum transfer from consumers to the producers. It therefore generates no deadweight loss.

The domestic private firm's profit function is

$$\pi_m = p(Q)q_m - c(q_m) + sq_m \tag{1}$$

and the profit of the foreign firm is

$$\pi_f = p(Q)q_f - c(q_f) \tag{2}$$

We assume that the public firm's objective is to maximize social welfare as the unweighted sum of the consumer surplus and producers' profits considering the lump-sum transfer,

$$W = \pi_n + \pi_m + CS - sq_m \tag{3}$$

where $\pi_n = p(Q)q_n - c(q_n)$ is the public firm's profit. Firms choose their output levels as the strategic choice variables and are involved in a quantity-setting game. The set of the domestic firm's strategies is $S_h = [0, \bar{Q}]$ and the foreign firm's strategy set is $S_f \equiv [0, \bar{q}_f]$ where $\bar{q}_f$ is the monopoly output of the foreign firm.

We take the total number of the firms in the domestic industry and the foreign firm as exogenously given. A two stage game is applied to model the interaction of firms in different settings. Three regimes are considered. In all regimes, the public authority uses a subsidy to the domestic private firms' output and/or the operation of the public sector in advance of the private firms' moves. We seek the subgame Nash perfect equilibrium outputs of firms in pure strategies. These regimes are described as follows.

- A state-owned industry: there is a public monopoly in home industry. The public sector acts as a Stackelberg leader. In the first stage of the game, the board with full information about the behaviour of the foreign firm calculates its output as a function of the total output of the home industry $q_f(Q^h)$ and, constructs the domestic welfare function as a function of the home industry output $W(Q^h, q_f(Q^h))$. The board maximizes this function with respect to Q^h and commits itself to it. In the second stage of the game, given the optimal level of the home industry output, the foreign firm chooses a level of output which maximizes its own profit.

- A regulated mixed structure: in the first stage of the game, having the full knowledge about the outcome of the Cournot-Nash equilibrium of the private firms' competition in the second stage ($q_m(q_n, s)$, and $q_f(q_n, s)$) the board and government move simultaneously to maximize the domestic welfare function. While government maximizes domestic welfare $W(s;q_n)$ with respect to the level of subsidy for any given level of the public sector's output, the board is instructed to choose a level of the public sector's output which maximizes the domestic welfare function $W(q_n;s)$ for a given level of subsidy. Taking the announced level of the public sector's output and the subsidy as given, in the second stage of the game, private firms move simultaneously to maximize their own profits.

- A regulated privatized industry: in stage 1, the government, with full knowledge about the equilibrium in private firms' interactions, announces the level of subsidy and commits itself to it for the rest of the game. In stage 2, the private firms observe the announced subsidy and move simultaneously choose their output levels to maximize their profits.

In all cases, using backward induction, first we solve the models for the second stage equilibrium expressions. Since all objective functions are smooth (twice differentiable) and strictly concave in their own strategy variables, the equilibrium expressions are obtained from the solution of the system of the first order conditions of the profit maximization problems $\frac{\partial \pi}{\partial q} = 0$ for private firms operating in the second stage of the game. Then, given the outcomes of the last stage as a function of the subsidy and/or the output of the public sector, we look for the SPNE level of s and q_n.

3 The Case of a Closed Economy

3.1 The Ideal Setting

Consider first a closed economy with 2 domestic firms in the market where there is no foreign competitor. In an ideal setting or the first best, the output of the home industry is produced efficiently. For a given Q, efficiency in production requires solving the following problem,

$$\underset{q_1,q_2 \in S_h}{Min} \; c(q_1) + c(q_2) \quad s.t. \quad q_1 + q_2 = Q \tag{4}$$

where q_1 , q_2 are the levels of firms' outputs regardless of their ownership. From the first order condition of this minimization problem we have $c'(q_1) = c'(q_2)$. This implies that productive efficiency requires the total industry output to be distributed among the firms evenly, due to the identical cost structure, such that $q_1 = q_2 = Q/2$. Thus, there exists an aggregate cost function

$$C(Q) = Min\{c(q_1) + c(q_2)\} \tag{5}$$

along which the allocation of production of any output level among the domestic firms is cost minimizing. Hence the welfare maximization problem can be written as,

$$\underset{Q \in S_h}{Max} W = \int_0^Q p(t)dt - C(Q). \tag{6}$$

From the solution of this problem, it turns out that the ideal setting or optimum requires two conditions to be met:

$$i) \;\; p(Q^*) = C'(Q^*) \qquad (6.a)$$
$$ii) \;\; q_i^* = Q^*/2, i = 1, 2 \qquad (6.b)$$

where the first condition corresponds to the marginal cost pricing (MCP) rule and the second condition guarantees the productive efficiency condition. Note that in this model convexity of the cost function guarantees the existence of a unique equilibrium output Q^* which solves (6).

3.2 A State-Owned Industry

In the state-owned industry regime, the optimum can be achieved either in a centralized setting where the board solves both (4) and (5) or in a decentralized setting where the manager of the public firm 1 maximizes its contribution in welfare considering the output of other public firm constant. Thus, it solves

$$\underset{q_1 \geq S_h}{Max} W = \int_0^{\hat{q}_2} p(t)dt + \int_{\hat{q}_2}^{q_1+\hat{q}_2} p(t + \bar{q}_2)dt - \sum_{i=1}^{2} c(q_i). \tag{7}$$

In welfare function the first term is a constant. The solution of this setting yields

$$p(Q^*) = c'(q_1^*). \tag{8}$$

Since the firms are symmetric (in the sense that their cost functions are the same) and follow an identical pricing rule, they produce the same level of output. Thus, following MCP rule by each public firm is sufficient to achieve at optimum (Beato and Mas-Colell 1984).

3.3 A Regulated Privatized Industry

In a regulated privatized industry, the game consists of two stages. At stage 1, government chooses the level of subsidy to maximize welfare. Then at stage 2, firms choose their outputs simultaneously to maximize their profits under the Cournot assumptions. The log-concavity of demand function and the strict convexity of the firms' identical cost functions guarantee the existence of a unique equilibrium in the second stage of the game[1]. The government still can impose MCP rule on firms, if it implements the subsidy as a strategic industrial policy instrument optimally.

Lemma 1) *In a regulated closed economy, a subsidy to the private firm's production is optimal if and only if* $s^* = -[p'(Q^*)]q_m^*$.

Proof. The domestic private firm chooses q_m to maximizes (1) taking the level of subsidy and the output of other firm as given. From the first order condition

$$p(Q) + p'(Q)q_m - c'(q_m) + s = 0. \tag{9}$$

Adding the first order conditions and with identical cost structure at optimum where total production is distributed equally we have

$$\frac{p'(Q^*)Q^*}{2} + s^* = [p(Q^*) - c'(q_m^*)]. \tag{10}$$

But the right hand side of this equation is equal to zero at optimum. Hence,

$$s^* = -[p'(Q^*)]q_m^*. \tag{11}$$

If the government implements s^*, then for both firms we have

$$p(\hat{Q}) - c'(\hat{q}_m) = 0 \tag{12}$$

[1]In fact, identical cost structure and convex technology together guarantee existence of Cournot equilibrium. Also without requiring identical cost or convex technology the log-concavity of inverse demand is sufficient to guarantee the existence of the equilibrium(Novshek, 1985).

where $\hat{q} = \frac{\hat{Q}}{2}$. Since welfare function is strictly concave and the marginal cost pricing leads to a unique optimum, thus $\hat{Q}$ is equal to Q^*. ■

This analysis has the following implications regarding the general value of the optimal subsidy(Eq.11). First, as the literature on strategic industrial policy asserts, the optimal subsidy in an imperfect competitive market is always positive because $p' < 0$. Additionally, since the inverse demand function is monotone and decreasing everywhere, the optimal output is unique. The optimal subsidy is unique too. Third, the optimal level of subsidy is decreasing in the (exogenously fixed) number of existing firms. Fourth, if we relax the identical cost structure assumption, the government would be required to implement different level of subsidies across the firms to regulate the market.

3.4 A Mixed Market Structure

In a regulated mixed structure regime, both regulatory devices are used at the same time. At the first stage, the government sets the level of optimal subsidy while the public firm chooses the level of output to maximize welfare, taking the reaction of the private firm into account. At the second stage of the game, the private firm chooses its output for any given level of subsidy and the output of the public firm. If in this regime again the MCP condition is met, we may claim the following irrelevance result in general for a closed economy.

Proposition 1) *The equilibrium levels of the optimal subsidy and welfare are identical for a closed economy irrespective of whether i)the firms in the industry are publicly owned and maximize welfare or ii) the industry is privatized and regulated by an optimal subsidy or iii) the public firm co-exists with a private firm in the industry where the government uses a subsidy to the private firm's output optimally.*

Proof. We have shown that in a regulated privatized industry and a state owned industry the MCP condition is met and the firms produce the same level of outputs. Now, we need to show that the MCP condition holds in equilibrium in the regulated mixed structure regime as well. The best response of the private firm from the last stage is $q_m = q_m(s, q_n)$. Since $Q = q_n + q_m(s, q_n)$, using the chain rule in anti-differentiation the objective of the government in the first stage of the game can be rewritten as

$$\int_0^s p(Q(t;q_n))(\partial q_m/\partial s)\partial t - c(q_m(s;q_n)) - c(q_n). \tag{13}$$

Taking q_n as given, s^* is optimal if $\frac{dW}{ds} = \frac{dq_m}{ds}[p(Q(s^*)) - c'(q_m(s^*))] = 0$. But $\frac{dq_m}{ds} > 0$. Therefore, setting s^* such that it follows MCP at equilibrium solves the government's problem. The public firm maximizes (3) at the first stage of the game. From Appendix A, q^* solves the public firm problem if

$$[p(Q(q_n^*) - c'(q_n^*)] + \frac{dq_m}{dq_n}[p(Q) - c'(q_m)] = 0. \tag{14}$$

Since the private firm is following MCP at equilibrium, it implies that the public firm also should follow MCP. Following the same pricing rule at equilibrium, they produce the same level of output. Hence, the conditions (6.a) and (6.b) are met. ■

Note that for a given number of firms, the level of the optimal output under all the regimes is the same. From the general expression for the optimal subsidy Eq.(11) we may conclude that s^* in a regulated mixed structure and in the regulated privatized market are the same. Following Myles(2002), we have shown that the irrelevance result found by White(1996) in a simple example can be extended to a more general framework. Section 4 extends the irrelevance result even further to the case of an open economy in the presence of a foreign competitor.

4 Privatisation in a Market Open to Foreign Competition

In this section we suppose that the domestic market is open to foreign competition. In the presence of a foreign competitor, as Fjell and Pal(1996) have noticed, the public sector does not follow the MCP rule. The domestic economy produces Q^h, but it consumes $Q = Q^h + q_f$ which partly is provided from abroad via imports. Since a foreign firm joins a regulated market and the regulator has the first mover advantage, the choice of a combination of the home industry output and the foreign firm's level of output (Q^h, q_f) can capture all possible combination of outputs along the best response of the foreign firm. We assume,

- **Assumption 4)** *There exists a unique* $(Q^{h*}, q_f^*) \in V \equiv \{(Q^h, q_f) \in R^2, Q^h \in S_h, q_f \in q_f(Q^h)\}$ *which maximizes the welfare function of the domestic economy.*

The point (Q^{h*}, q_f^*) indeed characterizes the optimum of the domestic economy[2]. The slope of the foreign firm's best response under assumptions (1-3) reveals some important properties.

Lemma 2) *The best response function of the foreign firm is decreasing and belongs to the interval (-1,0).*

Proof. Adopting the assumptions (1-3) and using the rule of implicit differentiation,

$$\frac{dq_f}{dQ^h} = -\frac{p''(Q)q_f + p'(Q)}{p''(Q)q_f + 2p'(Q) - c''(q_f)}. \tag{15}$$

Because of the log-concavity of demand function $p''(Q)q_f < -p'(Q)$ and $p''(Q)q_f < -2p'(Q)$. Hence the nominator and the denominator are both negative and the latter is greater in absolute value. Thus $\frac{dq_f}{dQ^h} \in (0, -1)$. ■

It can be checked that, indeed the best response of the any profit-maximizing firm under the assumptions (1-3) shares these properties.

4.1 The Ideal Setting

In an ideal setting, the economy produces up to a point where the benefit of an additional unit of output is equal to its social cost provided that it is produced efficiently. Assuming that in equilibrium the problem (4) is solved for the home industry, there exists an aggregate cost function for the home industry $C(Q^h)$ along which the allocation of production of any output level among the domestic firms is cost minimizing and the production is distributed evenly. Considering the effect of the domestic production on imports where the foreign firm acts as a follower, and using the chain rule in anti-differentiation, the domestic welfare can be rewritten as

$$W(Q^h, q_f) = \int_0^{Q^h} p(t)[1 + \frac{dq_f}{dQ^h}]dt - p(Q)q_f - C(Q^h). \tag{16}$$

[2]Although we could have established the existence results by imposing some other restrictions on the curvatures of the functions, we took it as given in the first place because this study deals with the comparison of the strategies of the regulator, rather than the existence problem.

The marginal benefit MB and the marginal cost MC of an additional unit of Q^h are as follows,

$$\begin{aligned} MB &= p(Q) + p(Q)\frac{dq_f}{dQ^h} \\ MC &= C'(Q^h) + p'(Q)[1 + \frac{dq_f}{dQ^h}]q_f + p(Q)\frac{dq_f}{dQ^h}. \end{aligned}$$

Thus, the benefit of the production of an additional unit of Q^h is not equal to its price, nor does the additional cost accruing from the last unit of Q^h equal exactly what has been spent on it in the home industry. Solving the first order condition of maximization problem (16) for the price yields the first order condition of an optimal allocation for the domestic economy,

$$p(Q^*) = C'(Q^{h*}) + p'(Q^*)[1 + \frac{dq_f}{dQ^h}]q^*_{f.} \tag{16.a}$$

The second condition of an optimal allocation refers to productive efficiency. This that requires that

$$q^{h*} = Q^{h*}/2 \tag{16.b}$$

where q^{h*} denotes the equilibrium output level of any domestic firm regardless of its ownership. Now, we can introduce a new pricing rule as a pricing rule that can lead the domestic economy to its optimum in the presence of a foreign firm.

Definition 1 *The* ***adjusted marginal cost pricing (AMCP)*** *rule is said to be a pricing rule which requires the domestic firms to produce at the point where the sum of the marginal reduction of the value of imports, due to change in price(caused by an additional unit of the domestic firm's production), and its marginal cost is equal to the market price.*

The intuition behind this new rule is simple. In an open market, the choice of an optimal level of the domestic production cannot be independent of the effect of domestic production on the import's price. Since the good is provided by the domestic firms and the foreign firm, the imports are in fact some part of the costs of providing it. The new rule includes an adjustment to capture the effect of the domestic output on costs of the provision of the good in total.

4.2 A State-Owned Industry

In the state-owned industry, a two stage game is applied. In the first stage, the board maximizes (16) with respect to Q^h considering the best response of the foreign firm in the second stage. The public sector leadership solution is (Q^{h*}, q_f^*) such that

$$\underset{(Q^h,q_f)\in S_h\times S_f}{W(Q^{h*}, q_f^*)} \geq W(Q^h, q_f^*) \text{ for all } Q^h \in S_h \text{subject to } q_f \in \arg\ \text{Max}\ \pi_f(Q^h, q_f)\ . \tag{17}$$

Assumption(4) ensures that there exists a unique point which is the tangency between the iso-welfare contours, the loci of output combinations of the home industry output and the foreign firm's output (Q^h, q_f) corresponding to the maximum welfare level for the domestic country. From the first order condition of the public sector, it is clear by definition that in the presence of a foreign competitor, where the public sector sets its output in advance of the foreign firm, it always follows AMCP rule. Since the cost function is identical, the board also needs to minimize the cost of providing Q^h by distributing the production in the home industry equally thus $C'(Q^{h*}) = c'(q_n^*)$.

In a decentralized setting, public firm 1 simply maximizes its contribution in welfare taking the level of output of other firm and the best response of the foreign firm as given. Hence we have,

$$\begin{aligned} \underset{q_1\in S_h}{Max}\ W &= \int_0^{\bar{q}_2} p(t)(1+\frac{dq_f}{dq_1}) - R_f'(t)]dt + \int_{\hat{q}_2}^{q_1+\hat{q}_2} p(t)(1+\frac{dq_f}{dq_1}) \\ &\quad -R_f'(t)]dt - \sum_{i=1}^{2} c(q_i) \end{aligned} \tag{18}$$

where $R_f' = \frac{\partial R}{\partial q_n}$ for $n = 1, 2$ and R denotes the revenue of the foreign firm or simply the level imports. The first term of the welfare function is constant. If both public firms maximize their contributions in welfare by maximizing (18), the equilibrium behaviour of each public firm can be characterized by

$$p(Q^*) = c'(q_n^*) + p'(Q^*)[1 + \frac{dq_f}{dq_n}]q_{f.}^* \tag{19}$$

that means they follow AMCP rule and therefore, they produce the same level of output.

Following AMCP ensures that the marginal cost of the public firm in equilibrium always exceeds the market price because $\frac{\partial q_f}{\partial Q^h} = \frac{dq_f}{dq_n} \in (-1, 0)$. This implies that, even at optimum, the operation of the public firm results in a loss which cannot be attributed to mismanagement of the public firm. The budget deficit of the public firm is a well-known problem in the presence of a fixed cost. But this result is particularly surprising because in our model there is no fixed cost and still the public firm should operate at a loss at the optimum.

4.3 A Regulated Privatized Industry

In a regulated privatized industry regime, a two stage-game is applied. At the stage 1, the government uses subsidies to regulate the market. Otherwise, the home industry produces an aggregate level of production Z which is less than the optimal level of output Q^*. At stage 2, taking the announced subsidy as given, firms choose their outputs simultaneously to maximize profits. Recall that the best response of each domestic private firm is monotone and decreasing in the output levels of its rivals and the slope of the best response functions of the domestic private firms also are limited to the interval $(-1, 0)$ for the same reason as stated in Lemma (2). Hence, the best response functions satisfy both Hahn's condition and Seade's stability conditions[3]. These ensure that with changes in subsidies the output of the home industry increases and imports falls such that it captures all possible combinations of the home industry outputs and the foreign firm's outputs which are along the best response function of the foreign firm and $Q^h(s) \in [Z, \bar{Q})$ as s changes in the interval $[0, \bar{s})$ where $\bar{s}$ is a level of subsidy of which $Q^h(\bar{s}) = \bar{Q}$. Now we claim that

Proposition 2) *In the presence of a foreign firm, if the government uses subsidies optimally, the equilibrium behaviour of the domestic private firms is such that the conditions of adjusted marginal cost pricing rule are met.*

Proof. The domestic private firm chooses q_m to maximizes (1) taking the level of subsidy and the outputs of other firms as given. From the first

[3]The Hahn condition requires a decreasing marginal revenue of each firm in its rival's output or simply the downward-sloping best response functions. Based on Seade's conditions the absolute value of the slope of the best response functions should be less than 1 then Cournot equilibrium of a homogeneous good is stable. For a strictly convex cost function with linear demand these conditions are always met(See Dixit,1986).

order condition

$$p'(Q)q_m + p(Q) + s - c'(q_m) = 0. \tag{20}$$

Assume the government sets the subsidy optimally. Summing the first order conditions of two identical domestic firms,

$$p(Q^*) + \frac{p'(Q^*)Q^{h*}}{2} + s^* = c'(q_m^*). \tag{21}$$

If we add and subtract $p'(Q^*)[1 + \frac{dq_f}{dQ^h}]q_f^*$ to both sides in Eq.(21), then we have

$$p(Q^*) + \{\frac{p'(Q^*)Q^{h*}}{2} + p'(Q^*)[1 + \frac{dq_f}{dQ^h}]q_f^* + s^*\} = c'(q_m^*) + p'(Q^*)[1 + \frac{dq_f}{dQ^h}]q_f^*. \tag{22}$$

Comparing Eq.(22) with the AMCP rule (16.a) shows that the terms in bracket must equal zero if we are at the optimum. Therefore, the optimal subsidy is

$$s^* = -p'(Q^*)[\frac{Q^{h*}}{2} + (1 + \frac{dq_f}{dQ^h})q_f^*]. \tag{23}$$

Since $\frac{dq_f}{dQ^h} \in (-1.0)$, $s^* > 0$. Also $s^* < \bar{s}$ because $Q^h(s) < \bar{Q}$ and it is monotone and increasing in s, hence $s^* \in [0, \bar{s})$. The uniqueness of the optimum from assumption (4) ensures that s^* in Eq.(23) is unique. Once it is implemented, the conditions of AMCP are met for each domestic firm in equilibrium. ■

At the optimum the output levels of firms in the home industry are the same and $q_m^* = \frac{Q^{h*}}{2}$ because of the identical cost structure. Taking the total number of the domestic firms as given, we can argue that s^* does not depend on how many of the domestic firms are private or publicly owned. To show this, suppose one of the firms in the home industry is changed to a welfare-maximizing firm due to the public acquisition. Based on the general term of the optimal subsidy, the public acquisition of the domestic firm would not change the optimal subsidy, if that firm still follows AMCP rule. In fact, there is a good reason to believe that in the mixed structure regime the public firm always follows AMCP rule.

4.4 A Mixed Market Structure

In a regulated mixed structure, the domestic industry consists of one public firm, one domestic private firm and one foreign firm. In the first stage of a two-stage game, the public firm chooses its output level to maximize welfare while the government chooses the level of subsidy to induce the welfare-maximizing behaviour of the domestic private firm. In the second stage the private firms, taking the level of subsidy and the output level of the public firm as given, choose outputs simultaneously to maximize their own profits. If the domestic firms in this setting also follow the AMCP rule in equilibrium, then the irrelevance result can be proved in general.

Proposition 3) *In the presence of a foreign competitor in the domestic market, if the operation of the public sector and/or a subsidy to home production are used completely and optimally to regulate the market, and both are implemented in advance of the private firms' actions, privatisation of one or all firms in the home industry will not change the optimal subsidy and welfare for the domestic country.*

Proof. In proposition (2) and section (4.2) we have shown that, in two extreme cases, domestic firms follow AMCP rule in the regulated privatized industry and in the publicly owned industry. To prove this proposition we need to show that in a regulated mixed market structure also firms follow AMCP. Maximizing (1) and (2) by the private firms in the second stage of the game yields the equilibrium conditions

$$\frac{\partial \pi_f}{\partial q_f} = p'(Q)q_f^* + p(Q) - c'(q_f^*) = 0 \qquad (24.\text{a})$$

$$\frac{\partial \pi_m}{\partial q_m} = p'(Q)q_m^* + p(Q) - c'(q_m^*) + s^* = 0. \qquad (24.\text{b})$$

From Appendix B the equilibrium conditions of the first stage requires

$$\frac{dW}{dq_n} = \frac{\partial W}{\partial q_n} + \frac{\partial W}{\partial q_f}\frac{\partial q_f}{\partial q_n} = p(Q^*) - c'(q_n^*) - p'(Q^*)[1 + \frac{dq_f}{dq_n}]q_f^* = 0 \qquad (24.\text{c})$$

and $\frac{dW}{ds} = 0$ implies that

$$\frac{\partial W}{\partial q_m} + \frac{\partial W}{\partial q_f}\frac{\partial q_f}{\partial q_m} = p(Q^*) - c'(q_m^*) - p'(Q^*)[1 + \frac{dq_f}{dq_m}]q_f^* = 0. \qquad (24.\text{d})$$

that means both domestic firms follow AMCP rule and in total they produce $Q^{h*} = q_n^* + q_m^*$. Since the public firm and the domestic private firm produce the same level of output which is associated with AMCP rule, the value of the optimal subsidy that guarantees conditions (24.b) and (24.d) is equal to Eq.(23). The uniqueness of optimum ensures that the level of optimal subsidy and welfare are the same for a regulated privatized industry and a regulated mixed industry regime. ■

If the irrelevance result holds even in the presence of a foreign firm in a regulated market by optimal subsidy, privatisation in such a market cannot improve welfare. The possibility of welfare improvement of privatisation crucially depends on how the interaction of firms in the market is modelled. For instance if following Pal and White(1998) we assume that the public firm moves simultaneously with the private firms, then

Lemma 3) *The equilibrium output of the public firm in a regulated mixed structure industry is higher if the public firm moves simultaneously with private firms than when it acts as a Stackelberg leader.*

Proof. Suppose in a regulated mixed market regime, that public firm decides to set its output level at the second stage and moves simultaneously with other firms. Then it maximizes (3) taking q_f , q_m and s as given. From the solution of the first order conditions of the firms' problems at the second stage of the game, we have

$$\frac{\partial \pi_f}{\partial q_f} = p'(Q^0)q_f^0 + p(Q^0) - c'(q_f^0) = 0 \tag{25.a}$$

$$\frac{\partial \pi_m}{\partial q_m} = p'(Q^0)q_m^0 + p(Q^0) - c'(q_m^0) + s^0 = 0 \tag{25.b}$$

$$\frac{\partial W}{\partial q_n} = p(Q^0) - c'(q_n^0) - p'(Q^0)q_f^0 = 0. \tag{25.c}$$

The objective of the government can be expressed in terms of the subsidy as $W(q_n(s), q_m(s), q_f(s))$. Hence, the equilibrium condition of the government behaviour in the first stage is

$$\frac{dW}{ds} = \frac{\partial W}{\partial q_m} + \frac{\partial W}{\partial q_f}\frac{\partial q_f}{\partial q_m} + \frac{\partial W}{\partial q_n}\frac{\partial q_n}{\partial q_m} = 0. \tag{25.d}$$

But from Eq.(25.c) $\frac{\partial W}{\partial q_n} = 0$. This shows that Eq.(25.d) is equal to Eq.(24.d) and the only difference in equilibrium conditions refers to the equilibrium condition of the public firm's behaviour. A comparison between Eq.(25.c) and Eq.(24.c) indicates that $q_n^0 > q_n^*$, because $\frac{dq_f}{dq_n} \in (-1, 0)$ and $p'(Q) < 0$. Thus the public firm produces more when it moves simultaneously. ■

When the public firm moves simultaneously with private firms, it does not follow AMCP rule. This violates the optimum condition. When the public sector acts as a Stackelberg leader, it can choose the output level of Cournot game from its strategy space. Since Cournot outcome differs from the public sector leadership outcome(Lemma 3), the uniqueness of the optimum implies that from the board's points of view, playing a public sector leadership game is preferable to moving simultaneously with other firms in the Cournot game[4].

5 Conclusion

This paper investigates the effect of change in the ownership of firms in a regulated domestic market open to foreign competition through different scenarios. While Pal and White(1998) argue that in an international mixed oligopoly the domestic country always is better off by privatizing the welfare-maximizing public firm even if it is just as efficient as the private firms, a comparison of the results under different regimes shows that if public authority uses the operation of the public sector or an optimal subsidy in advance of private firms' moves, the ownership of the domestic firms is irrelevant. The reason is that, in this setting, all the domestic firms always follow the adjusted marginal cost pricing(AMCP) in equilibrium. Therefore, anything a regulated privatized industry can achieve can always be obtained by a regulated mixed market structure.

In a closed economy, using the optimal subsidy in a privatized industry leads to MCP condition, which corresponds to the equilibrium behaviour of the public firm. Hence in this setting the ownership of firms does not matter. But if the domestic market is open to foreign competition, insisting

[4] I would like to emphasize that we do not claim the public firm leadership is always preferable. We are aware that, even in a closed economy, it might be preferable for the public firm to act as a follower(see Beato and Mas-colell(1984)). Also in the presence of a foreign firm, an example can be constructed in which the domestic country will be better-off, if the publicly owned industry acts as a follower. The main lesson of the present analysis is the dependence of the outcome on the timing structure of the game.

on the MCP rule or the simultaneous order of moves prevent the board of the public sector from the appropriate choice of output. In this case, following the AMCP rule is the best policy for the public firm to follow. The choice of output associated with AMCP is available for the public firm if we model the game by considering a sequential order of moves with the public firm as a Stackelberg leader. Under public sector leadership all regimes lead to the AMCP condition and again the irrelevance result can be established.

To summarise, this study suggests that in the first best world, we may concentrate on timing of the game as a source of inefficiency in a regulated mixed market structure rather than the ownership of the domestic firms in the home industry. Since the timing of the game has such a crucial impact on the results, it is desirable to provide a rationale for any assumption regarding the timing of the game. To address the question of choosing an appropriate assumption about the order of firms' moves, further research in this area can extend the game to a preplay stage, wherein firms can choose the time of action rather than acting in an ordered time.

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Appendix A.

The objective of the public firm in the first stage is to maximize the following function

$$\int_0^{\hat{q}_n} p(Q(t;s))(1+\frac{dq_m}{dq_n})dt - c(q_m(s;q_n)) - c(q_n) \tag{A.1}$$

with respect to q_n. From the first order condition

$$p(Q(q^*))(1+\frac{dq_m}{dq_n}) - c'(q_m)\frac{dq_m}{dq_n} - c'(q_n) = 0 \tag{A.2}$$

or

$$[p(Q(q_n^*)) - c'(q_n^*)] + \frac{dq_m}{dq_n}[p(Q) - c'(q_m)] = 0. \tag{A.3}$$

Appendix B.

In the first stage, the government maximizes $W(q_n, q_m(s, q_n), q_f(s, q_n))$ with respect to the subsidy. A subsidy to home production affects q_f through a change in the domestic private firm's output. Hence, using the chain rule, $\frac{dq_f}{ds} = \frac{\partial q_f}{\partial q_m}.\frac{\partial q_m}{\partial s}$. From the first order condition of the government problem

$$\frac{dW}{ds} = \frac{\partial W}{\partial q_m}\frac{\partial q_m}{\partial s} + \frac{\partial W}{\partial q_f}\frac{\partial q_f}{\partial q_m}.\frac{\partial q_m}{\partial s} = 0. \tag{B.1}$$

Since $\frac{\partial q_m}{\partial s} > 0$, hence

$$[\frac{\partial W}{\partial q_m} + \frac{\partial W}{\partial q_f}\frac{\partial q_f}{\partial q_m}] = 0. \tag{B.2}$$

The public firm are also instructed to maximize welfare with respect to q_n. From the first order condition,

$$\frac{dW}{dq_n} = \frac{\partial W}{\partial q_n} + \frac{\partial W}{\partial q_m}\frac{\partial q_m}{\partial q_n} + \frac{\partial W}{\partial q_f}\frac{\partial q_f}{\partial q_m}\frac{\partial q_m}{\partial q_n} + \frac{\partial W}{\partial q_f}\frac{\partial q_f}{\partial q_n} = 0 \tag{B.3}$$

or,

$$\frac{dW}{dq_n} = \frac{\partial W}{\partial q_n} + \frac{\partial q_m}{\partial q_n}[\frac{\partial W}{\partial q_m} + \frac{\partial W}{\partial q_f}\frac{\partial q_f}{\partial q_m}] + \frac{\partial W}{\partial q_f}\frac{\partial q_f}{\partial q_n} = 0. \tag{B.4}$$

If the optimal subsidy is used the public firms' equilibrium behaviour can be represented by

$$\frac{dW}{dq_n} = \frac{\partial W}{\partial q_n} + \frac{\partial W}{\partial q_f}\frac{\partial q_f}{\partial q_n} = 0. \tag{B.5}$$

Therefore, Eq.(B.2) and Eq.(B.5) together characterize the equilibrium condition in the first stage of the game.