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Abstract: This paper compares Bertrand and Cournot equilibria in a horizontally differentiated duopoly market with non-tournament R&D competition. We consider that success in R&D is uncertain. We show that whether firms invest more under Cournot competition or Bertrand competition is ambiguous and depends on the probability of success in R&D but does not depend on the degree of product differentiation. While 'static' welfare is higher under Bertrand competition, we find that 'dynamic' welfare may be higher under Cournot competition for moderate R&D productivities. Further, we show that whether the difference between the expected welfare under Cournot and Bertrand competitions increases with higher product differentiation is ambiguous and depends on the R&D productivity.

Key Words: Bertrand, Cournot, Uncertain R&D, Welfare

JEL Classification: D34, L13, O33

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Bertrand and Cournot competitions in a dynamic game

1 Introduction

There is an existing debate on the relationship between R&D investment and the intensity of competition in the product market. In a seminal work Schumpeter (1943) argued that market concentration is a stimulus to the innovation. So, a society might prefer to sacrifice 'static' efficiency for the 'dynamic' efficiency. The work by Arrow (1962) challenged this view. In his work Arrow (1962) argued that the incentive for innovation would be stronger for competitive industry than a monopolist.

Contrary to the earlier literature focusing on the competitive market and monopoly, the recent contributions examine the incentive for R&D under different types of competition in oligopolistic markets. More specifically, researchers examine whether firms invest more in R&D under Bertrand competition or Cournot competition (see, e.g., Delbono and Denicolo, 1990 and Qiu, 1997). The purpose of the present paper is to re-examine the relationship between the type of product market competition in the oligopolistic market and the incentive for R&D and its implication on welfare. In particular, we consider a two-stage duopoly model of R&D and production where the firms produce horizontally differentiated products and success in R&D is uncertain, which depends on the R&D investment. Employing a model of non-tournament R&D competition, we show that whether R&D investment and welfare are higher in a market with more competitive behavior depends on the R&D productivity.

We find that R&D investments are more (less) under Cournot competition compared to Bertrand competition if the equilibrium probabilities are greater (lower) than a critical level. While 'static' welfare is always higher under Bertrand competition, 'dynamic' welfare may be more under Cournot competition for R&D productivities.[1]

The present paper is closely related to Delbono and Denicolo (1990), Bester and Petrakis (1993), Qiu (1997) and Bonanno and Haworth (1998).[2] In a model of symmetric homogeneous oligopoly, Delbono and Denicolo (1990) argue that R&D investment is

[1] One may refer to Delbono and Denicolo (1990) for the meaning of 'static' and 'dynamic' welfare. By 'static' welfare we mean welfare ex-post R&D and by 'dynamic' welfare we mean the expected welfare ex-ante R&D.

[2] One may refer to Singh and Vives (1984), Vives (1985) and Cheng (1985) for works on Bertrand and Cournot competition in a static framework, i.e., where the firms face same demand and cost structure in both types of product market competition.

greater under Bertrand competition. But, with a horizontally differentiated product market Qiu (1997) argues that firms invest more in R&D under Cournot competition. However, unlike these papers, our analysis shows the possibility of higher R&D investment under Bertrand competition as well as under Cournot competition depending on the equilibrium probability of success. In contrast to Delbnono and Denicolo (1990) and Qiu (1997), our results also differ significantly for the analysis on welfare under Bertrand and Cournot competitions. For example, while Delbono and Deniciolo (1990) need sufficiently large numbers of firms for higher welfare under Cournot competition, we show that this is possible even in a duopoly market structure. Further, while the existence of knowledge spillover is necessary for Qiu (1997) to prove the possibility of higher welfare under Cournot competition, the present paper shows the possibility of higher welfare under Cournot competition in absence of knowledge spillover.

Though we consider a two-stage game with R&D and production like Delbono and Denicolo (1990) and Qiu (1997), our conclusions are significantly different from these two papers. Hence, at this point it is worth mentioning the main differences between the present paper and those papers. While Qiu (1997) considered a model of R&D where cost of production reduces with higher amount of R&D investment and success in R&D is certain, we consider a different type of R&D competition where success in R&D is uncertain and increases with R&D investments but the cost reduction from R&D is given. Contrary to the model of R&D race in Delbono and Denicolo (1990), we consider a model of non-tournament R&D competition. So, in our framework it is possible for both firms to have newly invented technology. Thus, our model of R&D competition is similar to the one considered in Choi (1993). Further, unlike Delbono and Denicolo (1990), we consider a model of product differentiation where homogeneous product is a special case of our framework.

Bester and Petrakis (1993), Bonanno and Haworth (1998) and Symeonidis (2003) also deal with similar questions addressed in the present paper. However, contrary to the present paper, Bonanno and Haworth (1998) and Symeonidis (2003) consider a model of vertical product differentiation and argue that the incentive for R&D is higher under Cournot competition. Though in their paper, Bester and Petrakis (1993) consider a model with horizontal product differentiation, they consider R&D investment by only one firm and the success in R&D is certain. They found that the incentive for R&D investment would depend on the degree of product differentiation. It will be evident from our analysis that even if only one firm does R&D in our framework, equilibrium R&D investment is higher under Bertrand competition.

In what follows, in section 2 we consider a general framework for a dynamic game of R&D competition and production. We assume that each firm's success in R&D is uncertain, which depends on the R&D investment of that firm. In our analysis we consider symmetric probability function for these firms and focus on drastic innovation only, i.e., if only one firm is successful in R&D then only the successful firm produces in the market.

Section 3 compares the R&D investments under Bertrand and Cournot competitions and shows that whether R&D investment is more under Cournot or Bertrand competition depends on the probability of success. Suppose the equilibrium probability of success is sufficiently low. Then, given the low probability of success of the competitor, it increases a firm's chance of becoming a single innovator. Since the gain from becoming a single innovator is lower under Cournot competition, this possibility encourages a firm to reduce its optimal R&D investment under Cournot competition compared to Bertrand competition, given the equilibrium R&D investment of the competitor firm. The symmetry of the firms implies that equilibrium R&D investments of these firms are lower under Cournot competition compared to Bertrand competition.

Now, consider the situation where the equilibrium probabilities of success are sufficiently high. Given the high probability of success of the competitor, it reduces the chance of becoming the single innovator and a duopoly market structure is the more likely outcome. Since duopoly profit is higher under Cournot competition compared to Bertrand competition, here the firms invest more under Cournot competition compared to Bertrand competition.

It is well known that lower prices under Bertrand competition helps to create higher 'static' welfare under Bertrand competition compared to Cournot competition. However, our analysis shows that R&D investments are more under Cournot competition when the equilibrium probability of success exceeds a critical level. Therefore, in this situation, the probability of getting the innovation is higher under Cournot competition compared to Bertrand competition. As a result, 'dynamic' welfare may be more under Cournot competition if the equilibrium probability of success is greater than a critical value. We show that if the R&D productivity is not sufficiently high then 'dynamic' welfare may be more under Cournot competition compared to Bertrand competition.

As the R&D productivity increases, it increases the R&D investment, which also increases the probability of success in R&D. If the R&D productivity is sufficiently large so that the equilibrium probability of success is greater than a critical value then we find that the probability of getting the innovation is higher under Cournot competition. It creates the possibility of higher 'dynamic' welfare under Cournot competition. But, if the R&D productivity is very high so that the difference between the R&D investments under Cournot and Bertrand competitions is significantly large then this higher amount of R&D investment under Cournot competition creates significantly large negative effect on welfare under Cournot competition. Hence, for very high R&D productivity, welfare is higher under Bertrand competition. Thus, for very low R&D productivity (where R&D investments are higher under Bertrand competition) and for very high R&D productivity, 'dynamic' welfare is higher under Bertrand competition. But, for intermediate R&D productivities, 'dynamic' welfare may be more under Cournot competition.

With an example we show that whether the difference between the expected welfare under Cournot and Bertrand competitions increases with higher product differentiation is ambiguous and depends on the R&D productivity. As the products are getting differentiated, it affects the welfare by changing the outputs of the firms and also by changing the R&D investments of these firms. Hence, the result depends on how these two effects are changing the welfare under Bertrand and Cournot competitions. The example shows that higher product differentiation increases (reduces) the difference between the expected welfare under Cournot and Bertrand competitions for sufficiently low (high) R&D productivity.

Rest of the paper is organized as follows. The next section gives a general framework for our analysis. In section 3, we compare the equilibrium R&D investments of these firms under Bertrand and Cournot competitions. Section 4 examines the welfare implications under these two types of market competitions. Section 5 concludes the paper.

2 A general framework

Consider an economy with two firms, called 1 and 2, producing horizontally differentiated products. Assume that the firms have similar technologies at the beginning and each of them faces constant average cost of production $\bar{c}$. Each firm can invest in R&D and can improve its technology. We assume that each of them tries to invent a technology corresponding to constant average cost of production c.[3] However, the R&D process is uncertain and firm i, $i=1,2$, can succeed with an unconditional probability p_i where the probability of success depends on the ith firm's R&D investment x_i. We consider that $p_i'(x_i)>0$, $p_i''(x_i)<0$, $p_i'(0)=\infty$ and $p_i'(\infty)=0$ for $i=1,2$. Since our purpose is to focus on the effects of product market competition, we assume that both firms face same probability function so that the results are not influenced by the asymmetry in probability functions. Hence, we do our analysis under the assumption that $p_i(x)=p_j(x)=p(x)$. Therefore, our model of R&D competition is similar to Choi (1993). Further, for simplicity, we assume that there are no costs of doing R&D except R&D investment.

We consider a two-stage game. In stage 1, both firms simultaneously invest in R&D. In stage 2, these firms compete in the product market and simultaneously take their decision in the product market. In the following analysis we will consider two types of product market competition, viz., Bertrand competition and Cournot competition.

For simplicity, in our analysis we will assume that the new technology is drastic in nature, i.e., only the successful firm will serve the product market in case of unilateral success in R&D. It is important to note that we need relatively higher amount of cost reduction from R&D to create drastic R&D as the degree of product differentiation increases. Further, the amount of cost reduction that creates drastic R&D also depends on the type of product market competition (see, e.g., Bester and Petrakis, 1993). Hence, a certain amount of cost reduction could imply drastic R&D for a particular degree of product differentiation or for a particular type of product market competition but might not imply drastic R&D for different degree of product differentiation or for different type of product market competition. However, as a simplification we assume that the cost reduction from R&D is sufficiently large to create drastic R&D for all degree of product differentiation except for the situation of isolated products.[4]

We consider the following demand structure for our analysis. We assume that the representative consumer's utility is a function of consumption $q=(q_1,q_2)$, where q_1 and q_2 are the outputs of firm 1 and firm 2, and the numeraire good m. It is given by $U(q)+m$ with

[3] The new technologies could be different but creating same cost of production.

$$U(q) = a(q_1 + q_2) - \frac{1}{2}(q_1^2 + 2\gamma q_1 q_2 + q_2^2), \tag{1}$$

where the term γ shows the degree of product differentiation and can take any value between 0 and 1. If $\gamma = 0$, this implies that the products of these firms are isolated but for $\gamma = 1$, the products are perfect substitutes. We will do our analysis for $\gamma \in (0,1]$.[5] We assume that $a \geq \bar{c}$. If $a = \bar{c}$ then the optimal output and profit of these firms are 0 even if neither of them is successful in R&D. We will do our analysis for $a > \bar{c}$ if not specified otherwise.

The above utility function given in (1) generates the following inverse demand function for the ith firm:

$$P_i = a - q_i - \gamma q_j, \tag{2}$$

where, $i, j = 1,2$, $i \neq j$.

Let us define the optimal profit level of the ith firm, $i = 1,2$, in the product market (i.e., revenue minus total cost of production) for a given degree of product differentiation by $\pi(c)$, $\pi_i(c,c;\gamma)$ and $\pi_i(\bar{c},\bar{c};\gamma)$ respectively for the situations where only the ith firm is successful in R&D, where both firms are successful in R&D and where neither firm is successful in R&D. The arguments in the profit functions are showing the constant average cost of production of these firms. Since the successful firm becomes a monopoly in case of unilateral success in R&D, we do not write the subscript and the degree of product differentiation for this situation.

In what follows, we will do a general analysis with these reduced form profit functions and then, in the next section, we will examine how the values depend on the particular type of the product market competition.

The net profit of the i th firm is given by

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[4] It is easy to understand that sufficiently higher pre-innovation marginal cost of production increases the possibility of drastic R&D. For example, if these firms do not have any technology to produce the product without innovation then it is trivial that any cost reduction from R&D implies drastic R&D.

[5] The assumption of drastic innovation is justifiable provided the products are not isolated.

$$p(x_i)p(x_j)\pi_i(c,c;\gamma)+p(x_i)(1-p(x_j))\pi(c)+(1-p(x_i))(1-p(x_j))\pi_i(\bar{c},\bar{c};\gamma)-x_i\,, \qquad (3)$$

where, $i, j = 1,2$ and $i \neq j$.Maximizing (3), we get the optimal R&D investment for firm i, given the R&D investment of firm j. So, the profit maximizing R&D investment for firm i is

$$p'(x_i)p(x_j)\pi_i(c,c;\gamma)+p'(x_i)(1-p(x_j))\pi(c)-p'(x_i)(1-p(x_j))\pi_i(\bar{c},\bar{c};\gamma)=1\,, \qquad (4)$$

or, $$p'(x_i)[\pi(c)-\pi_i(\bar{c},\bar{c};\gamma)]-p'(x_i)p(x_j)[\pi(c)-\pi_i(c,c;\gamma)-\pi_i(\bar{c},\bar{c};\gamma)]=1\,. \qquad (4')$$

Second order condition for maximization is satisfied. Due to the symmetry of the probability functions, we have similar reaction functions for these firms. Solving these two reaction functions, we can find the optimal R&D investments of these firms. We assume that the probability functions are such that we get a unique equilibrium for R&D investments. Define optimal R&D investments of these firms by x_1^* and x_2^* for firms 1 and 2 respectively. Further, the symmetric probability function considered in this section will lead to same level of R&D investments by these firms.

It is clear from (4') that the optimal R&D investment of the ith firm depends on the differences $[\pi(c)-\pi_i(\bar{c},\bar{c};\gamma)]$ and $[\pi(c)-\pi_i(c,c;\gamma)-\pi_i(\bar{c},\bar{c};\gamma)]$. It is easy to understand that the term $[\pi(c)-\pi_i(\bar{c},\bar{c};\gamma)]$ is always positive for any degree of product differentiation except for the isolated products but the sign of $[\pi(c)-\pi_i(c,c;\gamma)-\pi_i(\bar{c},\bar{c};\gamma)]$ is not clear and depends on the degree of product differentiation.

Lemma 1: *If the product differentiation is below (above) a critical level then* $[\pi(c)-\pi_i(c,c;\gamma)-\pi_i(\bar{c},\bar{c};\gamma)]$ *is positive (negative). This critical level of product differentiation is likely to be different under Bertrand competition and Cournot competition.*

Proof: If the products are almost isolated, i.e., $\gamma = \varepsilon$, where $\varepsilon > 0$ but very small, then the term $[\pi(c) - \pi_i(c,c;\gamma) - \pi_i(\bar{c},\bar{c};\gamma)]$ tends to be $[\pi(c) - \pi(c) - \pi(\bar{c})] < 0$ since each firm will be almost monopoly for its product.

Now, consider the other extreme where the products are perfect substitutes, i.e., $\gamma = 1$. Here, the term $[\pi(c) - \pi_i(c,c;\gamma) - \pi_i(\bar{c},\bar{c};\gamma)]$ reduces to $\pi(c) > 0$ under Bertrand competition. Also, we find that $[\pi(c) - \pi_i(c,c;\gamma) - \pi_i(\bar{c},\bar{c};\gamma)]$ is positive for Cournot competition if the products are perfect substitutes. Hence, we find that the term $[\pi(c) - \pi_i(c,c;\gamma) - \pi_i(\bar{c},\bar{c};\gamma)]$ is positive under Cournot competition when the products are perfect substitutes.

The profits are continuous for $\gamma \in [\varepsilon, 1-\delta]$, where $\delta > 0$ and very small, under Bertrand competition and for $\gamma \in [\varepsilon, 1]$ under Cournot competition. Therefore, we can find a critical value of γ for Bertrand competition and a critical value of γ for Cournot competition such that the term $[\pi(c) - \pi_i(c,c;\gamma) - \pi_i(\bar{c},\bar{c};\gamma)]$ is positive (negative) when the actual value of γ is more (less) than this critical value, i.e., when the degree of product differentiation is below (above) that critical level of product differentiation.

Since the term $[\pi(c) - \pi_i(c,c;\gamma) - \pi_i(\bar{c},\bar{c};\gamma)]$ takes different values under Cournot and Bertrand competition, the critical values under Cournot and Bertrand competition is likely to be different. Q.E.D.

We have found that R&D investment of the ith firm depends on the differences $[\pi(c) - \pi_i(\bar{c},\bar{c};\gamma)]$ and $[\pi(c) - \pi_i(c,c;\gamma) - \pi_i(\bar{c},\bar{c};\gamma)]$. The next result shows how these terms affect the equilibrium R&D investments of these firms.

Lemma 2: *Let x_i^* is the optimal R&D investment of the ith firm for any given R&D investment of the jth firm, $i, j = 1,2$ and $j \neq i$.*

(i) We find $\frac{\partial x_i^*}{\partial z} > 0$, where, $z = [\pi(c) - \pi_i(\bar{c},\bar{c};\gamma)]$.

(ii) We find that $\frac{\partial x_i^*}{dy} < 0$, where $y = [\pi(c) - \pi_i(c,c;\gamma) - \pi_i(\bar{c},\bar{c};\gamma)]$.

Proof: (i) Totally differentiating (4) for x_i^* and z, we find that $\frac{\partial x_i^*}{\partial z} = -\frac{p'(x_i^*)}{(zp''(x_i^*) - yp''(x_i^*)p(x_j^*))} > 0$, where $i \neq j$. This is because the denominator $(zp''(x_i^*) - yp''(x_i^*)p(x_j^*))$ is negative due to the second order condition of profit maximization with respect to the R&D investment.

(ii) Totally differentiating (4) for x_i^* and y, we find that $\frac{\partial x_i^*}{\partial y} = \frac{p'(x_i^*)p(x_j^*)}{(zp''(x_i^*) - yp''(x_i^*)p(x_j^*))} < 0$.

Q.E.D.

At this point, it is worth considering the implication of the above lemma. First, consider a situation where only the ith firm does R&D. This implies that $p(x_j) = 0$. So, the optimal R&D investment of firm 1 will satisfy the condition

$$p'(x_i)[\pi(c) - \pi_i(\bar{c},\bar{c};\gamma)] = 1. \tag{5}$$

Suppose, x_i^* is the optimal R&D investment that satisfies condition (5). So, from (5) it is clear that if only the ith firm does R&D then its optimal R&D investment, x_i^*, will reduce as the value of z reduces. Hence, the stand-alone incentive for investing in R&D, i.e., the incentive for investing in R&D by a firm when the competitor does not invest in R&D, reduces as the gain from R&D reduces. Therefore, due to the stand-alone incentive for doing R&D, one firm's R&D investment will be more under Bertrand competition, which is in contradiction to the Bester and Petrakis (1993).

Since firm j also does R&D, we need to consider the strategic effect on R&D investment as well. The second part of the above lemma shows that as the strategic gain from R&D reduces, a firm's incentive for R&D investment increases, i.e., given the R&D investment of the jth firm, the R&D investment of the ith firm increases due to the reduction in y only. Hence, due to the strategic incentive, a firm's R&D investment will be more under Cournot competition.

3 Comparison between Bertrand and Cournot competitions

The previous section has developed a model of R&D competition in a general framework and examined the effects of stand-alone incentive and strategic incentive. In this section we will see how different types of product market competition, viz., Bertrand and Cournot competitions, affect the R&D investments of these firms. The next section will compare the welfare effects of these product market competitions.

Before examining the effects of Bertrand and Cournot competitions, we will prove the following lemma comparing the outputs and the profits of these firms ex-post R&D, which will be helpful for our following analysis.

Lemma 3: *Consider that either both firms succeed in R&D or neither firm succeeds in R&D and* $\gamma \in (0,1]$.

(i) The optimal output of each firm is more under Bertrand competition than Cournot competition, i.e., $q_i^b(c,c;\gamma) > q_i^c(c,c;\gamma)$ *and* $q_i^b(\bar{c},\bar{c};\gamma) > q_i^c(\bar{c},\bar{c};\gamma)$, $i = 1,2$.

(ii) The profits are greater under Cournot competition compared to Bertrand competition for each of these firms, i.e., $\pi_i^c(c,c;\gamma) > \pi_i^b(c,c;\gamma)$ *and* $\pi_i^c(\bar{c},\bar{c};\gamma) > \pi_i^b(\bar{c},\bar{c};\gamma)$, $i = 1,2$.

Proof: Assume that $\gamma \in (0,1)$. Given the demand and cost specifications, the optimal output and profit of the ith firm, $i = 1,2$, under Cournot and Bertrand competitions are respectively

$$q_i^c = \frac{(a-k)}{(2+\gamma)} \quad \text{and} \quad \pi_i^c = \frac{(a-k)^2}{(2+\gamma)^2} \tag{6}$$

and $$q_i^b = \frac{(a-k)}{(2-\gamma)(1+\gamma)} \quad \text{and} \quad \pi_i^b = \frac{(a-k)^2(1-\gamma)^2}{(2-\gamma)^2(1-\gamma^2)}, \tag{7}$$

where, $k = c$ when both firms succeed in R&D and $k = \bar{c}$ when neither firm succeeds in R&D. It is easy to check from (6) and (7) that $q_i^c < q_i^b$ and $\pi_i^c > \pi_i^b$.

If $\gamma = 1$ then the expressions in (6) show the optimal outputs and profits under Cournot competition. But, in this situation, the optimal output and profit of the ith firm under Bertrand competition are given by $\frac{(a-k)}{2}$ and 0. Therefore, also in this situation, the optimal output is more under Bertrand competition and the profit is more under Cournot competition.

Thus, we prove the result of this lemma. Q.E.D.

Now, we are in a position to compare the R&D investments under Bertrand and Cournot competitions. From Lemma 3(ii), it is clear that both $[\pi(c) - \pi_i(\bar{c}, \bar{c}; \gamma)]$ and $[\pi(c) - \pi_i(c, c; \gamma) - \pi_i(\bar{c}, \bar{c}; \gamma)]$ are greater under Bertrand competition compared to the Cournot competition. Therefore, from Lemma 2 we find that both firms will invest more in R&D under Bertrand competition due to the stand-alone incentive. But, given the R&D investment of the competitor, strategic effect encourages a firm to invest less in R&D under Bertrand competition. So, total effect on the equilibrium R&D investment is ambiguous and as the following proposition shows it depends on the probability of success in R&D.

Proposition 1: *(i) The equilibrium probability of success in R&D under both Bertrand competition and Cournot competition is either greater than, less than or equal to* $\bar{p}$.
(ii) If the equilibrium probability of success under Bertrand and Cournot competitions is greater (less) than a critical value, i.e., say, $\bar{p}$*, the equilibrium R&D investments are more (less) under Cournot competition compared to Bertrand competition.*

Proof: Suppose, under Bertrand competition, x_1^{b*} and x_2^{b*} are the equilibrium R&D investments of these firms. So, x_1^{b*} and x_2^{b*} satisfy the condition (4) under Bertrand competition. Straightforward calculation will show that, given the R&D investment x_2^{b*}, the left hand side of (4) for firm 1 under Cournot competition will be greater than 1 provided

$$[\pi_1^c(\bar{c}, \bar{c}; \gamma) - \pi_1^b(\bar{c}, \bar{c}; \gamma)] < p(x_2^{b*})[\pi_1^c(\bar{c}, \bar{c}; \gamma) - \pi_1^b(\bar{c}, \bar{c}; \gamma) + \pi_1^c(c, c; \gamma) - \pi_1^b(c, c; \gamma)]. \qquad (8)$$

or, $$p(x_2^{b*}) > \frac{[\pi_1^c(\bar{c},\bar{c};\gamma) - \pi_1^b(\bar{c},\bar{c};\gamma)]}{[\pi_1^c(\bar{c},\bar{c};\gamma) - \pi_1^b(\bar{c},\bar{c};\gamma) + \pi_1^c(c,c;\gamma) - \pi_1^b(c,c;\gamma)]} = \bar{p}. \quad (8')$$

If $p(x_2^{b*})$ tends to 1 then the condition (8') is satisfied. But, if $p(x_2^{b*})$ tends to 0 then the condition (8') does not hold. Further, the left hand side of (8') is continuous and increasing in $p(x_2^{b*})$ over $[0,1]$. Therefore, if $p(x_2^{b*})$ is greater than a critical value, say $\bar{p}$, then, given the R&D investment x_2^{b*}, the optimal R&D investment of firm 1 under Cournot competition is more than x_1^{b*}. Since these firms are symmetric, we have a similar condition for firm 2 also. Further, symmetry of the firms imply that these firms will invest same amount in R&D. Therefore, we can say that if $p(x_2^{b*}) = p(x_1^{b*}) > \bar{p}$, the optimal R&D investments of these firms under Cournot competition, say x_1^{c*} and x_2^{c*}, will be more than the optimal R&D investments of these firms under Bertrand competition.

But, if the condition (8') does not hold, i.e., if $p(x_2^{b*}) = p(x_1^{b*}) < \bar{p}$, the optimal R&D investments of these firms are lower under Cournot competition compared to Bertrand competition.

(i) We have seen that if $p(x_2^{b*}) = p(x_1^{b*}) > \bar{p}$, the equilibrium R&D investment is more under Cournot competition and hence, the probability of success is higher under Cournot competition. But, for $p(x_2^{b*}) = p(x_1^{b*}) < \bar{p}$, the equilibrium R&D investment is lower under Cournot competition. Hence, in this situation, the equilibrium probability of success is lower under Cournot competition compared to Bertrand competition. We have same equilibrium R&D investment and probability of success under Bertrand and Cournot competition when $p(x_2^{b*}) = p(x_1^{b*}) = \bar{p}$. Therefore, the equilibrium probability of success in R&D under both Bertrand competition and Cournot competition is either greater than, less than or equal to $\bar{p}$.

(ii) It follows from the above argument that if $p(x_{2b}^*) = p(x_{1b}^*) > \bar{p}$, the probability of success and the optimal R&D investments of these firms are more under Cournot competition compared to Bertrand competition. The opposite situation arises for $p(x_{2b}^*) = p(x_{1b}^*) < \bar{p}$. This proves the result. Q.E.D.

Thus, we find that whether the firms invest more in R&D under a more or less competitive environment depends on the equilibrium probability of success in R&D. This contradicts the previous results of this literature where the authors have shown that equilibrium R&D investments are always higher under a particular mode of competition (see, e.g., Delbobo and Denicolo, 1990 and Qiu, 1997).

In Proposition 1 we have considered the effects of different types of competition on R&D investments for a given degree of product differentiation. Now, we will examine the influence of product differentiation on our result.

Proposition 2: *Whether R&D investment is higher under Cournot competition or Bertrand competition does not depend on the degree of product differentiation.*

Proof: We have seen in Proposition 1 that R&D investments are more under Cournot (Bertrand) competition if the equilibrium probability of success is greater (less) than the critical value $\bar{p}$. From (6), (7) and (8') we find that $\bar{p}$ is independent of the degree of product differentiation since

$$\frac{\pi_i^c(c,c;\gamma)-\pi_i^b(c,c;\gamma)}{\pi_i^c(\bar{c},\bar{c};\gamma)-\pi_i^b(\bar{c},\bar{c};\gamma)}=\frac{(a-c)^2}{(a-\bar{c})^2}. \tag{9}$$

Hence, it implies that the comparison of R&D investments under Cournot and Bertrand competitions does not dependent of the degree of product differentiation. Q.E.D.

Thus, in contrary to the previous result of this literature (see, Bester and Petrakis, 1993) we show that the comparison between the equilibrium R&D investments under Bertrand and Cournot competitions does not depend on the degree of product differentiation.

3.1 An example

In this subsection we consider a specific probability function faced by these firms and provide an example for Proposition 1. We consider that these firms face same probability function and is given by $p(x_i) = \mu x_i^{\frac{1}{2}}$, $i = 1,2$. Further, in this example we assume that the products are homogenous, i.e., $\gamma = 1$.

If the products are homogeneous then the critical value of the probability of success in R&D is given by

$$\overline{p}(\gamma = 1) = \frac{\pi_i^c(\overline{c}, \overline{c}; 1)}{\pi_i^c(\overline{c}, \overline{c}; 1) + \pi_i^c(c, c; 1)}. \tag{10}$$

Using the condition (4) and due to symmetry, we find that the ith firm's, $i = 1,2$, optimal R&D investment and the corresponding probability of success under Bertrand and Cournot competitions are respectively

$$x^b(\gamma = 1) = \left[\frac{\mu\pi(c)}{2 + \mu^2\pi(c)}\right]^2 \quad \text{and} \quad x^c(\gamma = 1) = \left[\frac{\mu[\pi(c) - \pi_i^c(\overline{c}, \overline{c}; 1)]}{2 + \mu^2[\pi(c) - \pi_i^c(\overline{c}, \overline{c}; 1) - \pi_i^c(c, c; 1)]}\right]^2 \tag{11}$$

$$p^b(\gamma = 1) = \frac{\mu^2\pi(c)}{2 + \mu^2\pi(c)} \quad \text{and} \quad p^c(\gamma = 1) = \frac{\mu^2[\pi(c) - \pi_i^c(\overline{c}, \overline{c}; 1)]}{2 + \mu^2[\pi(c) - \pi_i^c(\overline{c}, \overline{c}; 1) - \pi_i(c, c; 1)]}. \tag{12}$$

From (10) and (12) we find that both $p^b(\gamma = 1)$ and $p^c(\gamma = 1)$ are less (greater) than $\overline{p}(\gamma = 1)$ provided $\mu^2\pi(c)\pi_i^c(c, c; 1) < (>) 2\pi_i^c(\overline{c}, \overline{c}; 1)$. Therefore, optimal R&D investments are more (less) under Bertrand competition compared to Cournot competition provided $\mu^2\pi(c)\pi_i^c(c, c; 1) < (>) 2\pi_i^c(\overline{c}, \overline{c}; 1)$. The direct comparison of the expressions in (11) also provides the same conclusion.

Thus, we find that if the R&D productivity is sufficiently low (high), i.e., μ is low (high), the optimal R&D investment is more (less) under Bertrand competition compared to Cournot competition.

4 Welfare comparison

It is well known that 'static' welfare is higher under Bertrand competition compared to Cournot competition since the deadweight loss is lower under Bertrand competition compared to Cournot competition. However, the above analysis shows that if the probability of success is sufficiently high then R&D investments are more under Cournot competition compared to Bertrand competition. Hence, there might be a conflict between the 'static' and 'dynamic' welfare. We define the welfare of the economy as the summation of consumer surplus and the industry profit net of R&D investments.

If the probability of success is sufficiently low then the analysis of section 3 shows that the chance of getting the innovation under Bertrand competition is higher compared to Cournot competition. However, in this situation, a higher R&D investment under Bertrand competition has a negative impact on welfare. But, since we are considering a situation where the R&D productivity is very low, this negative impact is sufficiently low and the expected welfare is most likely to be higher under Bertrand competition compared to Cournot competition. Hence, if the probability of success is sufficiently low then both 'static' and 'dynamic' welfare are higher under Bertrand competition compared to Cournot competition.

If the probability of success is sufficiently high then we have found that the R&D investments are higher under Cournot competition compared to Bertrand competition. Hence, the probability of success is higher under Cournot competition compared to Bertrand competition. So, in this situation, the 'static' welfare is higher under Bertrand competition but the chance of getting the innovation is higher under Cournot competition. As a result, the 'dynamic' welfare may be more under Cournot competition while the 'static' welfare is more under Bertrand competition.

Generally speaking, the comparison of welfare under Bertrand and Cournot competitions is cumbersome in our framework. In the following analysis we will consider the specific probability function that has been considered in subsection 3.1 and will provide examples to show the effects of the R&D productivity (i.e., μ) and the degree of product differentiation (i.e., γ) on welfare.

The next proposition will consider an example of homogeneous product to show the possibility of higher and lower welfare under Cournot competition compared to Bertrand competition.

Proposition 3: *Assume that* $a = 1$, $c = 0$, $\bar{c} \in [.5,1]$, $\gamma = 1$, $\mu \in [0, 3\sqrt{2}]$ *and both firms have same probability function* $p(x_i) = \mu x_i^{\frac{1}{2}}$, *where* $i = 1,2$.

(a) If the value of $\bar{c}$, i.e., the pre-innovation cost, is not very high then the expected welfare is higher under Bertrand competition for all R&D productivities.

(b) Suppose the value of $\bar{c}$ is sufficiently high. The expected welfare is higher under Bertrand competition for very low and very high R&D productivities but the expected welfare is higher under Cournot competition for moderate R&D productivities.

Proof: When $\gamma = 1$, we find that the equilibrium probability of success in R&D under Cournot competition is equal to 1 for $\mu = 3\sqrt{2}$. Further, given $a = 1$, the innovation is drastic under Bertrand and Cournot competition when $c = 0$ and $\bar{c} \in [.5,1]$.

Given the assumptions of this proposition and the demand function mentioned in (2), the expected welfare under Cournot and Bertrand competition are respectively

$$W^c = \frac{\begin{array}{c}[16\mu^4(9-4(1-\bar{c})^2)^2 + 27\mu^2(9-4(1-\bar{c})^2)(72-4\mu^2) + 16(72-4\mu^2)(1-\bar{c})^2 \\ -72\mu^2(9-4(1-\bar{c})^2)^2]\end{array}}{36[72+\mu^2(5-4(1-\bar{c})^2)]^2} \tag{13}$$

and

$$W^b = \frac{[\mu^4 + 12\mu^2 + 64(1-\bar{c})^2 - 4\mu^2]}{2[8+\mu^2]^2}. \tag{14}$$

We plot the expression $(W^c - W^b)$ in Figure 1.[6] The inspection of the Figure 1 proves the result.

For better understanding, in Figure 2 and Figure 3 we consider the value of $\bar{c}$ as $.6$ and $.9$ respectively with all other assumptions of this proposition. Q.E.D.

The reasons for the above results are following. We know that 'static' welfare is always higher under Bertrand competition. If R&D productivity is very low so that R&D investments are higher under Bertrand competition then the probability of success is higher under Bertrand competition. But, higher R&D investment under Bertrand competition has

[6] We use 'The Mathematica 4' for the figures related to this proposition and Proposition 4 (i.e., figures 1, 2, 3, 4, 5 and 6).

a negative impact on the welfare. Since, the R&D productivity is very low, this negative impact on welfare is very small and the expected welfare is higher under Bertrand competition. On the other extreme, if the R&D productivity is very high then the R&D investment and the probability of success in R&D are higher under Cournot competition. Since, now we are considering a situation with sufficiently higher R&D productivity, the negative impact of higher R&D investment under Cournot competition can be sufficiently large and the expected welfare can be higher under Bertrand competition even if the R&D investment is higher under Cournot competition.

However, if R&D productivity is sufficiently large to create higher R&D investment under Cournot competition but if it is not very high to create sufficiently large negative impact on welfare, we find that whether the expected welfare is higher under Cournot competition depends on the pre-innovation cost. Given the probability function of Proposition 3, it is clear from (12) that the R&D investment under Bertrand competition does not depend on the pre-innovation cost but the R&D investment under Cournot competition increases with the pre-innovation cost. Therefore, the benefit from higher R&D investment under Cournot competition increases as the pre-innovation cost increases. Hence, if the pre-innovation cost is sufficiently low (not sufficiently low) then, for these R&D productivities, the probability of success is not sufficiently higher (sufficiently higher) under Cournot competition to outweigh the effect of higher 'static' welfare under Bertrand competition. Hence, in this situation, the expected welfare is higher under Cournot (Bertrand) competition for sufficiently higher (lower) pre-innovation costs.

In the above proposition we show that welfare under Cournot competition could be higher compared to Bertrand competition. In our model we did not consider the possibility of knowledge spillover and hence, this finding contradicts the result of Qiu (1997), which shows that welfare is always higher under Bertrand competition compared to Cournot competition when there is no knowledge spillover under R&D. Further, in contrary to Delbono and Denicolo (1990), which shows the possibility of higher welfare under Cournot competition if the number of firms in the industry is sufficiently large, we show that welfare under Cournot competition could be higher compared to Bertrand competition even if we consider a duopoly market. Further, unlike the previous contributions we find that welfare under Cournot competition is higher under moderate R&D productivities.

The above proposition shows the impact of R&D productivity on welfare under Bertrand and Cournot competition. In the next proposition, we will provide an example to show the importance of the product differentiation. We will show that whether the difference between the expected welfare under Cournot and Bertrand competitions increases or decreases with higher product differentiation is ambiguous and can depend on the R&D productivity.

Proposition 4: *Assume that* $a = \bar{c} = 1$, $c = 0$, $\mu \in (0, 2\sqrt{2}]$ *and consider that* $\gamma \in (0,1)$ *and both firms have same probability function* $p(x_i) = \mu x_i^{\frac{1}{2}}$, *where* $i = 1,2$. *The difference between the expected welfare under Cournot and Bertrand competitions increases (reduces) with higher product differentiation if the R&D productivity is sufficiently low (high).*

Proof: We find that if the products are isolated (i.e., $\gamma = 0$) then the probability of success in R&D under Cournot competition equals to 1 when $\mu = 2\sqrt{2}$. Therefore, if $\mu \in (0, 2\sqrt{2}]$ then, for any degree of product differentiation, probability of success in R&D is lower than 1 under Bertrand and Cournot competitions. Further, the innovation is drastic under Bertrand and Cournot competition when $a = \bar{c} = 1$ and $c = 0$.

Given the assumptions of this proposition and the utility function specified in (1), we find that the expected welfare under Cournot and Bertrand competitions are given by

$$W^c = \frac{[4\mu^4(2+\gamma)^2(3+\gamma) + 3\mu^2(2+\gamma)^2(8(2+\gamma)^2 - 4\mu^2) - 8\mu^2(2+\gamma)^4]}{4[8(2+\gamma)^2 + \mu^2\gamma(4+\gamma)]^2} \tag{15}$$

and

$$W^b = \frac{\begin{array}{l}[4\mu^4(2-\gamma)^2(1-\gamma)^2(1+\gamma)(3-2\gamma) + \mu^2(2-\gamma)^2(1-\gamma^2)(8(2-\gamma)^2(1-\gamma^2) \\ -4\mu^2(1-\gamma)^2) - 8\mu^2(2-\gamma)^4(1-\gamma^2)^2]\end{array}}{4[8(2-\gamma)^2(1-\gamma^2) + \mu^2((2-\gamma)^2(1-\gamma^2) - 4(1-\gamma)^2)]^2}. \tag{16}$$

Subtracting (16) from (15) we can find the expression for $(W^c - W^b)$. In Figure 4 we plot this expression $(W^c - W^b)$ for $\mu \in [0, 2\sqrt{2}]$ and $\gamma \in [0,1]$. The inspection of Figure 4 proves the result.

This finding is more prominent in Figure 5 and Figure 6 where we plot $(W^c - W^b)$ for $\mu = 1$ and $\mu = 2.5$ along with other assumptions of this proposition. Q.E.D.

If the products are getting differentiated then it changes the outputs and R&D investments of these firms under Bertrand and Cournot competitions. Given the assumptions of this proposition, it is easy to check from (4') that as the products are getting differentiated, the R&D investments increase. If the R&D productivity is sufficiently low so that the R&D investments are sufficiently low, the effects through R&D investments tend to be small. Hence, in this situation, the difference between the expected welfare under Cournot and Bertrand competition increases with higher product differentiation, as the difference of outputs between Bertrand and Cournot competitions reduces with higher product differentiation. But, if the R&D productivity is sufficiently high so that the R&D investments are sufficiently high, the effects through R&D investments tend to be larger. Due to the high R&D productivity, the effect on R&D investment is sufficiently large. This creates a negative impact on welfare due to the high R&D investment and a positive effect on welfare due to the higher chance of getting the innovation. Since, here the probability of success under Cournot competition is already sufficiently high, the latter effect is relatively week under Cournot competition. Further, higher product differentiation reduces the difference of outputs between Bertrand and Cournot competitions. On the balance, the negative impact of higher R&D investment under Cournot competition tends to dominate the result. Hence, the difference between the expected welfare under Cournot and Bertrand competition reduces with higher product differentiation.

5 Conclusion

Whether R&D investment increases under higher or lower competitive environment is an existing debate. While the initial works have looked at monopoly and competitive markets to find the answer, recent contributions have focused on oligopolistic markets. In this paper we examine this issue in a duopoly market and show that significantly different conclusions can be obtained if we consider non-tournament R&D competition between firms where each firm affects its probability of success in R&D through its own R&D investment.

We show that whether equilibrium R&D investment is more under Bertrand or Cournot competition depends on the equilibrium probability of success. Considering symmetric probability function we show that if the equilibrium probability of success is sufficiently low (high) then the equilibrium R&D investment is more under Bertrand (Cournot) competition. Hence, this finding may have important implications for designing government policies to encourage R&D activities in an economy.

Since the equilibrium R&D investment is more under Cournot competition when the probability of success is sufficiently high, here the chance of getting the innovation is higher under Cournot competition compared to Bertrand competition. Thus, the expected gain from the new innovation may create higher 'dynamic' welfare under Cournot competition compared to Bertrand competition. In this respect we show the importance of the R&D productivity and the degree of product differentiation.

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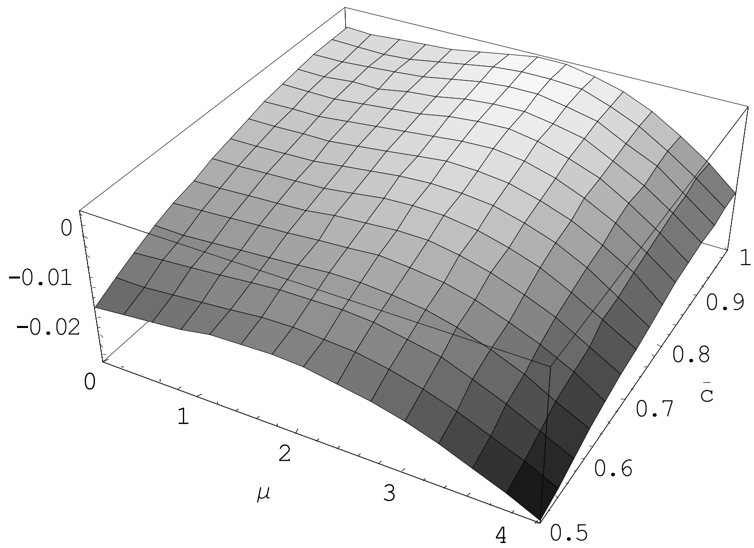

Figure 1: Subtracting (14) from (13).

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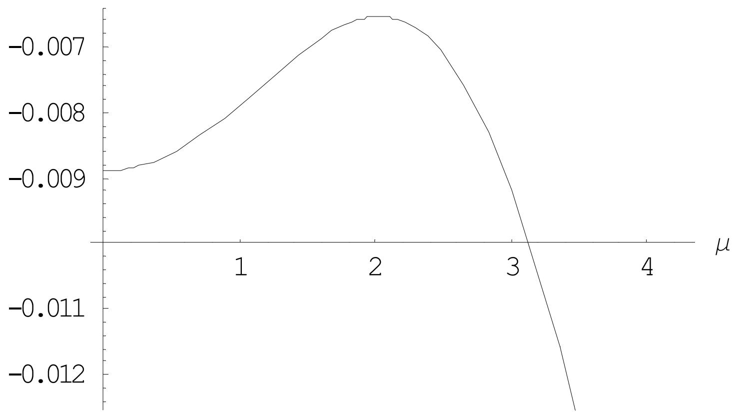

Figure 2: Subtracting (14) from (13) for $\bar{c} = .6$.

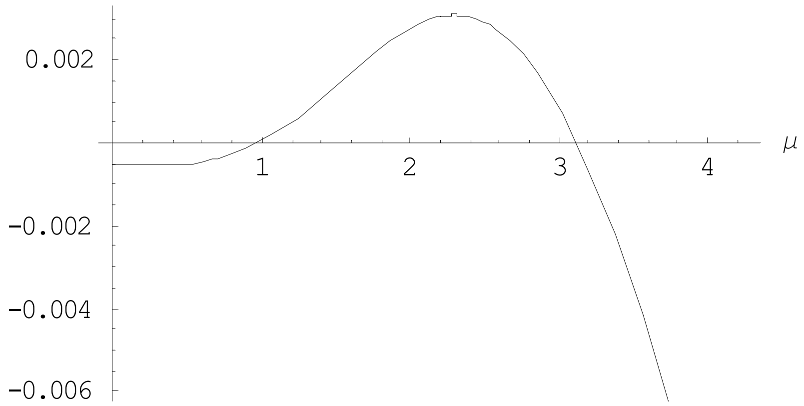

Figure 3: Subtracting (14) from (13) for $\bar{c} = .9$.

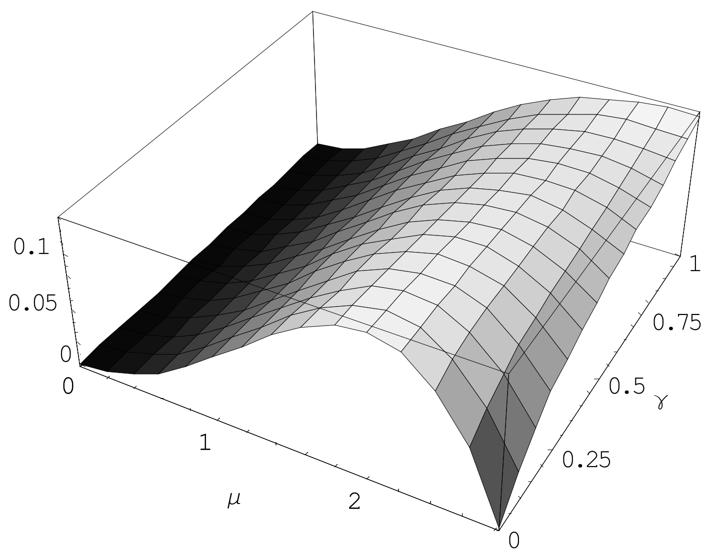

Figure 4: Subtracting (16) from (15) and for $\mu \in [0, 2\sqrt{2}]$ and $\gamma \in [0,1]$.

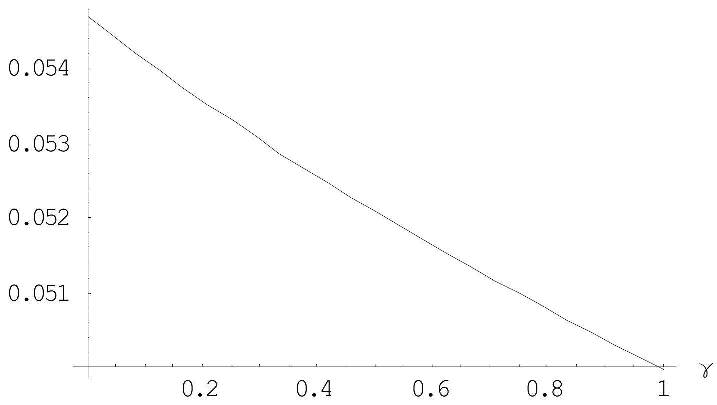

Figure 5: Subtracting (16) from (15) and for $\mu = 1$ and $\gamma \in [0,1]$.

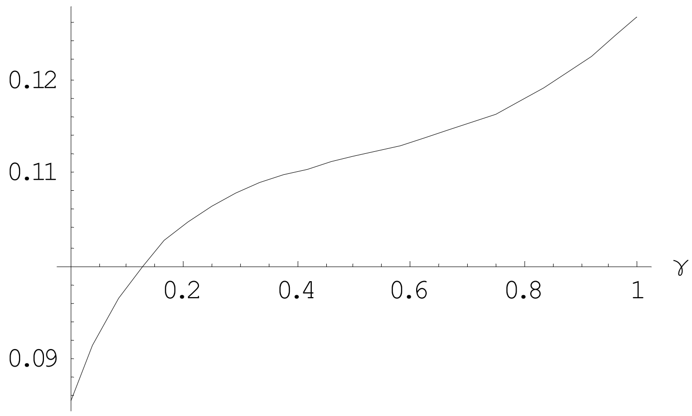

Figure 6: Subtracting (16) from (15) and for $\mu = 2.5$ and $\gamma \in [0,1]$.