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Cournot Vs Stackelberg Equilibria with a Public Enterprise and International Competition

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Abstract

This paper examines the validity of alternative assumptions about public enterprise strategies in the presence of both domestic and international competition. It extends the quantity-setting game to a preplay stage and endogenizes the firms' order of moves to show that *i*) Cournot competition is not the subgame perfect Nash equilibrium of the extended game, *ii*) the only SPN equilibria are sequential and *iii*) for some values of structural parameters, public Stackelberg leadership is the unique SPN equilibrium solution of the game. This has a significant consequence for debates over privatisation because with a public Stackelberg leadership game, there is no distortionary effect associated with the operation of a public enterprise in the domestic market.

JEL: D43, L33, L13
Keywords: Privatisation, International Mixed Oligopoly.

1 Introduction

In recent decades great efforts have been made to redirect the incentive structures in tune with a market-oriented economy. This is called marketisation and involves two major components: privatisation, which involves the transfer of ownership from government to private sector and trade liberalisation, which involves the opening of domestic market to competitive forces. The

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coordination of these policies also has been imposed by international organisations like World Bank through conditionalities for loans to many developing countries.

Nevertheless the majority of papers on privatisation have concentrated on the impact of public ownership on a single firm or at the most on a single country, and the effects of privatisation in the context of increased international/global competition have not been studied in depth so far. Exceptions to this are some contributions from the mixed oligopoly literature (Fjell and Pal 1996, Pal and White 1998, Fjell and Heywood 2001, Sepahvand 2002). They investigate the effects of privatisation by comparing the outcome of a mixed market structure with the results of a private oligopoly; that is presumably the structure of market after privatisation.

Concerning the effects of privatisation in the presence of international competition the outcomes are not conclusive. Comparison between two market structures under different assumptions about the order of the firms' moves reveals that the welfare effect of privatisation in these models is sensitive to the timing of the game. Under the Cournot-Nash assumption, privatisation in the home industry always improves welfare of the domestic country in the presence of a production subsidy (Pal and White, 1998). But if the firms play a Stackelberg public firm leadership game, privatisation leaves the levels of the optimal subsidy and domestic welfare unchanged. In short, it leads to an irrelevance result (Sepahvand, 2002).

This paper examines the validity of alternative assumptions about the firms' order of moves in an international mixed oligopoly model in the presence of a production subsidy. Following the earlier literature, we abstract from principal-agent issues and the other problems to concentrate on the public firm's objectives and its timing of play. We adopt the "extended game with observable delays" suggested by Hamilton and Slutsky (1988, 1990) and developed by Amir and Grilo (1999) in order to determine endogenously the choice of models (either Cournot or Stackelberg) in an international mixed oligopoly model with subsidisation. We argue that in this context firms' commitments to the simultaneous strategy are not credible and the self-enforcing equilibrium is a Stackelberg public firm leadership game. Thus Cournot competition is not an appropriate modelling assumption in this framework.

Most of the economic literature assumes Cournot competition with simultaneous play as the natural order of moves in a quantity-setting game[1]. However, recent advances in game theory argue that the assumed order of

[1] That is because without establishing a mechanism that provides a precommitment for the leader (and only for the leader) to a certain strategy, the only credible precommitment for the profit-maximizing firms is a simultaneous play strategy (*Wolfstetter, 1999, p. 79*).

play should also be consistent with the players' preferences over the time of actions. We add a preplay stage to the basic quantity-setting game to endogenise the order of play. The subgame-perfect Nash equilibrium of this extended game induces an endogenous order of moves in the basic game. As we rule out the possibility of multiple equilibria in the basic game, the timing choice of firms can be obtained by comparing the payoffs of each timing choice with the payoffs of any feasible timing of play at that subgame.

We show that a public firm leadership game Pareto dominates simultaneous play. That is because at the equilibrium of the simultaneous play game the public firm produces too much, thereby damaging competition in the market. With increasing marginal costs, the operation of the public firm under a Cournot competition is detrimental not only to the foreign competitors but also to the domestic economy - it deprives the home economy of possible gains from trade. Therefore, if firms have some flexibility in the timing of action, Cournot competition cannot be the natural order of play.

This paper is organised as follows. Section 2 presents a mixed duopoly model with two players: a domestic public firm and a foreign firm. This allows us to adopt the two-player extended game with observable delays as introduced by Hamilton and Slutsky directly into an international mixed market structure. Section 3 incorporates a domestic private firm in the model. Section 4 concludes.

2 The Extended Game of a Mixed Duopoly

Consider a single market for a homogeneous good supplied by a domestic public firm and a foreign private firm whose outputs are denoted by q_n and q_f respectively.

Demand behaviour in the domestic market can be characterised by an inverse demand function $p = p(Q)$ where p is the price that clears the market and $Q = q_n + q_f$ is the total output. $p(.)$ is assumed to be finite valued, non-negative, decreasing, twice continuously differentiable and log-concave function for any Q where $p(Q) > 0$. There exists a finite level of output $\bar{Q}$ where $p(\bar{Q}) = 0$.

The firms have access to an identical decreasing returns to scale technology represented by $c = c(q_j)$ which is a non-negative, strictly convex and twice continuously differentiable cost function with $c(0) = 0$.

The domestic public firm and the foreign firm choose their outputs from closed subsets of positive real numbers that are bounded above by $\bar{Q}$ and are denoted by A_n and A_f respectively.

The foreign private firm's profit is

$$\pi_f(Q^h, q_f) = p(Q^h + q_f)q_f - c(q_f) \tag{1}$$

where Q^h is the home industry output produced by the public firm, so that $Q^h = {q_n}^2$. The domestic public firm maximises social welfare, defined as the unweighted sum of home industry profits and consumer surplus:

$$W(Q^h, q_f) = \int_0^Q p(t)dt - p(Q)Q + \pi_n(Q^h, q_f) \tag{2}$$

where $\pi_n(Q^h, q_f) = p(Q^h + q_f)Q^h - c(Q^h)$ is the profit of the public firm. Thus $W(.)$ can be written as

$$W(Q^h, q_f) = \int_0^Q p(t)dt - p(Q)q_f - c(Q^h) \tag{3}$$

and the set of payoffs is $P = (W, \pi_f)$.

Following Hamilton and Slutsky (1990), we consider a hypergame consisting of two stages - a preplay stage and a basic stage - in order to study simultaneous versus sequential play.

In the preplay stage, both players decide at the same time whether to choose actions in the basic game at the first opportunity and move early, denoted by E, or to wait until observing their rival's action and move late, denoted by L. While each firm commits itself to the announced time of action, it does not need to specify the action it will take. In the basic stage after the announcement firms select their outputs knowing when the other will make its choice.

The timing choice of firms can be obtained by comparing the payoffs of each timing choice with the payoffs of any feasible timing of play at that subgame. Three possibilities arise: 1) only one of the sequential outcomes Pareto dominates the simultaneous move outcome and it will be the unique equilibrium of the extended game, 2) both sequential outcomes Pareto dominate the simultaneous move outcome, 3) neither of the sequential outcomes Pareto dominates the simultaneous moves. The possibility of the simultaneous play arises only in the third case.

This hypergame is called an extended game with observable delays because it is assumed that firms can observe if the other one delays at the basic stage. The associated game tree is shown in Figure 1.

In an extended game with observable delay, the assumption that the firms can commit themselves to move at the announced time is not restrictive. For

[2] The change in notation is helpful to extend the analysis further in the next section.

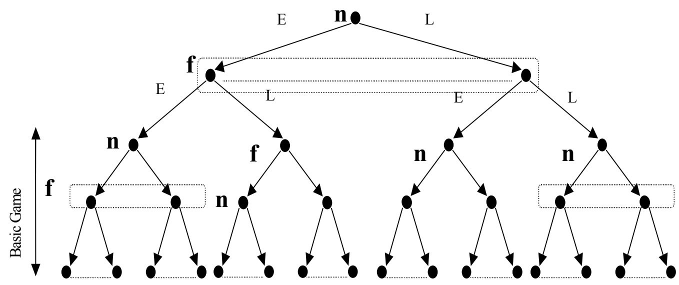

Figure 1: The extensive form of the extended game with observable delays.

instance, if both firms choose moving early, they have already rejected being a follower. Moreover, as we assume that delays are observable, no firm also can take advantage of changing its decision without affecting its rival's decision.

More formally, let $\Gamma \equiv (N, S, P)$ be the strategic form of an extended game with observable delay. As mentioned above, $N \equiv \{n, f\}$. Firms choose their outputs from their action set A_i (where $i \in N$), a positive finite interval of R_{++}. The set of possible times at which firms choose their actions is represented by $T \equiv \{E, L\}$ where E denotes moving early and L denotes moving late. The set of public firm strategies is $S_n = T \times \Phi_n$ where Φ_n is the set of functions that map $\{(E, E), (E, L), (L, E) \times A_f, (L, L)\}$ into A_n. Similarly, the set of foreign firm strategies is $S_f = T \times \Phi_f$ where Φ_f is the set of functions that map$\{(E, E), (E, L) \times A_n, (L, E), (L, L)\}$ into A_f. A strategy of the public firm is $s_n = (t, q_n)$ where $t \in T$ and $q_n \in \Phi_n$. For instance, in the case of the outcome of a simultaneous game where both prefer moving early, $s_n = (E, q_n^{(EE)})$ where $q_n^{(EE)}$ is the Cournot Nash equilibrium output of the public firm when the public firm's strategy is to move early, knowing the foreign firm also moves early. It chooses q_n from A_n to maximise welfare, taking the foreign firm output as given. The set of strategy profiles is $S = S_n \times S_f$. All information is common knowledge.

We do not need to consider all possible strategies of firms at the basic game in order to solve the game, because Nash equilibrium in subgames eliminates non-equilibrium output choices. Therefore, we can confine ourselves to Nash equilibria in subgames. Let $e^{(EL)}(\hat{s}_n, \hat{s}_f)$ denote the equilibrium in a subgame where $\hat{s}_n = (E, q_n^{(EL)})$ and $\hat{s}_f = (L, q_f^{(EL)})$. This indicates the public firm chooses to move early and acts as a Stackelberg leader while the foreign firm follows its decision and sets its output later. The set of payoffs is $P^{(EL)} \equiv (W^{(EL)}, \pi_f^{(EL)})$. It is worth mentioning that firms cannot choose to be leader or follower because that depends on other firm's timing choice. The payoffs of all equilibrium outcomes in subgames are shown in Table 1.

Table.1-The reduced strategic form of the game

		Foreign Firm	
		Early	*Late*
Public	*Early*	$(W^{(EE)}, \pi_f^{(EE)})$	$(W^{(EL)}, \pi_f^{(EL)})$
Firm	*Late*	$(W^{(LE)}, \pi_f^{(LE)})$	$(W^{(LL)}, \pi_f^{(LL)})$

As an example, let us take the linear forms for demand and marginal cost functions with the slopes equal to unity. Setting $p(Q) = 10 - Q$ and $c(q) = \frac{1}{2}q^2$ the game's outcomes are reduced to the following results,

Table.2-The results of the extended game in a simple example

		Foreign Firm	
		Early	*Late*
Public	*Early*	(263.8 41.6)	(264.7 46.7)
Firm	*Late*	(263.8 41.6)	(263.8 41.6)

Table 2 shows the Nash equilibria in the subsequent subgames in this simple example. The simultaneous timing choice at early time is not an SPN equilibrium because the foreign firm always has an incentive to deviate from this setting and postpone its timing of action. A situation in which both move at the later period also is not an SPN equilibrium because the public firm can improve welfare by changing its timing choice. Note that $P^{(EL)} \equiv (W^{(EL)}, \pi_f^{(EL)})$ dominates both $P^{(EE)}$ and $P^{(LL)}$. This implies that, if firms can choose their timing choice, the simultaneous game will not occur. To check whether this result holds in general, we need to identify the general features of the model.

The log-concavity of demand and strict convexity of cost functions implies that the best response function of the foreign firm, $q_f = q_f(Q^h)$, has the following properties:

Lemma 1) *The best response function of the foreign firm is monotone and decreasing in the home industry output and its slope belongs to the interval (-1,0).*

Proof. The best response of the foreign firm to Q^h is the unique solution to the first order condition

$$\frac{d\pi_f}{dq_f} = p'(Q)q_f + p(Q) - c'(q_f) = 0 \tag{4}$$

whenever the solution is positive. Using the rule of implicit differentiation,

$$b'_f = \frac{dq_f}{dQ^h} = -\frac{p''(Q)q_f + p'(Q)}{p''(Q)q_f + 2p'(Q) - c''(q_f)}. \quad (5)$$

Because of the log-concavity of demand function $p''(Q)q_f + p'(Q) < 0$ or $p''(Q)q_f < -p'(Q)$ and we may conclude that $p''(Q)q_f < -2p'(Q)$. The nominator and the denominator are both negative and the latter is greater in absolute value. Thus $b'_f \in (0, -1)$ whenever q_f is positive. ■

In fact, the best response function of any profit-maximising firm in the standard model shares these properties. But there are some distinguishing features of the mixed duopoly model introduced here that make it different from the standard private duopoly. In a private duopoly, the payoff of a private firm is decreasing in its rival's output. By contrast, the payoff of the public firm is increasing in q_f for any given level of the home industry output[3]. This has some implications for model building to which we shortly return.

Let V be the set of all points representing the best response of the foreign firm to any given level of the home industry output. We assume the welfare of the domestic country can be ranked along the best response function of the foreign firm and,

Assumption 1) *There exists a unique point,* $(Q^{h*}, q_f^*) \in V \equiv \{(Q^h, q_f) \in R^2, q_f \in arg\max \pi_f(Q^h, q_f), Q^h \in A_n\}$ *which maximises the welfare function of the domestic economy.*

Assumption 1 guarantees the existence of the most preferred combination of outputs along the best response of the foreign firm that maximises social welfare of the domestic country[4]. Although the aggregate output is increasing in the home industry production since $\frac{dQ}{dQ^h} = \frac{\partial[Q^h + q_f(Q^h)]}{\partial Q^h} > 0$ as $1 + b'_f > 0$ (recall that $b'_f \in (-1, 0)$ from Lemma 1), Assumption 1 indicates that there is a level of home production at which a further increase in home industry output reduces the domestic welfare. The presence of the foreign firm in the

[3] Note that the first derivative of the welfare with respect to q_f for a given level of Q^h is always positive.

[4] We take the existence of the equilibrium as given to focus on the purpose of this study that is a comparision of equilibria under different regimes. However one could establish the existence result by imposing some extra restrictions on the curvatures of the functions. For instance, the sufficient condition for existing a unique optimum can be derived from $p''(1+b')^2 + b''p' \geq 0$ where $b' = \frac{\partial q_f}{\partial Q^h}$ and $b'' = \frac{\partial^2 q_f}{\partial Q^{h2}}$. This garuantees the strict concavity of $W(Q^h, q_f(Q^h))$ as introduced in (2).

domestic market is associated with two contradicting effects. It increases consumers' surplus by providing the commodity at a lower price. At the same time, it reduces the home industry profits and transfers part of the profits to abroad. Thus, it is not in the domestic economy's interests to supply all the demand only from the domestic producer or the foreign firm when the technology in the home industry is decreasing returns to scale. This consideration forces the domestic government to leave some part of the home market to the foreign competitor. As the following lemma shows, this part will be higher under the sequential order of moves.

Lemma 2) *In an extended game of the mixed duopoly* $\Gamma(.)$*, the home industry production is lower under the public firm leadership than when it moves simultaneously with the foreign firm under Cournot competition.*

Proof. The public firm's problem under the public firm leadership is

$$\underset{(q_n,q_f)\in A_n\times A_f}{Max} W(q_n,q_f) \text{ subject to } q_f \in argmax\ \pi_f(q_n,q_f).$$

The existence of a unique solution is guaranteed by Assumption 1. Hence the necessary and sufficient conditions for the solution point $e^{(EL)*}$ are

$$\begin{cases} p(Q^*) = c'(q_n^*) + p'(Q^*)(1+b'_f)q_f^* \\ p(Q^*) = c'(q_f^*) - p'(Q^*)q_f^* \end{cases} \tag{6}$$

where $Q^* = q_n^* + q_f^*$. If the public firm takes the output of the foreign firm as given and chooses its output simultaneously with the foreign firm to maximise welfare then the first order condition would be

$$\begin{cases} p(\hat{Q}) = c'(\hat{q}_n) + p'(\hat{Q})\hat{q}_f \\ p(\hat{Q}) = c'(\hat{q}_f) - p'(\hat{Q})\hat{q}_f \end{cases} \tag{7}$$

where $\hat{Q} = \hat{q}_n + \hat{q}_f$. Comparing equations (6) and (7), where $b'_f \in (-1,0)$ (Lemma 1), shows that the equilibrium behaviour of the public firm under the public firm leadership is associated with a lower level of output than when it moves simultaneously. Thus $\hat{q}_n > q_n^*$. ■

Figure 2 illustrates the equilibrium outcomes of Cournot competition and the optimum in the firms' strategy space.

Lemma 2 indicates that the Cournot solution point is on the right-hand side of the optimum in Figure 2. ACB is an isoprofit contour of the foreign firm that is associated with the foreign firm's profit at Cournot equilibrium. Any combination of the home industry and foreign firm outputs, (Q^h, q_f), to the left of this contour for the foreign firm is preferable to C. In the strategy

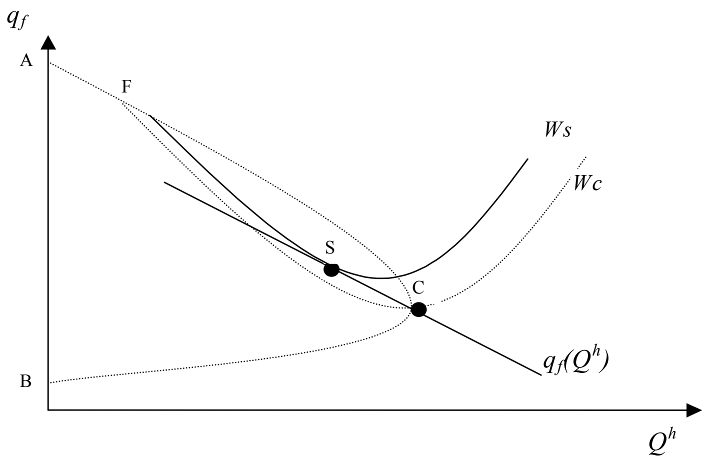

Figure 2: S and C depict the optimum point and Cournot solution respectively.

space, Wc and Ws are isowelfare contours of Cournot and Stackelberg solutions respectively. The set of all points that are trapped between Wc and ACB from C to F Pareto dominate the Cournot competition point. Now we can extend the result of the table to more general forms of demand and cost functions.

Proposition 1) *In an extended game with observable delay of a regulated mixed duopoly* $\Gamma(.)$, *the simultaneous order of moves is not a subgame Nash perfect equilibrium outcome but the public firm leadership is an SPN equilibrium.*

Proof. The home industry output is higher under a simultaneous game than when firms play a public firm leadership game (Lemma 2). But the foreign firm's profit is decreasing in the output level of the home industry. Thus, the foreign firm does not prefer to set its output simultaneously with the public firm at the early period where it can postpone its timing of action later. Since the simultaneous move strategy is also available when the public firm chooses to move early we may conclude that $W^{(EL)} \geq \{W^{(LL)}, W^{(EE)}\}$. As the public firm leadership Pareto dominates to simultaneous play in both periods, $P^{(EL)} \geq \{P^{(EE)}$ OR $P^{(LL)}\}$, it is the SPN equilibrium of the extended game. ∎

Proposition 1 asserts that the SPN equilibrium of the extended game of the mixed duopoly is a sequential order of moves rather than a simultaneous play. Therefore, in the present model, it is the Cournot assumption of simultaneous play that lacks credibility and is vulnerable to Wolfstetter's criticism.

3 An Extension to a Triopoly Model

We now extend the analysis to a triopoly model by adding a domestic private firm to the competition in the domestic market. All underlying assumptions made in the previous section hold.

In this triopoly model the homogeneous good is supplied by a domestic public firm, a domestic private firm and a foreign private firm where the output of the domestic private firm is denoted by q_m. The home industry output is $Q^h = q_n + q_m$. We assume that the government uses a subsidy per unit of the output of the domestic private firm to maximise welfare.

The announcement stage is divided into two periods. First, like the preplay stage in the previous section, firms move simultaneously to choose their time of action and announce it as a commitment. In the later period of the announcement stage, the government, knowing the timing choices of firms, announces the optimal level of subsidy to the domestic firm's production. These assumptions require some modifications of the game introduced in the previous section.

Let $\Gamma' \equiv (N', S', P')$ be the strategic form of the extended game with observable delay of the international mixed oligopoly with subsidisation. Players are the set of firms $N' \equiv \{n, m, f\}$ and the government of the domestic country where m stands for the domestic private firm. Firms choose their outputs from their action set A_i, (where $i \in N'$). The set of the domestic private firm's actions is the same as the public firm. The government chooses the subsidy from a non-negative finite subset of real numbers B in advance of the firms moves. So in action stage, firms know the timing of action and the announced level of subsidy associated with that order of moves. The set of possible times at which firms choose their actions is presented by $T \equiv \{E, L\}$ where E denotes moving early and L denotes moving late.

The set of public firm strategies is $S'_n = T \times \Phi_n$ where Φ_n is the set of functions that maps the information set of the public firm[5] into A_n. Similarly, the set of foreign firm's strategies is $S'_f = T \times \Phi_f$ where Φ_f is the set of functions that maps the information set of the foreign firm into A_f. Likewise $S'_m = T \times \Phi_m$. A strategy of the public firm is $s'_n = (t, q_n(s))$ where $t \in T$,

[5] The information set of the public firm is $\{\{(EEE), (EEL), (ELL),$ $(LLL), (LEE) \times A_m \times A_f, (LLE) \times A_f, (LEL) \times A_m\} \times B\}$ where B is the set of production subsidies.

$s \in B$ and $q_n \in \Phi_n$. For instance, in the case of the equilibrium outcome of a simultaneous game at the first period, $s'_n = (E, q_n^{(EEE)}(s))$ where $q_n^{(EEE)}(s)$ is the Cournot Nash equilibrium output of the public firm. Its strategy is to move early knowing that the foreign firm and the domestic private firm also are moving early and knowing also the announced level of subsidy. Thus it chooses q_n from A_n to maximise welfare, taking the outputs of private firms and the level of subsidy as given. In total, the set of strategies profile is $S' = S'_n \times S'_m \times S'_f$.

The payoff of the domestic private firm is

$$\pi_m = p(Q)q_m - c(q_m) + sq_m. \quad (8)$$

The welfare of the domestic country is the unweighted sum of the home industry profits, consumer surplus less the cost of subsidy. Hence the set of payoffs is $P' = (W', \pi_m, \pi_f)$ where the domestic welfare function W' is

$$\begin{aligned} W' &= \int_0^Q p(t)dt - p(Q)Q + \pi_n + \pi_m - sq_m \\ &= \int_0^Q p(t)dt - p(Q)q_f - c(q_n) - c(q_m). \end{aligned} \quad (9)$$

In the basic game, we may concentrate only on the Nash equilibria in subgames because perfectness rules out any non-equilibrium point in subgames. For a three-player game the reduced strategic form of the extended game is presented in Table 3. Let $e^{(ELL)} \equiv (\hat{s}'_n, \hat{s}'_m, \hat{s}'_f)$ denotes a possible outcome of a setting in the basic game where the public firm acts as a Stackelberg leader and chooses the early period to move while both the foreign private firm and the domestic private firm choose moving later. All payoffs for other equilibria in subgames are as follows.

*Table.*3 General case

I) Public firm chooses moving early (E)

		Foreign Firm	
		Early	*Late*
Domestic Private Firm	E	$(W^{(EEE)}, \pi_m^{(EEE)}, \pi_f^{(EEE)})$	$(W^{(EEL)}, \pi_m^{(EEL)}, \pi_f^{(EEL)})$
	L	$(W^{(ELE)}, \pi_m^{(ELE)}, \pi_f^{(ELE)})$	$(W^{(ELL)}, \pi_m^{(ELL)}, \pi_f^{(ELL)})$

II) Public firm chooses moving late (L)

		Foreign Firm	
		Early	*Late*
Domestic Private Firm	E	$(W^{(LEE)}, \pi_m^{(LEE)}, \pi_f^{(LEE)})$	$(W^{(LEL)}, \pi_m^{(LEL)}, \pi_f^{(LEL)})$
	L	$(W^{(LLE)}, \pi_m^{(LLE)}, \pi_f^{(LLE)})$	$(W^{(LLL)}, \pi_m^{(LLL)}, \pi_f^{(LLL)})$

A simultaneous play is the SPN equilibrium of the extended game if $P'^{(EEE)}$ or $P'^{(LLL)}$ dominate all other equilibria that are feasible at these points. Alternatively, public firm leadership with sequential play could be the equilibrium solution of the game if no firm at $e^{(ELL)}$ is better off by deviating from this order of moves. The results provide a ground to avoid a modelling assumption that has no rationale in the absence of any precommitment mechanism.

Let us first compare the simultaneous versus sequential order of play in a simple example. Consider the linear forms for demand and marginal cost functions with the slopes equal to unity as introduced in Table 2. All possible outcomes of the implied game $\Gamma'(.)$ are summarised in Table 4.

Table.4 A simple example

I) Public firm chooses moving early (E)

		Foreign Firm	
		Early	*Late*
Domestic Private Firm	*E*	[33.96 15.4 1.92]	[34 11.95 2.16]
	L	[34.16 14.39 2.03]	[**34 15.36 2.16**]

II)Public firm chooses moving late (L)

		Foreign Firm	
		Early	*Late*
Domestic Private Firm	*E*	[33.96 10.29 1.92]	[33.96 8.6 1.92]
	L	[34.06 15.25 1.92]	[33.96 15.4 1.92]

Table 4 shows that the simultaneous play in the early period of the basic game cannot be an SPN equilibrium. Under this setting, a comparison of payoffs shows that the foreign firm always prefers to postpone its action time. Also, a situation in which all firms move at the latter period cannot be an SPN equilibrium because the public firm could have improved welfare by deviating from the simultaneous game and moving at the earlier period of the game.

To investigate the timing choice of the firms in a more general framework, we need to go beyond the limitations of the above numerical example. Adopting a log-concave demand and strictly convex cost functions to an international mixed triopoly with subsidy we get the following results.

Proposition 2) *Suppose the government uses the operation of the public firm and a production subsidy to regulate the home industry in the presence of* *a foreign competitor where there exists a unique optimal combination of the home industry output and the foreign firm's output along the best response function of the foreign firm. Then,*

- *a) if the public firm sets its output in advance of the private firms' actions, the optimum is always attainable. Further, the domestic firms, irrespective of their ownership, always follow the adjusted marginal cost pricing (AMCP) condition that asserts*

$$p(Q^*) = c'(q_i^*) + p'(1 + b'_f)q_f^* \quad i = n, m \tag{10}$$

  and, the level of optimal subsidy is equal to

$$s^{*(ELL)} = -p'(Q^*)[q_m^* + (1 + b'_f)q_f^*]. \tag{11}$$

- *b) But if the public firm chooses its output simultaneously with other firms, the home industry output is relatively higher and deviates from the optimum.*

Proof. The proof of Proposition 2.a follows from the proof of Proposition 3 in Sepahvand (2002) where in it has also been proved that the output of the public firm is higher when the public firm moves simultaneously with other firms (Sepahvand 2002, Lemma 3). We need to show that the home industry output is increasing in q_n. We do this in two steps. First let have two domestic public firms. Then there is no subsidy and the equilibrium behaviour of the firms can be characterised by (7) and (6) where in each case we should add another AMCP condition for the new public firm (firm m). From Appendix A we have $\frac{dq_m}{dq_n} > -1$. Thus $\frac{dQ^h}{dq_n} = 1 + \frac{dq_m}{dq_n} > 0$. In the second step, from Sepahvand (2002, Lemma 3) we can argue that if one of the domestic private firm is replaced with a private one and the government uses a production subsidy, the pattern of the equilibrium behaviour of that firm still satisfies the AMCP condition. Therefore, the home industry output is increasing in q_n. Consequently, the simultaneous play in the basic game is associated with a higher level of the public firm's output and the home industry output. ■

Proposition 2 ensures that a simultaneous play with all firms moving late, $e^{(LLL)}$, cannot be an SPN equilibrium of the extended game. However we may still have a simultaneous play if all firms choose moving early, $e^{(EEE)}$. But we will show in the next lemma and proposition that the foreign firm has an incentive to deviate from this setting.

Lemma 3) *In the extended game $\Gamma'(.)$, if the foreign firm chooses moving late and the public firm chooses moving early, the equilibrium level of the home industry output is unchanged regardless of the domestic private firm's choice of action time.*

Proof. If q_m is the equilibrium output level of the domestic private firm at $e^{(EEL)}$, it is required to maximise (8) taking the best response function of the foreign firm, the level of subsidy, and the output of the public firm as given. Thus it should solve

$$\frac{d\pi_m}{dq_m} = p'(Q)(1 + b'_f)q_m + p(Q) - c'(q_m) + s = 0. \tag{12}$$

From the first order condition of the public firm's problem (maximising (9) with respect to q_n, while taking the best response of the foreign firm and the level of the subsidy and q_m as given) we know that it will follow the AMCP condition as introduced by (10). Comparing (12) and (11) we conclude that there exists an optimal subsidy equal to

$$s^{*(EEL)} = -p'(Q)(1 + b'_f)(q_m^* + q_f^*) \tag{13}$$

that belongs to the set of government choice of subsidy, B, and once implemented, it drives the equilibrium behaviour of the domestic private firm to the AMCP condition. From Proposition 2 the domestic firms in the public firm leadership game also follow the AMCP rule. Thus, we can conclude that the SPN equilibrium level of the home industry output in both settings are the same because of the uniqueness of the optimum along the best response of the foreign firm. Therefore, $Q^{h(EEL)} = Q^{h(ELL)}$. ■

Lemma 3 simply asserts that if the foreign firm plans to move late and the public firm chooses moving early, change in the timing of action of the domestic private firm just changes the level of subsidy. But the SPN equilibrium level of the home industry output and the share of each domestic firm in the total home industry output remains unchanged. The following proposition summarises our findings about the simultaneous order of moves.

Proposition 3) *In an extended game with observable delay of a regulated mixed oligopoly market $\Gamma'(.)$, the simultaneous order of moves is not a subgame Nash perfect equilibrium outcome.*

Proof. The simultaneous order of moves where all firms moving late is not an SPN equilibrium of the extended game $\Gamma'(.)$ because of Proposition 2. As Lemma 3 shows $Q^{h(EEL)} = Q^{h(ELL)}$. But from Proposition 2.b we know that $Q^{h(EEE)} > Q^{h(ELL)}$. Recall π_f is decreasing in Q^h. This ensures that the foreign firm always prefers moving late rather than moving simultaneously with domestic firms. Hence the simultaneous order of moves in the first period, $e^{(EEE)}$, is not also an SPN equilibrium outcome of the extended game. ■

So far, we have shown that simultaneous play will not occur in the extended model of the international mixed triopoly. Now we want to evaluate an alternative assumption about the order of moves, namely public firm leadership. The feasible alternatives for the firms in this setting are $e^{(LLL)}$, $e^{(EEL)}$ and $e^{(ELE)}$ (see Table 3). First, as Proposition 2 asserts, welfare of the domestic country is higher if the public firm acts as a Stackelberg leader rather than moving simultaneously with private firms. This implies that the public firm has no incentive to deviate from public firm leadership.

Second, from the domestic private firm point of views public firm leadership is preferable to $e^{(EEL)}$ because of the follow.

Lemma 4) *If the public firm moves first and the foreign firm moves later, the payoff of the domestic private firm is always higher when it chooses moving late.*

Proof. A comparison of (13) and (11) shows that

$$s^{*(ELL)} - s^{*(EEL)} = p'(Q^*)q_m^* b_f' > 0 \tag{14}$$

since $b' \in (-1, 0)$ and $p'(Q) < 0$ everywhere. Therefore, while the domestic private firm produces the same level of output, it will get more subsidy when it chooses moving late. ■

The third case, which concerns the evaluation of the foreign firm's incentives at $e^{(ELE)}$, is not straightforward. Assumption 1 ensures the uniqueness of optimum when a binding constraint on the welfare maximisation forces the equilibrium point to be along the best response function of the foreign firm denoted by V. But if there exists a solution for setting $e^{(ELE)}$, it can be above the best response function of the foreign firm. In this setting, we may speculate that the equilibrium output of the foreign firm will be higher than its output under public firm leadership because in the former one, the government can affect the equilibrium behaviour of the foreign firm indirectly. By "indirectly" we mean that, by changing the level of subsidy, the government can induce the foreign firm to produce a desired level of output via the effect of domestic private firm's output on the foreign firm's decision.

Let U denotes the set of all points along the best response of the domestic private firm that maximises the foreign firm's profit for different levels of subsidy, taking the output of the public firm as given:

$$U \equiv \{(q_n, q_m(s), q_f) \in R^3_+,\ q_f \in \underset{q_f \in A_f}{argmax}\ \pi_f(q_n, q_m(s), q_f)$$
$$s.t.\ q_m \in \underset{q_m \in A_m}{argmax}\ \pi_m(q_n, q_m(s), q_f),\ q_n = \hat{q}_n,\ s \in B\}$$

Figure 3 illustrates three possible points of U in the two-dimension space for a given level of q_n.

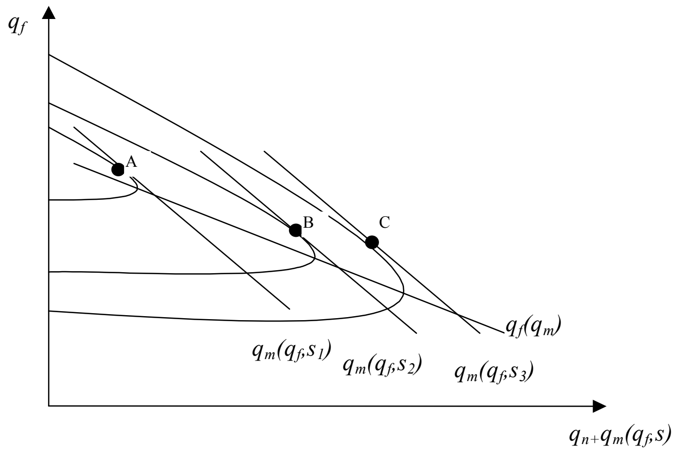

Figure 3: A,B and C are three elements of U for different levels of subsidy.

The graph of the best response of the domestic private firm is the tangency to the isoprofit contour at A if the level of subsidy is s_1. The government can choose points B or C if it sets the level of subsidy at s_2 or s_3. The following lemma explains the effect of the timing choice of the foreign firm on its equilibrium output when the public firm moves early and the domestic private firm chooses moving late.

Lemma 5) *Suppose* $(\tilde{Q}^h, \tilde{q}_f)$ *is an element of* V *where* $\tilde{Q}^h = \hat{q}_n + q_m(\tilde{s})$, *then there exists an element* $(\hat{q}_n, q_m(\hat{s}), \hat{q}_f) \in U$ *such that* $\tilde{Q}^h = \hat{Q}^h$. *Furthermore* $W(\tilde{Q}^h, \tilde{q}_f) \leq W(\hat{Q}^h, \hat{q}_f)$.

Proof. Let $(\tilde{Q}^h, \tilde{q}_f)$ be an element of V where $\tilde{Q}^h = \hat{q}_n + q_m(\tilde{s})$. Taking the public firm's output level given at $\hat{q}_n$, the government may choose a level of subsidy, $\hat{s}$, for which the domestic private firm's best response function is the tangency of the foreign firm's isoprofit contour and $\hat{Q}^h = \hat{q}_n + q_m(\hat{s})$ equals $\tilde{Q}^h$. Then $(\hat{q}_n, q_m(\hat{s}), \hat{q}_f) \in U$ by definition and $\tilde{q}_f \leq \hat{q}_f$ due to the decreasing slope of the domestic private firm's best response function. But the welfare function (9) is always increasing in q_f for any given level of the home industry output as $\frac{\partial W(\tilde{Q}^h, q_f)}{\partial q_f} = -p'(Q)q_f$ and $p'(Q) < 0$. Therefore $W(\tilde{Q}^h, \tilde{q}_f) \leq W(\hat{Q}^h, \hat{q}_f)$. ■

Lemma 5 simply asserts that for any given level of the home industry output along the graph of the foreign firm's best response function, the foreign firm with a first-mover advantage produces more output and this favours the home economy. Recall that, because of the log-concavity of the (inverse) demand function, the Cournot solution is on the right-hand side of the public firm Stackelberg leadership solution, (Q^{*h}, q_f^*). This implies that in the neighbourhood of (Q^{*h}, q_f^*), for a given level of the foreign firm's output, an increase in the home industry output promotes welfare for the domestic country. Therefore, we may conclude that if q_f^* is achievable at a higher level of the home industry output when the foreign firm moves early, that would be preferable for the domestic country. The following proposition summarises our results.

Proposition 4) *In the extended game with observable delay* $\Gamma'(.)$, $e^{(ELL)}$ *is an SPN equilibrium solution.*

Proof. Consider a situation in which the public firm chooses moving early and the other two move late. In the first place, the public firm cannot be better off if it chooses moving late since $W^{(ELL)} > W^{(LLL)}$ (Proposition 2). Second, the domestic private firm will not change its timing choice from the order of moves under public firm leadership as this results in a reduction in the level of optimal subsidy and consequently its profit (Lemma 4). Third, the foreign firm will also stick to moving late because if the foreign firm chooses moving early for any given level of the home industry output the domestic government can implement an appropriate level of subsidy at $e^{(ELE)}$ to induce a higher level of the foreign firm's output (Lemma 5). But given the home industry output at $e^{(ELL)}$, an increase in the foreign firm's output results in a reduction in its profit. Thus its profit falls if it moves early. As no firm is willing to deviate from the order of moves under public firm leadership, $e^{(ELL)}$ is the SPN equilibrium of the extended game with observable delay $\Gamma'(.)$. ■

Proposition 4 extends the outcomes of Table 4 to a fairly general framework. In that simple example, the public firm leadership setting is the only SPN equilibrium of the extended game with observable delays. Nevertheless, in general we may have other SPN equilibrium solutions with different orders of play. In the case of the existence of multiple equilibria, further assumptions are required to select one over the others. However, none of those multiple equilibria could be a simultaneous play strategy. Therefore this will not affect the general results that we obtained here.

4 Coordination Problem and Predictability

The subgame perfect Nash equilibrium concept rejects any strategy which is incorrect or inconsistent with the players' rationality in all subgames. But it may accept many equilibria that makes it difficult to predict the result of the game.

If the supergame in the extended model has a unique SPN equilibrium, the result of the game is predictable. In our model of a mixed duopoly this happens when the demand function is strictly concave. Then assuming that the welfare function is strictly concave in the home industry output, the best response function of the public firm is upward-sloping[6]. Thus it does not enter in the Pareto superior set relative to the simultaneous play solution.

When the demand function is not concave but log-concave, the graphs of domestic public firm and the foreign firm are downward-sloping and they both may enter in the Pareto superior set to the simultaneous play. Then the game has multiple equilibria. The public firm leadership equilibrium is at point S in Figure (4). In addition, point D where the isoprofit curve of the foreign private firm is tangent to the public firm's best response curve is another possible equilibrium solution that is associated with a game in which the private firm acts as a Stackelberg leader and the public firm acts as a follower.

With two sequential equilibria, firms face the coordination problem. How do the firms know which equilibrium strategy to play? One possibility is that these two equilibria could be ranked based on the Pareto-efficiency criterion. Then if the firms can talk beforehand, the choice of the Pareto superior outcome arises from this pregame communication. Nevertheless, even when firms do not communicate, but they repeat the game time after time, the same result is predictable.

It is not always possible to rank the equilibria. Both equilibria solutions could be Pareto efficient in the sense that it is not possible to increase the payoff of one firm without decreasing the payoff of the other firm. In this case, the game resembles the battle-of-the-sexes. Then, if by any mistake,

[6] If there exists a solution for the public firm's maximisation problem when $p''(Q) < 0$, then the slope of the public firm best response function which is $dQ^h/dq_f = -\frac{-p''(Q)q_f}{-p''(Q)q_f + p'(Q) - c''(Q^h)}$ is greater than zero.

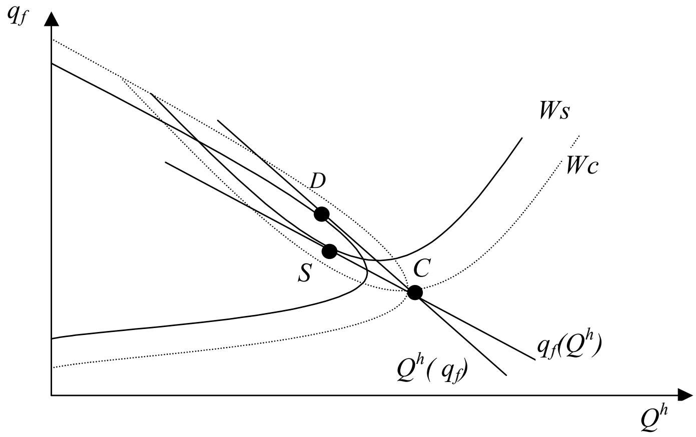

Figure 4: Points S and D are two possible equilibria of the extended game.

the game results in a simultaneous move, both firms are losers. Thus there is some room for bargaining between the firms. The result of the game then depends on the firms' bargaining power. For instance, when the domestic government is determined in having the foreign firm in the domestic market, it might cut the capacity-expanding subsidy to the public firms to show its commitment to the public firm leadership game and give the second-mover-advantage to the private firm.

As a result, the outcome of the game between domestic public enterprise and a foreign private firm with multiple equilibria may not be predictable. One may observe the simultaneous play outcome. But that only happens because of the coordination problem, and it indicates that the firms have mistaken about each other's strategies. Although we may not be able to predict the exact outcome of the game, we still know for sure that the most plausible assumption about the order of play is a sequential one.

5 Conclusion

In a quantity-setting game it is common to assume that firms are involved in a Cournot competition and play simultaneously. That is because without a precommitment mechanism which is available just to one of the profit-maximising firms, the only credible strategy (or the natural order of play) is a simultaneous play. But as Hamilton and Slutsky (1990) show, it is not only the type of strategic choice variable that determines the order of play. The natural order of play also depends on the firms' preferences and available information. In certain situations the Stackelberg competition could be the most plausible modelling assumption even when firms compete in quantities. We have shown that this applies for international mixed oligopoly models. In fact, industries with mixed market structure are typically dominated by former public monopolies and these industries more closely resemble Stackelberg public firm leadership rather than Cournot oligopoly with simultaneous order of play.

The result of this study has a crucial impact on the analysis of privatisation in this context. The previous studies built their model on Cournot competition and insist on the distortionary effect of the presence of a public enterprise in a domestic market that is opened to international competition even if it operates as efficient as its private counterparts. But modelling the firms' interactions by a more plausible assumption, Stackelberg public firm leadership game, indicates that the presence of a domestic public enterprise does not necessarily destrots competition in the home market. Furthermore, if privatisation does not bring about improvement in organisational (internal) efficiency of the public enterprise, it does not affect the competition in the market and entails no welfare effect.

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1. **Appendix A**

The public firm m chooses q_m to maximise welfare function (9). From the first order condition for an interior solution we have

$$\frac{dW(.)}{dq_m} = B(Q^h, q_f) - c'(q_m) = 0 \tag{15}$$

where $B(Q^h, q_f) = \partial[\int_0^Q p(t)dt - p(Q^h, q_f)q_f]/\partial q_m$ and $Q^h = q_m + q_n$. The rule of implicit differentiation can be applied to show that,

$$\frac{dq_m}{dq_n} = -\frac{\partial B(.)/\partial q_n}{\partial B(.)/\partial q_m - c''(q_m)}. \tag{16}$$

Note that $\partial B(.)/\partial q_n = \partial B(.)/\partial q_m$. If the solution for this maximisation problem exists then $\partial B(.)/\partial q_m - c''(q_m) < 0$. As $c''(q_m) > 0$ everywhere it follows that $\frac{dq_m}{dq_n} > -1$.