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# Loss Aversion and the Tullock Paradox

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**Abstract**

We show that the presence of loss aversion on the part of participants in a Tullock imperfectly discriminating contest will significantly reduce the proportion of the rent dissipated in the form of resources used up in the competition for that rent. We also suggest a simple experiment that can reveal whether contestants are, indeed, loss averse.

Key words: rent-seeking, contests, loss aversion, rent dissipation
JEL classification: C72, D72, D80

## 1 Introduction

Since the seminal discussion of rentseeking behavior by Tullock [14], a recurring theme in the literature on contests is the concern that, in some sense, the theoretical models explain too much [9]. They generally predict that, at least in the limit as the number of contestants becomes large, close to 100% of the value of an exogenous rent will be dissipated in the form of resources used up in the competition for that rent. Much of the recent anthology edited by Lockard and Tullock [7], subtitled 'Chronicle of an Intellectual Quagmire', can be read as a blow-by-blow account of attempts by various authors to escape this conclusion, only to be 'pushed back into the bog' by Tullock.

The puzzle arises because empirical evidence suggests, at least to Tullock, that the proportion of rents actually dissipated falls substantially short of the 100% predicted by the theory. Serious attempts to measure the extent of rent dissipation are few in number. Sobel and Garrett [11] provide an estimate of the total level of rentseeking activity in the US, and conclude that it is quantitatively significant. However, they do not identify specific contests and their associated rents, and their results do not allow one to infer the level of dissipation associated with an individual contest. Hazlett and Michaels [4] estimate that, in the FCC cellular telephone bandwidth lotteries, approximately 38% of the rents were dissipated by the expenses associated with the preparation of applications - a significant proportion, certainly, but far from full dissipation.

Various possible escape routes from the theoretical prediction have been explored - the existence of risk aversion, strategic effects, technological effects and asymmetry under the assumption of diminishing returns to scale have all been examined. Elsewhere, we have scrutinised these various suggestions and argued that they do not present serious challenges to the prevailing theoretical prediction [1], [3]. In this paper, we suggest an alternative route. We consider the implications of loss aversion on the part of contestants. Two significant conclusions emerge. First, if contestants are loss averse, then the Nash equilibrium of an imperfectly discriminating contest will reduce dissipation relative to the equilibrium level when loss aversion is absent. Moreover, the expression for the limiting extent of dissipation can be related in a very simple way to the strength of contestants' loss aversion and, by contrast with the reduction arising from returns to scale, it is not susceptible to further dissipation in small entry costs. Second, the contest model suggests a simple way of testing for the presence of loss aversion. We present two slightly different contests which, in the absence of loss aversion, have the same observational implications. However, if contestants are loss averse, the predictions of the two models differ in a way that can be specified very precisely, thereby providing a neat way of detecting loss averse behavior.

In the next section, we characterize and explore the properties of the Nash equilibria of Tullock contests with loss averse contestants. The analysis focuses on symmetric equilibria of symmetric contests but includes an informal discussion of various extensions of this model. In Section 3, we study a version of the contest with positive entry costs to investigate the effect of loss aversion on rent dissipation and, in the following section, we compare contests with divisible and indivisible rents of the same magnitude.

In a Technical Appendix, we determine the complete (not just symmetric) set of equilibria of symmetric contests with loss averse contestants. In particular, we show that multiple equilibria are possible but, if the kink in the value function attributable to loss aversion is not too sharp, the equilibrium will be unique.

# 2 Rentseeking by loss averse contestants

A loss averse individual is one who, starting from a given initial wealth, places a higher value on the loss of a given amount than on the gain of identical magnitude. Such an individual, expecting to find \$100 in her wallet, will be pleased, but only mildly so, to discover \$110. On the other hand, the discovery of only \$90 produces a relatively strong feeling of dismay and loss. A variety of observed behavioral responses that are difficult to reconcile with expected utility theory can, it has been argued, be readily explained by supposing that individuals are loss averse. When the contestants are firms, similar behavior could be explained by agency failure. A manager whose rentseeking activities fail to secure an indivisible rent may find it much harder to justify expenditure securing no tangible benefit than if they had won the rent. If this leads the manager to undervalue a gain relative to a loss of the same size, their behavior may be similar to that of a loss averse individual.

Loss aversion asserts that losses from some reference level figure larger than gains. To apply such a behavioral assumption requires specification of the reference level. Initially, we make the simplest assumption: *ex post* wealth is referenced to current income which, without loss of generality, we set to zero. Further discussions of loss aversion can be found in [5], [8], [12], [15] and [16]. Recent experiments by Schmidt and Straub [10] indicate the presence of loss aversion in non-strategic choice situations.

A simple and common way to capture loss aversion is to suppose that a gain in wealth is evaluated at $\theta$ times the same loss in wealth, where $0 < \theta < 1$. This can be applied to a standard Tullock rentseeking model with $n$ contestants in which contestant $i$ chooses a level of expenditure $x_i$ to devote to contesting an exogenously fixed and indivisible rent $R$. We consider the classic Tullock form of contest success function in which the probability that contestant $i$ wins is given by $x_i^r / \sum_{j=1}^n x_j^r$, where $r \leq 1$ captures the returns to scale of the rent-seeking technology. The contestants are therefore engaged in a symmetric simultaneous-move game in which the payoff of player $i$ is

$$\pi_i = \frac{x_i^r}{\sum_{j=1}^n x_j^r} \min\{\theta(R - x_i), R - x_i\} - \left[1 - \frac{x_i^r}{\sum_{j=1}^n x_j^r}\right] x_i, \tag{1}$$

recalling that the reference level of wealth is zero for each player. Note that any $x_i$ exceeding $R$ is strictly dominated (by $x_i = 0$) so the minimum can be replaced by $\theta(R - x_i)$ without changing the set of equilibria. The next result characterizes the symmetric equilibrium of this game.

**Theorem 1** *If* $0 < r \leq 1$, *there is a unique symmetric Nash equilibrium of the rent-seeking game in which the expenditure of each player is*

$$\widehat{x}^n(\theta) = \frac{\theta r R (n-1)}{n^2 - (1-\theta)(n + rn - r)}.$$

**Proof.** To analyze this game it is convenient to transform the expenditure variable $x_i$ into an input variable $y_i = x_i^r$ so that the payoff can be written

$$\widetilde{\pi}_i = \frac{\theta R y_i}{Y} - y_i^{1/r} + \frac{(1-\theta) y_i^{(r+1)/r}}{Y}, \tag{2}$$

where the sum of all contestants' inputs is denoted by $Y \equiv \sum_{j=1}^n y_j$. There will be a symmetric pure-strategy Nash equilibrium $y_i = \widehat{y}$ for all $i$ if and only if $y_1 = \widehat{y}$ is a best response to $y_2 = \cdots = y_n = \widehat{y}$. Writing $f(w, y) = \widetilde{\pi}_1(w, y, \ldots, y)$,

$$\frac{\partial f}{\partial w}(y, y) = 0$$

simplifies to

$$\theta r R (n-1) = \left\{n^2 - (1-\theta)(n + rn - r)\right\} y^{1/r}.$$

Transforming $\widehat{y}$, the unique solution to this equation, back into effort gives the expression in the theorem. To complete the proof, it is necessary to confirm that $f(\widehat{y}, \widehat{y}) \geq f(w, \widehat{y})$ for all $w$. Note that $f$ need not be a concave function of $w$. However, after some manipulation, we have

$$\frac{\partial^2 f}{\partial w^2}(\widehat{y}, \widehat{y}) = -\frac{(1-r)(n-1) + \theta(1+r)}{n r^2 \widehat{y}^{(2r-1)/r}} < 0,$$

where the inequality follows from $0 < r \leq 1$, $n \geq 2$ and $\theta > 1$. Since $f(0, \widehat{y}) = 0 > f(R^r, \widehat{y})$, we conclude that there is a unique stationary point which must be a global maximum. ■

We can make a number of observations on this theorem.

1. Although there is a unique symmetric equilibrium, there may be additional asymmetric equilibria. More specifically, we show in the Technical Appendix below that, if $r = 1$, there is a unique equilibrium if and only if $\theta \geq \frac{1}{2}$ (Corollary 7). When $r = 1$ and $\theta < \frac{1}{2}$, there will be alternative asymmetric equilibria for all large enough $n$ (Corollary 6). Even if $r < 1$, the equilibrium is unique provided $\theta r^2 \geq 1 + r$ (Corollary 7). We also have uniqueness when $r < 1$ for all large enough $n$ (Corollary 8).

2. The symmetric equilibrium may display counter-intuitive comparative statics with respect to the number of contestants. For example, if $r = 1$ and $\theta < \frac{1}{2}$, aggregate expenditure on rent-seeking: $n\widehat{x}^n(\theta)$ is decreasing in $n$ for all large enough $n$. Similar results can hold for $r < 1$. Under loss-neutrality, increasing aggregate expenditure implies that payoffs decrease in the number of players. In the technical appendix, we show that this result continues to hold when contestants are loss averse, even if aggregate expenditure is decreasing in $n$.

3. Comparative statics of loss aversion are as expected: increasing aversion to loss reduces equilibrium expenditure. This is readily seen by rewriting $\widehat{x}^n(\theta)$ in the form

$$\widehat{x}^n(\theta) = \frac{rR(n-1)}{n+rn-r}\left\{1 - \frac{(n-1)(n-r)}{n^2 - (1-\theta)(n+rn-r)}\right\}.$$

   which shows that, as $\theta$ falls, so does $\widehat{x}^n(\theta)$. In particular, loss averse contestants devote less effort to rent-seeking than otherwise identical loss neutral ($\theta = 1$) contestants.

4. As $n \longrightarrow \infty$, so $n\widehat{x}^n(\theta) \longrightarrow \theta rR$. Equilibrium expenditure devoted to rent seeking in a large contest is reduced by a factor equal to the loss aversion ratio. In the frequently studied case where $r = 1$, loss neutral contestants would exhaust the whole value of the rent whereas, if loss averse, only a proportion $\theta$ would be spent. Tversky and Kahneman [15] suggest that a loss aversion ratio of about $\frac{1}{2}$ is consistent with the much of the experimental and empirical evidence, at least for small or moderate changes in wealth (though much smaller values may also be observed, for example when health risks are involved ). With such a value, loss aversion can account for 'missing' rent-seeking effort up to half the value of the rent.

5. In some instances - for example, if a loss averse manager sets a positive revenue target - it may be appropriate to analyze a reference level, $a$, different from zero. If we maintain the same ratio of slopes: $\theta$, the payoff can be written

$$\pi_i = \frac{x_i^r}{\sum_{j=1}^n x_j^r}\theta\left(R^* - x_i\right) - \left[1 - \frac{x_i^r}{\sum_{j=1}^n x_j^r}\right]x_i - a,$$

where $R^* = R - 1 + 1/\theta$, provided winnings net of expenditure exceed $a$ and expenditures exceed $-a$. Hence, if $0 < a + x^* < R^*$, where $x^*$ is given by the formula for $\widehat{x}^n\left(\theta\right)$ in the theorem but with $R$ replaced by $R^*$, the profile $x_i = x^*$ is a Nash equilibrium. It follows that, if $0 \leq a < R^*$, aggregate equilibrium expenditure approaches $r\left[\theta R + \left(1 - \theta\right)a\right]$ as $n \longrightarrow \infty$: a positive reference level may offset some of the reduction arising from loss aversion.

The intuition behind these results rests on the observation that, since winning the rent increases wealth, loss aversion decreases its value by a factor of $\theta < 1$. However, loss aversion also effectively decreases the cost of expenditure on rent-seeking in the case (and only in the case) of a win. Hence, the aggregate expenditure is decreased relative to the case of risk neutrality but by a factor less than $\theta$. Furthermore, if $\theta$ increases, the increase in effective value of the rent leads to an increase in expenditure. Observation 3. shows that the partial offset due to the effective reduction in the cost of successful rent-seeking does not reverse this conclusion. When there are many contestants, the expenditure of each contestant is sufficiently small relative to the rent that it can be ignored, in which case the reduction in aggregate expenditure is equal to $\theta$ (cf. Observation 4).

These observations allow us to discuss several extension of these results to the case where the technological coefficient $r$ differs between the contestants, although we do not undertake a formal analysis. Even in this asymmetric case, the aggregate expenditure will be reduced (by a factor between $\theta$ and 1). When there are many contestants and all individual expenditures are small, for example in a contest in which there are many players of each of a finite number of types, the reduction in aggregate expenditure will be nearly equal to $\theta$.

Returning to the symmetric case, another generalization is to contest success functions of the form $f\left(x_i\right)/\sum_{j=1}^n f\left(x_j\right)$, where $f$ is a concave function satisfying $f\left(0\right) = 0$. In [2], we showed that, if the elasticity $xf'\left(x\right)/f\left(x\right)$

has a limit $\eta$ as $x \longrightarrow 0$, aggregate expenditure approaches $\eta R$ as $n \longrightarrow \infty$. The intuition is that Tullock's contest success function with $r = \eta$ is a good approximation for the small individual expenditures found in symmetric equilibria when $n$ is large. For the reasons set out above, loss aversion will again reduce limiting aggregate expenditure by the factor $\theta$.

We have been studying imperfectly discriminating contests in which any active contestant has a positive probability of winning the contest irrespective of the strategies chosen by her rivals. By contrast, the winner of a perfectly discriminating contest is the player who lays out the largest expenditure, with ties resolved in some arbitrary fashion. Under an assumption of perfect information, the only symmetric equilibrium entails the use of mixed strategies. Nevertheless, expected aggregate expenditure will be reduced by a multiplier equal to $\theta$ in a large contest and by less than this when there are few contestants. These results are also true of (mixed-strategy) symmetric Nash equilibria of symmetric Tullock contests with increasing returns: $r > 1$.

# 3 Entry costs

Early studies of rent seeking (e.g. Tullock [13] and Krueger [6]) generally assumed a competitive model with free entry leading to the conclusion that the value of rent seeking activities would be equal to the value of the rent. With decreasing returns, as embodied in the contest success function implicit in the payoff function (1), this suggests that all the rent would be dissipated in a Tullock contest for large $n$. This is indeed the case when $r = 1$, but for $r < 1$ a proportion $1 - r$ of the rent remains undissipated when contestants are risk neutral. In [2], we argued that, if there are entry costs to participation in the contest and these are taken into account in calculating the total outlay on rent-seeking, the full value of the rent will be dissipated even if $r < 1$ when these costs are small. It is appropriate to ask whether loss aversion as an explanation of incomplete dissipation is susceptible to the same argument.

A simple two-stage model to address this question has $N\,(\geq 2)$ identical potential contestants. In the first stage, these players decide whether or not to enter and pay a positive entry fee $\kappa$, which is less than the value of the rent. Those who choose not to enter receive a payoff of 0. The remaining players take part in the final stage: a contest with payoffs given by (1) but reduced by the entry fee $\kappa$. Specifically, if $n \geq 2$ potential contestants choose to enter and the expenditure of the $j$'th entrant in the final stage is $x_j$ for

$j = 1, \ldots, n$, the $i$'th entrant receives a payoff of

$$\widetilde{\pi}_i = \frac{x_i^r}{\sum_{j=1}^n x_j^r} \min \left\{ \theta \left( R - x_i - \kappa \right), R - x_i - \kappa \right\} - \left[ 1 - \frac{x_i^r}{\sum_{j=1}^n x_j^r} \right] \left( x_i + \kappa \right). \tag{3}$$

If there is only one entrant, we assume that the rent is awarded to that player without requiring her to make any further expenditure, resulting in a payoff of $\theta \left( R - \kappa \right)$. Note that final-stage expenditure exceeding $R - \kappa$ yields a smaller payoff than staying out (i.e. zero) and will not be played in a subgame perfect equilibrium. This means we can replace the minimum in (3) with $\theta \left( R - x_i - \kappa \right)$.

Our formulation assumes that loss-averse players measure *overall* wealth changes relative to the reference level of zero. In particular, entrants do not re-set their reference level between paying the entry fee and determining their expenditure on rent-seeking in the final stage. Re-setting the reference raises questions of dynamic inconsistency which cast doubts on the use of subgame perfection as a solution concept which we do not wish to pursue here. In any case, we focus principally on small entry fees so ignoring the effects of re-setting should be a good approximation. Alternatively, we can assume that entry fees are paid at the same time as expenditure on rent seeking.

To avoid problems with multiple equilibria and coordination, we suppose that players make their entry decisions in sequence. Thus potential contestants are arranged in order and all players after the first know the decisions taken by their predecessors. To prevent ambiguity, we assume that a player indifferent between entering and staying out chooses the latter. Our conclusions do not depend in an essential way on this assumption. Intuitively (and proved formally below), equilibrium individual payoffs net of entry cost decrease with $n$ and fall to zero as $n \longrightarrow \infty$. This means that, in equilibrium, all entrants precede non-entrants in the sequence. Furthermore, all entrants receive a positive payoff and non-entrants (if any) would have made a loss or received zero payoff if they had decided to enter. These conditions determine the equilibrium number of entrants, as set out in the next theorem, as well as the final-stage strategies.

**Theorem 2** *If $0 < r \leq 1$, there is a subgame perfect Nash equilibrium of the contest with entry costs, in which the first $m$ potential contestants enter the final stage, where $n = \min \left\{ \nu \left( \theta \right), N \right\}$ and $\nu \left( \theta \right)$ satisfies (uniquely)*

$$\frac{\left[ \nu \left( \theta \right) - 1 \right] \left( 1 - r \right) + \theta}{\left[ \nu \left( \theta \right) - 1 + \theta \right]^2} \theta R > \kappa \geq \frac{\nu \left( \theta \right) \left( 1 - r \right) + \theta}{\left[ \nu \left( \theta \right) + \theta \right]^2} \theta R. \tag{4}$$

*The expenditure of each entrant is* $\overline{x}^{\nu(\theta)}(\theta)$*, where*

$$\overline{x}^{n}(\theta) = \frac{r\left[\theta R + (1-\theta)\kappa\right](n-1)}{n^2 - (1-\theta)\left[n + rn - r\right]} \tag{5}$$

*and this equilibrium is unique amongst those with symmetric play in the final stage.*

**Proof.** If $\nu(\theta) = 1$, then $\overline{x}^{n}(\theta) = 0$ which makes the assertions in the final sentence trivial. (Recall that a single entrant is awarded the rent without further expenditure.) If $\nu(\theta) \geq 2$ these assertions can be justified by rearranging the expression for the payoff net of entry costs (3) as

$$\widetilde{\pi}_i = \frac{\theta x_i^r}{\sum_{j=1}^n x_j^r}(R' - x_i) - \left[1 - \frac{x_i^r}{\sum_{j=1}^n x_j^r}\right] x_i - \kappa,$$

where $R' = R + (1-\theta)\kappa/\theta$. Comparison with (1) shows that Theorem 1 can be applied with $R$ replaced by $R'$. This gives the expression in the theorem.

The payoff of each player who enters the final round is $\overline{\pi}^{\nu(\theta)}$, where

$$\overline{\pi}^n = \frac{n - 1 + r + \theta - rn}{D(n)}\theta R' - \kappa$$

and

$$D(n) = n^2 - n - rn + r + \theta n + \theta rn - \theta r.$$

Furthermore, $\overline{\pi}^n$ is strictly decreasing in $n$. This can be seen by direct computation:

$$\overline{\pi}^{n+1} - \overline{\pi}^n = -\frac{(1+r)\theta^2 + (2n-1)\theta + (1-r)n(n-1)}{D(n+1)D(n)}\left[\theta R + (1-\theta)\kappa\right] < 0.$$

(The inequality uses the fact that $D(n) \geq \theta > 0$ for $n \geq 1$.) In addition, $\overline{\pi}^n \longrightarrow 0$ as $n \longrightarrow \infty$. It follows that players will enter until either an additional entrant would reduce equilibrium payoffs in the final stage below the entry fee:

$$\overline{\pi}^{\nu(\theta)} > 0 \geq \overline{\pi}^{\nu(\theta)+1} \tag{6}$$

or until all players have entered. The latter holds in equilibrium if and only if $\overline{\pi}^N > 0$; that is $N < \nu(\theta)$. The inequalities (6) can be re-arranged using the expression for $\overline{\pi}^n$ to give (4). ■

We can make a number of observations on this theorem.

1. If the final stage equilibrium is unique, so is the subgame perfect equilibrium of the multistage game. From Corollary 7, this occurs if (but not only if) $\theta r^2 \geq 1 + r$.

2. The characterization of the number of entrants depends only on two conditions: (i) the payoff (net of entry costs) to entrants be positive and (ii) this payoff would become non-positive were one more player to enter. Any alternative model of the entry process in which these conditions were necessary in equilibrium would still have the same equilibrium number of entrants and therefore the same total outlay (including entry costs). For example, if entry decisions were made simultaneously and independently in the first stage of a two-stage game, the second stage being the contest played by entrants, these conditions are necessary for any pure-strategy equilibrium and therefore all consequences of the characterization in the theorem will apply. Of course, there will typically be multiple pure-strategy equilibria (for example, by permuting the set of players) and the sequential model can be seen as a simple way of resolving the resulting coordination problem.

3. The equilibrium number of entrants increases as entry costs fall. This follows from the fact, derived in the proof, that payoffs in the Nash equilibrium final stage decrease with the number of entrants. Furthermore, as $\kappa \longrightarrow 0$, so $\nu(\theta) \longrightarrow \infty$. This becomes obvious if the right hand inequality in (4) is re-written as
$$[\nu(\theta) + \theta]^2 \geq \frac{[\nu(\theta) + \theta](1 - r) + \theta r}{\kappa} \theta R.$$
Thus, the number of entrants grows without bound as the entry cost becomes small, unless restricted by the number of potential contestants, $N$. To focus on almost free entry, we analyze the *small cost limit* in which, as $\kappa \longrightarrow 0$ we change $N$ to exceed $\nu(\theta)$.

4. From the theorem, the small cost limit of the aggregate expenditure in the final (contest) stage is
$$\lim_{\kappa \longrightarrow 0} \nu(\theta) \overline{x}^{\nu(\theta)}(\theta) = \theta r R.$$
This is the same as the large game limit found in the previous section. The total spent on entry fees will satisfy:
$$\lim_{\kappa \longrightarrow 0} \kappa \nu(\theta) = \theta (1 - r) R > 0.$$

This limit can be established by multiplying (4) throughout by $\nu(\theta)$ and observing that the left and right hand sides both approach $\theta(1-r)R$ as $\nu(\theta) \longrightarrow \infty$. (If $r=1$, aggregate entry fees will vanish in the limit.) Adding entry costs to final stage expenditure, shows that total outlay approaches $\theta R$ as $\kappa \longrightarrow 0$, independent of $r$. When players are loss neutral ($\theta = 1$), the reduction in rent seeking arising from decreasing returns in the rent-seeking technology ($r < 1$) is entirely offset by expenditure on entry, leading to full dissipation when entry fees are small and positive. Under loss aversion, however, whilst that part of the incomplete dissipation arising from technological effects is again eliminated by entry costs, that part of the reduction in expenditure attributable to loss aversion remains unaffected.

5. Loss aversion results in incomplete dissipation, even with non-negligible entry costs. If $n$ players enter the final round in equilibrium, (4) places upper and lower bounds on the entry cost. For example, if $\kappa$ lies just below the upper bound, aggregate expenditure in the final stage is

$$n\overline{x}^n(\theta) \simeq \frac{rn(n-1)\theta R}{(n-1+\theta)^2}.$$

When entry costs are included, total outlay: $n[\overline{x}^n(\theta)+\kappa]$ becomes

$$\frac{n\theta R}{n-1+\theta}.$$

For example, if $\theta = \frac{1}{2}$ and there are two, (three, four) entrants then almost two thirds, (three fifths, five sevenths) of the rent is dissipated.

The intuition for these observations is straightforward. For loss neutral contestants, infinitesimal but positive entry fees induce entry until payoffs are driven almost to zero. Under the same conditions, in equilibrium, loss averse potential contestants will also choose to enter until payoffs become small. However, the probability of winning is small $[1/\nu(\theta)]$ which means that the value of winning is almost $\theta R$ as measured relative to costs of losing, whether these consist of entry fees or expenditure in the final stage. Hence, total expenditure is close to $\theta R$ and a proportion $1-\theta$ of the rent remains undissipated. As for the single stage model of the previous section, when entry fees are not small, the number of entrants is not large and the value of fees and expenditure in the final stage is reduced with non-negligible probability which has the potential to offset the reduction in value of the rent caused by loss aversion.

# 4 A comparison of two contests

The standard contest in which all players are risk and loss neutral is open to an alternative interpretation. Although we viewed the contest as one in which each player receives the full rent $R$ with a probability of $p_i = x_i^r / \sum_{j=1}^n x_j^r$, we could equally well have interpreted it as one which, with certainty, gives each contestant a share of the rent equal to $p_i$. The two games - the one with an indivisible rent, the other with a divisible rent - are strategically equivalent. In particular, both yield the same predicted relationship between the proportion of rent dissipated and the number of contestants. Under loss aversion, however, the two games have distinct equilibria.

The analysis of a contest in which a divisible rent of $R$ is shared in proportion to $x_i^r$ is straightforward. With no entry fees, contestant $i$'s payoff function is

$$\pi_i^D = \min \left\{ \theta \left( p_i R - x_i \right), \left( p_i R - x_i \right) \right\}$$

where $p_i = x_i^r / \sum_{j=1}^n x_j^r$. Equilibrium payoffs can never be negative, so the payoff function can be replaced with $\theta \left( p_i R - x_i \right)$ which is the payoff to a loss neutral contestant (up to a positive multiplicative constant). So, the Nash equilibrium can be found by applying Theorem 1 with $\theta = 1$. The following corollary of Theorem 1 can be exploited to compare equilibria of the two contests. The proof is given in Observations 3. and 4. following the theorem.

**Corollary 3** *Suppose $0 < r \leq 1$ and $0 < \theta < 1$, and $\widehat{x}_i(\theta)$ is defined as in Theorem 1. Then $\widehat{x}_i(\theta) < \widehat{x}_i(1)$ and $\widehat{x}_i(\theta) / \widehat{x}_i(1) \longrightarrow \theta$ as $n \longrightarrow \infty$.*

Equilibrium aggregate expenditure on rent seeking with a divisible rent and loss averse contestants exceeds that for an indivisible rent of the same magnitude. This conclusion assumes that probabilities in the latter case are equal to the proportions in the former and that value functions are linear except at the origin. This latter assumption will be particularly appropriate for small rents as found, for example, in experiments and leads us to predict that, in experimental contests, making the rent indivisible will reduce expenditure. With a large number of contestants, the reduction factor will be equal to the loss aversion parameter $\theta$ and independent of the technology.

For larger rents, non linear value functions for positive arguments may be appropriate to reflect risk aversion. For divisible rents, this makes no difference to the payoff function. However, if the rent is indivisible and there are many contestants, individual expenditures will be small so their value will be an approximately linear function of their magnitudes (assuming that the only kink in the value function is at the origin) whereas the value of winning will be reduced below $\theta R$. Hence, even in this case, equilibrium expenditure will be smaller for an indivisible rent as compared to a divisible rent. Indeed, even for loss-neutral but risk averse contestants equilibrium expenditure is reduced. This may lead to the objection that populations of risk averse contestants may exhibit behavior similar to those of loss averse contestants. However, in response to this, we should point out that risk aversion with a smooth utility function over wealth yields the prediction that, for small rents, the contestants should act as risk neutral players and the two contests would lead to the same levels of expenditure. By contrast, loss aversion implies that the utility function cannot be accurately approximated by a linear function. Experiments should therefore be capable of detecting a difference between the contests even for small rents.

When entry costs are positive and we maintain the assumption of linearity apart from the kink at the origin, similar arguments allow us to deduce that payoffs for a divisible rent are given by (3) with $\theta = 1$ and this allows us to apply the following corollary of Theorem 2.

**Corollary 4** *Suppose* $0 < r \leq 1$ *and* $0 < \theta < 1$, *and* $\nu(\theta)$ *and* $\overline{x}^n(\theta)$ *are defined as in Theorem 2. Then* $\nu(\theta) \leq \nu(1)$ *and, for all small enough* $\kappa$, *we have* $\overline{x}^{\nu(\theta)}(\theta) < \overline{x}^{\nu(1)}(1)$. *Furthermore,*

$$\lim_{\kappa \longrightarrow 0} \frac{\nu(\theta)\,\overline{x}^{\nu(\theta)}(\theta)}{\nu(1)\,\overline{x}^{\nu(1)}(1)} = \theta$$

*and, if* $r < 1$, *then* $\nu(\theta)/\nu(1) \longrightarrow \theta$.

**Proof.** To justify the first assertion, we note that both the right and left hand sides of (4) are decreasing in $\nu(\theta)$ (for $\nu(\theta) \geq 1$) and increasing in $\theta$.

The former assertion was established in the proof of Theorem 2. The partial derivative of the right hand side of (4) with respect to $\theta$, can be written

$$\frac{\nu(\theta)(1-r)+\theta(1+r)}{[\nu(\theta)+\theta]^3}\nu(\theta)[\nu(\theta)+\theta]R>0$$

and a similar inequality holds for the left hand side. These observations allow us to deduce from (4) that $\nu(\theta)$ is increasing in $\theta$; in particular $\nu(\theta) \leq \nu(1)$.

In the previous section (observation 5.), we proved that $\kappa\nu(\theta) \longrightarrow (1-r)\theta R$ and $\nu(\theta)\overline{x}^{\nu(\theta)}(\theta) \longrightarrow r\theta R$ as $\kappa \longrightarrow 0$ and the remaining assertions follow immediately. ■

The corollary implies that total outlay is smaller when the rent is indivisible than when it is divisible, provided the entry fee is small enough. The qualification is necessary. In Lemma 9 in the Technical Appendix, it is shown that $\overline{x}^n(\theta)$ is strictly increasing in $\theta$ for fixed $n$ and, in the Corollary, that the number of entrants, $\nu(\theta)$, in non-decreasing in $\theta$. However, as $\theta$ is increased through a value at which $\nu(\theta)$ increases by one, we cannot rule out the possibility that $\overline{x}^n(\theta)$ falls. (See Observation 2. in Section 2.)

# 5 Technical appendix

In this section, we discuss technical issues associated with symmetric contests in more detail. In particular, we characterize the complete set of pure strategy Nash equilibria. We establish that, for certain values of the parameters $r$ and $\theta$, the contest has a unique Nash equilibrium. For the remaining parameter values we show how asymmetric equilibria of the (symmetric) contest may be determined. Our analysis applies the method of share correspondences developed by the authors [3]. The image of $Y = \sum_j x_j^r$ under the share correspondence of player $i$ is the set of winning probabilities: $x_i^r/Y$ consistent with an equilibrium value of $Y$. In the next subsection, we investigate the properties of the share correspondence with particular emphasis on conditions for which the correspondence is single-valued. In the subsequent subsection, we apply these results to characterize the set of equilibria. In the final subsection, we extend these applications to comparative statics results.

## 5.1 Share correspondences

It proves convenient to work with the transformed variables: $y_i = x_i^r$ and the payoff function (2), which we re-write:

$$\widetilde{\pi}_i(y_i, Y) = \frac{\theta R y_i}{Y} - y_i^u + \frac{(1-\theta)\, y_i^{u+1}}{Y},$$

where $u = 1/r$. For any contestant $i$ and $Y > 0$, our approach first determines necessary and sufficient conditions on $\widehat{y}_i(< Y)$ for there to be a Nash equilibrium profile $(\widehat{y}_1, \ldots, \widehat{y}_n)$ satisfying $Y = \sum_{j=1}^n \widehat{y}_j$. This is equivalent to requiring

$$\psi_i(y) = \widetilde{\pi}_i(y, Y - \widehat{y}_i + y) \leq \widetilde{\pi}_i(\widehat{y}_i, Y) = \psi_i(\widehat{y}_i) \text{ for all } y \geq 0. \tag{7}$$

Observing that $\psi_i(0) = 0 > \psi_i(y)$ if $y \geq R^r$ (since expenditure then exceeds the rent), we see that $\psi_i$ must have at least one maximizer in the interval $[0, R^r)$. The first order condition for an interior solution to this maximization problem is

$$\psi_i'(y) = \frac{\partial \widetilde{\pi}_i}{\partial y_i}(\widehat{y}_i, Y - \widehat{y}_i + y) + \frac{\partial \widetilde{\pi}_i}{\partial Y}(\widehat{y}_i, Y - \widehat{y}_i + y) = 0. \tag{8}$$

Direct calculation shows that

$$\psi_i'(y) = \theta \frac{(Y - \widehat{y}_i) R}{(Y - \widehat{y}_i + y)^2} - u y^{u-1} + (1-\theta)\Delta,$$

where

$$\Delta = \frac{u y^u}{Y - \widehat{y}_i + y} + \frac{(Y - \widehat{y}_i) y^u}{(Y - \widehat{y}_i + y)^2}.$$

To confirm optimality, we need to examine second derivatives and find:

$$\psi_i''(y) = -2\theta \frac{(Y - \widehat{y}_i) R}{(Y - \widehat{y}_i + y)^3} - u(u-1) y^{u-2} + (1-\theta)\Delta'.$$

If $\psi_i'(y) = 0$, we can substitute for the first term in $\psi_i''(y)$ to obtain, after simplification, that

$$\psi_i''(y) = -2\frac{u y^{u-1}}{Y - \widehat{y}_i + y} - u(u-1) y^{u-2} + (1-\theta)\frac{u(u+1) y^{u-1}}{Y - \widehat{y}_i + y}.$$

Using $\theta > 0$, we obtain

$$\psi_i''(y) < u(u-1)\left[\frac{y^{u-1}}{Y - \widehat{y}_i + y} - y^{u-2}\right] < 0$$

since $\widehat{y}_i < Y$. We may deduce that $\psi_i$ has no interior local minima.

When investigating the possibility of boundary optimal, it is useful to distinguish between $r < 1$ and $r = 1$. In the former case, which implies $u > 1$, then $\psi_i'(y) > 0$ for all small enough positive $y$, and this rules out the boundary solution: $y = 0$. Hence, $\psi_i$ has a unique global maximum satisfying $\psi_i'(y) = 0$. If $r = 1$, then, as $y \longrightarrow 0$,

$$\psi_i'(y) \longrightarrow \frac{\theta R}{Y - \widehat{y}_i} - 1,$$

which implies that $y = 0$ is a local maximum of $\phi_i$ if and only if $Y > \widehat{y}_i + \theta R$. Then, since there are no local minima, $y = 0$ must be the global maximum. If $Y \le \widehat{y}_i + \theta R$, there will be a unique local maximum satisfying $\psi_i'(y) = 0$. Summarizing these results, (7) holds if and only if $\widehat{y}_i$ satisfies $\psi_i'(\widehat{y}_i) = 0$ except in the case that $r = 1$ and $Y > \theta R$, in which case $\widehat{y}_i = 0$.

For any contestant $i$ and $Y > 0$, we write $S_i(Y)$ for the corresponding set of values of $\widehat{y}_i/Y$, where $\widehat{y}_i$ satisfies (7). If $r < 1$ or $Y \le \theta R$ (or both), we can rearrange $\psi_i'(\sigma R) = 0$ to conclude that $\sigma \in S_i(Y)$ if and only if $\sigma > 0$ and $Y = \varphi(\sigma)$, where $\varphi = [\theta R / D(\sigma)]^r$ and

$$D(\sigma) = u\sigma^{u-1} - (1-\theta)\sigma^u + \frac{\theta u \sigma^u}{1-\sigma}.$$

If $r = 1$ and $Y > \theta R$, then $0 \in S_i(Y)$. We refer to $S_i$ as the *share correspondence* of contestant $i$ and $\varphi$ with domain $(0,1)$ as the *inverse share function*. Note that $\sigma \in (0,1) \cap S_i(Y)$ if and only if $Y = \varphi(\sigma)$: the share correspondence is obtained by reflecting the inverse share function in the 45° line. We now turn to a discussion of the properties of the inverse share function.

We start by considering the case $r = 1$. Then, $u = 1$ and

$$D(\sigma) = (1-\theta)(1-\sigma) + \frac{\theta}{1-\sigma}.$$

It is readily seen that $D \longrightarrow \infty$ as $\sigma \longrightarrow 1$ and has a unique minimum in $\sigma < 1$ at

$$\sigma = 1 - \sqrt{\frac{\theta}{1-\theta}}.$$

This minimum is positive if and only if $\theta < \frac{1}{2}$. If the latter holds, $\varphi \longrightarrow \theta R$ as $\sigma \longrightarrow 0$, increases and then decreases to 0 as $\sigma \longrightarrow 1$. We give a typical graph of $D(\sigma)$ in Figure 1a. and of the resulting inverse share function in Figure 1b. The share correspondence for $r = 1$ and $\theta < \frac{1}{2}$ is drawn in Figure 1c. (Recall our earlier argument that $0 \in S_i(Y)$ for $Y \geq \theta R$.) If $\theta \geq \frac{1}{2}$, then $D$ is strictly increasing in $\sigma$ for $0 < \sigma < 1$ and therefore the inverse share function decreases monotonically from $\theta R$ to 0 as $\sigma$ goes from 0 to 1. The graphs of $D$ and $\varphi$ are given in Figures 2a. and 2b. respectively and the share correspondence in Figures 2c. Note that, the latter is singleton-valued; the correspondence is actually a function which we refer to as a *share function*. Note further that this function is strictly decreasing, where positive.

When $r < 1$, then $u > 1$ and $u\sigma^{u-1} > \sigma^{u-1} > (1-\theta)\sigma^u$ for $0 < \sigma < 1$. This means that $D$ is positive in $(0,1)$ as well as having the limiting properties: $D \longrightarrow 0$ as $\sigma \longrightarrow 0$ and $D \longrightarrow \infty$ as $\sigma \longrightarrow 1$. Furthermore, it is readily checked that $D'(\sigma) = 0$ resolves into a cubic equation in $\sigma$. Given its other properties, we deduce that $D$ is either increasing throughout $(0,1)$ or has exactly two distinct turning points, a local maximum followed by a local minimum. A typical $D$ for the latter case, is graphed in Figure 3a., the corresponding inverse share function in Figure 3b. and the share correspondence in Figure 3c. The alternative case in which $D$ is increasing from 0 (at 0) to $\infty$ (at 1) results in an inverse share function which falls throughout $(0,1)$ and approaches $\infty$ as $\sigma \longrightarrow 0$. Typical graphs of $D$ and $\varphi$ are given in Figure 4a. and 4b. In this case, the share correspondence is a function which decreases strictly from 1 at 0 to 0 as $\sigma \longrightarrow \infty$ and is graphed in Figure 4c. Explicit necessary and sufficient conditions to distinguish between the two cases are hard to obtain, but a sufficient condition for the correspondence to be a function is that $\theta \geq (r+1)/r^2$. To see this, note that

$$\begin{aligned} D'(\sigma) &= \theta u \sigma^{u-1} + \frac{\theta u^2 \sigma^{u-1}}{1-\sigma} + \frac{\theta u \sigma^u}{(1-\sigma)^2} + (u-1)(u-\sigma)\sigma^{u-2} - \sigma^{u-1} \\ &> \left\{\left[1 + \frac{u}{1-\sigma}\right]\theta u - 1\right\}\sigma^{u-1} > \{\theta u(1+u) - 1\}\sigma^{u-1} \geq 0, \end{aligned}$$

using $u \geq 1 > \sigma > 0$ and $\theta \geq u + u^2$. Hence, $D$ is increasing in $(0,1)$. We summarize our conclusions in the next proposition.

**Proposition 5** *If $\sigma \in S_i(Y)$, then $\sigma = 0$ if and only if $r = 1$ and $Y \geq \theta R$. Positive elements in the correspondence are characterized by the continuous inverse share function $\varphi$ which has the following properties in $(0,1)$:*

1. *if $r = 1$ and $\theta \geq \frac{1}{2}$, it is strictly decreasing from $\theta R$ to $0$,*

2. *if $r = 1$ and $\theta < \frac{1}{2}$, it increases to a maximum from $\theta R$ and then decreases to $0$,*

3. *if $r < 1$ then either*

   (a) *it is strictly decreasing from $\infty$ to $0$, or*

   (b) *it is strictly decreasing from $\infty$ to a local minimum, then increases to a local maximum and finally decreases to $0$.*

*Case 3(a). holds if $\theta \geq (1 + r) / r^2$.*

In cases 1. and 3(a)., there is a share function which is strictly decreasing where positive.

## 5.2 Equilibria

We can use Proposition 5 to characterize the set of all equilibria. This is based on the consistency condition that the strategy profile $(\widehat{y}_1, \ldots, \widehat{y}_n)$ is a Nash equilibrium if and only if $\widehat{y}_i / \widehat{Y} \in S_i \left(\widehat{Y}\right)$ for $i = 1, \ldots, n$, where $\widehat{Y} = \sum_{j=1}^{n} \widehat{y}_j$. In particular, $\widehat{Y}$ is an equilibrium value of $Y$ if and only if

$$1 \in \sum_{j=1}^{n} S_j \left(\widehat{Y}\right), \tag{9}$$

using conventional set addition. The proof of these observations is straightforward and omitted here. (In our study, all contestants have the same share correspondence, but the preceding result holds even for asymmetric games.) We may conclude that $y_i = \widehat{y}$ for all $i$ is a symmetric equilibrium if and only if, for any $i$:

$$\frac{1}{n} \in S_i \left(n\widehat{y}\right).$$

This can be re-written as $\widehat{y} = \varphi \left(1/n\right) / n$, where $\varphi$ is the inverse share function and shows that there is a unique symmetric equilibrium. Transforming back to the original strategic variables gives $\widehat{x}_i \left(\theta\right) = \left[\varphi \left(1/n\right) / n\right]^u$ and, using the formula for $\varphi$ derived in the previous subsection, we have an alternative proof of Theorem 1.

A unique symmetric equilibrium does not preclude the possibility of alternative asymmetric equilibria. When $r = 1$, the strategy profile in which $m < n$ contestants choose $x_i = y = \varphi(1/m)/m$ and the remaining $n - m$ choose $x_i = 0$ is a Nash equilibrium, provided $\varphi(1/m) \geq \theta R$. The latter condition is required to ensure that the equilibrium requirement: $0 \in S_i(\varphi(1/m))$ is satisfied. For example, if $n = 3$ and $\theta = 0.2$, the symmetric equilibrium is $(0.08R, 0.08R, 0.08R)$. An alternative Nash equilibrium is $(0.125R, 0.125R, 0)$ since we can check that the requirement $\varphi(1/2) = 0.25R > \theta R$ is satisfied. This is illustrated in Figure 5. Note that such multiple equilibria will always occur for large enough $n$ (exceeding the smallest $m$ such that $\varphi(1/m) \geq \theta R$.)

**Corollary 6** *If $2\theta < r = 1$, the contest has multiple equilibria for all large enough $n$.*

When, as in cases 1. and 3a. in Proposition 5, the share correspondence is actually a function, strictly decreasing where positive, there is a unique $\widehat{Y}$ satisfying (9) and multiple equilibria are ruled out.

**Corollary 7** *If $r^2\theta \geq 1 + r$, the contest has a unique equilibrium.*

If $r < 1$, all players are active in equilibrium, but this does not rule out the possibility of more than one equilibrium. However, this cannot occur if $n$ is large enough. To see this, note that it follows from Proposition 5 that there is a value $\overline{Y}$ of $Y$ and $\overline{\sigma} \in (0, 1)$ such that (i) $S_i(Y) \subseteq [\overline{\sigma}, 1)$ for $0 < Y \leq \overline{Y}$ and (ii) for $Y > \overline{Y}$, the correspondence $S_i(Y)$ is single-valued: $S_i(Y) = \{s_i(Y)\}$ and the function $s_i$ is continuous and strictly decreasing. This is illustrated in Figure 6. It follows that, if $n > 1/\overline{\sigma}$, every element in $\sum_{j=1}^{n} S_j(Y)$ exceeds unity for $0 < Y \leq \overline{Y}$ whereas $\sum_{j=1}^{n} s_j(Y)$ is a strictly decreases to zero in $\left(\overline{Y}, \infty\right)$ and $\sum_{j=1}^{n} s_j\left(\overline{Y}\right) > 1$. We conclude from the equilibrium condition (9) that there is a unique equilibrium.

**Corollary 8** *If $r < 1$, the contest will have a unique equilibrium for all large enough $n$.*

## 5.3 Comparative statics

In this subsection, we apply Proposition 5 to deduce comparative statics properties of the contest with particular emphasis on those results which are useful for developments in the rest of the paper. We first observe a curious consequence of share correspondences with the shape shown in Figures 1. and 3. This is most easily seen when $r = 1$ for then the aggregate expenditure on rent seeking with $n$ contestants is $X^n = \varphi(1/n)$. If $\theta \geq \frac{1}{2}$, the existence of a share function, decreasing where positive allows us to deduce the intuitive result that $X^n$ is increasing in $n$. However, if $\theta < \frac{1}{2}$, then $X^n$ is decreasing (to $\theta R$) for all large enough $n$. More contestants *reduce* dissipation. When $r < 1$, we have $X^n = n^{1-u}[\varphi(1/n)]^u$ and we cannot deduce that an increase in $\varphi(1/n)$ increases $X^n$, but a decrease in $\varphi(1/n)$ does decrease $X^n$. If the share correspondence has the shape of Figure 3., there may be an interval of values of $n$ for which $\varphi(1/n)$, and therefore $X^n$, decreases with $n$. We will not pursue such results further, instead turning to the effect of changes in $n$ and $\theta$ on payoffs.

If $X^n$ increases with $n$, we can immediately deduce that the equilibrium payoff:

$$\widehat{\pi}^n = n^{-1}\left\{\theta R - \left[1 - \frac{1-\theta}{n}\right]X^n\right\}$$

is strictly decreasing in $n$. However, the same conclusion can be drawn even if $X^n$ decreases in $n$. This is most easily seen by direct calculation of first differences of $\widehat{\pi}^n$. After some manipulation, we find, for $n \geq 1$, that

$$\widehat{\pi}^{n+1} - \widehat{\pi}^n = -\frac{(1+r)\,\theta^2 + (2n-1)\,\theta + (1-r)\,n\,(n-1)}{D\,(n+1)\,D\,(n)}R,$$

where $D(n)$ is defined in the proof of Theorem 2. Since $D(n)$ is positive for $n \geq 1$, we conclude that $\widehat{\pi}^n$ is strictly decreasing in $n$.

Finally, we consider the equilibrium level of output, $\overline{x}^n(\theta)$, in the model with entry costs, defined in (5).

**Lemma 9** *For fixed $n$, $\overline{x}^n(\theta)$ is an increasing function of $\theta$.*

**Proof.** Note that

$$\frac{d\overline{x}^n}{d\theta} = \frac{R\,(n-1)\,(n-r) - \kappa n^2}{\{n^2 - (1-\theta)\,[n + rn - r]\}^2}r\,(n-1)$$

and that the derivative is zero if $n = 1$. For $n \geq 2$, note that the left hand inequality in (4) puts an upper bound on $\kappa$ which implies

$$\frac{d\overline{x}^n}{d\theta} > \frac{Ar + B\,(1-r)}{\{n^2 - (1-\theta)\,[n + rn - r]\}^2}rR\,(n-1)\,,$$

where

$$\begin{aligned} A &= (n-1)^2 - \left[\frac{\theta n}{n-1+\theta}\right]^2, \\ B &= \frac{(n-1)^2-\theta}{n-1+\theta} n. \end{aligned}$$

Since

$$n-1 = \frac{\theta n + (n-1)^2 - \theta}{n-1+\theta}$$

and $(n-1)^2 \geq 1 > \theta$, we have $A > 0$. Similarly, the numerator of $B$ is positive, so $B > 0$ and we can deduce that $\overline{x}^n$ is strictly increasing in $\theta$. ■

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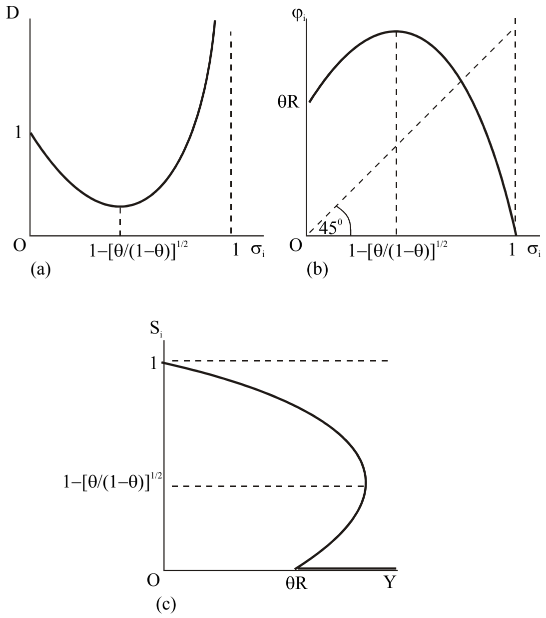


Figure 1: $D(\sigma_i)$, $\varphi_i(\sigma_i)$, $S(Y)$ for $r = 1$, $\theta < 1/2$.

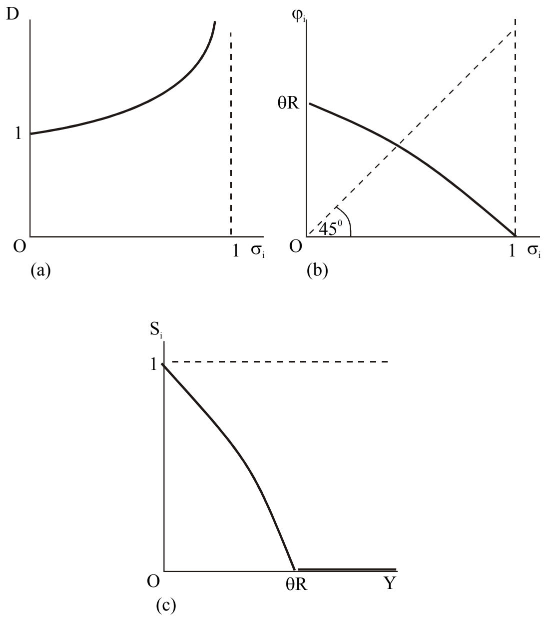


Figure 2: $D(\sigma_i)$, $\varphi_i(\sigma_i)$, $S(Y)$ for $r = 1$, $\theta > 1/2$.

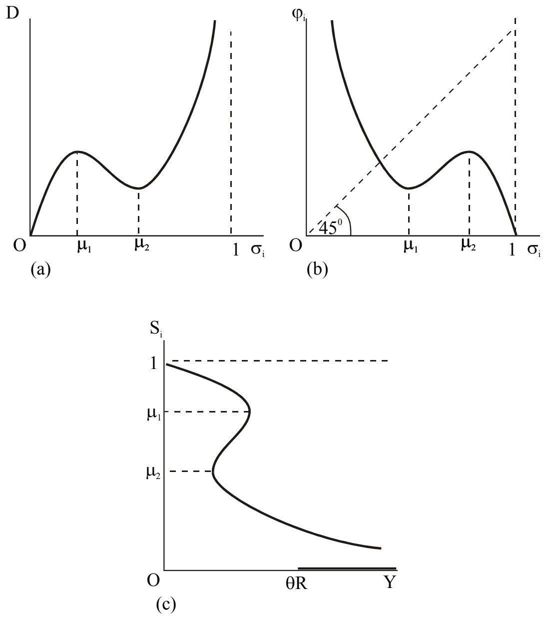


Figure 3: $D(\sigma_i)$, $\varphi_i(\sigma_i)$, $S(Y)$ for $r < 1$, $D$ non-monotonic.

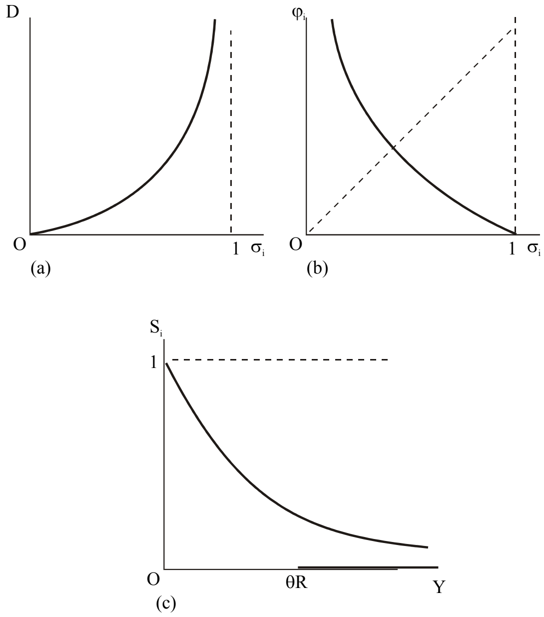


Figure 4: $D(\sigma_i)$, $\varphi_i(\sigma_i)$, $S(Y)$ for $r < 1$, $D$ strictly increasing.

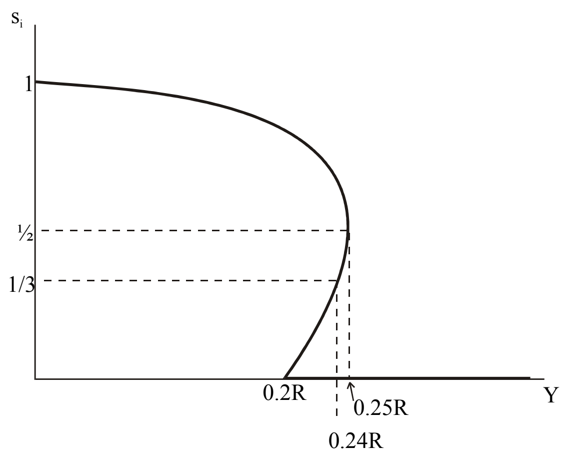


Figure 5: Multiple equilibria: $r = 1$, $\theta = 1/5$.

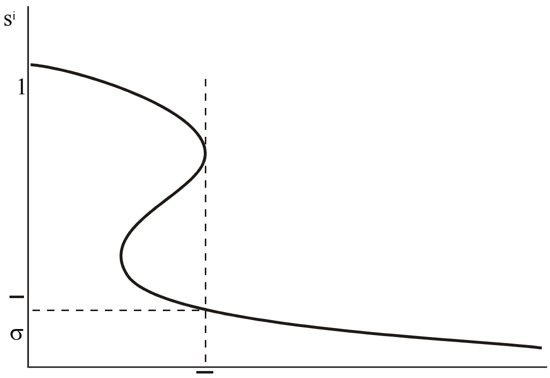


Figure 6: Definitions of $\overline{Y}$ and $\overline{\sigma}$ when $r < 1$.