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DIFFERENTIATED DUOPOLY WITH ASYMMETRIC COSTS: NEW RESULTS FROM A SEMINAL MODEL

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September 2003

Differentiated duopoly with asymmetric costs: new results from a seminal model

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Abstract

This paper re-considers the comparison of Bertrand and Cournot equilibria in a differentiated duopoly with linear demand and cost functions. It focuses on the case of substitute goods, and allows for cost asymmetries between firms. The main finding is that, when the degree of cost asymmetry is sufficiently high and/or the degree of product differentiation is sufficiently low, equilibrium profits of the efficient firm and the industry profits are higher under price than under quantity competition. This contrasts with a standard result by Singh and Vives (1984), that is, with substitute goods both firms earn higher profits under Cournot than under Bertrand competition.

JEL codes: D43, L13

Keywords: Cost asymmetry; product differentiation; duopoly; Bertrand; Cournot.

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1. INTRODUCTION

In this paper we re-consider the comparison of Bertrand and Cournot equilibria in a differentiated duopoly model with linear demand and cost functions (Dixit (1979)). Focusing on the case of substitute products, we study the role played by firms' asymmetry in costs on the comparison between the two forms of competition.

Singh and Vives (1984) provide a basic result for this model. When goods are substitutes, price competition entails higher quantities and larger social welfare than quantity competition, whereas prices and profits are higher under quantity competition.[1] Although Singh and Vives allow for cost asymmetry, they restrict the space of the model along that dimension by assuming that the "primary outputs" are always positive for both firms (i.e. each firm faces positive demand when both prices are set at marginal costs).

As noted by Amir and Jin (2001), this assumption is crucial (at least) for the Singh and Vives's ranking of the equilibrium quantities under the two forms of competition.[2] This fact is apparent in the special case of a homogeneous duopoly with linear cost functions. In this case, any asymmetry in costs implies that the less efficient firm's output is zero if both firms price at marginal costs. Hence, the assumption of positive primary outputs is violated unless the space of the model is restricted only to the case of symmetric costs. Maintaining the cost asymmetry implies that the inefficient firm is active in Cournot equilibrium (provided that the inefficiency gap between the two firms is not drastic) but is inactive in the limit-pricing equilibrium arising under Bertrand competition. Therefore, the Singh and Vives's ranking of the equilibrium quantities under the two forms of competition does not hold anymore.

[1] Singh and Vives's result is complete with respect to the nature of goods, in that they show that with complementary goods only the ranking of profits under the two forms of competition is reversed. Therefore, with complementary goods, price competition entails lower prices and higher productions, profits and social welfare than quantity competition.

[2] However, Amir and Jin (2001) maintain this assumption. Using an oligopoly model with linear demand functions and a mixture of complementary and substitute goods, they show that the ranking of equilibrium prices and quantities under the two forms of competition is ambiguous when strategic complementarity does not characterise either of the two games. Their main result is that price competition always leads to lower average prices, lower mark-up/otput ratios and larger average output.

In this paper we drop the assumption of positive primary outputs in a version of Singh and Vives's model with symmetric demand functions. The dimension of cost asymmetry is constrained only by the assumption that, for any degree of product differentiation, the efficiency gap between the two firms is not drastic, meaning that the most efficient firm can not monopolise the market irrespective of the form of competition. As a result, the relevant space of the model splits in two regions according to the market structure arising under the two forms of competition. With low degrees of cost asymmetry, and/or high degrees of product differentiation, both firms are active in the market under both forms of competition. The parameter space of the model considered by Singh and Vives coincides with the portion of this region where both firms produce more in Bertrand than in Cournot equilibrium. For high degrees of cost asymmetry, and/or low degrees of product differentiation, the less efficient firm is active in Cournot equilibrium, while it is inactive (but exerting a competitive pressure from outside the market) in the limit-pricing equilibrium prevailing under Bertrand competition.

The comparison of the two forms of competition over the entire parameter space of the model reveals that, for sufficiently high degrees of cost asymmetry and low degrees of product differentiation, the efficient firm profits and the industry profits are higher under Bertrand than under Cournot competition. Therefore, besides the ranking of the inefficient firm's production, the ranking of profits also differs from Singh and Vives's result over a significant portion of the model's parameter space. On the other hand, it is confirmed that price competition leads to lower prices and larger social welfare over the relevant parameter space of the model.

The intuition behind these results is as follows. When firms are asymmetric in costs, price competition not only entails lower prices (*price effect*) but also a stronger selective effect against the market share of the less efficient firm (*selection effect*) than quantity competition does. While the *price effect* works towards lower profits under Bertrand than under Cournot competition for both firms, the *selection effect* works in the opposite direction on the efficient firm's profits. Moreover, the *price effect* weakens while the *selection effect* gets stronger when either the degree of cost asymmetry increases (given any degree of product differentiation) or products are closer substitutes (for a sufficient degree of cost asymmetry). As a result, the efficient firm earns higher profits under price than under quantity competition when its efficiency advantage over the rival is sufficiently high and products are close substitutes. Further, the *selection effect* entails more productive efficiency under price than under quantity competition, which explains the reversal of the industry profits' ranking for high degrees of cost asymmetry and low degrees of product differentiation.

From the viewpoint of social welfare, while consumers always gain from the lower prices arising under Bertrand competition, the overall producers surplus is lower with price competition when the profits erosion due to the *price effect* dominates the efficiency gain due to the *selection effect*. In this case the social welfare (as measured by the total surplus) turns out to be larger under price competition since the increase in consumers surplus more than compensates the fall in producers surplus. However, with high degrees of cost asymmetry and low degrees of product differentiation, social welfare is higher with price competition, since both consumers surplus and industry profits are larger.

The inversion of the ranking of the efficient firm profits under the two forms of competition has been noted before by Boone (2001) for the case of a homogeneous duopoly. Also within a homogeneous duopoly model, Denicolo' and Zanchettin (2002) have shown that the ranking of industry profits reverses when the efficiency gap between the two firms is sufficiently high but not drastic. The present paper provides a generalisation of these results, since the framework of a differentiated duopoly allows us to show how the comparison of the equilibrium profits under the two forms of competition depends on both the degree of cost asymmetry and the degree of product differentiation.[3] Further, from the characterisation of Bertrand and Cournot equilibria over the entire parameter space of the model we obtain additional results on the behaviour of the equilibrium profits when firms are asymmetric in costs. Namely, under both forms of competition, while the inefficient firm profits always

[3] Our results are also related to Hackner (2000). In a version of Dixit (1979) model with both vertical and horizontal product differentiation and symmetric costs, Hackner shows that the "high-quality firms" may make higher profit with price than with quantity competition when there are more than two competitors in the market and the differences in quality are sufficiently large.

decrease as products become closer substitutes, the efficient firm profits are non-monotonic in the degree of products differentiation. Therefore, the efficient firm may have a local incentive to reduce the degree of product differentiation.[4] This contrasts with the standard result that arises with symmetric costs, that is, Bertrand and Cournot duopolists always gain from product differentiation (see, among others, Oz Shy (1995), pp.138-140).

The rest of the paper is organised as follows. The following section describes the model. Section 3 characterises Bertrand and Cournot equilibria over the relevant parameter space of the model. Section 4 compares the two forms of competition and proves the main results of the paper. Section 5 provides some concluding remarks. All proofs are relegated to an appendix.

2. THE MODEL

We consider the following version of the Singh and Vives (1984) model of a differentiated duopoly with linear demand and cost functions. [5]

On the demand side of the market, the representative consumer's utility is a symmetric-quadratic function of two products, q_1 and q_2, and a linear function of a numeraire good, m,

$$U = \alpha\left(q_1 + q_2\right) - \frac{1}{2}\left(q_1^2 + q_2^2 + 2\gamma q_1 q_2\right) + m \ .$$

As usual, the parameter γ measures the degree of product differentiation. We consider the case of substitute goods: $0 \leq \gamma \leq 1$. The extremes of the interval, $\gamma = 0$ and $\gamma = 1$, correspond to the cases of maximum (independent goods) and minimum (homogeneous goods) degree of differentiation, respectively.

[4] We also show that this is more likely to happen under price than under quantity competition. Indeed, under price competition, the efficient firm's profits are non-monotonic in the degree of products differentiation for any positive degree of asymmetry in costs. On the contrary, a sufficient degree of cost asymmetry is required under quantity competition.

[5] This specification is simpler than Singh and Vives (1984) in two respects. First we assume symmetric demand functions, second we normalise to one the coefficients of the squared terms in the utility function (i.e. the "own quantity slopes" of the inverse demand functions). Neither of these simplifications affect our results.

This utility function generates the linear system of inverse demand functions

$$\begin{aligned} p_1 &= \alpha - q_1 - \gamma q_2 \\ p_2 &= \alpha - q_2 - \gamma q_1 \quad , \end{aligned} \tag{1}$$

whose inversion (by imposing $\gamma < 1$) leads to the direct demand system

$$\begin{aligned} q_1 &= \tfrac{1}{1-\gamma^2}\left[(1-\gamma)\,\alpha - p_1 + \gamma p_2\right] \\ q_2 &= \tfrac{1}{1-\gamma^2}\left[(1-\gamma)\,\alpha - p_2 + \gamma p_1\right] \ . \end{aligned} \tag{2}$$

System (2) gives the direct demand functions provided that prices lead to positive demands for both goods. Discarding the trivial case with zero-demand for both goods, the region of prices where the demand of good 2 is nil while the demand of good 1 is positive, $\bar{R}_2$, is identified by

$$\bar{R}_2 = \left\{ \begin{array}{l} p_1, p_2 \geq 0 \\ (1-\gamma)\,\alpha - p_2 + \gamma p_1 \leq 0 \\ \alpha - p_1 > 0 \end{array} \right\} . \tag{3}$$

Inside region $\bar{R}_2$, the demand function of good 1 becomes $q_1 = \alpha - p_1$. An analogue region, $\bar{R}_1$, where the demand function of good 2 becomes $q_2 = \alpha - p_2$, can be obtained in the same way.

On the supply side of the market, products 1 and 2 are produced and supplied by firm 1 and firm 2, respectively. Both firms face linear cost functions, but firm 1 is more efficient than firm 2. More precisely, we set firm 1's marginal cost at c, where $0 < c < \alpha$ to avoid the trivial case in which neither firm has an incentive to produce.[6] Then, firm 2 marginal cost is set at λc, where $1 \leq \lambda \leq \frac{\alpha}{c}$.

The parameter λ measures the efficiency gap between the two firms, and its range is fixed to admit all the relevant cases. At the lower bound of the interval, $\lambda = 1$, the asymmetry in costs disappears. At the upper bound, $\lambda = \frac{\alpha}{c}$, firm 2's "primary mark-up", $\frac{\alpha - \lambda c}{\alpha}$, is nil,

[6] Obviously, if $c \geq \alpha$, firm 1 "primary mark-up" (i.e. the mark-up over price on the first unit sold when the rival's production is zero, $\frac{\alpha - c}{\alpha}$) is not positive. Hence firm 1 has no incentive to produce irrespective of the form of competition and the degree of product differentiation. The same is true for firm 2, since it is less efficient than firm 1, and the inverse demand functions are symmetric.

and hence firm 2 is not active in the market irrespective of the form of competition and the degree of products differentiation.

The ranges of γ and λ define the space of the model as $S(\lambda, \gamma) = \{0 \leq \gamma \leq 1; 1 \leq \lambda \leq \frac{\alpha}{c}\}$. Figure 1 (left diagram) depicts the space $S(\lambda, \gamma)$.

In what follows we find sometimes convenient to measure the efficiency gap between the two firms as the ratio of their primary mark-ups[7]

$$x = \frac{(\alpha - \lambda c)}{(\alpha - c)}. \tag{4}$$

From (4), as λ increases from 1 to $\frac{\alpha}{c}$, x decreases from 1 to 0. Hence the degree of asymmetry in costs increases with λ but decreases with x. Figure 1 (right diagram) depicts the space of the model in terms of the parameters x and γ, i.e. $S(x, \gamma) = \{0 \leq \gamma \leq 1 \; ; \; 0 \leq x \leq 1\}$.

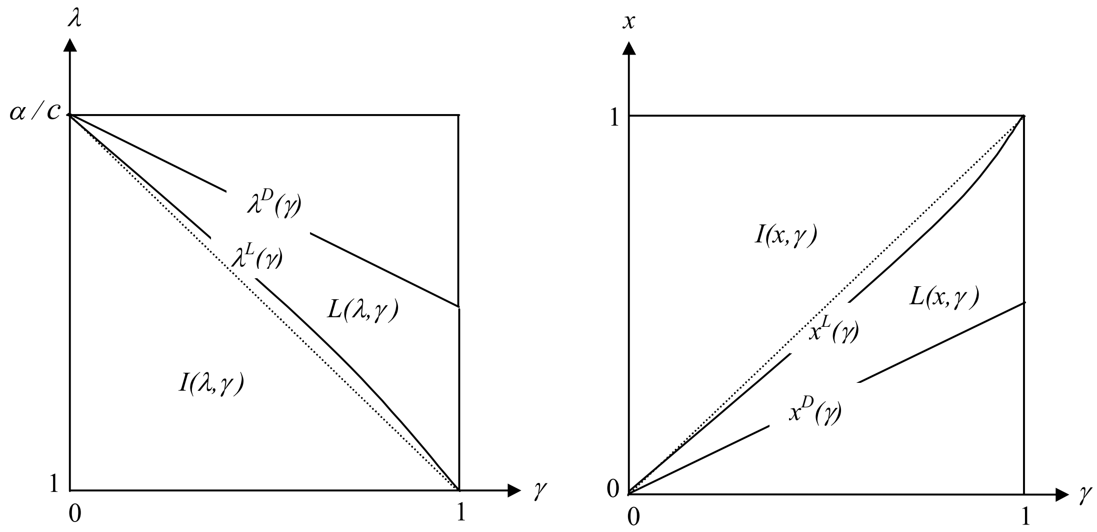

FIG. 1.

[7] As we will see in the following sections, most of the proofs are considerably simplified by using x instead of λ as the measure of the asymmetry in costs.

3. THE MARKET GAME WITHIN THE SPACE OF THE MODEL

We assume that firm 1 and firm 2 compete in prices or in quantities, alternatively. However, the form of competition between the two firms matters for the market game only in a sub-space of $S(\lambda, \gamma)$.

First, when $\gamma = 0$, firm 1 and firm 2 are monopolists on independent segments of the market. In this case, using the demand system (1) with $\gamma = 0$, we obtain

$$\begin{gathered} p_1^M = \tfrac{\alpha+c}{2}\ ; \quad q_1^M = \left(p_1^M - c\right) = \tfrac{\alpha-c}{2}\ ; \quad \pi_1^M = \left(\tfrac{\alpha-c}{2}\right)^2 \\ p_2^M = \tfrac{\alpha+\lambda c}{2}\ ;\ q_2^M = \left(p_2^M - \lambda c\right) = \tfrac{\alpha-\lambda c}{2}\ ;\ \pi_2^M = \left(\tfrac{\alpha-\lambda c}{2}\right)^2\ , \end{gathered} \tag{5}$$

where p_i^M, q_i^M and π_i^M $[i = 1, 2]$ are firm i's monopoly price, quantity and profit, respectively. Obviously, π_1^M and π_2^M are the highest profits for each firm over the entire space of the model.[8]

Second, for any $0 < \gamma \leq 1$, if λ is sufficiently high, then firm 1 can practise its monopoly solution without bearing any competitive pressure from firm 2, which is driven out of the market irrespective of the form of competition. This is the case when, if firm 1 prices at p_1^M and supplies q_1^M, the maximum price of good 2 leading to positive demand falls short firm 2's marginal cost. Formally, this requires that the price vectors $\left(p_1^M, p_2\right)$ belong to region $\bar{R}_2$ for any $p_2 \geq \lambda c$.

Let us denote as *drastic* any efficiency gap such that the latter condition holds. Then, by using (3), we get the following condition for λ to be *drastic*

$$\lambda \geq \lambda^D(\gamma) = \frac{\alpha}{c} - \frac{\gamma}{2}\frac{(\alpha - c)}{c}\ . \tag{6}$$

Equation (6) identifies a "drastic-frontier", $\lambda^D(\gamma)$, which is decreasing in γ over the

[8] For $\lambda = 1$, firms are symmetric and the monopoly solutions coincide. Then, as λ increases, p_2^M increaseas, while q_2^M and π_2^M decrease (p_1^M, q_1^M and π_1^M do not, of course, change with λ). For $\lambda = \frac{\alpha}{c}$, firm 2 is out of the market even in the case of independent products, i.e. $p_2^M = \lambda c$ and $q_2^M = \pi_2^M = 0$. Obviouosly, the assumption $\alpha > c$ guarantees that $\pi_1^M > 0$.

space $S(\lambda, \gamma)$ (see Figure 1, left diagram).[9] Namely, when products are closer substitutes, a lower threshold level of λ suffices for firm 1 to monopolise the market irrespective of the form of competition. Further, from equation (6) we see that $\lambda^D(0) = \frac{\alpha}{c} > 1$ and $\lambda^D(1) = \frac{1}{2}\left(\frac{\alpha}{c} + 1\right) > 1$. This confirms our claim that, for any $\gamma \in (0,1]$, there is a drastic threshold level of λ within the interval $(1, \frac{\alpha}{c})$.[10]

Therefore, we must characterise the market equilibrium, separately for quantity and price competition, only over the sub-space of the model $S_0(\lambda, \gamma) = \{0 < \gamma \leq 1;\, 1 \leq \lambda < \lambda^D(\gamma)\}$.

3.1. Cournot competition

Under quantity competition, firm i chooses q_i to maximise

$$\pi_i = \left(\alpha - q_i - \gamma q_j - \lambda^{i-1} c\right) q_i \quad [i, j = 1, 2;\, i \neq j] \ ,$$

by taking q_j as given.

The best response functions of firm 1 and firm 2 are (respectively)

$$\begin{aligned} q_1 &= \max\left\{\tfrac{1}{2}(\alpha - c - \gamma q_2)\ ; 0\right\} \\ q_2 &= \max\left\{\tfrac{1}{2}(\alpha - \lambda c - \gamma q_1)\ ; 0\right\} \ . \end{aligned} \tag{7}$$

Assuming an interior solution for system (7), the market equilibrium under Cournot competition is fully characterised as follows

$$\begin{aligned} q_1^C &= (p_1^C - c) = \tfrac{2(\alpha - c) - \gamma(\alpha - \lambda c)}{4 - \gamma^2}; \quad \pi_1^C = \left\{\tfrac{2(\alpha - c) - \gamma(\alpha - \lambda c)}{4 - \gamma^2}\right\}^2 ; \\ q_2^C &= (p_2^C - \lambda c) = \tfrac{2(\alpha - \lambda c) - \gamma(\alpha - c)}{4 - \gamma^2}; \quad \pi_2^C = \left\{\tfrac{2(\alpha - \lambda c) - \gamma(\alpha - c)}{4 - \gamma^2}\right\}^2 \ . \end{aligned} \tag{8}$$

Equation (8) shows that production, unit-profit and profits are always higher for firm 1

[9] Using equation (4), the "drastic frontier" can be converted in the space $S(x, \gamma)$ as $x^D(\gamma) = \frac{\gamma}{2}$. Clearly $x^D(\gamma)$ is incresing in γ over $S(x, \gamma)$ (see Figure 1, right diagram).

[10] In contrast, the less efficient firm can never practices its monopoly solution unless $\gamma = 0$. Indeed, given any $\gamma \in (0,1]$, if λ is drastic, then for every $p_2 > \lambda c$ (e.g., including p_2^M) firm 1 has always a price above its marginal cost leading to positive demand for good 1, namely p_1^M. Alternatively, λ is not drastic. Therefore, even firm 2 can price above its marginal cost and face positive demand when the rival prices at p_1^M. Then, since $p_2^M \geq p_1^M$ and $\lambda c \geq c$, the symmetry of the demand system ensures that the same is true for firm 1.

when $\lambda > 1$.[11] For $\lambda = 1$ the equilibrium is symmetric. Then, as λ increases, q_1^C and π_1^C increase, while q_2^C and π_2^C decrease. This leads to the following lemma.

Lemma 1 *Under quantity competition, for any $\gamma \in (0,1]$ the less efficient firm is active in equilibrium provided that $1 \leq \lambda < \lambda^D(\gamma)$. For $\lambda = \lambda^D(\gamma)$, the most efficient firm reaches its monopoly solution (eq.(4)), while the less efficient firm is out of the market.*

Note that Lemma 1 and the shape of the "drastic frontier" imply immediately that firm 1 equilibrium profits are non-monotonic in γ, provided that the efficiency gap between the two firms is not too small. In fact, consider any level of λ, say λ_0, which can become *drastic* as γ reaches a critical level $\gamma_0^D \in (0,1)$ on frontier $\lambda^D(\gamma)$. Then, given λ_0, firm 1 profits equal the monopoly levels at both $\gamma = 0$ and $\gamma = \gamma_0^D$. Since profits change continuously with γ, this means that they are not monotonic in γ over the interval $[0, \gamma_0^D]$. The following lemma characterises more precisely the behaviour of the equilibrium profits of the two firms as functions of the degree of products differentiation.

Lemma 2 *Under quantity competition, given any $\lambda \in [1, \frac{\alpha}{c})$, the equilibrium profits of the less efficient firm decrease as γ increases from 0 to the minimum between 1 and $\gamma^D(\lambda) \equiv \lambda^{D^{-1}}(\gamma)$. On the contrary, there exist a threshold level $\lambda_C^* \in (1, \lambda^D(1))$ and a critical locus $0 < \gamma_C^*(\lambda) < \min\{\gamma^D(\lambda), 1\}$, such that the equilibrium profits of the most efficient firm increase with γ if $\lambda > \lambda_C^*$ and $\gamma > \gamma_C^*(\lambda)$, and decrease with γ otherwise.*[12]

According to Lemma 2, the conventional result that, in a Cournot duopoly, firms have always a strong incentive to differentiate their products should be amended whenever a sufficient degree of asymmetry in costs is allowed. Indeed, if the efficiency gap is high and products are initially close substitutes, the efficient firm may have locally a stronger incentive to reduce the degree of product differentiation. The intuition is that a higher substitutability

[11] On the contrary, $p_2^C \geq p_1^C$ over $S_0(\lambda, \gamma)$, with strict inequality holding for $\lambda > 1$ and $\gamma < 1$. Further, both prices increase with λ.

[12] From equation (8) it is also clear that the equilibrium price and production of each firm behave exactly as the equilibrium profits with respect to γ over the entire space $S_0(\lambda, \gamma)$.

of products lowers the inverse demand functions of both goods, as consumers value less any bundle of the two products in terms of the numeraire good. If firms are symmetric in costs, this leads to a new symmetric equilibrium in which both firms' production and profits are lower. However, in the presence of asymmetric costs, the inverse demand function of the inefficient firm is shifted more since consumers substitute the higher priced good for the lower priced one (i.e., by eq.(1), given an initial couple of equilibrium quantities, the inverse demand function of each firm is shifted down proportionally to the rival's production). Hence, the difference in the equilibrium quantities of the two firms increases with γ. As a result, the efficient firm benefits from the larger reduction of the rival's production, which counteracts the negative effect of the increase of γ on its inverse demand function. With a sufficient degree of asymmetry in costs, when products are close substitutes this positive effect will eventually prevail, inverting the sign of the overall effect exerted by further increases of γ on the efficient firm profits.

3.2. Bertrand competition

In order to characterise the equilibrium under price competition we must account of the discontinuity of the demand system (2) when $\gamma = 1$. Further, for $\gamma < 1$, the demand functions are kinked when the price vector enters region $\bar{R}_2$ or $\bar{R}_1$, kinking the best response function of the firm with positive demand in that region.

Assume first that $\gamma < 1$, and suppose that both firms are active in equilibrium. Then firm i chooses p_i to maximise

$$\pi_i = \left(p_i - \lambda^{i-1} c\right) \frac{(1-\gamma)\,\alpha - p_i + \gamma p_j}{1-\gamma^2} \qquad [i, j = 1, 2;\ i \neq j] \ ,$$

by taking p_j as given.

The best response functions of firm 1 and firm 2 are (respectively)

$$\begin{aligned} p_1 &= \tfrac{1}{2}\left[(1-\gamma)\,\alpha + c + \gamma p_2\right] \\ p_2 &= \tfrac{1}{2}\left[(1-\gamma)\,\alpha + \lambda c + \gamma p_1\right] \ . \end{aligned} \tag{9}$$

Solving system (8), we get the following characterisation of the internal equilibrium

$$q_1^B = \frac{p_1^B - c}{(1-\gamma^2)} = \frac{(2-\gamma^2)(\alpha-c)-\gamma(\alpha-\lambda c)}{(1-\gamma^2)(4-\gamma^2)};\ \pi_1^B = \frac{1}{1-\gamma^2}\left\{\frac{(2-\gamma^2)(\alpha-c)-\gamma(\alpha-\lambda c)}{4-\gamma^2}\right\}^2;$$
$$q_2^B = \frac{p_2^B - \lambda c}{(1-\gamma^2)} = \frac{(2-\gamma^2)(\alpha-\lambda c)-\gamma(\alpha-c)}{(1-\gamma^2)(4-\gamma^2)};\ \pi_2^B = \frac{1}{1-\gamma^2}\left\{\frac{(2-\gamma^2)(\alpha-\lambda c)-\gamma(\alpha-c)}{4-\gamma^2}\right\}^2. \tag{10}$$

Equation (10) shows that production, unit-profit and profits are higher for firm 1 provided that $\lambda > 1$.[13] For $\lambda = 1$ the equilibrium is symmetric. As λ increases, both production, unit-profit and profits increase for firm 1 but decrease for firm 2. This leads to the following lemma.

Lemma 3 *Under Bertrand competition, for any* $\gamma \in (0,1]$ *the less efficient firm is active in equilibrium provided that* $1 \leq \lambda < \lambda^L(\gamma)$, *where*

$$\lambda^L(\gamma) = \frac{\alpha}{c} - \frac{\gamma}{2-\gamma^2}\frac{(\alpha-c)}{c} < \lambda^D(\gamma). \tag{11}$$

Equation (11) defines a frontier, $\lambda^L(\gamma)$, which limits the region of $S_0(\lambda,\gamma)$ where price competition leads to an internal equilibrium (region $I(\lambda,\gamma)$ in Figure 1, left diagram).[14] Note that $\lambda^L(\gamma)$ is decreasing in γ, meaning that any efficiency advantage of firm 1 exerts a stronger effect on the rival 's market share when products are closer substitutes. Further, $\lambda^L(\gamma)$ lies below the "drastic frontier" $\lambda^D(\gamma)$, meaning that, for any degree of products differentiation (but $\gamma = 0$), price competition has a stronger selective effect against the market share of the less efficient firm than quantity competition.[15]

For $\lambda^L(\gamma) \leq \lambda < \lambda^D(\gamma)$, price competition leads to a limit-pricing equilibrium where the most efficient firm drives the rival out of the market by pricing below the monopoly price p_1^M.

[13] In contrast, for $\lambda > 1$ and $\gamma < 1$, $p_2^B \geq p_1^B$ with strict inequality. Further, both equilibrium prices increase with λ.

[14] Similarly, from equations (4) and (11) we get $x^L(\gamma) = \frac{\gamma}{2-\gamma^2}$ as the lower boundary of the region of $S(x,\gamma)$ where price competition leads to an internal equilibrium (region $L(x,\gamma)$ in Figure 1, right diagram). Clearly, $x^L(\gamma)$ increases with γ, and $x^L(\gamma) > x^D(\gamma)$ for any $\gamma \in (0,1]$.

[15] For $\gamma = 0$, the two frontiers, $\lambda^L(\gamma)$ and $\lambda^D(\gamma)$, intersect at $\lambda = \frac{\alpha}{c}$. At the opposite extreme of the range of γ, $\lambda^L(1) = 1$, and the region of the internal equilibrium shrinks to a single point (see Figure 1, left diagram).

To see this, consider Figure 2. Recall that, for any λ and γ, firm 1 best response function is given by equation (9) provided that the rival is active in the market (i.e. $(p_1, p_2) \notin \bar{R}_2$). On the other hand, when p_2 is sufficiently high, the best response function of firm 1 reaches the boundary of the price region $\bar{R}_2$, where the demand for good 2 is zero, it kinks thereafter and continues along the boundary. We call this boundary the "$q_2 = 0$ *line*", whose expression, $p_1 = \frac{1}{\gamma}[p_2 - (1-\gamma)\alpha]$, comes from equation (3).[16] Moreover, from (9) it is clear that the best response function of firm 2 shifts outward as λ increases, while firm 1 best response function is independent of λ. Then, for $\lambda = \lambda^L(\gamma)$, firm 2 best response function is shifted just enough to cross firm 1 best response function where the latter kinks. For $\lambda > \lambda^L(\gamma)$ the intersection lies along the $q_2 = 0$ *line*.[17]

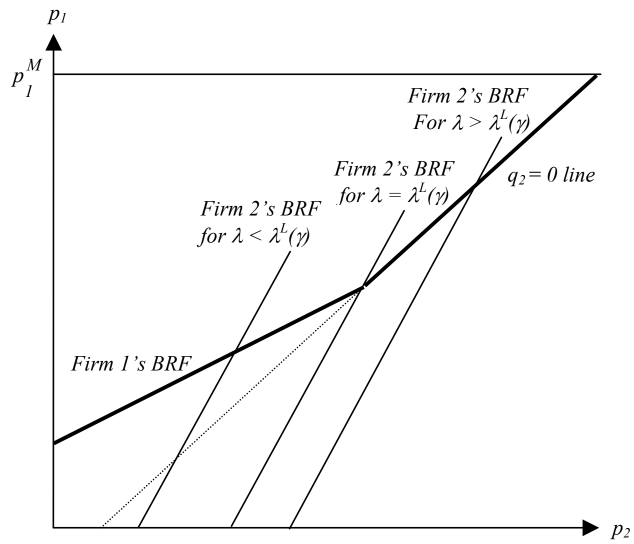

FIG. 2.

Next, by solving the system between the $q_2 = 0$ *line* and the best response function of

[16] For any p_2, the $q_2 = 0$ *line* gives the maximum p_1 which allows firm 1 to drive the rival out of the market. Since firm 1 best response function (as given by eq.(9)) lies below the $q_2 = 0$ *line* line after their intersection, it is obviously optimal for firm 1 to price exactly at that level.

[17] Note that, the assumption that firm 1 is always at least as efficient as firm 2 rules out that the equilibrium prices could ever lie in region $\bar{R}_1$ (i.e. firm 1 is always active in equilibrium).

firm 2,

$$p_1 = \frac{1}{\gamma}\left[p_2 - (1-\gamma)\,\alpha\right]$$
$$p_2 = \frac{1}{2}\left[(1-\gamma)\,\alpha + \lambda c + \gamma p_1\right] \ ,$$

and using $q_1 = \alpha - p_1$ as the demand function of good 1, we obtain the characterisation of the limit-pricing equilibrium

$$\begin{aligned} p_1^L - c &= \tfrac{1}{\gamma}\left[\gamma\,(\alpha - c) - (\alpha - \lambda c)\right];\ q_1^L = \tfrac{1}{\gamma}\,(\alpha - \lambda c)\,; \\ \pi_1^L &= \tfrac{1}{\gamma^2}\left[\gamma\,(\alpha - c)\,(\alpha - \lambda c) - (\alpha - \lambda c)^2\right]; \\ p_2^L - \lambda c &= q_2^L = \pi_2^L = 0\ . \end{aligned} \tag{12}$$

From equations (10) and (12), it is relatively easy to derive the following properties of Bertrand equilibrium over the space $S_0(\lambda,\gamma)$. First, for any $\gamma \in (0,1)$, when λ reaches the frontier $\lambda^L(\gamma)$ the limit-pricing equilibrium replaces the internal equilibrium without discontinuity. Then, as λ increases from $\lambda^L(\gamma)$ to $\lambda^D(\gamma)$, firm 1's price and profits increase (and its production decreases) until they reach the monopoly levels on the "drastic frontier".[18] Second, for $\gamma = 1$ and $\lambda^D(\gamma) \geq \lambda > \lambda^L(1)$, equation (12) gives the standard limit-pricing equilibrium of an homogeneous duopoly with linear asymmetric cost functions (i.e. $p_1^L = p_2^L = \lambda c$; $\pi_1^L, q_1^L > 0$; $\pi_2^L, q_2^L = 0$). Third, for $\gamma = 1$ and $\lambda = 1$, the equilibrium prices and profits of equation (12) replicate the standard Bertrand equilibrium of an homogenous duopoly with linear symmetric cost functions (i.e. $p_1^L = p_2^L = c$; $\pi_1^L = \pi_2^L = 0$), but the total production is entirely assigned to firm 1. By contrast, assuming $\lambda = 1$ in equation (10) and taking the limit of the internal equilibrium for $\gamma \to 1$, we obtain the standard Bertrand equilibrium with total production equally shared between the two firms.[19]

As in the case of quantity competition, the characterisation of the Bertrand equilibrium over the entire space $S_0(\lambda,\gamma)$ implies that firm 1 equilibrium profits are non-monotonic in γ. The following lemma shows more precisely the behaviour of the equilibrium profits of the two firms as functions of the degree of product differentiation.

[18] It is easy to verify that, given any $\gamma \in (0,1]$, equation (12) implies $c < p_1^L < p_1^M$ for $\lambda \in [\lambda^L(\gamma); \lambda^D(\gamma))$, and $p_1^L = p_1^M$ for $\lambda = \lambda^D(\gamma)$.

[19] The discontinuity of Bertrand equilibrium quantities for $\gamma = \lambda = 1$ will force us to treat this point separately in most of the proofs of the next section.

Lemma 4 *Under price competition, given any $\lambda \in [1, \frac{\alpha}{c})$, the equilibrium profits of the less efficient firm decrease as γ increases from 0 to $\gamma^L(\lambda) \equiv \lambda^{L^{-1}}(\gamma)$. On the contrary, for any $\lambda \in (1, \frac{\alpha}{c})$ there exists a critical value of γ, $0 < \gamma^*_B(\lambda) < \gamma^L(\lambda)$, such that the equilibrium profits of the most efficient firm decrease as γ rises from 0 to $\gamma^*_B(\lambda)$, while they increase as γ increases from $\gamma^*_B(\lambda)$ to the minimum between 1 and $\gamma^D(\lambda) \equiv \lambda^{D^{-1}}(\gamma)$.*[20]

Like Lemma 2 for quantity competition, Lemma 4 suggests that the asymmetry in costs can modify the conventional result on firms' incentives to differentiate products under Bertrand competition, providing the efficient firm with a local incentive to reduce the degree of differentiation when products are initially close substitutes. The main difference between the two Lemma is that, under quantity competition, the equilibrium profits of the efficient firm may increase with γ only if the efficiency gap is sufficiently high, while under price competition firm 1 profits are non-monotonic in γ for any (positive) degree of asymmetry in costs.

4. COMPARISON OF BERTRAND AND COURNOT EQUILIBRIA

The limit-price frontier $\lambda^L(\gamma)$ partitions the relevant space of the model, $S_0(\lambda, \gamma)$, into two regions, $I(\lambda, \gamma)$ and $L(\lambda, \gamma)$ (see Figure 1, left diagram).[21] According to Lemma 1 and

[20] It can also be proved that the equilibrium price of both firms decrease with γ over the region where the equilibrium is internal, while the price of the efficient firm increases with γ when the limit pricing equilibrium prevails. The behaviour of the equilibrium quantities with respect to γ is more complicated. For the efficient firm, for any $\lambda \in [1, \frac{\alpha}{c})$ there exists a critical level of γ inside the region of the internal equilibrium such that the equilibrium production decreases (increases) for γ below (above) the critical level. However, as γ crosses the locus $\gamma^L(\lambda)$ and enters the region of the limit-pricing equilibrium, the efficient firm's production starts decreasing again. For the inefficient firm, if the efficiency gap is not too small then the equilibrium production is always decreasing in γ. On the contrary, for a small degree of asymmetry in costs, the inefficient firm's production first decreases and then increases with γ, decreasing again when γ is sufficiently close to the locus $\gamma^L(\lambda)$. Finally, if firms are symmetric in costs, their equilibrium production first decreases and then increases as γ rises from 0 to 1.

[21] The right diagram of Figure 1 shows the analogous partition of the space $S_0(x, \gamma)$ into the regions $I(x, \gamma)$ and $L(x, \gamma)$

3, these regions differ in the structure of the market equilibrium prevailing under the two alternative forms of competition. In region $I(\lambda,\gamma)$, which lies below $\lambda^L(\gamma)$, both Cournot and Bertrand competition lead to an internal equilibrium in which the less efficient firm is active in the market. In region $L(\lambda,\gamma)$, lying between $\lambda^L(\gamma)$ and $\lambda^D(\gamma)$, the less efficient firm is active in the market under Cournot competition, while it is inactive, but exerting a competitive pressure on the rival from outside the market, under Bertrand competition.

Singh and Vives (1984) restrict the comparison of Bertrand and Cournot equilibria to a sub-region of $I(\lambda,\gamma)$. They assume that the "primary output" is positive for both firms, meaning that each firm can sell a positive quantity when prices are set at the marginal costs.[22] In our version of the model, this assumption is binding only for the inefficient firm and, formally, it implies the parametric restriction

$$\lambda < \frac{\alpha}{c} - \gamma\frac{\alpha - c}{c} \tag{13}$$

Comparing (13) and (11), it is easy to see that (13) is a stronger condition than the requirement for an internal equilibrium under price competition. Indeed, (13) identifies an upper bound for λ that lies below the frontier $\lambda^L(\gamma)$ for any $\gamma \in (0,1)$ (i.e. the dotted line in Figure 1).[23]

In this section we extend the comparison of Bertrand and Cournot equilibria over the entire space $S_0(\lambda,\gamma)$. We start by comparing the equilibrium prices and quantities under the two alternative forms of competition. We have the following lemma.

Lemma 5 (prices) *The equilibrium prices of both firms are higher under Cournot than under Bertrand competition over the entire space $S_0(\lambda,\gamma)$.*

[22] We are adopting here a terminology introduced by Amir and Jin (2001).

[23] Intuitively, when $\gamma < 1$, the efficient firm is pricing above its marginal cost along the frontier $\lambda^L(\gamma)$, leaving the rival a maximum price compatible with a non-negative demand equal to the rival's marginal cost (i.e. prices are on the boundary of the region $\bar{R}_2$). Obviously, if the efficient firm priced at its marginal cost, the inefficient firm would still remain out of the market (i.e, prices would enter the region $\bar{R}_2$). This means that (12) cannot be satisfied along frontier $\lambda^L(\gamma)$ when $\gamma < 1$. By continuity, there must exist a portion of region $I(\lambda,\gamma)$, just below $\lambda^L(\gamma)$, where (13) does not hold as well.

Lemma 6 (quantities) *The efficient firm produces more under Bertrand than under Cournot competition over the entire space* $S_0(\lambda, \gamma)$*. For the inefficient firm, Bertrand production exceeds Cournot production iff* $\lambda < \frac{\alpha}{c} - \gamma\frac{\alpha-c}{c}$ *(i.e. the same condition as equation (13)).*

Lemma 5 confirms the Singh and Vives's ranking of Bertrand and Cournot equilibrium prices over the entire space $S_0(\lambda, \gamma)$. Namely, under price competition firms perceive a more elastic demand than under quantity competition, and this refrains them from increasing prices. This is true either when the inefficient firm is active in the market under both forms of competition (i.e. over region $I(\lambda, \gamma)$), or when the asymmetry in costs and the degree of products substitutability allow the efficient firm to engage in limit pricing (i.e. over region $L(\lambda, \gamma)$). Lemma 6 emphasises the more selective effect of price competition against the market share of the inefficient firm. While the efficient firm always produces more under Bertrand competition, the inefficient firm produces less under Bertrand than under Cournot competition for high values of λ (given any $\gamma > 0$), or for high values of γ (given any $\lambda > 1$), inside region $I(\lambda, \gamma)$, as well as over the entire region $L(\lambda, \gamma)$ (see, again, Figure 1). Therefore, this aspect of the comparison between the two forms of competition can be captured only by allowing for a sufficient degree of asymmetry in costs and/or for a sufficient degree of products substitutability. Finally, Lemma 6 shows that the Singh and Vives's restriction of the model space (eq.(13)) coincides with the portion of region $I(\lambda, \gamma)$ where both firms produce more under price than under quantity competition.

Turning to the comparison of the equilibrium profits, we first show that the Singh and Vives's ranking of Bertrand and Cournot equilibrium profits extends over the entire region $I(\lambda, \gamma)$.

Proposition 1 (Singh and Vives 1984) *Inside region* $I(\lambda, \gamma)$*, both firms earn higher profits under quantity than under price competition, i.e.* $\pi_1^C > \pi_1^B$ *and* $\pi_2^C > \pi_2^B$*.*

Consider now region $L(\lambda, \gamma)$. Obviously, in this region firm 2 profits are always higher with quantity competition (i.e. positive rather than zero). Nevertheless, in the following proposition we prove that the efficient firm obtains higher profits under Bertrand than under Cournot competition in a significant portion of region $L(\lambda, \gamma)$.

Proposition 2 (firm 1 profits) *For any $\gamma \in (0,1]$ there exists a threshold level of λ inside region $L(\lambda,\gamma)$, $\widehat{\lambda}(\gamma) = \frac{\alpha}{c} - \frac{\gamma}{2-\gamma^2+\frac{\gamma^4}{4}}\frac{\alpha-c}{c}$, such that $\lambda \gtreqless \widehat{\lambda}(\gamma)$ implies $\pi_1^L \gtreqless \pi_1^C$. Similarly, since $\widehat{\lambda}(\gamma)$ is monotonically decreasing in γ, for any $\lambda \in (\widehat{\lambda}(1), \frac{\alpha}{c})$ we can identify a critical value of γ inside region $L(\lambda,\gamma)$, namely $\widehat{\gamma}(\lambda) = \widehat{\lambda}^{-1}(\gamma)$, such that $\gamma \gtreqless \widehat{\gamma}(\lambda)$ implies $\pi_1^L \gtreqless \pi_1^C$.*

According to Proposition 2, for any degree of products differentiation (except $\gamma = 0$), the equilibrium profits of the efficient firm are higher with price than with quantity competition whenever the efficiency gap between the two firms is sufficiently high. Moreover, the same ranking of firm 1 profits arises if products are sufficiently close substitutes and the efficiency gap exceeds the critical value $\widehat{\lambda}(1) = \frac{1}{5}(4 + \frac{\alpha}{c}) > 1$. Figure 3 illustrates.

The interpretation of this result relies on the comparison of the effects on prices and market shares exerted by the two forms of competition in regions $I(\lambda,\gamma)$ and $L(\lambda,\gamma)$. On one hand, both firms face lower prices with Bertrand competition over the entire space $S_0(\lambda,\gamma)$, and this tends to make their profits lower under Bertrand than under Cournot competition (*price effect*). On the other hand, when firms are asymmetric in costs, price competition tends to reduce more the market share of the less efficient firm than does quantity competition. Obviously this tends to make the efficient firm profits higher, and the inefficient firm profit lower, under price than under quantity competition (*selection effect*).

Both effects work in the same direction for the inefficient firm, while they operate in opposite directions for the efficient firm. Moreover, given any positive degree of products substitutability, the *selection effect* gets stronger as the efficiency gap between the two firms increases, reaching its maximum intensity on the limit-pricing frontier $\lambda^L(\gamma)$, where the inefficient firm's market share is zero under Bertrand but still positive under Cournot. By contrast, as the efficiency gap between the two firms rises, the *price effect* weakens, vanishing as λ reaches the "drastic frontier" where the prices of both firms are identical under the two forms of competition (i.e., the monopoly price, p_1^M, for the efficient firm, and the marginal cost, $\lambda^D(\gamma)c$, for the inefficient firm).

Turning to the profits of the efficient firm, Proposition 1 shows that the *price effect* dominates the *selection effect* over region $I(\lambda, \gamma)$, while Proposition 2 assures that the *selection effect is* stronger than the *price effect* when the efficiency gap is sufficiently close to the "drastic frontier". Intuitively, along the "drastic frontier" the efficient firm is pricing at the monopoly level and the inefficient firm is out of the market under both price and quantity competition. For a given degree of product differentiation, a slight decrease of the efficiency gap below the drastic frontier causes the efficient firm to be constrained by outside competition under Bertrand, meaning that it is now forced to price below the monopoly level. However, since the efficiency gap is almost drastic, the equilibrium price p_1^L is just below the monopoly price p_1^M, and the effect of outside competition on the efficient firm's profit is second order. By contrast, a slight decrease of the efficiency gap below the drastic frontier allows the inefficient firm to hold a positive market share under Cournot. While firm 1 equilibrium price decreases less than under Bertrand competition, the reduction in firm 1 market share exerts a first order negative effect on its profits. This is formally confirmed by the following lemma, which also suggests a similar interpretation of Proposition 2 along the dimension of products differentiation.

Lemma 7 *Along the drastic frontier, $\lambda^D(\gamma)$, the partial derivatives of the efficient firm profits with respect to λ and γ are strictly positive under Cournot, while they are zero under Bertrand competition. Further, under Cournot competition, the partial derivatives of the inefficient firm profits with respect to λ and γ are zero along the drastic frontier.*

As a direct consequence of Proposition 2 and Lemma 7, we can now prove that the industry profits are higher under Bertrand than under Cournot competition in a portion of region $L(\lambda, \gamma)$ close to the drastic frontier. Indeed, along the drastic frontier, the industry profits are identical under the two forms of competition. Then, Proposition 2 assures that, starting from the drastic frontier, a slight decrease of λ (or γ) makes firm 1 profits higher under Bertrand than under Cournot, while the second part of Lemma 2 shows that such a movement of λ (or γ) from the drastic frontier has only a second order effect on firm 2 profits under Cournot competition. The reason is that, although firm 2 gains a (slightly) positive market share under Cournot competition, it sells its production at a price level almost equal to its marginal cost since the efficiency gap is just below the drastic frontier. Therefore the overall effect of a small decrease of λ (or γ) from the drastic frontier is to make the industry profits higher under price than under quantity competition. By continuity, the same ranking of the industry profits under the two forms of competition must hold over a sub-region of $L(\lambda, \gamma)$ sufficiently close to the drastic frontier. The next proposition gives a precise statement of this result.

Proposition 3 (industry profits) *Denote with $\pi^C = \pi_1^C + \pi_2^C$ and $\pi^L = \pi_1^L$ the industry profits under Cournot and Bertrand competition, respectively, over region $L(\lambda, \gamma)$. Then, for any $\gamma \in (0, 1]$ there exists a threshold level of λ inside region $L(\lambda, \gamma)$, $\widetilde{\lambda}(\gamma) = \frac{\alpha}{c} - \frac{\gamma(4+\gamma^2)}{8-2\gamma^2+\gamma^4} \frac{\alpha-c}{c}$, such that $\lambda \gtreqqless \widetilde{\lambda}(\gamma)$ implies $\pi^L \gtreqqless \pi^C$. Similarly, since $\widetilde{\lambda}(\gamma)$ is monotonically decreasing in γ, for any $\lambda \in (\widetilde{\lambda}(1), \frac{\alpha}{c})$ we can identify a critical value of γ inside region $L(\lambda, \gamma)$, namely $\widetilde{\gamma}(\lambda) = \widetilde{\lambda}^{-1}(\gamma)$, such that $\gamma \gtreqqless \widetilde{\gamma}(\lambda)$ implies $\pi^L \gtreqqless \pi^C$. Figure 3 illustrates.*

The explanation of this result is linked to our interpretation of Proposition 2. When firms are asymmetric in costs, a switch from Cournot to Bertrand competition exerts two opposite effects on industry profits. The decrease in the equilibrium prices works towards a reduction of the industry profits (*price effect*), while the stronger selective mode of price competition relocates production from the inefficient to the efficient firm, working towards an increase of the industry profits (*productive efficiency effect*). According to Proposition 3, the *productive efficiency effect* dominates the *price effect* either when, given any positive degree of product substitutability, the degree of costs asymmetry is sufficiently high, or when products are close substitutes and the efficiency gap exceeds the critical level $\widetilde{\lambda}(1) = \frac{1}{7}(5 + 2\frac{\alpha}{c}) > 1$

We conclude this section by comparing Bertrand and Cournot equilibria from the viewpoint of social welfare over the entire space $S_0(\lambda, \gamma)$. Using the consumer's utility function, we obtain the following expression for the total surplus

$$TS(\lambda, \gamma) = (\alpha - c) q_1 + (\alpha - \lambda c) q_2 - \left[\frac{1}{2}(q_1 + q_2)^2 - (1 - \gamma) q_1 q_2\right] + m, \qquad (14)$$

while the consumer surplus is given by the difference between the total surplus and the industry profits. Clearly, as far as the marginal utility of each good exceeds the respective

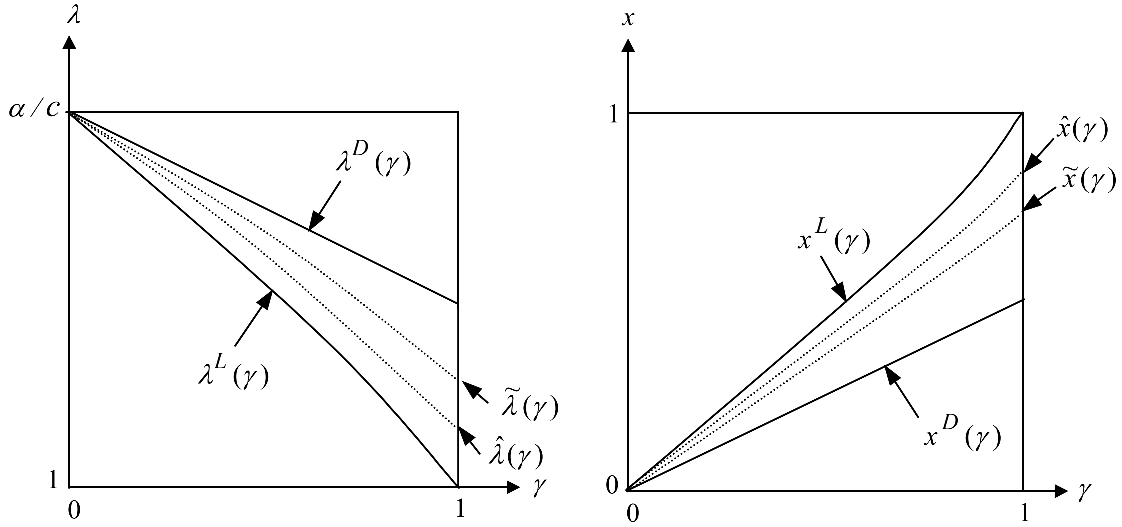

FIG. 3.

marginal cost, the total surplus increases with q_1 and q_2, while the consumer surplus is always decreasing in prices. As we have shown in Lemma 6, Singh and Vives consider only the region of the model where both firms produce more under price than under quantity competition. Since the equilibrium prices are lower with price competition, they can easily conclude that both total surplus and consumer surplus are higher under Bertrand than under Cournot over that region.

Looking at the other regions of the space $S_0(\lambda, \gamma)$, we know by Lemma 5 that the equilibrium prices are always lower under Bertrand than under Cournot. This assures that the Singh and Vives's ranking of the consumer surplus extends over the entire space $S_0(\lambda, \gamma)$. Obviously, this is also sufficient to prove that the total surplus is larger with price competition over the sub-region of $L(\lambda, \gamma)$ where price competition entails larger industry profits (see Proposition 3). By contrast, the ranking of the total surplus under the two forms of competition is not immediate either over the sub-region of $L(\lambda, \gamma)$ where the industry profits are higher under quantity competition, or over the portion of region $I(\lambda, \gamma)$ where the inefficient firm produces less under price than under quantity competition. However, the next proposition shows that the Singh and Vives's ranking of total surplus extends over the entire space $S_0(\lambda, \gamma)$.

Proposition 4 (welfare) *Consumer surplus and total surplus are higher under Bertrand than under Cournot competition over the entire space* $S_0(\lambda, \gamma)$.

Summarising, while consumers always gain from a switch from Cournot to Bertrand competition, for a low degree of asymmetry in costs (and/or a high degree of products differentiation) both firms lose profits since the *price effect* dominates the *productive efficiency effect.* The total surplus rises because the increase of consumer surplus exceeds the decrease of industry profits. By contrast, when the asymmetry in costs is high (and/or products are close substitutes), a switch from Cournot to Bertrand competition increases both consumer surplus and industry profits, since now the overall effect on industry profits reflects the dominance of the *productive efficiency effect* over the *price effect*.

5. CONCLUSIONS

In this paper we have re-considered the comparison of price and quantity competition within the standard model of a horizontally differentiated duopoly with linear demand and cost functions. Our main innovation with respect to the previous analysis of Singh and Vives (1984) has been to enlarge the parameter space of the model by allowing for any relevant combination of firms' cost asymmetry and products differentiation.

The main result of the paper is that, for high degrees of cost asymmetry and/or low degrees of product differentiation, price competion entails higher profits for the efficient firm, and for the industry, than quantity competition. The interpretation of this result is based on the stronger selective effect of price competition against the inefficient firm (*selection effect*), which also leads to more productive efficiency under this form of competition relative to quantity competiton (*productive efficiency effect*). These two effects work in the direction of higher profits for the efficient firm and for the industry under Bertrand than under Cournot competition, which contrasts the opposite effect due to the lower equilibrium prices arising with the former mode of competition (*price effect*). With large cost asymmetries and low degrees of product differentiation, the *selection* and the *productive efficiency effects* dominate the *price effect*, inverting the standard ranking of profits under the two forms of competition.

The interaction of the *price*, *selection* and *productive efficiency effects* is well known in the literature that analyses the intensity of market competition and its effect on firms' incentive to innovate.[24] Our paper contributes to this literature by offering a complete map of the composition of these effects along the two dimensions of firms' cost asymmetry and horizontal products differentiation, within a model which is widely employed as a "building block" in many fields of the literature.

[24] Among others, Vickers (1995); Aghion and Shankerman (2000); Boone (2000) and (2001); Delbono and Denicolo' (1990); Bester and Petrakis (1993); Bonanno and Haworth (1998); Qui (1998).

Appendix

Proof of Lemma 1. Using (8), it is easy to verify that the inequalities $\pi_2^C \geq 0$ and $\pi_1^C \leq \pi_1^M$ lead to the same condition $\lambda \leq \frac{\alpha}{c} - \frac{\gamma}{2}\frac{(\alpha-c)}{c} = \lambda^D(\gamma)$

Proof of Lemma 2. From equation (8) it is clear that $\frac{\partial \pi_2^C}{\partial \gamma} = 2\sqrt{\pi_2^C}\frac{\partial q_2^C}{\partial \gamma}$ and $\frac{\partial \pi_1^C}{\partial \gamma} = 2\sqrt{\pi_1^C}\frac{\partial q_1^C}{\partial \gamma}$. Hence, as far as profits are positive, the sign of the derivative of profits corresponds to the sign of the derivative of the equilibrium quantity for both firms.

Consider first firm 2. Re-stating q_2^C in terms of the parameter x, and taking the derivative with respect to γ, we get $\frac{\partial q_2^C}{\partial \gamma} = (\alpha - c)\frac{\left[-(4+\gamma^2)+4\gamma x\right]}{(4-\gamma^2)^2}$. The latter expression is always negative since γ, $x \in [0,1]$ over the space of the model.

Consider now to firm 1. Re-stating q_1^C in terms of x and differentiating with respect to γ leads to $\frac{\partial q_1^C}{\partial \gamma} = (\alpha - c)\frac{\left[-x(4+\gamma^2)+4\gamma\right]}{(4-\gamma^2)^2}$. Since the sign of this expression corresponds to the sign of the term in the square brackets, we have $\frac{\partial \pi_1^C}{\partial \gamma} \gtreqless 0 \Longleftrightarrow x \lesseqgtr \frac{4\gamma}{4+\gamma^2} \equiv x_C^*(\gamma)$. It is easy to show that $x_C^*(\gamma)$ describes a monotonic increasing locus in the space $S(x,\gamma)$, taking the values $x_C^*(0) = 0$ and $x_C^*(1) = \frac{4}{5} < 1$ at the extremes of the range of γ. Further, for any $\gamma \in (0,1)$, $x_C^*(\gamma) > x^D(\gamma) = \frac{\gamma}{2}$. Hence, provided that $0 < x < x_C^*(1)$, the equilibrium profits of the efficient firm are non-monotonic in γ, decreasing as γ rises from 0 to the inverse of the locus $x_C^*(\gamma)$ (that is $\gamma_C^*(x)$), and increasing as γ rises from $\gamma_C^*(x)$ to the minimum between 1 and $\gamma^D(x)$. Finally, the conversion of this result from the space $S(x,\gamma)$ to the space $S(\lambda,\gamma)$ requires only to use equation (4).

Proof of Lemma 3. Using equation (10), the inequality $\pi_2^B \geq 0$ implies $\lambda \leq \frac{\alpha}{c} - \frac{\gamma}{2-\gamma^2}\frac{(\alpha-c)}{c} = \lambda^L(\gamma)$. Then, by comparing equations (6) and (11), we find that $\lambda^L(\gamma) < \lambda^D(\gamma)$ for any $\gamma \in (0,1]$, while $\lambda^L(0) = \lambda^D(0) = \frac{\alpha}{c} > 1$. Finally, $\lambda^L(\gamma)$ is decreasing in γ and takes value $\lambda^L(1) = 1$ for $\gamma = 1$. This means that $\lambda^L(\gamma) > 1$ for any $\gamma \in (0,1]$.

Proof of Lemma 4. Consider first firm 1. Using equations (4) and (10), firm 1's Bertrand profits under an internal equilibrium can be re-stated in terms of the parameter x as

$$\pi_1^B = (\alpha - c)\frac{\left[2-\gamma^2-\gamma x\right]^2}{(4-\gamma^2)^2(1-\gamma^2)}.$$

Differentiating π_1^B with respect to γ, after some manipulations we get

$$\frac{\partial \pi_1^B}{\partial \gamma} = \frac{2(\alpha-c)\left(2-\gamma^2-\gamma x\right)}{(4-\gamma^2)^3(1-\gamma^2)^2}\left[\gamma\left(4-2\gamma^2+\gamma^4\right) - x\left(4+\gamma^2-2\gamma^4\right)\right].$$

The first term of this expression is always positive over the space of the model. Therefore, the sign of the derivative of profits corresponds to the sign of the term in the square brackets, that is

$$\frac{\partial \pi_1^B}{\partial \gamma} \gtreqless 0 \Longleftrightarrow x \lesseqgtr \frac{\gamma\left(4-2\gamma^2+\gamma^4\right)}{\left(4+\gamma^2-2\gamma^4\right)} \equiv x_B^*(\gamma).$$

It can be shown that $x_B^*(\gamma)$ describes a monotonic increasing locus in the space $S(x,\gamma)$, taking the values $x_B^*(0) = 0$ and $x_B^*(1) = 1$ at the extremes of the range of γ. Further, for any $\gamma \in (0,1)$, the locus $x_B^*(\gamma)$ lies above frontier $x^L(\gamma) = \frac{\gamma}{2-\gamma^2}$, and the two loci intersect at the extremes of the range of γ. Denote with $\gamma_B^*(x)$ and $\gamma^L(x)$ the inverse functions of $x_B^*(\gamma)$ and $x^L(\gamma)$, respectively. Then, for any $x \in (0,1)$, $\gamma_B^*(x)$ identifies a critical level of γ between 0 and $\gamma^L(x)$, such that firm 1's profits decrease as γ rises from 0 to $\gamma_B^*(x)$, while they increase as γ rises from $\gamma_B^*(x)$ to the boundary of the region of the internal equilibrium.

Finally, using equations (4) and (12), firm 1's Bertrand profits under a limit pricing equilibrium can be restated as $\pi_1^L = \frac{(\alpha-c)^2}{\gamma^2}[\gamma x - x^2]$. Differentiating π_1^L with respect to γ, we get $\frac{\partial \pi_1^L}{\partial \gamma} = \frac{(\alpha-c)^2}{\gamma^3}[x(2x-\gamma)]$. This expression is positive for $x > \frac{\gamma}{2} = x^D(\gamma)$, and hence over the region between frontiers $x^L(\gamma)$ and $x^D(\gamma)$ where the limit pricing equilibrium occurs. Notice that $\frac{\partial \pi_1^L}{\partial \gamma}$ is nil along the drastic frontier $x^D(\gamma)$.

<u>Consider now firm 2</u>. By re-stating π_2^B (eq.(10)) in terms of x, and differentiating with respect to γ, we get

$$\frac{\partial \pi_2^B}{\partial \gamma} = \frac{2(\alpha-c)\left((2-\gamma^2)x-\gamma\right)}{(4-\gamma^2)^3(1-\gamma^2)^2}\left[x\gamma\left(4-2\gamma^2+\gamma^4\right)-\left(4+\gamma^2-2\gamma^4\right)\right].$$

The first term of this expression is positive for $1 \leq x < \frac{\gamma}{2-\gamma^2} \equiv x^L(\gamma)$, namely as far as firm 2 is active in equilibrium. Hence, the sign of the derivative of profits corresponds to the sign of the term in square brackets, that is

$$\frac{\partial \pi_2^B}{\partial \gamma} \leq 0 \Longleftrightarrow x \leq \frac{\left(4+\gamma^2-2\gamma^4\right)}{\gamma(4-2\gamma^2+\gamma^4)} \equiv \frac{1}{x_B^*(\gamma)}.$$

From the part of the proof concerning firm 1, we know that $x_B^*(\gamma) \in [0,1]$. Therefore its reciprocal is always greater than 1 (it is equal to 1 only for $\gamma = 1$). This means that the inequality above is always verified over the space of the model, and firm 2's profits are always decreasing in γ over the region of the internal equilibrium.

The translation of this results from the space $S(x,\gamma)$ to the space $S(\lambda,\gamma)$ is straightforward.

Proof of Lemma 5. Using equations (8) and (10), we get the following expressions for the differences in equilibrium prices over region $I(\lambda, \gamma)$

$$p_1^C - p_1^B = \frac{\gamma^2}{4-\gamma^2}(\alpha - c)$$
$$p_2^C - p_2^B = \frac{\gamma^2}{4-\gamma^2}(\alpha - \lambda c).$$

It is immediate to verify that both expressions are positive for $\gamma \in (0,1]$ and $\lambda < \frac{\alpha}{c}$, and therefore at any point inside region $I(\lambda, \gamma)$.

Over region $L(\lambda, \gamma)$, the inefficient firm is pricing at marginal cost under Bertrand, and above the marginal cost under Cournot. For the efficient firm, using equations (8) and (12) we get

$$p_1^C - p_1^L = \frac{(2-\gamma^2)}{\gamma(4-\gamma^2)}\left[2(\alpha - \lambda c) - \gamma(\alpha - c)\right]$$

This expression is positive for $\gamma \in (0,1]$ and $\lambda < \lambda^D(\gamma) = \frac{\alpha}{c} - \frac{\gamma}{2}\frac{(\alpha-c)}{c}$, and therefore, $p_1^C > p_1^L$ everywhere inside region $L(\lambda, \gamma)$. More precisely, from the expression above we can see that $p_1^C - p_1^L$ is positive and decreasing in λ over $L(\lambda, \gamma)$. As λ reaches the boundary $\lambda^D(\gamma)$, the efficient firm prices at p_1^M, while the inefficient firm prices at λc, irrespective of the form of competition. Hence the difference in equilibrium prices disappears for both firms.

Proof of Lemma 6. From equations (8) and (10), the differences in equilibrium quantities take the following expressions in region $I(\lambda, \gamma)$

$$q_1^B - q_1^C = \frac{\gamma^2}{(1-\gamma^2)(4-\gamma^2)}\left[(\alpha - c) - \gamma(\alpha - \lambda c)\right]$$
$$q_2^B - q_2^C = \frac{\gamma^2}{(1-\gamma^2)(4-\gamma^2)}\left[(\alpha - \lambda c) - \gamma(\alpha - c)\right]$$

The first expression is positive for $\gamma \in (0,1)$ and $\lambda > 1$. For $\lambda = 1$ and $\gamma \longrightarrow 1$, the model approaches a standard homogeneous duopoly with linear-symmetric cost functions, and both expressions converge to the positive limit $\frac{\gamma^2(\alpha-c)}{(1-\gamma^2)(4-\gamma^2)}$. Finally, from the second expression, we find immediately that, for any $\gamma \in (0,1)$, $q_2^B - q_2^C \gtreqless 0 \Longleftrightarrow \lambda \lesseqgtr \frac{\alpha}{c} - \gamma\frac{(\alpha-c)}{c}$.

Over region $L(\lambda, \gamma)$, the inefficient firm's production is nil under Bertrand but positive under Cournot. For the efficient firm, using equations (8) and (12) we get

$$q_1^L - q_1^C = \frac{2}{\gamma(4-\gamma^2)}\left[2(\alpha - \lambda c) - \gamma(\alpha - c)\right]$$

This expression is positive for $\gamma \in (0,1]$ and $\lambda < \lambda^D(\gamma) = \frac{\alpha}{c} - \frac{\gamma}{2}\frac{(\alpha-c)}{c}$. Therefore $q_1^L > q_1^C$ everywhere inside region $L(\lambda,\gamma)$. Further, $q_1^L - q_1^C$ is decreasing in λ over region $L(\lambda,\gamma)$. As λ reaches the boundary $\lambda^D(\gamma)$, the efficient firm produces its monopoly quantity, q_1^M, while the inefficient firm is out of the market, irrespective of the form of competition. Hence the difference in equilibrium quantities disappears for both firms.

Proof of Proposition 1. Consider firm 1. We first reformulate firm 1's profits under Cournot (eq.(8)) and under Bertrand (eq.(10)) in terms of the parameter x (as defined by (4)):

$$\pi_1^B = \frac{(\alpha-c)^2}{(1-\gamma^2)(4-\gamma^2)^2}[2-\gamma^2-\gamma x]^2$$
$$\pi_1^C = \frac{(\alpha-c)^2}{(4-\gamma^2)^2}[2-\gamma x]^2.$$

Recall that $(\alpha - c) > 0$ by assumption, and notice that $\gamma \in (0,1)$ and $x \in (0,1)$ over region $I(x,\gamma)$ by ignoring the limit point $(x = 1, \gamma = 1)$. Obviously at that point firm 1's profits are higher under Cournot, since products are perfect substitutes and costs are symmetric. Then, from the expressions above we see that the inequality $\pi_1^C \geq \pi_1^B$ leads to the inequality in γ and x

$\left(1-\gamma^2\right)\{2-\gamma x\}^2 \geq \left\{\left(2-\gamma^2\right)-\gamma x\right\}^2$, or after some manipulations, $\gamma^3[\gamma x^2 - 2x + \gamma] \leq 0$.

Given any $\gamma \in (0,1)$, this inequality is satisfied for $x \in [\underline{x}(\gamma), \overline{x}(\gamma)]$, where $\underline{x}(\gamma) = \frac{1}{\gamma}(1 - \sqrt{1-\gamma^2})$ and $\overline{x}(\gamma) = \frac{1}{\gamma}(1+\sqrt{1-\gamma^2})$. However, only $\underline{x}(\gamma)$ lies in the admissible range $(0,1)$, while $\overline{x}(\gamma) > 1$ for any $\gamma \in (0,1)$. Therefore we can say that $\pi_1^C \geq \pi_1^B$ for any $x \in [\underline{x}(\gamma), 1)$, with equality holding only for $x = \underline{x}(\gamma)$. To complete the proof, it remains only to show that the threshold level $\underline{x}(\gamma)$ lies below frontier $x^L(\gamma)$ for any $\gamma \in (0,1)$, i.e. $\frac{\gamma}{2-\gamma^2} > \frac{1}{\gamma}(1-\sqrt{1-\gamma^2})$ for $\gamma \in (0,1)$. The latter inequality can be written as $\sqrt{1-\gamma^2}(1-\sqrt{1-\gamma^2})^2 > 0$, which is indeed true for any $\gamma \in (0,1)$.

The part of the proposition concerning firm 2's profits can be proved along the same lines. Indeed, once reformulated firm 2's profits in terms of x and γ,

$$\pi_2^B = \frac{(\alpha-c)^2}{(1-\gamma^2)(4-\gamma^2)^2}[\left(2-\gamma^2\right)x-\gamma]^2$$
$$\pi_2^C = \frac{(\alpha-c)^2}{(4-\gamma^2)^2}[2x-\gamma]^2,$$

the inequality $\pi_2^C \geq \pi_2^B$ leads exactly to the same inequality $\gamma^3[\gamma x^2 - 2x + \gamma] \leq 0$.

Proof of Proposition 2. As in the proof of proposition 1, we consider firm 1's profits under Bertrand (eq.(12)) and under Cournot (eq.(8)) in terms of the parameter x (eq. (4)):

$$\pi_1^L = \frac{(\alpha-c)^2}{\gamma^2}[\gamma x - x^2]$$

$$\pi_1^C = \frac{(\alpha-c)^2}{(4-\gamma^2)^2}[2-\gamma x]^2.$$

Recall that $\gamma, x \in (0,1]$ over region $L(x,\gamma)$. Then, from the expressions above we see that the inequality $\pi_1^L \geq \pi_1^C$ leads to the following inequality in γ and x

$\left(4-\gamma^2\right)^2[\gamma x - x^2] \geq \gamma^2[2-\gamma x]^2$, or after some manipulations,

$\left(16-8\gamma^2+2\gamma^4\right)x^2 - \gamma\left(16-4\gamma^2+\gamma^4\right)x + 4\gamma^2 \leq 0.$

Given any $\gamma \in (0,1]$, we find that this inequality is satisfied for $x \in \left[x^D(\gamma), \widehat{x}(\gamma)\right]$, where $x^D(\gamma) = \frac{\gamma}{2}$ is the "drastic frontier" in the space $S(x,\gamma)$, and $\widehat{x}(\gamma) = \frac{\gamma}{2-\gamma^2+\frac{\gamma^4}{4}}$. Notice that $\widehat{x}(\gamma) > x^D(\gamma)$ for any $\gamma \in (0,1]$. Moreover, $\widehat{x}(\gamma) < x^L(\gamma) = \frac{\gamma}{2-\gamma^2}$ for any $\gamma \in (0,1]$. This means that the locus $\widehat{x}(\gamma)$ always lies in region $L(x,\gamma)$ (see Figure 3). Finally, using equation (4) it is immediate to translate this result to the space $S\{\lambda,\gamma\}$, i.e. $\pi_1^L \geq \pi_1^C$ for $\lambda \in [\widehat{\lambda}(\gamma), \lambda^D(\gamma)]$, where $\widehat{\lambda}(\gamma) = \frac{\alpha}{c} - \widehat{x}(\gamma)\frac{\alpha-c}{c}$. Obviously, the reverse inequality, $\pi_1^L \leq \pi_1^C$, holds for $\lambda \in [\lambda^L(\gamma), \widehat{\lambda}(\gamma)]$, while the equality $\pi_1^L = \pi_1^C$ holds only for $\lambda = \widehat{\lambda}(\gamma)$ and $\lambda = \lambda^D(\gamma)$.

Proof of Lemma 7. Firm 1. From the proof of Lemma 2 it follows directly that $\frac{\partial \pi_1^C}{\partial \gamma} > 0$ for $x = x^D(\gamma)$, since $x^D(\gamma) < x_C^*(\gamma)$. From equation (8) we get the partial derivative of firm 1's Cournot profits with respect to λ, $\frac{\partial \pi_1^C}{\partial \lambda} = 2\sqrt{\pi_1^C}\frac{\gamma c}{4-\gamma^2}$. This expression does not depend on λ, and it is positive for any $\gamma \in (0,1]$.

Consider now the partial derivatives of firm 1's Bertrand profits. From the proof of Lemma 4 we already know that $\frac{\partial \pi_1^L}{\partial \gamma} = 0$ along the drastic frontier. Using equations (4) and (12) we get the partial derivative with respect to λ, $\frac{\partial \pi_1^L}{\partial \gamma} = \frac{c(\alpha-c)}{\gamma^2}[2x-\gamma]$. This expression is nil for $x = x^D(\gamma) = \frac{\gamma}{2}$.

Firm 2. From the proof of Lemma 2 it follows directly that $\frac{\partial \pi_2^C}{\partial \gamma} = 0$ along the drastic frontier, where firm 2's Cournot profits vanish. The same is true for the partial derivative of firm 2's Cournot profits with respect to λ, since from equation (8) we get $\frac{\partial \pi_2^C}{\partial \lambda} = -2\sqrt{\pi_2^C}\frac{c}{4-\gamma^2}$.

Proof of Proposition 3. Using equations (4), (8) and (12), we first formulate the industry profits under the two forms of competition over region $L(\lambda, \gamma)$ in terms of the parameter x

$$\pi^L = \frac{(\alpha-c)^2}{\gamma^2}[\gamma x - x^2]$$
$$\pi^C = \frac{(\alpha-c)^2}{(4-\gamma^2)^2}\left[x^2\left(4+\gamma^2\right) - 8\gamma x + \left(4+\gamma^2\right)\right]$$

From these expressions we see that $\pi^L \geq \pi^C$ implies the inequality in x and γ

$\left(4-\gamma^2\right)^2[\gamma x - x^2] \geq \gamma^2\left[x^2\left(4+\gamma^2\right) - 8\gamma x + \left(4+\gamma^2\right)\right]$, or after some manipulations,

$$\left[16 - 4\gamma^2 + 2\gamma^4\right]x^2 - \gamma\left(16+\gamma^4\right)x + \gamma^2\left(4+\gamma^2\right) \leq 0.$$

Solving the latter inequality in x, we find it verified for any $x \in \left[x^D(\gamma), \widetilde{x}(\gamma)\right]$, where $x^D(\gamma)$ is the drastic frontier in the space $S(x,\gamma)$, and $\widetilde{x}(\gamma) = \frac{\gamma(4+\gamma^2)}{8-2\gamma^2+\gamma^4}$. As it can be easily verified, $\widetilde{x}(\gamma)$ describes a monotonically increasing locus in the space $S(x,\gamma)$, taking values $\widetilde{x}(0) = 0$ and $\widetilde{x}(1) = \frac{5}{7}$ at the extremes of the range of γ. Further, $\widetilde{x}(\gamma)$ lies above frontier $x^L(\gamma) = \frac{\gamma}{2-\gamma^2}$ for any $\gamma \in (0,1]$ (see Figure 3, right diagram). Therefore, for any $\gamma \in (0,1]$, $\widetilde{x}(\gamma)$ identifies a threshold level of x inside region $L(x,\gamma)$ such that $x \lesseqqgtr \widetilde{x}(\gamma)$ implies $\pi^L \gtreqqless \pi^C$. The conversion of this result from the space $S(x,\gamma)$ to the space $S(\lambda,\gamma)$ require only to use equation (4), i.e. $\widetilde{\lambda}(\gamma) = \frac{\alpha}{c} - \widetilde{x}(\gamma)\frac{\alpha-c}{c}$.

Proof of Proposition 4. Consider first the total surplus over the region $I(x,\gamma)$. Using equations (4), (8) and (14) we find the following expression for the total surplus under Cournot competition

$$TS^C = \left(\frac{\alpha-c}{4-\gamma^2}\right)^2\left[6 - \tfrac{1}{2}\gamma^2 - 8\gamma x + x\gamma^3 + 6x^2 - \tfrac{1}{2}x^2\gamma^2\right].$$

Similarly, by equations (4), (10) and (14), the total surplus under Bertrand equilibrium can be written as

$$TS^B = \left(\frac{(\alpha-c)}{(1-\gamma^2)(4-\gamma^2)}\right)^2\left[\begin{array}{c} 6x^2 - \frac{21}{2}\gamma^2x^2 - 8\gamma x + 6 - \frac{21}{2}\gamma^2 + \frac{11}{2}\gamma^4x^2 + \\ +11\gamma^3x + +\frac{11}{2}\gamma^4 - \gamma^6x^2 - 3\gamma^5x - \gamma^6 \end{array}\right].$$

Taking the difference between TS^B and TS^C, we get

$$TS^B - TS^C = \left(\frac{(\alpha-c)}{(1-\gamma^2)(4-\gamma^2)}\right)^2\left[\begin{array}{c} \left(2\gamma^2 - \frac{1}{2}\gamma^6 - \frac{3}{2}\gamma^4\right)x^2 + \left(-6\gamma^3 - \gamma^7 + 7\gamma^5\right)x + \\ + \left(2\gamma^2 - \frac{1}{2}\gamma^6 - \frac{3}{2}\gamma^4\right) \end{array}\right]$$

From this expression it is clear that the inequality $TS^B - TS^C \geq 0$ is equivalent to the second-degree inequality in x

$$\left(2\gamma^2 - \frac{1}{2}\gamma^6 - \frac{3}{2}\gamma^4\right)x^2 + \left(-6\gamma^3 - \gamma^7 + 7\gamma^5\right)x + \left(2\gamma^2 - \frac{1}{2}\gamma^6 - \frac{3}{2}\gamma^4\right) \geq 0.$$

It is easy to verify that $\left(2\gamma^2 - \frac{1}{2}\gamma^6 - \frac{3}{2}\gamma^4\right) > 0$ for any $\gamma \in (0,1)$. The discriminant of the inequality, $\Delta = \gamma^4\left(60\gamma^2 + 55\gamma^6 - 85\gamma^4 + \gamma^{10} - 15\gamma^8 - 16\right)$, is negative for any $\gamma \in (0,1)$. This suffices to prove that the inequality is satisfied for any $x \in [0,1]$.

For $\gamma = 1$ we must consider only the point $(\gamma = 1, x = 1)$ on the boundary of region $I(\lambda, \gamma)$. This point corresponds to the standard homogeneous duopoly with symmetric costs, where we already know that the total surplus is larger under Bertrand.

Let us turn now to region $L(x, \gamma)$. The total surplus under Cournot is still given by the expression above. Using equations (4), (12) and (14) we find the following expression for the total surplus under Bertrand

$$TS^L = \frac{(\alpha - c)^2}{\gamma^2}\left[\gamma x - \frac{1}{2}x^2\right].$$

Taking the difference, we get

$$TS^L - TS^C = \frac{(\alpha - c)^2}{\gamma^2(4-\gamma^2)^2}\left[-\left(8 + 2\gamma^2\right)x^2 + 16\gamma x - \frac{1}{2}\gamma^2(12 - \gamma^2)\right].$$

Imposing $TS^L - TS^C \geq 0$ implies the inequality in x and γ

$$-\left(8 + 2\gamma^2\right)x^2 + 16\gamma x - \frac{1}{2}\gamma^2(12 - \gamma^2) \geq 0.$$

Solving for x, we find the inequality satisfied for any $x \in [x^D(\gamma), x^{TS}(\gamma)]$, where $x^D(\gamma)$ is the drastic frontier in the space $S(x, \gamma)$ and $x^{TS}(\gamma) = \frac{\gamma}{2}\frac{12-\gamma^2}{4+\gamma^2}$. Since $x^{TS}(\gamma)$ lies above the frontier $x^L(\gamma) = \frac{\gamma}{2-\gamma^2}$, the inequality is satisfied over the entire region $L(\lambda, \gamma)$.

The part of Proposition 4 concerning the consumer surplus has already been proved in the text.

REFERENCES

[1] Aghion, P. and M. Shankerman, 2000, A model of market enhancing infrastructure, CEPR Discussion Paper No.2461.

[2] Amir, R. and J.Y. Jin, 2001, Cournot and Bertrand equilibria compared: substitutability, complementarity and concavity, International Journal of Industrial Organization 19, 303-317.

[3] Bester, H. and E. Petrakis, 1993, The incentives for cost reduction in a differentiated industry, International Journal of Industrial Organization 11, 519-534.

[4] Bonanno, G. and B. Haworth, 1998, Intensity of competition and the choice between product and process innovation, International Journal of Industrial Organization 16, 495-510.

[5] Boone, J., 2000, Competitive pressure: the effects on investments in product and process innovation, Rand Journal of Economics 31, 549-569.

[6] Boone, J., 2001, The intensity of competition and the incentive to innovate, International Journal of Industrial Organization 19, 705-726.

[7] Delbono, F. and V. Denicolo', 1990, R&D investment in a symmetric and homogeneous oligopoly, International Journal of Industrial Organization 8, 297-313.

[8] Denicolo', V. and P.Zanchettin, 2002, Competition and growth in a neo-schumpeterian model, mimeo.

[9] Dixit, A.K.,1979, A model of duopoly suggesting a theory of entry barriers, Bell Journal of Economics 10, 20-32.

[10] Hackner, J., 2000, A note on price and quantity competition in differentiated oligopolies, Journal of Economic Theory 93, 233-239.

[11] Oz Shy, 2000, Industrial Organization: Theory and Applications, The MIT Press, Cambridge.

[12] Qui, L.D., 1997, On the dynamic efficiency of Bertrand and Cournot equilibria, Journal of Economic Theory 75, 213-229.

[13] Singh, N. and X.Vives, 1984, Price and quantity competition in a differentiated duopoly, Rand Journal of Economics 15, 546-554.

[14] Vickers, J.,1995, Concepts of competition, Oxford Economic Papers 47, 1-23.