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Price and Quantity competition under free entry

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Abstract: This paper shows that if number of firms is determined *endogenously*, Cournot competition generates higher welfare compared to Bertrand competition when products are sufficiently differentiated. If products are close substitutes, welfare is higher under Bertrand competition. We show that the qualitative results are robust with respect to different demand formulations. Therefore, our results are in sharp contrast to the previous literature considering Bertrand and Cournot competition with *given* number of firms.

Key Words: Cournot competition, Bertrand competition, Free entry

JEL Classification: D34, L13, O33

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Price and Quantity competition under free entry

1 Introduction

This paper compares welfare under Bertrand and Cournot competition when there is *free entry* and firms produce *imperfect substitutes*. We show that welfare is higher under Cournot competition for sufficiently differentiated products. But, if products are close substitutes, welfare is higher under Bertrand competition.[1]

This paper contributes to the debate on welfare impacts of the nature of product market competition. Singh and Vives (1984) discuss the choice of product market competition between Bertrand and Competition in a duopoly set up. Using a duopoly model, they show that welfare is always higher under Bertrand competition than Cournot competition. Recently, using the utility function of Singh and Vives (1984), Häckner (2000) shows that welfare may be higher under Cournot competition in an oligopolistic market when products are *complements* but welfare is always higher under Bertrand competition when products are substitutes. Singh and Vives (1984) and Häckner (2000) consider a given number of firms.

We show that if number of firms is determined *endogenously*, Cournot competition generates higher welfare than Bertrand competition when the products are sufficiently imperfect *substitutes*.

The reason for our result is as follows. For any degree of product differentiation, the profit of each firm is higher under Cournot compared to Bertrand competition. Therefore, more firms enter the market under Cournot competition than under Bertrand competition. Sufficient product differentiation reduces competition between the firms significantly and also increases number of firms in the market. So, if products are sufficiently differentiated, the benefit from fierce competition under Bertrand competition is dominated by sufficiently large number of firms under Cournot competition and generates higher welfare under Cournot compared. But, if products are close substitutes, the difference between number of firms under Cournot and Bertrand competition is not sufficiently large but lower product differentiation creates fierce competition between the firms. So, in this situation, the benefit from higher competition under Bertrand competition exceeds the benefit from the larger

[1] We show that number of firm under Bertrand competition needs to exceed one for welfare to be higher under Bertrand competition.

number of firms under Cournot competition. Therefore, welfare is higher under Bertrand compared to Cournot competition. We show that our qualitative results also hold under a different demand formulation due to Shubik and Levitan (1980).

The remainder of the paper is organized as follows. Section 2 provides the basic model and results. Section 3 considers a different demand formulation due to Shubik and Levitan (1980). Section 4 concludes.

2 The basic model and results

Assume that there is a large number of firms in an economy. These firms have the same production technology. Assume that the marginal cost of production of each firm is constant and is assumed to be zero, for simplicity. However, each firm must incur a cost of entry K. Products of these firms are imperfect substitutes and there is free entry in the market. So, entry will occur until profit of the new entrant is equal to the cost of entry.

We assume that the utility function of the consumer is of type given in Singh and Vives (1984) and Häckner (2000):

$$U(q,I) = \sum_{i=1}^{n} aq_i - \frac{1}{2}\left(\sum_{i=1}^{n} q_i^{\,2} + 2\gamma \sum_{i \neq j} q_i q_j\right) + I . \tag{1}$$

So, utility function depends on the consumption of q-goods and the numeraire good I. The parameter γ measures the substitutability between the products. If $\gamma = 0$, the products are isolated and if $\gamma = 1$ they are perfect substitutes.

From the utility function (1), we get the inverse market demand curve as

$$p_i = a - q_i - \gamma \sum_{i \neq j} q_j . \tag{2}$$

We consider the following game. In stage 1, firms decide whether to enter or not. To avoid strategic entry decision, which is not the purpose of this paper, we assume that firms enter the market sequentially. After the entry decision is over, in stage 2, these firms compete in the product market as either Cournot oligopolists or Bertrand oligopolists. We solve the game through backward induction. So, first, we look at the market outcome, given that n firms have entered. Then we derive the equilibrium values of n under both Cournot and Bertrand competition.

A1: $\frac{a^2}{4} > K$.

Our analysis assumes that assumption A1 holds. This ensures that at least one firm has the incentive to enter the market in both Bertrand and Cournot competition. Further, for simplicity, we will consider the number of firms as a continuous variable.

In our analysis we will concentrate on the values of $\gamma \in (0,1)$. If $\gamma = 0$ then given the assumption A1, all firms will enter the market and since the products are isolated, here the type of product market competition has no effect. On the other hand, if $\gamma = 1$, there is the well-known 'Bertrand paradox' and firms' profit functions become discontinuous under Bertrand competition. We avoid this problem in our analysis by ignoring the extreme situation of $\gamma = 1$, though it does not affect the main purpose of this paper.

2.1 Cournot competition

Under Cournot competition, firms choose quantities to maximize profits. Given that n^c have entered the market,[2] the ith firm maximizes the following expression to maximize its profit in the product market (i.e., excluding the cost of entry):

$$\underset{q_i}{Max}(a - q_i - \gamma \sum_{i \neq j} q_j) q_i \,. \tag{3}$$

So, the reaction function of firm i equals to

$$q_i = \frac{a - \gamma \sum_{i \neq j} q_j}{2} \,. \tag{4}$$

Symmetry implies that in equilibrium each (of the n) firm produces

$$q_i^c = \frac{a}{(2 + (n^c - 1)\gamma)} \,. \tag{5}$$

So, net profit, which includes the cost of entry, of each of the n^c firms is

$$\pi_i^c = \frac{a^2}{(2 + (n^c - 1)\gamma)^2} - K \,. \tag{6}$$

Since each firm will earn zero net profit in the free entry equilibrium, the equilibrium number of firms under Cournot competition will satisfy

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[2] We use superscripts c and b to mean Cournot and Bertrand competition respectively.

$$\frac{a^2}{(2+(n^c-1)\gamma)^2} = K\,. \tag{7}$$

2.2 Bertrand competition

From the utility function (1), we can derive the demand function for the i th firm as[3]

$$q_i = \frac{a(1-\gamma) - p_i(1+(n-2)\gamma) + \gamma\sum_{i\neq j} p_j}{(1-\gamma)(1+(n-1)\gamma)}\,. \tag{8}$$

Under Bertrand competition, the firms choose prices to maximize profits. Given that n^b firms have entered the market, the i th firm maximizes the following expression in the product market:

$$\underset{p_i}{Max}\; p_i\left(\frac{a(1-\gamma) - p_i(1+(n^b-2)\gamma) + \gamma\sum_{i\neq j} p_j}{(1-\gamma)(1+(n^b-1)\gamma)}\right). \tag{9}$$

So, the reaction function of firm i equals to

$$p_i = \frac{a(1-\gamma) + \gamma\sum_{i\neq j} p_j}{2(1+(n^b-2)\gamma)}\,. \tag{10}$$

Symmetry again implies that in equilibrium each (of n^b) firm charges

$$p_i^b = \frac{a(1-\gamma)}{(2+(n^b-3)\gamma)}\,. \tag{11}$$

So, each of them produces

$$q_i^b = \frac{a(1+(n^b-2)\gamma)}{(1+(n^b-1)\gamma)(2+(n^b-3)\gamma)}\,. \tag{12}$$

Therefore, net profit of each of the n^b firms is

$$\pi_i^b = \frac{a^2(1-\gamma)(1+(n^b-2)\gamma)}{(1+(n^b-1)\gamma)(2+(n^b-3)\gamma)^2} - K\,. \tag{13}$$

Since each firm will earn zero net profit in the free entry equilibrium, optimal number of firms under Bertrand competition will satisfy

$$\frac{a^2(1-\gamma)(1+(n^b-2)\gamma)}{(1+(n^b-1)\gamma)(2+(n^b-3)\gamma)^2} = K\,. \tag{14}$$

[3] See Häckner (2000) for the derivation of the demand curve.

2.3 Comparison between Bertrand and Cournot

Let us now compare Bertrand and Cournot competition.

Proposition 1: *The number of firms that makes further entry unprofitable under Cournot competition cannot be lower than the number of firms that makes further entry unprofitable under Bertrand competition.*

Proof: Assume that there are the same number of firms, n, under Bertrand and Cournot competition. Then gross profits (i.e., profits including the cost of entry) under Bertrand and Cournot competition of each firm are $\frac{a^2(1-\gamma)(1+(n-2)\gamma)}{(1+(n-1)\gamma)(2+(n-3)\gamma)^2}$ and $\frac{a^2}{(2+(n-1)\gamma)^2}$ respectively. Following Häckner (2000), it is easy to check that $\frac{a^2}{(2+(n-1)\gamma)^2} > \frac{a^2(1-\gamma)(1+(n-2)\gamma)}{(1+(n-1)\gamma)(2+(n-3)\gamma)^2}$. This implies that for the same number of firms under Bertrand and Cournot competition, net profit of each firm is higher under Cournot competition compared to Bertrand competition. This implies that the number of firms that equates firms' net profits to zero under Cournot competition cannot be lower than the number of firms that equates their net profits to zero under Bertrand competition. Q.E.D.

The above proposition shows that incentive for entry is higher under Cournot competition compared to Bertrand competition.

Now we compare social welfare under Bertrand and Cournot competition. First, note that the condition of free entry equilibrium implies that all firms earn zero net profit in equilibrium. So, only consumer surplus determines whether welfare is higher under Bertrand or Cournot equilibrium.

Since the product market equilibrium is symmetric, each firm produces same amount of output, i.e., $q_1^c = q_2^c = ... = q_{n^c}^c$ and $q_1^b = q_2^b = ... = q_{n^b}^b$. It follows from (2) that $a - q_i - \gamma\sum_{i \neq j} q_j > 0$ for $p_i > 0$, where $i = 1,...,n^c$ and $i = 1,...,n^b$ under Cournot and Bertrand competition respectively and, therefore, we find from (1) that consumer surplus increases with total output. Therefore, it is enough for us to compare total outputs under Bertrand and Cournot competition.

Proposition 2: *Assume* $\gamma \in (0,1)$. *Welfare is higher under Cournot (Bertrand) competition if and only if products are sufficiently (not sufficiently) differentiated.*[4]

Proof: We find from condition (7) that, given the value of K, optimal output of the i th firm under Cournot competition is

$$q_i^c = \frac{a}{(2+(n^c-1)\gamma)} = \sqrt{K}\,. \tag{15}$$

Due to symmetry, we get total output under Cournot competition as

$$n^c q_i^c = n^c \sqrt{K}\,. \tag{16}$$

From (12) and (14) we find that, given the value of K, optimal output of the i th firm under Bertrand competition is

$$q_i^b = \frac{K(2+(n^b-3)\gamma)}{a(1-\gamma)}, \tag{17}$$

since $\gamma < 1$. Due to symmetry, we get total output under Bertrand competition as

$$n^b q_i^b = \frac{n^b K(2+(n^b-3)\gamma)}{a(1-\gamma)}. \tag{18}$$

Comparing (16) and (18) we find that $n^c q_i^c \underset{<}{\overset{\geq}{-}} n^b q_i^b$ if and only if

$$n^c \sqrt{K} \underset{<}{\overset{\geq}{-}} \frac{n^b K(2+(n^b-3)\gamma)}{a(1-\gamma)}$$

or

$$n^c a(1-\gamma) \underset{<}{\overset{\geq}{-}} n^b (2+(n^b-3)\gamma)\sqrt{K}\,. \tag{19}$$

The left hand side (LHS) and the right hand side (RHS) of (19) are continuous in γ over $[0,1-\delta]$ (where $\delta \to 0$).[5] Further, we get LHS of (19) is greater than RHS of (19) at $\gamma = \varepsilon$ (where $\varepsilon \to 0$), since $\frac{a^2}{4} > K$ (see A1) and $n^c \geq n^b$ (see Proposition

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[4] We write this proposition by assuming number of firms as continuous variable. See our remark in footnote 8 if number of firms is considered as discrete variable.

[5] We consider γ up to $(1-\delta)$ since we are not including $\gamma = 1$ in our analysis.

1). But LHS of (19) is less than RHS of (17) at $\gamma = 1-\delta$ for $n^b > 1$,[6] whiled assumption A1 ensures that $n^b > 1$.[7] This implies that there exists a value of $\gamma = \gamma^*$ such that total output under Cournot competition is greater (less) than the total output under Bertrand competition for $\gamma \in (0, \gamma^*)$ ($\gamma \in (\gamma^*, 1)$). Q.E.D.

While the number of firms is higher under Cournot competition, competition is fierce under Bertrand competition. However, the effect of competition reduces with higher degree of product differentiation. So, if the products are sufficiently differentiated, effect of the relatively large number of firms under Cournot competition dominates the effect of fierce competition under Bertrand competition. Hence, in this situation, welfare is higher under Cournot competition. But, when the products are close substitutes, effect of fierce competition under Bertrand competition dominates the effect of the relatively large number of firms under Cournot competition and generates higher welfare under Bertrand competition.[8]

3 Different demand formulation

In this section we see whether our qualitative results hold under a different type of demand function due to Shubik and Levitan (1980).[9] They consider the inverse market function for the i th firm as

$$p_i = \frac{\alpha}{\beta} - \frac{(n+\theta)}{\beta(1+\theta)}\left(\frac{\theta}{n+\theta}q_1 + \frac{\theta}{n+\theta}q_2 + \ldots + q_i + \ldots + \frac{\theta}{n+\theta}q_n\right) \tag{20}$$

and therefore, the market demand function for the i th firm is

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[6] Proposition 1 ensures that $n^c \geq 1$ if $n^b > 1$.

[7] Note that assumption A1 implies $n^b > 1$, since we have assumed number of firm as continuous variable. If number of firm is treated as discrete variable, the market under Bertrand competition may be monopoly even if with the assumption A1. However, it is trivial that if number of firm under Bertrand competition is 1, the market is monopoly under Bertrand competition. Therefore, in this situation, welfare is likely to be higher under Cournot competition when number of firm is greater than 1 under Cournot competition and K is sufficiently large.

[8] After writing this paper, we came to know that Cellini et al. (1999) found similar result independently in their working paper. However, unlike them, the present paper also considers a different demand formulation due to Shubik and Levitan (1980) and shows the effects of free entry under Bertrand and Cournot competition. Further, we prove the main result of this paper analytically while Cellini et al. (1999) provide a graphical illustration of this possibility.

[9] Though the demand functions considered by Singh and Vives (1984) and Shubik and Levitan (1980) give similar results for the same number of firms, they have different implications when number of firms changes (see Martin, 2002).

$$q_i = \frac{\beta}{n}\left(\frac{\alpha}{\beta} - \left(1 + \frac{(n-1)\theta}{n}\right)p_i - \theta\sum_{j\neq i}^{n} p_j\right). \tag{21}$$

In this section we use a different notation θ for the degree of product differentiation, as this will help us to distinguish between these two types of demand functions.

Note that in the formulation of Shubik and Levitan, when $\theta = 0$, the inverse demand function (20) becomes $p_i = \frac{\alpha}{\beta} - \frac{n}{\beta}q_i$ and implies that the products are isolated. But, the products are perfect substitutes when $\theta \to \infty$, since here the inverse demand function (20) becomes $p_i = \frac{\alpha}{\beta} - \frac{1}{\beta}(q_1 + q_2 + ... + q_i + ... + q_n)$ and implies that the products are perfect substitutes.

3.1 Cournot competition

If there are n^c firms under Cournot competition, we find that the equilibrium output of each of these firms is

$$q_i^c = \frac{\alpha(1+\theta)}{(2n^c + \theta n^c + \theta)} \tag{22}$$

and the corresponding price is

$$p_i^c = \frac{\alpha(n^c + \theta)}{\beta(2n^c + \theta n^c + \theta)}. \tag{23}$$

So, the net profit of the i th firm is

$$\pi_i^c = \frac{\alpha^2(1+\theta)(n^c+\theta)}{\beta(2n^c + \theta n^c + \theta)^2} - K. \tag{24}$$

Since, each firm earns zero profit in the free entry equilibrium, the equilibrium number of firms under Cournot competition will satisfy

$$q_i^c = \frac{K\beta(2n^c + \theta n^c + \theta)}{\alpha(n^c + \theta)}. \tag{25}$$

3.2 Bertrand competition

If there are n^b firms under Bertrand competition, we find that the equilibrium price of each of these firms is

$$p_i^b = \frac{\alpha n^b}{\beta(2n^b + \theta {n^b}^2 + \theta n^b - 2\theta)} \tag{26}$$

and the corresponding output is

$$q_i^b = \frac{\alpha(n^b + \theta n^b - \theta)}{n^b(2n^b + \theta n^b + \theta n^{b^2} - 2\theta)}. \tag{27}$$

So, the net profit of the i th firm is

$$\pi_i^b = \frac{\alpha^2(n^b + \theta n^b - \theta)}{\beta(2n^b + \theta n^b + \theta n^{b^2} - 2\theta)^2} - K. \tag{28}$$

Since, each firm earns zero profit in the free entry equilibrium, the equilibrium number of firms under Bertrand competition will satisfy

$$q_i^b = \frac{K\beta(2n^b + \theta n^{b^2} + \theta n^b - 2\theta)}{\alpha n^b}. \tag{29}$$

Proposition 3: *Proposition 1 remains valid.*

Proof: (24) and (28) imply that $\pi_i^c + K > \pi_i^b + K$, which proves the result. Q.E.D.

Proposition 4: *Welfare is higher under Cournot (Bertrand) competition for sufficiently (not sufficiently) differentiated products.*

Proof: Given n^c number of equilibrium firms under Cournot competition, total output under Cournot competition is

$$n^c q_i^c = \frac{Kn^c\beta(2n^c + \theta n^c + \theta)}{\alpha(n^c + \theta)}. \tag{30}$$

Given n^b firms at the Bertrand equilibrium, total output under Bertrand competition is

$$n^b q_i^b = \frac{Kn^b\beta(2n^b + \theta n^{b^2} + \theta n^b - 2\theta)}{\alpha n^b}. \tag{31}$$

We find that $n^c q_i^c - n^b q_i^b \gtreqless 0$ if and only if

$$n^c(\theta + \theta n^c + 2n^c) - (\theta + n^c)(2n^b - 2\theta + \theta n^b(1 + n^b)) \gtreqless 0. \tag{32}$$

The LHS of (32) is concave and continuous in θ. Further, we find that LHS of (32) is positive at $\theta = 0$, since $n^c > n^b$ (see Proposition 3) but it is negative as $\theta \to \infty$. This implies that if the products are sufficiently differentiated (i.e., θ is low), LHS of (32) is positive and therefore, total output and welfare is higher under Cournot competition compared to Bertrand competition. But, for close substitutes (i.e., for sufficiently higher θ), welfare is higher under Bertrand competition. Q.E.D.

Proposition 4 gives result similar to proposition 2 and therefore, suggests that our qualitative results are robust with respect to different demand formulations.

4 Conclusion

This paper compares welfare under Bertrand and Cournot competition under free entry. We show that while welfare is higher under Cournot competition for sufficiently large product differentiation, welfare is higher under Bertrand competition for sufficiently small product differentiation. We show that these results are robust with respect to different demand formulations.

Therefore, if number of firms is determined *endogenously*, results of the previous literature comparing Bertrand and Cournot competition can be altered significantly. So, it is not always true that fierce product market competition always generates higher welfare since it may imply a sufficiently small number of firms in the market.

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