PARETO-IMPROVING PENSION REFORM THROUGH TECHNOLOGICAL INNOVATION

by Mark A. Roberts
UNIVERSITY OF NOTTINGHAM

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Mark Roberts is Lecturer, School of Economics, University of Nottingham

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Pareto-improving pension reform through technological innovation\(^1\)

Mark A. Roberts
University of Nottingham

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**Abstract.**
The abolition or reform of unfunded pensions will generally make members of a transitional generation worse-off, because of the "double burden" of funding their own retirement along with that of paying off the unfunded pension liability. Reform will also lower the time-path of interest rates, which will reduce both firms' costs of capital and the rates at which they discount their future profits. Each of these two effects will raise the present value of the gains from technological innovation. These may then cause an immediate supply-side response, which, in turn, may solve the double-burden problem. We demonstrate this possibility numerically in a simple overlapping-generations model.

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1. **Introduction.**
Recent demographic factors, such as increased longevity and declining birth rates, as well as the economic phenomena of early retirement and declining growth rates, threaten the fiscal sustainability of "pay-as-you-go" (PAYG) pension schemes, which rest on a viable **support ratio** of contributors to net recipients. The possibility that existing pension schemes, especially those with generous benefit replacement ratios, may break down at some time in the future has placed the issue of reform firmly on the current agenda.

Pension reform also has known benefits, based on the prediction of life-cycle models that unfunded pensions reduce savings and capital accumulation. Feldstein (1975) first raised this point thirty years ago before demographic factors were an issue. Even before then, Samuelson (1958) pointed out that the implicit return on pensions is equivalent to the growth rate of the income tax base, which coincides with the growth rate in the steady-state. This implied that unfunded pensions are poor substitutes for private saving or for the state funded alternative, because, empirically, growth rates lie well below the equivalent rates of return on equity.

Although there are clear economic arguments for reform, there is also considerable political opposition to this possibility, because of the costs of transition. A PAYG pension scheme that has a finite life may be characterised as an intertemporal redistribution, favouring most the initial generation, which receives benefits but does not make contributions, while hurting most the final generation, which makes contributions but does not receive benefits. Abolition of the system imposes a "double burden" problem on members of this last transitional generation, who are required to pay off the existing social security liability and yet make independent provision for their own retirement, thus raising the question of intergenerational equity. This welfare issue, however, is likely to be settled democratically, since a member of the transitional generation is also likely to be the median voter.

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2 A counter-example is provided in Roberts (2003), where a PAYG pension reduces financial sector profits and thereby "unproductive saving" in financial sector equity in favour of "productive saving" in physical capital.
3 Feldstein (1998) gives respective figures of 2.6% and 9.3% for the US.
The juxtaposition of the clear long-run benefits with the considerable short-run obstacles to reform has led researchers to look at the possible circumstances in which reform is actually Pareto-improving. This requires the presence of some additional side-benefit, which would arise from the removal of some distortion, externality or market failure running alongside the unfunded pension scheme. Distortionary taxation is a prime candidate, because of its effect on labour supply. Breyer (1989) argues that no reform can be Pareto-improving, in the absence of other externalities, if labour supply is exogenous. Subsequent papers by Homburg (1990), Breyer and Straub (1993) and Raffelhueschen (1993) have demonstrated possible Pareto-improvements where labour supply is endogenous. Demmel and Keuschnigg (2000) also show that a reform can also be Pareto-improving, if employment is determined as the outcome of wage bargaining between firms and unions. Belan, Michel and Pestieau (1998) present an endogenous growth model where a Pareto-improvement may arise because of production externalities.

This present paper continues this approach by considering another possible supply-side benefit pension reform. In the model, firms have the choice of implementing a new technology, which would yield higher profits but which also involves switching costs, as in Roberts (2002). Life-cycle models predict that the abolition of an unfunded pensions scheme will raise saving and the capital stock, and, thus, reduce interest rates under the assumption of diminishing marginal productivity. Firms will innovate, if they evaluate the long-run gains to be in excess of some up-front, non-pecuniary and, hence, non-refundable, implementation cost. The long-run gains will depend in two ways on interest rates, as they constitute both the costs of capital and the rates at which future profits are discounted. Consequently, pension reform will raise the gains from technological implementation.

The first proposition of this paper is that if the resulting increase in innovation gains is sufficiently large to cover the costs to firms, reform may trigger an immediate supply response as firms adopt new, improved technologies. The second and final proposition is that, for certain parameter values, the productivity gains may swamp the transition costs to households, so that the abolition of an unfunded pension
scheme is Pareto-improving. In short, pension reform promotes technology implementation, which in turn may render the reform Pareto-improving.

The notion of a "Pareto-improving pension reform", if tenable, is highly controversial, because the possibility of an improvement is obtained from the accompanying side-benefits and not from the pension reform per se. This suggests that it may be more appropriate to gear policy towards eradicating the underlying distortion rather than to tinker with a pension system that is intrinsically dynamically-efficient. However, there still remains the possibility that a distortion-targeted policy may not be a feasible option, leaving pension reform as the only recourse. To give an example, it may be rightly argued that distortionary taxes should be replaced with non-distortionary taxes, but if all taxes are inevitably distortionary, then the only real scope may be for some other policy that facilitates a reduction in taxation. Then, although pension reform cannot be Pareto-improving in the stricter theoretical sense, it could be regarded as such in a looser practical sense.

The rest of the paper is set up as follows. In Sections 2 and 3 the model is presented. Section 2 lays out the behaviour of the household in determining savings and in bargaining with the firm over the wage. The firm's investment decision is then determined for technologies at individual and aggregate levels. In Section 3 the firm chooses a technology. Section 4 analyses the strategic interactions that arise out of these technology choices. The main analysis is contained in Sections 5 and 6 which consider the possibility of a general equilibrium switch to an improved technology in the respective cases where there is not and where there is a designated pension reform. In the latter case, if there is a switch, we ask whether it would render the reform Pareto-improving. Sections 7 and 8 then extend and conclude the analysis.

2. The model.

The model is a version of the Diamond (1962) overlapping generations model with the exception that there is wage bargaining with a fixed (full-) employment level in

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4 That is even without a compensation scheme.
5 One reason is income effects on the labour supply decision.
each firm. This gives rise to positive profits where firms have some degree of bargaining power and, consequently, positive present value gains from implementing a new technology. Firms weigh these present value gains against the non-zero switching costs that are incurred immediately when installing a new technology.

**Households.**

There is a representative household, which buys a single consumption good in each period of a two-period lifetime and has the rate of time preference, \( \delta \), \( 0 \leq \delta < \infty \). It also supplies a fixed unit of labour without incurring a disutility cost of effort. The utility function is:

\[
V_i = \exp \left( \frac{1 + \delta}{2 + \delta} \left( \ln c_i^Y + \frac{1}{1 + \delta} \ln c_i^O \right) \right) \tag{1}
\]

where \( c_i^Y \) and \( c_i^O \) are the consumption levels of a household, which is young at time \( t \) and old at time \( t + 1 \). The specification is of a logarithmic utility function, implying a unitary elasticity of substitution, which has been transformed in order to give risk-neutrality.

The household budget constraint is:

\[
c_i^Y + \frac{1}{1 + r_{t+1}} c_i^O = x_i^Y + \frac{1}{1 + r_{t+1}} x_i^O \tag{2}
\]

where \( x_i^Y \) and \( x_i^O \) are the respective disposable income levels at \( t \) and \( t + 1 \) when young and old; \( r_{t+1} \) is the rate of interest received at time \( t + 1 \) on saving made at \( t \).

There is no population growth, everyone lives for two periods, but works for only one, so that the support ratio is unity. The income taxes of the young workers are paid to finance pay-as-you-go pensions payments, \( b_t \), to the old by paying taxes at the rate \( \beta_t \). The old do not pay taxes on their pension benefits, so that the PAYG pension system is given by

\[
x_i^Y = (1 - \beta_t) w_t, \quad x_i^O = \beta_{t+1} w_{t+1} \tag{3}
\]

The young expect to receive \( \beta_{t+1} \) of future labour income as a pension in the next period.
There are diminishing returns to capital, so that both the capital stock and its rate of return, the market rates of interest, converge to positive, finite values. In the absence of technology growth, the long-run growth rate, which is also long-run rate of return on social security, is zero, ensuring that this economy is dynamically efficient.

Maximisation of the utility function in (1), subject to the budget constraint in (2) and the pension rule in (3), determines the household savings function is solved as:

\[
s_t = \frac{1}{2 + \delta} \left( (1 - \beta_t)w_t - \frac{1 + \delta}{1 + \beta_{t+1}} \beta_{t+1} w_{t+1} \right)
\]

As \( c_t^y = (1 - \beta_t)w_t - s_t \) and \( c_{t+1}^D = \beta_{t+1} w_{t+1} + (1 + \beta_{t+1}) s_t \), the indirect utility from (1) and (4) is

\[
V_t = (2 + \delta)^{-1} \left( 1 + \delta \right)^{\frac{1 + \delta}{2 + \delta}} \left( 1 + \beta_{t+1} \right)^{\frac{1}{2 + \delta}} \left( (1 - \beta_t)w_t + \frac{1}{1 + \beta_{t+1}} \beta_{t+1} w_{t+1} \right)
\]

Assuming a lagged response from saving to investment and 100% depreciation of the capital stock within the period gives investment as

\[
k_{t+1} = s_t = \frac{1}{2 + \delta} \left( (1 - \beta_t)w_t - \left( \frac{1 + \delta}{1 + \beta_{t+1}} \beta_{t+1} w_{t+1} \right) \right)
\]

**The firm**

Each firm, indexed \( z \), maximises an intertemporal profit function:

\[
v_t(z) = \pi_t(z) + \frac{\pi_{t+1}(z)}{1 + \beta_{t+1}} + \frac{\pi_{t+2}(z)}{(1 + \beta_{t+1})(1 + \beta_{t+2})} + \frac{\pi_{t+3}(z)}{(1 + \beta_{t+1})(1 + \beta_{t+2})(1 + \beta_{t+3})} + ..
\]

where \( \pi_{t+i}(z) \) is instantaneous profit at time \( t + i \) and future profits are discounted by the interest factors. As there is a 100% within-period depreciation rate, the capital stock does not survive beyond the single period of its life, so that the long-run objective of the firm is obtained by maximising instantaneous profit in each period. Production is determined by a Cobb-Douglas function of capital, \( k_i(z) \), and labour, \( l_i(z) \), under constant returns to scale. Instantaneous profit is given by

\[
\pi_t(z) = A(z)k_i(z)^{\theta}l_i(z)^{1-\theta} - r_t k_i(t) - w_t l_i(z)
\]
The firm makes the following sequence of three actions: (i) it chooses the level of technology; (ii) it decides on the level of the capital stock; and (iii) it bargains with its workforce over the wage. The model is solved in the standard way by reversing this sequence of actions. In this present section, first, the wage and, then, the capital stock are determined; in the following section, the firm's technology choice is considered.

**Wage bargaining**

The firm and union bargain over the wage only. To keep the model tractable, we assume that the employment level, \( \bar{I}_t(z) \), in each firm is fixed.

\[
\bar{I}_t(z) = \bar{I} \quad \forall z
\]

(9)

The firm is also held to this employment level before and after the bargain. The model is one of full employment and workers are assumed to be immobile between firms.\(^6\)

In the event of an agreement in bargaining, the firm receives the profit income in (8) and each of the workers receives the wage, \( w_t \). There are strikes in the event of the agreement, so that production is completely closed down and the firm is not obliged to pay the strikers. We consider binding contracts, so that the firm is not committed to paying off the cost of capital in the event of a strike.\(^7\) These assumptions imply that the firm's disagreement profit is zero, so that its bargaining surplus is the agreement profit level in equation (8). Also, workers' pension rights are not affected by whether they work or strike. The indirect utility function in equation (5) implies that the union bargaining surplus is a linear function of the wage, \( \Omega_t(z)w_t(z) \), because of risk-neutrality, where \( \Omega_t(z) = (2 + \delta)^{-1}(1 + \delta)^{\frac{1+\delta}{2+\delta}}(1 + r_{t+1})^\frac{1}{2+\delta}(1 - \beta_t)\bar{I}_t(z) \),

\[
\Omega_t(z) = (2 + \delta)^{-1}(1 + \delta)^{\frac{1+\delta}{2+\delta}}(1 + r_{t+1})^\frac{1}{2+\delta}(1 - \beta_t)\bar{I}_t(z) 
\]

It is well established that the outcome of the Nash bargain is equivalent to maximising the following *Nash function*, comprising the geometrically weighted sum of the two bargaining surpluses,

\[
N_t(z) = (\Omega_t(z)w_t(z))^{1-\sigma} [A(z)k_t(z)^{\theta} \bar{I}_t(z)^{1-\theta} - r_t(k_t(t) - w_t(z)\bar{I}_t(z))]^\omega
\]

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\(^6\) The focus on symmetric equilibrium in the analysis provides some justification for this assumption.
where the weights, $\varpi$ and $1 - \varpi$, representative the bargaining powers of the workers and union. The term, $\Omega_i(z)$, given above, has no bearing on the solution as a factor in the Nash function. The fact that each side of the labour market has been specified to be risk-neutral generates wage bill and profits solutions that are each proportional to an efficient joint surplus, where the proportions are determined by the respective bargaining weights:

\[
\begin{align*}
    w_i(z) &= \varpi \left( A_i(z)k(z)^\theta \tilde{I}_i(z)^{-\theta} - r_i k_i(z) \tilde{I}_i(z)^{-1} \right) \\
    \pi_i(z) &= (1 - \varpi) \left( A_i(z)k(z)^\theta \tilde{I}_i(z)^{1-\theta} - r_i k_i(z) \right)
\end{align*}
\]

(10)  (11)

**Capital stock determination**

At an earlier stage, the capital stock is chosen to maximise the anticipated instantaneous profit level in (11). The first-order condition determines the firm's demand for capital as

\[
    k_i(z) = \left( A_i(z) r_i^{-1} \right)^{1-\theta} \tilde{I}_i
\]

(12)

Apart from imposing employment symmetry across firms in equation (9), we also normalise the common employment level to unity:

\[
    \tilde{I} = 1 \ \forall z
\]

(13)

**Interest rate determination**

There is a competitive market for capital and its total demand is obtained by aggregating the firms' demands in (12), using (13), to get

\[
    k_i \equiv \int_0^1 k_i(x) dx = \left( A_i r_i^{-1} \right)^{1-\theta} \text{ where } A_i \equiv \left( \int_0^1 A_i(x) \frac{1}{A_i(x)} \right)^{1-\theta}
\]

(14)

The latter part, $A_i$, is defined as "aggregate technology".

Inversion of the aggregate demand for capital in equation (14) gives the solution for the interest rate:

\[
    r_i = \frac{1}{A_i k_i^{1-\theta}}
\]

(15)

The alternative where contracts are non-binding would not affect the qualitative nature of the results, but is quantitatively equivalent to raising the cost of capital.
Equations (12), (13) and (15) into (10) and then (11) give profits and the wage for each firm as
\[ w_i(z) = \omega (1 - \theta) A_i(z) \frac{1}{1 - \theta} \frac{-\theta}{k_i^{\theta}} \]
(16)
\[ \pi_i(z) = (1 - \omega)(1 - \theta) A_i(z) \frac{1}{1 - \theta} \frac{-\theta}{k_i^{\theta}} \]
(17)

**The aggregate capital stock**

In order to determine the aggregate capital stock, we need to return to the saving-investment equilibrium in equation (6). The final form of this requires an expression for the average wage, which is obtained by the aggregation of firm-specific wages in (17), using the definition of aggregate technology in (14):
\[ w_i \equiv \int_0^1 w_i(z) dz = \omega (1 - \theta) A_i k_i^{\theta} \]
(18)

Equations (6), (15) and (18) then give the aggregate capital stock adjustment process as
\[ k_{t+1} = \frac{\omega (1 - \theta)}{2 + \delta} \left( (1 - \beta_i) A_i k_i^{\theta} - \left( \frac{1 + \delta}{1 + \delta A_{t+1} k_{t+1}^{\theta-1}} \right) \beta_{t+1} A_{t+1} k_{t+1}^{\theta} \right) \]
(19)

**The steady-state of the capital stock and the rate of interest**

The steady-state of the capital stock is solved as a quadratic from equation (19), which with equation (15) gives an equivalent solution for the interest rate:
\[ k^{1 - \theta} = \Theta A \left( -\frac{g}{h} + \left( \frac{g}{h} \right)^2 - \frac{1}{h} \right)^{1/2} \]
\[ r = g + \left( g^2 + h \right)^{1/2} \]

where
\[ g \equiv \frac{1}{2(1 - \beta)} \left( 2 + \delta \left( \beta + \frac{\theta}{\omega(1 - \theta)} \right) - 1 \right) \]
\[ h \equiv \frac{2 + \delta}{(1 - \beta)} \frac{\theta}{\omega(1 - \theta)} \]

The steady-state interest rate is positively related to the benefit-replacement rate, \( \beta \), the rate of time-preference, \( \delta \), and the Cobb-Douglas exponent on capital, \( \theta \). It is
negatively related to relative union bargaining power, $\sigma$, because higher wages, lead to more saving and capital, reducing its rate of return.\(^8\)

Important features of the model are the interest rate dynamics. A change in aggregate technology has no effect on the interest rate in the steady-state, because of the full adjustment of the capital stock to its new steady-state. However, there is a very powerful, one-to-one response in the interest rate on impact, since the aggregate capital stock is predetermined by the previous periods' savings decisions. Consequently, the interest rate will overshoot its long-run value.

3. Technology choice

Each firm has a choice between two possible technologies, parameterised by $A(z) = A_L, A_H$, $A_L < A_H$, "low" and "high" or "old and "new". Firms start off with the low technology, $A_L$. Defining $\lambda$, where $0 \leq \lambda \leq 1$, as the proportion of "innovating" firms which choose, $A_H$, the expression for aggregate technology in (14) can be presented as

$$A_t = \left(\lambda A_H \frac{1}{1-\theta} + (1-\lambda)A_L \frac{1}{1-\theta}\right)^{1-\theta}$$

The instantaneous gain in profit to the firm from choosing the new technology is the difference in equation (17) where $A(z) = A_H$ and $A(z) = A_L$. The result using equation (21) is

$$(1-\sigma)(1-\theta) \left( A_H \frac{1}{1-\theta} - A_L \frac{1}{1-\theta} \right) \lambda A_H \frac{1}{1-\theta} + (1-\lambda)A_L \frac{1}{1-\theta} \right)^{-\theta} k_t \theta$$

The intertemporal profit gain, $G_t$, where $G_t = V_t(A_H) - V_t(A_L)$, for a stationary value of $\lambda$ is

$$G_t = (1-\sigma)(1-\theta) \left( A_H \frac{1}{1-\theta} - A_L \frac{1}{1-\theta} \right) \lambda A_H \frac{1}{1-\theta} + (1-\lambda)A_L \frac{1}{1-\theta} \right)^{-\theta} \sum_{j=0}^{\infty} \prod_{i=1}^{j} (1+r_{i+j}) k_{i+j} \theta$$

\(^8\) This is a feature of life-cycle models with full employment, because union bargaining power
The intertemporal gain function is presented for a stationary proportion of innovating firms, \( \lambda \). This is justified as follows. First, \( \lambda \) will never fall, because, even with zero costs of switching back, innovating firms will never revert to an inferior technology. Secondly, it is a feature of the model that \( \lambda \) will never rise gradually over time. The intertemporal gains of innovation are greatest at the moment the new technology is available, so that delay would lead to either lower or lost gains for the firm. The only possible reason for delay, consistent with optimality, is imperfect information that reaches firms at different speeds, while the model considered is one of homogeneous, perfect information. Even with imperfect information, it is implausible that delays would amount to the half life-span length that measures the time-periods of this model.

A major component of the model is the assumption of an up-front implementation or switching cost, \( C \), for each firm. There are various possible rationales for this. The workforce may have to be retrained by spending time away from production or they may reduce their normal activity levels in trying to get to grips with a new mode of production. It could also be regarded as the psychic costs to owner-managers, who may weigh up the profit gains they receive as owners against stress involved in implementing the new technology as managers. Either way, the minimal requirements of the model are that innovation is costly and that these are costs are sunk.

Implementation costs are translated into real monetary value, so that they be may measured against the profit gains. Net profit maximisation implies that the firm will implement the new technology, \( A(z) = A_H \), if \( G > C \) and stick with the old, \( A(z) = A_L \), if \( G \leq C \).

The potential entry of new firms is precluded by also assuming entry costs, \( D \). New entrants would immediately adopt the better technology and would thus not incur switching costs. The no-entry condition is \( V(A_H) < D \) where \( V(A_H) \) is effectively redistributes income to working households at the start of the life-cycle.
intertemporal profit under the new technology. For the possibility that the new technology is taken up by some incumbent firms, that \( G \equiv V(A_{hf}) - V(A_L) > C \), and they alone, also requires the condition \( C < D - V(A_L) \), that switching costs are sufficiently lower than entry costs.

4. Strategic substitutability and complementarity

An implicit form of the capital stock adjustment equation (18) is \( k_{t+1} = f(A_{t+1}, k_t) \), where \( f_1(A_{t+1}, k_t) > 0 \), \( f_2(A_{t+1}, k_t) > 0 \) and where \( A_{t+1} = A_t \). Substitution into equation (15) gives

\[
\frac{\partial r_{t+1}}{\partial A_t} = \theta f(A_{t+1}, k_t)^{-(1-\theta)} \left( 1 - (1-\theta) \frac{f_1(A_{t+1}, k_t)}{f(A_{t+1}, k_t) A_{t+1}} \right)
\]

An increase in aggregate technology has a positive direct effect - for a given capital stock - on the interest rate and a negative indirect effect through the capital accumulation process. The net effect is positive on impact, because the capital stock is predetermined by past savings, while the change in technology directly raises productivity. It is zero in the long-run as evident from the steady-state properties of equation (19). The net effect is also positive in the medium term, because of the property of monotonic convergence of the capital stock. Moreover, it is medium-term interest rates that are important for determining the present value profit gains in equation (22). This implies there is strategic substitutability in technological implementation, because a higher aggregate technology raises medium-term interest rates, which constitute the costs of capital and the rates at which future values are discounted. Strategic substitutability generally implies the uniqueness of equilibrium, which includes the possible case of an interior solution where \( 0 < \lambda < 1 \).

In the case of a pension reform, there is also the possibility of strategic complementarity. There is a then a change in saving behaviour that alters the form of the \( f(\ldots) \) function in equation (23) in a way which enhances the second indirect effect also working through capital accumulation. Strategic complementarity gives rise to the possibility of dual symmetric technological equilibria, \( \lambda = 0 \) and \( \lambda = 1 \), for
certain configurations. However, the simulation results show that the effects of these strategic aspects of the model are relatively small compared with the overall increase in the present value profit gain that arises from the designated pension reform.

5. Technology choice in the absence of pension reform

This section and the one following give the present value profit gains, respectively, without and with a pension reform. In both cases, the initial base is of a general equilibrium where all firms use the old technology. Two questions are then asked. Can a designated pension reform cause a general equilibrium switch to the new technology? Then, if so, will the resultant productivity gains render the social security reform Pareto-improving? We answer this by simulation, because of the nonlinearity of the model.

Parameter values

The starting point is of a symmetric general equilibrium with the following parameter values. There is a generous pension replacement rate: \( \beta = 0.5 \). The Cobb-Douglas capital exponent on capital is close to its stylised empirical value: \( \theta = 0.3 \). Firms and workers have equal bargaining powers, so that \( \varpi = 0.5 \). The rate of time preference is zero: \( \delta = 0 \).\(^9\) We set the initial technology value at 14.125, normalise the capital stock, \( k = 1 \), which implies that \( r = 4.238 \), given equation (14) and that \( \theta = 0.3 \). To summarise, initially: \( \beta = 0.5, \theta = 0.3, \varpi = 0.5, \delta = 0, A_L = 14.125 \). Then, we consider the effect of a new technology represented by a value of \( A \) that is 20% higher, giving \( A_H = 16.95 \).

Evaluation of the gain function

If the gain is a positive function of \( \lambda \), there is strategic complementarity; a negative function implies strategic substitutability. The gain function is evaluated only at the two extreme points where \( \lambda = 0 \) and \( \lambda = 1 \). These represent the respective cases facing a single firm where no other firm innovates and all other firms innovate. The single firm has zero weight on the aggregate outcome, \( \lambda \), so it takes this variable as
given. It is not needed to evaluate the gain function at the interior, $0 < \lambda < 0$, for the following reason. If strategic complementarity/substitutability is apparent in moving from $\lambda = 0$ to $\lambda = 1$, it will also be present in moving from $\lambda = \lambda'$ to $\lambda = \lambda''$, where $\lambda' < \lambda''$. This is because raising the value of $\lambda$ is equivalent either to raising that of $A_H$ or to the lowering that of $A_L$. The three terms, $\lambda$, $A_L$, and $A_H$, only affect interest rates through determining the level of aggregate technology in equation (21), so that changes in any of their values merely constitute scale effects without any qualitative significance. Gain, therefore, is a monotonic function of $\lambda$ and strategic complementarity/substitutability is a global phenomenon for any configuration of values for the other variables.

The first case of $\lambda = 0$ can be solved analytically, because the aggregate capital stock and interest rate do not change in the case of an unchanged aggregate technology. The gain is calculated where the aggregate economy remains in the initial steady-state, with the assigned parameter values as

$$G|_{\lambda=0} = 1.818$$

Next, we calculate the same where all other firms adopt the new technology where $\lambda = 1$. We consider the case where the value of the relevant aggregate technology parameter is 20% higher. This causes convergence to a new capital stock valued at 1.288 and to an unchanged interest rate. Initially, however, the interest rate will overshoot its new long-run equilibrium, because while the technology can be changed immediately, the capital stock changes gradually according to the saving-investment dynamics of the model. The time paths of capital and the interest rate for both polar cases are presented in Table 1.

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9 The low value assigned for the rate of time preference ensures a reasonable level of saving in a model
Table 1: Time paths of capital and the interest rate
for $\lambda = 0$ and $\lambda = 1$ without pension reform

<table>
<thead>
<tr>
<th>Time</th>
<th>Capital $\lambda = 0$</th>
<th>Interest rate $\lambda = 0$</th>
<th>Capital $\lambda = 1$</th>
<th>Interest rate $\lambda = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$</td>
<td>1</td>
<td>4.238</td>
<td>1</td>
<td>5.085 (4.238)</td>
</tr>
<tr>
<td>$t + 1$</td>
<td>1</td>
<td>4.238</td>
<td>1.197</td>
<td>4.484</td>
</tr>
<tr>
<td>$t + 2$</td>
<td>1</td>
<td>4.238</td>
<td>1.265</td>
<td>4.313</td>
</tr>
<tr>
<td>$t + 3$</td>
<td>1</td>
<td>4.238</td>
<td>1.287</td>
<td>4.261</td>
</tr>
<tr>
<td>$t + 4$</td>
<td>1</td>
<td>4.238</td>
<td>1.295</td>
<td>4.243</td>
</tr>
<tr>
<td>$t + \infty$</td>
<td>1</td>
<td>4.238</td>
<td>1.298</td>
<td>4.238</td>
</tr>
<tr>
<td>Sum of PV Profits</td>
<td>1.818</td>
<td></td>
<td>1.729</td>
<td></td>
</tr>
</tbody>
</table>

Using the simulated values for capital and the interest rate, the intertemporal gain has been calculated at $G|_{\lambda=1} = 1.729$. Comparison with $G|_{\lambda=0} = 1.818$ indicates a small amount of net strategic substitutability throughout, as predicted, since the gain falls by 4.90% from $\lambda = 0$ to $\lambda = 1$.

Equation (22) shows that the direct effect a rise in aggregate technology of 20% reduces the gain by 8.1%\(^{10}\), so that the indirect effect of a rising capital stock - discounted by higher interest rates - must increase the gain by 3.05%. To conclude, strategic substitutability is not quantitatively significant issue for the model specification\(^{11}\), given common implementation costs. There is uniqueness of equilibrium; with solutions at $\lambda = 1$, if $C < 1.729$; at $\lambda = 0$ if $C > 1.818$; and at $\lambda^*$, where $0 < \lambda^* < 1$, if $1.729 < C < 1.818$.

\(^{10}\) $(1.2)^{0.3/(1-0.7)} - 1 = 0.0812$

where the only motive for saving is consumption smoothing.
6. **Technology choice with a pensions reform**

We define a pension reform where the currently old generation still receives their pre-reform pension entitlement. They will be better off by a higher rate of return on their savings, if the reform triggers an improvement in aggregate technology of any measure from \( A_L \) to \( A \) (\( A > A_L \) where \( \lambda > 0 \)). The currently young are taxed to pay for pension transfers, but at a reduced rate, if aggregate technology also rises by any measure, because taxable income, namely wages, rise by the factor of \( A/A_L \). The young lose the pension benefits they would have expected for their own old age. All future generations neither pay taxes nor receive pensions.

This reform is defined as

\[
\beta_t = \left( \frac{A_t}{A} \right) \beta, \quad \beta_{t+j} = 0 \quad \text{for } j \geq 1
\]

which implies capital accumulation according to (19) is:

\[
k_{t+1} = \frac{\sigma(1-\theta)}{2+\theta} \left( 1 - \beta \left( \frac{A_L}{A} \right) \right) A k_t^\theta = \frac{1}{2+\theta} \phi(A - \beta A_L) k_t^\theta,
\]

\[
k_{t+1+j} = \frac{\sigma(1-\theta)}{2+\theta} A k_{t+j}^\theta \quad \text{for } j \geq 1, \quad \text{where } A_L < A < A_H
\]

We consider the time paths of capital and the interest rate for this kind of reform and again the two polar cases where \( \lambda = 0 \) and \( \lambda = 1 \). The results for the time paths of capital and the interest rate are reported in *Table 2*.

---

11 Strategic substitutability becomes stronger, if the labour market were also competitive, for each firm’s wage would then depend on aggregate technology rather than firm-specific technology.
Table 2: Time paths of capital and the interest rate for $\lambda = 0$ and $\lambda = 1$ with pension reform

<table>
<thead>
<tr>
<th>Time</th>
<th>Capital $\lambda = 0$</th>
<th>Interest rate $\lambda = 0$</th>
<th>Capital $\lambda = 1$</th>
<th>Interest rate $\lambda = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$</td>
<td>1</td>
<td>4.238</td>
<td>1</td>
<td>5.085 (4.238)</td>
</tr>
<tr>
<td>$t+1$</td>
<td>1.236</td>
<td>3.654</td>
<td>1.730</td>
<td>3.465</td>
</tr>
<tr>
<td>$t+2$</td>
<td>2.624</td>
<td>2.151</td>
<td>3.496</td>
<td>2.117</td>
</tr>
<tr>
<td>$t+3$</td>
<td>3.306</td>
<td>1.835</td>
<td>4.317</td>
<td>1.827</td>
</tr>
<tr>
<td>$t+4$</td>
<td>3.539</td>
<td>1.750</td>
<td>4.600</td>
<td>1.747</td>
</tr>
<tr>
<td>$t+5$</td>
<td>3.612</td>
<td>1.725</td>
<td>4.688</td>
<td>1.724</td>
</tr>
<tr>
<td>$t+6$</td>
<td>3.633</td>
<td>1.718</td>
<td>4.715</td>
<td>1.717</td>
</tr>
<tr>
<td>$t+7$</td>
<td>3.640</td>
<td>1.715</td>
<td>4.723</td>
<td>1.715</td>
</tr>
<tr>
<td>$t+8$</td>
<td>3.642</td>
<td>1.715</td>
<td>4.725</td>
<td>1.715</td>
</tr>
<tr>
<td>$t+9$</td>
<td>3.643</td>
<td>1.714</td>
<td>4.726</td>
<td>1.715</td>
</tr>
<tr>
<td>$t+\infty$</td>
<td>3.643</td>
<td>1.714</td>
<td>4.727</td>
<td>1.714</td>
</tr>
</tbody>
</table>

Sum of PV profits | **2.318** | **2.471**

The calculations show that the sum of present value profit gains from implementing a new technology - with an accompanying pension reform - are $G^R|_{\lambda=0} = 2.318$ and $G^R|_{\lambda=1} = 2.471$ for the two polar cases. There is now a small degree of strategic complementarity shown by the result that $G^R|_{\lambda=1}$ is 6.60% greater than $G^R|_{\lambda=0}$.

Strategic complementarity emerges through the effect of increased saving on capital accumulation, following the pension reform, since it enhances the second indirect effect of technology on interest rates in equation (23), because this also works through the capital accumulation process.
There is a unique equilibrium at $\lambda = 0$, if $C > 2.471$; multiple equilibria at $\lambda = 0, \lambda^*, 1$, where $0 < \lambda^* < 1$, for a relatively narrow range of possible cost values, $2.318 < C < 2.471$; and a unique equilibrium at $\lambda = 1$ if $C < 2.318$. The most important finding is that pension reform has increased the overall size of the present value profit gains by 35% through reducing interest rates. This suggests that, for a reasonable and intermediate range of implementation costs, pension reform will take the economy from a low to a high technology general equilibrium. This possibility is presented in the first proposition below.

**Proposition 1: Pension reform can take the economy from a low to a high technology general equilibrium.**

Using the calculated values above we find that if the costs fall within the intermediate range, $G^N|_{\lambda=0}=1.818 < C < 2.318 = G^R|_{\lambda=1}$, we find that $\lambda = 0$ before and $\lambda = 1$ after the designated pension reform.

In the rest of the paper we assume that costs do fall within the intermediate range required for Proposition 1, that the pension reform does trigger a widespread switch of technology.\(^13\)

Before proceeding to the second proposition, some light is shed by considering the necessary condition for young households not to be worse off. Equation (5) gives their pre-reform, steady-state indirect utility as

$$V^N_t = (2 + \delta)^{-1} \left(1 + \delta\right)^{\frac{1+\delta}{2+\delta}} \left(1 + r^N\right)^{2+\delta} \left(1 - \beta + \frac{1}{1 + r^N} \delta\right)w^N$$

(5.N)

Equations (24) into (5) gives their post-reform equivalent as

$$V^R_t = (2 + \delta)^{-1} \left(1 + \delta\right)^{\frac{1+\delta}{2+\delta}} \left(1 + r^R_{t+1}\right)^{2+\delta} \left(\frac{A_h}{A_L} - \beta\right)w^N$$

(5.R)

\(^{12}\) The equilibrium, $\lambda^*$, is unstable in the sense that a perturbation, $\varepsilon$, where $\varepsilon > 0$, would take the economy to the equilibrium $\lambda = 1$.

\(^{13}\) This case where all firms use the old and new technology before and after the pension reform - along with the simplifying assumption of a common employment level - implies that there is also a common wage - before and after. This provides some justification for the assuming that there is no inter-firm employment mobility.
The necessary condition that $V_i^R \geq V_i^N$ requires

$$\frac{A_H}{A_L} \geq \left(1 + \frac{r_i^N}{1 + r_{t+1}^R}\right)^{\frac{1}{2+\delta}} + \left[1 + \left(1 + r_{t+1}^R\right)^{\frac{1}{2+\delta}} \left(1 + r_N \right)^{\frac{1}{2+\delta}} \left(1 + r_N \right)^{-1} \right] \beta$$

$$= \left(1 + \frac{r_i^N}{1 + r_{t+1}^R}\right)^{\frac{1}{2+\delta}} + \left[1 + \left(1 + r_{t+1}^R\right)^{\frac{1}{2+\delta}} \left(1 + r_N \right)^{\frac{1}{2+\delta}} \left(1 + r_N \right)^{-1} \right] \beta$$

This, at least, requires a switch from a low to a high technology general equilibrium. The wage of every single worker then rises on impact by the same factor as the aggregate technology increase, $A_H/A_L$. The productivity gains must be sufficiently high to compensate them for the loss of pension benefits along with any adverse movement in the interest rate.

Suppose that there is no change in the relevant interest rate, $r_{t+1}^R = r_N$, the condition then becomes one that there is no fall in young household wealth:

$$A_H/A_L \geq 1 + \beta/(1 + r_N)$$

This inequality highlights the significance of the scale of the short-run productivity gain, $A_H/A_L$, relative to the generosity of the pension system, $\beta$, for the young not to be hurt during the transition of reform. If there is no gain, $A_H/A_L = 1$, the young suffer a utility loss for any pension reform, $\beta > 0$.

If it does, it is clearly not a sufficient condition, because of the possibility of an adverse change in the interest rate at time $t+1$. In principle, the movement can go either way, because while the capital stock becomes more productive, households also save more. In the simulation, interest rates fall from 4.238 to 3.465 [See Table 2]. Thus, a significant wealth increase is necessary, because young households will receive a lower rate of return on their saving.

Applying equation (25) to (5) gives the post-reform utility of all future generations as

$$V_{i+j}^R = (2 + \delta)^{-1} (1 + \delta)^{\frac{1}{2+\delta}} \left(1 + r_{t+j}^N \right)^{\frac{1}{2+\delta}} w_{i+j}^R \quad j \geq 1 \quad (5.RJ)$$
It is likely that if currently young households are better off with the reform, they will be too. They, like currently young households, lose access to a state pension in later life, but they will receive higher wages, because of increased productivity through the process of capital accumulation, and they will not pay any taxes on them, because the PAYG system will have ended. Against these relative gains, future generations receive lower rates of return on their saving.

**Proposition 2: Pension reform is Pareto-improving with the assigned parameter values, particularly the scale of the technology gain, $A_H/A_L$, relative to the generosity of the PAYG system, $\beta$.**

Indirect utility calculations for the initial steady-state and for the post-reform sequence are $V^N = 3.369$ and $V^R = 3.656$, $V^R_{t+1} = 6.173$, $V^R_{t+\infty} = 6.190$. The long-run utility gains from reform are 84% of the original utility. The immediate transitional generation receives a much lower gain of 8.5%, because they are still taxed to pay for current pension commitments, while most of the long-run gains will accrue to the succeeding generation with a utility gain of 83% of the original steady-state level. Furthermore, those who are currently old become better off: although they receive the same pension benefits, the return on their saving is now 5.085 instead of 4.238 [See Table Two]. Calculation shows their second-period wealth, consumption and utility all rise by 12.8%.

7. **Further analysis**

An example has been given where a pension reform triggers technological innovation and is Pareto-improving. It is obvious to consider whether other forms of pension reform might also achieve the same objective? There are two possible dimensions to reform: (i) the distribution of net transfers between the two generations that are alive at the moment the reform is initiated and (ii) the distribution of net transfers over time, if the reform is staged.

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14 The succeeding generation receives a lower wage but enjoys a higher saving rate of interest than the generation that comes to life when the new steady-state is reached. Evidently, these two differences roughly cancel out.
With reference to the first point, calculations from this model show that it would be possible to cut benefits to current retirees by 34% and leave them indifferent to the reform, because of the increased rate of return on their savings. A Pareto improvement also depends on the assumption of equality in assets holdings among the old. The young would then gain more by paying lower taxes. They would also save more, so that the capital stock would be higher for all succeeding generations, which in turn would also save more. This alternative policy would redistribute the gains of reform away from current retirees in favour of current taxpayers and all generations.

Secondly, the model predicts that there are no benefits to a pension reform that is staged over time. Partial reform at any time would lead to lower gains that might be too small to achieve the "big push" required for a sufficiently high number of firms to switch technologies. The rise in present value profits that would arise from an earlier stage of reform would become a bygone the next time round, leaving lower gains in the pipeline that may not be sufficient to achieve the objective.

Suppose, however, that the an earlier stage of a reform does induce a general switch in technologies. The implication then is that the later stages of reform cannot be Pareto-improving, because the benefit of productivity gains from technological innovation will have been spent. The Pareto criterion would then preclude the completion of the reform.

The model works, because of the assumption of switching costs: otherwise, there would be no role for reform in raising the gains from technological implementation. Might there be alternative policies that could be practically employed to encourage innovation by firms, negating the role of a pension reform? Obvious candidates are the taxation of low-tech firms and/or the subsidisation of high-tech ones. Taxing lower technology firms at a higher rate not only runs against the principle of progressive taxation, but it could also lead to the closure of weaker firms where there is a wider distribution of characteristics other than the binomial possibility for technology.
Subsidising hi-tech firms leads clearly to the problem of dynamic-inconsistency. Once a sufficiently high number of firms have implemented the new technology, it would become *ex post* optimal to withdraw the subsidy, especially as the subsidy would necessarily be financed by voting households paying higher taxes. Rational firms would anticipate the withdrawal of subsidies in the absence of a credible mechanism for precommitment. Furthermore, the dynamic-inconsistency problem would be even more severe, if subsidies were also required over the longer term beyond the period of involvement of current policy-makers.

The merit of a pension reform is that it is dynamically consistent in the sense that if the economy is dynamically-efficient, there would be no incentive to reverse the reform by the re-introduction of an unfunded pension scheme. Dynamic-efficiency is a necessary - but not sufficient - condition for the reform to be Pareto-improving and it is also a Pareto condition that would rule-out the reintroduction of an unfunded pension scheme.

### 8. Concluding comments

Firms will invest in a new technology if the long-run gains exceed the costs. A major effect of pension reform in any model with decreasing returns to capital would be to reduce long-run interest rates, which in turn would raise the net present value gains from technological innovation. Total factor productivity is immediately increased, while the capital stock is fixed at a level determined by past aggregate saving decisions. Investment in new technology raises may then raise current wages sufficiently to make members of the transitional generation even better off.

The existence of current resource constraints generally precludes the prospect of a Pareto-improving pension reform, because a transitional generation must bear the "double burden" without receiving the income benefits that only accrue later from capital accumulation. A model has been presented that overcomes this resource constraint problem by allowing for the plausible possibility that pension reform may also trigger technological innovation. If, alternatively, the parameter values would
not support a full Pareto-improvement, there would at least be some mitigation in the transitional costs of reform.
References


Roberts, M.A (2003), "Can pay-as-you-go pensions raise the capital stock?" The Manchester School, Supplement, 1-20