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# AN INVESTMENT CONTEST TO INFLUENCE ENVIRONMENTAL POLICY

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September 2004

# An investment contest
# to influence environmental policy

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**Abstract.** In an investment contest for environmental policy, polluters and victims of pollution invest in an increase of their marginal benefits of pollution and environmental quality, respectively. These investments influence time-consistent environmental policy. Investments will exceed their optimal level. Coordination of the victims' investments results in higher aggregate investment, higher payoffs for the victims and lower welfare. Coordination of the polluters' investments results in lower aggregate investment and higher payoffs for polluter and victims.

**JEL Code:** D78, Q28

**Key words:** contest, environmental policy, time consistency, coordination

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# 1 Introduction

Many government policies are effective only if the government can credibly commit. Take for instance the Dutch government's policy to contain aircraft noise from Schiphol airport in the 1990s. The airport was given an annual noise quota, but it chose to schedule so many flights that it had consumed this quota by October. Compliance would imply shutting down the airport for the rest of the year, which was clearly not a credible option. In the end, the government would tolerate Schiphol's violation of the noise limits.

The example suggests that when the government cannot commit to regulation directly or indirectly limiting a firm's size, the firm will grow faster than when the government can commit. The firm knows that the government will set more lenient regulation after the investment has been made. The firm may then invest just to obtain more lenient regulation.

The finding that the government is better off when it can commit, or that in the absence of government commitment, other agents can influence government's policy to their advantage, is well-established in the literature.[1] The analysis of time consistency and commitment, started by Kydland and Prescott [13] and Fischer [6], has been applied to trade policy by a.o. Leidy [15] and Leahy and Neary [14], and to environmental policy by Yao [22], Malik [18] and Gersbach and Glazer [8]. Dijkstra [5] analyzes the case of a single polluter who can invest in extra capacity and thereby get the government to allow more output. It is shown that this will lead to too much investment from the government's point of view.

In this paper, we analyze the case where the polluters as well as the victims of pollution can invest in their marginal benefits of pollution and environmental quality, respectively. Whereas it may be easy to think of investments by polluters (e.g. in extra capacity), this may be more difficult for the victims of pollution. Investments by victims include:

- Investment in accommodation and facilities for tourism and recreation.
- Campaigns to protect wildlife and increase environmental awareness.

[1] However, Dijkstra [5] shows that with taxation of a firm making an indivisible investment, time consistency can lead to higher welfare than commitment. Petrakis and Xepapadeas [19] and Arguedas and Hamoudi [2] come to similar conclusions.

- Agriculture specializing in crops and livestock that are sensitive to environmental quality.
- Building houses.

If the government could commit to allowed pollution levels, the polluters' and victims' investment levels would be optimal. However, when government policy has to be time-consistent, it can only be set after the polluters and victims have made their investments. Then the government cannot reach the welfare optimum. The agents will now invest too much in an attempt to influence government policy in their favour with their investments. The victims of pollution are involved in an investment contest against the polluters. The polluters are involved in a contest not only against the victims, but also against each other, since higher investment by one polluter will increase her allowed pollution at the expense of the other polluters.

There is a large literature on contests[2] with some applications in environmental policy. Some authors have analyzed lawsuits over environmental issues as contests. Baik and Shogren [3] look at reimbursement for citizen suits against polluting firms. Heyes [11] extends their analysis to include asymmetric information. Hurley and Shogren [12] examine the effect of SLAPP (Strategic Lawsuits Against Public Participation). The contest framework has also been applied to lobbying in environmental policy. Dijkstra [4] analyzes the choice between direct regulation and a financial instrument in environmental policy as a two-stage contest for instrument choice and revenue division. Liston-Heyes [16] and Graichen et al. [9] endogenize the environmental damage of a firm's proposed project in anticipation of a contest against an environmental group. The present paper distinguishes itself in that the efforts made by the contestants consist of investments in their benefits of a favourable decision, rather than legal or lobbying expenditures. Moreover, the contestants' expenditures influence the decision taken by the welfare-maximizing government. This provides an explicit foundation for the so-called contest success function, which gives the contestants' success as a function of their efforts. This is an advantage over the contest literature so far, where the contest success function has remained a black box.[3]

[2] For a collection of papers, see Lockard and Tullock [17].

[3] The menu auction or common agency model of politics (Grossman and Helpman [10]), where interest

The rest of the paper is organized as follows. The model is introduced in Section 2. In Section 3, we derive the welfare optimum and show that the government can implement the optimum with commitment. In Section 4, we look at the case where the agents can influence environmental policy with their investments. Single as well as multiple polluters and victims are considered. We find that there will always be overinvestment. Multiple victims invest less than a single victim, because environmental quality is a public good. Multiple polluters invest more than a single polluter, because pollution is a private good. Section 5 concludes the paper.

## 2 The model

There are $m$ polluters, $j = 1, \cdots, m$. Polluter $j$ derives benefits from production $q_j$ and investment $y_j^q$. Polluter $j$'s payoff $U_j^q$ is given by

$$U_j^q(q_j, y_j) = \frac{1}{m} B(Q_j, Y_j) - y_j \qquad Q_j \equiv m q_j \qquad Y_j \equiv m y_j \tag{1}$$

where the benefit function $B(Q, Y)$ satisfies $B_Q(1, Y) = 0$, and for $Q < 1$:[4]

$$B_Q, B_Y > 0,\ B_{QQ},\ B_{YY} < 0,\ B_{QY} > 0 \tag{2}$$

Investment is defined such that investment cost is unitary. The marginal benefits of investment are positive and decreasing in allowed production $q_j$ and investment $y_j$. Investment increases the marginal benefits of production. The function (1) is chosen such that $mU_j^q(q_j, y_j) = B(Q_j, Y_j) - Y_j$. Thus, when each of the $m$ polluters invests $Y_j/m$ and produces $Q_j/m$, their aggregate payoff would be the same as when there were one polluter, with investment cost $Y_j$ and production of $Q_j$. As a result, increasing the number of polluters, say from one to two, does not amount to adding another polluter, but rather to "splitting" the polluter in two. In this way, we isolate the pure strategic effect of an increase in $m$, without introducing a size effect in which we are not interested.

groups offer campaign contributions in return for favourable policies, can also be seen as a contest, and one where the incumbent government maximizes an objective function. Aidt [1] and Fredriksson [7], among others, apply this model to environmental policy making.

[4] Subscripts to the benefit function denote partial derivatives.

There are $n$ victims of pollution, $i = 1, \cdots, n$. Victim $i$ derives benefits from environmental quality $e$ and investment $X_i$. Victim $i$'s payoff $U_i^e$ is given by:

$$U_i^e(e, x_i) = \frac{1}{n} B(e, X_i) - x_i \qquad X_i \equiv n x_i \tag{3}$$

Investment is defined such that investment cost is unitary. The function $B$ in (3) is the same as in (1). Thus, conditions (2) apply here as well. The function (3) is chosen such that $nU_i^e(e, x_i) = B(e, X_i) - X_i$. Thus, when all $n$ polluters invest $X_i/n$, their aggregate payoff of environmental quality $e$ would be the same as when there were one polluter, with investment cost $X_i$.

Finally, the relation between production and the environment is given by:

$$e = 1 - \sum_{j=1}^{m} q_j \tag{4}$$

Thus, every extra unit of production from any producer results in an equal decrease in environmental quality. There is no possibility to reduce emissions per unit of output.

As an example, we shall often use the following specification for the benefit function:

$$B(Q, Y) = \sqrt{2Y} \left(1 - \frac{1}{2} Q\right) Q$$

Defining effective investment $A_j^q \equiv \sqrt{2my_j}$ and $A_i^e \equiv \sqrt{2x_i/n}$ and substituting into (1) and (3) yields:

$$\begin{aligned} u_j^q(q_j, A_j^q) &= A_j^q q_j \left(1 - \frac{m}{2} q_j\right) - \frac{1}{2m} \left(A_j^q\right)^2 & (5) \\ u_i^e(e, A_i^e) &= A_i^e e \left(1 - \frac{1}{2} e\right) - \frac{n}{2} \left(A_i^e\right)^2 & (6) \end{aligned}$$

which yields the following marginal benefit functions:

$$\begin{aligned} \frac{\partial u_j^q}{\partial q_j} &\equiv MB_j^q = A_j^q (1 - m q_j) \\ \frac{\partial u_i^e}{\partial e} &\equiv MB_i^e = A_i^e (1 - e) \end{aligned}$$

This is the specification upon which the Figures in this paper are based, with effective investment $A_j^q$ and $A_i^e$ the intercepts of the polluter's and victim's $MB$ curves with the vertical axis.

# 3 The welfare optimum

In this section, we derive the optimum for the welfare-maximizing government. We shall also see that the government can implement the optimum when it can commit to environmental policy.

The government maximizes total social benefits.[5] From (1), (3) and (4):

$$\max\ TB = \frac{1}{m}\sum_{j=1}^{m} B(mq_j, my_j) - \sum_{j=1}^{m} y_j + \frac{1}{n}\sum_{i=1}^{n} B(e, nx_i) - \sum_{i=1}^{n} x_i \qquad s.t.\ \ e = 1 - \sum_{j=1}^{m} q_j \tag{7}$$

Differentiating with respect to the output levels $q_j$ yields:[6]

$$\frac{\partial B(mq_j, my_j)}{\partial Q_j} = \frac{1}{n}\sum_{i=1}^{n} \frac{\partial B(e, mx_i)}{\partial e} \tag{8}$$

Differentiating with respect to the investment levels $x_i$ and $y_j$ yields:

$$\frac{\partial B(mq_j, my_j)}{\partial Y_j} = \frac{\partial B(e, mx_i)}{\partial X_i} = 1 \tag{9}$$

Combining (8) and (9), we find that the optimal values for environmental quality, output and investment are given by:

$$e^* = \frac{1}{2} \qquad q_j^* = \frac{1}{2m} \qquad\qquad \frac{\partial B\left(\frac{1}{2}, Y_j^*\right)}{\partial Y_j} = \frac{\partial B\left(\frac{1}{2}, X_i^*\right)}{\partial X_i} = 1 \tag{10}$$

By construction, the optimal values for environmental quality, aggregate output and aggregate investment on either side are independent of the number of polluters and victims, optimal environmental quality equals optimal aggregate output and optimal aggregate investment on either side is equal.

If the government would be able to commit to its optimal environmental policy $q_j = 1/2m$, the interest groups would set their investment at the optimal level given by (10). This is because each polluter $j$ chooses the investment level $y_j$ that maximizes her payoff $U_j^q$, given the allowed output level of $q_j = 1/2m$. From (1):

$$\frac{\partial B\left(\frac{1}{2}, Y_j\right)}{\partial Y_j} = 1$$

[5] Alternatively, the government might maximize a politically-motivated objective function. The analysis is similar, as long as the objective function contains polluters' and victims' payoffs.

[6] For the payoff functions (5) and (6) and $n = m = 1$, this becomes $q = A^q/(A^q + A^e) = \sqrt{y}/(\sqrt{y} + \sqrt{x})$, which is the Tullock [21] contest success function with exponent $r = \frac{1}{2}$.

This condition is equal to the welfare-maximizing condition (10) for $y_j$. In the same way, the investment level $x_i^*$ that maximizes victim $i$'s benefits given $e = \frac{1}{2}$ maximizes welfare. The reason why the government can reach its first best with commitment is that given allowed output, each agent's investment only affects his own payoff (Dijkstra [5]).

# 4 The investment contest

In this section, we will see what happens when the government's environmental policy has to be time consistent. This means that environmental policy must be optimal, given investments by polluters and victims. Thus, the agents' investments influence government policy: The more they invest, the more favourable environmental policy will be for them.

Each agent will set the investment level that maximizes his benefits, given the investments by all other agents, but taking into account that his investment influences subsequent environmental policy. In this section we determine the subgame perfect Nash equilibrium of the investment game for several cases. In subsection 4.1, there is only one polluter and one victim. In subsection 4.3, there are multiple victims, but only one polluter. Finally, in subsection 4.4, there are multiple polluters, but only one victim. Before we can analyze the multiple-victim and polluter cases, we have to analyze the single opponent's reaction curve. We do this in subsection 4.2.

## 4.1 One polluter, one victim

In this subsection, we determine the equilibrium of the investment contest between one polluter and one victim. In stage one of the game, the polluter and the victim make their investments. In stage two, the government sets allowed output.

The government will set allowed output $q$ at the level that is optimal given the polluter's and the victim's investments, $y$ and $x$ respectively. From (8) with $n = m = 1$:

$$B_q = B_e \tag{11}$$

In stage one, the polluter sets the investment level $y$ that maximizes her benefits, taking the victim's investment level $x$ as given, but realizing that her investment influences allowed output. Totally differentiating the polluter's benefits (1) with $m = 1$ with respect to $y$ yields:

$$\frac{dU^q(q,y)}{dy} = B_y + B_q \frac{dq}{dy} - 1 = 0 \tag{12}$$

where the value of $dq/dy$ follows from the total differentiation of (11):

$$\frac{dq}{dy} = \frac{-B_{qy}}{B_{qq} + B_{ee}} > 0 \tag{13}$$

The inequality follows from (2). Similarly, totally differentiating the victim's benefits (3) with $n = 1$ with respect to $x$ yields:

$$\frac{dU^e(e,x)}{dx} = B_x + B_e \frac{de}{dx} - 1 = 0 \tag{14}$$

where the value of $de/dx$ follows from the total differentiation of (11):

$$\frac{de}{dx} = \frac{-B_{ex}}{B_{qq} + B_{ee}} > 0 \tag{15}$$

Let us now examine how the polluter's investment incentives differ from those under commitment, where she sets $y$ according to (9).[7] The first term on the RHS of (12) denotes the polluter's marginal benefits of investment under the previously allowed output level. This term also appears under commitment, when $q$ is given, but the second term on the RHS of (12) only appears with time consistency. This term denotes the polluter's benefits of extra allowed output, brought about by her extra investment. Since allowed output depends positively on the polluter's investment level, the polluter's marginal benefits of investing are higher when she can influence government policy. Thus, the polluter invests more than the socially optimal amount. The victim invests the same amount as the polluter, so that environmental policy will be the same as in the optimum ($e = q = \frac{1}{2}$).

Figure 1 illustrates the polluter's incentive to overinvest in the investment contest against the victim. The polluter's (victim's) marginal benefits from effective investment $OA$ ($RS$) are given by $MB^q$ ($MB^e$). Given these investment levels, the government sets allowed output at $OD$. When the polluter increases her investment to $OA'$, her extra benefits given allowed output $OD$ are $AA'FG$. If the polluter only took account of these benefits, she would set the optimal investment level. However, in a contest against the victim who invests $RS$, the polluter's extra investment also results in an increase $DD'$ in allowed output. This yields extra benefits $DFF'D'$ for the polluter.

[7] The analysis for the victim's investment incentives is analogous.

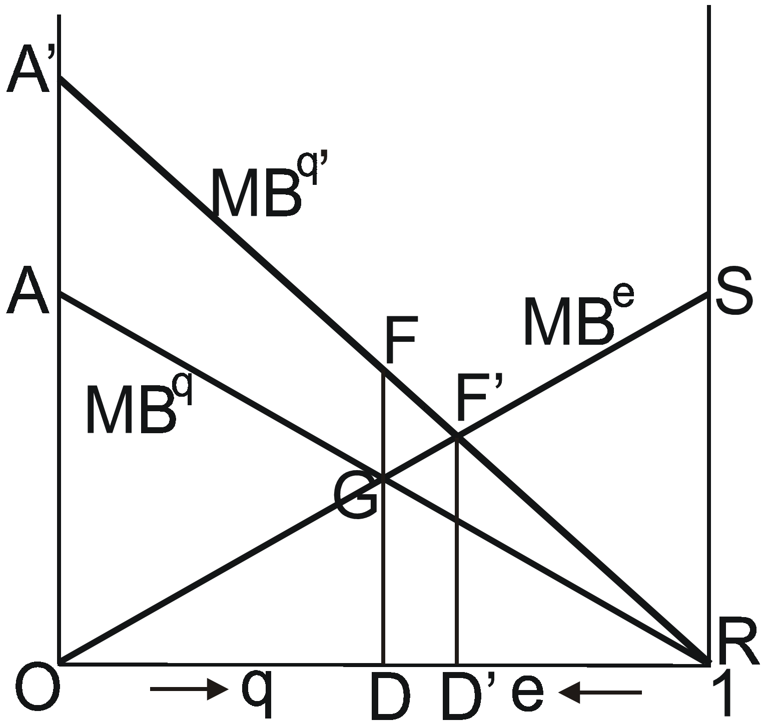


Figure 1: The polluter's incentive to overinvest in the contest

## 4.2 The polluter's reaction curve

In this subsection, we examine the slope of the single polluter's reaction curve (the analysis for the single victim is obviously analogous). The slope of the reaction curve plays an important role in subsections 4.3 and 4.4. From the total differentiation of (12) with respect to $x$ we find:

$$\frac{dy}{dx} = \frac{-\left[B_{qy} + B_{qq}\frac{dq}{dy}\right]\frac{dq}{dx} + B_q \frac{d^2q}{dydx}}{B_{yy}(B_{qq} + B_{ee}) - B_{qy}^2} \tag{16}$$

The denominator on the RHS of (16) is negative because it is a second-order condition for welfare maximization. The expression between square brackets is positive by (13) and (2):

$$B_{qy} + B_{qq}\frac{dq}{dy} = \frac{B_{qy}B_{ee}}{B_{qq} + B_{ee}} > 0$$

For the value of $d^2q/dydx$, we find from (13):

$$\frac{d^2q}{dydx} = \frac{-B_{qqy}\frac{dq}{dx}\left(B_{qq} + B_{ee}\right) + B_{qy}\left(B_{qqq}\frac{dq}{dx} + B_{eee}\frac{de}{dx} + B_{eex}\right)}{\left[B_{qq} + B_{ee}\right]^2}$$

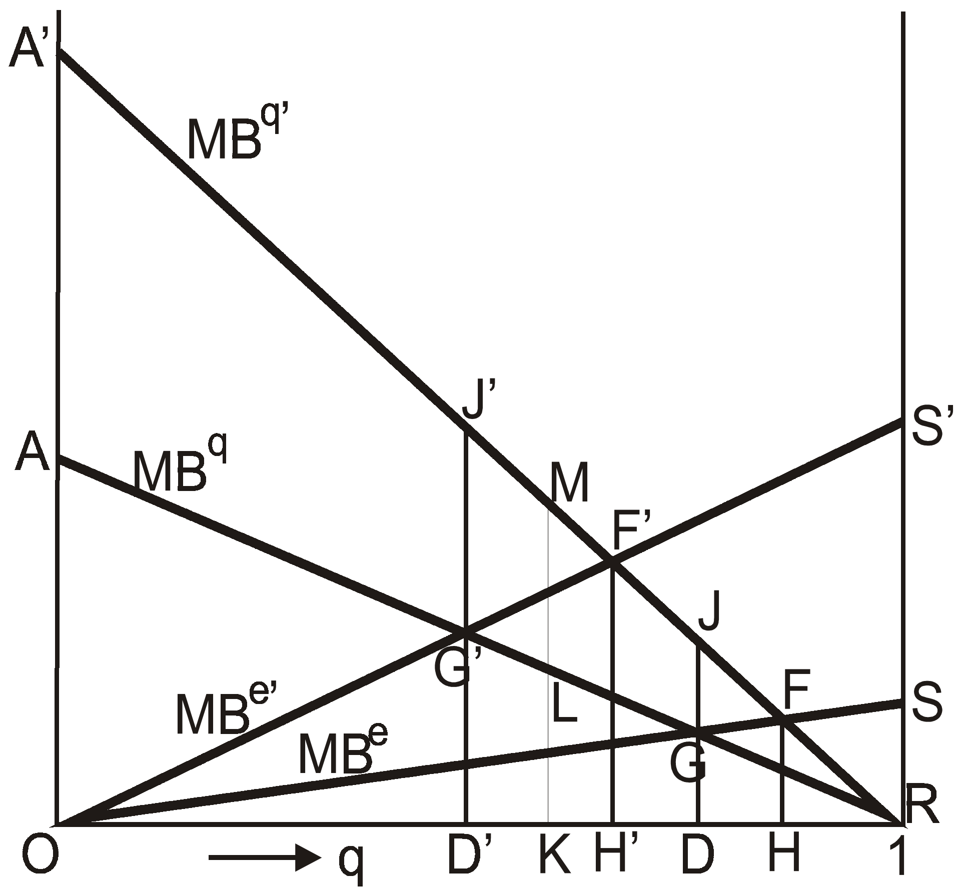


Figure 2: The effect of the victim's investment on the polluter's marginal benefits of investment

Substituting (15) yields:

$$\frac{d^2q}{dydx} = \frac{-B_{qy}B_{ex}}{(B_{qq}+B_{ee})^3}\left([B_{eee}-B_{qqq}] - \left[\frac{B_{qqy}}{B_{qy}} - \frac{B_{eex}}{B_{ex}}\right](B_{qq}+B_{ee})\right) \tag{17}$$

We see that $d^2q/dydx = 0$ for $y = x$. Thus, in the single-polluter, single-victim equilibrium, the reaction curves are downward sloping. It is beyond the scope of this paper to give a general characterisation of the whole reaction curve. For the purpose of this paper, we shall simply assume that it is downward sloping in the relevant domain:

**Condition 1** *The polluter's reaction function is downward sloping in the Nash equilibrium for every number $n$ of victims. The victim's reaction function is downward sloping in the Nash equilibrium for every number $m$ of polluters.*

Figure 2 illustrates our finding. To find out how the polluter responds to an increase in the victim's investment, we need to look at the change in her marginal benefits of investment. If the marginal benefits decrease (increase), the polluter's reaction curve will have a negative (positive) slope.

In Figure 2, we compare the polluter's extra benefits of raising her $MB$ curve from $MB^q$ to $MB^{q\prime}$ when the victim's marginal benefits are $MB^e$ and when they are $MB^{e\prime}$. We see that the increase in the victim's investment leads to a change in the polluter's extra benefits from $AA'FHDG$ to $AA'F'H'D'G'$. This change can be decomposed into three elements.

First, there is a decrease in marginal benefits given the prevailing output level, due to the fact that the victim's investment increase has pushed output back from $OD$ to $OD'$. Given the respective output levels, the marginal benefit of investment has decreased from $AA'JG$ to $AA'J'G'$, a decrease of $G'J'JG$. This decrease corresponds to the first term within square brackets on the RHS of (16).

The second effect is that a given increase in output will lead to more extra benefits for the polluter, the higher the victim's investment. This is because the victim's investment pushes the polluter's marginal benefits of output up. In Figure 2, let us take the output increase $DH$ with the low level of victim investment as the benchmark. If output increased by the same amount with the high level of victim investment, the increase would be $D'K = DH$. With the high level of victim investment, the benefit of extra output is $D'J'MK$, which is more than the increase $DJFH$ with low victim investment. This increase in marginal benefits corresponds to the second term within square brackets on the RHS of (16).

We know from our analysis of (16) that the term between square brackets on the RHS of (16) contributes negatively to $dy/dx$. This means that on balance, the negative first effect and the positive second effect lead to a reduction in the polluter's marginal benefits from investment. This can also be seen with the aid of Figure 2. The net effect of the first two changes is:

$$D'J'MK - DJFH - G'J'JG = D'G'LK - LMJG - DJFH < 0$$

Finally, we come to the third effect of the increase in the victim's investment, which is that it changes $dq/dy$. In Figure 2, the increase in output from extra polluter investment is $D'H'$ with high victim investment, which is more than the increase $DH$ with low victim investment. This leads to extra benefits $KMF'H'$ with high victim investment (remember that $D'K = DH$). While the output increase is higher for higher victim investment in Figure 2, this is not always the case. The reader can verify that the output increase eventually becomes smaller and smaller as $MB^e$ gets steeper and steeper. As equation (17) indicates, the output increase reaches a maximum when the victim's investment is equal to the polluter's investment.

Thus, we see that the first and second effect taken together lead to a decline in the polluter's investment as the victim's investment rises, whereas the third effect leads to an increase for $y$ when $x < y$. Figure 3 shows the polluter's and the victim's reaction curves, $A^q(A^e)$ and $A^e(A^q)$ for the benefit functions (5) and (6).[8] When the victim spends nothing, the polluter invests $A^q = \frac{1}{2}$. From there, the reaction curve is increasing slightly, to a maximum of $14/27 \approx 0.518$ for $A^e = 7/27 \approx 0.259$. Then the reaction curve is decreasing until it reaches $A^q = 0$ for $A^e = 2$. The unique Nash equilibrium of the investment stage occurs at $A_1$ in Figure 3, with $A^e = A^q = \frac{1}{2}$.[9] While the reaction curves are downward sloping in the equilibrium $A_1$, they are not monotonically decreasing. However, we shall see that Condition 1 is satisfied: The reaction curves are downward sloping in the range where the equilibria occur.

## 4.3 One polluter, multiple victims

Let us now analyze what happens when there are $n > 1$ victims of pollution, but only one polluter. In stage one of the game, the polluter and the victims make their investments. In stage two, the government sets allowed output.

The government will set allowed output $q$ at the level that is optimal given the polluter's and the victims' investments, $y$ and $x_i$, $i = 1, \cdots, n$, respectively. From (8) with $m = 1$:

$$B_q = \frac{1}{n}\sum_{i=1}^{n}\frac{\partial B(e, nx_i)}{\partial e} \tag{18}$$

[8]The polluter's reaction function is implicitly defined by $(A^q)^2 + 3A^qA^e + 4(A^e)^2 = 2(A^q + A^e)^3$.
[9]By contrast, the welfare-maximizing effective investment levels are $A^e = A^q = 3/8$.

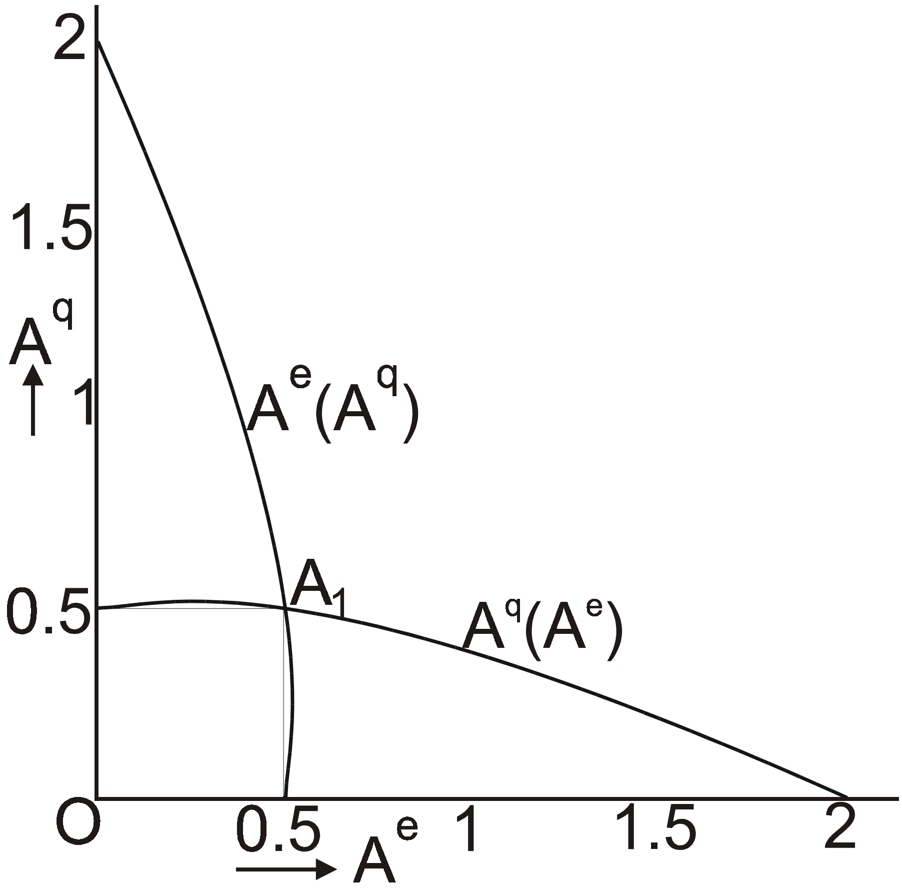


Figure 3: Polluter's and victim's reaction curves

In stage one, each agent sets the investment level that maximizes their payoff, taking all other agents' investments as given, but realizing that their investment influences environmental policy. The polluter sets $y$ according to (12). Totally differentiating victim $i$'s payoff $U_i^e$ from (3) with respect to $x_i$, we find:

$$\frac{dU_i^e}{dx_i} = \frac{\partial B}{\partial X_i} + \frac{1}{n}\frac{\partial B}{\partial e}\frac{de}{dx_i} - 1 = 0 \tag{19}$$

where the value of $de/dx_i$ follows from the total differentiation of (18) with $x_1 = \cdots = x_n$ with respect to $x_i$:

$$\frac{de}{dx_i} = \frac{-B_{eX}}{B_{qq} + B_{ee}} > 0 \tag{20}$$

Comparing (19) and (20) with (14) and (15) for the single-victim case, we see that for $x_i = x/n$, a victim's marginal benefit of investment, given environmental quality, and the increase in environmental quality due to the extra investment are equal in both cases.

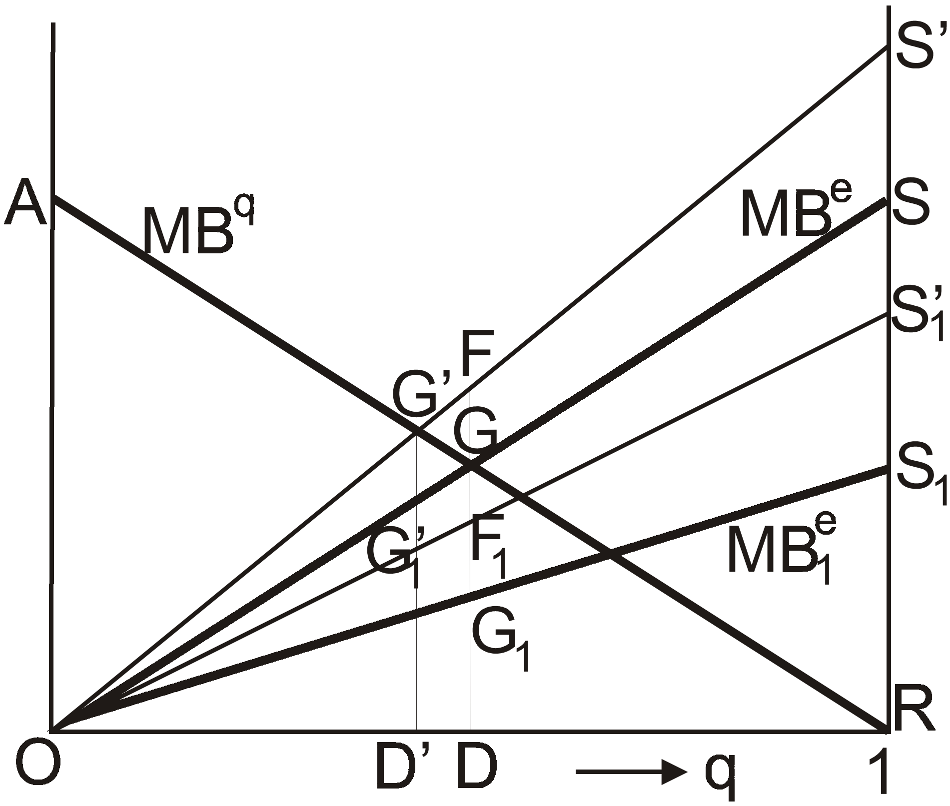


Figure 4: Victims’ investment incentives, single- and two-victim cases compared

However, with multiple victims, a victim’s marginal benefit from higher environmental quality (the second term on the RHS of (19) and (14)) is only a fraction $1/n$ of his marginal benefit in the single-victim case.

Figure 4 illustrates a victim’s investment incentives for the case of one and two victims. When there is one victim investing $RS$, his marginal benefits of environmental quality are given by the $MB^e$ curve $OS$. Given the polluter’s investment level of $OA$, the victim’s benefits of extra investment $SS'$ are $SGFS'$ given environmental quality $DR$ plus $D'G'FD$ from the increase $DD'$ in environmental quality. Now let there be two victims, both investing $RS_1 = \frac{1}{2}RS$, so that aggregate marginal benefits from environmental quality are again given by $MB^e$.[10] Victim 1 contemplates increasing investment by $S_1S_1' = SS'$. This would cost the same as the single victim’s extra investment of $SS'$ and, like the single victim’s extra investment, increases environmental quality by $DD'$. Victim 1’s extra benefits $S_1G_1F_1S_1'$ given environmental quality equal the single victim’s extra benefits

[10]Since environmental quality is a public good, we have to add up the victims’ $MB$ curves vertically.

$SGFS'$. However, victim 1's benefits $D'G_1'F_1D$ from increased environmental quality are less than the single victim's benefits $D'G'FD$. Thus, with multiple victims, a victim's marginal benefits of investment are lower and the victims will invest less.

Let us now perform comparative statics with respect to the number of victims $n$, starting with the investment levels. Writing the (aggregate) reaction functions as $nx_i = X(y,n)$ and $y(X)$, we find:

$$\frac{dX}{dn} = \frac{\partial X/\partial n}{1 - \left[\frac{dy}{dX}\frac{\partial X}{\partial y}\right]} \tag{21}$$

Since the term between square brackets should be below one by stability of the equilibrium, we have:

$$sign\frac{dX}{dn} = sign\frac{\partial X}{\partial n}$$

Totally differentiating (19) with respect to $n$ yields:

$$\frac{\partial X}{\partial n} = \frac{B_e de/dx_i}{n^2 d^2 U_i^e/dx_i} < 0 \qquad \Rightarrow \frac{dX}{dn} < 0 \tag{22}$$

The inequality follows from (2), (20) and the fact that $d^2U_i^e/dx_i^2 < 0$ as the second order condition for maximization of $U_i^e$.

For the polluter's investment, we find:

$$\frac{dy}{dn} = \frac{dy}{dX}\frac{dX}{dn} > 0 \tag{23}$$

The inequality follows from (22) and Condition 1. For total investments, we find from (21), (22) and (23):

$$\frac{d(X+y)}{dn} = \left(1 + \frac{dy}{dX}\right)\frac{\partial X/\partial n}{1 - \left[\frac{dy}{dX}\frac{\partial X}{\partial y}\right]} > 0 \tag{24}$$

The inequality follows from the fact that $dy/dX > -1$ by stability.

Obviously, since the victim's aggregate investments decrease and the polluter's investment increases, environmental quality decreases as the number of victims rises.

It is clear that the polluter's payoff increases as the number of victims rises. The more victims there are, the less aggressive they are, and the polluter benefits from that. Formally, writing the polluter's payoff $U^q$ as $V^q = V^q(y,X)$, we find:

$$\frac{dV^q(y,X)}{dn} = \frac{\partial V^q}{\partial y}\frac{dy}{dn} + \frac{\partial V^q}{\partial X}\frac{dX}{dn} = B_q\frac{dq}{dX}\frac{dX}{dn} > 0$$

The second equality follows from $\partial V^q(y,X)/\partial y = dU^q(q,y)/dy = 0$ by (12). The inequality follows from (22).

It is also straightforward to see that the victims' aggregate payoff is decreasing in $n$. Writing the victims' aggregate payoff as $V^e(X,y) = mU(e,X)$, we find:

$$\frac{dV^e(X,y)}{dn} = \frac{\partial V^e}{\partial X}\frac{dX}{dn} + \frac{\partial V^e}{\partial y}\frac{dy}{dn} = \frac{n-1}{n}B_e\frac{de}{dX}\frac{dX}{dn} + B_e\frac{de}{dy}\frac{dy}{dn} < 0 \tag{25}$$

The second equality follows from (19). Both terms on the RHS of equality (25) are negative for $n > 1$. The first term reflects the fact that when there is one victim, this victim's investment level is optimal given the polluter's investment $y$. However, the more victims there are, the less they invest, so that the victims' payoff decreases in $n$, given $y$. On top of that, the polluter's investment increases in $y$, which leads to an additional reduction in the victims' aggregate payoff (the second term on the RHS of (25)).

Given that the victims' aggregate payoff decreases and the polluter's payoff increases with $n$, it is not clear a priori what happens to aggregate payoff. From (1) and (3), we can write:

$$\frac{d(nU_i^e + U^q)}{dn} = (B_e - B_q)\frac{de}{dn} + \frac{1}{2}(B_X + B_y - 2)\frac{d(y+X)}{dn} + \frac{1}{2}(B_y - B_X)\frac{d(y-X)}{dn} \tag{26}$$

The first term on the RHS is equal to zero by (18). The second term on the RHS gives the volume effect of reducing total investment. This term is positive by (12), (19) and (24). As the number of victims increases, total investment decreases. This has a positive impact on aggregate payoff, because both polluter and victims tend to invest too much in an attempt to influence environmental policy. The third term on the RHS of (26) is the skewness effect: the polluter's investment rises while the victims' investment decline. This has a negative impact on aggregate payoff, because ideally, investments on both sides should decline equally fast, as they are equally productive.[11] At $n = 1$, the skewness effect does not exist yet (since $B_y = B_X$), and thus aggregate payoff initially increases due to the positive volume effect. As $n$ increases, however, the negative skewness effect takes over and aggregate payoff declines in the number of victims.[12] This follows from (19) and

[11] Formally, $B_y < B_X$ at $B_q = B_e$ if and only if $y > X$.

[12] It is possible that the skewness effect already dominates the volume effect when $n$ rises from 1 to 2. However, we shall ignore that possibility.

(26) with $n \to \infty$:

$$\lim_{n \to \infty} \frac{d(nU_i^e + U^q)}{dn} = -B_q \frac{dq}{dy}\frac{dy}{dn} < 0$$

When the number of victims is very large, each victim only has a negligibly small benefit from an increase in environmental quality. Therefore the victims don't try to influence environmental policy. They take environmental policy as given, which as we know from subsection 4.1 yields the optimal investment level. The polluter, on the other hand, still invests too much, in her attempt to influence environmental policy. Thus when $n$ is very large and increasing, aggregate welfare falls because excessive investment by the polluter increases. To summarize the effects of an increase in $n$:

**Proposition 1** *As the number of victims $n$ increases, the victims' aggregate investment decreases, the polluter's investment increases and total investment decreases. Environmental quality declines. The victims' aggregate payoff decreases and the polluter's payoff increases. Total payoff increases for low $n$, reaches a maximum for a finite $n$ and declines for high $n$.*

We can apply this result to the analysis of cooperation. Rather than acting on their own, the victims could try to cooperate and coordinate their investment decisions. One form this cooperation could take is that the group of all victims acts as a single victim. Most effects of this kind of cooperation follow straightforwardly from Proposition 1. In addition, let us assume:

**Condition 2** *Total payoff is higher for $n \to \infty$ than for $n = 1$.*

Then we find:

**Corollary 1** *When multiple victims coordinate their investment behaviour to act as a single victim, the victims' aggregate investment increases and the polluter's investment decreases. Total investment increases and environmental quality rises. The victims' aggregate payoff increases, while the polluter's payoff decreases. Total payoff decreases under Condition 2.*

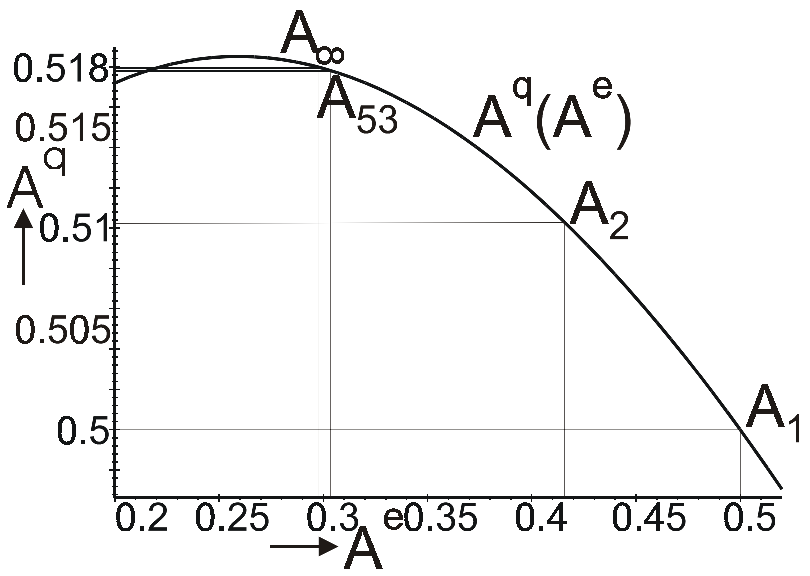


Figure 5: Effective investment levels depending on the number of victims

When the victims cooperate, they take into account that an increase in one victim's investment leads to a higher environmental quality for all. They will thus invest more, pushing the polluter's investment back. This will increase their own payoff, at the expense of the polluter and of total payoff.

Finally, let us illustrate our findings with the specific functions (5) and (6). The changes in effective investment levels as the number of victims increases are illustrated in Figure 5 which shows the relevant portion of the polluter's reaction curve $A^q(A^e)$. With a single victim, polluter and victim invest the same amount: $A^q = A^e = \frac{1}{2}$. This is point $A_1$ in the Figure. As the number of victims increases, their aggregate investment level decreases. The polluter's investment level increases, as she is on the decreasing branch of her reaction curve. For two victims, the equilibrium is $A_2$ with $(A^e, A^q) = (0.417; 0.510)$ and $q = 0.55$. As the number of victims keeps increasing, the equilibrium approaches $A_\infty = (0.299; 0.518)$ and $q = 0.63$. This is still on the decreasing branch of the polluter's reaction curve. Thus, Condition 1 holds in this example. Since the polluter's reaction curve is quite flat, total investment cost falls as $n$ increases.

We find that total payoff is maximal for $n = 53$, with $(A^e, A^q) = (0.305; 0.518)$. This is point $A_{53}$ in Figure 5. Finally, total payoff is higher for $n \to \infty$ than for $n = 1$ (0.135 and 0.125, respectively). Thus, Condition 2 holds in the example as well.

## 4.4 Multiple polluters, one victim

Let us now analyze what happens when there are $m > 1$ polluters, but only one victim. In stage one of the game, the polluters and the victim make their investments. In stage two, the government sets the allowed output levels.

The government will set allowed output $q_j$ for polluter $j$ at the level that is optimal given the polluters' and the victim's investments, $y_j$, $j = 1, \cdots, m$, and $x$, respectively. From (8) with $n = 1$, the government sets polluter $k$'s allowed output according to:

$$\frac{\partial B_k^q}{\partial Q_k} = B_e \tag{27}$$

In stage one, each agent sets the investment level that maximizes their payoff, taking all other agents' investments as given, but realizing that their investment influences environmental policy. The victim sets $x$ according to (14). Polluter $j$ sets the investment level that maximizes her payoff (1):

$$\frac{dU_j^q}{dy_j} = \frac{\partial B_j^q}{\partial Y_j} + \frac{\partial B_j^q}{\partial Q_j}\frac{dq_j}{dy_j} - 1 = 0 \tag{28}$$

To obtain the expression for $dq_j/dy_j$, we have to differentiate the $m$ conditions (27) for $q_k$, $k = 1, \cdots, m$, and the environmental constraint (4) totally with respect to $y_j$. For all $l = 1, \cdots, m$, $l \neq j$:

$$\frac{\partial^2 B_j^q}{\partial Q_j \partial Y_j} + \frac{\partial^2 B_j^q}{\partial Q_j^2}\frac{dq_j}{dy_j} = \frac{\partial^2 B_l^q}{\partial Q_l^2}\frac{dq_l}{dy_j} = -\frac{\partial^2 B^e}{\partial e^2}\sum_{k=1}^{m}\frac{dq_k}{dy_j}$$

Solving for $dq_j/dy_j$ with $q_1 = \cdots = q_n$, $y_1 = \cdots = y_n$:

$$\frac{dq_j}{dy_j} = -\frac{B_{QY}}{B_{QQ}}\left[\frac{B_{QQ} + (m-1)B_{ee}}{B_{QQ} + mB_{ee}}\right] > 0 \tag{29}$$

Now we can compare the polluter's investment incentives for single and multiple polluters with $y_j = y/m$, $q_j = q/m$. From (12) and (28), we see that in both cases, the polluter has the same marginal benefits of extra investment, given the allowed output level, and the same marginal benefits of extra output. The difference lies in the effect of extra investment on allowed output. Totally differentiating (29) with respect to $m$:

$$\frac{d^2 q_j}{dy_j dm} = \frac{B_{QY}}{(B_{QQ} + mB_{ee})^2} > 0 \tag{30}$$

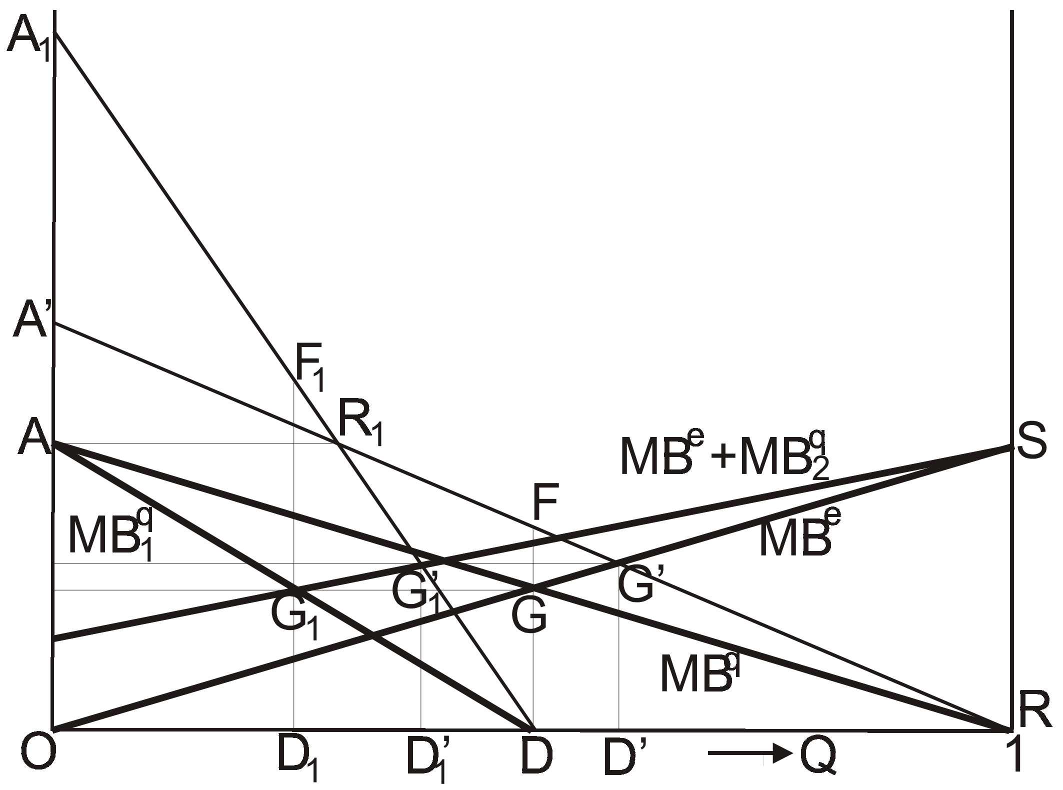


Figure 6: Polluters' investment incentives, compared for one and two polluters

We see that the extra investment leads to a larger increase in allowed output, the larger the number of polluters. This is because a single polluter can only increase output at the expense of the victim. With multiple polluters, a polluter can also take allowed output away from other polluters.

Figure 6 illustrates the comparison of a polluter's investment incentives for the single- and two-polluter cases. When there is one polluter, her marginal benefits when investing $OA$ are given by the $MB^q$ curve $AR$. Given the victim's investment of $RS$, the polluter's benefits of extra investment $AA'$ are $AA'FG$ given allowed output $OD$ plus $DFG'D'$ from the increase $DD'$ in allowed output.

Now let there be two polluters, both investing $OA$, so that aggregate marginal benefits from allowed output are again given by $MB^q$.[13] Polluter 1's allowed output is $OD_1 = \frac{1}{2}OD$. When polluter 1 marginally increases her investment spending by the same amount as the single polluter above, this must result in the same aggregate $MB^q$ curve for $q_2 > 0$,

[13]Since output is a private good, we have to add up the polluters' $MB$ curves horizontally.

i.e. $R_1R$. This means that polluter 1's $MB_1^q$ curve shifts up from $AD$ to $A_1D$ and the aggregate $MB^q$ curve becomes $A_1R_1R$. As in the single-polluter case, aggregate allowed output increases by $DD'$. However, polluter 1 does not only benefit from an increase in aggregate allowed ouput, but also takes away allowed output from polluter 2. To find out by how much polluter 1's allowed output increases, we construct the marginal benefit curve $MB^e + MB_2^q$. These are aggregate marginal benefits for the victim and polluter 2. We see that polluter 1's allowed output increases by $D_1D_1' > DD'$. This is because the $MB^e + MB_2^q$ curve that polluter 1 faces is more elastic than the $MB^e$ curve that the single polluter faces. Polluter 1's extra benefits $AA_1F_1G_1$ given allowed output $OD_1$ equal the single polluter's extra benefits $AA'FG$ given $OD$. However, since polluter 1 enjoys a larger increase in allowed output, her benefits from increased output $D_1F_1G_1'D_1'$ exceed the single polluter's extra benefits $DFG'D'$.

Let us now perform comparative statics with respect to the number of polluters $m$. Analogous to the multiple-victim case, we have:

$$sign\frac{dY}{dm} = sign\frac{\partial Y}{dm}$$

Totally differentiating (28) with respect to $m$ yields, from (30):

$$\frac{\partial Y}{\partial m} = \frac{-B_{QY}}{d^2U_j^q/dy_j^2\left(B_{QQ} + mB_{ee}\right)^2} > 0 \qquad \Rightarrow \frac{dY}{dm} > 0 \tag{31}$$

Thus, the polluters' aggregate investments are increasing in $m$. For the victim's investment level, we find:

$$\frac{dx}{dm} = \frac{dx}{dY}\frac{dY}{dm} < 0$$

The inequality follows from (31) and Condition 1. For the level of total investments, we find:

$$\frac{d(x+Y)}{dm} = \left(1 + \frac{dx}{dY}\right)\frac{dY}{dm} > 0 \tag{32}$$

The inequality follows from (31) and $dx/dY > -1$ by stability.

Since the polluters' aggregate investments increase and the victim's investment decreases, it is clear that environmental quality declines as the number of polluters increases.

Let us now look at total payoff. Analogous to (26), we find:

$$\frac{d(U^e + mU_j^q)}{dm} = \frac{1}{2}(B_x + B_Y - 2)\frac{d(x+Y)}{dm} - \frac{1}{2}(B_Y - B_x)\frac{d(Y-x)}{dm} \tag{33}$$

The first term on the RHS is the volume effect, which is negative from (14), (28) and (32). Total investments increase, which is bad for aggregate payoff, because investments were already too high in the first place. The second term on the RHS of (33) is the skewness effect: the polluter's investment rises while the victims' investment decline. This has a negative impact on aggregate payoff for $m > 1$, because ideally, investments on both sides should be equal. We conclude that total payoff declines as the number of polluters increases.

It is obvious that the victim's payoff decreases as $m$ rises. The more polluters there are, the more aggressive they will be, and the victim is harmed by that. Formally, defining $V^e(x, Y) = U^e(x, e)$, we find:

$$\frac{dV^e}{dm} = \frac{\partial V^e}{\partial x}\frac{dx}{dm} + \frac{\partial V^e}{\partial Y}\frac{dY}{dm} = B_e\frac{de}{dY}\frac{dY}{dm} < 0$$

The second equality follows from $\partial V^e(x, Y)/\partial x = dU^e(e, x)/dx = 0$ by (14).

Writing aggregate polluters' payoff $mU_j^q(q_j, y_j) = V^q(Y, x)$, we find:

$$\frac{dV^q}{dm} = \frac{\partial V^q}{\partial Y}\frac{dY}{dm} + \frac{\partial V^q}{\partial x}\frac{dx}{dm} = \frac{B_Q B_{QY}(m-1)B_{ee}}{B_{QQ}\left(B_{QQ} + mB_{ee}\right)}\frac{dY}{dm} + B_Q\frac{dQ}{dx}\frac{dx}{dm}$$

The second equality follows from (28) and (29). The first term on the RHS denotes the effect of the polluters' investments. At $m = 1$, the polluters' investments are optimal given the victim's investment, but as $m$ rises, polluters' investments rise and grow too large. The second term denotes the effect of the victim's investment. Since the victim's investment decreases with $m$, this raises the polluters' payoff. For $m = 1$, only the second effect exists, and polluters' aggregate payoff is rising in $m$. However, this does not imply that aggregate polluter payoff rises as $m$ increases from 1 to 2, because this is a discrete increase. In fact, we shall see that in our example, aggregate polluter payoff decreases. For simplicity, we generalize this finding:

**Condition 3** *In equilibrium, the polluters' aggregate payoff with $m = 2$ is lower than the polluter's payoff with $m = 1$.*

Under this condition, the polluters' aggregate payoff is decreasing in $m$. Summing up, we find:

**Proposition 2** *As the number of polluters $m$ increases, the polluters' investment increases, the victim's investment decreases and total investment increases. Environmental quality declines. The polluters' aggregate payoff decreases under Condition 3. The victim's payoff decreases, as does total payoff.*

Again, we can apply this result to cooperation. Instead of acting on their own, the polluters could try and cooperate to coordinate their individual investment decisions. This cooperation could take the form of the polluters acting as a single polluter. The consequences follow straightforwardly from Proposition 2:

**Corollary 2** *When multiple polluters coordinate their investment behaviour to act like a single polluter, the polluters' aggregate investment decreases and the victim's investment increases. Total investment decreases. Environmental quality improves. Total payoff and the victim's payoff both increase. The polluters' aggregate payoff increases under Condition 3.*

When the polluters coordinate their investments, they will take into account that an increase in any one polluter's allowed output comes partly at the expense of other polluters' allowed output. Thus, they will invest less when they coordinate. Both the polluters and the victim will benefit from this reduction in polluters' investment.

Investment levels depending on the number of polluters for our example functions (5) and (6) are illustrated in Figure 7 which shows the relevant portion of the victim's reaction curve $A^e(A^q)$. The single-polluter equilibrium point is $A_1$, with $A^q = A^e = \frac{1}{2}$. As the number of polluters increases, their effective investment level $A^q$ increases. In response, the victim's effective investment level decreases, because he is on the decreasing branch of his reaction curve. Since the victim's reaction curve is quite flat, total investment effort is decreasing in the number of polluters. For two polluters, the equilibrium is $A_2$ with $(A^q, A^e) = (0.558; 0.491)$ and $e = 0.468$. As the number of polluters keeps increasing, the

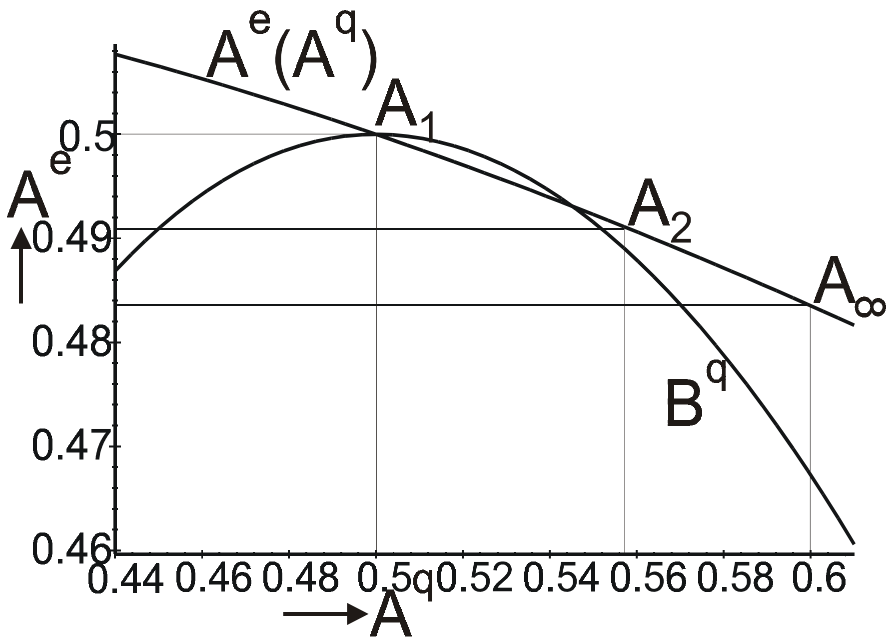


Figure 7: Investment levels depending on the number of polluters

equilibrium approaches $A_\infty = (0.600; 0.484)$ with $e = 0.45$.[14]

In Figure 7, the polluter's isopayoff curve through the single-polluter equilibrium $A_1$ is drawn as $B^q$. By definition, this isopayoff curve has zero slope at $A_1$, so that points on the victim's reaction curve $A^e(A^q)$ immediately to the right of $A_1$ result in higher payoffs for the polluters. The fact that the polluters' payoffs are increasing in $m$ at $m = 1$ does not imply yet that they will increase as the number of polluters rises discretely from 1 to 2. Indeed, as Figure 7 shows, the two-polluter equilibrium $A_2$ lies outside the isopayoff curve through $A_1$ and therefore leads to lower payoffs for the polluters. Thus, Condition 3 holds and the polluters' aggregate payoff is decreasing in $m$.

# 5 Conclusion

In this paper, we have analyzed an investment contest for environmental policy. The polluters invest in their marginal benefits of output and the victims of pollution invest in their marginal benefits of environmental quality. These investments influence environmental policy, because policy is time-consistent: it is optimal, given the polluters' and

[14]Note that environmental quality is higher for $m$ polluters versus one victim than for one polluter versus $m$ victims.

victims' investment levels.

The government would be able to reach the social optimum if it could commit to environmental policy. Then the agents take allowed output or environmental quality as given when they decide upon their investment level. In the investment contest, the agents invest more than socially optimal. This is because an agent does not only look at his payoff given environmental policy, but also sees investment as a way to influence environmental policy in his favour.

Environmental quality is a public good. A victim does not take into account that other victims benefit from the increase in environmental quality due to his investment. Coordination of the victims' investments leads to more investment by the victims. The polluter's investment would decline and total investment would rise. Coordination by victims increases victims' aggregate payoff and decreases the polluter's payoff as well as total welfare.

Pollution is a private good. A polluter does not take into account that her increase in allowed output is achieved partly at the expense of other polluters' allowed output levels. Coordination of the polluters' investments leads to less investment by polluters. The victim's investment would rise and total investment would decline. Coordination by polluters obviously increases the victim's payoff and less obviously also increases the polluters' aggregate payoff. Thus, total welfare also rises.

We conclude that the effects of coordination by victims and polluters are asymmetric. Everyone benefits from coordination by polluters. However, the victims benefit from their own coordination, but the polluter is harmed and total welfare declines.

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