UNIVERSITY OF NOTTINGHAM

Discussion Papers in Economics

Discussion Paper
No. 05/05

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June 2005

DP 05/05

ISSN 1360-2438

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June 2005

Entry in a Stackelberg perfect equilibrium

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Abstract: This paper considers welfare effects of entry when the incumbent firm behaves like a Stackelberg leader in the product market. In contrast to previous work (Klemperer, 1988, Journal of Industrial Economics), we show that entry may always increase welfare. Using general demand function, we show the condition for welfare improving entry.

Key Words: Cournot competition; Entry; Stackelberg competition; Welfare

JEL Classification: D43 ; L13 ; O34

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Entry in a Stackelberg perfect equilibrium

1. Introduction

Does entry always increase welfare? In an interesting paper Klemperer (1988) has shown that entry reduces welfare if the marginal cost of the entrant is sufficiently higher than the incumbent.[1] While this is an interesting finding, Klemperer (1988) has assumed that the incumbent and the entrant behave like Cournot duopolists in the product market, i.e., the firms produce their outputs simultaneously. However, it is well known that often the incumbency advantage allows the incumbent to behave like a Stackelberg leader in the product market. In this paper, we relax the assumption of Cournot competition and allow the incumbent firm to behave like a Stackelberg leader.[2] We show that, in this situation, entry may increase welfare always. Using general demand function, we show the condition for welfare improving entry in a Stackelberg perfect equilibrium.[3] So, entry is likely to increase welfare in an economy where the incumbent is a dominant firm in the product market. This may be important for competition policies designed to attract new firms in the industry.

The rest of the paper is organized as follows. Section 2 shows the sufficient condition for a welfare improving entry. Section 3 shows when this sufficient condition for welfare improving entry is satisfied in a Stackelberg model. Section 4 concludes.

[1] There is another literature that shows entry may reduce welfare due to the existence of fixed costs (see, e.g., Stiglitz, 1981, Spence, 1984, Tandon, 1984, Schmalensee, 1976, von Weizsäcker, 1980a, b, Mankiw and Whinston, 1986 and Suzumura and Kiyono, 1987).

[2] See, e.g., Spulber (1981) and Basu and Singh (1990) for works with *exogenous* Stackelberg structure on entry deterrence. One may refer to Dixit (1980), Basu (1995), Hamilton and Slutsky (1990) and van Damme and Hurkens (1999) for the works on *endogenous* Stackelberg structure.

2. The sufficient condition for welfare improving entry

This section uses a diagram to show the sufficient condition for welfare improving entry. Let us consider an economy with an incumbent and an entrant. Both firms can produce a homogeneous product with the marginal cost of production c_i. However, each consumer has a 'switching cost' $s > 0$ for switching from the incumbent's product to the entrant's product (see, Klemperer, 1988). Therefore, the effective marginal cost of the entrant is $(c_i + s) \equiv c > c_i$.[4] For simplicity, assume that there is no other cost of production or entry. We consider two situations to see the effects of entry: (i) when there is no-entry and the incumbent is monopolist, and (ii) when there is entry and the market structure is duopoly.

Let us now consider Figure 1.

Figure 1

Assume that the market demand curve is AA, which is considered to be linear for simplicity. In the next section, we will use general demand function for our analytical solution. Assume that the monopoly output is qm and therefore, the price under no-entry is pm. The area Ac_igd shows social welfare, which is the summation of consumer surplus and industry profit, under no-entry.

Now consider entry of a firm with the marginal cost c. Suppose, the industry output under entry is qe and therefore, the price is pe. Also assume that the incumbent produces qie. Hence, the entrant produces $(qe - qie)$. Therefore, the area Ac_izytd' shows social welfare under entry. Comparison of welfare under entry and no-entry shows

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[3] It follows from Clarke and Collie (2003) that entry in an economy with a linear demand function increases welfare if the firms compete in prices.

[4] Instead of switching cost, the different marginal cost of the firms may be the outcome of technological differences. For example, if the incumbent is a technology leader, the expiration of patent on one of its old technologies may create entry of a firm with relatively higher marginal cost of production.

that entry increases welfare compared to no-entry if and only if the area $dg'td'$ is greater than the area $yzgg'$, which has been created due to the output reduction of the incumbent and the higher cost of production of the entrant. Given other things constant, the area $yzgg'$ decreases with lower c and higher qie, whereas the area $dg'td'$ increases with higher qe.[5] It is clear from Figure 1 that, if $qie \geq qm$, the area $yzgg'$ vanishes and entry always increases welfare.[6] Note that this argument holds for any demand function.

Hence, the following proposition is immediate.

Proposition 1: *The sufficient condition for welfare improving entry is the higher output of the incumbent under entry than its output under no-entry.*

In the next section, we will show that if the marginal revenues of the firms are decreasing with respect to the outputs of the competitors (which is true under the linear demand function considered in Klemperer, 1988), the above result never holds under Cournot competition though it may hold always under Stackelberg competition. Hence, in this situation, entry creates the possibility of lower welfare under Cournot competition, but entry may increase welfare always under Stackelberg competition.

3. Cournot vs. Stackelberg competition

In the following analysis, we will concentrate on the situation where the marginal revenues of the firms are decreasing with respect to the outputs of the competitors, since it easily follows form Bulow et al. (1985) that if the marginal revenues of the firms are

[5] It must be noted that though the outputs are dependent on the parameter values, they also depend on the type of product market competition, e.g., Cournot and Stackelberg competition. So, for a given c, the outputs qie and qe are different for Cournot and Stackelberg competition.

increasing with respect to the outputs of the competitors[7], the incumbent produces more under entry than no-entry *even* under Cournot competition. Hence, in this situation, entry always increases welfare *even* under Cournot competition.

Let us assume that the inverse market demand function is $p(q)$, with $p' < 0$ and $p'' \leq 0$, and the notations have usual meanings. The assumption on the second derivative of the demand function is sufficient to ensure that the marginal revenues the firms are decreasing with respect to the outputs of the competitors, and it will also be enough to satisfy the second order conditions for maximization in our analysis. Like the previous section, assume that the marginal costs of the incumbent and the entrant are c_i and c respectively.

If the incumbent is monopolist, it maximizes the following expression to determine its optimal output:

$$\underset{q_i^m}{Max}(p(q_i^m) - c_i)q_i^m . \tag{1}$$

The optimal output is determined form the following equation:

$$p(q_i^m) - c_i + q_i^m p' = 0 . \tag{2}$$

3.1 Cournot competition

Now, consider the output choice under entry when the firms compete like Cournot duopolists.

The incumbent and the entrant maximize the following expressions respectively to determine their optimal outputs:

$$\underset{q_i^e}{Max}(p(q_i^e + q_e^e) - c_i)q_i^e \tag{3}$$

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[6] I would like to thank an anonymous referee for suggesting me this method for welfare comparison.
[7] For example, this happens for constant elasticity demand functions.

$$\underset{q_e^e}{Max}(p(q_i^e + q_e^e) - c)q_e^e . \qquad (4)$$

The optimal outputs for the incumbent and the entrant are determined by solving the following expressions:

$$p(q_i^e + q_e^e) - c_i + q_i^e p' = 0 \qquad (5)$$

$$p(q_i^e + q_e^e) - c + q_e^e p' = 0 . \qquad (6)$$

It is easy to see from (2), (5) and (6) that, if the entrant produces positive output, the output of the incumbent under entry will be lower than its output under no-entry. Hence, Proposition 1 does not hold under Cournot competition if the marginal revenues of the firms are decreasing with respect to the outputs of the competitors, and therefore, in this situation, entry may reduce welfare.

The industry output can be found by adding the equations (5) and (6), which gives the condition:

$$2p(q^e) - (c_i + c) + q^e p' = 0 , \qquad (7)$$

where $q^e = q_i^e + q_e^e$.

3.2 Stackelberg competition

If entry occurs but the incumbent behaves like a Stackelberg leader, it maximizes the following expression to determine its optimal output:

$$\underset{q_i^e}{Max}(p(q_i^e + q_e^e(q_i^e)) - c_i)q_i^e , \qquad (8)$$

since the incumbent internalizes the output strategy of the entrant, which can be found from the entrant's reaction function $q_e^e(q_i^e)$. So, the optimal output of the incumbent is determined from the following equation:

$$p(q_i^e + q_e^e(q_i^e)) - c_i + q_i^e p' + q_i^e p' q_e^{e\,\prime} = 0 , \qquad (9)$$

where ${q_e^e}'$ is the slope of the reaction function of the entrant.

Given the output choice of the incumbent, the entrant maximizes the following expression to determine its optimal output:

$$\underset{q_e^e}{Max}(p(q_i^e + q_e^e) - c)q_e^e. \tag{10}$$

The optimal output of the entrant is determined from the following equation:

$$p(q_i^e + q_e^e) - c + q_e^e p' = 0. \tag{11}$$

The implicit function $q_e^e(q_i^e)$ can be found from equation (10), and the slope of this implicit function, i.e., ${q_e^e}'$, is negative. Given that the entrant produces positive output under both Cournot and Stackelberg competition, we get from (5), (6), (9) and (11) that the optimal output of the incumbent (entrant) is higher (lower) under Stackelberg competition than Cournot competition. The industry output under Stackelberg competition can be found by using the equations (9) and (11). It is easy to see that the industry output is higher under Stackelberg competition than Cournot competition, which is also higher than the incumbent's monopoly output.

So, the outputs of the incumbent and the industry are higher under Stackelberg competition compared to Cournot competition, which implies that the possibility of welfare reducing entry is lower under the former competition than the latter. However, it is yet to see whether entry always increases welfare under Stackelberg competition.

Proposition 2: *Assume that $c = c_i$. If the absolute slope of the entrant's reaction function corresponding to the incumbent's monopoly output, q_i^m, is not lower than the absolute slope of the incumbent isoprofit curve at the output combination $(q_i^m, q_e^e(q_i^m))$,* *i.e.,* $-q_e^{e'}(q_i^m) \geq \frac{p(q_i^m + q_e^e(q_i^m)) - c_i + q_i^e p'}{q_i^m p'}$, *the incumbent's output under Stackelberg competition is always higher than its monopoly output as long as the entrant produces positive output in the market. Therefore, in this situation, entry always increases welfare under Stackelberg competition.*

Proof: It is easy to check that as the marginal cost of the entrant increases, it reduces the entrant's output and increases the output of the incumbent. So, if the incumbent's output under Stackelberg competition is greater than or equal to its monopoly output when the entrant's marginal cost is c_i, the incumbent always produces more under entry than no-entry for $c > c_i$ as long as the entrant produces positive output in the market.

However, if the entrant's marginal cost is c, the incumbent, under Stackelberg competition, does not produce lower than its monopoly output if and only if

$$p(q_i^m + q_e^e(q_i^m)) - c_i + q_i^e p' + q_i^m p' q_e^{e'} \geq 0$$

or $$-q_e^{e'}(q_i^m) \geq \frac{p(q_i^m + q_e^e(q_i^m)) - c_i + q_i^e p'}{q_i^m p'}, \tag{12}$$

where the left hand side (LHS) of (12) shows the absolute slope of the entrant's reaction function and the right hand side (RHS) of (12) shows the absolute slope of the incumbent's isoprofit curve.[8] Q.E.D.

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[8] The isoprofit curve of the incumbent is $(p(q_i^e + q_e^e(q_i^e)) - c_i)q_i^e = K$, where K is a constant. Slope of the isoprofit curve is $\frac{\partial q_e^e}{\partial q_i^e} = \frac{-(p - c_i + q_i^e p')}{q_i^e p'}$.

Using (2) and the expression for $q_e^{e'}$, which is $\dfrac{-(p'+q_e^e p'')}{2p'+q_e^e p''}$ (where the denominator is negative due to the second order condition for entrant's profit maximization), and after rearrangement, we can re-write the condition (12) as:

$$\frac{q_i^m}{q_e^e(q_i^m)} \leq 2 + \frac{q_e^e p''}{p'}. \tag{13}$$

If the demand function is linear, i.e., $p''=0$, the LHS of (13) is equal to the RHS of (13). For example, if the inverse demand function is $p = a - bq$, we get $q_i^m = \dfrac{(a-c_i)}{2b}$ and $q_e^e(q_i^m) = \dfrac{(a-c_i)}{4b}$. So, both the LHS and RHS of (13) are 2. Hence, entry always increases welfare for the demand function considered in Klemperer (1988). It should also be clear that condition (12) or (13) can also hold for non-linear demand function since, given that $q_e^{e'}$ is negative and the entrant's optimal output is positive, it follows from (2) and (9) that $p(q_i^m) - c_i + q_i^m p' + q_i^m p' q_e^{e'} > 0$. Since the incumbent internalizes the output strategy of the entrant, the incumbent's optimal output corresponding to the zero output of the entrant is greater than its monopoly output. Though the positive optimal output of the entrant reduces the output of the incumbent, the incumbent's output under entry can still be higher than its monopoly output.

It must be noted that the above proposition provides a strong sufficient condition for the welfare improving entry. Because, even if (12) does not hold, welfare increases with entry since the marginal cost of the entrant equals to c and eliminates the area $yzgg'$ in Figure 1. Since, in Figure 1, the area $yzgg'$ is concave with respect to c and the area $dg'td'$ decreases with higher c, it must be clear that the possibility of welfare improving entry is higher under a weaker sufficient condition, which says that condition (12) holds for that value of c at which welfare under entry becomes minimal.[9]

The intuition for our result is as follows. Under entry, the industry output will be more than the incumbent's monopoly output. Since, under Stackelberg competition, the incumbent chooses its output before the entrant's production, it internalizes the output strategy of the entrant and may have the incentive to commit to a higher output level than its monopoly output, because higher production of the incumbent reduces output of the entrant. The incentive for higher production by the incumbent increases with the higher marginal cost of the entrant, since the higher marginal cost of the entrant induces the entrant to react less aggressively for a given output of the incumbent. Further, lower output of the entrant under Stackelberg competition compared to Cournot competition creates lower production inefficiency under the former competition than the latter.

4. Conclusion

Assuming Cournot competition, Klemperer (1988) has shown that entry may reduce welfare if the entrant is sufficiently cost inefficient than the incumbent. However, the incumbency advantage often allows the incumbent to behave like a Stackelberg leader in the product market. We show that if the incumbent behaves like a Stackelberg leader, it is possible that entry always increases welfare. Using general demand function, we show the condition for welfare improving entry under Stackelberg competition.

[9] Since, the area $yzgg'$ is concave with respect to c and the area $dg'td'$ decreases with higher c, there is a critical value of c at which welfare under entry becomes minimal. Whether this critical value is lower than the value of c at which the entrant stops producing in the market depends on the parameter values and the type of product market competition. For example, in case of linear demand function, this critical value under Stackelberg competition is lower than the value of c at which the entrant stops producing.

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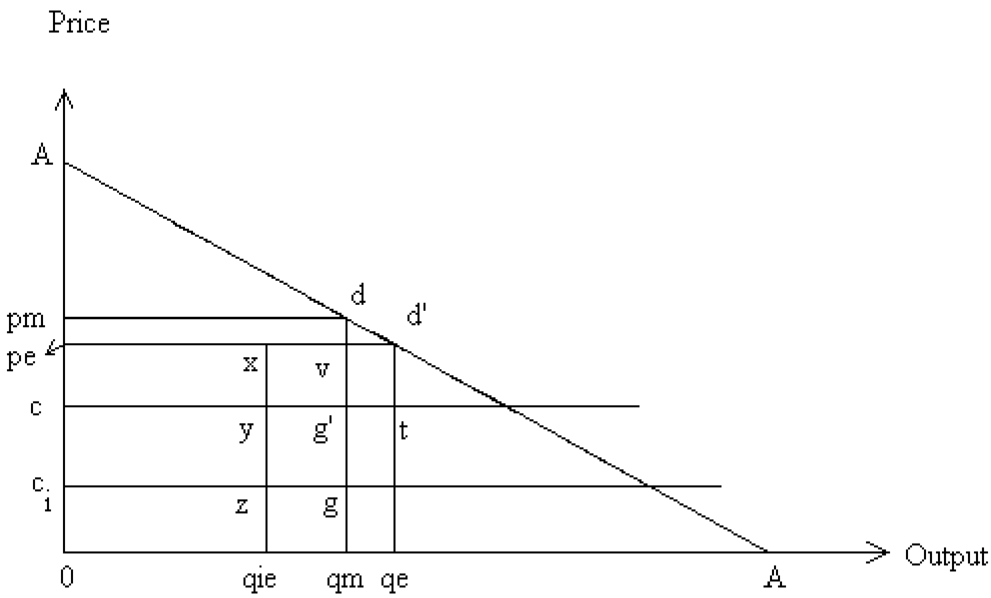

Figure 1: The sufficient condition for welfare improving entry