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Competition, innovation and welfare*

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Abstract: We show the effects of Bertrand and Cournot competition on R&D investment and social welfare in a duopoly with R&D competition where success in R&D is probabilistic. We show that R&D investments are higher under Bertrand (Cournot) competition when R&D productivities are sufficiently low (high), and this holds for both drastic and non-drastic R&D. We also show that Cournot competition can generate higher social welfare in absence of knowledge spillover and this happens if R&D is drastic, difference between the pre-innovation and the post-innovation costs is sufficiently large and the R&D productivities are moderate. So, our results differ significantly from both the deterministic R&D model and the patent race model.

JEL Classification: D43; L13; O33

Key Words: Bertrand; Cournot; Uncertain R&D; Welfare

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Competition, innovation and welfare

1. Introduction

What is the effect of competition on welfare? This debate goes back to Schumpeter (1943) and Arrow (1962). While both the papers focus on competitive market and monopoly, recent literature considers oligopolistic markets and examines the effects of different types of competition (i.e., Bertrand competition and Cournot competition) on profits, R&D investments and welfare.

There are two main lines of the recent research in the industrial organization literature addressing the effects of competition on welfare. One line of research assumes that firms have the same marginal costs under both types of competition and compares welfare under different types of product market competition (see, e.g., Singh and Vives, 1984, Vives, 1985, Cheng, 1985, Acharyya and Marjit, 1998, Häckner, 2000 and Mukherjee, 2003). So, these papers do 'static' welfare[1] comparison and this literature, except Mukherjee (2003), concludes that welfare is higher under Bertrand competition when the products are substitutes.[2]

The other line of research focuses on 'dynamic' welfare comparison where firms do R&D before production either to reduce the costs of production or to increase the degree of product differentiation (see, e.g., Delbono and Denicolò, 1990, Bester and Petrakis, 1993, Qiu, 1997, Bonanno and Haworth, 1998 and Symeonidis, 2003). These papers also look at the effect of competition on R&D investments. However, one common feature of these papers, except Delbono and Denicolò (1990), is to consider a deterministic R&D process

[1] One may refer to Delbono and Denicolò (1990) for the meaning of 'static' and 'dynamic' welfare. By 'static' welfare we mean welfare ex-post R&D and by 'dynamic' welfare we mean the expected welfare ex-ante R&D.

and they conclude that R&D investments are *always* higher under Cournot competition and welfare is higher under Bertrand competition if either knowledge spillover is weak and the products are sufficiently differentiated. In contrast, Delbono and Denicolò (1990) uses a patent race model to show that R&D investments are *always* higher under Bertrand competition and welfare can be higher under Cournot competition provided there is large number of firms in the industry.[3]

The purpose of the present paper is also to analyze the effects of Bertrand and Cournot competition in a 'dynamic' model of innovation and production. However, innovation in our model differs from both the deterministic R&D model and the patent race model. We consider R&D competition with probabilistic success in R&D but include the possibility of successful R&D by both firms.[4] So, unlike the deterministic R&D model, we consider probabilistic success in R&D, and, unlike the patent race model, we allow both firms to succeed in R&D. Therefore, our analysis is suitable for industries where the patent system allows firms to generate similar costs of production with different processes.

In a duopoly model with no knowledge spillover, we show that both R&D investments and social welfare can be higher under Bertrand competition or Cournot competition, and the answer depends on the R&D productivity and difference between the pre-innovation and the post-innovation costs of production. R&D investments are higher under Cournot (Bertrand) competition for higher (lower) R&D productivity and this is true

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[2] Mukherjee (2003) shows that when the firms engage in cooperative actions such as technology licensing, welfare may be higher under Cournot competition with homogeneous products, and the result depends on the cost differences between the firms.

[3] In a patent race model, Boone (2001) considers the effects of different types of product market competition on R&D incentives where the R&D firms and the producers are different. Boone (2000) and Lin and Saggi (2002) focus on the effects of competition on product and process R&D, but neither of them consider welfare implications of competition. Recently, López and Naylor (2004) and Mukherjee et al. (2004) examine the effects of different types of product market competition in a vertical structure with upstream and downstream agents.

[4] One may refer to Marjit (1991) and Choi (1993) for other works on R&D competition where success in R&D is uncertain.

for both ‘drastic’ and ‘non-drastic’ innovations.[5] Social welfare is higher under Cournot competition if R&D is ‘drastic’, the pre-innovation cost is sufficiently higher than the post-innovation cost and the R&D productivities are moderate; otherwise, welfare is higher under Bertrand competition.

These conclusions contrast with both the deterministic R&D model and the patent race model. Unlike the deterministic R&D model, R&D investments in this paper can be higher under Bertrand competition and welfare can be higher under Cournot competition without knowledge spillover. In contrast to the patent race model, we show that both R&D investment and welfare can be higher under Cournot competition in a duopoly model. Further, unlike both those models, social welfare in our analysis is higher under Cournot competition if the R&D productivities are moderate and difference between the pre-innovation and the post innovation costs is sufficiently large.

Our results also contrast with a related literature on endogenous growth showing the relationship between the intensity of competition and the incentive to innovate. For example, van de Klundert and Smulders (1997) and Peretto (1999) consider deterministic R&D model and show the positive association between the intensity of competition and growth. On the other hand, Aghion et al. (1997), Encaoua and Ulph (2000), Aghion et al. (2001) and Aghion et al. (2002) consider the patent race models with technology leaders and followers. Results of these papers for the comparable situations, i.e., when all firms in an industry are neck-and-neck *ex ante* in their analysis, show that R&D and growth increase with the intensity of competition and our conclusions are in contrast to them. However, our result on R&D investment is akin to the recent paper by d’Aspremont et al. (2002), which shows non-monotonic relationship between the intensity of competition and the incentive to innovate.

[5] In case of drastic innovation, if only one firm is successful in R&D and has lower cost of production, only this firm produces positive output in the product market and charges monopoly price for its product. But, in

We derive our result in a one-shot duopoly model, whereas they consider an overlapping generation model with endogenous number of firms. Further, while R&D investments and growth rate are positively related to their analysis, we show that higher R&D investments under Cournot competition may correspond to lower welfare under Cournot competition.

The remainder of the paper is organized as follows. The next section gives a general framework for our analysis under drastic innovation and, in section 3, we compare equilibrium R&D investments and corresponding welfare under Bertrand and Cournot competition. Section 4 discusses the implications of non-drastic innovations on our results. Section 5 concludes.

2. A general framework

Consider an economy with two risk-neutral firms, 1 and 2, producing homogeneous products. Assume that the firms have similar technologies[6] at the beginning and each of them faces constant average cost of production $\bar{c}$. Each firm can improve its technology through own R&D. The innovated technology corresponds to the constant average cost of production c.[7] Now, we consider that R&D is drastic in nature and we will consider the case of non-drastic innovation in section 4. However, the R&D process is uncertain and firm i, $i = 1,2$, can succeed with an unconditional probability p_i where probability of success depends on the ith firm's R&D investment x_i with $p_i'(x_i) > 0$, $p_i''(x_i) < 0$, $p_i'(0) = \infty$ and $p_i'(\infty) = 0$ for $i = 1,2$.[8] Since our purpose is to focus on the effects of product market competition, we assume that both firms face the same probability function so that the results

case non-drastic innovation, both firms always produce positive outputs even if only one firm is successful in RD and has lower cost of production.

[6] We define technology by the cost of production. Lower cost of production implies better technology.

[7] The new technologies could be different but creating the same cost of production.

are not influenced by the asymmetry in probability functions. Hence, we consider $p_i(x) = p_j(x) = p(x)$. For simplicity, we also assume that there are no fixed costs of R&D or production.

We consider a two-stage game. At stage 1, both firms simultaneously invest in R&D. At stage 2, the firms compete in the product market and take their decisions simultaneously.

Assume that the inverse market demand function is

$$P = 1 - q\,, \tag{1}$$

where the notations have usual meaning.

Let us define the optimal profit of the ith firm, $i = 1,2$, in the product market (i.e., revenue minus total cost of production) by $\pi(c)$, $\pi_i(c,c)$ and $\pi_i(\bar{c},\bar{c})$ respectively for the situations where only the ith firm is successful in R&D, where both firms are successful in R&D and where neither firm is successful in R&D. The arguments in the profit functions are showing the constant average cost of production of the firms. Since the successful firm is a monopolist under unilateral success in R&D, we omit the subscript in this situation.

In what follows, we will do a general analysis with the reduced form profit functions and then, in the next section, we will examine how Bertrand and Cournot competition affect the profits.

Net expected profit of the ith firm is

$$p(x_i)p(x_j)\pi_i(c,c) + p(x_i)(1-p(x_j))\pi(c) + (1-p(x_i))(1-p(x_j))\pi_i(\bar{c},\bar{c}) - x_i\,, \tag{2}$$

where, $i, j = 1,2$ and $i \neq j$. Maximizing (2), we get the optimal R&D investment of the ith firm for a given R&D investment of the jth firm. The profit maximizing R&D investment of the ith firm is

[8] The primes define respective derivatives with respect to R&D investment of the ith firm, $i = 1,2$.

$$p'(x_i)p(x_j)\pi_i(c,c) + p'(x_i)(1-p(x_j))\pi(c) - p'(x_i)(1-p(x_j))\pi_i(\bar{c},\bar{c}) = 1\,, \qquad (3)$$

or, $$p'(x_i)[\pi(c) - \pi_i(\bar{c},\bar{c})] - p'(x_i)p(x_j)[\pi(c) - \pi_i(c,c) - \pi_i(\bar{c},\bar{c})] = 1\,. \qquad (3')$$

Second order condition for maximization is satisfied. Due to symmetry of the probability functions, we have similar reaction functions for both firms. We can find the optimal R&D investments of the firms by solving the reaction functions. We assume that the probability functions are such that there is a unique equilibrium for R&D investments. Define the optimal R&D investments by x_1^* and x_2^* for firms 1 and 2 respectively. Further, symmetry of the firms implies that the firms have the same equilibrium R&D investments.

Equation (3') shows that the optimal R&D investment of the ith firm, $i = 1,2$, depends on the differences $[\pi(c) - \pi_i(\bar{c},\bar{c})]$ and $[\pi(c) - \pi_i(c,c) - \pi_i(\bar{c},\bar{c})]$. It is easy to see that both $[\pi(c) - \pi_i(\bar{c},\bar{c})]$ and $[\pi(c) - \pi_i(c,c) - \pi_i(\bar{c},\bar{c})]$ are always positive.

Lemma 1: *Let* x_i^{**} *is the optimal R&D investment of the* i*th firm for a given R&D investment of the* j*th firm,* $i, j = 1,2$ *and* $j \neq i$.

(i) We find that $\dfrac{\partial x_i^{**}}{\partial z} > 0$, *where,* $z = [\pi(c) - \pi_i(\bar{c},\bar{c})]$.

(ii) We find that $\dfrac{\partial x_i^{**}}{\partial y} < 0$, *where* $y = [\pi(c) - \pi_i(c,c) - \pi_i(\bar{c},\bar{c})]$.

Proof: (i) Totally differentiating (3) for x_i^{**} and z, and rearranging, we find that $\dfrac{\partial x_i^{**}}{\partial z} = -\dfrac{p'(x_i^{**})}{(zp''(x_i^{**}) - yp''(x_i^{**})p(x_j^{**}))} > 0$, where $i \neq j$. This is because the denominator $(zp''(x_i^{**}) - yp''(x_i^{**})p(x_j^{**}))$ is negative due to the second order condition of profit maximization with respect to the R&D investment.

(ii) Totally differentiating (3) for x_i^{**} and y, and rearranging, we find that $\frac{\partial x_i^{**}}{\partial y} = \frac{p'(x_i^{**})p(x_j^{**})}{(zp''(x_i^{**}) - yp''(x_i^{**})p(x_j^{**}))} < 0$. Q.E.D.

The expressions $[\pi(c) - \pi_i(\bar{c}, \bar{c})]$ and $[\pi(c) - \pi_i(c,c) - \pi_i(\bar{c}, \bar{c})]$ capture the incentives to innovate. Let us first consider the situation where only the i th firm does R&D. This implies $p(x_j) = 0$. So, the optimal R&D investment of firm 1 satisfies

$$p'(x_i)[\pi(c) - \pi_i(\bar{c}, \bar{c})] = 1. \tag{4}$$

Suppose, x_i^* is the optimal R&D investment that satisfies condition (4). Then, it follows from (4) that x_i^* reduces as z reduces. Hence, the stand-alone incentive for investing in R&D, i.e., the incentive for investing in R&D by a firm when the competitor does not invest in R&D, reduces as the gain from R&D reduces.

However, since firm j also does R&D, there is a possibility that firm j also succeeds in R&D. The expression $[\pi(c) - \pi_i(c,c) - \pi_i(\bar{c}, \bar{c})]$ shows the benefit of unilateral success in innovation over no innovation and bilateral success in innovation. Hence, it captures the effect of rivalry and shows the strategic incentive for innovation. The second part of the above lemma shows that as the strategic incentive for R&D reduces, it increases the incentive for R&D investment, i.e., given the positive R&D investment of the j th firm, the R&D investment of the i th firm increases due to the reduction in y only.

3. Comparison between Bertrand and Cournot competitions

The previous section has shown the effects of stand-alone and strategic incentives on R&D investments. Now, we see how Bertrand and Cournot competition affect these incentives and the overall R&D investments of the firms.

Let us first compare outputs and profits of the firms ex-post R&D, which will be helpful for our following analysis.

Lemma 2: *Assume that either both firms succeed in R&D or neither firm succeeds in R&D.*

(i) Optimal output of each firm is higher under Bertrand competition compared to Cournot competition, i.e., $q_i^b(c,c) > q_i^c(c,c)$ *and* $q_i^b(\bar{c},\bar{c}) > q_i^c(\bar{c},\bar{c})$, $i = 1,2$.

(ii) Optimal profit of each firm is higher under Cournot competition compared to Bertrand competition, i.e., $\pi_i^c(c,c) > \pi_i^b(c,c)$ *and* $\pi_i^c(\bar{c},\bar{c}) > \pi_i^b(\bar{c},\bar{c})$, $i = 1,2$.

Proof: Given the demand and cost specifications, optimal output and profit of the i th firm, $i = 1,2$, under Cournot and Bertrand competition are respectively

$$q_i^c = \frac{(1-k)}{3} \quad \text{and} \quad \pi_i^c = \frac{(1-k)^2}{9} \tag{5}$$

and

$$q_i^b = \frac{(1-k)}{2} \quad \text{and} \quad \pi_i^b = 0, \tag{6}$$

where, $k = c$ when both firms succeed in R&D and $k = \bar{c}$ when neither firm succeeds in R&D. It is easy to check from (5) and (6) that $q_i^c < q_i^b$ and $\pi_i^c > \pi_i^b$. Q.E.D.

It follows from Lemma 2(ii) that both $[\pi(c) - \pi_i(\bar{c},\bar{c})]$ and $[\pi(c) - \pi_i(c,c) - \pi_i(\bar{c},\bar{c})]$ are greater under Bertrand competition compared to Cournot competition. Therefore, Lemma 1 implies that both firms' R&D investments are higher under Bertrand competition due to the stand-alone incentive, but their R&D investments are lower under Bertrand competition due to the strategic effect. So, total effect on the equilibrium R&D investment is ambiguous and as the following proposition shows it depends on the probability of success in R&D.

Proposition 1: *(i) If the equilibrium probability of success under Bertrand and Cournot competition is greater than, equal to or less than* $\bar{p}$*, the equilibrium R&D investments are greater than, equal to or less than under Cournot competition compared to Bertrand competition.*

(ii) The equilibrium probability of success in R&D under both Bertrand and Cournot competition is either greater than, less than or equal to a critical value, say $\bar{p}$.

Proof: (i) Suppose, x_1^{b*} and x_2^{b*} are the equilibrium R&D investments of the firms under Bertrand competition. So, x_1^{b*} and x_2^{b*} satisfy condition (3) under Bertrand competition. Straightforward calculations show that, given the R&D investment x_2^{b*}, the left hand side (LHS) of (3) for firm 1 under Cournot competition is greater than 1 provided

$$\pi_1^c(\bar{c},\bar{c}) < p(x_2^{b*})[\pi_1^c(\bar{c},\bar{c}) + \pi_1^c(c,c)] \tag{7}$$

or
$$p(x_2^{b*}) > \frac{\pi_1^c(\bar{c},\bar{c})}{[\pi_1^c(\bar{c},\bar{c}) + \pi_1^c(c,c)]} = \bar{p}, \tag{7'}$$

where $0 < \bar{p} < 1$.[9]

If $p(x_2^{b*})$ tends to 1, condition (7') is satisfied. But, if $p(x_2^{b*})$ tends to 0, condition (7') does not hold. Further, LHS of (7') is continuous and increasing in $p(x_2^{b*})$ over $[0,1]$.

[9] Note that $\pi_1^b(c,c) = \pi_1^b(\bar{c},\bar{c}) = 0$.

Therefore, if $p(x_2^{b*})$ is greater than the critical value $\overline{p}$, then, given the R&D investment x_2^{b*}, the optimal R&D investment of firm 1 under Cournot competition is higher than x_1^{b*}. Since the firms are symmetric, we have a similar condition for firm 2 also. Further, symmetry of the firms implies that the firms invest the same amount in R&D. So, if $p(x_2^{b*}) = p(x_1^{b*}) > \overline{p}$, the optimal R&D investments of the firms under Cournot competition, say x_1^{c*} and x_2^{c*}, are higher to that of under Bertrand competition.

But, if condition (7') does not hold, i.e., if $p(x_2^{b*}) = p(x_1^{b*}) < \overline{p}$, the optimal R&D investments of the firms are lower under Cournot competition compared to Bertrand competition.

(ii) If $p(x_2^{b*}) = p(x_1^{b*}) > \overline{p}$, the equilibrium R&D investments are higher under Cournot competition and hence, the probabilities of success are higher under Cournot competition. If $p(x_2^{b*}) = p(x_1^{b*}) < \overline{p}$, the equilibrium R&D investments and the equilibrium probabilities of success are lower under Cournot competition compared to Bertrand competition. Equilibrium R&D investment and probability of success are the same under Bertrand and Cournot competition when $p(x_2^{b*}) = p(x_1^{b*}) = \overline{p}$. Q.E.D.

The reason for the above finding is as follows. Suppose the equilibrium probabilities of success are sufficiently low. Given the low probability of success of the competitor, it increases a firm's chance of becoming the single innovator. Since the gain from becoming the single innovator is lower under Cournot competition, it induces lower R&D investments under Cournot competition compared to Bertrand competition.

On the other hand, if the equilibrium probabilities of success are sufficiently high then, given the high probability of success of the competitor, it reduces a firm's chance of becoming the single innovator. Hence, in this situation, a duopoly market structure is the more likely outcome. Since, duopoly profit is higher under Cournot competition compared to Bertrand competition, here R&D investments are more under Cournot competition.

We find that whether the firms invest more in R&D under a more competitive environment or a less competitive environment depends on the equilibrium probability of success in R&D. This contradicts the previous results of this literature where the equilibrium R&D investments are always higher either under Bertrand competition or under Cournot competition (see, e.g., Delbobo and Denicolò, 1990 and Qiu, 1997).

3.1 An example

Now, we consider a specific probability function to provide an example for Proposition 1. We consider that each firm faces the same probability function $p(x_i) = \mu x_i^{\frac{1}{2}}$, $i = 1,2$.

The critical value of the probability of success in R&D is

$$\bar{p} = \frac{\pi_i^c(\bar{c},\bar{c})}{\pi_i^c(\bar{c},\bar{c}) + \pi_i^c(c,c)}. \tag{8}$$

Condition (3) and symmetry imply that the ith firm's, $i = 1,2$, optimal R&D investment and the corresponding probability of success under Bertrand and Cournot competition are respectively

$$x^b = \left[\frac{\mu\pi(c)}{2 + \mu^2\pi(c)}\right]^2 \quad \text{and} \quad x^c = \left[\frac{\mu[\pi(c) - \pi_i^c(\bar{c},\bar{c})]}{2 + \mu^2[\pi(c) - \pi_i^c(\bar{c},\bar{c}) - \pi_i^c(c,c)]}\right]^2 \tag{9}$$

$$p^b = \frac{\mu^2\pi(c)}{2 + \mu^2\pi(c)} \quad \text{and} \quad p^c = \frac{\mu^2[\pi(c) - \pi_i^c(\bar{c},\bar{c})]}{2 + \mu^2[\pi(c) - \pi_i^c(\bar{c},\bar{c}) - \pi_i(c,c)]}. \tag{10}$$

It follows from (8) and (10) that both p^b and p^c are less (greater) than $\overline{p}$ provided $\mu^2\pi(c)\pi_i^c(c,c) < (>) 2\pi_i^c(\overline{c},\overline{c})$. Therefore, optimal R&D investments are higher (lower) under Bertrand competition compared to Cournot competition provided $\mu^2\pi(c)\pi_i^c(c,c) < (>) 2\pi_i^c(\overline{c},\overline{c})$. The direct comparison of the expressions in (9) also provides the same conclusion.

So, if the R&D productivity is sufficiently low (high), i.e., μ is sufficiently low (high), the optimal R&D investments are higher (lower) under Bertrand competition compared to Cournot competition.

3.2. Welfare comparison

It follows from Lemma 2(i) that industry output is lower under Cournot competition compared to Bertrand competition. Therefore, ex-post R&D, deadweight loss is higher under the former and 'static' welfare is higher under Bertrand competition. However, it follows from subsection 3.1 and Proposition 1 that if the R&D productivity is sufficiently high, R&D investments and probabilities of success in R&D are higher under Cournot competition. Hence, there might be a conflict between the 'static' and the 'dynamic' welfare, where welfare is the summation of consumer surplus and industry profit net of R&D investments.

Welfare comparison under Bertrand and Cournot competition is cumbersome in our general framework. Now, we will consider the specific probability function of subsection 3.1 for welfare comparison and will show the effects of R&D productivity, μ, on our results. Further, for analytical convenience we assume that $c = 0$. However, this simplification does not affect our qualitative results at all.

Proposition 2: *Assume that both firms have the same probability function* $p(x_i) = \mu x_i^{\frac{1}{2}}$, *where* $i = 1,2$, $c = 0$, $\bar{c} \in [.5,1]$ *and* $\mu \in [0, 3\sqrt{2}]$.

(a) If $\bar{c}$ *(i.e., the pre-innovation cost) is not very high, expected welfare is higher under Bertrand competition for all R&D productivities.*

(b) If $\bar{c}$ *is sufficiently high, expected welfare is higher under Bertrand competition for very low and very high R&D productivities but it is higher under Cournot competition for intermediate R&D productivities.*

Proof: It follows from (10) that, if $c = 0$, the equilibrium probability of success in R&D is always less than 1 under Bertrand competition but it is equal to 1 under Cournot competition for $\mu \geq 3\sqrt{2}$. Further, if $c = 0$, R&D is drastic (which is the assumption of this section) under Bertrand and Cournot competition provided $\bar{c} \in [.5,1]$.

Given the assumptions of this proposition and the demand function mentioned in (1), expected welfare under Cournot and Bertrand competition are respectively

$$W^c = \frac{\begin{array}{c}[16\mu^4(9-4(1-\bar{c})^2)^2 + 27\mu^2(9-4(1-\bar{c})^2)(72-4\mu^2) \\ +16(72-4\mu^2)^2(1-\bar{c})^2 - 72\mu^2(9-4(1-\bar{c})^2)^2]\end{array}}{36[72+\mu^2(5-4(1-\bar{c})^2)]^2} \tag{11}$$

and

$$W^b = \frac{[\mu^4 + 12\mu^2 + 64(1-\bar{c})^2 - 4\mu^2]}{2[8+\mu^2]^2}. \tag{12}$$

We plot the expression $(W^c - W^b)$ in Figure 1.[10]

Figure 1

Inspection of Figure 1 proves the result.

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[10] We use 'The Mathematica 4' (see Wolfram, 1999) for the figures of this paper.

For better understanding of Figure 1, we also look at $(W^c - W^b)$ for $\bar{c} = .6$ (i.e., relatively low pre-innovation cost) and $\bar{c} = .9$ (i.e., relatively high pre-innovation cost) respectively, in Figures 2 and 3, with all other assumptions of this proposition.

Figures 2 and 3

Q.E.D.

Ex-post R&D, the industry output is higher under Bertrand competition and creates higher 'static' welfare under Bertrand competition. If R&D productivity is sufficiently low, the R&D investments are higher under Bertrand competition. So, the probabilities of success in R&D are higher under Bertrand competition, which has a positive impact on welfare. However, higher R&D investment under Bertrand competition also has a negative impact on welfare under Bertrand competition since it acts as a cost. Because the R&D productivity is very low, the effects of R&D investments are negligible and expected welfare is higher under Bertrand competition. On the other extreme, if the R&D productivity is very high, both the R&D investments and the probabilities of success in R&D are higher under Cournot competition. High R&D productivity creates large difference in R&D investments between Cournot and Bertrand competition and imposes large negative impact on welfare. In this situation, the negative effects of deadweight loss and the cost of higher R&D investments under Cournot competition outweigh the benefit of higher probabilities of success under Cournot competition and therefore, expected welfare is higher under Bertrand competition.

However, if the R&D productivity is not very high to create sufficiently large difference in R&D investments between Cournot and Bertrand competition, but it is high enough to create higher R&D investment under Cournot competition, then whether expected welfare is higher under Cournot competition depends on the pre-innovation cost. It is clear from (9) that R&D investment under Bertrand competition does not depend on the pre-innovation cost but R&D investment under Cournot competition increases with the pre-innovation cost. Therefore, the benefit from higher R&D investment under Cournot competition increases with the pre-innovation cost. If the pre-innovation cost is sufficiently low (not sufficiently low), the probability of success in R&D is not sufficiently high (sufficiently high) under Cournot competition to outweigh the negative effects of higher deadweight loss and the cost of higher R&D investments under Cournot competition. So, in this situation, expected welfare is higher under Cournot (Bertrand) competition for sufficiently high (low) pre-innovation costs.

The above proposition shows the possibility of higher welfare under Cournot competition in a duopoly market structure and without knowledge spillover. Hence, this result is in contrast to the results of the deterministic R&D models (e.g., Qiu, 1997), where welfare is always higher under Bertrand competition without knowledge spillover, and also to the patent race model of Delbono and Denicolò (1990), where welfare is higher under Cournot competition provided the number of firms in the industry is sufficiently large. Moreover, unlike those models, we find that welfare under Cournot competition is higher for moderate R&D productivities.

So far we have considered the situation where probability of success in R&D is less than 1 under both types of market competition except for $\mu = 3\sqrt{2}$. Now, we briefly discuss the situation for $\mu > 3\sqrt{2}$.

The probability of success in R&D is 1 under Cournot competition for $\mu = 3\sqrt{2}$. So, the optimal R&D investment of the i th firm, $i = 1,2$, under Cournot competition is $x_i^c = \frac{1}{\mu^2}$ when $\mu > 3\sqrt{2}$. In this situation, expected welfare under Cournot competition is

$$W^c(\mu > 3\sqrt{2}) = \frac{4}{9} - \frac{2}{\mu^2}. \tag{13}$$

So, as μ increases from $3\sqrt{2}$, expected welfare under Cournot competition increases.

Since probability of success under Bertrand competition is always less than 1, expected welfare under Bertrand competition is given by (12). Further, we find from (9) that the relationship between R&D investment and R&D productivity under Bertrand competition is concave with a maximum at $\mu = 2\sqrt{2}$. So, as μ increases from $3\sqrt{2}$, R&D investment under Bertrand competition reduces, though probability of success in R&D increases.[11] Both these effects increase expected welfare under Bertrand competition.

We subtract (12) from (13) and plot the difference in Figure 4 for $\mu \in [3\sqrt{2},100]$ (i.e., for sufficiently high R&D productivities) and find that the difference (13) – (12) is negative for all the values of R&D productivities.

Figure 4

So, for R&D productivities greater than $3\sqrt{2}$, the positive effects of lower deadweight loss and the cost of lower R&D investments under Bertrand competition dominate the positive effect higher probabilities of success under Cournot competition. Hence, the following proposition is immediate.

[11] Remember that the probability of success is the product of R&D investment and R&D productivity.

Proposition 3: *Assume that both firms have the same probability function* $p(x_i) = \mu x_i^{\frac{1}{2}}$*, where* $i = 1,2$*,* $c = 0$ *and* $\bar{c} \in [.5,1]$*. If R&D productivity is sufficiently large (i.e.,* $\mu > 3\sqrt{2}$*), expected welfare is higher under Bertrand competition.*

4. Non-drastic R&D

Now, we see how non-drastic R&D influences the results of the previous section. In case non-drastic R&D, both firms produce in the product market even under unilateral success in R&D. So, in this situation, the i th firm maximizes the following expression:

$$\begin{aligned} &p(x_i)p(x_j)\pi_i(c,c) + p(x_i)(1-p(x_j))\pi_i(c,\bar{c}) + (1-p(x_i))p(x_j)\pi_i(\bar{c},c) + \\ &(1-p(x_i))(1-p(x_j))\pi_i(\bar{c},\bar{c}) - x_i. \end{aligned} \tag{14}$$

Optimal R&D investment of the i th firm is

$$\begin{aligned} &p'(x_i)p(x_j)\pi_i(c,c) + p'(x_i)(1-p(x_j))\pi_i(c,\bar{c}) - p'(x_i)p(x_j)\pi_i(\bar{c},c) \\ &- p'(x_i)(1-p(x_j))\pi_i(\bar{c},\bar{c}) = 1. \end{aligned} \tag{15}$$

Following the analysis of section 3, we find that the R&D investments are higher under Cournot competition provided

$$p(x_i^{b*}) = p(x_j^{b*}) > \frac{[\pi_i^c(\bar{c},\bar{c}) + \pi_i^b(c,\bar{c}) - \pi_i^c(c,\bar{c})]}{[\pi_i^b(c,\bar{c}) - \pi_i^c(c,\bar{c}) + \pi_i^c(\bar{c},\bar{c}) + \pi_i^c(c,c) - \pi_i^c(\bar{c},c)]} = \bar{\bar{p}},\text{[12]} \tag{16}$$

where $i, j = 1,2$ and $i \neq j$. It is easy to check that $0 < \bar{\bar{p}} < 1$.

Proposition 4: *Even if R&D is non-drastic, we have the result similar to Proposition 1.*

Since the proof of the above proposition is similar to Proposition 1, we are omitting it here.

Now, we compare welfare under Bertrand and Cournot competition. Like the previous section, we assume that both firms have the same probability function $p(x_i) = \mu x_i^{\frac{1}{2}}$, where $i = 1,2$, and $c = 0$. Hence, the assumption of non-drastic R&D implies $\bar{c} \in (0,.5)$.

Given the demand function and $c = 0$, equilibrium probability of success under Cournot and Bertrand competition are respectively

$$p(x_1^c) = p(x_2^c) = \frac{4\mu^2 \bar{c}}{(18 + 4\mu^2 \bar{c}^2)} \tag{17}$$

and

$$p(x_1^b) = p(x_2^b) = \frac{\mu^2 \bar{c}(1-\bar{c})}{(2 + \mu^2 \bar{c}(1-\bar{c}))}. \tag{18}$$

Though the equilibrium probabilities of success under Bertrand competition is less than 1, the equilibrium probabilities of success under Cournot competition is less than 1 provided $\mu < \frac{3}{\sqrt{2\bar{c}(1-\bar{c})}}$.

We find that expected welfare under Cournot competition is

$$W^c = \frac{\begin{array}{c}[64\mu^4 \bar{c}^2 + 4\mu^2 \bar{c}(18 - 4\mu^2 \bar{c}(1-\bar{c}))(8 - 8\bar{c} + 11\bar{c}^2) \\ + 4(18 - 4\mu^2 \bar{c}(1-\bar{c}))^2 (1-\bar{c})^2 - 288\mu^2 \bar{c}^2]\end{array}}{9[18 + 4\mu^2 \bar{c}^2]^2}, \text{ for } \mu < \frac{3}{\sqrt{2\bar{c}(1-\bar{c})}} \tag{19}$$

$$W^c = \frac{4}{9} - \frac{2}{\mu^2}, \qquad \text{for } \mu > \frac{3}{\sqrt{2\bar{c}(1-\bar{c})}}, \tag{20}$$

while expected welfare under Bertrand competition is

$$W^b = \frac{[\mu^4 \bar{c}^2 (1-\bar{c})^2 + 4\mu^2 \bar{c}(1-\bar{c})^2 (1+\bar{c}) + 4(1-\bar{c})^2 - 4\mu^2 \bar{c}^2 (1-\bar{c})^2]}{2[2 + \mu^2 \bar{c}(1-\bar{c})]^2}. \tag{21}$$

[12] Note that $\pi_i^b(c,c) = \pi_i^b(\bar{c},\bar{c}) = \pi_i^b(\bar{c},c) = 0$.

Let us first consider the situation where R&D productivities are such that equilibrium probabilities of success in R&D are less than 1 for both competition and for all $\bar{c} \in [0,.5)$. Since $\frac{3}{\sqrt{2\bar{c}(1-\bar{c})}}$ reduces with $\bar{c}$, the equilibrium probabilities of success under Cournot competition is always less than 1 for $\mu < 3\sqrt{2}$. Hence, consider $\mu \in (0,3\sqrt{2}]$. Here, the relevant expression are (19) and (21). We subtract (21) from (19) and plot the difference in Figure 5.

Figure 5

Figure 5 shows that welfare is always higher under Bertrand competition.

Now, we consider the opposite situation where the R&D productivities are very high such that equilibrium probabilities are always 1 under Cournot competition for all $\bar{c} \in [0,.5)$, i.e., $\mu > \frac{3}{\sqrt{2\bar{c}(1-\bar{c})}}$ for all $\bar{c} \in [0,.5)$. It is clear that we have to exclude $\bar{c} = 0$ for this comparison and we do our analysis for $\bar{c} \in [.00001,.5)$ (i.e., starting from a very low value of $\bar{c}$ but excluding 0). Therefore, the relevant R&D productivities are $\mu > \frac{3}{\sqrt{.00447211}}$. Now, (20) and (21) are the relevant expressions for this case. We subtract (21) from (20) and plot the difference in Figure 6 for $\bar{c} \in [.00001,.5)$ and $\mu \in [\frac{3}{\sqrt{.00447211}},10000]$.

Figure 6

Figure 6 shows that welfare is always higher under Bertrand competition.

The remaining case is to consider the R&D productivities $\mu \in [3\sqrt{2},\frac{3}{\sqrt{.00447211}}]$. In this situation, we have to consider two situations, viz., $\mu \in [3\sqrt{2},\frac{3}{\sqrt{2\bar{c}(1-\bar{c})}}]$ and

$\mu \in [\frac{3}{\sqrt{2\bar{c}(1-\bar{c})}}, \frac{3}{\sqrt{.00447211}}]$, separately for each $\bar{c}$. Following the above procedure, we find that welfare under Bertrand competition is higher compared to Cournot competition even for these R&D productivities.[13]

Hence, the above discussion gives us the following proposition.

Proposition 5: *If R&D is non-drastic, welfare is always higher under Bertrand competition compared to Cournot competition.*

In case of non-drastic R&D, difference between the pre-innovation and the post-innovation costs is not very large. So, even if R&D investment may be higher under Cournot competition, it does not increase the probabilities of success in R&D significantly to outweigh the negative effects of higher deadweight loss and the higher cost of R&D under Cournot competition. Therefore, welfare is always higher under Bertrand competition for non-drastic R&D.

5. Conclusion

Whether R&D investments and welfare are higher under a more competitive environment is an existing debate. While the initial works have looked at monopoly and competitive markets, recent contributions have focused on oligopolistic markets and consider Bertrand and Cournot competition. We re-examine this issue in a duopoly market with R&D competition where success in R&D is probabilistic and there is no knowledge spillover. We show that our results differ significantly from the existing literature.

[13] To avoid repetition, we are not showing the figures for these intermediate R&D productivities.

We show that both R&D investments and social welfare can be higher either under Bertrand competition or under Cournot competition. So, there is non-monotonic relationship between competition and, R&D investments and welfare. Our results differ significantly from both the deterministic R&D model and the patent race model, and depend on the R&D productivities and difference between the pre-innovation and the post-innovations costs.

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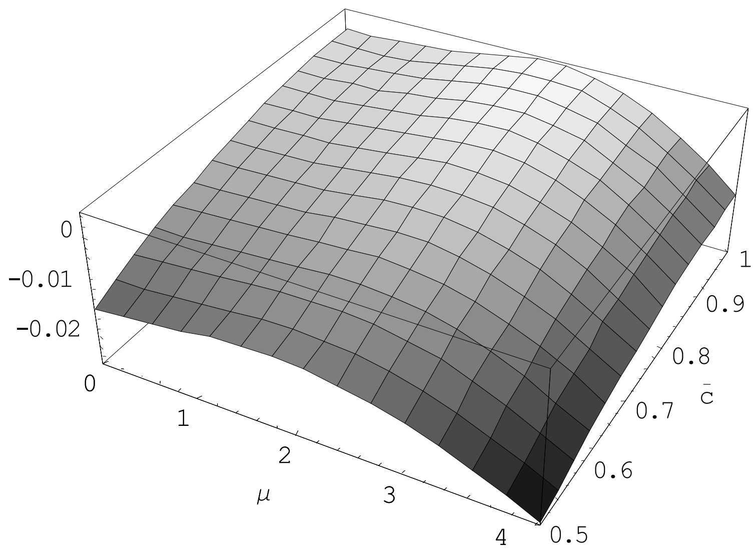

Figure 1: Subtracting (12) from (11) for $\bar{c} \in [.5,1]$ and $\mu \in [0,3\sqrt{2}]$.

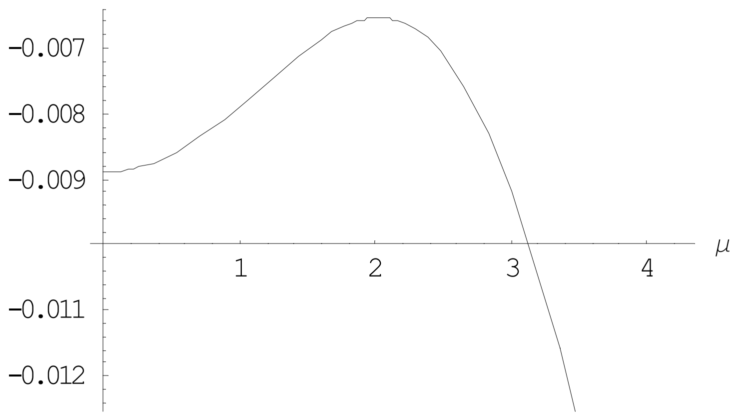

Figure 2: Subtracting (12) from (11) for $\bar{c} = .6$ and $\mu \in [0,3\sqrt{2}]$.

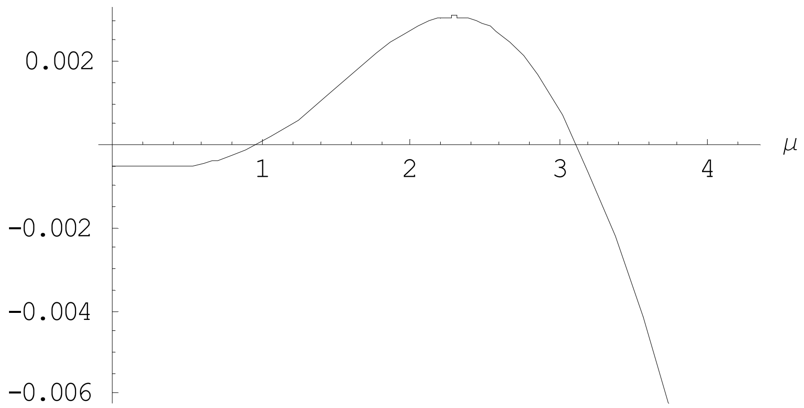

Figure 3: Subtracting (12) from (11) for $\bar{c} = .9$ and $\mu \in [0, 3\sqrt{2}]$.

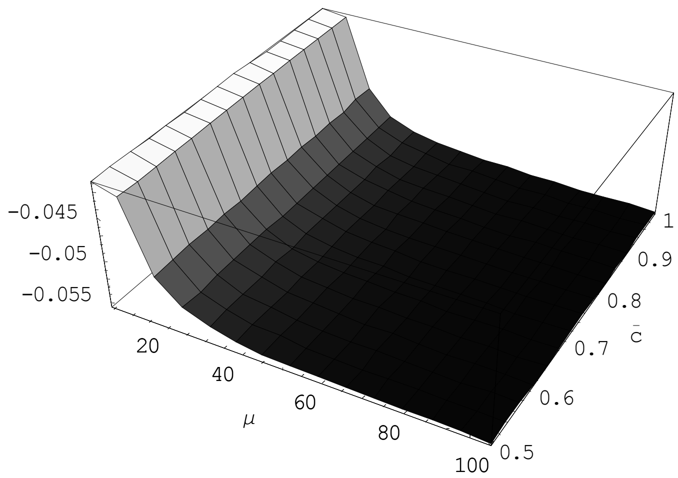

Figure 4: Subtracting (12) from (13) for $\mu \in [3\sqrt{2}, 100]$ and $\bar{c} \in [.5, 1]$.

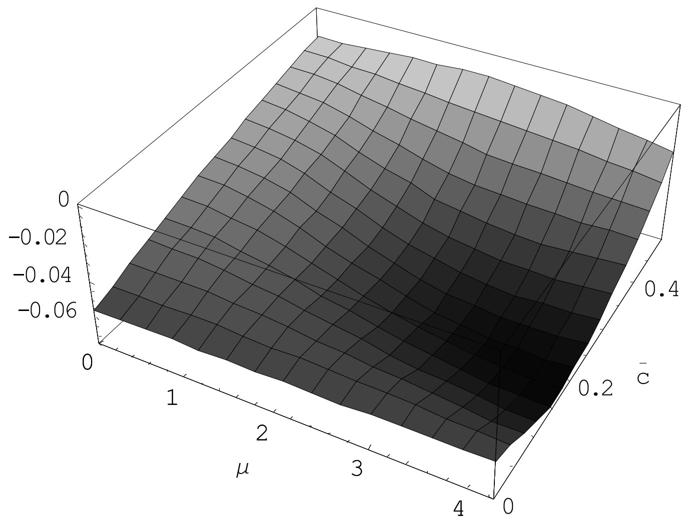

Figure 5: Subtracting (21) from (19) for $\bar{c} \in [0,.5]$ and $\mu \in [0,3\sqrt{2}]$.

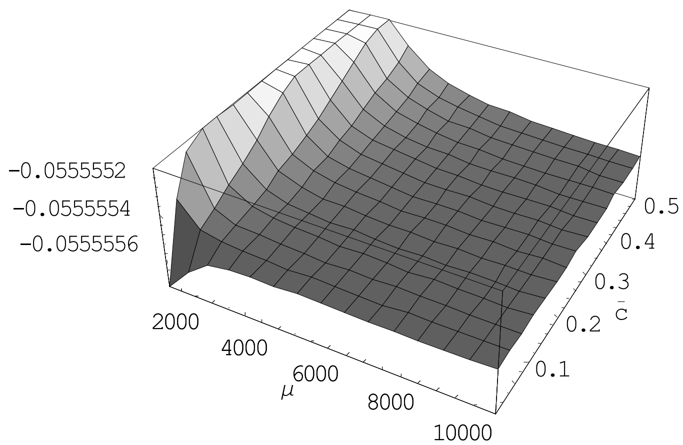

Figure 6: Subtracting (21) from (20) for $\bar{c} \in [.00001,.5)$ and $\mu \in [\frac{3}{\sqrt{.00447211}},10000]$.