UNIVERSITY OF NOTTINGHAM

Discussion Papers in Economics

Discussion Paper
No. 05/12

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December 2005

DP 05/12
ISSN 1360-2438

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Abstract

We develop a microeconomic banking framework to analyze how liquidity management decisions influence the availability of liquidity to depositors, the viability of the asset transformation process and the solvency of banking firms. In particular, we focus on how the optimal allocation of deposits between reserves and a risky investment portfolio by a profit maximizing bank affects the likelihood of each type of bank failure in an environment where there is uncertainty on both sides of the balance sheets. Our main aim in this paper, is to examine the risk-management trade-offs between those risks arising because of the uncertainty in the behaviour of depositors and those arising on the asset side of their balance sheet associated with the risk attached to their portfolio investment. By distinguishing between illiquidity and insolvency as causes of bank failures we hope to understand how the interaction between these two types of risk affects the bank's asset transformation process.

JEL: G21
Key words: banks, liquidity, solvency

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1. Introduction

We develop a microeconomic banking framework to analyze how liquidity management decisions influence the availability of liquidity to depositors, the viability of the asset transformation process and the solvency of banking firms. In particular, we focus on how the optimal allocation of deposits between reserves and a risky investment portfolio by a profit maximizing bank affects the likelihood of each type of bank failure in an environment where there is uncertainty on both sides of the balance sheets. Our main aim in this paper, is to examine the risk-management trade-offs between those risks arising because of the uncertainty in the behaviour of depositors and those arising on the asset side of their balance sheet associated with the risk attached to their portfolio investment. By distinguishing between illiquidity and insolvency as causes of bank failures we hope to understand how the interaction between these two types of risk affects the bank's asset transformation process. In order to address these kinds of issues, we need a framework that explicitly includes both types of uncertainty something that, we argue, is missing from the banking literature.

The theoretical literature on banking has developed along two distinct paths. Early work attempts were based on the Monti-Klein deterministic model[1] where the decision-maker is a profit-maximizing bank. Uncertainty in that framework has been considered by Dermine (1986) who introduces risky long-term portfolio investment, and by Prisman, Slovin and Sushka (1986) who allow for random deposit withdrawals. However, the majority of the literature has evolved around the work of Diamond and Dybvig (1983) that was a development of ideas first expressed by Bryant (1980). In this literature, that attempts to explain the liquidity provision role of banks, the decision-maker is the depositor. Uncertainty only arises in the Diamond and Dybvig (1983) model because each depositor faces an idiosyncratic liquidity risk, however, there is no aggregate uncertainty because the liquidity needs of the economy as a whole are public knowledge. Chari and Jagannathan (1988) and Jacklin and Bhattacharya (1988) have also restricted their attention to the case where there is no aggregate liquidity uncertainty but have allowed for uncertainty on the asset side

[1] See Klein (1971) and Monti (1972).

while Bhattacharya and Gale (1987) have considered the case of aggregate uncertainty but the bank's portfolio return in their model is deterministic.[2]

We choose to follow the first branch of the banking literature because we want to focus on the behaviour of banks, in particular their liquidity management decisions. Our framework combines elements from Dermine (1986) and Prisman, Slovin and Sushka (1986) so that both types of uncertainty can be incorporated in a single framework. Our approach reflects our thesis that both sides of the balance sheet are equally relevant to understand the institutional risk management process. Our emphasis on risk management follows the suggestions of Stiglitz and Greenwald (2003) who emphasize that in order to understand the functions of a modern banking system it is important that we treat banks as risk-averse agents. This can be accomplished in two distinct ways; the first is a direct one whereby we endow banks with a concave objective function while the second one is an indirect way that induces a risk-averse behaviour on banks by introducing bankruptcy costs. We have chosen the second option.

To keep things simple the only choice that the bank in our model needs to make is how to allocate its deposits between reserves and portfolio investment. In our model both a shortage and a surplus of reserves are costly. In the former case the bank needs to liquidate its portfolio investment but such liquidation is costly. In the latter case the bank needs to hold excess reserves that offer a return that is lower than the exogenous return that the bank offers on its liabilities. We believe that these conditions capture part of the actual environment within which banks operate. Banks are also limited in their choice of portfolio investments. We impose an upper-bound on the return of a bank's portfolio investment reflecting the fact that the portfolio mainly consists of loans that are fixed-return assets.

Using our framework we study the inter-temporal relationships among risk-management practices, banking fragility, solvency and profitability. We identify three causes of bank failure. Two of these causes, namely illiquidity (when the demand for liquidity cannot be satisfied even when the whole portfolio return is liquidated) and insolvency (when the value of assets falls short of the value of liabilities) have been the main focus of the existing literature. This is because they can be independently

[2] This is only a small sample of this fast expanding literature. See Bhattacharya and Thakor (1994) for a relatively early review.

analyzed by focusing on only one type of uncertainty. Lack of liquidity is a failure of the liabilities side while insolvency reflects a bad performance of banking assets. However, we also identify a third possible cause where despite having sufficient liquidity the bank fails because even under the best asset performance scenario it is clear that the bank will not be able to meet its long-term obligations to depositors. To understand this type of failure we need to jointly consider the two types of uncertainty.

2. The Banking Model

In this section, we describe our banking framework paying special attention to its temporal character that will allow us to investigate the linkage between risk management and financial fragility. More specifically, we develop the framework keeping in mind that liquidity and risk management practices are motivated by the existence of both liquidity and solvency risks.

Consider a three period framework (T=0,1,2). At T=0, the bank allocates its deposits D between liquid reserves, R (short-term securities and money), and illiquid portfolio investments, L (loans and securities,), to maximise long-term expected profits.[3] In making these decisions the bank takes into account elements of uncertainty on both sides of its balance sheet. A liquidity shock will affect the demand for withdrawals by depositors at T=1, while the return on the bank's portfolio investments remains unknown until T=2. On the liabilities side, we assume that the proportion, x, of deposits withdrawn at T=1 is a random variable with support on [0,1] and known density function $f(x)$. Those depositors that withdraw at T=1 receive the amount that they have invested, xD, while those depositors that wait until T=2 to withdraw their funds they receive $(1-x)rD$, where $r>1$.[4] On the assets side, we assume that the bank's investment portfolio offers a gross return, y, that is a continuous random variable with support on $[0,Y]$, known density function $g(y)$ and $E(y)>1$; where E denotes the expectations operator. We have set an upper bound on the support of the distribution

[3]To keep our model tractable we will concentrate on the simple case where the bank's only choice is its level of reserves. Both the level of deposits and their interest rate are treated as exogenous parameters. In the absence of deposit insurance, moral hazard is not an important issue and therefore in order to keep the model simple we ignore bank equity. For an introduction to these issues, see Mishkin (2000).

[4] Following the traditional financial crisis literature we assume that at T=0 when depositors invest their funds at the bank are uncertain about when they will need to make withdrawals. Nevertheless, they understand that only if they keep their funds at the bank until T=2 will receive the higher return. To keep things simple we have normalized the gross return on early withdrawals to unity.

since the bank's portfolio consists of loans and securities that have the characteristics of debt contracts; that is their maximum returns are fixed by the interest rate specified in the contracts. In addition, we assume that the bank's net return on reserves is zero.

We introduce costly liquidation to allow the bank to honour its obligations with depositors in the case of unexpected liquidity shortages ($xD>R$). Suppose that at T=1 the liquidation value of a unit of portfolio investment is equal to $1/c$ ($c>1$).[5] As long as shortages are sufficiently small the bank will be able to meet the demand for withdrawals. The bank can ensure the viability of the asset transformation process by holding a sufficiently high amount of reserves. When there are surpluses ($xD<R$) the bank holds excess reserves that yield a lower return relative to its portfolio. Therefore, liquidity surpluses can also threat the asset transformation process because they increase the risk that the value of the final assets will not match the bank's long-term liabilities.

In order to rationalize risk management we introduce bankruptcy costs. More specifically, we assume that when the bank fails it incurs a loss of size B.[6] The bank can fail either because of lack of liquidity at T=1 or because of insolvency at T=2. In the absence of bankruptcy costs the bank would maximize expected returns ignoring any risk arising by its decisions. Effectively, bankruptcy costs introduce non-convexities in its payoff and as a result the bank acts in a risk-averse manner. Finally, we assume that bank owners are protected by limited liability that implies bankruptcy costs do not include any pecuniary penalties.

3. Liquidity Management

Liquidity management refers to the bank's choice of its level of reserves. Reserves ($R=D-L$) are held to cope with early deposit withdrawals at T=1. As we noted above the bank faces a trade-off when it makes this choice. A too low level of reserves and the bank risks failure at T=1 because of being unable to meet the demand for early withdrawals. A too high level of reserves and the bank risks insolvency at T=2 because of being unable to meet the demand for late withdrawals. Let x_R denote the

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[5] These liquidation costs are related to selling securities, calling in loans and selling off loans.

[6] These costs might be associated to the loss of the right to perform banking activities in the future and the loss of credibility. Alternative rationales for risk management in the context of financial intermediaries are managerial self-interest, non-linear taxes and capital market imperfections. However, several authors agree that the costs of financial distress offer the best alternative. See Allen and Santomero (1997), Scholtens and Wensdeen (2000).

threshold level of the proportion of deposits withdrawn at T=1 that exactly matches the ratio of reserves to deposit outflows; formally,

$$x_R(R,D) = \frac{R}{D} \tag{1}$$

If the realized value of x is equal to x_R then the levels reserves and payments to early depositors will be identical and long-term banking outcomes will only depend on portfolio investment.

If the initial reserve provision does not match the level of early withdrawals, the availability of contingent liquidity strategies must be considered. Inadequate reserve provisions can be managed through the costly liquidation of illiquid assets while reinvesting remaining reserves is the option for liquidity surpluses. When the realized proportion of early withdrawals is higher than the threshold value, $x>x_R$, there is a liquidity shortage, $R\text{-}xD<0$. In the opposite case, $x<x_R$ there is a liquidity surplus, $R\text{-}xD>0$, and the excess reserves are held until the last period. Let S denote the difference $R\text{-}xD$.

When there is a surplus revenues at T=2 will be augmented by the corresponding excess reserves. In contrast, when there is a shortage the portfolio investment will be reduced by cS. The provision of liquidity for the short- term by holding reserves has a negative effect on future profits because it reduces investment. However, inadequate liquidity provision does not only increase the risk of illiquidity at T=1 but also reduces future profits because of the liquidation of investment. Let x_L be the value of x such that $L\text{-}c(x_LD\text{-}R)=0$. In words, when the proportion of deposits withdrawn at T=1 is equal to x_L, the liquidation value of the bank's portfolio is equal to the liquidity shortage. Solving for x_L we get

$$x_L(R,D) = \frac{L+cR}{cD} = \frac{D+R(c-1)}{cD} \tag{2}$$

If $x>x_L$ the bank will not be able to meet the demand for withdrawals and will fail because of lack of liquidity. The asset transformation process ends because liquidity requirements exhaust the banking assets. Notice that the two cut-offs defined above are functions of both reserves and deposits and therefore depend on liquidity management decisions.

4. Liquidity and Solvency

The liquidity management decisions adopted by the bank together with the realized return of its portfolio investment determine the financial status (liquid, illiquid, solvent and insolvent) of the bank and conditional on being solvent at T=2 its level of profits. The latter are given by:

$$\pi = [L + cMin(0,S)]y + Max(0,S) - (1-x)rD \qquad (3)$$

The first-term captures the bank's portfolio return. Notice that in the case of a shortage (S<0) the investment is reduced by amount of liquidation. The second term shows the level of excess reserves when there is a surplus (S>0) and the last term shows the total promised payments to those depositors that withdraw their funds at T=2.

Given our analysis of the model so far it is clear that a liquidity management policy that excludes the possibility of financial failure does not exist. If the bank does not make any portfolio investment (R=D) will never fail because of lack of liquidity but since r_D>1 it will become insolvent with certainty as long as x<1. It is also clear that insolvency cannot be avoided for any R<D since the probability that y is less than the gross return on reserves, which is equal to 1, is strictly positive.

Given that $x<x_L$, i.e. the bank did not fail at T=1 because of liquidity problems, it can still fail at T=2 because of insolvency if y is below a threshold level that we refer to as the bank solvency threshold. When at T=2 the return on its portfolio investment is below this threshold, the value of its portfolio will be less than its liabilities. Put differently, the bank solvency threshold corresponds to the minimum investment return necessary for the bank to meet its long-term obligations. However, given that these liabilities depend on the realization of x, i.e. the demand for early withdrawals, and the bank's liquidity management decisions, we need to evaluate bank solvency threshold values for both liquidity shortages and liquidity surpluses. Let y_{S-} and y_{S+} denote the corresponding threshold values. These thresholds will depend on R and D, the bank's liquidity management decisions, and x, the proportion of deposits withdrawn at T=1.

To obtain the threshold $y_{S-}(x,R,D)$ that determines the value of y such that bank profits vanish, for given initial deposits and their allocation between reserves and portfolio investment, and conditional on a liquidity shortage and the realized demand for early withdrawals, set (3) equal to zero and solve for y:

$$y_{S-}(x,R,D)=\frac{(1-x)rD}{L+cS}=\frac{(1-x)rD}{D-R-c(xD-R)} \tag{4}$$

where S=R-xD<0.

In a similar way we obtain the threshold $y_{S+}(x,R,D)$ that determines the value of y such that bank profits vanish conditional on a liquidity surplus:

$$y_{S+}(x,R,D)=\frac{(1-x)rD-S}{L}=\frac{(1-x)rD-(R-xD)}{D-R} \tag{5}$$

where S=R-xD>0.

Notice that in the benchmark case where $x=x_R$, i.e. the demand for early withdrawals exactly matches the level of reserves the threshold level is given by the ratio of long-term obligations to portfolio investment.

5. Viability of the Asset Transformation Process

Up to this point, we have found that the bank can fail either because of liquidity problems at T=1 or because of insolvency at T=2. In this section, we are going to identify another reason for bank failure that can take place at T=1 that is related to the viability of the asset transformation process. In the previous section, we have derived the bank solvency thresholds that identify minimum values for the bank's portfolio return that are consistent with bank solvency. These thresholds values depend on the bank's choice of its level of deposits, the bank's liquidity management decisions and the realized early demand for withdrawals and therefore their exact values become known at T=1. However, given that the bank's portfolio return is bounded from above, if these thresholds are larger than the maximum return then at T=1 it will be known with certainty that the bank will be insolvent at T=2. In such cases, the bank fails because the long-term asset transformation process is not viable.

We consider first the case of liquidity shortages. Using (4) we find that

$$\frac{dy_{S-}}{dx}=\frac{rD(D-R)(c-1)}{(D-R-c(xD-R))^2}>0 \tag{6}$$

The higher the realized value of the proportion of deposits withdrawn early the larger the shortage will be that implies a higher proportion of portfolio investment will be liquidated. Then, since both c and r are higher than 1, the higher the return on the bank's portfolio investment need to be in order for the bank to be able to honour its long-term obligations. By setting the left-hand side of (4) equal to Y, the upper bound of the support of the portfolio return's distribution, we derive a cut-off value for x, denoted by x_{S-}, such that for realized values of the proportion of early withdrawals higher than this cut-off, the asset transformation process fails to be viable.

$$x_{S-}(R,D) = \frac{Y(D+(c-1)R)-rD}{D(cY-r)} \tag{7}$$

Notice that $x_{S-}<x_L$, which suggests that when there are shortages the asset viability transformation condition is stricter than the liquidity one. Put differently, a bank that fails at T=1 because of illiquidity problems also fails because its asset transformation process is not viable. This would suggest that we only need to concentrate on the viability of the asset transformation process since the bank's payoff when it fails is independent of the cause of failure. However, for practical reasons, we must examine these two cases separately. When the liquidity constraint is not satisfied the bank's only option is to liquidate its portfolio investment to satisfy the demand for liquidity. In contrast, when only the viability constraint is not satisfied there are two alternatives. The bank can either be liquidated at T=1 and distribute the proceeds to those depositors that were planning to withdraw their funds late or wait until T=2 for its investments to mature and then distribute the proceeds to the same depositors. In both cases late depositors will receive less than what they were promised and which option is followed it will depend on the expectations at T=1 about the performance of the bank's portfolio.

Next, we consider the case of surpluses. Using (5) we find that

$$\frac{dy_{S+}}{dx} = \frac{D(1-r)}{D-R} < 0 \tag{8}$$

The lower the realized value of the proportion of deposits withdrawn early the higher excess reserves will be. But since the gross return on excess reserves is 1 which is less than the gross interest rate offered on deposits, r, the higher the return on the bank's portfolio investment need to be in order for the bank to be able to honour its long-term obligations. By setting the left-hand side of (5) equal to Y, the upper bound of the support of the portfolio return's distribution, we derive a cut-off value for x, denoted by x_{S+}, such that for realized values of the proportion of early withdrawals lower than this cut-off, the asset transformation process fails to be viable.

$$x_{S+}(R,D) = \frac{Y(D-R)-rD+R}{D(1-r)} \tag{9}$$

Since $Y>r>1$ if r is sufficiently close to the 1, i.e. the gross return on reserves then the above ratio will be negative and the transformation process will be viable with certainty.

The above analysis clearly suggests that because illiquidity and insolvency are intertemporally linked these two causes of bank failure must be considered together. Either a too high or a too low level of liquidity provision can cause the bank to fail because they reduce the chances that the bank will be able meet its obligations to depositors.

6. Banking Regimes

The bank's performance depends on its choice of the level of deposits, on its liquidity management decisions, on the realization of the demand for early withdrawals and, given that at T=1 the viability of the asset transformation process is assured, on the realization of its portfolio return. The various threshold levels derived in the previous sections define 7 banking regimes related to the short-term and long-term status of the bank's balance sheet. The first 4 scenarios correspond to the case of a shortage of reserves at T=1 while the last 3 correspond to the case of a surplus of reserves.

***Illiquidity* (*L*)**: $x>x_L>x_{S-}>x_R$

The demand for early withdrawals is so high that cannot be satisfied even after the whole portfolio investment is liquidated. Bank losses are equal to $-B$.

***Shortage-Failure* (*ShF*)**: $x_L>x>x_{S-}>x_R$

The demand for early withdrawals is satisfied, however, the asset transformation process is not viable. Even if the realized portfolio return is at its maximum the bank will not be able to meet its long-term obligations. Bank losses are equal to $-B$.

***Shortage-Insolvency* (*ShI*)**: $x_L>x_{S-}>x>x_R$ and $y<y_{S-}<Y$

The demand for early withdrawals is sufficiently small to ensure the viability of the asset transformation process but the realized portfolio return is too low for the bank to be able to meet its long-tem obligations. Bank losses are equal to $-B$.

***Shortage-Solvency* (*ShS*)**: $x_L>x_{S-}>x>x_R$ and $y_{S-}<y<Y$

The demand for early withdrawals is sufficiently small to ensure the viability of the asset transformation process and the realized portfolio return is sufficiently high and thus the bank is able to meet its long-tem obligations. Bank profits are equal to π, where S<0.

***Surplus-Failure* (*SuF*)**: $x<x_{S+}<x_R$

The volume of excess reserves is so high that the asset transformation process is not viable. Even if the realized portfolio return is at its maximum the bank will not be able to meet its long-term obligations. Bank losses are equal to –*B*.

***Surplus-Insolvency* (*SuI*)**: $x_{S+}<x<x_R$ and $y<y_{S+}<Y$

The demand for early withdrawals is sufficiently high to ensure the viability of the asset transformation process but the realized portfolio return is too low for the bank to be able to meet its long-tem obligations. Bank losses are equal to –*B*.

***Surplus-Solvency* (*SuS*)**: $x_{S+}<x<x_R$ and $y_{S+}<y<Y$

The demand for early withdrawals is sufficiently high to ensure the viability of the asset transformation process and the realized portfolio return is sufficiently high and thus the bank is able to meet its long-tem obligations. Bank profits are equal to π, where S>0.

Figure 1 shows the 7 banking regimes. On the horizontal axis we measure the proportion of deposits withdrawn at T=1 and on the vertical axis we measure the portfolio return. Notice that if the portfolio return is less than the interest rate on deposits then the bank will fail with certainty. In the limiting case where the two rates are equal and the demand for early withdrawals exactly matches the volume of reserves the bank breaks even. What happens when the portfolio return is higher than the interest rate on deposits it depends on the proportion of deposits withdrawn early.

Our graphical analysis highlights how important it is to consider illiquidity and insolvency as causes of bank failure jointly. For example, consider the benchmark case of our model where the bank's portfolio return is certain and its value is higher than the interest rate on deposits. In this case, the bank can fail only because of severe liquidity problems; that is the demand for early withdrawals can not be met even when the bank liquidates its whole portfolio. Then, the probability of bank failure is equal to 1-F(x_L), where F(.) denotes the distribution function of x. Now, consider the other benchmark case where the demand for early withdrawals is exactly equal to reserves but the portfolio return is random. In this case, the bank fails when $y<r$ or with probability $G(r)$, where G(.) denotes the distribution function of y. When we consider

the two cases of uncertainty together, we find that the bank can also fail when $y>r$ and $x<x_L$ as long as the pair of values lies outside the two solvency scenarios in figure 1.

7. Optimal Banking Policies

The bank's objective at T=0 is to choose *R* to maximize expected profits. Using (3) expected profits are given by:

$$\begin{aligned}
&\int_{x_{S+}}^{x_R}\int_{y_{S+}}^{Y}[(D-R)y+(R-xD)-(1-x)rD]g(y)f(x)dydx+\\
&\int_{x_R}^{x_{S-}}\int_{y_{S-}}^{Y}[(D-R-c(xD-R))y-(1-x)rD]g(y)f(x)dydx- \qquad (10)\\
&B\left(1-\int_{x_{S+}}^{x_R}\int_{y_{S+}}^{Y}g(y)f(x)dydx-\int_{x_R}^{x_{S-}}\int_{y_{S-}}^{Y}g(y)f(x)dydx\right)
\end{aligned}$$

The first double integral is equal to expected profits in the *Surplus-Solvency* (*SuS*) regime, the second integral is equal to expected profits in the *Shortage-Solvency* (*ShS*) regime, and the last expression is equal to expected losses given that the bank has failed. The choice of the level of deposits will determine the scale of banking activities (measured by total liabilities) and the interest rate on deposits. In what follows, we are going to concentrate on the choice of the level of reserves taking as given the level of deposits and the corresponding rate. We do this to keep things simple and because for all practical purposes, it is the choice of reserves that primarily will determine the likelihood of each of the seven banking regimes.

As a preliminary step, we derive the effects of a marginal increase in reserves on the various threshold levels. It is straightforward to show that:

$$\frac{dx_R}{dR}>0,\ \frac{dx_L}{dR}>0,\ \frac{dy_{S+}}{dR}>0,\ \frac{dy_{S-}}{dR}<0,\ \frac{dx_{S+}}{dR}>0,\ \frac{dx_{S-}}{dR}>0 \qquad (11)$$

Other things equal, an increase in the level of reserves increases the likelihood of a surplus. It follows that an increase in the level of reserves reduces the likelihood that the bank will fail because of illiquidity. The last four effects imply that the solvency area in figure 1 that comprises of the *Surplus-Solvency* and the *Shortage-Solvency* regimes shifts to the right. Figure 2 shows how the 7 banking regimes are affected by an increase in the level of reserves.

Next, we turn our attention to the comparative static results of our model. Given the complexity of the expected profit function (10) we are going to restrict our attention to the case where the two random variables of our model *x* and *y* are uniformly distributed. Then we have 5 exogenous variables; namely the interest rate on long-term deposits, *r*, the cost of early liquidation, *c*, the cost of bankruptcy, *B*, the level of deposits, *D*, and the upper bound of the distribution of return of the bank's investment portfolio, *Y*. Our interest is to examine how changes in these variables affect the bank's optimal liquidity management policy, its profitability and the likelihood of each of the 7 banking regimes.

We have obtained these comparative static results by calibrating our model. Then, we have used numerical optimization to find for each set of values for the 5 exogenous variables an approximate value for the level of reserves, *R*, that maximizes expected profits.[7] Table 1 provides a summary of all the comparative static results. While performing the numerical calculations one clear pattern emerged. For the parameter values that we have chosen in all equilibria the *Surplus-Solvency* banking regime vanishes (the value for x_{S+} was negative in every configuration). The intuition is that it is never optimal for the bank to hold such a high level of reserves so that there is a risk that its investment return would be insufficient to cover its long-term obligations when the realized demand for early liquidity is very low. Given that observation whenever the parameter x_{S+} appears as a limit on the integrals in (11) we have to replace it with a 0.

Changes in the interest rate on deposits, r

An increase in the interest rate on deposits increases the bank's expected long-term obligations. The bank reacts by increasing its portfolio investment that offers a positive net expected return by decreasing its level of reserves. Although the decrease in the level of reserves decreases the probability of a surplus, the probability of the *Surplus-Insolvency* regime increases. This is because the effect of an increase in the interest rate on deposits on long-term obligations and thus on insolvency dominates the expected liquidity effect. An increase in the interest rate on deposits also increases the three probabilities attached to the three other regimes associated with bank failures. This is because the decrease in reserves increases the probability of a shortage. It is clear that the above also imply that an increase in the interest rate on deposits increases the overall probability of a bank failure, *p*(*B*), and decreases the

[7] The numerical calculations were performed using MATHEMATICA 4. The detailed calculations are available from the authors upon request.

probabilities of the two banking regimes associated with solvency. Finally, it also has a negative effect on expected profits.

Changes in the liquidation cost, c

We are interested in this type of change because it might be capturing variations in economic activity. As Shleifer and Vishny (1992) have pointed out during recessions, when economic activity is low, liquidation costs are high because of low resale values. In our model, an increase in the liquidation cost forces the bank to choose a higher level of reserves. Given that the increase in reserves increases the probability of a surplus it also increases the probability of the *Surplus-Insolvency* regime and decreases the *Shortage-Insolvency* regime. However, when the liquidation cost increases the probability of the *Shortage-Failure* regime declines. This is because given that there is a shortage the higher liquidation costs has a negative effect on the viability of the transformation process. The increase in reserves also reduces the probability of a failure because of lack of liquidity. Overall, the probability of failure increases. In addition, the increase in reserves implies that the probability of the *Surplus-Solvency* regime increases while the probability of the *Shortage-Solvency* regime decreases. Finally, an increase in the liquidation cost has a negative impact on expected profits.

Changes in Y

An increase in *Y* corresponds to a change in the distribution of the return of the bank's investment portfolio such that the new distribution stochastically dominates in the first-order sense the old one. The increase in the expected return offers incentives to the bank to increase its portfolio investment by decreasing its reserves. Clearly, the decline in reserves increases the probability of failure because of lack of liquidity. However, the probabilities of all the other banking regimes associated with a bank failure decrease while the probabilities of the two regimes associated with a solvent bank increase. Expected profits also increase.

Changes in bankruptcy costs, B

An increase in bankruptcy costs offers incentives to the bank to behave in a more risk-averse manner by increasing its reserves. The increase in reserves has a positive effect on the probabilities of all regimes associated with a surplus and a negative effect on those regimes associated with a shortage. Overall, the probability of failure decreases, however, expected profits decline because the direct effect on costs dominates the indirect effect that works through the increase in reserves.

Changes in the level of deposits, D

The bank's reaction to an increase in the level of deposits is to decrease its level of reserves. The intuition behind this result is that an increase in the level of deposits, keeping bankruptcy costs the same, offers incentives to the bank to behave in a less risk-averse manner. This will always be the case as long as the expected return on the bank's investment portfolio is higher than the interest rate the bank needs to pay on its long-term liabilities. Given that an increase in the level of deposits has the opposite effect on the bank's behaviour relative to an increase in bankruptcy costs, it is not surprising that all comparative static results are reversed.

Scale effects, B&D

A change in the level of deposits can be interpreted as a kind of a scale effect. Under this interpretation the above results would suggest that larger banks set their reserves to deposits ratio at a lower level and are subject to a higher overall probability of failure ($p(B)$). However, what induces this behaviour is the fact that part of the costs has not been scaled at all. Specifically, in the above exercise we have kept bankruptcy costs fixed. The last column of table 1 shows the comparative static results when deposits and bankruptcy costs increase by the same proportion. In this case the model suggests that there are not any scale effects as the only variable affected is expected profits. Of course, it is doubtful that what we, in our model, capture by bankruptcy costs increases proportionately with size. If what actually happens is between the two extreme cases, then our model would still predict that, ceteris paribus larger banks are more prone to risk-taking.

8. Conclusion

We have developed a banking framework to study the optimal liquidity management decisions of a profit-maximizing bank. We have defined the possible short- and long-term scenarios that banks may face given their management decisions taking into account the presence of both liquidity and solvency risks and the availability of liquidity-risk management strategies. Using our framework we have described how the banking decisions define the scenarios and outcomes that the intermediaries could eventually face. In particular, we have clarified the inter-temporal relationships between liquidity and solvency risks and between the viability of the asset transformation process and profitability.

Our framework can be extended in a number of directions. In this paper we have only allowed the bank to choose the allocation of deposits between liquid reserves and a risky investment portfolio. The next step would be to let banks also choose the return distribution of their portfolio. Given the complexity of the framework this extension is not that straightforward especially if we want to stick to continuous distributions. Nevertheless, our understanding of risk-management can be substantially improved by allowing the bank this double choice given that it is facing two types of risks.

In this paper we have treated both the level of deposits and the interest rate on deposits parametrically. We believe that as long as we restrict our attention to a monopolistic banking system this simplification is appropriate. However, it would be interesting to introduce competition in the deposit market in order to examine its effects on the behaviour of banks and in that case we would have to allow the interest rate on deposits to be endogenously determined.

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Figure 1: Banking Regimes

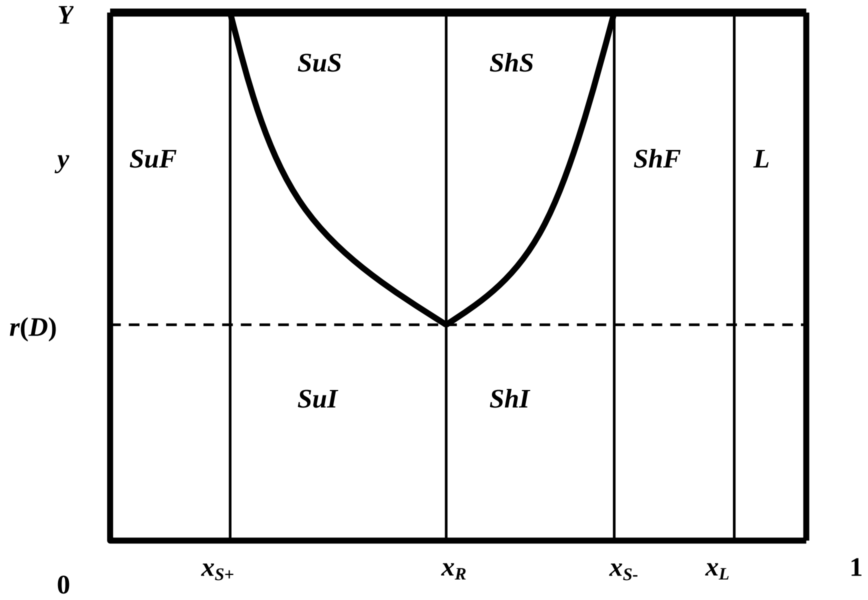

Figure 2: Effects of a Change in Reserves on the Banking Regimes

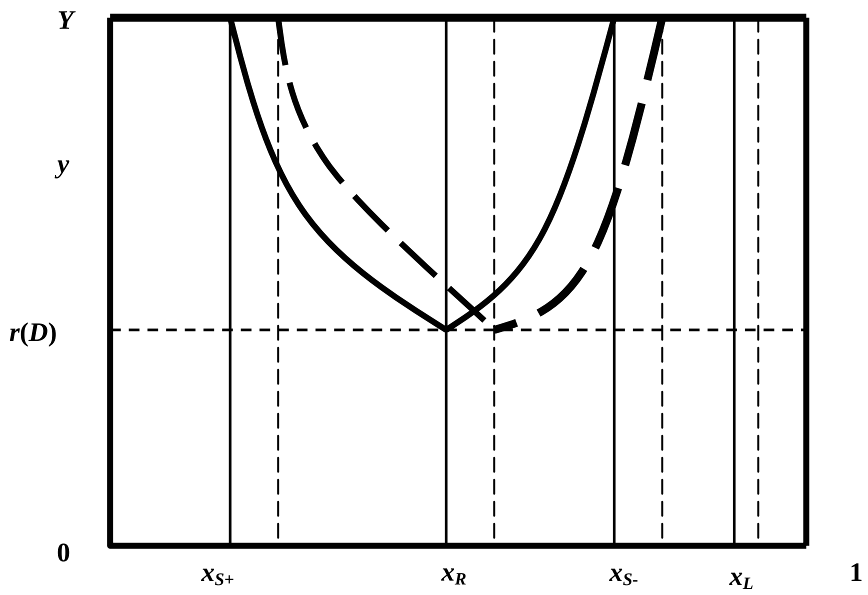

Table 1: Comparative Statics

	r	c	Y	B	D	$B\&D$
R/D	-	+	-	+	-	0
$p(SuI)$	+	+	-	+	-	0
$p(ShI)$	+	-	-	-	+	0
$p(ShF)$	+	+	-	-	+	0
$p(L)$	+	+	+	-	+	0
$p(B)$	+	+	-	-	+	0
$p(SuS)$	-	+	+	+	-	0
$p(ShS)$	-	-	+	-	+	0
$E(Profits)$	-	-	+	-	+	+