Specialization in the bargaining family

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July 2010
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July 1, 2010

Abstract

We develop a two period family decision making model in which spouses bargain over their contributions to a family public good and the distribution of private consumption. In contrast to most models in the literature, specialization within the couple emerges endogenously from the production of the public good, and is not caused by exogenous differences between the spouses. Increasing marginal benefits of labour market experience make specialization efficient, even if both spouses have equal market and household productivities on the outset. If spouses are not able to enter into a binding contract governing the distribution of private consumption in the second period, the spouse specialized in market labour cannot commit to compensate the other spouse for foregone investments in earnings power. As a consequence, this spouse may withdraw part of his/her contribution and the provision level of the household good is likely to be inefficiently low.

Keywords: Family bargaining, specialization, private provision of public goods

JEL Classification: D19, H41

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1 Introduction

Do couples always make efficient decisions regarding labour supply, fertility and consumption? For the pioneer of family economics, Gary Becker, the answer was clearly yes. Since, in his “unitary model”, the altruistic head of the household has the power to determine intrafamily allocation, there is no need to deal with differing preferences within the household. The family decision making problem collapses to the decision making problem of one rational individual and is therefore efficient.\footnote{Among the many contributions of Becker to the theory of the family, see e. g. Becker (1965, 1981).}

In this paper, we show how allowing for a distributional conflict between the spouses can generate an inefficient outcome of the family decision making process. We develop a simple two period model, in which spouses bargain over the provision level of a family public good and the division of private consumption between the spouses. The family public good is produced at home, with spouses’ time as the only input, while private consumption results from labour market work. We emphasize the importance of labour market experience as a determinant of wage rates. Based on recent findings in empirical labour economics, we make an assumption about the specific functional form linking labour market experience and wage rates. This allows us to show that specialization in household and market production within the couple is efficient, even if spouses are equally productive in both spheres ex-ante. Unlike most models in the literature, we do not need to rely on biological differences or labour market discrimination against women to account for the widespread gender division of labour still observed in most industrialized countries. This division of labour arises endogenously as a consequence of the provision of the public good. It leads to different levels of investment in human capital within the couple, which in turn leads to spouses having different wage rates in the second period.

Provided the couple can reach a binding agreement about the distribution of their resources over the course of their whole married life, the spouse who suffers from a lower wage rate due to the provision of the family public good is compensated by the other spouse. In this case, the outcome of the family decision making process is efficient.

If, on the other hand, binding agreements across periods are not feasible, the efficient degree of specialization may not be time-consistent. There is an asymmetry between market and household work. Working in the labour market leads to a higher wage rate in the second period while producing the public good does not. Therefore, the public good provider suffers from a lower consumption level in the second period. This may cause an underprovision of the public good, because the spouse specialized in market work cannot credibly commit to compensate her or his partner for their lower wage rate in the second period.

The model starts with “young marriage”, when the couple establishes their first joint
household, and family public goods become relevant.\footnote{While we use the term “marriage” throughout the text, the model applies to any form of cohabitation or partnership that involves a major joint project. Also other settings outside the economics of the family are imaginable, like a partnership of firms contributing to R & D as a public good in a first stage of a 2-stage-game.} Not only every-day household work is a public good for the couple, but also the benefits of long-term investments like the rearing of children or the building of a house are enjoyed jointly by both spouses. This is what we have in mind when we talk about the production of a family public good, which requires the spouses’ time as an input. Naturally, the time devoted to household production goes at the expense of other activities, most prominently the building of a professional career. The age at which most couples have children and build their houses coincides with the stage in life in which most market related human capital is accumulated and careers progress in a most decisive way, playing a crucial role in lifetime earnings and in later income patterns. We incorporate this tradeoff by assuming learning by doing in the labour market. The more an individual works in the market and invests in his or her professional career, the more productive she/he becomes. Or, conversely, the more time an individual spends producing the household public good, the more she/he forgoes present and future income at the labour market. In line with empirical findings, we assume this effect to become weaker the longer the absence from the labour market.

In the second period, the couple is established (e.g., the children have grown up and left home, the house is built). We assume that there is no household public good consumption and that both spouses devote themselves entirely to their careers.

In both periods, spouses determine the distribution of private consumption via Nash bargaining. In this solution concept, the utilities the spouses could guarantee themselves if they were unable to reach an agreement - the threat or disagreement points - are of crucial importance for the distribution of private consumption goods. We devote special attention to spouses’ threat points, since there is no consensus in the literature as to what is likely to be used as a threat point in family bargaining. Two quite different specifications have been proposed so far. In the classic models of Manser and Brown (1980) and McElroy and Hornay (1981), spouses divorce and live as singles. Alternatively, they may stay together, but resort to non-cooperative behaviour within marriage (Lundberg and Pollak (1993) and Konrad and Lommerud (2000) are examples of this strand of literature). We compare both threat point specifications to assess their relevance. It turns out that the two different threat points “favor” different spouses, so the choice of the threat point does have distributional consequences.

The paper proceeds as follows. In the next section, we discuss the related literature. Then we present the model and its main assumptions. Section 4 derives the efficient Nash Bargaining Solution (NBS) if binding agreements across periods are feasible and discusses the two threat point specifications as well as their implications for the distribution
of resources within the couple. Section 5 analyzes the model if spouses cannot commit across time periods. We describe possible inefficient outcomes and discuss potential policy implications of the model. Section 6 summarizes our results and concludes.

2 Related Literature

The family bargaining literature was pioneered by Manser and Brown (1980) and McElroy and Horney (1981). They allowed for a conflict of interest within the household, and proposed that spouses resort to Nash Bargaining to settle their differences.\(^3\) In Nash bargaining models, household decision making is efficient by assumption - this is often motivated by the presumption that spouses can make binding and enforceable agreements that enable them to reach efficient outcomes.

While it is arguable that family decision making is efficient on an everyday basis, it is not clear how families should make binding contracts about intra-household distribution over long time horizons.\(^4\) Recent literature shows that inefficient family decision making can arise in models that contain more than one period and evolve around decisions that influence future “bargaining power”. Lundberg and Pollak (2003) discuss the decision whether to migrate or to stay put. They find that if the couple is unable to reach a binding agreement over the division of marital resources before making a location decision, inefficient divorce or inefficient migration choices by the couple may be the outcome. Another decision that is likely to influence future “bargaining power” is the educational choice. If one spouse has a higher education, and therefore a higher wage rate, than the other, efficiency demands that this spouse specializes in market work, while the other performs most of the “housework” (the production of family public goods). But specialization decisions are typically not neutral with respect to “bargaining power”. Being more productive in the provision of a public good is not necessarily an advantage in a non-cooperative setting (Buchholz and Konrad, 1995). In Konrad and Lommerud (2000) spouses invest non-cooperatively in education before marriage. Then they marry and in the second period they may behave non-cooperatively or they may cooperate with the equilibrium of the non-cooperative game as the threat point. Both spouses have an incentive to inefficiently overinvest in education (i. e., their productivity in market work), because a higher wage rate improves their bargaining position in the second stage (whether directly in the private provision game or indirectly via the fall-back utility in the Nash bargaining game).\(^5\)

\(^3\)Another important strand of the family economics literature stresses the importance of household production and trade within the household, see Apps and Rees (1988, 1996). The “collective” model of Chiappori (1988) does not specify a decision making process, but suggests that family decision making leads to efficient outcomes. In this paper we focus on the bargaining approach.

\(^4\)Note that prenuptial agreements fix the distribution of resources in the case of divorce, but rarely within marriage.

\(^5\)Vagstad (2001) proposes a similar model where the spouses do not invest in education valuable on the
In these models, investment in earnings power is investment in education that predates marriage. The educational choice is purely strategic, it is taken in order to improve one’s bargaining position by aiming to alter relative productivities within the couple to one’s favor. In our model, in contrast, specialization within the couple occurs as a consequence of the provision of the public good, and not in its strategic anticipation. Unequal investment in human capital results endogenously as a consequence of the private provision of the household public good. Also, the main reason for the inefficiency of the bargaining outcome in our model is not the different effect of household- and market skills on bargaining power as in these models, but the fact that spouses are credit constrained.

Ott (1992) was one of the first to analyze a dynamic model of intrafamily distribution that produces an inefficient outcome. Her model is related to ours because it also incorporates learning by doing on the labour market. In a similar way, Gugl (2006) analyzes the effects of the taxation of couples on labour supply and intra-family distribution when there is learning by doing in the labour market. She also considers different threat point specifications. In a more recent contribution, Gugl (2009) revisits the issue of joint versus individual taxation of couples in a dynamic bargaining model, where spouses choose their labour supply non-cooperatively in the first period, and use the Nash Bargaining solution only in the second period. She also examines the role of control over labour supply - i.e. if both spouses have the power to chose their own labour supply - for intra-household distribution. In these two contributions, the reason for a subefficient provision level of the public good is that spouses manipulate their bargaining position in the second period by working too much in the labour market in the first period to enhance their productivity. Lundberg (2002) considers a similar model with learning by doing in the labour market. Her focus is on family policy and she does not explicitly model family decision making, but household utility is a weighted average of individual utilities.

Reiner (2008) uses the framework of Konrad and Lommerud (2000) to analyze whether specialization between men and women in the household and in the labour market are reinforced by gender discrimination in the labour market. He assumes that women have an exogenous advantage in household production. This exogenous difference in relative productivity between the spouses leads to unequal intra-household bargaining positions; the husband’s advantage in intrahousehold bargaining may lead to an inefficient household allocation. Our paper complements this approach by showing that inefficiency can be due to long-term commitment problems, even if husband and wife have equal productivities on the outset.

All the models quoted above assume exogenously that spouses have differing productivities in household and market work (otherwise, specialization within the couple would not be efficient). We do not need such an assumption. In contrast, our model, specialization arises endogenously due to the provision of the public good even if both spouses labour market, but in household production skills. Asymmetric household productivity is assumed.
have identical productivities at the beginning of marriage. Thus, our model extends the existing literature by analyzing specialization between spouses that are *ex-ante* identical and leading endogenously to an asymmetric ex-post outcome. This suggests a different source of inefficiency - contract and credit constraints - as opposed to strategic motives (influencing future threat points), that have dominated the literature so far.

3 The Model

Consider a household consisting of two spouses, $i = f, m$, whose lifespan stretches over two periods. In both periods, the spouses’ time endowment is fixed and normalized to 1. In period 1, which can be thought of as “young marriage”, spouses $f$ and $m$ allocate their time between household work producing the family public good $G$, which can be thought of as raising children or building a house, and market work. In the second period, spouses devote all their time to market work and only consume the private good.  The spouses’ lifetime utility function is additively separable and is given by

$$U^i = c^i_1 + v(G) + c^i_2, \quad i = f, m,$$  

where $c^i_j$, $i = f, m$, $j = 1, 2$ denotes the private consumption of spouse $i$ in period $j$, and $v(G)$ is the utility the spouses derive from the public good $G$. The function $v(G)$ is monotonically increasing, concave, and twice continuously differentiable. Thus, lifetime utility is quasilinear and intertemporally linear. For the sake of simplicity there is no leisure and no discounting, therefore lifetime utility is the sum of the utilities of both periods. Further, we assume that spouses do not have access to the capital market, there is no borrowing and no saving.

The public good is produced in the first period with the spouses’ time as the only input according to the linear technology $G = h^f g^f + h^m g^m$, where $g^i, i = f, m$ are her and his contributions to $G$, and $h^i, i = f, m$ are her and his productivities in household production, respectively. Throughout the paper we will assume, without loss of generality, that the

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6 For the sake of a complete analysis, we also consider the case of unequal partners.

7 Having no public good in the second period allows focusing on the commitment effect of the public good provision in the first period. Konrad and Lommerud (2000) have analyzed the case of public good provision in the second period and the corresponding strategic behaviour in the first period.

8 This special form of utility is quite restrictive, but it is commonplace in the literature to restrict the analysis to special utility functions for the sake of analytical tractability. Lundberg and Pollak (1993) assume a Stone-Geary utility function, Konrad and Lommerud (2000) work with a quasilinear “payoff” function and Vagstad (2001) analyzes the case of Cobb Douglas preferences with equal coefficients for the public and the private good. We chose this quasilinear formulation because in our Nash bargaining setting the utility possibility frontier (the locus of all Pareto efficient utility pairs) is linear and utility is easily transferable between the spouses, which greatly simplifies the formal analysis, see Bergstrom (1997).

9 For a discussion of this assumption, see section 5.3.

10 This household production technology where time inputs of husband and wife are perfect substitutes
wife is at least as productive in the household as the husband, $h^f \geq h^m$ (note that this involves the case of equal household productivities as a border case).\footnote{We want to stress that we do not resort to any kind of exogenous or biological argument here. We want to allow for the case that one partner has a comparative advantage in household production and we consider equal productivities as the border case. The direction of the inequality is a matter of indexation only and could as well be reversed.}

The private consumption good $c_i$ is purchased with the income the spouses earn on the labour market whenever they are not busy producing the family public good. On the outset, spouses have the same market productivity given by the wage rate $w$. Private consumption in the first period is

\begin{align*}
c_f^1 &= w \cdot (1 - g^f) + P^1, \quad (2) \\
c_m^1 &= w \cdot (1 - g^m) - P^1 \quad (3)
\end{align*}

where $1 - g^i$ is the time spent on market work, and $P^1$ is a transfer from husband to wife in period 1 (which of course can also be negative). In period 2, spouses devote all their time to market work and only consume the private good. Second period consumption is

\begin{align*}
c_f^2 &= w(g^f) + P^2, \quad (4) \\
c_m^2 &= w(g^m) - P^2 \quad (5)
\end{align*}

where their wage rate depends on their first period labour supply and $P^2$ is the second period transfer.

### 3.1 Learning by doing in the labour market

Labour market experience acquired in period 1 increases the wage rate in period 2. Equivalently, time spent in household production in the first period decreases the market productivity in the second period, so the wage function $w(g^i)$ is decreasing. Furthermore, the (negative) marginal productivity effect of an additional time unit spent in household production (i.e. not on the labour market) decreases with the total duration of that individual’s absence from the labour market, i.e., the marginal productivity effects of $g^i$ are diminishing.

To illustrate this assumption, consider the marginal effect of one additional year of maternity leave on the mother’s wage rate. Whether she takes one or two years off following the birth of her child has a relatively big (adverse) effect on her career, but whether she stays away from the labour market for seven or eight years does not make a big difference was proposed by Becker (1981), who argued that “at the beginning everyone is identical; differences in efficiency are not determined by biological or other intrinsic differences” (Becker, 1981, p.32).
If an individual devotes all her/his time to market work, the highest possible wage in period 2 is given by a fixed, finite wage $\overline{w} := w(0)$. The lowest possible wage is denoted by $\underline{w} := w(1)$.

Thus, $w(g^i)$ is a monotonically decreasing, convex and twice continuously differentiable function, $w'(g^i) < 0$, $w''(g^i) > 0$ for $g^i \in [0, 1]$. This is a crucial assumption of our model, which we believe to be both plausible and supported by the empirical data.

### 3.2 Time structure

We consider a two period model with the following structure.

1. The first period is young marriage. In this phase, the couple chooses the contributions to $G$. We assume that the couple decides cooperatively via Nash bargaining on the size of this investment, on the spouses’ labour supply and on the distribution of the private consumption good between the partners. Spouses have two possible threat points when bargaining:
   
   (a) Non-cooperative marriage as threat point: Spouses contribute privately to the family public good.
   
   (b) Divorce as threat point: Spouses evaluate their utilities as being life-time singles.

2. In the second period there is no public good to be provided and both spouses devote all their time to market work, i.e., to private consumption. Again, Nash bargaining determines the allocation of goods within marriage.

We will consider two versions of this model. First, as a benchmark, we look at the outcome if spouses can enter into a binding contract at the beginning of marriage that

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12 The assumption that the wage decreases at a decreasing rate in the time devoted to household production corresponds to the assumption that the returns to learning by doing increase at an increasing rate. It is based on several recent empirical contributions that suggest that the relationship between wages and time invested in one’s earnings power (be it in education or work experience) is non-linear. Regarding education, Mincer (1997), Deschênes (2006) and Lemieux (2006b,a) all find a convex wage function of education. Lemieux (2006a) argues that especially time invested in postsecondary education increases the wage rate in an overproportional way. With respect to work experience, Beblo and Wolf (2002) analyze the effect of job leaves on wages by modeling a convex wage function, which is supported by their data. Schönberg and Ludsteck (2007) analyze the effect of several expansions in the duration of maternity leave in Germany on women’s wages. They find that an expansion in job-protected leave from 2 to 6 months that took place in 1979 significantly decreased wages, while subsequent expansions from 6 to 10 months in 1986 and from 18 to 36 months in 1992 did not lower wages significantly. Although some of these papers focus on the effects of education and not on work experience, the relevant aspect in the context of our model is whether an individual invests her/his time in her/his own career and human capital (be it work experience or further education) or whether (s)he contributes to the household public good instead.
stipulates private consumption in both periods. Then, in section 5, we relax this assumption and assume that spouses have to renegotiate the distribution of private consumption in period 2.

4 Binding agreements are feasible

As a reference point for our analysis, in this section we look at the model if the couple can enter into a binding contract at the beginning of period 1, that determines both the level of the public good provision (including who provides it) and the distribution of private consumption in periods 1 and 2.

The NBS is, by construction, efficient, and lies on the boundary of the couple’s utility possibility set, the Utility Possibility Frontier (UPF). Quasilinear utility implies that the UPF is linear in the relevant region where both spouses consume positive quantities of public and private goods. The couple’s UPF and the NBS are depicted in Figure 1, where the efficient NBS is found at the tangency point of the UPF and the curve representing the Nash product.\textsuperscript{13} The optimal provision level of the public good is unique, while the division of the private consumption good is determined by spouses’ threat point utilities.

4.1 Efficient outcome

We can treat the two periods of the model as one because we assume binding contracts across periods. The efficient outcome is found by maximizing the sum of the spouses’ intertemporal utilities, the couple’s joint utility, with respect to their contributions $g^f$ and $g^m$:

\[
U(g^f, g^m) = U^f(g^f, g^m) + U^m(g^f, g^m) = \]
\[
= w \cdot (1 - g^f) + w \cdot (1 - g^m) + 2v(h^f g^f + h^m g^m) + w(g^f) + w(g^m). \]

In order to have a non-trivial problem, we assume that each spouse consumes a positive amount of both the public and the private good if (s)he were single, $0 < g^i < 1$ for $i = f, m$. Equivalently, the marginal utility of the first unit of time devoted to the public good $G$ is sufficiently large and the marginal wage effect of the last unit of time devoted to market

\textsuperscript{13}The UPF is non-linear in its border regions, where private consumption is zero for one spouse. If one of the spouses controls all private consumption goods, he or she can further increase his or her utility only by decreasing the level of public good provision below the socially optimal level. Any reduction beyond his or her individually optimal provision level decreases both utilities. This results in the concave borders of the UPF as depicted in Figure 1. Because both spouses consume public and private goods at the threat points and control their own income, the NBS will never lie on these borders.
work is sufficiently small for both spouses \( i = f, m \):

\[
v'(0) h^i \ > \ w - w'(0) \tag{7}
\]

\[2 \cdot v'(h^i) h^i \ < \ w - w'(1). \tag{8}\]

Note that the second of these Inada conditions states that, at the solution of the couple’s joint maximization problem, the sum of the contributions to the public good \( g^f + g^m \) is smaller than 1. This condition also implies that each spouse on her or his own would never spend all available time in household production. Additionally, we assume that 

\[ U_i(g^f, g^m), i = f, m, \] are globally concave:

\[-w''(g^i) > v''(h^i g^i)(h^i)^2 \quad \forall g^i \in [0, 1]. \tag{9}\]

These assumptions guarantee that the individual maximization problem does not have a trivial (corner) solution and that the second order conditions for both individual and joint utility maximization are satisfied. We say that an individual \( i \) is fully specialized if (s)he devotes all of her/his time in the first period to one type of work (market or household work), that is, if \( g^i = 0 \) or \( g^i = 1 \).
Proposition 1 (Efficiency)

In the efficient solution, one spouse is fully specialized on market work while the other spouse divides his or her time between market work and household production. If \( h_f > h_m \), it is the husband who specializes in market work. If \( h_f = h_m \) either spouse can specialize in market work.

Proof. See Appendix.

The intuition behind this result is that, in the first period, both spouses have the same wage rate, so they face the same direct cost of providing the public good. In terms of second period consumption, though, the first unit of the public good is the most expensive. This is due to the convex wage function. Therefore, it cannot be optimal to equally split the time they want to devote to household production, since both spouses would then suffer the expensive first units of wage loss. This is true for couples with identical productivities. If, on top of that, the wife has a comparative advantage in household production, it is optimal that she becomes the sole provider of the household public good and that the husband specializes in market work.

This full specialization result also depends on the simple payoff functions assumed. One can think of many, more sophisticated payoff functions that do not lead to full specialization; e. g. a payoff function where spouses’ contributions to the public good are not perfect substitutes. However, it is our aim to keep the specification as simple as possible to focus on the spouses’ inability to sign a binding agreement. Anticipating the discussion in Section 5.2, the chosen specification makes, in general, an inefficient outcome less, and not more likely. In addition, our approach avoids exogenous assumptions on gender specific productivity differences. Instead we offer an explanation as to why specialization according to gender persists, even though both genders are equally capable at performing household and labour market tasks.

Proposition 1 shows that specialization is efficient because it minimizes the joint cost of the provision of the public good. Although it does not matter which spouse specializes if spouses are equally productive, for notational convenience and without loss of generality we will assume that the wife is the public good provider, i. e., \( G^* = h_f g^{f*}, g^{m*} = 0 \).

The first order condition for the efficient provision level of the public good is found by maximizing the couple’s joint utility (6) with respect to \( g^f \) to obtain

\[
2 \cdot h_f v'(h_f g^{f*}) = 2 \cdot h_f v'(G) = w - w'(g^{f*}).
\]

(10)

The distribution of private consumption in the NBS is determined by spouses’ relative threat point utilities, denote them by \( T^f \) and \( T^m \). The higher a spouses’ utility in the event of disagreement, the larger the share of resources that spouse can claim. Along the linear region of the UPF, the socially optimal amount of the public good is provided, and only the distribution of the private consumption good varies. In this region, spouses bargain over the partition of a “gains from marriage cake” of fixed size. In such a setting,
the NBS guarantees each spouse their threat point utility, and the remaining surplus is split equally among them (see, e.g., Muthoo, 1999, p. 25). Formally, the NBS \( (U^f, U^m) \) is given by

\[
U^f = T^f + \frac{U^* - T^f - T^m}{2} \quad \text{and} \quad U^m = T^m + \frac{U^* - T^f - T^m}{2}. \tag{11}
\]

Unlike the husband, the wife works in household production, so she bears the resulting cost alone. At the NBS, she has to receive a transfer payment to ensure that she reaches the utility level guaranteed to her in (11). In the first period, she receives a transfer \( P^1 \) to make up for her direct income loss, while in the second period, a payment \( P^2 \) compensates her for her lower wage rate. In the following, let \( \Delta U^{f-m} := U^f - U^m \) denote the utility difference between wife and husband and \( \Delta T^{f-m} := T^f - T^m \) denote the difference between the threat point utilities of wife and husband, which we will also call the wife’s utility edge at the threat point. The transfers are calculated by subtracting the wife’s utility level at the optimal household production level \( g^f \) without the transfer, \( U(g^f) \), from the utility level she reaches in the NBS, \( U^f \) (as given in (11)).\(^\text{14}\)

\[
P^1 + P^2 = \frac{1}{2}(U^* + \Delta T^{f-m}) - U^f(g^f) = \frac{1}{2}(U^f(g^{f*}) + U^m(g^{f*}) + \Delta T^{f-m}) - U^f(g^f) = \frac{1}{2}(-\Delta U^{f-m} + \Delta T^{f-m}), \tag{12}
\]

where in expression (12), \( \Delta U^{f-m} \) is evaluated at the efficient level of \( g^{f*} \).

In the following, we will lay out in detail the two threat point specifications we consider, non-cooperative marriage (NC) and divorce (D), and calculate the transfers corresponding to each threat point. As will become clear, the two specifications “favor” different spouses.\(^\text{15}\)

### 4.2 Non-cooperative marriage as threat point

In the non-cooperative marriage threat point the couple live together without coordinating their actions. They play a non-cooperative private-provision of a public good game, that is, each spouse maximizes his/her utility, taking the behaviour of their partner as given, there are no transfers. This game has a Cournot-Nash equilibrium. Spouses maximize their individual utilities

\[
U^f(g^f, g^m)|_{NC} = w \cdot (1 - g^f_{NC}) + v(h^f g^f_{NC} + h^m g^m_{NC}) + w(g^f_{NC}) \tag{13}
\]

\[
U^m(g^f, g^m)|_{NC} = w \cdot (1 - g^m_{NC}) + v(h^f g^f_{NC} + h^m g^m_{NC}) + w(g^m_{NC}) \tag{14}
\]

\(^\text{14}\) Remember that \( g^{f*} \) denotes the efficient contribution to the public good and that \( U^{f*} \) denotes not only the efficient outcome, but additionally the NBS. So, if the wife provides the public good, \( U(g^{f*}) < U^{f*} \) because of the transfer.

\(^\text{15}\) Note that the threat point is exogenously given in our setting, and cannot be chosen by the spouses.
The FOCs implicitly describing the reaction functions are

\[
v'(h^f g_{NC}^f + h^m g_{NC}^m) \leq \frac{1}{h^f} (w - w'(g_{NC}^f)), \tag{15}
\]

\[
v'(h^f g_{NC}^f + h^m g_{NC}^m) \leq \frac{1}{h^m} (w - w'(g_{NC}^m)), \tag{16}
\]

with equality if \( g^i > 0 \). The left hand side (LHS) is the marginal utility of an additional unit of the public good, while the right hand side (RHS) represents the marginal cost of that unit in forgone units of private consumption. Spouses not only take the direct cost of their home time into account (\( w \) on the RHS), but also the lower wage rate a marginal unit of household work has as a consequence in the second period (\( w'(g_{NC}^i) \) on the RHS).

**Lemma 1 (Non-cooperative marriage as threat point)**

For equal household productivities \( h^f = h^m \), the contributions to the household public good are equal, \( g_{NC}^f = g_{NC}^m \). If the wife is more productive in the household than her husband, \( h^f > h^m \), she is the only contributor and he free-rides on her public good provision, \( h^f g_{NC}^f = G_{NC}, g_{NC}^m = 0 \). Moreover, the wife spends less time in household production in the non-cooperative threat point than she does at the efficient outcome, \( g_{NC}^f < g^f \).

\[ g_{NC}^f < g^f \]

**Proof.** See Appendix.

Now we can explicitly calculate the transfer payments the wife receives at the NBS if the threat point is non-cooperative marriage. Because the equilibrium contributions \( g_{NC}^f \) and \( g_{NC}^m \) differ for the cases \( h^f > h^m \) and \( h^f = h^m \), we will consider them separately.

**Asymmetric productivities \( h^f > h^m \).** The wife’s provision level of the public good at the non cooperative threat point \( g_{NC}^f \) is determined by (15) holding with equality, while the husband’s contribution is zero. The wife’s utility edge is

\[
\Delta T_{NC} = T_{NC} - T_{NC} = \left( w \cdot (1 - g_{NC}^f) + v(h^f g_{NC}^f + h^m g_{NC}^m) + w(g_{NC}^f) \right) \tag{17}
\]

\[ - \left( w \cdot (1 - g_{NC}^m) + v(h^f g_{NC}^f + h^m g_{NC}^m) + w(g_{NC}^m) \right) \]

\[ = w \cdot (g_{NC}^m - g_{NC}^f) + (w(g_{NC}^f) - w(g_{NC}^m)) \]

\[ = -w \cdot g_{NC}^f + (w(g_{NC}^f) - w(0)) < 0 \]

where the last equality results from the husband free-riding on his wife’s public good provision in the non-cooperative marriage game. Expression (17) is negative, reflecting the fact that with non-cooperative marriage as the threat point, the husband is at least as well off as his spouse. Explicit calculation of the transfer yields

\[
P_{NC}^1 = \frac{1}{2} [w - w \cdot (1 - g^f) + w \cdot (1 - g^f) - w] = \frac{1}{2} w \cdot (g^f - g_{NC}^f) > 0 \tag{18}
\]

\[
P_{NC}^2 = \frac{1}{2} [w(0)L - w(g^f)L + (w(g_{NC}^f) - w(0))] = \frac{1}{2} [w(g_{NC}^f) - w(g^f)] > 0. \tag{19}
\]

13
The first period transfer $P_{1NC}^{1}$ is positive, because the efficient contribution to the public good is larger than the non-cooperative contribution, $g^{f*} > g_{NC}^{f}$. The second period transfer $P_{2NC}^{2}$ is also positive because the wage is decreasing in the amount of time invested in the public good, $w(g_{NC}^{NC}) > w(g^{f*})$. With $P_{1NC}^{1}$, the husband compensates the wife for half of the time she worked more in the cooperative outcome than she would have done in the absence of an agreement, thus foregoing labour income. $P_{2NC}^{2}$ compensates her for half of her associated wage loss.

The outcome is not “fair” in the sense that spouses share equally in the provision of the public good. This is because the husband knows that, if he paid his wife nothing, she would nevertheless contribute something to the public good. Therefore, he does not have to compensate her for that contribution.

Equal productivities $h^{f} = h^{m}$. In this case, both spouses spend the same amount of time in household production at the non-cooperative threat point. Hence, both have the same level of threat point utility, and so their utility at the bargaining outcome is equal - the costs of the public good provision is shared in a “fair” way. The wife receives a payment of

$$P_{NC}^{1} = \frac{1}{2}w \cdot g^{f*}$$ \hspace{1cm} (20)

$$P_{NC}^{2} = \frac{1}{2}(w(0) - w(g^{f*})).$$ \hspace{1cm} (21)

### 4.3 Divorce as threat point

In the event of a divorce, spouses live as singles in both periods. There are no transfers between them, and the public good becomes a private good. The wife maximizes her single utility as in

$$U^{f}(g_{D}^{f})|_{D} = w \cdot (1 - g_{D}^{f}) + v(h^{f}g_{D}^{f}) + w(g_{D}^{f}).$$ \hspace{1cm} (22)

Her first order condition reads

$$h^{f}v'(h^{f}g_{D}^{f}) = w - w'(g_{D}^{f}).$$ \hspace{1cm} (23)

and analogously his first order condition is

$$h^{m}v'(h^{m}g_{D}^{m}) = w - w'(g_{D}^{m}).$$ \hspace{1cm} (24)

**Lemma 2 (Divorce as threat point)**

*For equal household productivities $h^{f} = h^{m}$, both spouses devote the same amount of time to the public good, $g_{D}^{f} = g_{D}^{m}$, but their contributions are duplicated because the good is now private. If the wife is more productive in the household than her husband $h^{f} > h^{m}$, she consumes more of the public good than he does, $g_{D}^{f} > g_{D}^{m}$. Moreover, the wife’s contribution to the public good is the same in the divorce as in the non-cooperative marriage threat point, therefore it is below the socially optimal level, $g_{D}^{f} = g_{NC}^{f} < g^{f*}$.**
Proof. See Appendix.

As above, we can now calculate the transfers at the NBS, if divorce is the threat point, again discriminating between spouses with equal and asymmetric productivities.

Asymmetric productivities $h^f > h^m$. The wife’s utility edge $\Delta T^f_D = T^f_D - T^m_D$ is given by

$$\Delta T^f_D = \left( w \cdot (1 - g^f_D) + v(h^f g^f_D) + w(g^m_D) \right) - \left( w \cdot (1 - g^m_D) + v(h^m g^m_D) + w(g^f_D) \right)$$

$$= \frac{w \cdot (g^m_D - g^f_D)}{?} + \left( v(h^f g^f_D) - v(h^m g^m_D) \right) > 0. \tag{25}$$

The middle term is positive, since because of her higher household productivity, the wife consumes more effective units $h^f g^f_D$ of the public good at the divorce threat point than the husband does.

The sign of the first and the last term depend on who devotes more time to household production at the divorce threat point. Our assumptions about $v(G)$ and $h^f/h^m$ do not directly answer this question, the signs of the first and the last term are therefore undetermined. Still, the wife’s utility edge $\Delta T^f - m$ is strictly positive, because she is better endowed than the husband.

In the same way as in the preceding section, the transfer payment from husband to wife at the NBS is given by

$$P^1_D = \frac{1}{2} \left[ w - w \cdot (1 - g^f_D) \right.$$

$$\left. + w \cdot (g^m_D - g^f_D) + (v(h^f g^f_D) - v(h^m g^m_D)) \right] > 0, \tag{26}$$

$$P^2_D = \frac{1}{2} \left[ w(0) - w(g^f_D) \right.$$

$$\left. + (w(g^f_D) - w(g^m_D)) \right] > 0. \tag{27}$$

Again, $P^1_D$ compensates the wife for her forgone earnings, while $P^2_D$ makes up for part of her wage loss due to the she time spent in household production. Spouses do not share equally in the provision of the public good. In the divorce threat point, the wife’s higher household productivity is an advantage, her better overall endowment enables her to claim a higher share of marital resources in the NBS.

Equal productivities $h^f = h^m$. In this situation we obtain $g^f_D = g^m_D$, and therefore $v(h^f g^f_D) = v(h^m g^m_D)$, and the utility edge is zero. The transfer payments are the same as in the non-cooperative threat point case, $P^1_D = P^1_{NC}$, $P^2_D = P^2_{NC}$. This is due to the fact that, with equal productivities, both spouses make the same threat point contributions...
to the public good (their first order conditions coincide) regardless of the threat point specification. The threat point specification only matters if the wife has a comparative advantage in household production. A symmetric couple bears the cost of the public good provision in equal shares; the wife is not harmed because she is the sole public good provider at the bargaining outcome. She is fully compensated by her husband.

4.4 Comparison of the two threat point specifications

In the NBS, spouses’ relative threat point utility levels determine the distribution of the gains of cooperation within the couple. So to assess the importance of the distinction between non-cooperative marriage and divorce, we have to look at who is better off in each threat point.

Asymmetric productivities $h^f > h^m$. The husband is better off than the wife in the non-cooperative threat point, while in the divorce threat point, spouses’ relative utility levels tilt to her favour. With non-cooperative marriage as the threat point, the wife’s comparative advantage in household production allows the husband to free-ride on her public good provision. Since he can work for private consumption while she is busy providing the family public good, his utility is higher than hers. The wife has exactly the same utility in divorce as she has in non-cooperative marriage (her FOCs (15) and (23) coincide and $g_{NC}^f = g_D^f$), while his utility in the non-cooperative threat point is strictly higher. He consumes more of the public good, and, since he does not have to provide it, can devote all of his time to his private consumption.

If, on the other hand, divorce is the threat point in bargaining, the wife is better off than the husband. Since both are equally productive in market work, but she is more productive in household production, her utility if single is higher than his. Her comparative advantage in household production, that was a disadvantage in the non-cooperative marriage threat point, is an asset in the divorce threat point.

At the NBS, this difference in threat point utilities translates into the transfers from the husband to the wife. Table 1 summarizes these transfer payments. Both threat point settings are broken down according to periods. In period 1, the wife receives more if divorce, than if non-cooperative marriage were the threat point in bargaining. With non-cooperative marriage as the threat point, the husband must only persuade the wife to lift her contribution from $g_{NC}^f$ to $g_{NC}^f$. He does not need to compensate her for a contribution she would as well have made if there were no agreement ($g_{NC}^f$). But when divorced, he cannot free-ride on her public good provision. Not only must he remunerate her for lifting her contribution from $g_D^f$ to $g_{D}^f$, he also has to compensate her for half of the time he would have contributed to the public good if divorced. Additionally, she is able to extract an extra payment because she would enjoy more of the public good than he would. For the same reasons, the transfer corresponding to the second period is higher in the divorce
Table 1: Comparison of transfers $P^1_i$ and $P^2_i$ for both threat point specifications $i = NC, D$

<table>
<thead>
<tr>
<th></th>
<th>$P^1_i$</th>
<th>$P^2_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>NC</td>
<td>$\frac{1}{2} w \cdot (g_f - g_{fNC})$</td>
<td>$\frac{1}{2} (w(g_{fNC}) - w(g^{fs}))$</td>
</tr>
<tr>
<td>D</td>
<td>$\frac{1}{2} [w \cdot (g_f - g_{fD} + g_{mD}) + (v(h_f g_{fD}^m) - v(h_m g_{mD}))]$</td>
<td>$\frac{1}{2} [(w(0) - w(g_{fD}^m)) + w(g_{fD} - w(g^{fs}))]$</td>
</tr>
</tbody>
</table>

case than in the case of non-cooperative marriage as the threat point.

Equal productivities $h_f = h_m$. If the spouses are equally endowed, they have the same utility both in the divorce and in the non-cooperative threat point. Hence, the utility edge is zero for both threat point specifications, and the spouses split the gains to cooperation equally among them. The husband pays the wife a transfer of $\frac{1}{2} g_f$ in the first and $w(0) - w(g^{fs})$ in the second period - he compensates her for half of the direct cost of household production in the first, and for half of the associated wage loss in the second period. This outcome is “fair” in the sense that both partners share the cost of the public good provision in equal parts. Although the wife contributes more to the public good, she is fully compensated by her husband.

Thus, if the spouses had the possibility to choose between the two threat point specifications, the husband would opt for non-cooperative marriage if he had the comparative advantage in market work, while the wife would prefer divorce in this case. A symmetric couple would be indifferent between the two options.

We summarize our results in the following

**Proposition 2 (Efficiency under binding agreements)**
Under binding agreements across periods, there is full specialization within the couple. Efficiency is reached via monetary transfers, the sizes of which depend on the threat point specification. For both threat point specifications, the first and the second period transfer payments do not violate the husband’s budget constraints. A wife with a comparative advantage in household production is better off (and the husband is correspondingly worse off) if the threat point is divorce than if it is non-cooperative marriage. With equal productivities, the choice of the threat point does not matter for intra-household distribution.

*Proof. See Appendix.*

5 Binding agreements are not feasible

In a more realistic version of our model, we suppose that the couple can commit to labour supply and monetary transfers only within but not across periods. That is, at the beginning of marriage, only monetary transfers in the first period can be credibly assured, whilst payments in the second period are determined at the beginning of period 2. This reflects the fact that transfers within marriage typically cannot be legally enforced.
In the model without binding agreements, the spouses play two successive Nash bargaining games, one in each period. The outcome of the game in the second period depends on the NBS of the first period, because second period wage rates depend on first period labour supply. Since the spouses anticipate that their agreement in the first period will influence their agreement in the second period through their wage rates, this game combines cooperative and non-cooperative elements. Ott (1992) solves this problem by first deriving a “conditional solution” for the second period for given wage rates and then using the resulting indirect utility functions to find the NBS in the first period. We use a more direct approach. We start by determining the transfer in the second period, and let the couple then bargain on a subset of the original utility possibility set, that only contains utility pairs that can be reached given the size of the second period transfer.

5.1 The second period

In the second period, there is no public good, no gains to specialization and therefore no surplus to be divided. The second period payoffs of husband and wife for given contributions \(g_f^t\) and \(g_m^t\) in the first period are:

\[
U_f^t(g_f^t, g_m^t, P^2)_{t=2} = w(g_f^t) + P^2 \\
U_m^t(g_f^t, g_m^t, P^2)_{t=2} = w(g_m^t) - P^2,
\]

where \(P^2\) denotes the transfer payment from the husband to the wife in the second period.

Spouses’ threat points are also determined by their first period contributions \(g_f^t\) and \(g_m^t\). Note that in the second period, the distinction between the non-cooperative and the divorce threat point becomes obsolete, since there is no public good provision. In the threat point, each spouse simply controls his or her labour market income, there are no transfers:

\[
T_f^t(g_f^t)_{t=2} = w(g_f^t) \\
T_m^t(g_m^t)_{t=2} = w(g_m^t).
\]

From the maximization of the Nash product in the second period with respect to \(P^2\)

\[
(w(g_f^t) + P^2 - w(g_f^t)) \left( w(g_m^t) - P^2 - w(g_m^t) \right)
\]

it follows that the second period transfer without binding agreements is zero. In the absence of a public good, the husband has no reason to share his private consumption with his wife. If the second period transfer cannot be legally enforced, the wife will not trust in her husband making such a payment, and she will change her actions in the first period accordingly.
5.2 The first period

Let $P^1$ denote the transfer payment from the husband to the wife in the first period. Given that $P^2 = 0$ by the previous section, the sum of the spouses’ payoffs over both periods are given by:

\[ U_f(g^f, g^m, P^1) = w \cdot (1 - g^f) + v(h^f g^f + h^m g^m) + P^1 + w(g^f), \]  
\[ U_m(g^f, g^m, P^1) = w \cdot (1 - g^m) + v(h^f g^f + h^m g^m) - P^1 + w(g^m). \]  

(33)  

(34)  

The constraint $P^2 = 0$ alters the couple’s UPF. The UPF of the original problem (as illustrated in Figure 1) is a straight line whenever both spouses consume a strictly positive amount of the private good. Along this line, the sum of spouses’ utilities is constant and given by:

\[ 2v(h^f g^{f*}) + w \cdot (2 - g^{f*}) + w(0) + w(g^{f*}). \]  

(35)  

Any movement along this line only redistributes private consumption between the spouses while the provision level of the public good remains constant. At the north end of this line, the wife gets all the private consumption, while at the south end the husband does. The condition $P^2 = 0$ means that this redistribution is not possible for income earned in the second period. Thus, the linear part of the UPF is shortened to the line between the point where the wife controls all first period labour income and the husband only consumes the public good, and the point where the distribution is tilted to the other extreme and the husband gets all first period consumption. Figure 2 illustrates this modified UPF for the area in which the wife controls all first period consumption.\(^{16}\)

The modified UPF starts at point $A$ where the wife has the highest utility she can obtain while providing the socially optimal level of the public good, $\hat{U}_f = v(h^f g^{f*}) + w \cdot (2 - g^{f*}) + w(g^{f*})$. Because of the constraint, the only way to increase her utility at that point is to reduce her level of public good provision, boosting her private consumption. Reducing $g^f$ below $g^{f*}$ increases her utility up to point $B$ where her contribution equals her individual optimum $g^f_B$, $\underline{U}_f = v(h^f g^f_B) + w \cdot (2 - g^f_B) + w(g^f_B)$. At point $B$, the wife receives all the couple’s private consumption in the first period, and the consumption earned by herself in the second period. Because she provides less than the socially optimal level of the public good, the UPF of the model without binding agreements lies below the original UPF for all $U_f \geq \hat{U}_f$. It is non-linear because in this segment of the UPF, utility can only be redistributed between the spouses by varying $g^f$ (because the wife already controls the couple’s entire first period consumption, and second period consumption cannot be redistributed), and the $U_i$ are strictly concave in $g^f$. The liquidity constraint preventing second period income from being distributed means that the concave region of the UPF is enlarged compared to the original problem.

\(^{16}\)The north area of the modified UPF is not depicted because our assumption that the wife provides the public good renders it impossible that the NBS could lie on this segment.
Formally, the new segment of the UPF where the liquidity constraint is binding can be described as follows. For all values of $g^f$ such that $g^f \in [g^f_D, g^f_\ast]$, spouses’ utilities along the UPF are given by

\begin{align}
U^f &= v(h^f g^f) + w \cdot (2 - g^f) + w(g^f), \\
U^m &= v(h^f g^f) + w(0),
\end{align}

so the wife controls the couple’s entire first period labour income, and each spouse controls the second period labour income earned by themselves. The maximum utility the wife can obtain given that $P^2 = 0$ is at the maximum of this segment of the UPF, at the point where the public good provision level is the wife’s individual optimum: $\bar{U}^f = v(h^f g^f_D) + w \cdot (2 - g^f_D) + w(g^f_D)$. The wife’s lowest utility on this segment of the UPF is the point where she provides the socially optimal level of the public good $\hat{U}^f = v(h^f g^f_\ast) + w \cdot (2 - g^f_\ast) + w(g^f_\ast)$.

Since the wife controls the couple’s entire first period consumption in this area of the UPF, spouses’ utility only depends on the provision level of the public good. Define the set of all utility levels the husband can obtain in this region,

$$
\Omega = \left\{ u^m : U^m = v(h^f g^f) + w(0), \ g^f \in [g^f_D, g^f_\ast] \right\}.
$$

Because $U^m(g^f)$ is strictly increasing in $\Omega$, its inverse $U^{m^{-1}}(u^m) = g^f(u^m)$ exists. This
allows us to write the wife’s utility, $U_f(g^f)$ as a function of her husband’s, $f(u^m) \equiv U_f(g^f(u^m)), u^m \in \Omega$.

**Lemma 3**

The function $f(u^m)$ is strictly decreasing and strictly concave for all $u^m \in \Omega$.

*Proof.* See Appendix.

The intuition behind this result is simple. From (36) it follows that the husband’s utility increases in $g^f$. The wife’s utility must be decreasing for $g^f > g^f_D$ because $U(g^f)$ is concave and $g^f_D$ is its maximum. Hence, $f(u^m)$ must be a decreasing and strictly concave function of $u^m$ in this region.

The difference between the model with and without binding agreements is that the straight segment of the UPF is shorter in the latter model, because only first period labour income can be freely distributed between spouses. If the NBS of the model with binding agreements still lies on the straight segment of the UPF of the model without binding agreements, the NBS remains unchanged if the assumption of binding agreements is abolished. Put differently, if in the model with binding agreements the transfers $P^1$ and $P^2$ satisfy the condition

$$P^1 + P^2 \leq w, \quad (39)$$

the NBS of the model without binding agreements is the same as in the model with binding agreements. If this the case, the husband is rich enough in the first period to compensate the wife not only for her direct wage loss, but also for her lower wage rate in the second period. He can afford to compensate his wife in advance and thereby sidesteps the commitment problem. Therefore, the level of public good provision is also efficient. This situation is depicted in Figure 2.

If, on the other hand, the transfers of the model with binding agreements exceed the husband’s first period budget, i.e. $P^1 + P^2 > w$, the original NBS cannot be reached in the model without binding agreements and the outcome is not efficient as illustrated in Figure 3.

The Nash product is

$$(U_f(g^f, g^m, P^1) - T_{i^f}^f)(U_m(g^f, g^m, P^1) - T_{i^m}^m), \quad i \in D, NC, \quad (40)$$

where $T_{i^f}^f$ and $T_{i^m}^m$ denote the threat point utilities for the divorce and non-cooperative threat point as determined in Sections 4.2 and 4.3. To find the NBS, we maximize the Nash product (40) subject to the constraints that the transfer $P^1$ in the first period must be feasible, $P^1 \leq w \cdot (1 - g^m)$, and that the transfer $P^2$ in the second period is zero, $P^2 = 0$, as demanded by subgame perfection. If the constraint $P^2 = 0$ is binding, the new NBS must lie on the new segment of the UPF and can be found by maximizing (40) on $f(u^m)$. We summarize our results in the following
Figure 3: Inefficient outcome due to the liquidity constraint

\[ d(U_f - T_f)(U_m - T_m) = 0 \]

**Proposition 3 (Binding agreements not feasible)**

If the couple cannot commit to a transfer in the second period at the beginning of period 1, there will be no monetary transfers in period 2. Two cases can be distinguished:

1. The payments determined in section 4 satisfy the condition \( P^1 + P^2 \leq w \). The husband fully compensates the wife in the first period, and the outcome of the model is unaffected by the absence of binding agreements.

2. The husband cannot afford to fully compensate the wife in period 1, the payments determined in section 4 are such that \( P^1 + P^2 > w \). The wife receives the couple’s entire first period consumption. The first order condition for the wife’s public good provision when binding agreements are not feasible, \( g_{NB}^f \), is given by:

\[
 w - w'(g_{NB}^f) = \left(1 + \frac{U_f(g_{NB}^f) - T_f}{U_m(g_{NB}^f) - T_m}\right) h_f v'(h_f g_{NB}^f). \tag{41}
\]

In this case, the wife’s contribution to the public good, \( g_{NB}^f \), is increasing in the husband’s threat point utility and decreasing in the wife’s.

**Proof.** See Appendix.
Table 2: Comparison of first order conditions

<table>
<thead>
<tr>
<th>Situation</th>
<th>FOC</th>
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</thead>
<tbody>
<tr>
<td>Efficiency</td>
<td>$w - w'(g^f) = 2 \cdot h^f v'(h^f g^f)$</td>
</tr>
<tr>
<td>Threat point D or NC</td>
<td>$w - w'(g^f_D) = h^f v'(h^f g^f_D)$</td>
</tr>
<tr>
<td>No binding agreements</td>
<td>$w - w'(g^f_{NB}) = \left(1 + \frac{U^f(g^f_{NB}) - T^f}{U^m(g^f_{NB}) - T^m}\right) h^f v'(h^f g^f_{NB})$</td>
</tr>
</tbody>
</table>

If the constraint $P^2 = 0$ is binding, spouses’ threat point utilities not only influence the distribution of resources within the couple, but also the efficiency of the public good provision. Changes in his and her threat point utility push $g^f_{NB}$ in different directions. To see why, note that in the NBS, the wife’s utility must increase if her threat point utility goes up. Since the wife’s utility is decreasing in $g^f$ on $f(u^m)$, it is clear that $g^f_{NB}$ must go down with an increase in the wife’s threat point. Because the wife’s utility is at least as high in the divorce threat point as it is in the non-cooperative threat point, $g^f_{NB}$ is at least as large if non-cooperative marriage is the threat point in bargaining as it is if spouses use divorce as the disagreement point.

This inefficiency is entirely due to the lack of a commitment device and is robust to the specification of the payoff function (in contrast to the full specialization efficiency result of Section 4.1). In fact, assuming more sophisticated payoff function would in most cases increase the probability of an inefficient outcome.

**Corollary 1**

*If binding agreements are not feasible and the constraint $P^2 = 0$ is binding, the wife’s contribution to the public good $g^f_{NB}$ lies between her individual optimum $g^f_D$ and the couple’s joint optimum $g^f$:

$$g^f_D < g^f_{NB} < g^f.$$ (42)*

Her individually optimal contribution $g^f_{NB}$ must lie in this interval because we found it by maximizing the Nash product on $f(u^m)$ which is defined on this interval. The first inequality stems from the fact that the wife would contribute $g^f_D$ without a transfer of $w$ from her husband, the second is due to the concavity of $f(u^m)$ and the fact that $P^2 = 0$ is binding.

### 5.3 Policy implications

The inefficiently low provision of the family public good in our model arises because the spouse specialized on a labour market career (in our model the husband) cannot credibly commit to compensate his or her partner for foregone career opportunities later in life. The most direct way to eliminate this inefficiency would hence be for the husband to sign a binding contract at the beginning of marriage and therein pledge himself to compensate his wife later. If that were possible, our model would predict an efficient provision level...
of the public good. Unfortunately, such contracts are generally not legally enforceable. In this section, we quickly review other policies that could mitigate the inefficiency. If we imagine the family public good to be the quantity and quality of children, it might also be in the government’s interest to step in.

**Borrowing and saving**

One way for the husband to avoid the commitment problem is to “pay” his wife up front to compensate her for her loss of second period earnings in the first period. This is what he does in our model if his first period income is big enough for such a payment. Empirically, we observe substantial returns to labour market experience. Given the importance of seniority as a determinant of the hourly wage rate, it seems unlikely that a young man should be able compensate his wife for a lower wage rate (that she has to suffer throughout her entire working life) before starting a family.

He could however try to obtain the necessary funds on the capital market. We believe that it would be very difficult if not impossible for the average husband to take out a loan of the necessary magnitude to compensate his wife ex ante, even more so if he has young children and needs credit for other things, like buying a house. Another theoretical possibility would be for the husband to issue a debt certificate to his wife at the beginning of marriage, payable at a later date. This would allow him to avoid the capital market, borrowing from his wife instead. However, the signing of debt certificates in new couples is nearly unheard of. Also, this would create a new moral hazard incentive problem both for the wife (an unconditional payment is agreed for future services) and the husband (he agrees to share future earnings that depend on effort). Therefore we think that ruling out borrowing and saving in our model aptly describes the situation of young partnerships.

**Divorce law and alimony**

While the distribution of resources within marriage is generally not regulated, monetary transfers after a divorce are stipulated by law in most industrialized countries. Indeed, the existence of divorce legislation that incorporates alimony payments contingent on the time spent child on rearing suggests that the law acknowledges the inadmissibility of binding agreements within marriage over long periods of time. It assures compensation for foregone labour market earnings in the event of divorce that can not be guaranteed in an intact marriage.

Since divorce law determines the distribution of resources only when divorced, it influences our model only if the threat point is assumed to be divorce. With an alimony payment $P_D^2$ assured to her by law, the wife’s threat point utility in the second period rises from $w(g')$ to $w(g') + P_D^2$, while the husband’s is diminished by the same amount. By plugging this into the objective function (32) it can easily be seen that the second period
transfer without binding agreements is the compensation that would be due to the wife in the event of divorce, \( P_2^{D} \). This extends the part of the UPF where the socially optimal amount of the public is produced, and hence increases the probability that the NBS lies on it. Ideally, the legal alimony payment should depend on the time spent out of the labour market to bring it as close as possible to the original \( P^2 \) that is probably unknown to policy makers, but can be estimated. Also, divorce legislation should not be changed quickly, so individuals can rely on future payments. Observe that in the context of our model, spousal support legislation also benefits the husbands (at least ex-ante).

**Public child care**

The inefficiency within our model originates in the couple’s inability to effectively exploit the gains to specialization, that household production and learning by doing on the labour market offer them. Instead of trying to eliminate the lack of commitment, policy makers could also try to get rid of the need for specialization within families and promote specialization between households instead, since child care professionals do not suffer from foregone labour market experience.\(^{17}\) The public provision of child care eliminates the need for specialization within couples and thereby avoids its harmful effect on the spouse who falls short of acquiring labour market experience. Many industrialized countries heavily subsidize childcare facilities, that is, many governments provide child care as a benefit in kind. Translated into our framework, this amounts to an exogenous increase in \( G \). Because of quasilinear utility, the couple withdraws their private contribution in response to the public provision of \( G \) in order to maintain the level \( h^f g^f_{NB} \). All private contributions have to be crowded out before the total amount of \( G \) begins to rise, but a sufficiently generous subsidy to child care facilities can indeed improve efficiency within our framework.

Because it enables young mothers to focus more on their careers, public provision of child care reduces not only direct, but also indirect costs of children, and can therefore stimulate fertility.\(^{18}\)

With respect to intra-household distribution, the public provision of child care is to the advantage of the wife in the non-cooperative marriage threat point. It crowds out her contribution, enabling her to work more on the labour market like her husband. So, the introduction of free public child care facilities tilts intra-household distribution to the

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\(^{17}\)In fact, we would expect the emergence of private markets for child care. But market provided child care has to use taxed labour while family provided care is tax free. Which one is cheaper overall depends on the tax system and the importance of learning by doing (which is likely to differ between individuals). We think that tax wedges are important enough in most industrialized countries to inhibit the emergence of legal and completely unsubsidized markets for childcare, that offer prices affordable to all families (for a similar argument, see Konrad and Lommerud, 1995).

\(^{18}\)Indeed, there is anecdotal evidence that countries that generously provide child care like France and Sweden have higher birth rates than countries that concentrate on monetary transfers to support families, like Germany and Austria.
wife’s favor, if the threat point is non-cooperative marriage. With a divorce threat point, both spouses equally benefit from free child care in the threat point, so intra-household distribution in the first period remains unchanged. The wife of course benefits in both threat point scenarios from her higher wage rate in the second period.

6 Conclusions

This paper highlights that, as in many other areas of economics, commitment problems do exist within the household and that they may cause inefficient outcomes. Following sensible assumptions about the effect of career interruptions on wage rates, we illustrate with a simple two period model that it is efficient that only one spouse contributes to a family public good like housework or child care, while the other spouse concentrates on her/his career. This is also true if husband and wife are ex-ante equally productive in household and market work. Specialization arises endogenously as a consequence of the provision of the family public good in our model, and not in its strategic anticipation or because of an exogenous productivity asymmetry, as in most models in the literature.

But this specialization within the family can result in inefficient household allocations: due to lack of commitment devices, too little of the joint household public good may be provided. If the spouse specialized in market work (who is typically the husband) can credibly commit her/himself to share the fruits of her/his labour market experience with their partner later in life, the couple achieves the efficient provision level of the public good. But, if such an intertemporal commitment is not feasible, the wife withdraws part of her contribution to the public good below the socially optimal level. She anticipates that, once there is no more public good production (e.g. because children have grown up and left home) the husband has no reason to make any payments to her. This inefficiency is a direct consequence of the liquidity constraint faced by the couple, and is not caused by the spouses trying to manipulate their threat points in later stages, as in other models in the literature.

We devote special attention to the specification of the threat point that is used in the NBS, since there is yet no consensus in the literature on what is a sensible threat point in a family bargaining setting. We find that the choice of the threat point does have important implications for the distribution of resources within the couple. If the threat point is non-cooperative behaviour within marriage, the husband can still enjoy the wife’s public good provision, which is impossible if the couple is divorced. Hence, the spouse that has the comparative advantage in public good provision benefits from a switch of the threat point from non-cooperative marriage to divorce. Which threat point is more likely to be used in marital bargaining can only be determined by empirical studies that can clearly discriminate between the two; unfortunately, to date we are not aware of such a study. However, divorce is unlikely to be the threat point in marital bargaining if terminating a
marriage is not really an option in the social environment of a couple. We may speculate that, as divorce becomes more common in the Western world, wives are more and more able to credibly threat with divorce in marital disputes.

Appendix

Proof of Proposition 1 (Efficiency).

It may be optimal that one spouse fully specializes on market work, so we have to maximize the joint intertemporal utility \( U(g^f, g^m) \) subject to the non-negativity constraints \( g^f \geq 0 \) and \( g^m \geq 0 \). The Kuhn-Tucker first order conditions are

\[
\frac{\partial U}{\partial g^f} = -w + 2v'(G)h^f + w'(g^f) \leq 0, \quad \frac{\partial U}{\partial g^m} = 0, \quad g^f \geq 0, \quad g^m \geq 0. \tag{43}
\]

Our Inada assumptions regarding the spouses’ preferences imply that the solution to the joint maximization problem (6) involves a strictly positive provision level of the family public good \( \bar{G} > 0 \), so we cannot obtain \( g^f = g^m = 0 \). They also imply that both \( g^f < 1 \) and \( g^m < 1 \). Thus, it suffices to show that only one spouse contributes to the family public good and that, in the case of asymmetric productivities, the only contributor is the spouse with the higher productivity.

Joint utility \( U \) can be expressed as a function of \( g^f \) and \( \bar{G} \), since \( g^m = \frac{1}{h^m} \bar{G} - \frac{1}{h^m} h^f g^f \):

\[
U(g^f, \bar{G}) = F(g^f, \bar{G}) + w(g^f) + w(\bar{G} - g^f), \tag{45}
\]

where \( F(g^f, \bar{G}) := w \cdot (1 - g^f) + w \cdot (1 - g^m(\bar{G})) + 2v(h^f g^f + h^m g^m(\bar{G})) \).

Consider first the case of equal productivities, \( h^f = h^m \). The contributions \( g^f \) and \( g^m \) in the term \( F(\cdot) \) are perfect substitutes and thus spouses’ individual contributions are irrelevant as long as their sum equals \( \bar{G} \). Therefore, for a given \( \bar{G} \), \( U(g^f, \bar{G}) \) is maximized whenever the sum \( w(g^f) + w(\bar{G} - g^f) \) is maximized. Since \( w \) is a convex function, this sum is maximized when the values \( g^{f*} \) and \( g^{m*} = \bar{G} - g^{f*} \) are on the border of the interval at a corner solution where \( g^{f*} = \bar{G} \) and \( g^{m*} = 0 \) or, alternatively, \( g^{m*} = \bar{G} \) and \( g^{f*} = 0 \).

Consider now the case \( h^f > h^m \). By the same argument, the sum \( w(g^f) + w(\bar{G} - g^f) \) is maximized at the corners of the interval, that is at \( g^{f*} = \bar{G} \), \( g^{m*} = 0 \) and \( g^{m*} = \bar{G} \), \( g^{f*} = 0 \). But \( g^f \) and \( g^m \) are no perfect substitutes in the term \( 2v(h^f g^f + h^m g^m) \) any more. If \( h^f > h^m \), the term \( 2v(h^f g^f + h^m g^m) \) is greater if \( g^{f*} = \bar{G} \) than if \( g^{m*} = \bar{G} \). Therefore, if \( h^f > h^m \), the only efficient outcome is the corner solution \( g^{f*} = \bar{G} \) and \( g^{m*} = 0 \). QED.
Proof of Lemma 1 (Non-cooperative marriage).

For each spouse, the first and second order conditions (FOC and SOC) are

\[
\frac{\partial U^i}{\partial g^i} = -w + v'(h^i g_{NC}^i + h^{m} g_{NC}^m)h^i + w'(g_{NC}^i) \leq 0, \\
\frac{\partial U^i}{\partial g^i} \cdot g_{NC}^i = 0, \quad g_{NC}^i \geq 0, \\
\frac{\partial U^i}{\partial g^i} \cdot g_{NC}^i = 0, \quad g_{NC}^i \geq 0, \\
\frac{\partial U^i}{\partial g^i} \cdot g_{NC}^i = 0, \quad g_{NC}^i \geq 0,
\]

Consider first the case \( h^i = h^m \). We will show by contradiction that \( g_{NC}^i = g_{NC}^m \). Suppose \( g_{NC}^i \neq g_{NC}^m \), and without loss of generality let \( g_{NC}^i > g_{NC}^m \). Then, because of the convexity of \( w(g^i) \), the RHS of (15) (the wife’s first order condition) is larger than the RHS of (16) (the husband’s). Since the LHS of both conditions are equal, equality can only hold for one spouse. If it holds for the husband, the concavity of \( v(h^i g_{NC}^i + h^{m} g_{NC}^m) \) would induce the wife to withdraw part of her contribution, since her marginal cost would otherwise exceed her marginal gain from the public good. If it holds for the wife, the husband would increase his contribution, because his marginal gain from the public good exceeds his marginal cost. Hence, \( g_{NC}^i = g_{NC}^m \). Observe that our Inada assumptions (7) guarantee that \( g_{NC}^i = g_{NC}^m \) are strictly positive.

Now consider the case \( h^i > h^m \). First, observe that \( g_{NC}^i \neq g_{NC}^m \). If they were equal, both conditions could not hold simultaneously, because the marginal utility from the public good has to be the same for both spouses. Then, assume that \( g_{NC}^m \in (g_{NC}^i, 1] \). If this were true, \( w - w'(g_{NC}^m) > w - w'(g_{NC}^i) \) because of the convexity of \( w(\cdot) \), and both conditions cannot hold because \( \frac{1}{h^m} > \frac{1}{h^i} \). Finally, consider \( g_{NC}^m \in (0, g_{NC}^i) \). Rearrange (15) and (16) such that

\[
h^i \cdot v'(h^i g_{NC}^i + h^{m} g_{NC}^m) = w - w'(g_{NC}^i), \\
h^m \cdot v'(h^i g_{NC}^i + h^{m} g_{NC}^m) = w - w'(g_{NC}^m).
\]

Clearly, the LHS of (48) is larger than the LHS of (47). If both conditions hold with equality we have: \( w - w'(g_{NC}^m) > w - w'(g_{NC}^i) \), rearranging yields \( w'(g_{NC}^m) > w'(g_{NC}^i) \). Because \( w(g^i) \) is decreasing and convex, this implies that \( g_{NC}^m > g_{NC}^i \), which leads to a contradiction. Thus, both spouses cannot be in an interior solution, and \( g_{NC}^m = 0 \).

What remains to be shown is that \( g_{NC}^i < g_{NC}^{i*} \). First, observe that \( g_{NC}^i \neq g_{NC}^{i*} \), for if it were, the RHS of (10) and (48) would coincide, resulting in their LHS to be equal as well which clearly cannot be the case. To see that \( g_{NC}^i < g_{NC}^{i*} \), multiply the RHS of (10) by two. Because the RHS is positive, the RHS must then be larger than the LHS:

\[
2 \cdot h^i v'(h^i g_{NC}^{i*}) < 2 \cdot (w - w'(g_{NC}^{i*})).
\]
By (48), this condition holds with equality at $g^f = g^f_{NC}$, so if $g^{f*} < g^f_{NC}$, it would have to be increased to equal $g^f_{NC}$. Increasing $g^f$ reduces both sides of the condition, but the first derivative of the RHS must be larger than the first derivative of the LHS since $U^i(g^f, g^m)$ is globally concave and (9) has to hold in $g^{f*}$. Therefore increasing $g^f$ would would shrink the RHS of (49) slower than the LHS, aggravating the equality. Hence, $g^{f*}$ cannot be smaller than or equal to $g^f_{NC}$, and must therefore be larger. QED.

Proof of Lemma 2 (Divorce).

That $g^f_D = g^m_D$ if $h^f = h^m$ follows from the first order conditions (23) and (24). For the case of asymmetric productivities, one cannot say whether a higher household productivity leads to more or less time spent on household work. The first order conditions only allow us to state that the “expenditure” $h^f g^f_D$ on $G$ by the more productive spouse must be larger than the “expenditure” of the less productive spouse $h^m g^m_D$. That the wife’s contribution to the public good is the same in both threat point specifications is obvious since the first order conditions (15) and (23) coincide for both threat point specifications. QED.

Proof of Proposition 2 (Efficiency under binding agreements).

The NBS is efficient by construction, so by Proposition 1 there is full specialization within the couple. Lemmas 1 and 2 and Table 1 imply that, if the wife has a comparative advantage in household production, she is better off (and the husband is correspondingly worse off) if the threat point is divorce than if it is non-cooperative marriage, and that with equal productivities, the choice of the threat point does not matter for intra-household distribution. It only remains to show that, for both threat point specifications, the first and the second period transfer payments do not violate the husband’s budget constraints.

Consider first the individual budget constraints when the threat point is non-cooperative marriage. In the first period, the husband has an income of $w$, which is greater than the first period transfer $\frac{1}{2}w : (g^{f*} - g^f_{NC})$, since $g^{f*} - g^f_{NC} < 1$. His second period income is $w(0)$, which again is greater than the second period transfer $\frac{1}{2}(w(g^f_{NC}) - w(g^{f*}))$, since both $g^f_{NC} > 0$ and $g^{f*} > 0$, therefore $w(0) > w(g^f_{NC})$ and $w(0) > w(g^{f*})$ and thus $w(0) > \frac{1}{2}(w(g^f_{NC}) - w(g^{f*}))$.

For the divorce threat point, direct calculation shows that if $T^f_D + T^m_D < U^{f*} + U^{m*}$, then $P^f_D < U^{m*}$. QED.

Proof of Lemma 3 ($f(w^m)$ is decreasing and strictly concave).

To see that $f(w^m)$ is decreasing, fix two arbitrary points $u^m_1$ and $u^m_2$ with $u^m_1 \in \Omega$ and $u^m_2 \in \Omega$ such that $u^m_1 > u^m_2$. Since $u^{m-1}(w^m) = g^f(u^m)$ is strictly increasing, $g^f(u^m_1) > g^f(u^m_2)$. $u^f(g^f)$ must be strictly decreasing for $g^f \in (g^f_D, g^f_{NC})$ because $g^{f*} > g^f_D$ and $g^f_D$ is

29
the maximum of this function. Hence, \( U^f(g^f(u^m_1)) < U^f(g^f(u^m_2)) \), which is equivalent to stating that \( f(U^m_1) < f(U^m_2) \).

We now show that \( f(U^m) \) is strictly concave. Again, fix \( u^m_1 \) and \( u^m_2 \) with \( u^m_1 \in \Omega \) and \( u^m_2 \in \Omega \) such that \( u^m_1 > u^m_2 \). Fix any \( \alpha \in [0, 1] \) and define \( g^f_1 = g^f(u^m_1) \), \( g^f_2 = g^f(u^m_2) \) and \( g^f_3 = \alpha g^f(u^m_1) + (1 - \alpha)g^f(u^m_2) \). Since \( u^f(g^f) \) is strictly concave, we have

\[
U^f(g^f_3) > \alpha U^f(g^f_1) + (1 - \alpha)U^f(g^f_2).
\]

Define \( u^m_3 = \alpha u^m_1 + (1 - \alpha)u^m_2 \). Since \( g^f(U^m) \) is strictly convex (because \( U^m(g^f) \) is strictly concave),

\[
g^f(u^m_3) < \alpha g^f(u^m_1) + (1 - \alpha)g^f(u^m_2),
\]

and hence \( g^f(u^m_3) < g^f_3 \). Moreover, since \( U^f(g^f) \) is strictly decreasing for \( g^f \in (g^f_1, g^f_3) \), (51) implies that \( U^f(g^f(u^m_3)) > \alpha U^f(g^f(u^m_1)) + (1 - \alpha)U^f(g^f(u^m_2)) \), which, since \( f(u^m) = U^f(g^f(u^m)) \), establishes that \( f(u^m) \) is strictly concave. QED.

**Proof of Proposition 3 (Binding agreements not feasible).**

If the sum of the payments determined in section 4 is lower than the husband’s first period income, \( P^1 + P^2 \leq w \), the constraint \( P^2 = 0 \) is not binding in the model without binding agreements - the husband can afford to pay the wife both \( P^1 \) and \( P^2 \) in the first period. Consequently, it does not influence the maximization problem, which establishes case 1.

Now consider the case where the parameters of the model are such that the solution of the original problem, as discussed in section 4, incorporates payments from husband to wife that exceed the husband’s first period income. Then, all utility pairs on the UPF where \( U^f \in [\widehat{U}^f, \overline{U}^f] \) are of the form described in (37) and (36), and only depend on \( g^f \) (because the wife controls all income that can be distributed between the spouses). \( U^m \) can be written as a function of \( g^f \), and since \( U^m(g^f) \) is strictly increasing in \( g^f \), its inverse exits:

\[
U^{m^{-1}}(u^m) = g^f(u^m) = \frac{1}{h^f} \cdot v^{-1}(u^m - w(0)).
\]

Hence, \( U^f(g^f) \) can be written as a function of \( u^m \):

\[
f(u^m) \quad \equiv \quad U^f(g^f(u^m)) = u^m - w(0) + \frac{1}{h^f} \cdot v^{-1}(u^m - w(0))
\]

\[
+ w \left( \frac{1}{h^f} \cdot v^{-1}(u^m - w(0)) \right).
\]

To find the NBS, we have to maximize the Nash product, \( (U^f - T^f)(U^m - T^m) \) on the UPF, \( U^f(u^m) = f(u^m) \). This is equivalent to maximizing \( (f(u^m) - T^f)(U^m - T^m) \) over \( u^m \). The first order condition is

\[
-f'(u^m)(U^m - T^m) = (f(U^m) - T^f).
\]

30
Plugging in the first derivative of (53) and rearranging yields condition

$$w - w'(g_{NB}^f) = \left(1 + \frac{U^f(g_{NB}^f) - T^f}{U^m(g_{NB}^f) - T^m}\right) h^f v'(h^f g_{NB}^f).$$

(55)

To establish that $g_{NB}^f$ decreases with a rise in $T^f$, fix the NBS of a bargaining problem that lies on $f(u^m)$ and consider a small increase of $T^f$ in the NBS. This diminishes $\frac{U^f(g_{NB}^f) - T^f}{U^m(g_{NB}^f) - T^m}$ on the RHS of condition (41). Since the RHS is positive, the LHS must now be larger than the RHS. To arrive at the new NBS, $g_{NB}^f$ must be lifted or reduced. Increasing $g_{NB}^f$ results in $v'(h^f g_{NB}^f)$ diminishing faster than $-w'(g_{NB}^f)$ by the concavity condition (9).

Also, $U^f(g_{NB}^f)$ is decreasing in $g_{NB}^f$, while $U^m(g_{NB}^f)$ is increasing in $g_{NB}^f$. This reduces $\frac{U^f(g_{NB}^f) - T^f}{U^m(g_{NB}^f) - T^m}$. Both effects aggravate the inequality. Hence, $g_{NB}^f$ must be reduced to arrive at the new NBS. To prove that $g_{NB}^f$ rises with an increase in $T^m$, reverse this argument.

QED.

References


