Firm formation and economic growth:  
the effects of labour union bargaining power  
and of worker mobility  

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September 2010
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19 September 2010

Key words: 
Labour unions, wage bargaining, entrepreneurs, overlapping generations, economic 
growth, labour mobility, income distribution.

JEL Numbers: E10, J50

Abstract
A model is presented where economic growth is co-determined with the number of 
entrepreneurial firms as functions of union wage bargaining power and of inter-firm 
labour mobility. There is an inverse-U relationship between economic growth and the 
number of firms, if they are both heterogeneous and operate under decreasing returns 
to scale. If labour is immobile, economic growth is greatest where unions have a 
moderate degree of wage bargaining power, because this deters less able 
entrepreneurs from setting-up firms without discouraging too many of the more able 
ones. However, if labour is highly mobile, economic growth is greatest where union 
wage bargaining power is very weak - although not necessarily greater than in the 
immobility case - because the anticipation that workers can switch from lower to 
higher ability/wage firms acts as a very powerful entry deterrent for all but the highest 
ability entrepreneurs. Between these extremes, the model points to two empirical 
findings, a the positive correlation between the wage and firm-size, and, for some 
parameter values, a negative cross-country relationship between economic growth and 
income inequality, because countries with more labour mobility should have less 
wage inequality as well as greater allocative efficiency.

1 Thanks are due to seminar participants at an Economic Theory Workshop at the University of 
Nottingham, particularly to Arijit Mukherjee and Fabrice Defever, and at the Money, Macro and 
Finance Conference at the Cyprus University of Technology, September 2010. The usual disclaimer 
remains.

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1. Introduction
The main purpose of this paper is to investigate the effects of labour union wage bargaining power on macroeconomic activity through its effect on firm formation. The number of firms in an economy becomes an important consideration, once they are acknowledged either as being heterogeneous or as operating outside constant returns to scale, both of which are here assumed. If firms differ in productivity, but commonly have constant returns to scale production functions, aggregate output will be highest with the fewest and most productive firms present. Conversely, if firms are symmetric, but subject to decreasing returns, aggregate output will be increasing in their number. So if firms are both heterogeneous and subject to decreasing returns, there may be an inverted U-shaped relationship between their number and aggregate output, which would peak at an interior value. The position of an economy on this inverted-U curve will depend, not least, on the degree of union bargaining power, as a key variable governing the entry of firms.

If the economic growth rate, as well as the number of firms, is also endogenous, following an investment externality, according to Romer (1986) or Lucas (1988), the accumulative effects of their number on trend output will, over the course of time, come to dominate any business cycles effects. Thus, we focus on the economic growth effect of firm entry, but abstracting from the dynamics aspects of that might drive a Schumpeterian process, as considered by Aghion and Howitt (1992). Nor do we consider the effects of entrepreneurship, whether innovative as in Aghion and Howitt or imitative as in Schmitz (1989), but merely the number of entrepreneurs as being synonymous with the number of firms, where each is owned with a single entrepreneur. One aspect of the analysis is to show how endogenous firm formation might alter the comparative statics of the relationship between union wage bargaining power and economic growth. An analysis of corporate taxation with Laffer Curve effects would be analogous, especially if aggregate tax revenue should depend on the number of firms.
Modelling entry and exit means necessary means eschewing the more usual treatment of firms as impersonal, mechanistic entities that merely return factor incomes - wages, interest and rents - to their recipients - workers, lenders and landlords. The inclusion of entrepreneurs, as the owners of firms’ organisational structures rather than of firms’ capital stocks, suggests they are motivated by a desire to earn entrepreneurial profits rather than merely to return residual profits. In this two-period OLG model, entrepreneurs are deemed to be equivalent to other young households, namely, workers, to the extent they engage in economic activity and save in the first period of their lives. Each young household decides to be one or the other on the basis of weighing up the expected prospective income returns, entrepreneurial profits and wages. Consequently, there are three broad classes of household, young workers, young entrepreneurs and old households, the last having previously supplied either a labour or an entrepreneurial input. The intra-generational distribution of income between wages and entrepreneurial profits is important for economic growth through affecting firm formation, while the inter-generational distribution of income between the young and old plays no role in the basic set-up with binding labour contracts.

Firm formation and wage determination are closely knit for several reasons. First, both the incentive for firms to enter and the scope for wage bargaining are predicated on the existence of surplus profits, making it difficult to think of one activity without the other. Secondly, payments to labour, in comprising the major part of total production costs, are of obvious importance for the employer-employee income differentials as the basis for the entry decisions. Wage pressure from strong labour unions should limit the number of firms, and, as this model shows, this may be either for good or bad in terms of economic growth. Furthermore, in a setting of general equilibrium, the aggregated effects of decentralized wage bargaining are generally known to feedback on the local conditions facing the atomistic firm and labour union [Moene, Wallerstein and Hoel (1993)].

Much research has already been devoted to looking at the possible effects of labour union wage bargaining on economic activity, which is both too extensive and intensive too review here. Thematically, however, if economic activity \( (y) \) depends
on employment \((l, \partial y/\partial l > 0)\), and if the essential purpose of labour unions is to raise wages \((w)\) above existing competitive or reservation levels, the main union effect may be gauged by the sign of the derivative \(\partial l/\partial w\). If, according to a more mainstream view, firms exert their right-to-manage, where employment is confined to a downward-sloping demand curve, \(\partial l/\partial w < 0\), unions are evidently harmful to economic activity. Conversely, in monopsony models with upward-sloping labour supply curves, \(\partial l/\partial w < 0\), a more positive view of union bargaining power may be taken. And, where there is full-employment at a maximum participation level \((m)\), \(l = m\), it follows, trivially, that \(\partial y/\partial w = 0\), unless wage bargaining elicits some other productivity response.

In addition, as mentioned, there has been increasing attention given to a more recent class of dynamic models, initiated by Romer (1986) and Lucas (1988), containing the property of non-decreasing returns in a general measure of the capital stock, so that aggregate saving, \(s\), drives long-run economic growth. In models such as these the sign of the derivative \(\partial s/\partial w\) trumps that of \(\partial l/\partial w\) for the long-term consequences.

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3 There are other union effects beyond the scope of this paper that are considered extensively in Booth (1995).

4 A moral hazard problem then arises if governments rather than unions take responsibility for providing unemployment insurance by causing the cost of aggressive wage bargaining to be externalized. There is then another, detrimental effect on economic activity throughout the economy, if these public unemployment benefits are financed through distortionary taxes.

5 Generally, apart from the possibility of restrictive practices discouraging investment by the firm, positive cross-productivity effects in the production function mean that the aggressive wage bargaining will also reduce the firm’s demand for capital as well as its level of employment.

6 Layard and Nickell (1990) show that this result extends to the model of McDonald and Solow (1991) where firms and unions bargain wages and employment simultaneously, if the reservation wage, which determines the position of an efficient contract curve, is indexed to aggregate wages.

7 A moral hazard problem then arises if governments rather than unions take responsibility for providing unemployment insurance by causing the cost of aggressive wage bargaining to be externalized. There is then another, detrimental effect on economic activity throughout the economy, if these public unemployment benefits are financed through distortionary taxes.

8 The consideration of labour market monopsony has been rehabilitated by Manning (2003) in emphasizing the role of imperfect information as a basis rather than the traditional view that it rested on single city-firms. Since union bargaining power will then raise employment and output, there are further potential general equilibrium benefits as benefit payments and distortionary taxes are reduced.

9 For example, Booth (1995) suggests that the reduction of X-inefficiency [Leibenstein (1966)] provides a possible benefit for union bargaining power.
of factors like union bargaining power. Decomposing aggregate saving, $s$, into 
$\sigma \equiv s/y$ and $y$, wage bargaining effects may work by affecting the average saving 
rate, $\sigma$, for given aggregate incomes or by altering these, $y$, for a given $\sigma$. The 
decomposition implies

$$\frac{\partial s}{\partial w} \equiv y (\frac{\partial \sigma}{\partial w}) + \sigma (\frac{\partial y}{\partial l})(\frac{\partial l}{\partial w}).$$

The first effect constitutes the well known Kaldorian growth effect, which is 
demonstrated most clearly within a full employment context: capital accumulates 
faster, if the income redistribution favours individuals with higher than average saving 
rates [See Kaldor (1957)]. Although in Kaldor, by assumption, “capitalists” save 
proportionally more than “workers”, the opposite arises as a result in the basic two-
period form of the Diamond (1965) life-cycle model, where “young households” work 
and save, while “old households” own the capital stock and dis-save by running down 
their accumulated assets.

The second effect depends, $\partial l/\partial w$, clearly depends on the same factors relevant in to 
static models but with consequences for rates of change in dynamic models instead of 
levels in static ones. Since the focus of this present paper is on impact of union 
bargaining power through influencing the number of firms, we choose to isolate this 
particular effect by modelling an economy at full employment, so that $\partial l/\partial w = 0$, 
and also preclude the Kaldorian redistribution effect in its basic form, so that 
$\partial \sigma/\partial w = 0$.

The paper thus isolates an additional route, the entry decision of entrepreneurial firms 
by assuming that, initially, each young household chooses to become either an 
entrepreneur or a worker on the basis of the relative expected incomes, depending on

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11 Although this is assumed in this earlier non-optimizing model, it arises as a result under 
optimization within an infinite-horizon version of Romer’s (1986) endogenous growth model in Bertoli 
(1993), provided that labour is regarded strictly as a non-accumulated factor in the sense there is no 
accumulation of human capital.

and/or sufficiently rising wage-profiles within multi-period models.

12 Bequest saving by the old modifies this result, as would the admission of financial market 
imperfections and/or of sufficiently rising wage-profiles within multi-period models.
known own entrepreneurial ability, determining a personal prospective profit income, relative to the expected or actual of all prospective entrepreneurs, out of which wages are determined. Households then engage in the production process when young, either as entrepreneurs or workers, and then retire in the second period to live off their accumulated asset income.

Entrepreneurs receive a profit income after they have paid both wages to workers, their contemporaries, and the interest income to the old. So, while the feature that old households collectively own the economy’s capital stock is maintained, it is implicitly assumed that young entrepreneurs as the initiators of firms own them as organizational structures. Entrepreneurial profits are bargained shares of the income that remains after payments to capital.

The distinctive feature of the model is that wage bargaining affects the intra- rather than the inter-generational distribution of income. Under full employment increased bargaining power would affect aggregate saving if workers and entrepreneurs had different saving rates. As common saving rates are assumed, wage bargaining can only influence aggregate saving through interest rates changes affecting the inter-generational redistribution of income. In the discussion that follows the main analysis, we point that this would occur for a case of non-binding labour contracts, as considered by Deveureux and Lockwood (1991).

Behaviour consists of the following sequence of actions. Initially, young households decide whether to become entrepreneurs or workers, and the latter locate to entrepreneur/firms and form local labour unions. On location, workers then come to realise the entrepreneurial abilities of their own and of all other employers. There are then two absolute possibilities: an immobility case where workers are stuck with their initial choice of entrepreneur-firm and a mobility case where workers may freely move from lower ability to higher ability ones. For each possibility, there are configurations of wages and profits. Each young household then saves a portion of his or her income, and, finally, the saving of all households are aggregated to form the economy’s capital stock.
The main analysis covers two polar and manageable cases. One is the complete immobility case. Here labour is allocated equally across disparate firms based on the initial ignorance of relative entrepreneurial abilities, and a wage distribution follows commensurate with these realised abilities. The main result is that an intermediate degree of relative labour union power in wage bargaining is beneficial for economic activity and growth, because this is maximized at an interior number of firms because of a trade-off between the average ability of firms and decreasing returns to scale. For the second polar case of full labour mobility, an arbitraging process leads to firms paying the same wage but with the more able ones being able to attract a more than proportional supply of workers. The associated and anticipated feature of a corresponding distribution of profits deters all but the most able firms from establishing firms. In these circumstances, even a moderate degree of relative union bargaining may be very harmful for economic growth.

There are also two offshoots from the main analysis. An intermediate case of labour mobility would imply that more able entrepreneurs would both pay higher wages and employ more workers. This replicates the robust empirical relationship found that is defined as the “employer-size wage effect”. Secondly, then distributional aspect of the model implies there may be a relationship between income inequality and economic growth, which is negative where union bargaining power is relatively weak, because labour mobility leads both to wage equalization and allocative efficiency.

The set-up of the paper is as follows. In Section 2 aggregate saving and capital accumulation is determined from the last two. In section 3 deals with the production sector, the factor demands and the wage bargain. The main analysis is considered in Sections 4 and 5 that consider the respective cases of no and full labour mobility. Section 6 extends the discussion and Section 7 gives a brief summary of the analysis.

2. The basic model
2.1 Households

Households, indexed \( h \), live for and derive utility from consumption, \( c_{h,t}^Y \) and \( c_{h,t+1}^O \), in two periods. They have Cobb-Douglas utility functions,

\[
U_{h,t} = c_{h,t}^Y \frac{1}{(1+\theta)} c_{h,t+1}^O \frac{1}{(1+\theta)}, \quad \forall h, \text{ where } \theta, \; 0 < \theta < 1, \text{ is a time-preference parameter with a common value for all households.}
\]

Their budget constraints are

\[
c_{h,t}^Y + c_{h,t+1}^O / R_{t+1} = z_{h,t}^Y, \quad \forall h, \text{ where } R_{t+1} \text{ is a common interest return factor accruing in the second period on first period saving; } z_{h,t}^Y \text{ is a first-period, heterogeneous income from activity. All households are retired in the second period when they live off previous period savings and do not receive any endowment or pension income. Utility is maximized by the individual household by the choice of saving at,}
\]

\[
s_{h,t} = (\theta/1+\theta)z_{h,t}^Y, \quad \forall h
\]

which in aggregate is

\[
s_t = (\theta/1+\theta) \int_0^1 z_{h,t}^Y dh, \quad \text{where } s_t \equiv \int_0^1 s_{h,t} dh
\]

Symmetric preferences and the linearity of the saving-income relationship imply that aggregate saving is immune from intra-generational income distribution effects, thus precluding a possible Kaldorian effect on economic growth. The indirect utility of each household is solved as

\[
U_{h,t} = (1+\theta)^{-1} (\theta R_{t+1})^{\theta/(1+\theta)} z_{h,t}^Y \quad \forall h
\]

With full depreciation of the capital stock within one period and with the same lag length for aggregate financial saving, \( s_t \), to transform into physical investment, capital accumulation is given by

\[
k_{t+1} = s_t = (\theta/1+\theta)z_t^Y, \quad \text{where } y_t^Y = \int_0^1 y_{h,t}^Y dh
\]
We may skimp on notation by hatting the two future variables, $k_{t+1}$ and $R_{t+1}$, as $\hat{k}$ and $\hat{R}$, and by presenting current variables, $x_t$, without time-subscripts as $x$. The notation for the economic growth factor is $G$ where $G \equiv \hat{k}/k$.

2.2 Production

Output in firm $i$ is given by the following production function

$$y_i = B e_i^{1-\alpha} m_i^\beta k_i^{\alpha}, \quad \alpha < \beta, \quad 0 < \alpha + \beta < 1, \quad \text{for } m_i \geq 1$$

(4)

It contains two parameters, general total factor productivity, $B$, and firm-specific entrepreneurial ability, $e_i$, and two variable inputs, labour, $m_i$, and capital, $k_i$, each of which is now considered in turn.

2.2.1 Total factor productivity

The common term for total factor productivity, $B$, is exogenous to the individual firm, but depends on a general learning-by-doing externality, following Arrow (1962) and Romer (1986). Knowledge is a by-product of private investment, $k_i$, which becomes a public good relating to the aggregate stock of capital, $k$,

$$B = Ak^{1-\alpha} \quad \text{where } k \equiv \int k_idi$$

(5)

2.2.2 Entrepreneurial ability

The output of each firm also depends on the innate ability, $e_i$, of its single entrepreneur, $i$. The distribution of entrepreneurial abilities is important for the general equilibrium, and we assume a uniform distribution,

$$e_i \sim U(\bar{e} - \sqrt{3}\sigma, \bar{e} + \sqrt{3}\sigma).$$

(6)

to obtain analytical solutions to the model. Its supports are presented in terms of the mean and standard deviation, $\sigma$ - with the mean of the distribution, $\bar{e}$ - in order to focus on the dispersion. The population mean may be negative, $\bar{e} < 0$, which would imply that less than half of the population would be able to make a positive profit when
playing an entrepreneurial role. The condition $\bar{e} > -\sqrt[3]{3}\sigma$ is necessary to ensure that least some of them would be so able to do.

According to equation (6), the person who is ranked $i$th has the ability level, $e(i) = \bar{e} + \sqrt[3]{3}\sigma(2i - 1)$. Designating $\lambda$ as the proportion of firms, a key endogenous variable and assuming that the more able are the first to start firms, the individual who is indifferent between being an entrepreneur and a worker would have the critical level of ability,

$$e_C(\lambda) = \bar{e} + \sqrt[3]{3}\sigma(1 - 2\lambda),$$

(7)

The uniform nature of the distribution implies that the average ability of all those who choose to start firms is $1/2 e_C(\lambda) + 1/2(\bar{e} + \sqrt[3]{3}\sigma)$ or

$$E(e) = \left( e_C(\lambda) + (\bar{e} + \sqrt[3]{3}\sigma) \right)/2 = \bar{e} + \sqrt[3]{3}\sigma(1 - \lambda)$$

(8)

2.2.3 Labour

The whole workforce belongs to a firm-specific unions each with a membership of $m_i$, which must be at least as large as employment, $m_i \geq l_i$. The assumptions, (i) that household utilities are linear in income, (ii) that unions have utilitarian objectives, and (iii) that unemployment incomes are zero, together imply that the objective of labour unions is to maximize the wage bill. Since this is decreasing in the wage, since the Cobb-Douglas production generates an elastic demand for labour, the union and the firm will always concur upon wage reduction wherever $l_i \geq m_i$. Consequently, there will always be bargaining at full employment where $l_i = m_i$.13

The term $m_i$ is defined as being exclusive of the single entrepreneur, so that equation (4) shows that it takes at least two individuals, one worker plus the single entrepreneur in order to produce an output. Thus, the production function exhibits

13 This technology gives rise to a constant elasticity of labour demand, implying that in the event of unemployment the wage would be determined as a multiple of the unemployment utility. As this is assumed to be zero, the result of a zero wage implies that unemployment cannot be a possibility.
increasing returns locally where \( m_i \) is small, but decreasing returns ensue where \( m_i \) is large, because \( \beta + \alpha < 1 \) and since \( e_i \) is a parameter unrelated to the scale of output.\(^{14}\)

### 2.2.4 Capital

The analysis considers the case where the firm’s investment decision is simultaneous to the wage bargain. The alternative where wage contracts are not binding, so that the union may renegotiate after the capital stock has been installed, following Grout (1984), is taken up in the subsequent discussion rather than in the main analysis in the interest of brevity.

### 2.2.5 Entrepreneurial profit

Entrepreneurial profit is given by

\[
\pi_i = Ak^{1-\alpha}e_i^{1-\alpha}m_i^{-\beta}k_i^{-\alpha} - w_i m_i - Rk_i
\]

where \( w_i \) is the wage of firm \( i \) and \( R \) is the total and common cost of capital, comprising the sum of the interest and depreciation rates under the 100% depreciation assumption.

### 2.3 Wage bargaining and investment

The generalized Nash bargaining solution is applied to determine the wage. It is well known that this may be obtained by maximizing a Nash function, a geometrically weighted sum of the two bargaining surpluses,

\[
N_i = \left( U_i^\pi - \hat{U}_i^\pi \right)^{1-\sigma} \left( U_i^w - \hat{U}_i^w \right)^{\sigma},
\]

and, in this case, with respect to the wage. The weight, \( \sigma \), \( 0 \leq \sigma \leq 1 \), represents the relative bargaining power of the union, and is the key parameter of the model.

\(^{14}\) \( y_i = 0 \), if the firm employs a single household - either where \( m_i = 1 \) but with no entrepreneur or with one entrepreneur where \( m_i = 0 \); but that \( y_i = Be_i^{1-\alpha}k_i^{-\alpha} > 0 \) if the firms employs two households, an entrepreneur and a worker.
Equation (2) implies that if the two parties agree, they will each receive a payoff linearly related to their income, $U_{h,i} = \Omega z_{h,i}^Y$, $\Omega = (1 + \theta)^{-1} \left( \frac{\hat{R}}{\theta} \right)^{\theta/(1+\theta)}$, whether a wage or a profit income, $z_{h,i}^Y = w_i$, $Ak^{1-\alpha}e_i^{1-\alpha}m_i^{\beta}k_i^{\alpha} - w_im_i - Rk_i$. In the event of disagreement, each party is assumed to receive zero, $\hat{U}_i^w = 0$, $\hat{U}_i^\pi = 0$.

The Nash bargaining solutions for the wage and profit on each firm are

$$w_i = \frac{\sigma}{m_i} \left( Ak^{1-\alpha}e_i^{1-\alpha}m_i^{\beta}k_i^{\alpha} - Rk_i \right)$$

(10)

$$\pi_i = (1-\sigma) \left( Ak^{1-\alpha}e_i^{1-\alpha}m_i^{\beta}k_i^{\alpha} - Rk_i \right)$$

(11)

The wage bill and profit are proportional to output net of the costs of capital with the proportions given by the bargaining weights, $\sigma$ and $1-\sigma$.

Simultaneously, the firm determines the profit-maximizing investment level,

$$k_i = \left( \alpha Am_i^{\beta}R^{-1} \right)^{1/\alpha} e_i k$$

(12)

This equation is the basis for a solution for the interest factor. First, aggregation gives the aggregate or mean investment level across firms as

$$k = \left( \alpha AR^{-1} \right)^{1/\alpha} E(m^{\beta/(1-\alpha)}e-k)$$

then, after removing the common factor and inverting the remaining expression,

$$R = \alpha A E(m^{\beta/(1-\alpha)}e)^{1-\alpha}$$

(13)

is obtained. Substituting this back into equations (10)-(12) gives

$$w_i = \sigma (1-\alpha) A \left( E(m^{\beta/(1-\alpha)}e) \right)^{\alpha} m_i^{-(1-\alpha-\beta)/(1-\alpha)} e_i k$$

(14)

$$\pi_i = (1-\sigma)(1-\alpha) A \left( E(m^{\beta/(1-\alpha)}e) \right)^{-\alpha} m_i^{\beta/(1-\alpha)} e_i k$$

(15)

$$k_i = \frac{m_i^{\beta/(1-\alpha)} e_i}{E(m^{\beta/(1-\alpha)}e)} k$$

(16)
Applying the aggregate forms of these to equation (3) for capital accumulation gives a general expression for the growth factor,

$$G = \left(\theta / 1 + \theta \right) (1 - \alpha) \lambda A \left( E \left( m^{\beta/(1-\alpha)} e \right) \right)^{-\alpha}$$

(17)

### 3. Occupational choice and the allocation of workers to firms

#### 3.1 The time structure of information

First, irrespective of the time-structure, the standard rational expectations assumption is maintained: individuals know the economic model, including its aggregate parameter values and the functional relationship. Consequently, they know, according to equations (14) and (15), that any firm’s wage $w_i$, is increasing in the ability of its entrepreneur, $\partial w_i / \partial e_i > 0$, but decreasing in the number of workers it employs, $\partial w_i / \partial m_i < 0$, while that the profit of any firm, $\pi_i$, is increasing both in the level of the ability of its entrepreneur, $\partial \pi_i / \partial e_i > 0$ and in the number of its workers, $\partial \pi_i / \partial m_i > 0$.

However, at an initial stage, individuals have limited information of individual parameters, namely, the potential entrepreneurial abilities of other households. It is at this stage that they must choose either to establish firms, becoming employers, or to allocate themselves to entrepreneur-firms as employees, where also and collectively they will establish local labour unions. Their knowledge of the distributional parameters implies they also know the equilibrium solution in the proportion of employers, $\lambda$, and the average ability, $E(e)$, which allows them to correctly anticipate the average wage, $E(w)$, but not a specific wage pertaining to any particular firm, $w_{ih}$, which pertains to the unknown ability level, $e_{ih}$, of a particular household. Thus, while they know the actual profit income they themselves would receive as a prospective employer, according to the exact form of equation (15), they only know the average wage they might receive as an employee based on an expectational version of equation (14).
\[
E(w) = \sigma (1 - \alpha ) A \left( E\left( m^{\beta/(1-\alpha)} e \right) \right)^{-\alpha} E\left( m^{-(1-\alpha-\beta)/(1-\alpha)} e \right) k
\] (18)

### 3.2 The indifference condition

The choice of occupation is based on the expected income of being a worker \( E(w) \) relative to that of the known income of being an employer \( \pi_i \), because of the linearity of the income-utility relationship and because of the absence of any other factors affecting utility. An individual will become an entrepreneur if \( \pi_i > E(w) \) and a worker, if \( \pi_i < E(w) \). An individual indifferent between these two occupations, for whom \( \pi_C = E(w) \), will have characteristics that satisfy,

\[
(1 - \sigma)m_C^{\beta/(1-\alpha)} e_C = \omega E\left( m^{-(1-\alpha-\beta)/(1-\alpha)} e \right)
\] (18)

This is obtained by setting equations (15) and (18) into equality and by removing the common factors. The term \( m_C \) is the number of workers in a firm that is led by an entrepreneur who himself is just indifferent to becoming a worker. The proportion of entrepreneur/firms, \( \lambda \), may eventually be solved by combining this equation with (7) and (8), but after solving also for the terms, \( m_C \) and \( E\left( m^{-(1-\alpha-\beta)/(1-\alpha)} e \right) \).

The solutions for these constituent terms depend on the particular mobility assumptions of the model, of which there are two polar possibilities. One is that workers (and entrepreneurs) are completely immobile between firms, so that they are stuck with their initial allocation and occupation decisions they made based on imperfect information. The other is where workers are completely mobile throughout, so that their initial allocation choices are immaterial to the ultimate equilibrium solution of the model. These two will be treated separately in the following two Sections. An intermediate case of imperfect mobility may also be considered but is more easily included in the later discussion as a (non-linear) combination of the two polar cases under consideration rather than in the main analysis as a separate case.

### 4. With no ex post labour mobility
4.1 The allocation of households to firms and the proportion of firms

In the absence of any degree of \textit{ex post} labour mobility, the \textit{ex ante} allocation of households to firms, by definition, persists as the \textit{ex post} allocation. Then, we suggest that the actual allocation will be according to the following result.

\textbf{Result One:} In the absence of \textit{ex post} labour mobility, (i) there is an equal allocation of workers across firms, $m_i = \overline{m} \forall i$, with the proportion of entrepreneurs/firms at $\lambda = (1 + \overline{m})^{-1}$; and (ii) a wage distribution with $w_i = w(e_i, \overline{m})$, $\partial w_i / \partial e_i > 0$.

Proof: By definition, the initial allocation persists, based on individuals not having prior knowledge of the various abilities levels of prospective employers. (i) We show that $m_i = \overline{m} \forall i$, if there is no reason for the allocation decision other than considerations of prospective relative incomes. Suppose by contradiction that $m_p > m_q$, the derivative sign, $\partial w_i / \partial m_i < 0$ then implies households would believe that $\hat{w}_p < \hat{w}_q$, since there is no basis on which they would believe anything other than $\hat{w}_p = \hat{w}_q$. This would cause an arbitraging movement of workers from firm $q$ to firm $p$ - within the initial allocation process - until beliefs that $\hat{w}_p = \hat{w}_q$ emerged whereat $m_p = m_q$. In addition, if $m_i = \overline{m} \forall i$, then $\lambda = (1 + \overline{m})^{-1}$ under the assumption of a single entrepreneur per firm. (ii) Equation (14) then implies that wages \textit{ex post} will be positively (linearly) related to firm specific-abilities.

Thus in the absence of \textit{ex post} labour mobility, employment is equalized across firms but with a wage distribution that reflects the distribution of entrepreneurial abilities.

4.2 The solution with no \textit{ex post} labour mobility (case A)

Applying Result One and equations (7) and (8) to equations (13)-(17) and using the subscript $A$ to indicate this particular immobility case, gives
\[ R_A = \alpha A \left( \bar{\varepsilon} + \sqrt{3} \sigma (1 - \lambda_A) \right)^{-\alpha} \left( \lambda_A^{-1} - 1 \right)^{\beta} \]  
(13A)

\[ w_{A,i} = \sigma (1 - \alpha) A \left( \bar{\varepsilon} + \sqrt{3} \sigma (1 - \lambda_A) \right)^{-\alpha} \left( \lambda_A^{-1} - 1 \right)^{\beta - 1} e_i k \]  
(14A)

\[ \pi_{A,i} = (1 - \sigma)(1 - \alpha) A \left( \bar{\varepsilon} + \sqrt{3} \sigma (1 - \lambda_A) \right)^{\alpha} \left( \lambda_A^{-1} - 1 \right)^{\beta} e_i k \]  
(15A)

\[ k_{A,i} = e_i / E(e) k \]  
(16A)

\[ G_A = (\theta/1 + \theta)(1 - \alpha) A \left( \bar{\varepsilon} + \sqrt{3} \sigma (1 - \lambda_A) \right)^{-\alpha} \lambda_A \left( \lambda_A^{-1} - 1 \right)^{\beta} \]  
(17A)

Note there is generally a non-monotonic relationship between the economic growth and the proportion of firms, \( \lambda_A \), which enters the equation thrice. Its first appearance has an unambiguously negative effect, because the presence of more firms spells a lower average ability level with the least able individuals being the last to become entrepreneurs. The remaining two appearances \( \lambda_A \) in equation (17A) indicate the variable scale effect. Returns are decreasing, \( \lambda_A < 1 - \beta \), where the that the proportion of firms is relatively small or where the number of workers per firm is relatively large, \( m < \beta/(1 - \beta) \). Here, the growth-maximizing proportion of firms, \( \lambda_A^* \), may lie within the interior, \( 0 < \lambda_A^* < 1 - \beta \). The following result gives a more precise statement of the possibilities.

**Result 2:** With no ex post labour mobility, economic growth is highest where the proportion of firms is \( \lambda_A^* \), where \( \lambda_A^* = \min(\tilde{\lambda}_A, 0.5) \),

\[ \tilde{\lambda}_A = \tilde{C}_A - \sqrt{\tilde{C}_A^2 - \tilde{D}_A}, \quad \tilde{C}_A = \frac{3 - \alpha - \beta + \bar{\varepsilon}/\sqrt{3} \sigma}{2(2 - \alpha)} \]

\[ \tilde{D}_A = \frac{(1 - \beta)(1 + \bar{\varepsilon}/\sqrt{3} \sigma)}{2 - \alpha}, \quad \frac{\partial \tilde{\lambda}_A}{\partial \sigma} < 0, \]  
where an interior solution,

---

\[ (\lambda^{-1} - 1)w_i + \pi_i \]  is the combined income of all young households attached to firm \( i \), and \( (\lambda^{-1} - 1)w + \pi \) for the average firm and \( \lambda \left( (\lambda^{-1} - 1)w + \pi \right) \) for the economy average with \( \lambda \) firm. The solutions with equation (2) give \( G_A \).
\[ \lambda_A^* = \tilde{\lambda}_A < 0.5, \text{ requires } \beta > 0.5 \left( \frac{\bar{e} + \alpha \sqrt{3\sigma}}{\bar{e} + \sqrt{3\sigma}} \right) \text{ for which } \beta > 0.5 \text{ is a sufficient condition since } \alpha < 1. \]

If firms are symmetric, so that \( \sigma = 0 \), then \( \tilde{\lambda}_A = 1 - \beta \), provided that \( \beta > 0.5 \) and then \( \lambda_A^* = \tilde{\lambda}_A = 1 - \beta < 0.5 \). This is at the point of constant returns to scale. Raising the dispersion of ability levels (\( \sigma > 0 \)) then introduces a negative effect, \( \partial G_A / \partial \lambda < 0 \), on top of this, because the average entrepreneurial ability level is lower with a greater number of firms. Growth is then highest where \( \lambda < 1 - \beta \), which falls within the region of decreasing returns, where the scale benefit of having more firms each employing fewer workers just offsets the reduction in the average ability purchased by this increase.

In a model with constant returns to scale, the number of firms would not matter for aggregate activity, if they all happened to be identical, so union power in affecting \( \lambda \) would be neutral in the present model as it stands. However, under constant returns and for any degree of dispersion in ability, aggregate output would be monotonically decreasing in the number of firms and, hence, strictly increasing in the relative degree of union wage bargaining power. In general, however, to recap, restricting the entry of firms may raise aggregate activity for two reasons: first, there are positive scale effects under increasing returns; and, secondly, average entrepreneurial ability is greatest, since the least able individuals are the last set up firms.

Also, applying Result One and equations (7) and (8) to (19) determines the proportion of firms,

\[ \lambda_A = C_A - \sqrt{C_A^2 - D_A}, \text{ where } C_A = \frac{1}{2} \left( 3 + \frac{\bar{e}/\sqrt{3\sigma}}{2 - \sigma} \right) \]

\[ D_A = \left( \frac{1 - \sigma}{2 - \sigma} \right) \left( 1 + \frac{\bar{e}}{\sqrt{3\sigma}} \right), \quad \frac{\partial \lambda_A}{\partial \sigma} < 0 \]  

(19A)
Naturally, the proportion of firms is decreasing in the relative union wage bargaining power, since $\partial \lambda_A / \partial \sigma < 0$. Also, note that while technically as $\sigma \rightarrow 1$, $\lambda_A \rightarrow 0$ and as $\sigma \rightarrow 0$, $\lambda_A \rightarrow 1$, if production requires at least one worker per firm, then $0 < \lambda_A \leq \lambda_{A, \min} = 0.5$ becomes an additional requirement, which places a minimum value for the union bargaining weight at $\bar{\omega}_A = 2\bar{\epsilon} / \left(4\bar{\epsilon} + \sqrt{3\sigma}\right) \leq 0.5$ as $\sigma \geq 0$ and $\partial \bar{\omega}_A / \partial \sigma < 0$.

Finally, we may establish the first main result that concerns the effect of union bargaining power by combining equation (19A) with Result Two in the following.

**Proposition 1**: In the absence of ex post labour mobility, provided $\beta$ and/or $\sigma$ are not too small, economic growth is maximized at an intermediate degree of relative wage bargaining power, $\bar{\omega}_A^*, \bar{\omega}_A^* > \bar{\omega}_A^{\min}$, where (ii) $\bar{\omega}_A^*$ and $\bar{\omega}_A^{\min}$, where $\bar{\omega}_A^* / \partial \sigma > 0$ and $\partial \bar{\omega}_A^{\min} / \partial \sigma < 0$.

Proof of Proposition 1: (i) Consider, $\sigma = 0$ and $\beta > 0.5$, then Result Two implies that $\lambda_A^*(0) = 1 - \beta < 0.5$ and equation (20A) that $\bar{\omega}_A^*(0) = \beta > 0.5$. Equation (19A) then gives $\bar{\omega}_A^{\min}(0) = 0.5$, so that $\bar{\omega}_A^*(0) > \bar{\omega}_A^{\min}(0)$. (ii) As Result Two and equation (20A) together imply $\partial \bar{\omega}_A^* / \partial \sigma > 0$ [because $\partial \lambda_A^* / \partial \sigma < 0$ and $\partial \lambda_A / \partial \sigma < 0$] and equation (19A) implies $\partial \bar{\omega}_A^{\min} / \partial \sigma < 0$, it follows that if $\beta > 0.5$, $\partial(\bar{\omega}_A^* - \bar{\omega}_A^{\min}) / \partial \sigma > 0$ and $\bar{\omega}_A^* > \bar{\omega}_A^{\min}$, $\forall \sigma$.

We note that where this proposition does not always hold, say, for example, where $\sigma = 0$ and $\beta < 0.5$. Economic growth is then an increasing function of $\lambda$ over its relevant range $(0,0.5)$ and so is highest where $\lambda_A(0)^* = 0.5$. The obverse is that growth is a decreasing function of $\sigma$, union bargaining power, and highest where unions are feasibly weakest at the feasible minimum, $\sigma = \bar{\omega}_A^{\min}$. The following Table furnishes some numerical results for the purpose of illustration.
Table One: The proportion of firms, economic growth and the growth-maximizing and maximal levels with no ex post labour mobility [case (A)]

<table>
<thead>
<tr>
<th>$\varpi$</th>
<th>$\lambda_A$</th>
<th>$G_A$</th>
<th>$\lambda_A$</th>
<th>$G_A$</th>
<th>$\lambda_A$</th>
<th>$G_A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.50</td>
<td>0.500X</td>
<td>0.42</td>
<td>0.702X</td>
<td>0.33</td>
<td>0.381X</td>
</tr>
<tr>
<td>0.7</td>
<td>0.30</td>
<td>0.499X</td>
<td>0.27</td>
<td>0.643X</td>
<td>0.23</td>
<td>0.396X</td>
</tr>
<tr>
<td>0.9</td>
<td>0.10</td>
<td>0.374X</td>
<td>0.095</td>
<td>0.577X</td>
<td>0.09</td>
<td>0.338X</td>
</tr>
</tbody>
</table>

Growth maximizing and maximal values

<table>
<thead>
<tr>
<th>$\varpi_A^*$</th>
<th>$\lambda_A^*$</th>
<th>$G_A^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.60</td>
<td>0.40</td>
<td>0.51X</td>
</tr>
<tr>
<td>$\varpi_A^*$</td>
<td>$\lambda_A^*$</td>
<td>$G_A^*$</td>
</tr>
<tr>
<td>0.64</td>
<td>0.31</td>
<td>0.723X</td>
</tr>
</tbody>
</table>

$X \equiv (\theta/1+\theta)(1-\alpha)A\mu^{1-\alpha}$

The proportion of firms is also decreasing in the population dispersion of ability levels relative to their mean. The assigned values imply that economic growth is highest where union bargaining power is roughly twice as great as that of employers.

5. **Ex post labour mobility**

5.1 **The allocation of households to firms and the proportion of firms**

At a later stage, following the initial allocation, all entrepreneurial ability levels become known. There is then an incentive for workers who initially attached themselves to lower ability entrepreneurs to relocate to high ability ones, because [from equation (14)] $\partial w_i/\partial e_i > 0$. There is also an incentive for prospective employers to accept them - and for existing employers to be reluctant to lose them - because [from equation (15)] $\partial \pi_i/\partial m_i > 0$. The fact that incumbent workers would not want to accept later arrivals, because $\partial w_i/\partial m_i < 0$, is immaterial to this mobility case. Thus, if insider power is the basis for the immobility assumption of the previous Section, it is absent by construction in the present one.
**Result Three:** Ex post labour mobility implies (i) a common wage, $w_i = w_B = \sigma(1-\alpha)A\left\{E(e^{\phi})\right\}^{1-\alpha-\beta}(\lambda_B^{-1}-1)^{\beta-1}k$, \(\forall i\), where $E(e^{\phi}) = \frac{1}{1+\phi}\left(\frac{e + \sqrt{3}\sigma}{2\sqrt{3}\sigma}\right)^{\alpha+\phi} - \left(\frac{e + \sqrt{3}\sigma(1-2\lambda)}{2\sqrt{3}\sigma\lambda}\right)^{\alpha+\phi}$ and (ii) a distribution of employment, \(m_i/m_j = (e_i/e_j)^{\phi}\ \forall i, j\), where $\phi \equiv (1-\alpha)/(1-\alpha-\beta) > 1$.

There is now no wage distribution, but, for a given $\lambda$, a greater dispersion of employment since $\phi \equiv (1-\alpha)/(1-\alpha-\beta) > 1$.

### 5.1 The bargaining outcome with ex post labour mobility

Applying Result 2 for full ex post labour mobility instead of Result 1 to equations (13)-(17), now indexed with $B$, to denote full labour mobility, gives the solutions

\[
R_B = \alpha A \left\{E(e^{\phi})\right\}^{1-\alpha-\beta}(\lambda_B^{-1}-1)^{\beta}
\]

(13B)

\[
w_B = \sigma(1-\alpha)A\left\{E(e^{\phi})\right\}^{1-\alpha-\beta}(\lambda_B^{-1}-1)^{\beta-1}k, \ \forall i
\]

(14B)

\[\pi_{B,i} = (1-\sigma)(1-\alpha)A\left\{E(e^{\phi})\right\}^{-(\alpha+\beta)}(\lambda_B^{-1}-1)^{\beta}ke_i^{\phi}, \ \forall i\]

(15B)

\[k_{B,i} = \left(e_i^{\phi}/E(e^{\phi})\right)k
\]

(16B)

\[G_B = (\theta/(1+\theta))\lambda_B(1-\alpha)A\left\{E(e^{\phi})\right\}^{1-\alpha-\beta}(\lambda_B^{-1}-1)^{\beta}
\]

(17B)

**Result Four:** If $\sigma > 0$, ex post labour mobility leads to a higher growth factor, $G_B > G_A$, for a given proportion of firms, $\lambda = \lambda_A = \lambda_B$.

Proof: Comparing equations (13A-15A) and (17A) with (13B-156B) and (17B) shows that all factor returns and growth are higher, $R_B > R_A$, $w_B > E(w_A)$, $E(\pi_B) > E(\pi_A)$ and $G_B > G_A$, because of the application of Jensen’s inequality, $\left(E(e^{\phi})\right)^{1/\phi} > E(e)$, since $\phi \equiv (1-\alpha)/(1-\alpha-\beta) > 1$. 

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This is an allocative efficiency result. With the full mobility of labour, for a given number of firms, the more able ones are able to obtain a larger number of workers than in the previously immobility case, and, with cross-effects in the production function, also acquire commensurately more capital. Wage equality is consistent with an unequal distribution of employment across heterogeneous firms. Cross-effects in the production function also cause a greater inequality of investment productivities across firms, so that the more able will also have a more than proportional allocation of capital, $k_i/k_j = (e_i/e_j)\phi\forall i, j, \phi > 1$. For example, if employer $j$ is more able than employer $i$ by a factor of 1.2 and if $\alpha = 0.3$ and $\beta = 0.6$, instead of $j$ having relatively more of each input by a factor of 1.2, as in the immobility case, it is now has more by a factor of 3.6.

**Result Five:** If $\sigma > 0$, the growth-maximizing proportion of firms is higher under ex post labour mobility, $\lambda_B^* > \lambda_A^*$.

Proof: Equations (17A) and (17B) have a number of common factors, including $\lambda(\lambda^{-1} - 1)^\beta$, but are differentiated by the respective terms $(E(e))^{1-\alpha}$ and $(E(e^\phi))^{1-\alpha-\beta}$. As $\lambda \to 0$, $(E(e))^{1-\alpha} \to (E(e^\phi))^{1-\alpha-\beta} \to e_{\text{max}}^{1-\alpha}$; but if $\lambda > 0$, $(E(e))^{1-\alpha} < (E(e^\phi))^{1-\alpha-\beta} < e_{\text{max}}^{1-\alpha}$, it then follows that $\partial(E(e))^{1-\alpha}/\partial \lambda < \partial(E(e^\phi))^{1-\alpha-\beta}/\partial \lambda < 0$, so that $\partial G_A/\partial \lambda < \partial G_B/\partial \lambda$ and that at $\partial G_A/\partial \lambda = 0$ where $\lambda = \lambda_A^*$, $\partial G_B/\partial \lambda > 0$, so that $\lambda_B^* > \lambda_A^*$.

The intuition for this particular Result is less obvious than that for the previous one, but is related. Under labour immobility, the fact that lower ability firms have the same number of workers as higher ability is costly in terms of economic growth. The only way for the more productive firms to obtain more workers is for there to be fewer firms in total. If the more productive firms may obtain more workers by the process of labour mobility instead, there is not the same advantage in having less firms, so the growth-maximizing proportion of firms rises. The question remains,
however, of the determination of the equilibrium proportion of firms in the full mobility case?

### 5.2 The proportion of firms with ex post labour mobility

Although economic growth will be higher under labour mobility for a given proportion of firms, we now show that the proportion of firms will lower for a given bargaining power parameter. The indifference condition is obtained by setting equations (14B) and (15B) for the marginal firm into equality. After removing the common factors, 

\[
(1 - \sigma)\phi_C^\phi (\lambda_B^{-1} - 1) = \sigma E(e^\phi),
\]

which, according to the distributional assumption underlying equations (7) and (8) gives,

\[
\frac{1 - \sigma}{\sigma} = \frac{1}{1 + \phi} \left( \frac{\bar{\varepsilon} + \sqrt{3}\sigma}{\bar{\varepsilon} + \sqrt{3}\sigma(1 - 2\lambda_B)} \right)^\phi - \frac{(\bar{\varepsilon} + \sqrt{3}\sigma(1 - 2\lambda_B))}{2\sqrt{3}\sigma(1 - \lambda_B)} \quad (19B)
\]

**Result Six:** If there is full ex post labour mobility, the proportion of firms is lower, \( \lambda_B < \lambda_A \), for any given degree of union relative bargaining power, \( \sigma = \sigma_A = \sigma_B \).

**Proof:** Using the definition \( x = \frac{\bar{\varepsilon} + \sqrt{3}\sigma}{\bar{\varepsilon} + \sqrt{3}\sigma(1 - \lambda)} \), equation (19B) is

\[
F \equiv \frac{1 - \sigma}{\sigma} - \frac{1}{1 + \phi} \left( x^{1+\phi} - 1 \right) \left( \frac{\bar{\varepsilon} + \sqrt{3}\sigma(1 - 2\lambda)}{2\sqrt{3}\sigma(1 - \lambda)} \right)
\]

and

\[
\frac{\partial F}{\partial \phi} = -\frac{1}{(1 + \phi)^2} \left( (1 + \phi)x^{1+\phi} \ln x - (x^{1+\phi} - 1) \right) \left( \frac{\bar{\varepsilon} + \sqrt{3}\sigma(1 - 2\lambda)}{2\sqrt{3}\sigma(1 - \lambda)} \right)
\]

Consider the central term \( z \equiv (1 + \phi)x^{1+\phi} \ln x - (x^{1+\phi} - 1) \). We show that \( \frac{\partial F}{\partial \phi} \leq 0 \) as \( \lambda \geq 0 \). First, as \( \lambda \to 0, x \to 1, z \to 0 \), whereat \( \frac{\partial F}{\partial \phi} \to 0 \).

Second, because \( \frac{\partial x}{\partial \lambda} > 0 \) and \( \frac{\partial z}{\partial x} = (1 + \phi)^2 x^\phi \ln x \), if \( \lambda > 0 \) so that \( x > 1 \), \( \frac{\partial z}{\partial x} = (1 + \phi)^2 x^\phi \ln x > 0 \), \( z > 0 \) and \( \frac{\partial F}{\partial \phi} < 0 \). Then it is apparent by inspection that \( \frac{\partial F}{\partial \lambda} < 0 \), so that \( \frac{\partial \lambda}{\partial \phi} < 0 \).
The intuition happens to be more obvious than the proof. Under labour mobility, prospective entrepreneurs of a lower ability anticipate they would subsequently lose workers to higher ability ones, which implies a cost in terms of lower profits, since \( \frac{\partial \pi_i}{\partial m_i} > 0 \); and this acts as a major entry deterrent.

There are now three Results for the mobility case. Results Five and Six together imply the following main result.

**Proposition Two:** The growth-maximizing level of union relative bargaining power is lower under ex post labour mobility, \( \sigma_B^* < \sigma_A^* \).

Proof: If the growth-maximizing proportion of firms were the same in each case, \( \lambda_A^* = \lambda_B^* = \lambda^* \), Result Six would then imply \( \sigma_B^*(\lambda^*) < \sigma_A^*(\lambda^*) \) in order to compensate for the deterrent effect of labour mobility on firm entry. However, Result Five also shows that \( \partial G_A / \partial \lambda_A > \partial G_B / \partial \lambda_B \) where \( \lambda_A = \lambda_B \), so that if \( \lambda_A^* \) maximizes growth \( G_A \), then \( \lambda_B^* \), where \( \lambda_B^* > \lambda_A^* \), maximizes \( G_B \), which requires that \( \sigma_B^*(\lambda_B^*) < \sigma_B^*(\lambda_A^*) < \sigma_A^*(\lambda_A^*) \).

If the growth-maximizing proportion of firms were the same across cases, union bargaining power would need to be lower in the mobility case to compensate for the deterrent effect of mobility. However, the growth maximizing proportions of firms is higher in the mobility case, so that the union bargaining power parameter is even lower.

However, Results Four and Six together show that, because of conflicting effects, it is not possible to state definitively under which regime economic growth will be higher - for a given bargaining power parameter. Although growth will be higher for a given proportion of firms [Result Four], it is also apparent that the latter must fall
[Result Six]. An additional Table with some numerical values is added to throw some light on this issue in comparison with Table One. It implies the final, main result.

**Proposition Three**: If $\sigma > 0$, ex post labour mobility may bring about higher growth if union bargaining power is low, but lower growth if it is high.

The intuition for this result is that union bargaining power and ex post labour mobility are substitutes to the extent that they each reduce proportion of firms. Consequently, even after accounting for some adjustment in the growth-maximizing proportion, either too much or too little of the two together where there is either powerful unions with highly mobile workers or weak unions with very immobile workers tends to be bad for growth.

Finally, a comparison of Tables One and Two also provides numerical examples in support of Proposition Two. If the ratio of the population standard deviation to the population mean is unity, economic growth is maximized where relative union bargaining power is 0.64 under immobility but 0.22 under mobility. If this same ratio is extremely high, the respective figures are calculated at 0.69 to 0.005 where this ratio is extremely high. Thus, strong unions are found to be beneficial for economic growth only where there are effective impediments to the mobility of labour. The greater the degree of heterogeneity of firms’ total factor productivities, the stronger is this result.

**Table Two**: The effect of ex post labour mobility on the proportion of firms, economic growth and on the growth-maximizing and maximal levels (case B)

<table>
<thead>
<tr>
<th>$\sigma$</th>
<th>$\lambda_B$</th>
<th>$G_B$</th>
<th>$\tilde{\lambda}_B$</th>
<th>$\tilde{G}_B$</th>
<th>$\frac{\lambda}{\lambda_B}$</th>
<th>$\frac{G}{G_B}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.50</td>
<td>0.500X</td>
<td>0.24</td>
<td>0.721X</td>
<td>0.158</td>
<td>0.391X</td>
</tr>
<tr>
<td>0.7</td>
<td>0.30</td>
<td>0.499X</td>
<td>0.17</td>
<td>0.676X</td>
<td>0.120</td>
<td>0.372X</td>
</tr>
<tr>
<td>0.9</td>
<td>0.10</td>
<td>0.374X</td>
<td>0.076</td>
<td>0.538X</td>
<td>0.062</td>
<td>0.303X</td>
</tr>
<tr>
<td>Growth</td>
<td>$\sigma_B^* = 0.60$</td>
<td>$\tilde{\sigma}_B^* = 0.22$</td>
<td>$\tilde{\sigma}_B^* = 0.005$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
maximal values
\[
\begin{array}{ccc}
\lambda_B^* = 0.40 & \lambda_B^* = 0.345 & \lambda_B^* = 0.33 \\
G_B^* = 0.51X & G_B^* = 0.723X & G_B^* = 0.427X
\end{array}
\]

\[X \equiv (\theta/1+\theta)(1-\alpha)A\mu^{1-\alpha}\]

A further comparison of the Tables allows us to make a final statement.

6. Further considerations

6.1 The intermediate mobility case.

It is, of course, implausible to assume that workers are all either perfectly mobile or immobile. The consideration of an intermediate case would require the specification of mobility costs, which are neither so high that workers would never move nor so low that immobility is no longer a consideration. Furthermore, if these costs vary across workers, some might readily switch between firms with small wage differences, while others might not even move at all from very low to very high wage firms. A fully fledged model of an intermediate case, containing distributions both for entrepreneurial ability and for mobility, would necessarily entail considerable complexity. A reasonable conjecture, however, is that any solution would be constitute some combination of the results for the two polar cases considered above. There should be distributions both for wages and for employment - with each, as well as investment, each being positively correlated with the ability of the entrepreneur.

6.2 The wage firm-size relationship

An intermediate case is of some interest, because it implies a positive relationship between the wage and the firm-size, a robust empirical finding. [See, for example, Main and Reilly (1993). Consequently, a generalization of this model would furnish a theoretical reason for an empirical relationship that has to some extent eluded
explanation, although Brown and Medoff (1989) provide some other possible reasons.\textsuperscript{16}

6.3 Income equality and economic growth

Proposition Three states that ex post labour mobility may bring about higher growth if union bargaining power is low, but lower growth if it is high. A tighter wage distribution also emerges where labour is more mobile, so that, if the wage distribution adequately proxies the general income distribution, the case of relatively weak unions may explain the positive cross-country association between income equality and economic growth. It should be noted that the income distribution here is not a causal factor for growth, for which various factors are considered in the survey by Aghion et al (1999). The point here is that labour mobility increases allocative efficiency, while reducing wage inequality. Furthermore, if unions are very strong, the present model also predicts the correlation would have a different sign.

6.4 Where investment precedes the wage bargain

Grout (1984) presented an alternative case of non-binding labour contracts where investment effectively precedes the wage bargain, because wages could always be renegotiated ex post. In an OLG model related to the present one - except without an entrepreneurial class - Deveureux and Lockwood (1991) showed that the effect of non-binding contracts is to raise the rate of economic growth through an inter-generational redistribution favouring young savers. The effect of non-binding contracts is tantamount to unions having greater bargaining power, so that while their basic result would carry over into the present model, the growth effects would be modified either by enhancing or by offsetting effects with an endogenous number. Both inter- and intra-generational redistribution effects would be at play.

Furthermore, the model may be played out over more than two stages, since investment may also precede the allocation of households to firms as well as the wage bargain. Roberts (2009) considered this extension to show that heterogeneous firms may over-invest in order to signal their relative levels to prospective employees. The

\textsuperscript{16} Another theoretical reason is provided by Green, Machin and Manning (1996).
possibility of over-investment by firms in a generally costly signalling game would discourage firms from entering, then suggesting, other things being equal, that economic activity would benefit from weaker labour unions. Furthermore, investment could be higher when it precedes the wage bargain, in contrast with the standard result, because the signalling effect could even dominate that of the hold-up.

6.5 **Insider unions**
The source of mobility costs has not been specified. One possibility is that these are imposed by “insiders”, those who had allocated to firms at the initial stage. Those who would, if they could, re-allocate later, may be designated the “outsiders”. There is an incentive for insiders to exclude outsiders, because wages are decreasing in employment. Snower and Lindbeck (2002) provide a summary of their body of work, giving reasons why insider-power might hold sway. The insider-outsider interpretation is interesting, because the present model predicts that if unions are strong in one dimension, in being able to control entry, it may be beneficial in terms of economic growth for them to also be reasonably strong in the other dimension of wage bargaining power. Although, insider-power is a possible explanation for the immobility case, the model does not depend on any particular cause, and the costs of geographical relocation are an obvious alternative among others.

6.6 **Political economy considerations**
Legislation addressed towards the labour market should affect the outcome of wage bargaining, of which the outlawing of the closed-shop in the UK in 1984 is a prime example. Households may also determine economic outcomes through engaging in the political process, particularly, by voting in democratic elections. The range of possibilities for political equilibria in the present context is far too wide to consider, because the model contains three types of household - with income distributions pertaining to each – as well as general equilibrium effects and non-monotonic relationships. However, as an illustration, for the case of no labour mobility mobility, the old, having left the labour market, would have an incentive to vote for greater union power - and, so, in favour of the closed-shop, as this would reduce the proportion of firms and, thus, increase the interest return on their predetermined
saving, according to equation (13A). This indicates the possibility of a possible voting coalition between workers and retirees, however, previously employed, but one which may not be sustained in the alternative scenario of very mobile labour.

6.7 Extending the model
There are of course a number of ways in which the model could be extended. The focus on wage bargaining has enabled us to regard relative income, as in Lucas (1978), as the driving factor for occupational choice and the number of firms. Kihlstrom and Laffont (1979) suggest that varying attitudes to risk may be a relevant criterion in an environment of uncertainty, while Evans and Jovanovic (1989) point to the significance of initial wealth holdings where financial markets are imperfect. While these alternatives provide a richer way of modelling occupational choice, they should not overturn the basic effect of union bargaining power on this decision.

An obvious extension would be to allow for unemployment. Households would then weigh up the probability of periodic unemployment - certainly in the lifetime of the worker and possibly in that of the potential entrepreneur too - when they come to make the initial allocation decision. Another possibility might be to allow for self-employment as a third option as a first resort and also as a consequence of unemployment.

Furthermore, corporations could be considered alongside individual-owned enterprises as well as corporations, where the present assumption of decreasing returns to scale is relaxed in some way to accommodate this richer corporate landscape. Individuals of high entrepreneurial ability might then face an additional occupational choice between being hired as a manager by others in a generally owned incorporation or and in ploughing their own furrow in a smaller but personal enterprise.

Finally, there will be other deterrent effects on firm formation, affecting economic growth, besides the power of labour unions to bargain wages. The relative taxation of wages and entrepreneurial profit would be a very close substitute to the parameter of
relative union bargaining power, and the model, as it stands, presents an economy that resides on something akin to a Laffer Curve. It is straightforward to predict that profits should be taxed less aggressively, where inter-firm labour mobility is greater.

8. Summary
A model has been presented that integrates key labour market considerations, firm formation with economic growth. The focus has been on the wage effect of unions on the entry of firms and, thence, on aggregate activity levels and growth, because of a trade-off between deterring less able firms from entering and encouraging the entry of firms in general that operate under decreasing returns. The main results are that if labour is immobile, the best outcome may be in the interior, where reasonably strong unions confer macroeconomic benefits; while in the opposite case of high labour mobility, the presence of all but the strongest firms may be so discouraged that even a small degree of union power may be harmful with decreasing returns. So, although the degree of labour union wage bargaining power has an uncertain effect on economic growth, its effect is more likely to be adverse, the greater the mobility of labour.

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