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Technology licensing in a differentiated oligopoly

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Abstract: We show the effects of product differentiation and competition on technology licensing by an outside innovator. Both the innovator and the society are better off under royalty licensing compared to auction (or fixed-fee) if the number of potential licensees is sufficiently large, irrespective of Cournot and Bertrand competition. We find that the relationship between product differentiation and the minimum number of potential licensees that is required to make royalty licensing profitable to the innovator is non-monotonic under Cournot competition, while it is positive under Bertrand competition. Hence, there are degrees of product differentiation for which neither the innovator nor the antitrust authority requires information about the type of product market competition while deciding on the licensing contract. It follows from our analysis that the innovator prefers auction plus royalty licensing (or fixed-fee plus royalty) over either royalty licensing or auction.

Key Words: Auction; Licensing; Royalty; Product Differentiation
JEL Classification: D43; L13

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Technology licensing in a differentiated oligopoly

1. Introduction

Technology licensing is an important element of conduct in many industries and has attracted a fair amount of attention in the industrial organization literature. The seminal papers by Kamien and Tauman (1984 and 1986) show that, if an innovator, who is not a producer,[1] licenses a technology to the final goods producers and the product market is characterized by Cournot competition, licensing with output royalty generates lower profit to the innovator compared to fixed-fee licensing and auction, regardless of the industry size and/or magnitude of the innovation.[2] In view of this theoretical result, the wide prevalence of output royalty in the licensing contracts (see, e.g., Taylor and Silberstone, 1973 and Rostoker, 1984) has remained a puzzle, and has generated a significant amount of interest in explaining the superiority of royalty licensing over auction or fixed-fee licensing. The factors attributed to the presence of output royalty in a licensing contract offered by an outside innovator[3] are asymmetric information (Gallini and Wright, 1990, Beggs, 1992, Poddar and Sinha, 2002 and Sen, 2005b), Bertrand competition (Muto, 1993), spatial competition (Poddar and Sinha, 2004), moral hazard (Macho-Stadler et al., 1996 and Cho, 2001), risk aversion (Bousquet et al., 1998), incumbent innovator (Shapiro, 1985, Kamien and Tauman, 2002 and Sen and Tauman, 2007), leadership structure (Kabiraj, 2004), strategic incentive delegation (Saracho, 2002), integer constraint on the number of licenses

[1] Licensing by the Universities or independent research labs to the producers may be the examples of this scenario.
[2] See Kamien (1992) for a nice survey of this literature.
[3] Outside innovator refers to the situation were the innovator (who is the licenser) and the licensees do not compete in the product market.

(Sen, 2005), input market power (Mukherjee, 2010) and convex costs (Mukherjee, 2010).[4]

An important feature of the above mentioned papers, except Muto (1993) and Poddar and Sinha (2004), is their focus on homogeneous goods, *thus ignoring the effects of product variety*. Even Muto (1993) and Poddar and Sinha (2004) fail to capture the effects of product market concentration by restricting their attention to *duopoly final goods markets*.

Muto (1993), which is most closely related to our paper, shows that royalty licensing can dominate either fixed-fee licensing or auction under Bertrand competition if the opportunity cost of winning a license depends on the type of licensing contract (or in other words, if a non-licensee's profits depend on the type of licensing scheme for the same number of licenses offered).[5] The working paper version of Muto (1993), i.e., Muto (1988), shows that royalty licensing cannot dominate fixed-fee licensing or auction under Cournot competition, irrespective of whether the opportunity cost of winning a license depends or not on the type of the licensing contract. Hence, his work suggests that the optimality of the licensing contracts depend on the type of product market competition. Therefore, among other things, an outside innovator needs to know the nature of the product market competition while choosing the patent licensing contract.

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[4] There is a related literature which shows the superiority of royalty licensing and licensing with a combination of fixed-fee and royalty when the licenser and the licensees compete in the product market (see, e.g., Rockett, 1990, Wang, 1998 and 2002, Wang and Yang , 1999, Filippini, 2001, Mukherjee and Balasubramanian, 2001, Faulí-Oller and Sandonis, 2002, Fosfuri, 2004, Kabiraj, 2005, Poddar and Sinha, 2005 and Mukherjee, 2007). In this literature, the competition softening effect of output royalty may make the royalty licensing preferable than fixed-fee licensing if the licenser and the licensees compete in the product market.

[5] In the terminology of Muto (1993), this is the case of a non-drastic innovation. Since we have experienced that some readers are not comfortable with the terms non-drastic and drastic innovations under product differentiation, we avoid using those terms. Instead we will say whether the opportunity cost of winning a license depends or not on the type of the licensing contract.

Using a widely used demand structure due to Bowley (1924), in which higher product differentiation increases value of the product to the consumers and enlarges the final goods market, and with $n \geq 2$ potential licensees who face the same (assumed to be zero, for simplicity) opportunity costs of licenses for different licensing contracts, we show that if the products of the licensees are imperfect substitutes, royalty licensing can dominate auction under both Cournot and Bertrand competition if the number of potential licensees is not very small. We find that while the relationship between product differentiation and the minimum number of potential licensees that is required to make the royalty licensing profitable to the innovator is non-monotonic under Cournot competition, it is positive under Bertrand competition.

Hence, we show that *(i)* royalty licensing can dominate auction under Cournot competition with imperfect substitutes, *(ii)* royalty licensing can dominate auction under Bertrand competition when the opportunity cost of winning a license does not depend on the type of the licensing contract , and *(iii) perhaps most importantly*, if the number of potential licensees is not very small, royalty licensing dominates auction for moderate levels of product differentiation irrespective of Cournot or Bertrand competition.

We also show welfare implications of the licensing contracts. We show that if the number of potential licensees is not very small, social welfare is higher under royalty licensing compared to auction, under both Cournot and Bertrand competition. Hence, neither the innovator nor the antitrust authority may require information about the type of product market competition while deciding on the licensing contract. Further, there may not be a disagreement between the innovator and the antitrust authority on the preference for a licensing contract.

We have written our results in terms of auction and royalty licensing. It is worth mentioning that, since the opportunity costs of the licensees do not depend on the type of the licensing contract, there is no difference between auction and fixed-fee licensing in our analysis. Hence, whatever we report under auction is also relevant for a fixed-fee licensing. It may worth mentioning that the implications of licensing with both fixed-fee and royalty follow easily from our analysis.[7]

In a Hotelling duopoly model with maximum product differentiation and price competition, Poddar and Sinha (2004) show that an outside innovator prefers royalty licensing than both auction and fixed-fee licensing. However, their analysis does not shed light on other degrees of product differentiation, other types of product market competition such as quantity competition, and on oligopolistic product market competition where the number of licensees is more than two. We analyze all these issues here. Moreover, demand is perfectly inelastic in Poddar and Sinha (2004), while quantity demanded varies in our analysis. Our results on price competition are different from theirs. We show that royalty licensing is dominated by auction if the products of the licensees have maximum differentiation. However, for other degrees of product differentiation, royalty can dominate auction (or fixed fee) if the number of potential licensees is not very small.

The remainder of the paper is organized as follows. We consider the case of Cournot competition in Section 2. Section 3 shows the implications on Bertrand competition. Section 4 concludes.

[7] The fixed fee can be either a take-it-or-leave-it offer from the seller or it can be the bids of an auction.

2. Cournot competition

Assume that there is an innovator, called I, who has invented a technology for a new product. However, I cannot produce the good. There are $n \geq 2$ symmetric potential licensees of the product, and I can license its technology to the potential licensees. To avoid analytical complexity, we ignore the integer constraint and consider the number of licensees as a continuous variable, unless specified otherwise.

We assume that no licensee has a technology to produce this product. Hence, if a licensee does not win a license for this product, it cannot produce the product irrespective of the licensing contract, and is assumed to earn zero profit, for simplicity. Hence, the opportunity costs of the licensees are the same and equal to zero irrespective of the licensing contract.[8] However, we assume that a potential licensee can produce the product at a constant marginal cost, which is assumed to be zero for simplicity, if it wins a license.

It is assumed that the products of the licensees are imperfect substitutes, due to the factors such as customer switching costs, differences in after-sales service, brand name, packaging, etc.[9] We consider a demand function due to Bowley (1924), and assume that the inverse market demand function for the ith licensee is

$$P_i = a - q_i - \gamma Q_{-i} \tag{1}$$

[8] More generally, zero opportunity costs of licenses occur if the potential licensees either do not produce other products or produce other products under perfect competition. Alternatively, it occurs even if the potential licensees earn profits on other products, and their production decisions on the new product do not affect their existing profits.

[9] As an example of product differentiation, consider the sale of iPhones in the US. Apple (the maker of iPhones) does not offer cellphone services directly and hence contracts with phone companies for the sale of iPhones. As per a previous agreement, iPhone users had to subscribe to AT&T but from 2011, iPhones will also be made available to Verizon subscribers. Notice that iPhones offered through these two cellphone providers is likely to be imperfect substitutes, due to the different level of services provided by the service providers.

where P_i is price of the ith product, q_i is the output of the ith licensee, Q_{-i} is the total output of all licensees other than the ith licensee and $\gamma \in [0,1]$ shows the degree of product differentiation. The products are perfect substitutes if $\gamma = 1$, while the products are completely differentiated for $\gamma = 0$. In this section, we assume that the competition in the produce market is characterized by Cournot competition.

We consider the following licensing contracts designed by I:

(i) Royalty licensing, where a fixed royalty payment r per unit of output is charged by I, and any licensee that wishes to can purchase the license at this royalty rate.

(ii) Auctioning k licenses, $1 \le k \le n$, by I through a first price sealed bid auction. The highest bidders get the license. The ties are resolved by I.

The innovator can also adopt a fixed-fee licensing contract, where the innovator charges a flat pre-determined license fee F, and any licensee that wishes to can purchase the license at this fixed-fee. However, it is immediate from Kamien et al. (1992) that the essential difference between auction and fixed-fee licensing stems from the difference in licensees' opportunity costs of having a license. Since we are considering a situation with zero opportunity costs of the licensees, it is immediate that auction and fixed-fee licensing provide the same solution. Therefore, we focus on auction and do not consider the case of fixed-fee licensing separately.

We consider the following games for our analysis. Under royalty licensing, at stage 1, I announces the uniform royalty rate r. At stage 2, the licensees simultaneously and independently decide whether or not to purchase a license. At stage 3, the licensees choose their outputs simultaneously. If only one licensee purchases the technology at stage 2, he produces like a monopolist at stage 3.

Under auction, at stage 1, I announces an auction of k licenses, where $1 \le k \le n$. At stage 2, the licensees simultaneously and independently decide whether or not to purchase a license, and how much to bid. At stage 3, the licensees choose their outputs simultaneously. If I auctions only one license, the licensee produces like a monopolist at stage 3. We solve these games by backward induction.

2.1. Royalty licensing

Under royalty licensing, each licensee always prefers to purchase a license for $r < a$, since a licensee has the option of producing nothing after purchasing a license, thus earning 0, which is the opportunity cost of having a license.

First, we determine the product market equilibrium under royalty licensing. If I licenses the technology to n licensees and each of the n licensees pays a per-unit royalty r, where $r < a$, the ith licensee, $i = 1, 2, ..., n$, chooses his output to maximize the following expression:

$$\underset{q_i}{Max}(a - q_i - \gamma Q_{-i} - r)q_i . \tag{2}$$

The equilibrium output of the ith licensee can be found as $q_{i,C}^R = \frac{a - r}{2 + \gamma(n-1)}$. Hence, I maximizes the following expression to determine the equilibrium royalty rate:

$$\underset{r}{Max}\frac{nr(a-r)}{2+\gamma(n-1)} . \tag{3}$$

The equilibrium royalty rate is $r_C^* = \frac{a}{2}$. The equilibrium output of the ith licensee is $q_{i,C}^R = \frac{a}{4 + 2\gamma(n-1)}$, and the equilibrium payoff of I is

$$\Pi_C^R = \frac{na^2}{4\left[2+\gamma(n-1)\right]}. \tag{4}$$

2.2. Auction

Now consider the game under auction. If I auctions k licenses, where $1 \le k \le n$, the output of the ith licensee is $q_{i,C}^A = \frac{a}{2+\gamma(k-1)}$. The profit of the ith licensee is $\frac{a^2}{[2+\gamma(k-1)]^2}$. Therefore, in the Nash equilibrium of the bidding game, each potential licensee bids $\frac{a^2}{[2+\gamma(k-1)]^2}$. As mentioned in Kamien et al. (1992), if $k = n,\ I$ can guarantee this equilibrium bid by specifying a minimum bid. However, for $k < n$, the potential licensees bid these amounts even if I does not specify the minimum bid.

If I auctions k licenses, his payoff is $\Pi_C^A = \frac{ka^2}{[2+\gamma(k-1)]^2}$, and the number of licenses to auction is determined by maximizing the following expression:

$$\underset{k}{Max} \frac{ka^2}{\left[2+\gamma(k-1)\right]^2}. \tag{5}$$

The equilibrium number of licenses is given by

$$k_C^* = \min\left\{\frac{2-\gamma}{\gamma}, n\right\}. \tag{6}$$

The profit of I when k_C^* licenses are being sold under auction is either

$$\Pi_C^A = \frac{na^2}{\left[2+\gamma(n-1)\right]^2}, \qquad \text{for } n \le \frac{2-\gamma}{\gamma} \tag{7a}$$

or $$\Pi_C^A = \frac{a^2}{4\gamma(2-\gamma)}, \qquad \text{for } n > \frac{2-\gamma}{\gamma}. \qquad (7b)$$

2.3. Comparing auction with royalty licensing

First, consider $n \le \frac{2-\gamma}{\gamma}$. In this situation, the profits of the innovator under royalty licensing and under auction are given by (4) and (7a) respectively. The comparison shows that $\Pi_C^R \gtrless \Pi_C^A$ if

$$n \gtrless \frac{2}{\gamma} + 1. \qquad (8a)$$

Since we are considering a situation where $n \le \frac{2-\gamma}{\gamma}$, it is then immediate that $n > \frac{2}{\gamma} + 1$ cannot hold. In other words, if $n \le \frac{2-\gamma}{\gamma}$, *the innovator is always better off under auction than under royalty licensing.*

The reason for the above result is as follows. If $n \le \frac{2-\gamma}{\gamma}$, the number of licenses is the same under auction and under royalty licensing. However, royalty licensing distorts the output of the licensees and also does not allow the innovator to extract the entire profits of the licensees. Hence, an auction generates higher profit to the innovator compared to royalty licensing, if the number of licenses is the same under auction and royalty licensing.

Now consider the case where $n \geq \frac{2-\gamma}{\gamma}$. In this situation, the profits of the innovator under royalty licensing and under auction are given by (4) and (7b) respectively. The comparison shows that $\Pi_C^R \underset{<}{\overset{>}{-}} \Pi_C^A$ if

$$n \underset{<}{\overset{>}{-}} \frac{2-\gamma}{\gamma(1-\gamma)}. \tag{8b}$$

Since $\frac{2-\gamma}{\gamma(1-\gamma)} > \frac{2-\gamma}{\gamma}$, and we are now considering a situation where $n > \frac{2-\gamma}{\gamma}$, it is then immediate that innovator prefers royalty licensing to auction if $n > \frac{2-\gamma}{\gamma(1-\gamma)}$, but it prefers auction to royalty licensing if $n \in (\frac{2-\gamma}{\gamma(1-\gamma)}, \frac{2-\gamma}{\gamma})$.

We have seen that if $n > \frac{2-\gamma}{\gamma(1-\gamma)}$, the innovator prefers royalty licensing compared to auction. However, it follows from (8b) and has also been shown in Figure 1 that this condition cannot be satisfied for a finite *n* if the products are either very much differentiated (i.e., $\gamma \to 0$) or are close substitutes (i.e., $\gamma \to 1$). Further, we get that right hand side (RHS) of (8b) attains the minimum value at $\gamma = .59(approx.)$, and the minimum value of RHS of (8b) is *5.8 (approx.)*. Hence, in order for royalty licensing to generate a higher profit for the innovator compared to the auction (i.e., to satisfy the condition $n > \frac{2-\gamma}{\gamma(1-\gamma)}$), it is *necessary* that the number of potential licensees be at least *6*.

Figure 1 below shows the condition $n > \frac{2-\gamma}{\gamma(1-\gamma)}$ for $\gamma \in (0,1)$ and *n* > *6*. It follows from the diagram that the innovator earns higher profit under royalty licensing compared to auction for the intermediate levels of product differentiation, while auction is more profitable at the extremes.

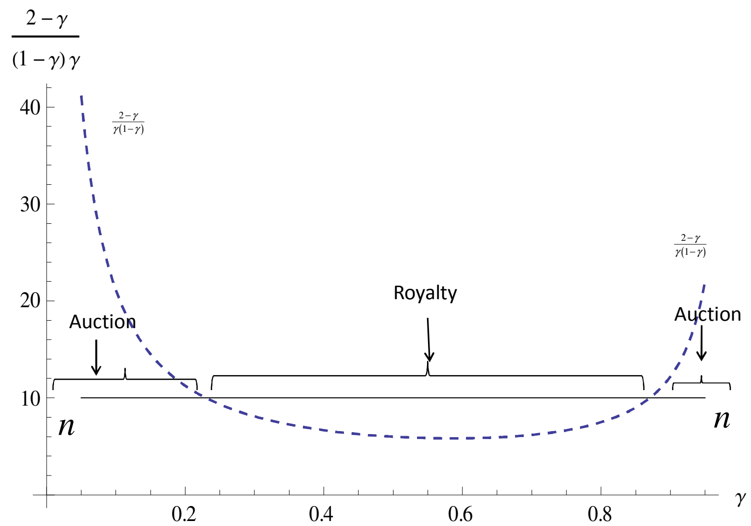

Figure 1: Comparison of *n* and $\dfrac{2-\gamma}{\gamma(1-\gamma)}$ for $\gamma \in (0,1)$ and n=10.

The following result summarizes the above discussion.

Proposition 1: *Consider Cournot competition in the product market. Unless the number of licensees is very small (which is 6 in our analysis), the innovator prefers royalty licensing compared to auction for intermediate values of γ, but it prefers auction for low and high values of γ. As n increases, it increases the range of γ over which the innovator's profit is higher under royalty licensing compared to auction.*

The intuition for the above result is as follows. The comparison between the innovator's profits depends on two factors:

1. **Output distortion effect:** A royalty imposes additional marginal costs on the licensees and results in a reduction of output compared to auction. This factor makes the royalty contract less attractive to the innovator compared to auction.
2. **Market enlargement effect:** Given our demand structure, it follows from Martin (2002) that an increase in the number of producers enlarges the market size. Hence, if the number of licensees is higher under royalty licensing than under auction, the market size is also larger under the royalty contract. As a result, if the number of licensees is higher under royalty, then the market enlargement effect tends to make the royalty contract more attractive from the perspective of the innovator.

Overall, the attractiveness of the royalty contract relative to an auction depends on the relative strength of the two above-mentioned factors. For the discussion below, recall that the number of licenses under the royalty contract is n but the number of licenses under an auction is given by $k_C^* = \min\left\{\frac{2-\gamma}{\gamma}, n\right\}$. First, consider the case when the products are significantly differentiated (i.e., when γ is close to 0). In this situation, the number of licenses is n under both contracts and the market enlargement effect does not play any role. However, the output distortion effect operates and this makes the royalty contract less profitable for the innovator compared to an auction. Second, we consider the other extreme, that is the case in which the products are sufficiently similar (i.e., when γ is close to 1). In this situation, the number of licenses is substantially lower under auction compared to royalty licensing. However, since the products are close substitutes, the profits of the licensees are low. Hence, the additional fee that the innovator can extract under the royalty contract is low as well, i.e., the market enlargement effect is weak in this case. Hence, the output distortion effect again dominates the market enlargement effect and the innovator's profit is higher under auction. Finally, consider the case of moderate product differentiation. In this situation, the number of licenses is higher under royalty licensing compared to auction, and the market enlargement effect under royalty licensing is strong enough to outweigh the royalty licensing's output distortion effect, thus making the innovator better off under royalty licensing.

We have allowed for the innovator to use either auction (or fixed-fee) or royalty licensing. Since the net profits of the licensees are positive under royalty licensing, it follows immediately that the innovator prefers to use fixed-fee along with royalty to extract the entire profits of the licensees. While more licenses under royalty licensing helps to create the market enlargement effect, auction helps to extract the entire surplus from the licensees. *It is then intuitive that the innovator prefers auction plus royalty licensing (or fixed-fee plus royalty) over either royalty licensing or auction*. Since the dominance of the auction plus royalty licensing is straightforward from the above analysis, we do not go into the details of that analysis.

2.4. Welfare

Now we look at the welfare comparison between royalty licensing and auction. Welfare is the sum of consumer surplus and the total profits of the innovator and the licensees.

It can be shown that consumer surplus under the royalty licensing is

$$CS_C^R = \frac{a^2\left[1+\gamma(n-1)\right]n}{8\left[2+\gamma(n-1)\right]^2} \tag{7}$$

and consumer surplus under auction is

$$CS_C^A = \frac{a^2[1+\gamma(n-1)]n}{2[2+\gamma(n-1)]^2}, \qquad \text{for } n \le \frac{2-\gamma}{\gamma}. \tag{8a}$$

$$CS_C^A = \frac{a^2(3-2\gamma)}{8\gamma(2-\gamma)}, \qquad \text{for } n \ge \frac{2-\gamma}{\gamma}. \tag{8b}$$

Further, welfare under royalty licensing is

$$W_C^R = CS_C^R + \Pi_C^R + n\left(q_{i,C}^R\right)^2 \tag{9}$$

and under auction is

$$W_C^A = CS_C^A + \Pi_C^A. \tag{10}$$

Note that there are three terms in the right hand side of (9) but only two terms in (10). This is because the net profits of the licensees are positive under royalty licensing, while their net profits are zero under auction.

First, consider the case where $n \le \frac{2-\gamma}{\gamma}$. By comparing (8a) and (10), it follows that the consumer surplus is higher under auction than under royalty licensing. Further, it has been discussed above that the profit of the innovator is also higher under auction in this case, since $\frac{2-\gamma}{\gamma} < \frac{2-\gamma}{\gamma(1-\gamma)}$. However, the profit of each licensee is positive under royalty licensing while it is 0 under auction. The comparison of (9) and (10) shows that the higher total net profits of the licensees under royalty licensing compared to auction are not high enough to outweigh the lower consumer surplus and the lower profit of the innovator under royalty licensing compared to auction. Hence, welfare is higher under auction compared to royalty licensing for $n \le \frac{2-\gamma}{\gamma}$.

Now consider the case where $n > \frac{2-\gamma}{\gamma}$. In this situation, consumer surplus is higher under auction than under royalty licensing, and the profit of the innovator may be higher or lower under auction than under royalty licensing depending on whether *n* is lower or higher than $\frac{2-\gamma}{\gamma(1-\gamma)}$. However, the total net profits of the licensees are higher under royalty licensing than under auction. It can be shown that, for a given γ, welfare is higher under royalty licensing (auction) for

$$n > (<) \frac{(2-\gamma)(3-\gamma\sqrt{29+\gamma(-34+9\gamma)})}{2\gamma(1-\gamma)} \equiv n_C^*. \qquad (13)$$

It can be checked that $n_C^* \geq \frac{2-\gamma}{\gamma}$.[10] It follows from n_C^* that (13) cannot hold for $\gamma = 1$ and $\gamma = 0$,[11] while it may hold for intermediate values of γ. Further, we get that in order for *n* to be greater than n_C^* , it is *necessary* that $n \geq 17$.

The next proposition summarizes the result on welfare.

Proposition 2: *Consider Cournot competition in the product market. Unless the number of potential licenses is very small (which is 17 in our analysis), welfare is higher under royalty licensing compared to auction for moderate values of* γ*, but it is higher under auction for high and low values of* γ*. As n increases from 17, it increases the range of* γ *over which welfare is higher under royalty licensing compared to auction.*

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[10] It may worth noting that the demand intercept, *a*, does not arise in (13) since it gets cancelled under comparison.

[11] In fact, (13) does not hold for reasonable values of *n* if γ is either close to *0* or it is close to *1*. For example, if $\gamma = .1$, (13) holds provided *n* is at least *85*.

The intuition for the above result is as follows. The effect of different licensing contracts on welfare also depends on the trade off created by the output distortion effect and the market enlargement effect. The output distortion effect tends to reduce welfare under the royalty licensing compared to auction, and the market enlargement effect tends to create the opposite effect. If the products of the firms are sufficiently differentiated (i.e., if γ is close to 0), the number of licenses under auction is close to the number of licenses under auction and hence the market enlargement effect is small. Hence, in this case, the output distortion effect is the dominant effect and this results in a higher welfare under an auction. Conversely, if the products are sufficiently similar (i.e., if γ is close to 1), even if the number of licenses are higher under royalty licensing compared to auction, the market enlargement effect is very weak since the products are close substitutes. In this situation, again the output distortion effect dominates the market enlargement effect, and results in a higher welfare under auction compared to royalty licensing. It is only for intermediate values of γ that the market enlargement effect can be strong enough to outweigh the output distortion effect, thus resulting in higher welfare under royalty licensing compared to auction.

Propositions 1 and 2 together show that if the number of potential licensees is sufficiently large and if the degree of product differentiation is moderate, both the innovator and the society prefer royalty licensing. Thus, we show the effects of product market competition and product variety in determining the privately and socially preferred licensing contracts.

3. Bertrand competition

The purpose of this section is to show that our main results of the previous section occur even under Bertrand competition. Thus, we show that if the number of potential licensees is not very small, both the innovator and society can be better off under royalty licensing irrespective of the type of product market competition.

Given the inverse demand function (1), the demand function for firm *i* is:

$$q_i = \frac{a(1-\gamma) - \left[1+(n-2)\gamma\right] p_i + \gamma \sum_{j \neq i} p_j}{(1-\gamma)\left[1+(n-1)\gamma\right]}. \tag{14}$$

In order to avoid the well known "Bertrand paradox", we will mainly concentrate on $\gamma \in [0,1)$ in this section.

3.1 Royalty licensing

First, determine the product market equilibrium under royalty licensing. If *I* licenses the technology to n licensees and each of the n licensees pays a per-unit output royalty *r*, where *r* < *a*, the equilibrium output of the ith licensee, $i = 1, 2, ..., n$, is $q_{i,B}^R = \frac{(a-r)\left[1+(n-2)\gamma\right]}{(1-\gamma)\left[1+(n-1)\gamma\right]}$. Hence, I maximizes the following expression to determine the equilibrium royalty rate:

$$\underset{r}{Max} \frac{nr(a-r)\left[1+(n-2)\gamma\right]}{(1-\gamma)\left[1+(n-1)\gamma\right]}. \tag{15}$$

The equilibrium royalty rate is $r_B^* = \frac{a}{2}$. The equilibrium output of the *i*th licensee is

$q_{i,B}^R = \frac{a\left[1+(n-2)\gamma\right]}{2(1-\gamma)\left[1+(n-1)\gamma\right]}$, and the equilibrium payoff of I is

$$\Pi_B^R = \frac{na^2\left[1+(n-2)\gamma\right]}{4\left[1+(n-1)\gamma\right]\left[2+(n-3)\gamma\right]}. \tag{16}$$

3.2. Auction

Now consider the game under auction. If I auctions k licenses, where $1 \le k \le n$, the output of the ith licensee is $q_{i,B}^A = \frac{a\left[1+(k-2)\gamma\right]}{\left[1+(k-1)\gamma\right]\left[2+(k-3)\gamma\right]}$ and the profit of the ith licensee is $\frac{a^2(1-\gamma)\left[1+(k-2)\gamma\right]}{\left[1+(k-1)\gamma\right]\left[2+(k-3)\gamma\right]^2}$. Therefore, in equilibrium, each potential licensee bids $\frac{a^2(1-\gamma)\left[1+(k-2)\gamma\right]}{\left[1+(k-1)\gamma\right]\left[2+(k-3)\gamma\right]^2}$. If $k=n$, I can guarantee this equilibrium bid by specifying a minimum bid. However, for $k<n$, the licensees bid these amounts even if I does not specify the minimum bid.

If I auctions k licenses, his payoff is $\Pi_B^A = \frac{a^2k(1-\gamma)\left[1+(k-2)\gamma\right]}{\left[1+(k-1)\gamma\right]\left[2+(k-3)\gamma\right]^2}$, and the number of licenses to auction is determined by maximizing the following expression:

$$\frac{a^2k(1-\gamma)\left[1+(k-2)\gamma\right]}{\left[1+(k-1)\gamma\right]\left[2+(k-3)\gamma\right]^2}. \tag{17}$$

Since we cannot find a closed form solution for the above maximization problem, in the following analysis, we solve for the optimal value of k numerically.

We consider three values of γ: $\gamma=.1$, $\gamma=.5$ and $\gamma=.9$. Thus, we consider very low, very high and intermediate values of γ. We get that the equilibrium number of licenses as $k_B^*(\gamma=.1)=18$, $k_B^*(\gamma=.5)=2$ and $k_B^*(\gamma=.9)=1$.[12] Hence, as product differentiation increases, i.e., as γ reduces, the equilibrium number of licenses under auction increases. The corresponding profits of I are $\prod_B^A(\gamma=.1)=1.27347a^2$, $\prod_B^A(\gamma=.5)=.296296a^2$ and $\prod_B^A(\gamma=.9)=.25a^2$.

3.3. Comparing auction with royalty licensing

Now compare the profits of the innovator under auction and royalty licensing for $\gamma=.1$, $\gamma=.5$ and $\gamma=.9$. We get that the corresponding profits of the innovator under royalty licensing are $\Pi_B^R(\gamma=.1)=\dfrac{na^2\left[1+(.1)(n-2)\right]}{4\left[1+(.1)(n-1)\right]\left[2+(.1)(n-3)\right]}$,

$\Pi_B^R(\gamma=.5)=\dfrac{na^2\left[1+(.5)(n-2)\right]}{4\left[1+(.5)(n-1)\right]\left[2+(.5)(n-3)\right]}$ and

$\Pi_B^R(\gamma=.9)=\dfrac{na^2\left[1+(.9)(n-2)\right]}{4\left[1+(.9)(n-1)\right]\left[2+(.9)(n-3)\right]}$. In all these cases, the profit of the innovator is higher under royalty licensing than under auction if the number of the potential licensees is sufficiently large. Royalty licensing is better for the innovator for $n\geq 20$ if $\gamma=.1$, for $n\geq 4$ if $\gamma=.5$ and for $n\geq 3$ if $\gamma=.9$. *Therefore, given a degree of product differentiation, the innovator prefers royalty licensing compared to auction if the number of potential licensees is sufficiently large.*

Further, as the degree of product differentiation increases, i.e., as γ reduces, the minimum number of potential licensees required to make royalty licensing more profitable to the innovator compared to an auction increases as well. Under Cournot competition, the minimum number of potential licensees required to make royalty

[12] We are considering the number of firms as integers.

licensing more attractive compared to auction depends on $\frac{2-\gamma}{\gamma(1-\gamma)}$. Notice that this term is non-monotonic in γ. Hence, the result obtained under Bertrand competition contrasts with the one obtained under Cournot competition.

The following proposition summarizes the above discussion.

Proposition 3: *Consider Bertrand competition in the product market. For a given* $\gamma \in (0,1)$*, the innovator is better off under royalty licensing compared to auction if the number of potential licensees is sufficiently large. Further, the minimum number of potential licensees that is required to make the innovator better off under royalty licensing increases if* γ *reduces.*

The intuition for the above result is similar to Proposition 1, where the output-distortion and the market enlargement effects play important roles. Further, as in Section 2.3, *the innovator prefers auction plus royalty licensing (or fixed-fee plus royalty) over either royalty contracts or auction*.

Proposition 3 does not incorporate the cases of $\gamma = 0$ and $\gamma = 1$. If $\gamma = 0$, that is, if the products of the licensees are completely differentiated, the innovator licenses to all of the potential licensees under both auction and royalty licensing, and is better off under auction. On the other hand, if $\gamma = 1$, i.e., if the products are perfect substitutes, the innovator will sell one license under auction, and will earn $\frac{a^2}{4}$. However, if $\gamma = 1$, the profit of the innovator under royalty licensing will also be $\frac{a^2}{4}$.

Hence, the innovator is indifferent between auction and royalty licensing if $\gamma = 1$. This is in line with Kamien (1992).

3.4 Welfare

Now we compare welfare under royalty licensing and auction, and show that welfare can be higher under royalty licensing.

It can be shown that consumer surplus under royalty licensing is

$$CS_B^R = \frac{n\left[1+\gamma(n-1)\right]\left(q_{i,P}^r\right)^2}{2} \tag{18}$$

and under auction is

$$CS_B^A = \frac{k\left[1+\gamma(k-1)\right]\left(q_{i,P}^F\right)^2}{2}. \tag{19}$$

Further, welfare under the royalty licensing is

$$W_B^R = CS_B^R + \Pi_B^R + n\left(q_{i,B}^R\right)^2 \tag{20}$$

and under auction is

$$W_B^A = CS_B^A + \Pi_B^A. \tag{21}$$

In general, welfare can go either way. In Figure 2 below, we plot welfare under royalty licensing and under auction for $a = 10$ and $n = 30$. Figure 2 shows that welfare is higher under auction for low values of γ but it is higher under royalty licensing for high values of γ.

Figure 2 along with Proposition 3 shows that if the number of potential licensees is sufficiently large, both the innovator and society can be better off under royalty licensing compared to auction if the products are not very much differentiated.

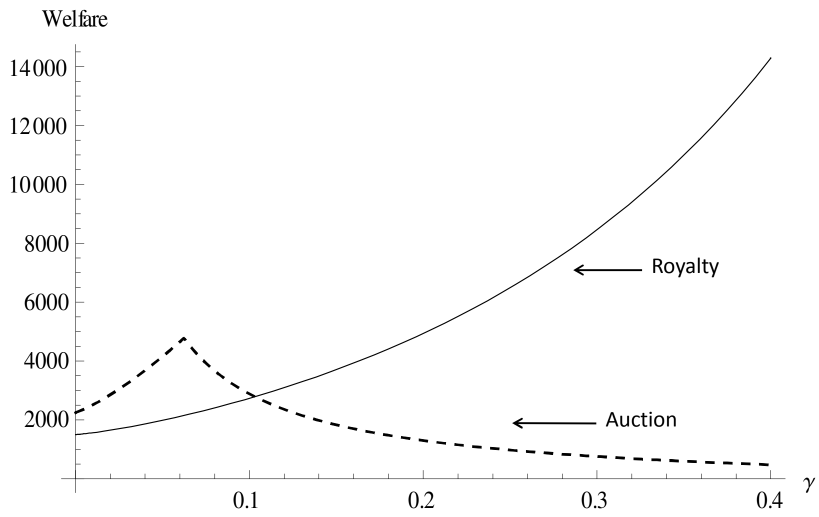

Figure 2: Welfare comparison under Bertrand competition for $a = 10$ and $n = 30$.

4. Conclusion

We consider technology licensing by an outside innovator, and show the effects of product differentiation and competition (given by the number of licensees producing in the market) on the innovator's profit and social welfare. We show that both the innovator and the society can be better off under royalty licensing compared to auction if the number of potential licensees is sufficiently large. We find that the relationship between product differentiation and the minimum number of potential licensees that is required to make the royalty licensing profitable to the innovator is non-monotonic under Cournot competition, while it is positive under Bertrand competition.

Hence, our analysis suggests that if the number of potential licensees is large, there is a wide range of the product differentiation parameter in which the innovator and the antitrust authority both prefer the royalty contract, regardless of the type of product market competition. It also follows from our analysis that the innovator prefers auction plus royalty licensing (or fixed-fee plus royalty) over either royalty licensing or auction.

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