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STOCHASTIC PRODUCTION AND THE LAW OF SUPPLY

by

INDRANEEL DASGUPTA

Abstract

This paper analyzes the problem of deriving the law of supply from plausible restrictions on observed input-output choices of a competitive firm, when the firm makes random input-output decisions. It models such random production behavior in terms of a stochastic supply function, and introduces a restriction on a stochastic supply function which leads to a stochastic version of the law of supply. Our analysis thus allows one to derive testable predictions about a competitive firm's response to price changes even in a class of cases where the firm's observed behavior violates the profit maximization postulate.

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I. INTRODUCTION

This paper analyzes the problem of deriving the law of supply from plausible restrictions on observed input-output choices of a competitive firm. It extends the literature by allowing the firm to make random input-output decisions, which need not be profit-maximizing. It introduces a restriction on such production decisions which leads to a stochastic version of the law of supply.

The standard theory of firm behavior assumes that, given a vector of output and input prices, a competitive firm chooses an output-input combination, in a deterministic manner, so as to maximize it's profit. The basic testable prediction generated by this theory is the so-called Law of Supply (LS): the supply of any output by a competitive firm must be non-decreasing in its own price. Profit-maximization implies a well-known restriction on firms' input-output behavior: the so-called Weak Axiom of Profit Maximization (WAPM). If competitive firms satisfy WAPM, they must also satisfy LS.[1]

Suppose however that, even with invariant prices and technology, a firm is observed to alter its input-output combination in such a way that its profitability is altered as well. Thus, for example, suppose that under identical price configurations, say *p*, a competitive firm is seen to be choosing some net output vector, say *v*, some of the time, and a less profitable net output vector, say *v'*, at other times, thereby violating WAPM. An intuitively plausible way to model such situations would be to ascribe *stochastic*, as opposed to deterministic, output supply and input demand functions to the firm. Thus, one may represent such random production decisions by saying that, given the price vector *p*, the firm produces the net output vector *x* with some probability *t*, and the (less profitable) net output vector *y* with the probability *(1-t)*, where $0< t <1$. In such random production situations, due to the apparent violation of the profit maximization hypothesis, standard theory no longer seems to provide any guidance for predicting the firm's response to price changes without imposing additional assumptions.

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[1] Consider any two price vectors $\langle p, p' \rangle$; suppose that the firm chooses the net output vectors *v* and *v'*; under *p* and *p'*, respectively. Then, the firm's choices satisfy WAPM if and only if, for all such pairs of price vectors, $[pv \geq pv']$. Samuelson (1947) showed that this restriction implies the Law of Supply and noted that it is necessary for profit maximization; while Hanoch and Rothschild (1972) pointed out that it is also sufficient. The name, WAPM, is due to Varian (1984).

Even if firms maximize their profits, one could observe such *seemingly* contrary behavior, if there are factors that are not taken into account by an external observer, but nevertheless influence firms' profitability. Random changes in these "missing" factors would generate randomness in the observed data. The external observer may typically be expected to have incomplete information about the firm's input output decisions, the nature of the constraints facing the firm and its cost structure, and may fail to take into account some aspects in his calculations. For example, suppose a firm gets a discount on its electricity rates during weekends. Then, if all other prices remain invariant throughout the week, the firm may be expected to produce a higher output on weekends. Now, suppose the observer fails to take into account the weekend discount in his reconstruction of the firm's production decisions. Clearly, in that case, the data would be interpreted as exhibiting a violation of WAPM, and the firm's behavior may be modelled in a stochastic manner by ascribing a probability of 2/7 to the higher weekend output.

Randomness and apparent violations of WAPM of this type are frequently encountered in empirical studies. A standard practice in econometric analysis is to attribute "small" violations of this kind to measurement error, rather than to violation of the profit-maximization hypothesis itself.[2] For such small violations, deriving predictions about the firm's responses to price changes on the basis of LS would seem to be a reasonable procedure. If, however, the violations are "large", then such predictions are unpersuasive. It has been argued that large departures from the profit maximization hypothesis, and, indeed, from maximizing behavior per se, are in fact routine in reality, and that this seriously reduces the scope and usefulness of the standard theory of producers' behavior.[3]

The problem of reconciling LS with a competitive firm's supply responses to price changes, when it appears to violate the profit-maximization hypothesis, seems to have generated two different types of responses at the theoretical level. As mentioned earlier, it is possible to

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[2] Varian (1985) provides a formal treatment of the notion of a "small" violation in this context.

[3] See, for example, Leibenstein (1976, 1979) and Simon (1979).

hypothesize that such behavior is reconcilable with the profit maximization postulate by ascribing it to a perception failure on part of the observer.[4] Thus, it has been hypothesized that such observations may be largely due to mis-specification of the firm's input output decisions, cost or return structure. The proper procedure, in this view, is to identify some alternative specification that would make the firm's observed behavior consistent with profit maximization.[5] Once such a specification has been identified, a version of *LS which conforms to that specification* can be used to generate testable predictions in the standard way. A second approach has been to argue that, even if individual firms do not necessarily maximize profits, the law of supply may be expected to hold in the aggregate as a market-wide phenomenon.[6]

Neither of the two approaches outlined above answers the following question. Suppose a competitive firm exhibits random input-output decisions, thereby violating the deterministic WAPM. Is it then possible to formulate a stochastic interpretation of LS, and to identify a plausible restriction on the firm's observed input-output behavior that will predict such a response to price changes? A large literature exists on stochastic choice and preference.[7] However, this literature has also neglected the problem of random choice behavior of a competitive firm. Yet, the identification of a restriction such as specified above would seem to be of considerable interest, since, in that case, one can develop testable predictions along standard lines entirely on the basis of observed firm behavior even when such behavior seems to violate the profit maximization hypothesis.

The purpose of the present paper is to address this issue. We model a competitive firm which may violate WAPM by ascribing a *stochastic*, as opposed to deterministic, input demand and output supply functions to it. We formulate a stochastic version of LS and introduce an

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[4] Such arguments are advanced, for example, by Stigler (1976) in his critique of the notion of X-efficiency proposed by Leibenstein.

[5] This understanding, in large measure, seems to motivate the transactions cost approach to firm behavior. For a survey, see de Alessi (1983a, 1983b). Leibenstein (1983) provides a response.

[6] See, for example, Becker (1962).

[7] See Bandyopadhyay, Dasgupta and Pattanaik (1999) and the references therein.

intuitively plausible restriction on the firm's input-output decisions that we term the Axiom of Stochastic Profit Maximization. We show that this restriction implies our stochastic version of LS.

The plan of the paper is as follows. Section II introduces the basic notation and definitions. Section III introduces the Axiom of Stochastic Profit Maximization and discusses its intuitive basis. Section IV presents our formal result. Section V concludes the discussion.

II. NOTATION AND DEFINITIONS

Let n be the number of commodities and let $N = \{1,2,...n\}$. Let R and R_{++} denote, respectively, the set of real numbers and that of positive real numbers. A competitive firm faces n-dimensional vectors of commodity prices and produces n-tuples of net outputs. We shall denote price vectors by p, p' etc. and net output vectors by v. We assume that all prices are finite and positive.

Definition 2.1. *A stochastic supply function* (SSF) is a rule *s*, which specifies, for every price vector *p*, exactly one probability measure[8] t over the class of all subsets of R^n, such that $[t(\text{t}\ ) = 1]$ for some compact subset t of R^n.

Given an SSF, *s*, let $t = s(p)$, and let A be a subset of R^n. Then $t(A)$ represents the probability that the firm will choose a net output vector from the set A, under the price vector p. Since all prices are positive by assumption, the definition of an SSF also incorporates the intuitively obvious restriction that a firm can only incur finite profits or losses.

Definition 2.2. A stochastic supply function *s* is *degenerate* iff, for every price vector p, there exists $v(p) \in R^n$ such that, for every subset A of R^n, $\lfloor if\ v(p) \in A,\ then\ t(A) = 1 \rfloor$ and $\lfloor if\ v(p) \notin A,\ then\ t(A) = 0 \rfloor$.

Remark 2.3. Formally, a degenerate SSF is different from a standard deterministic net supply function, since the former specifies a probability measure for every price vector, while the latter specifies a net output vector for every price vector. Intuitively, however, one can be identified with the other.

Notation 2.4. Given an SSF, s, and any pair of price vectors, $\langle p, p' \rangle$, let

$$\bar{\pi}(p, p') = \left[\inf\left\{\pi \in R^1 \mid t\{v \in R^n \mid pv \le \pi\} = t'\{v \in R^n \mid pv \le \pi\} = 1\right\}\right],$$

and let

$$\bar{\pi}'(p, p') = \left[\inf\left\{\pi \in R^1 \mid t\{v \in R^n \mid p'v \le \pi\} = t'\{v \in R^n \mid p'v \le \pi\} = 1\right\}\right],$$

where $t = s(p),\ t' = s(p')$.

Remark 2.5. It follows from Definition 2.1 that a finite infimum must necessarily exist.

Notation 2.6. Given an SSF, s, and any pair of price vectors, $\langle p, p' \rangle$, define

$$E(p, p') = \{v \in R^n \mid [pv \le \bar{\pi}(p, p')] \ and \ [p'v \le \bar{\pi}'(p, p')]\}.$$

Remark 2.7. By construction, $t(E(p, p')) = t'(E(p, p')) = 1$ (see Notation 2.4).

Given any pair of price vectors, the corresponding set E is to be interpreted as a convex representation of the firm's *production set*, recovered on the basis of the firm's observed choice behavior under the two price situations.

Notation 2.8. Given any price vector, p, and any non-empty set of net output vectors, A, let

$$q(p, A) = \sup\{pv \mid v \in A\}.$$

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[8] Let X be a given set and let F be an algebra, i.e., a non-empty class of subsets of X, such that, for all $A, B \in F$, $[A \cup B \in F \ and \ A - B \in F]$ and $[X \in F]$. Then, a probability measure defined on F is a countably additive function $u : F \to [0,1]$ such that $u(X) = 1$ (see, for example, Adams and Guillemin (1996), p. 42).

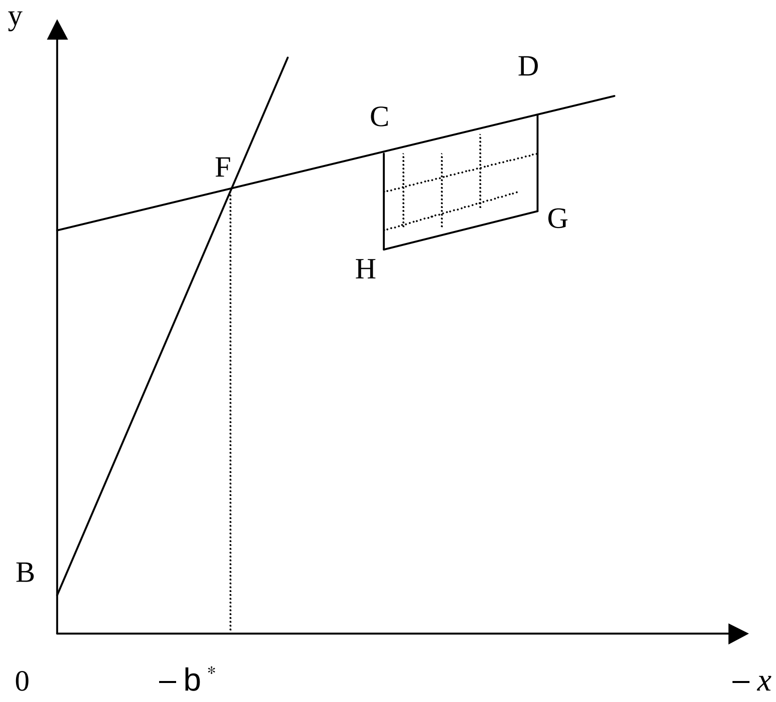
Notation 2.9. Given any price vector, *p*, and any non-empty set of net output vectors, *A*, let

$$\mathrm{a}(p, A) = \left[\left\{v \in R^n \mid pv \geq q(p, A)\right\} - A\right].$$

Intuitively, q is the maximum profit that may be obtained by choosing a net output vector from within the set *A*, while a is the set of all net output vectors outside *A* which provide a profit of at least *q*, given a price vector *p*.

In Figure 1 below, we represent the case for a firm that produces one output, y, using one input, x. Suppose that the firm chooses the net output vectors B and C with probability ½ under the price vector *p*, and it chooses the net output vectors B and D with probability ½ under the price vector p'. Suppose further that the iso-profit line through B, under *p*, is BF, and that through D, under p', is FD. Clearly, the firm is violating WAPM. Then, the profit achieved when the firm chooses some net output vector on the line BF, under the price configuration *p*, is $\bar{\mathrm{p}}(p, p')$. The profit achieved when the firm chooses some net output vector on the line FD, under the price vector p', is $\bar{\mathrm{p}}'(p, p')$. The set bounded above by, (and including) the line FD, and bounded on the left by (and including) the line BF is $E(p, p')$. If A is the set CDGH, then the set of all points on or above the line FD, except the line segment CD, is the corresponding set $\mathrm{a}(p', A)$.

FIGURE 1



III. THE AXIOM OF STOCHASTIC PROFIT MAXIMIZATION

We now introduce a restriction on the behavior of the firm and discuss its intuitive justification.

Definition 3.1. A stochastic supply function, *s*, satisfies the *axiom of stochastic profit maximization* (ASPM) iff, for every pair of price vectors $\langle p, p' \rangle$, and for every non-empty $A \subseteq E(p, p')$ we have:

$$[t(A) - t'(A) \leq t'(\alpha(p', A)) - t(\alpha(p', A))], \tag{3.1}$$

where $t = s(p)$ and $t' = s(p')$ (see Notation 2.6 and Notation 2.9).

Putting $A = \{v \in R^n \mid p'v = \pi\}$, the following can be seen to follow immediately.

Remark 3.2. ASPM implies that, for every $\pi \in R$,

$$[t'\{v \in R^n \mid p'v \geq \pi\} \geq t\{v \in R^n \mid p'v \geq \pi\}].$$

Given any arbitrary profit level π, when A is either (i)$\{v \in E(p, p') \mid p'v \leq \pi\}$ or (ii)$\{v \in E(p, p') \mid p'v < \pi\}$, it must be that:

$$[t(A) + t(\alpha(p', A)) = t'(A) + t'(\alpha(p', A)) = 1].$$

Clearly, (3.1) must necessarily hold in such cases. Thus, (3.1) imposes restrictions only in that it also requires that the SSF satisfy (3.1) for all non-empty proper subsets of (i) except (ii).

To see that the restriction has intuitive plausibility, first suppose, when prices change from p to p', the probability that the firm will choose a net output vector from the set *A* falls. Then, the fall in probability should not be because of a switch to an input vector which provides profit less than $q(p', A)$: a higher profit than that is attainable (assuming that the production set is convex) within the set A itself. If factors other than observed prices, which influence a firm's decision, are independent of observed prices, then if the firm switches a probability mass away from A in response to a price change, it should be in order to acquire higher profits. Hence, this probability mass will be passed on to the set $\mathrm{a}(p', A)$. The upper bound for such transfer is, naturally, $[t'(\mathrm{a}(p', A)) - t(\mathrm{a}(p', A))]$.

Now suppose instead the probability of choosing in A increases. Then it must be the case that, either, probability has fallen in the set $\mathrm{a}(p', A)$, or, it has fallen in the complement of $[A \cup \mathrm{a}(p', A)]$. If the latter is the case, then (3.1) does not imply any restriction on the firm's choice. So, suppose the former is the case. First note that, given the axiom, if $q(p', A)$ is not attainable in A, i.e., if $[\mathrm{a}(p', A) = \{v \in R^n \mid p'v \geq q(p', A)\}]$, (3.1) implies that probability cannot fall in $\mathrm{a}(p', A)$ under p' (see Remark 3.2). So, if it does fall, it must be the case that $q(p', A)$ is attainable in A. By construction, the complement of $[A \cup \mathrm{a}(p', A)]$ can only provide, at the most, a profit less than $q(p', A)$. Therefore, if there is a fall in the probability of choosing in $\mathrm{a}(p', A)$, it should be the case that this probability mass has been transferred to A. Therefore, the increase in probability of the firm choosing in A should be at least as much as the decrease in probability of the firm choosing in $\mathrm{a}(p', A)$, which is the restriction imposed by (3.1). This argument shows that our restriction has at least some a priori plausibility.

Remark 3.3. Formally, WAPM for a deterministic supply function is different from ASPM, since, formally, a deterministic supply function is different from a degenerate SSF (see Remark 2.3). However, if a degenerate SSF is identified with a deterministic supply function, then, in light of Remark 3.2, it can be seen that the restrictions imposed by the two axioms are identical.

IV. THE LAW OF SUPPLY

In order to develop the implications of ASPM, we first need to introduce a stochastic version of the familiar law of supply.

Notation 4.1. Given any $i \in N$, any $\mathrm{b} \in R$, any pair of price vectors $\langle p, p' \rangle$ and the corresponding set $E(p, p')$, let $E^i_{\geq \mathrm{b}}(p, p') = \{v \in E(p, p') \mid v_i \geq \mathrm{b}\}$, where v_i is the amount of the ith output in v. The sets $E^i_{> \mathrm{b}}(p, p'), E^i_{< \mathrm{b}}(p, p')$ and $E^i_{\leq \mathrm{b}}(p, p')$ are defined analogously.

Notation 4.2. For all $i \in N$, and all $p, p' \in R^n_{++}$, p, p' are said to be i-variants iff (i) $\lfloor p_j = p'_j$ for all $j \in N - \{i\}\rfloor$, and (ii) $[p_i \neq p'_i]$.

Notation 4.3. For every $i \in N$, let K_i denote the set of all ordered pairs of price vectors $\langle p, p' \rangle$ such that (i) p, p' are i-variants, and (ii) $p_i > p'_i$.

Definition 4.4. A stochastic supply function, s, satisfies the *law of supply* (LS) iff, for all $b \in R$, for all $i \in N$, and for all $\langle p, p' \rangle \in K_i$, we have

$$\lfloor t'(E^i_{\geq b}(p, p')) \leq t(E^i_{\geq b}(p, p'))\rfloor \text{ and } \lfloor t'(E^i_{> b}(p, p')) \leq t(E^i_{> b}(p, p'))\rfloor.$$

Suppose the price of a commodity falls. The law of supply requires that, for every real number b, neither the firm's probability of producing at least b amount of the commodity, nor the firm's probability of producing more than b amount of the commodity, can increase.

Remark 4.5. If the stochastic supply function is degenerate, and if a degenerate stochastic supply function is identified with a deterministic supply function (see Remark 2.3), then our law of supply is equivalent to the standard deterministic version.

We now state and prove our result.

Proposition 4.6. *Suppose an SSF satisfies the axiom of stochastic profit maximization. Then, it must also satisfy the law of supply.*

Proof. Let the SSF satisfy ASPM. Consider any $i \in N$ and any pair of price vectors $\langle p, p' \rangle$ such that $\langle p, p' \rangle \in K_i$. We need to show that, for all $b \in R$:

$$t'(E_{\geq b}(p, p')) \leq t(E_{\geq b}(p, p')), \tag{4.1}$$

and

$$t'\left(E_{>\beta}(p,p')\right) \le t\left(E_{>\beta}(p,p')\right). \tag{4.2}$$

Let $\bar{\pi} = \bar{\pi}(p,p'), \bar{\pi}' = \bar{\pi}'(p,p')$, and define β^* as the solution to $\bar{\pi} = \bar{\pi}' + \beta^*(p_i - p_i')$.

First consider any $\beta \le \beta^*$, and the corresponding set $E_{\ge\beta}$. It can be checked that

$$\left\{v \in R^n \mid pv = \bar{\pi}, v_i < \beta^*\right\} \cap E(p,p') = \emptyset . \tag{4.3}$$

From (4.3), it follows that

$$t\left(\alpha(p, E_{\ge\beta})\right) = t'\left(\alpha(p, E_{\ge\beta})\right) = 0 . \tag{4.4}$$

Since $E_{\ge\beta}$ is clearly non-empty, ASPM and (4.4) together imply that

$$t\left(E_{\ge\beta}(p,p')\right) \ge t'\left(E_{\ge\beta}(p,p')\right). \tag{4.5}$$

Now suppose $\beta > \beta^*$. It can be checked that

$$\left\{v \in R^n \mid p'v = \bar{\pi}', v_i > \beta^*\right\} \cap E(p,p') = \emptyset . \tag{4.6}$$

Then, in a similar way, (4.6) and ASPM together imply

$$t\left(E_{<\beta}(p,p')\right) \le t'\left(E_{<\beta}(p,p')\right).$$

It follows that

$$t\left(E_{\ge\beta}(p,p')\right) \ge t'\left(E_{\ge\beta}(p,p')\right). \tag{4.7}$$

Together, (4.5) and (4.7) yield (4.1).

An argument identical to the one used to establish (4.5) also implies, for all $\beta < \beta^*$,

$$t\left(E_{>\beta}(p,p')\right) \ge t'\left(E_{>\beta}(p,p')\right). \tag{4.8}$$

An argument identical to the one used to establish (4.7) also implies, for any $\beta \ge \beta^*$,

$$t\left(E_{>\beta}(p,p')\right) \ge t'\left(E_{>\beta}(p,p')\right). \tag{4.9}$$

Together,(4.8) and (4.9) yield (4.2). ◊

V. CONCLUDING REMARKS

This paper extends the basic result in the standard choice-based approach to the theory of firms' behavior, by permitting random input-output decisions by a competitive firm. Using the analytical framework of a stochastic supply function, we have shown that ASPM, which constitutes a stochastic counterpart of the standard Weak Axiom of Profit Maximization, implies a stochastic version of the familiar law of supply. Our analysis thus allows one to derive testable predictions about a competitive firm's responses to price changes even in a class of cases where the firm's observed behavior violates the profit maximization postulate.

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