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Mean Reversion of Real Exchange Rates in High-Inflation Countries[1]

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Abstract

We test for mean reversion in real exchange rates using data from five countries, four of which have experienced episodes of high inflation. We use monthly data for Argentina, Brazil, Chile, Colombia and Israel from 1972 to 1993 and find that in all cases except Brazil a stochastic unit root model is appropriate. Kalman filter estimates of the stochastic unit roots show sharp deviations from unity, associated with high inflation episodes. We conclude that stochastic unit root models are a more appropriate way to model mean reversion in real exchange rates for high inflation countries than models with fixed rates of mean reversion.

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1. Introduction

Are exchange rate dynamics different in high inflation periods? Surprisingly, there has been relatively little work done on this issue. Yet there is a strong *prima facie* case for the proposition that the dynamics of real exchange rates are influenced by inflationary circumstances. First, at high rates of inflation, nominal shocks dominate real shocks, whereas at low rates the opposite is true. This suggests that mean-reverting tendencies in real exchange rates are likely to be more evident at higher inflation rates. Second, when inflation is high, demand for domestic money as an asset falls, and it begins to be displaced by foreign currencies. This currency substitution implies flows across the exchanges, which are likely to affect both the short-run dynamics and the long-run equilibrium in real exchange rates.

In the many empirical tests of purchasing power parity (PPP) reported in the literature, there is a clear pattern of greater support for PPP in episodes of high inflation. The classic example is the German hyperinflationary experience of 1922-3, documented by Frenkel (1978), Edison (1985) and Taylor and McMahon (1988), but the same phenomenon is apparent for pegged exchange rates in Latin American countries in the post-war period (see McNown and Wallace, 1989; and Liu, 1992). It is particularly striking that the estimated coefficients of cointegrating regressions between exchange rates and relative prices are much closer to the PPP-predicted value of unity in high inflation cases (e.g. compare Liu (1992) with Cheung and Lai (1993) for OECD countries).

Empirical evidence from countries that have experienced varying inflation rates is, however, more ambiguous. Zhou (1997) cannot reject a unit root in the real exchange rate for five countries with episodes of high inflation, but concludes in favor of stationarity after allowing for structural breaks that represent shifts in the level and/or the time trend of the estimated equilibrium real exchange rate. These shifts, the dates of which are estimated endogenously, appear to be associated with changes in the inflation regime. In this paper, we pursue this line of inquiry using monthly data from four economies that have experienced episodes of high inflation. We investigate fixed-parameter mean reversion models of the real exchange rate using the standard ADF unit root tests and compare these with the Leybourne-McCabe test which takes stationarity as the null and has a unit root alternative. Further tests allow for a *stochastic* unit root, and permit the deviations of the root from unity to follow a noise process or a random walk. We find possible evidence of both kinds of behavior and use a Kalman filter to estimate the root trajectories through time. These show that large spikes occur in the mean reversion process at times of high inflation.

We conclude that a stochastic unit root model provides a more suitable econometric method to model mean reversion in real exchange rates than fixed coefficient unit root processes like ADF tests. Intuitively the stochastic unit root process is preferred because it captures the spikes in the unit root often associated with jumps in the real exchange rate arising from episodes of high inflation. Since stochastic unit root models are able to allow for jumps in real exchange rates (which might otherwise cause standard unit root tests to spuriously reject the null of nonstationarity) they are to be preferred when assessing the evidence for purchasing power parity in high inflation countries.

2. Data Issues

End of month data for the exchange rate and the wholesale price index were collected for Argentina, Brazil, Chile and Israel for the period 1972(1) - 1993(5) from *International Financial Statistics*. The countries chosen are similar to those covered by McNown and Wallace (1989); however, our data period is approximately twice as long as that used by McNown and Wallace. For reasons of comparison we also analyze data from a Latin American country that has not experienced a high inflation episode, i.e. Colombia. The inflation rate in the five countries is plotted in Figure 1, which shows the monthly change in the logarithm of the wholesale price index. Several features stand out.

- There appears to be a positive association between the mean and the variance of inflation: periods of high inflation are also characterized by high volatility of the inflation rate. This is a familiar finding.

- Periods of very high inflation are not very persistent. Rather, they tend to appear as spikes in the data, and even when the average inflation rate is high for a relatively long period, there are months of quite moderate inflation rates within that period. Thus the inflation rate has a highly skewed distribution, with below-average observations far more frequent than above-average ones.

- The historical behavior of the inflation rate varies considerably from country to country. This is not surprising, but it is a useful reminder that high-inflation experiences are not necessarily identical to one another (as our empirical results confirm).

We can identify certain historical features from the graphs since most of the major reforms have often been in response to inflationary circumstances getting out of hand. In the case of Argentina, the two most dramatic spikes occur in 1989 and 1990, as a result of two particularly traumatic hyperinflations, that resulted from ten years of public finance problems. They were brought under control by two stabilization programs, the first of which was unsuccesful, resulting in a further brief inflationary surge, and a second that successfully implemented a fiscal reform package in February 1991 and a convertibility program overseen by a currency board in April 1991. Similarly, in the case of Brazil, the spikes in 1986, 1987, and 1989 represent the three Cruzado plans which attempted to reform the financial and legal structure of society to ‘set inflation to zero’. The spikes in 1990 and 1991 indicate where the two Collor plans where implemented in order to change the rules on the central bank’s purchases of indexed bonds and to prevent retroactive short-term indexation. For Chile, the only spike occurred during the large hyperinflation in 1974; thereafter Chile was remarkably stable by Latin American standards, as was Colombia. Finally, the data for Israel demonstrates that there was a willingness to live with inflation for much of the period, but this came to a sudden halt in June 1985 when a disinflation program was adopted to remove indexation and set nominal targets for inflation. The change in regime is clearly evident.

The plots of the logarithm of the real exchange rate against the US dollar based on wholesale prices (in units of domestic currency against the dollar, so that a fall in the index indicates a real appreciation) are shown in Figure 2. In each case the range of fluctuation is quite wide, and Chile stands out because of the enormous appreciation in 1972-73. A second characteristic of all the plots is the frequency of spikes. Such spikes do not occur in the real exchange rates of major OECD countries over the same period. The difference almost certainly lies in the exchange rate regime. All four of the countries examined here pegged their nominal exchange rates, and a pegged nominal rate combined with a high inflation rate is a recipe for sharp variations in real exchange rates, unless the government is operating a policy of smooth daily devaluations with the deliberate aim of stabilizing the real exchange rate. Although we believe these spikes to be mostly genuine, there may also be a significant problem of measurement error, since the price data are unlikely to have been calculated on the same date as the exchange rate data. This is not an important issue when inflation is low, but it could become quite significant at high inflation rates.

Some basic statistics for inflation, the real exchange rate and the first difference of the real exchange rate are given in Table 1. These confirm the positive correlation of the mean and variance of inflation across countries. The severity of the inflationary experience is shown by the magnitude of the mean and the variance, and in all cases the magnitude of the mean is commensurate with the magnitude of the variance. It can also be seen that the inflationary experience of Argentina and Brazil has been far more extreme than that of Chile or Israel.

The fact that these economies have experienced periods of relatively moderate inflation as well as episodes of high inflation suggests that the history of their real exchange rates may be unusually informative, because of the variation in the relative importance of real and monetary shocks over the data set. If it is the case that the evidence for purchasing power parity is greater when monetary shocks are relatively large, then this should show up in the data. For this reason, we investigate models in which the unit root is a stochastic process which allows the real exchange rate to be mean reverting in some periods of the sample and subject to large jumps in others. This process is more characteristic of the real exchange rate in these countries than a standard unit root process, with a fixed rate of mean reversion, which an ADF test might aim to detect.

3. Models of Mean Reversion

We take the nominal end-of-month exchange rate for the high inflation countries vis-à-vis the US dollar (e_t), the domestic price level (p_t) and the US price level (p_t^*), where all variables are in natural logarithms. The real exchange rate (s_t) can be defined as

(1) $$s_t = e_t - p_t + p_t^*$$

We first of all consider the time series properties of s_t, employing both unit root tests and stationarity tests. Where applicable, we then test for the presence of stochastic unit roots in the data. Series which appear to contain stochastic unit roots are estimated using the Kalman filter. This allows us to examine the trajectory of the root through time.

i) Linear models: unit roots and stationarity.

As a starting point, we consider the following simple model

(2) $$s_t = (1+\delta)s_{t-1} + \varepsilon_t$$

where ε_t is a stationary process. Consider testing $H_0{:}\delta = 0$ against $H_1{:}\delta < 0$. Then, under H_0 the real exchange rate s_t contains a unit root and there is a zero rate of mean reversion; under H_1, s_t is stationary and there is (a constant rate of) mean reversion. A test of H_0 against H_1 can be performed using the augmented Dickey-Fuller test (ADF). This is essentially a replication of McNown and Wallace's (1989) testing approach, but on a longer data set.

As an alternative to (2), however, we might specify the model

(3) $$\begin{aligned} s_t &= w_t + \varepsilon_t, \\ \Delta w_t &= \eta_t, \quad \eta_t \sim i.i.d.(0, \sigma_\eta^2) \end{aligned}$$

and consider testing $H_0{:}\sigma_\eta^2 = 0$ against $H_1{:}\sigma_\eta^2 > 0$. Now it is under H_0 that s_t is stationary and exhibits (a constant rate of) mean reversion, whilst under H_1 there is a zero rate of mean reversion. In the model (3) a test of H_0 against H_1 can be carried out using the stationarity test suggested by Leybourne and McCabe (1994) (hereafter LM).

(ii) Nonlinear models: unit roots and stochastic unit roots.

If stationarity of the data is not implied by the above test procedures, we might further consider the following generating model for s_t:

(4) $s_t = (1+\delta_t)s_{t-1} + \varepsilon_t,$

(4a) $\delta_t \sim i.i.d.(0,\sigma_\delta^2)$

In the terminology of Leybourne, McCabe and Tremayne (1996) (hereafter LMT) and Granger and Swanson (1997), the nonlinear time series model (4)-(4a) is said to contain a *stochastic unit root*. In this way, we can think that the process allows for mean reversion in some periods and mildly explosive behavior in others, which is very much a characteristic of the real exchange rate data in these countries. A test of $H_0: \sigma_\delta^2 = 0$ against $H_1: \sigma_\delta^2 > 0$ is derived in LMT. This is a test of the fixed unit root null against a stochastic unit root alternative with the deviations from the unit root being noise. Notice, however, that under *both* regimes the real exchange rate is actually nonstationary. Moreover, it is only *difference* stationary under the null; under the alternative it is not stationary after any order of differencing.

As it stands, the model (4)-(4a) is somewhat constrained as it does not allow any persistence in the deviations from the unit root under the alternative. In view of this, Leybourne, McCabe and Mills (1996) (hereafter LMM) replace (4a) with the random walk process

(4b) $\Delta\delta_t = \mu_t, \quad \mu_t \sim i.i.d.(0,\sigma_\mu^2), \quad \delta_0 = 0$

LMM then derive a test of $H_0: \sigma_\mu^2 = 0$ against $H_1: \sigma_\mu^2 > 0$ in the model (4)-(4b). In reality, however, it is quite feasible that the actual process generating deviations from the unit root lies somewhere between the extreme cases represented by (4a) and (4b). In terms of testing, a rejection by LMT might actually be caused by δ_t being a near- noise stationary autoregressive process, rather than pure noise. Equally, a rejection by LMM may be indicating that δ_t is a near unit root stationary autoregressive process as opposed to a random walk. To incorporate both these possibilities, when we estimate the model (4), we allow δ_t to follow an unrestricted first order autoregressive process.

Finally we note that, as pointed out by Granger and Swanson (1997), standard ADF-type unit root tests will tend to indicate the presence of a unit root rather than stationarity when s_t is generated by a stochastic unit root process. This is not surprising since the process is, in some sense, still a unit root process "on average" and is therefore, in the eyes of the ADF test, a closer relative of a unit root process than a stationary one. By a similar argument, stationarity tests such as LM will also tend to indicate that unit roots are present in this situation.

For the series which show evidence of a stochastic unit root, we estimate the following state space model

$$\begin{aligned}(5)\qquad & (s_t - \alpha - \sum_{i=1}^{l} \phi_i s_{t-i}) = (1+\delta_t)(s_{t-1} - \alpha - \sum_{i=1}^{l} \phi_i s_{t-i-1}) + \upsilon_t, \quad \upsilon_t \sim i.i.d.(0, \sigma_\upsilon^2),\\ & \delta_t = \rho\delta_{t-1} + \omega_t, \quad \omega_t \sim i.i.d.(0, \sigma_\omega^2)\end{aligned}$$

Thus, the stochastic deviations from the unit root, δ_t, follow an unrestricted first order autoregressive process. A fixed unit root process is a special case of this model, and arises if $\sigma_\omega^2 = 0$ (on the basis of the above stochastic unit root tests, we would not, however, expect to find our estimates of σ_ω^2 insignificantly different to zero). If we assume that υ_t and ω_t are independent and normally distributed, then the model (5) is conditionally Gaussian, and its likelihood function can be constructed via the Kalman filter algorithm and the prediction error decomposition. All the parameters of the model, and sequential estimates of the stochastic deviations from the unit root δ_t, can be calculated using maximum likelihood methods. For brevity, we omit details of the Kalman filter algorithm and estimation procedure here; these may be found in Harvey (1989). The Kalman filter estimates of the δ_t obtained in this manner only use sample information up to time *t*-l. However, a fixed-interval smoothing algorithm may then be applied to the Kalman filter output to revise these estimates using full-sample information (again, see Harvey (1989) for details).

4. Empirical Results

The extended data set gives us about 250 observations. Our linear mean-reversion tests for the real exchange rates are based on equations (2) and (3), both augmented by linear deterministic terms and lagged differences in the dependent variable to account for stationary dynamics. The values of the ADF unit root tests and LM stationarity tests are reported in the second and third columns of Table 2, respectively. The ADF statistics show that for four of the five series (Argentina, Brazil, Colombia and Israel) the null of a (fixed) unit root cannot be rejected by the data in favour of stationarity, but for Chile the null is rejected. Examining the graph of the real exchange rate for Chile in Figure 2c, the series is clearly dominated by a steep fall early in the sample. The presence of a break at the beginning of an (otherwise) integrated series is known to cause the ADF test to spuriously reject the null of a unit root in favour of stationarity; see Leybourne, Mills and Newbold (1998). We consider this outcome to be a manifestation of this kind of phenomenon. Moreover, this view is supported when we test for a stationary null against a unit root alternative using the LM test. According to this test all five series clearly reject the null hypothesis of stationarity in favour of a unit root. Thus, our evidence rather overwhelmingly suggests that these real exchange rates do not exhibit mean reverting behaviour.

Given the above findings that these series do not appear stationary, we proceed to test whether they are better modelled as fixed or stochastic unit root processes. The LMT and LMM test results are given in the fourth and fifth columns of Table 2, respectively. The LMT test allows a stochastic unit root process which is i.i.d. distributed whilst the LMM test allows the stochastic unit root to be a random walk. The presence of a fixed unit root is not rejected for Brazil, thus, only the Brazil series appears to be adequately described by a conventional unit root process with a fixed rate of mean reversion. For Argentina and Colombia, we find the LMT test rejects but the LMM test does not, which suggests that these series have stochastic unit roots that are i.i.d. distributed. In view of our discussions above, we might *a priori* expect stochastic deviations from the unit root process to be low persistence (near noise) in these cases. In the case of Chile only the LMT test rejects and hence we might expect to find that the stochastic deviations from unity are a high persistence, near unit root process themselves. Both tests reject in the case of Israel, but in fact the rejection is more emphatic for the LMT test (1% level) than for the LMM test (5% level), which may indicate that a low persistence stochastic unit root is more likely than one which follows a random walk.

In Table 3 we give the parameter estimates from the model (5) for each country (except Brazil). The associated t-ratios are given in parentheses. The important characteristic which rules out a fixed mean reversion parameter is the estimate of s_w^2, which is significantly different from zero in each case. This is the basis of the LMT and LMM tests reported above and is evidence of stochastic unit root behaviour, as we would expect. The estimated coefficients, r, in the AR(1) models for the stochastic components, d_t, are very small for Argentina, Colombia and Israel (although significant in the first and third cases). In the case of Chile this coefficient is positive, large and highly significant. These findings are consistent with what we implied from the LMT and LMM test results.

Graphs of the smoothed estimates of the $(1+d_t)$ are given in Figure 3. The most notable feature of these is the fact that there are the characteristic spikes in the trajectories of $(1+d_t)$ relating to large departures from the otherwise moderate variation around a unit root mean. The historical interpretation of these spikes has already been discussed in section 2. Examining the timing of these spikes, it is evident that they are associated with points of time when inflation was mildly explosive and the real exchange rate series jumped. This is easiest to see on the figure for Chile, Figure 1c, which has a single large spike at exactly the same time as the large fall in the real exchange rate in Figure 2c. This corresponds to the spike in the stochastic unit root trajectory in Figure 3b. For two other countries, Argentina and Israel, the timing of the spikes also corresponds to outbursts of inflation which caused the real exchange rate to jump. In the case of Argentina the spikes in the stochastic unit root trajectory in the mid 1970s, early 1980s and 1990s tie in very closely with the jumps in the real exchange rate and inflation. For Israel, although inflation had a lower mean and standard deviation than in Argentina, there is still a close association between real exchange rate jumps and spikes in the inflation series. The only exception to this pattern is Colombia, but there were no episodes of high inflation in Colombia in our sample period; see Figure 1e. We conjecture that the absence of spikes in the stochastic unit root (Figure 3d) acts as a control case to confirm that the spikes occur because of the jumps in inflation and the real exchange rate. It would thus appear reasonable to suggest that the nature of mean reversion (as measured by $1+d_t$) in Figures 3 is altered by the occurrence of high inflation. Tests which impose a constant rate of mean reversion will fail to pick out the spiky nature of the mean reversion process caused by high inflation episodes, and may spuriously reject the null of nonstationarity.

5. Conclusions

The evidence from recent studies of the behavior of the real exchange rate in high inflation countries suggests significant mean reversion. However, under extreme and rapidly changing monetary conditions, the assumption that the rate of mean reversion is a fixed parameter over the sample period is somewhat difficult to accept. Our results emphatically confirm this conjecture on data from four high inflation economies over a twenty-year period. Simple (ADF) tests of mean reversion are not as conclusive as has been suggested in previous research, with only Chile decisively rejecting a unit root. This result is a common one for time series with a break early in the sample and does not necessarily imply mean reversion since the result can be spurious. This is confirmed by the LM tests for which the null of stationarity is strongly rejected. Moreover, with the sole exception of the Brazilian series, we find evidence of stochastic variation in the unit root models for real exchange rates.

Modeling these stochastic unit root process using a Kalman filter to examine the time-varying mean reversion parameter shows evidence that there are large spikes in the parameter associated with jumps in the real exchange rate. Given that the nominal exchange rate is pegged these jumps often arise because of outbursts of inflation. The conclusion we draw from the paper is that mean reversion in exchange rates is strongly influenced by high inflation and has characteristic spikes corresponding to these episodes. As a result we suggest that stochastic unit root processes are an appropriate way to model these effects, and almost certainly more appropriate than models which need to impose fixed rates of mean reversion over the sample period. Tests of mean reversion based on these simple models are very liable to induce incorrect inferences about purchasing power parity in high inflation countries.

References

Cheung, Y.-W. and Lai, K. S. (1993) ‘Long-run purchasing power parity during the recent float’, *Journal of International Economics*, 34, 181-92.

Edison, H. J. (1985) ‘Purchasing power parity: a quantitative reassessment of the evidence from the 1920's’, *Journal of International Money and Finance*, 4, 361-72.

Frenkel, J. A. (1978) ‘Purchasing power parity: doctrinal perspectives and evidence from the 1920's’, *Journal of International Economics*, 8, 169-91.

Granger, C.W.J. and Swanson, N.R. (1997) ‘An introduction to stochastic unit root processes’, *Journal of Econometrics*, 80, 35-62

Harvey, A.C. (1989) *Forecasting, structural time series models and the Kalman filter*, Cambridge University Press, Cambridge.

Leybourne, S. J. and McCabe, B. (1994) ‘A consistent test for a unit root’, *Journal of Business and Economic Statistics*, 12, 157-166.

Leybourne, S.J., McCabe, B. and Mills, T.C. (1996) ‘Randomized unit root processes for modeling and forecasting financial time series’, *Journal of Forecasting*, 15, 253-70.

Leybourne, S.J., McCabe, B. and Tremayne, A. (1996) 'Can economic time series be differenced to stationarity?’, *Journal of Business and Economic Statistics*, 14, 435-46.

Leybourne, S.J., Mills, T.C. and Newbold, P. (1998) ‘Spurious rejection by Dickey Fuller tests in the presence of a break under the null’, *Journal of Econometrics* (forthcoming)

Liu, P.C. (1992) ‘Purchasing power parity in Latin America: a cointegration analysis’, *Weltwirtschaftliches Archiv*, 128, 4, 66-79.

McNown, R and Wallace, M.S. (1989) 'National price levels, purchasing power parity, and cointegration: a test of four high inflation economies', *Journal of International Money and Finance*, 8, 533-45.

Taylor, M.P. and McMahon, P.C. (1988). 'Long-run purchasing power parity in the 1920', *European Economic Review*, 32, 179-197.

Zhou, S. (1997) 'Purchasing power parity in high-inflation countries: A cointegration analysis of integrated variables with trend breaks', *Southern Economic Journal*, 64, 450-67.

Table 1a. Inflation rates (monthly, in logs)

	mean	std deviation
Argentina	0.110	0.138
Brazil	0.093	0.099
Chile	0.045	0.085
Colombia	0.018	0.009
Israel	0.040	0.043

Table 1b. Real exchange rates (monthly, in logs)

	std deviation
Argentina	0.387
Brazil	0.146
Chile	0.614
Colombia	0.178
Israel	0.124

Table 1c. First difference of the real exchange rate (monthly, in logs)

	std deviation
Argentina	0.159
Brazil	0.047
Chile	0.078
Colombia	0.012
Israel	0.034

Table 2. Tests for unit roots, stationarity and stochastic unit roots

	ADF	LM	LMT	LMM
Argentina	-2.481 (1)	**0.408 (2)**	**0.109 (1)**	0.032 (1)
Brazil	-2.557 (0)	**1.139 (1)**	0.049 (4)	0.018 (4)
Chile	**-3.677 (3)**	**2.831 (2)**	0.043 (3)	**0.065 (3)**
Colombia	-2.052 (2)	**5.304 (1)**	**0.177 (2)**	0.021 (2)
Israel	-2.417 (0)	**2.997 (0)**	**0.312 (0)**	**0.078 (0)**
10% crit. value	-3.130	0.119	0.104	0.064

Notes: The regression models underlying each test include a constant and linear trend. Entries in bold are significant at the10% level. The figure in brackets represents the number of lagged differences of the dependent variable included in the regression; selected using general-to-specific testing at the 10% level.

Table 3. Parameter estimates from model (5)

	Argentina	Chile	Colombia	Israel
s_w^2	0.043 (2.49)	0.002 (3.10)	0.039 (2.13)	0.030 (4.56)
r	-0.049 (-2.63)	0.816 (15.6)	-0.466 (-5.76)	-0.017 (-3.16)
f_1	-0.169 (2.65)	-0.437 (-5.06)	0.721 (7.71)	-
f_2	-	-0.182 (-2.93)	-0.109 (1.30)	-
f_3	-	0.090 (1.80)	-	-
a	-1.126 (-9.82)	6.04 (8.69)	4.96 (0.602)	-2.240 (-84.5)
s_u^2	0.024	0.006	0.001	0.001

Notes: The parameter estimates above correspond to equation (5).