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Welfare Analysis in a Cournot Game with a Public Good

by

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Abstract

This note develops a method of recovering individual preferences, and of obtaining money-metric individual welfare comparisons, from demand functions generated by Cournot-Nash equilibria in games with public goods.

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I. Introduction

A number of different economic contexts have been modeled as Cournot games between two or more agents with one or more public goods[1]. In order to assess the welfare implications of alternative state policies in such contexts, it is necessary to derive a measure of the changes in each agent's welfare brought about by the relevant policy. However, the demand functions in these models are generated by Cournot-Nash equilibria, and therefore do not express any one agent's preferences completely. Hence, the standard method, which proceeds by reconstructing an individual's preferences from her observed demand functions, can no longer be used. Can one generalize the standard approach to take into account strategic interaction amongst agents, and thereby derive a money-metric measure of the welfare impact of distortionary interventions, in such contexts? The purpose of this note is to address this issue.

Changing the extent of distortion in a mixed subsidy-transfer scheme, while keeping total expenditure by the state constant, can be expected to change equilibrium consumption and induce welfare gains or losses for each individual. Intuitively, it is clear that the extent of welfare change will depend on the interaction between the change in the extent of the deadweight loss and the change in other agents' expenditures on the public good. To assess the deadweight loss, however, we need to know the agent's utility function. How can the policy-maker recover an agent's utility function from demand functions generated by Cournot-Nash equilibria? Second, how can the policy-maker derive a money-metric measure of the welfare loss/gain by modifying the standard notion of equivalent variation to our strategic context? These are our questions.

In Section II, we first develop a simple method of recovering the utility function of an agent who makes a positive contribution to the public good from Cournot-Nash demand functions. We then establish the following: a modified notion of equivalent variation can be used to derive a complete money-metric ranking of schemes in terms of an agent's individual welfare from the Cournot-Nash demand functions when that agent makes a positive contribution to the public good.

II. The Result

Consider two agents i and j, each of whom derives utility from the consumption of some private goods and a public good. Purely for notational simplicity we assume that the private goods are identical

[1] Applications of this model to the problem of intra-household resource allocation include Kanbur (1995), Lundberg and Pollack (1993), Ulph (1988) and Woolley (1988). Applications to private charity are much more extensive, the seminal contribution being Roberts (1984). Kemp (1984) analyzes international transfers.

for the two agents. Therefore, holding their prices fixed, they can be treated as a single Hicksian composite commodity with a normalized price of 1. Agent i is characterized by a strictly quasi-concave utility function $U^i = U(x_i, y)$ and individual income I_i, where x_i is the amount of this composite private good consumed by agent i and y is the total amount of the public good. Similarly, agent j has a strictly quasi-concave utility function $U^j = W(x_j, y)$ and income I_j, where x_j is the amount of the private good consumed by j. Let $I = I_i + I_j$. The price of the public good is p. The two agents take the prices as given and play a Cournot game with respect to the public good. Quantities of the public good purchased by agents i and j are, respectively, y_i and y_j. A *contributory agent* in any Nash equilibrium is one who purchases a positive amount of the public good at that equilibrium. We shall assume that a unique Nash equilibrium exists in this game.

Proposition 2.1: *Consider any Nash equilibrium where the agent j has personal expenditure* x_j^**, total consumption of the public good is* $y^\bullet$*, and the contributory agent i has a utility level* $U^* = U(x_i^*, y^*)$*. Then* $U^* = V(p, I - x_j^*)$*, where* V *is i's indirect utility function.*

Proof: Let $y_j^\bullet$ be the amount of the public good provided by agent j in the initial Nash equilibrium. Starting from this initial Nash equilibrium, suppose agent i is given a transfer from j equal to py_j^*. Then her personal income is $I_i' = I - x_j^*$. Since i is contributory, the consumption bundles remain unchanged in the new Nash equilibrium, the contributions to the public good being now $y_i' = y^*$, $y_j' = 0.$[2] If, alternatively, this agent was maximizing utility, in the absence of j, and with income I_i', i's maximization problem would be identical to that in the two-person Cournot game when she has the same income I_i' and j is non-contributory. Therefore, if j is non-contributory, i's consumption would be identical. In the Nash equilibrium after the income redistribution, by the neutrality property, j would indeed be non-contributory. The utility of i in the individual maximization case is given by $V(p, I - x_j^*)$, and since i has the same consumption in the original Nash equilibrium, $V(p, I - x_j^*) = U^*$.

◊

Proposition 2.1 implies that the demand functions $y(p, I_i, I_j), x_i(p, I_i, I_j)$ generated by Cournot-Nash equilibria can also be generated by a standard single-consumer utility maximization exercise by the contributory agent i when this agent is endowed with a variable personal income $I_i' = I - x_j\left(p, I_i, I_j\right)$. Since this transformation allows one to derive the individual Marshallian demand functions of i from the market demand functions if private demand functions and individual incomes are known, it also allows the utility function of i to be recovered in the standard way when i is a contributory agent. Proposition 2.1 can be extended in a straightforward fashion to cover the case of multiple public goods by using the corresponding neutrality property[3].

Suppose now that the state has a budgetary surplus (deficit) which it can transfer to (raise from) agent i either as a cash lump-sum or as subsidy (tax) on the public good.[4] It has a given budgetary support B and can choose alternative combinations of cash transfer T_i and subsidy rate *t*. The post-intervention demand for the public good is given by $\hat{y} = y\left(p(1-t), I_i + T_i, I_j\right)$, and the expenditure on the subsidy is

$S = pt\,\hat{y} = B - T_i$.

(2.1)

Let the private expenditure of agent j in the post-intervention equilibrium be $\hat{x}_j$. The utility derived by agent i in the post-intervention equilibrium is $\hat{U} = U\left(\hat{x}_i, \hat{y}\right)$. Let the minimum expenditure required at original price p to purchase a consumption bundle that yields the same utility to agent i be e, while the actual cost of the post-subsidy consumption bundle $\left(\hat{x}_i, \hat{y}\right)$ at the original price is $e + L$, where $L \geq 0$. Thus, $\hat{U} = V(p, e)$, where *V* is i's indirect utility function, and *L* is the (i-specific) money-metric deadweight loss associated with the subsidy.

Given B and the budget constraint for the state (2.1), consider any two alternative schemes 1 and 2. Denote the relevant magnitudes for each scheme by its corresponding number.

Corollary 2.2: *The contributory agent i prefers scheme 1 to scheme 2 if and only if* $\left[\hat{x}_j^1 + L_1\right] < \left[\hat{x}_j^2 + L_2\right]$.

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2 For analysis of this neutrality property of Cournot games with public goods, see Bergstrom et al. (1986).
3 Kemp (1984) and Bergstrom et. al. (1986) investigate the neutrality property with multiple public goods.
4 The treatment can be easily generalized to allow a tax or subsidy on one or both of the private goods.

Proof: Consider the general case for any such scheme. Since i is a contributory agent, by Proposition 2.1, we get $\hat{U} = V\left(p(1-t), \hat{E}\right)$, where her effective income $\hat{E}$ is given by

$$\hat{E} = I + T_i - \hat{x}_j = I + B - S - \hat{x}_j$$ (using (2.1)).

(2.2)

However, $e = e_i\left(p, \hat{U}\right)$ is the minimum expenditure required at original price p to provide the post-intervention utility level to i. Thus, the subsidy component transfers to i the equivalent variation

$$R = e - \hat{E} = S - L..$$

(2.3)

Equations (2.2) - (2.3) together yield

$$e = I + B - \left(\hat{x}_j + L\right).$$

(2.4)

Since $\hat{U} = V(p, e)$ and *V* is strictly increasing in expenditure, the result follows from (2.4). ◊

Equation (2.4) completely specifies each contributory agent's cardinal ranking over all possible intervention schemes involving lump-sum transfers to that agent and/or subsidy on the public good. The deadweight loss can be measured by recovering the utility function from the Cournot-Nash demand functions, as discussed earlier. Our results thus provide a simple method of deriving a money-metric measure of the individual-specific welfare impact of alternative distortionary interventions in contexts that may be modeled as Cournot games between two or more agents with one or more public goods. Aggregate welfare calculations can then proceed along standard lines.

It is clear from Corollary 2.2 that, if a subsidy increases the spending by the other agent on the public good, then the standard result regarding the superiority of lump-sum transfers vis-à-vis subsidies may get reversed in our strategic context.

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