THE WATTS' POVERTY INDEX WITH EXPLICIT PRICE VARIABILITY

Christophe MULLER

Centre for Research in Economic Development and International Trade,
School of Economics, University of Nottingham,
Nottingham NG7 2RG, United Kingdom

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Abstract

We derive an explicit formula of the Watts' poverty index, in terms of parameters of bivariate lognormal distributions of price indices and nominal living standards. This result enables us to: analyse the contributions of the distributions of prices and nominal living standards in poverty; interpret the e¤ects on poverty of changes in price distributions; estimate poverty when only means and variances of price indices and nominal living standards are known.

Using data from peasants in Rwanda, we test and estimate bivariate lognormal distributions of price indices and nominal living standards in four quarters of an agricultural year, and we calculate MLE of Watts' indices using the model, as well as estimates directly derived from the sampling scheme. The results show that despite frequent rejections of the lognormality assumptions, the Watts' poverty index estimated using the model is not signi…cantly di¤erent from sampling estimates. Moreover, estimates of the Watts index, using MME based on empirical means and variances of prices and nominal living standards, are not signi…cantly di¤erent. This allows the estimation of poverty without the direct availability of household survey data. Finally, we present simulations of e¤ects on poverty of changes in levels and changes in price variability.

Résumé

Nous dérivons une formule explicite pour l'indice de pauvreté de Watts, en termes des paramètres de distributions lognormales bivariées des prix et des niveaux de vie nominaux. Ce résultat nous permet d'analyser les contributions des distributions des prix et niveaux de vie nominaux dans la pauvreté; d'interpréter les e¤ets sur la pauvreté des changements des distributions des prix; d'estimer la pauvreté quand seulement moyennes et variances empiriques des indices de prix et niveaux de vie nominaux sont connus.

A partir de données pour les paysans au Rwanda, nous testons et estimons des distributions bivariées lognormales d'indices de prix et de niveaux de vie nominaux, pour les trimestres d'une année agricole. Nous calculons des estimateurs par la méthode du maximum de vraisemblance, de l'indice de pauvreté de Watts en utilisant le modèle, ainsi que des estimateurs dérivés directement du plan de sondage. Les résultats montrent que malgré de fréquentes rejections de l'hypothèse de lognormalité, l'indice de pauvreté de Watts, estimé en utilisant le modèle n'est pas signi…cativement di¤érent de l'estimateur du sondage. De plus, des estimations de

l'indice de Watts, faisant appel à la la méthode des moments basée sur les moyennes et variances empiriques des prix et des niveaux de vie, ne sont pas signi…cativement di¤érentes. Ceci permet l'estimation de la pauvreté sans la disponibilité directe de données d'enquête auprès des ménages. Finalement, nous présentons des simulations des e¤ets sur la pauvreté de changements du niveau de l'indice de prix et de la variabilité des prix.

1 Introduction

The structural adjustment plans in developing countries have raised misgivings associated with temporary rises in poverty (The World Bank (1990), Bourguignon, De Melo, Morrisson (1991), Sahn and Sarris (1991)). The implementation of these plans or other economic policy measures, are frequently accompanied by large temporal and geographical movements of relative prices[1], which may be one important channel of changes in living standards.

Moreover, geographical and seasonal di¤erences in prices that households face is a typical feature of LDCs, much explained by agricultural ‡uctuations of output, imperfect markets, high transport and commercialisation costs, and information problems. As discussed by Sen (1981), particularly in periods of famines, di¤erences in prices that household face can dramatically a¤ect their entitlements relations and their capacity to acquire food. The World Bank (1992) presents seasonal price ratios for twenty rural developing countries in the 1980^{s} and …ve products (rice, maize, wheat, sorghum, millet). These statistics show a generally high sensitivity of agricultural prices to seasons. These variations imply serious consequences for poor peasants that are often limited in their access to capital markets. In Africa, Baris and Couty (1981) suggest that the seasonal price ‡uctuations may aggravate the social di¤erentiation.

In these situations, the measured poverty index may incorporate substantial errors caused by unaccounted large price di¤erences between households or seasons (Jazairy, Alamgir and Panuccio (1992), Muller (1998)). Thus, the knowledge of the contribution of the price distribution in the assessment of poverty, is of outmost importance for welfare policies. Muller (1998) shows for a large range of poverty indicators that local and seasonal price di¤erences have a statistically signi…cant and large impact on the measurement of poverty in Rwanda.

Atkinson (1987), Lipton and Ravallion (1993) and Ravallion (1994) among others, insist on the use of accurate and axiomatically sound poverty indices.

[1] Sahn, Dorosh and Youngs (1997)) argue that in Ghana the market liberalisation during the adjustment program of the end 1980es has lead to price decreases (or moderate increases) despite a devaluation of 100 percent. Between 1984 and 1990 the prices of major staple foods fell and the ratio of decline was more rapid than in the 1970es and early 1980es. This was accompanied by substantial movements of relative prices.

One of the most popular axiomatically sound poverty index is the Watts' index (Watts (1968)). We focus in the present paper on this indicator.

The theoretical literature about price indices is extensive[2]. It has been notably used in applied welfare studies (Muellbauer (1974); Glewwe (1990), Grootaert and Kanbur (1996)). Theoretical price indices are defined as ratios of cost functions representing the preferences of households. In practice, applied price indices are generally Laspeyres or Paasche price indices, much ignoring the responses of households to price movements[3].

Though, the role of price index variability in the estimation of poverty indices has not been studied from a theoretical point of view, and there are no explicit results about the contribution of price distribution[4] to poverty. The present paper attempts to fill this lacuna by using a bivariate distribution model. In agricultural contexts, the prices of certain goods show large seasonal price fluctuations[5], and these fluctuations may have a substantial local component. This suggests using local price indices rather than national or regional inflation indicators, and to treat the seasonal variability of prices.

Finally, in many situations the only available information from publications about price indices and nominal living standards are means and standard deviations. A distribution model might help in dealing with poverty analysis in these cases and we study in this paper three different estimators of the Watts' index that are based on very different requirements of empirical information.

Can we derive an explicit formula of the Watts' poverty indicator, using a bivariate distribution model of price indices and nominal living standards? Can we interpret systematic effects induced by price level and variability in this model? Are estimates of poverty based on this model reliable? Can we extrapolate poverty from the sole observation of empirical means and variances of nominal welfares and prices? Is it possible to simulate effects

[2] Fisher and Shell (1972); Pollak (1978); Diewert (1981); Foss, Manser, Young (1982), Baye (1985); Pollak (1989); Diewert (1990), Selvanathan and Rao (1995).

[3] Braitwaith (1980) found in U.S. that the bias of the Laspeyres index due to differences in tastes is very moderate (about 1.5 percent for 25 years). Diewert (1998) provides an estimate of the upward bias of the Laspeyres index in the U.S equal to 0.41 percent for one year. However, the situation may be quite different in LDCs.

[4] Nonetheless, Muller (1998) provides a theoretical analysis in terms of directions of the bias due to the non correction for prices for several poverty indices.

[5] That is also well known for industrial countries in general (Riley (1961)).

on poverty of changes in prices? The aim of this article is to answer these questions firstly by analysing a distribution model, secondly by estimating the model using data from Rwanda.

We define in section 2 the Watts' poverty index. Then, under lognormality assumptions, we derive an expression of the Watts' index in terms of the parameters of the joint distribution of price indices and nominal living standards. We decompose this index and we analyse its sensitivity. We derive in section 3, theoretical estimators of the distribution parameters and of the Watts' index, as well as their asymptotic covariance matrices. In section 4, we describe the data used in the estimation. We test and estimate lognormal distributions of nominal living standards and price indices in section 5. In section 6, we compare maximum likelihood estimates of Watts' indices calculated using the model, with estimates using the sampling scheme, calculated directly from observed living standards, as well as poverty estimates based only on observed empirical means and variances. We present simulation results of effects of changes in levels and variability of prices, in section 7. Finally, section 8 concludes.

2 Watts' poverty index

The living standard indicator for household i at period t is

$$y_{it} = \frac{c_{it}}{es_i \, I_{it}} = \frac{w_{it}}{I_{it}} \tag{1}$$

where c_{it} is the value of the consumption of household i at period t ; w_{it} is the standard of living of household i at date t; es_i is the equivalence scale of household i and I_{it} is the price index associated with household i and period t. We denote $w_{it} = c_{it}/es_i$, the living standard indicator non corrected for price variability (nominal living standard, or n.l.s.). This variable is of first empirical importance, since it corresponds to what can be obtained from most statistical reports of household surveys, therefore from official statistics and from many articles.

The Watts' poverty index (Watts (1968)) is defined as

$$W = \int_0^z -\ln(y/z)\, d\mu(y) \tag{2}$$

where μ is the cumulative probability distribution of living standards y, and z is the poverty line.

The Watts' index satisfies the monotonicity, transfer and transfer sensitivity axioms. It is the only poverty index defined under absolute form from a social welfare function that satisfies monotonicity, continuity, decomposability and scale invariance (Zheng (1993)). These attractive properties enhances the interest of focusing on this index. In practice due to its axiomatic properties, it yields much better results than the head-count index (P_0), or even the poverty gap index (P_1). For example, Muller (1998) using data from Rwanda finds that most axiomatically sound indices, allowing some importance to the severity of poverty, lead to qualitatively similar results, by contrast with P_0 and P_1.

We now rewrite eq. 2 in terms of the joint distribution of w and I (denoted by the joint cumulative distribution function, F).

$$W = \int\int_{\Omega} -\ln((w/I)/z)\, dF(w, I) \tag{3}$$

where $\Omega = \{(w, I) | w > 0, I > 0, w/I < z\}$.

For general price variability, the poverty index cannot be simply decomposed in contributions of non corrected living standards and prices[6]. However, we shall show that it is possible to obtain explicit expressions by approximating F with bivariate lognormal distributions.

The choice of the lognormal distribution is supported by the fact that histograms of nominal living standards and price indices have unimodal asymmetrical and leptokurtic shapes, and the observations of these variables are always positive.

The lognormal approximation has been frequently used in applied economics for living standards (e.g. Alaiz and Victoria-Feser (1990), Slesnick (1993)). The assumption of lognormality of income has as well been exploited in theoretical economics (e.g. Hildenbrand (1998)). Log-wage or log-price equations are frequently estimated, implicitly relying on error terms

[6] Eq. (3) implies that the poverty line, z, is defined independently of the distributions of nominal living standards and price indices. The methods for calculating poverty lines are very varied, and the latter assumption may not always be satisfied. In that case, z should be replaced by an explicit function $z(F)$ and complementary terms are to be added to the expressions obtained in this paper. Since no general result can be derived for these very varied specifications, we do not pursue this direction in this paper.

related to normality assumptions, sometimes asymptotically. Eaton (1980), Deaton and Grimard (1992), for example, assume lognormality for price distributions. Other distribution models for living standards or incomes (Singh and Maddala (1976), Hirschberg and Slottje (1989)), such that the Pareto distribution or the Gamma distribution (Salem and Mount (1974)) or other distribution models for prices (Creedy and Martin (1994)) can also be used, but will not lead to an explicit expression for the Watts' index.

The reason why we adopt a lognormal specification is not that it corresponds to an almost perfect adequation to the data, but rather because we search for a bivariate distribution model for nominal living standards and price indices, which would have the well-behaved characteristics evoked above and which will lead to an explicit expression of the poverty index. Thus, the question of statistical adequation is here secondary in comparison with the use of the distribution model as an analytical tool. Therefore, even in the case of imperfect statistical adequation with the data, we would like to know if poverty estimates using the model are statistically close to the best poverty estimates without the model.

We present now the expression of the Watts' index under lognormality assumption.

Proposition 1

If the nominal living standards and the price indices follow a bivariate lognormal distribution law, $LN\left(\begin{pmatrix} m_1 \\ m_2 \end{pmatrix}; \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix}\right)$, the Watts' index is equal to:

$$W = (\ln(z) - m_1 + m_2) \cdot \Phi\left(\frac{\ln z - m_1 + m_2}{\sqrt{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2}}\right) + \sqrt{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2} \cdot \phi\left(\frac{\ln z - m_1 + m_2}{\sqrt{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2}}\right) \quad (4)$$

where ϕ and Φ are respectively the p.d.f. and c.d.f. of the standard normal distribution. The knowledge of

$$Z = \frac{\ln z - m_1 + m_2}{\sqrt{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2}} \quad (5)$$

and

$$S = \sqrt{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2} \quad (6)$$

is sufficient for the knowledge of W.

$$W = S.[Z.\Phi(Z) + \phi(Z)] = S.G(Z) \quad (7)$$

Proof: In appendix.

Eq. 4 shows that unless all price indices are very concentrated around 1, they should not be neglected in the estimation of the Watts' poverty index. It is also clear that the parameters associated respectively with distributions of w and I play similar roles. Note that m_1 and σ_1 (resp. m_2 and σ_2) are the mean and the standard deviation of the logarithms of living standards (resp. of price indices).

Eq. 7 shows that the Watts index can be decomposed in terms of two sufficient statistics, S, that is the standard deviation of the logarithm of the real living standards, that we call "global variability" ; and Z, which is the standardised logarithm of the poverty line expressed in real terms.

$\Phi(Z)$ is equal to the probability of incidence of poverty (or head-count index) under lognormality. Function G(Z) is a primitive function with respect to Z of the head-count index (with value $\frac{1}{\sqrt{2\pi}}$ at Z = 0), that we call "cumulating (lognormal) poverty incidence". It is also the Watts' poverty at unitary global variability and can itself be considered as a poverty index. Thus, eq. 7 provides an interpretation of the Watts' index as the product of the global variability and the cumulated poverty incidence. Consequently, the elasticity of W with respect to any variable is the sum of the elasticity of the global variability and the elasticity of the cumulative poverty incidence.

The gradient of W with respect to parameters can easily be calculated.

Proposition 2 The marginal variations of W with respect to S and Z are

$$\frac{\partial W}{\partial S} = Z.\Phi(Z) + \phi(Z) \tag{8}$$

$$\frac{\partial W}{\partial Z} = S.\Phi(Z) > 0 \tag{9}$$

The gradient of W with respect to distribution parameters has the following components.

$$\frac{\partial W}{\partial m_1} = -\Phi(Z) < 0 \tag{10}$$

$$\frac{\partial W}{\partial m_2} = \Phi(Z) > 0 \tag{11}$$

$$\frac{\partial W}{\partial \sigma_1} = \left(\frac{\sigma_1 - \rho\sigma_2}{S}\right)\phi(Z) \text{ of the sign of } \sigma_1 - \rho\sigma_2 \tag{12}$$

$$\frac{\partial W}{\partial \sigma_2} = \left(\frac{\sigma_2 - \rho\sigma_1}{S}\right)\phi(Z) \text{ of the sign of } \sigma_2 - \rho\sigma_1 \tag{13}$$

$$\frac{\partial W}{\partial \rho} = \left(\frac{-\sigma_1\sigma_2}{S}\right)\phi(Z) < 0 \tag{14}$$

Proof: Elementary differential calculus.

The marginal variations of W with respect to Z or G(Z) are positive. The latter illustrates the consistent link between the two notions of poverty (W and G). By contrast, an increase in the global variability S may be beneficial or noxious to poverty.

The gradient of W with respect to distribution parameters is shown, in eqs. 10 through 14. Poverty measured using the Watts' index decreases in the mean level of the logarithm of nominal living standards, m_1, and increases in the mean level of the logarithm of price indices, m_2. The corresponding gradients are respectively equal to minus and plus the incidence of poverty, and are therefore bounded (respectively in [-1,0] and [0,1]). The marginal rate of substitution of m_2 to m_1 is equal to -1, showing the perfect substitutability in W of a decrease in the mean of logarithms of nominal living standards (mean of "l.n.l.s.") and an increase in the mean of logarithms of price indices (mean of "l.p.i."). These variations are caused by the variation of the cumulated incidence of poverty, and not by the global variability that remains constant.

The evolution of poverty with the variance of the l.n.l.s., $¾_1^2$, or the variance of the l.p.i., $¾_2^2$, is less elementary, these variations being associated both with changes in the cumulated incidence of poverty and in the global variability. The change in Z is through the modi…cation of scale due to a change in variability of the logarithm of real living standards. We …rst examine the case of ½ positive (then ½ < 1=½), in which three regimes are possible, then the case of ½ negative.

a) If $¾_1$=$¾_2$ < ½, then poverty increases with the variance of l.p.i. and decreases with the variance of the l.n.l.s. Under relatively high correlation between l.p.i. and l.n.l.s., the e¤ects of the variances of logarithms of both variables have the same direction than the e¤ects of the levels of logarithms.

b) If 1=½ > $¾_1$=$¾_2$ > ½, then poverty increases with both variances. Under relatively average positive correlation between l.p.i. and l.n.l.s., an increase in variability of the logarithms of both variables increases poverty.

c) If $¾_1$=$¾_2$ > 1=½, then poverty decreases with the variance of the l.p.i. and increases with the variance of the l.n.l.s. Under relatively low positive correlations, the e¤ects of levels and variances of the logarithms of variables have opposite direction.

d) In the case of ½ negative, the order of ½ and 1/½ is reversed, and $¾_1$=$¾_2$ is greater than ½ and 1/½: The poverty increases with both variances.

An increase in the correlation between l.n.l.s. and l.p.i., is associated with a decrease in poverty. The marginal rate of substitution of ½ to $¾_1$ is equal to 1/$¾_2$ - 1/(½$¾_1$) and is negative for $¾_1$=$¾_2$ < 1=½, i.e. for small positive correlations. In that case, an increase in variability[7] of l.n.l.s. (often associated with an increase in inequality) can be compensated by higher correlations between l.p.i. and l.n.l.s., for example with higher prices for rich

[7] To shorten, we call "variabilities" the standard-deviation parameters $¾_1$ and $¾_2$:

households.

3 Estimators of the bivariate distribution and the Watts' index

3.1 MLE

We can estimate the parameters of the joint distribution, using samples of price indices and living standards. This estimation is interesting on several grounds. First, it informs about the shape of the considered distributions. Second, it helps to quantify the respective effects of both levels and variabilities of the l.n.l.s. and of the l.p.i., directly or directly by using eq. 4. Finally, the estimates can be incorporated in an estimator of W.

Proposition 3

If the distributions of w and I are jointly lognormal, then the maximum likelihood estimators (MLE) of (m_i, σ_i), i = 1 and 2, and ρ; are consistent, efficient and invariant. They are:

$$\hat{m}_1 = \frac{1}{n}\Sigma_i \ln(w_i) \text{ and } \hat{m}_2 = \frac{1}{n}\Sigma_i \ln(I_i) \tag{15}$$

$$\hat{\sigma}_1^2 = \frac{1}{n}\Sigma_i (\ln(w_i) - m_1)^2 \text{ and } \hat{\sigma}_2^2 = \frac{1}{n}\Sigma_i (\ln(I_i) - m_2)^2 \tag{16}$$

$$\hat{\rho} = \frac{\frac{1}{n}\Sigma_i (\ln(w_i) - m_1).(\ln(I_i) - m_2)}{\hat{\sigma}_1 \hat{\sigma}_2} \tag{17}$$

The Fisher information matrix associated with $(\hat{m}_1, \hat{m}_2, \hat{\sigma}_1; \hat{\sigma}_2; \hat{\rho})$ calculated from a sample of size n, is

$$
I_F = \begin{bmatrix}
\frac{n}{\sigma_1^2(1-\rho^2)} & \frac{-n\cdot\rho}{\sigma_1\sigma_2(1-\rho^2)} & 0 & 0 & 0 \\
\frac{-n\cdot\rho}{\sigma_1\sigma_2(1-\rho^2)} & \frac{n}{\sigma_2^2(1-\rho^2)} & 0 & 0 & 0 \\
0 & 0 & \frac{n\cdot(2-\rho^2)}{\sigma_1^2(1-\rho^2)} & \frac{-\rho^2\cdot n}{\sigma_1\sigma_2(1-\rho^2)} & \frac{n\cdot\rho}{\sigma_1^2(1-\rho^2)} \\
0 & 0 & \frac{-\rho^2\cdot n}{\sigma_1\sigma_2(1-\rho^2)} & \frac{n\cdot(2-\rho^2)}{\sigma_2^2(1-\rho^2)} & \frac{n\cdot\rho}{\sigma_2^2(1-\rho^2)} \\
0 & 0 & \frac{n\cdot\rho}{\sigma_1^2(1-\rho^2)} & \frac{n\cdot\rho}{\sigma_2^2(1-\rho^2)} & \frac{n\cdot(1+\rho^2)}{(1-\rho^2)^2}
\end{bmatrix} \tag{18}
$$

and the MLEs are asymptotically normal with

$$
\sqrt{n}\begin{pmatrix} \hat{m}_1 - m_1 \\ \hat{m}_2 - m_2 \end{pmatrix} \to N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}; \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix}\right) \tag{19}
$$

and independently

$$
\sqrt{n}\begin{pmatrix} \hat{\sigma}_1 - \sigma_1 \\ \hat{\sigma}_2 - \sigma_2 \\ \hat{\rho} - \rho \end{pmatrix} \to N\left(\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}; \begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{12} & B_{22} & B_{23} \\ B_{13} & B_{23} & B_{33} \end{bmatrix}\right). \tag{20}
$$

where

$$
B_{11} = \frac{\sigma_1^4\cdot(2\sigma_2^2 + \rho^2\cdot(1-\rho^2)\cdot(-1+\sigma_2^2))}{\Delta}
$$

$$
B_{12} = \frac{\rho^2\cdot\sigma_1^2\sigma_2^2(1 + \sigma_1\sigma_2 + \rho^2\cdot(-1 + \sigma_1\sigma_2)}{\Delta}
$$

$$
B_{13} = \frac{\rho\cdot(\rho^2 - 1)\cdot\sigma_1^2\sigma_2(2\sigma_2 + \rho^2\cdot(\sigma_1 - \sigma_2))}{\Delta}
$$

$$
B_{22} = \frac{\sigma_2^4\cdot(2\sigma_1^2 + \rho^2\cdot(1-\rho^2)\cdot(-1+\sigma_1^2))}{\Delta}
$$

$$B_{23} = \frac{\rho.(\rho^2 - 1).\sigma_1\sigma_2^2((-2+\rho^2).\sigma_1 - \rho^2\sigma_2)}{\Delta}$$

$$B_{33} = \frac{4.(\rho^2 - 1)^2.\sigma_1^2\sigma_2^2}{\Delta}$$

with

$$\Delta = \rho^4.(\sigma_1 - \sigma_2)^2 + 4\sigma_1^2\sigma_2^2 + \rho^2\left(4\sigma_1^2\sigma_2^2 - 2\sigma_1^2 - 2\sigma_2^2\right) \tag{21}$$

We denote the corresponding covariance matrix of $(\hat{m}_1, \hat{m}_2, \hat{\sigma}_1; \hat{\sigma}_2; \hat{\rho})$ as Σ_L.

Moreover, the $\hat{m}_1$ and $\hat{m}_2$ are unbiased estimators.

Proof: in appendix.

The MLE of the means $(\alpha_i; i = 1; 2)$, variances $(\gamma_i; i = 1; 2)$, and the correlation coefficient R of w and l (respectively with i=1,2), can be derived.

Proposition 4 :

For i = 1, 2 :

$$\hat{\alpha}_i = e^{\hat{m}_i + \frac{\hat{\sigma}_i^2}{2}} \tag{22}$$

$$\hat{\gamma}_i = \hat{\alpha}_i^2.(e^{\hat{\sigma}_i^2} - 1) \tag{23}$$

$$\hat{R} = \hat{\rho}(w; l) = \frac{\exp(\hat{\rho}\hat{\sigma}_1\hat{\sigma}_2) - 1}{\sqrt{(\exp(\hat{\sigma}_1^2) - 1).(\exp(\hat{\sigma}_2^2) - 1)}} \tag{24}$$

and they are asymptotically normal with

$$\sqrt{n}\begin{pmatrix} \hat{\alpha}_1 - \alpha_1 \\ \hat{\gamma}_1 - \gamma_1 \\ \hat{\alpha}_2 - \alpha_2 \\ \hat{\gamma}_2 - \gamma_2 \end{pmatrix} \longrightarrow N(0; C) \tag{25}$$

where $C = \nabla g' \cdot \Sigma \cdot \nabla g$, $\Sigma = N \cdot \Sigma_L$ is the asymptotic variance-covariance matrix of $\sqrt{n} \cdot (\hat{m}_1; \hat{m}_2; \hat{\sigma}_1; \hat{\sigma}_2; \hat{\rho})'$, and g the function vector defining $(\hat{\alpha}_1; \hat{\beta}_1; \hat{\alpha}_2; \hat{\beta}_2)$ from $(\hat{m}_1; \hat{m}_2; \hat{\sigma}_1; \hat{\sigma}_2; \hat{\rho})$ using equations 22 and 23.

A corresponding limit central theorem can be stated with $(\hat{\alpha}_1; \hat{\beta}_1; \hat{\alpha}_2; \hat{\beta}_2; \hat{R})$.

Due to the invariance property of the MLE, the MLE of W, denoted WL (for "likelihood"), can be defined using eq. 4 and substituting the MLEs for the distribution parameters.

$$WL = (\ln(z) - \hat{m}_1 + \hat{m}_2) \cdot \Phi\left(\frac{\ln z - \hat{m}_1 + \hat{m}_2}{\sqrt{\hat{\sigma}_1^2 + \hat{\sigma}_2^2 - 2\hat{\rho}\hat{\sigma}_1\hat{\sigma}_2}}\right) + \sqrt{\hat{\sigma}_1^2 + \hat{\sigma}_2^2 - 2\hat{\rho}\hat{\sigma}_1\hat{\sigma}_2} \cdot \phi\left(\frac{\ln z - \hat{m}_1 + \hat{m}_2}{\sqrt{\hat{\sigma}_1^2 + \hat{\sigma}_2^2 - 2\hat{\rho}\hat{\sigma}_1\hat{\sigma}_2}}\right) \quad (26)$$

The asymptotic variance of WL, that is asymptotically normal, is $V(WL) = \nabla WL' \cdot \Sigma \cdot \nabla WL$, where ∇WL denotes the gradient vector of WL with respect to $(\hat{m}_1; \hat{m}_2; \hat{\sigma}_1; \hat{\sigma}_2; \hat{\rho})$.

Proof: See appendix.

The different asymptotic variance-covariance matrices can be consistently estimated by replacing parameters m_i, σ_i^2 $(i = 1; 2)$ and ρ with consistent estimates, for example with the MLEs. Then, confidence regions of parameter estimates can be easily derived.

3.2 MME

As we said above, poverty indicators are not systematically published in household survey documents. Generally only the mean and standard deviation of nominal living standards are available, accompanied sometimes of price statistics. We propose to investigate the use of observed mean and standard-deviations of w and l (denoted $\alpha_1; \nu_1; \alpha_2; \nu_2$) to produce an estimator of poverty, denoted WM (for "moments"). We have shown above that eqs. 22 and 23 can be used to connect the MLE of parameters of the lognormal distributions to the MLE of means and standard-deviations of the two univariate distributions of x and l. Similarly, we can define estimators $\tilde{\mu}_i$ and $\tilde{\sigma}_i^2$ of the distribution parameters, using the method of moments (MME), as:

Definition 5 (MME) $i = 1, 2$

$$\tilde{\sigma}_i = \sqrt{\ln\left(1 + \frac{\beta_i}{\alpha_i^2}\right)} \tag{27}$$

$$\tilde{\mu}_i = \ln(\alpha_i) - \tilde{\sigma}_i^2/2 \tag{28}$$

Estimators $(\tilde{\mu}_1, \tilde{\mu}_2, \tilde{\sigma}_1^2; \tilde{\sigma}_2^2)$ and $(\hat{\mu}_1, \hat{\mu}_2, \hat{\sigma}_1^2; \hat{\sigma}_2^2)$ are consistent, although not asymptotically equivalent. $(\hat{\mu}_1, \hat{\mu}_2, \hat{\sigma}_1^2; \hat{\sigma}_2^2)$ is efficient if the lognormal assumption is valid, by contrast with the MME that is generally not efficient. $(\tilde{\mu}_1, \tilde{\mu}_2, \tilde{\sigma}_1^2; \tilde{\sigma}_2^2)$ can be as well considered as an asymptotic least square estimator (Gouriéroux, Monfort, Trognon (1985)) using eqs. 27 and 28 to define the link between the parameters of interest and a consistent and asymptotically normal estimator.

The MME can be substituted in eq. 4 under the hypothesis ($\rho = 0$) to define the estimator WM. Note that the latter hypothesis, which is not necessary to the definition of $(\tilde{\mu}_1, \tilde{\mu}_2, \tilde{\sigma}_1^2; \tilde{\sigma}_2^2)$ is important because firstly it may correspond to a plausible situation, secondly it eliminates the need for estimates of ρ or of R, which are typically not available in usual survey publications.

Definition 6

$$\begin{aligned} WM = {} & (\ln z - \tilde{\mu}_1 + \tilde{\mu}_2) \cdot \Phi\left(\frac{\ln z - \tilde{\mu}_1 + \tilde{\mu}_2}{\sqrt{\tilde{\sigma}_1^2 + \tilde{\sigma}_2^2}}\right) \\ & + \sqrt{\tilde{\sigma}_1^2 + \tilde{\sigma}_2^2} \cdot \phi\left(\frac{\ln z - \tilde{\mu}_1 + \tilde{\mu}_2}{\sqrt{\tilde{\sigma}_1^2 + \tilde{\sigma}_2^2}}\right) \end{aligned} \tag{29}$$

The associated moment conditions are

$$F_i(\theta) = \begin{pmatrix} f_1 : w_i - e^{m_1+\sigma_1^2/2} = 0 \\ f_2 : w_i^2 - e^{2m_1+2\sigma_1^2} = 0 \\ f_3 : l_i - e^{m_2+\sigma_2^2/2} = 0 \\ f_4 : l_i^2 - e^{2m_2+2\sigma_2^2} = 0 \end{pmatrix} = \begin{pmatrix} F_i^1(\theta) \\ F_i^2(\theta) \end{pmatrix}$$

where i is the index of the observation.

F denotes the vector of the $F_i(\theta)$.

Proposition 7

The asymptotic variance-covariance matrix of the MME $(\tilde{m}_j; \tilde{\sigma}_j)$; $j = 1; 2$ is

$$\tilde{\Sigma}_j = [D_j' \Omega_j^{-1} D_j]^{-1}/N \tag{30}$$

with $M^j = \underset{N \to +\infty}{\text{plim}} \{\frac{1}{N} F^{j\prime}.e\}$; $j = 1; 2$; where e is a vector of ones, and

$$D_j = \frac{\partial M^j}{\partial (m_j \ \sigma_j)'} = \begin{bmatrix} -e^{(m_j+\sigma_j^2/2)} & -\sigma_j e^{(m_j+\sigma_j^2/2)} \\ -2e^{2m_j+2\sigma_j^2} & -4\sigma_j e^{2(m_j+\sigma_j^2)} \end{bmatrix}, \text{ and}$$

$\Omega = \begin{pmatrix} \Omega_1 \\ \Omega_2 \end{pmatrix}$ where $\Omega_{kj} = \underset{N \to +\infty}{\text{plim}} \left(\frac{1}{N} \sum_{i=1}^{n} f_{ik}.f_{ij} \right)$ that can be estimated by $\hat{\Omega}_{kj} = \frac{1}{N} \sum_{i=1}^{n} f_{ik}.f_{ij}$.

Then, the asymptotic variance-covariance matrix of $(\tilde{m}_1, \tilde{\sigma}_1; \tilde{m}_2; \tilde{\sigma}_2)$ under the hypothesis $\rho = 0$ is

$$\Sigma_0 = \begin{pmatrix} \tilde{\Sigma}_1 & 0 \\ 0 & \tilde{\Sigma}_2 \end{pmatrix}$$

Moreover, the asymptotic variance-covariance matrix of $(\hat{m}_1; \hat{\sigma}_1; \tilde{m}_2; \tilde{\sigma}_2)$ is, under the hypothesis $\rho = 0$:

$$\Sigma_3 = \begin{pmatrix} \frac{1}{N} IF_1^{-1} & 0 \\ 0 & \tilde{\Sigma}_2 \end{pmatrix}$$

where IF_1^{-1} is the inverse of the bloc of IF corresponding to $(\hat{m}_1; \hat{\sigma}_1)$. Note that IF is the total information matrix for the whole sample.

Finally, $V(WM) = \nabla WM'.\Sigma_0.\nabla WM$, where ∇WM can be calculated using the formula of the gradient of W.

Proof: It is clear that $E[F_i] = 0$, which defines the "estimating equations" for the MME. Formula 30 is the asymptotic covariance matrix of the MME (Davidson and McKinnon (1993)). Matrices D and Φ are calculated in the case considered using the estimating equations.

Moreover, under $\rho = 0$, the two distributions of w and l are independent since those of lnw and lnl are. Then, $cov(\hat{\mu}_1;\tilde{\mu}_2) = cov(\hat{\mu}_1, \tilde{\sigma}_2) = cov(\hat{\sigma}_1;\tilde{\mu}_2) = cov(\hat{\sigma}_1;\tilde{\sigma}_2) = 0$.

The variance of WM is obtained by application of the "delta" operator (See for example Gouriéroux and Monfort (1989)). []

3.3 Simulators

The simulated Watts' poverty indices, WS and WMS, corresponding to changes in price indices distribution, are defined as follows.

Definition 8

$$\begin{aligned} WS = {} & (\ln z - \hat{\mu}_1 + \theta_1\hat{\mu}_2).\Phi\left(\frac{\ln z - \hat{\mu}_1 + \theta_1\hat{\mu}_2}{\sqrt{\hat{\sigma}_1^2 + \theta_2^2\hat{\sigma}_2^2 - 2\hat{\rho}\hat{\sigma}_1\theta_2\hat{\sigma}_2}}\right) \\ & + \sqrt{\hat{\sigma}_1^2 + \theta_2^2\hat{\sigma}_2^2 - 2\hat{\rho}\hat{\sigma}_1\theta_2\hat{\sigma}_2}.\phi\left(\frac{\ln z - \hat{\mu}_1 + \theta_1\hat{\mu}_2}{\sqrt{\hat{\sigma}_1^2 + \theta_2^2\hat{\sigma}_2^2 - 2\hat{\rho}\hat{\sigma}_1\theta_2\hat{\sigma}_2}}\right) \end{aligned} \tag{31}$$

$$\begin{aligned} WMS = {} & (\ln z - \hat{\mu}_1 + \tilde{\mu}_2(\theta_1'\alpha_2;\theta_2'\gamma_2)). \\ & \Phi\left(\frac{\ln z - \hat{\mu}_1 + \tilde{\mu}_2(\theta_1'\alpha_2;\theta_2'\gamma_2)}{\sqrt{\hat{\sigma}_1^2 + \tilde{\sigma}_2^2(\theta_1'\alpha_2;\theta_2'\gamma_2) - 2\hat{\rho}\hat{\sigma}_1\tilde{\sigma}_2(\theta_1'\alpha_2;\theta_2'\gamma_2)}}\right) \end{aligned} \tag{32}$$

$$\begin{aligned} & + \sqrt{\hat{\sigma}_1^2 + \tilde{\sigma}_2^2(\theta_1'\alpha_2;\theta_2'\gamma_2) - 2\hat{\rho}\hat{\sigma}_1\tilde{\sigma}_2(\theta_1'\alpha_2;\theta_2'\gamma_2)} \\ & .\phi\left(\frac{\ln z - \hat{\mu}_1 + \tilde{\mu}_2(\theta_1'\alpha_2;\theta_2'\gamma_2)}{\sqrt{\hat{\sigma}_1^2 + \tilde{\sigma}_2^2(\theta_1'\alpha_2;\theta_2'\gamma_2) - 2\hat{\rho}\hat{\sigma}_1\tilde{\sigma}_2(\theta_1'\alpha_2;\theta_2'\gamma_2)}}\right) \end{aligned} \tag{33}$$

where θ_1 and θ_2, θ_1' and θ_2' are simulation parameters describing the changes in the price distribution, and $\hat{m}_2(\theta_1'\alpha_2;\theta_2'\gamma_2)$ and $\hat{\sigma}_2(\theta_1'\alpha_2;\theta_2'\gamma_2)$ are the MME deduced from means and variances of price indices respectively equal to $\theta_1'.\alpha_2$ and $\theta_2.\gamma_2$.

The asymptotic covariance matrices of WS and WMS are derived from a combination of the asymptotic covariance matrices Σ_L and Σ_3 using the delta operator decomposed in several matrices of change in variables.

Proposition 9

$V(WS) = \nabla WS' \sum_L \nabla WS$

where ∇WS can be deduced straightforwardly from the formula of the gradient of W defined with the components in the order of proposition 2 and including the effects of parameters θ_1 and θ_2.

$V(WMS) = G.\Sigma_3.G'$ where

$$G = \left[\frac{\partial W}{\partial(m_1\ \sigma_1\ m_2\ \sigma_2)'}\right]\left[\frac{\partial(m_1\ \sigma_1\ m_2(\theta_1'\alpha_2;\theta_2'\gamma_2)\ \sigma_2(\theta_1'\alpha_2;\theta_2'\gamma_2))'}{\partial(m_1\ \sigma_1\ \theta_1'\alpha_2\ \theta_2'\gamma_2)}\right].$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \theta_1' & 0 \\ 0 & 0 & 0 & \theta_2' \end{bmatrix} . \left[\frac{\partial(m_1\ \sigma_1\ m_2\ \sigma_2)'}{\partial(m_1\ \sigma_1\ \alpha_2\ \gamma_2)}\right]^{-1}$$

is the delta operator corresponding to the mapping $(\hat{m}_1; \hat{\sigma}_1; \hat{m}_2; \hat{\sigma}_2) \mapsto$ WMS. Each Jacobian matrix corresponds to specific changes in variables and is calculated using consistent estimators at appropriate values of parameters.

$$\left[\frac{\partial(m_1\ \sigma_1\ m_2\ \sigma_2)'}{\partial(m_1\ \sigma_1\ \alpha_2\ \gamma_2)}\right] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{\partial m_2}{\partial \alpha_2} & \frac{\partial m_2}{\partial \gamma_2} \\ 0 & 0 & \frac{\partial \sigma_2}{\partial \alpha_2} & \frac{\partial \sigma_2}{\partial \gamma_2} \end{bmatrix}$$

where $\frac{\partial m_2}{\partial \alpha_2} = \frac{1}{\alpha_2}\left[1 + \frac{\gamma_2}{\alpha_2^2+\gamma_2}\right]$

$\frac{\partial m_2}{\partial \gamma_2} = -\frac{1}{2(\alpha_2^2+\gamma_2)}$

$$\frac{\partial \sigma_2}{\partial \alpha_2} = - \frac{1}{\sqrt{\ln\left(1+\frac{\gamma_2}{\alpha_2^2}\right)}} \cdot \frac{1}{\left(\alpha_2^2+\gamma_2\right)} \cdot \frac{\gamma_2}{\alpha_2}$$

$$\frac{\partial \sigma_2}{\partial \gamma_2} = \frac{1}{2\sqrt{\ln\left(1+\frac{\gamma_2}{\alpha_2^2}\right)}} \cdot \frac{1}{\left(\alpha_2^2+\gamma_2\right)}$$

Proof: The mapping g transforming $(\hat m_1, \hat\sigma_1, \hat m_2, \hat\sigma_2)$ whose covariance matrix, Σ_M, is known, into WMS can be decomposed into

$$(\hat m_1, \hat\sigma_1, \hat m_2, \hat\sigma_2) \overset{g_1}{\mapsto} (\hat m_1, \hat\sigma_1, \hat\alpha_2, \hat\gamma_2) \overset{g_2}{\mapsto} (\hat m_1, \hat\sigma_1, \mu_1'\hat\alpha_2, \mu_2'\hat\gamma_2)$$

$$\overset{g_3}{\mapsto} (\hat m_1, \hat\sigma_1, m_2(\mu_1'\hat\alpha_2; \mu_2'\hat\gamma_2), \sigma_2(\mu_1'\hat\alpha_2; \mu_2'\hat\gamma_2)) \overset{g_4}{\mapsto} WMS .$$

Then, the covariance matrix to calculate is $Jg.\Sigma_M.(Jg)'$ where $g = g_4 \circ g_3 \circ g_2 \circ g_1$, and J denotes the "Jacobian matrix of ". This yields using the chain rule:
$Jg_4\, Jg_3\, Jg_2\, Jg_1.\Sigma_M.(Jg_1)'(Jg_2)'(Jg_3)'(Jg_4)'$.

Note that to simplify $\hat\rho$ in WMS has been assumed here non random, which is for example the case when $\rho = 0$ is accepted from the above test results. AV□

The model developed so far for W is useful on several grounds. It first clarifies the effects of levels and variabilities in prices and nominal living standards. It can be used to extrapolate the poverty measure, from the sole observation of means and standard deviations of price indices and nominal living standards. It can finally be useful to simulate effects of changes in these variabilities and levels on the aggregate poverty. We investigate these applications in the next sections.

4 The Data

Rwanda in 1983 is a small rural country in Central Africa. At this period, it is relatively preserved from extreme economical, political or climatic shocks. Its population is 5.7 million, nearly half under 15 years of age. Rwanda is one of the poorest country in the world, with per capita GNP of US $ 270 per annum. More than 95 percent of the population live in rural areas (Bureau National du Recensement (1984)) and agriculture is the cornerstone of the economy, accounting for 38 percent of GNP and most of the employment.

Data for the estimation is taken from the Rwandan national budget-consumption survey, conducted by the Government of Rwanda and the French

Cooperation and Development Ministry, in the rural part of the country from November 1982 to December 1983 (Ministère du Plan (1986a))[8]. 270 households were surveyed about their budget and their consumption. The consumption indicators are of very high quality[9].

Agricultural year 1982-83 is a fairly normal year in terms of climatic fluctuations (Bulletin Climatique du Rwanda (1982, 1983, 1984)). The agricultural year can be split up into four climatic seasons and two cultural seasons. The collection of the consumption data was organised in four rounds, corresponding to four quarters (A, B, C, D) of the agricultural year 1982-83.

The sampling scheme[10] has four sampling levels (communes, sectors, districts and households). The drawing of the communes was stratified by prefectures, agro-climatic regions and altitude zones. One district was drawn in each commune and one cluster of three neighbouring households was drawn in each district. From this information, we have calculated sampling weights that reflect the probabilities of drawings of units at every stage of the sample scheme.

The average household size has 5.22 members, including 2.67 children or adolescents (less than 18 years old), and 2.55 adults (18 years old and more). The average land area is very small (1.24 ha). Table 1 shows that for the sample used in estimations, it corresponds to an average production of 57 158 Frw (Rwandan Francs) of agricultural output, that is close to 51 176 Frw of average consumption (10613 Frw per capita).

Several studies of price surveys in Rwanda have revealed the existence of

[8] The main part of the collection has been designed with the help of INSEE (French National Statistical Institute).

[9] Indeed, every household was visited at least once a day, during two weeks for every quarter. Daily and retrospective interviews and food weighting were carried out, and every household had also to register much information in a diary between the quarterly survey rounds. This enabled a thorough cleaning of the data, by more than thirty ex-enumerators after the collection, under our supervision. Sophisticated verification algorithms have been designed using the many redundancies present in the data. Finally, the consumption indicators are based on algorithms reducing measurement errors, from the combination of several information sources. The quality of consumption indicators seems to us a crucial requirement in poverty analysis, notably because of the non-robustness of some poverty indicators to data contamination, which has been analysed by Cowell and Victoria-Feser (1996).

[10] The sampling scheme has been modelled in Roy (1984) and completed by our own investigations during our stay at the Direction Générale de la Statistique du Rwanda)

considerable both geographical and seasonal price variability (Niyonteze and Nsengiyumva (1986), O.S.C.E. (1987), Ministère du Plan (1986b), Muller (1988)).

We have calculated elementary price indicators of the main categories of goods, for every season and every cluster of the sample. The prices of each category of goods are represented by the price of the main product, which enables us to compare prices across seasons and clusters with little quality bias.

True price indices could be derived from estimating a complete agricultural household model, from which shadow prices could be calculated. However, such procedure would incorporate much noise due to the inaccuracy of the estimates with small sample size. Muller (1998) discusses the type and the sample of prices used, the price index and the difficulty of the shadow prices approach (See also Singh, Squire and Strauss (1986), de Janvry, Sadoulet, Fafchamps (1991)). Nonetheless, the methods of the present paper can be applied to welfare indicators based on estimated price indices and estimated adult-equivalent scales.

We approximate the theoretical price index with a Laspeyres price index (I_{it}) specific to each household and each period, in which the basis is the annual national average consumption.

$$I_{it} = \sum_j \omega^j \frac{p^j_{gt}}{p^j_{::}} \quad \text{where } \omega^j = \frac{\sum_i \sum_t p^j_{it} q^j_{it} POND_{it}}{\sum_j \sum_i \sum_t p^j_{it} q^j_{it} POND_{it}} \tag{34}$$

where p^j_{it} (resp. p^j_{gt}) is the price of good j at date t for household i (resp. in cluster g where is observed household i), q^j_{it} is the consumed quantity of good j at date t by household i in cluster g, $POND_{it}$ is the sampling weight of household i at date t, corrected for missing values.

The annual national prices are calculated as follows:

$$p^j_{::} = \frac{\sum_i \sum_t p^j_{it} q^j_{it} POND_{it}}{\sum_i \sum_t q^j_{it} POND_{it}} \tag{35}$$

We therefore consider simultaneously geographical and seasonal price variability, although without modelling temporal and spatial autocorrelations of prices.

5 Tests and Estimates of Distributions

5.1 Tests of Lognormality

The lognormal assumptions of section 2 constitute practical approximations. It is likely that these approximations are consistent with results of statistical tests of the distribution shapes only in some cases. If the considered distributions are very close to lognormality, the derived formula WL will clearly be adequate. However, it is also interesting to explore its usefulness for distributions not statistically equal to lognormal distributions, and to compare poverty estimates derived from our model to estimates directly using the sampling scheme. In these cases, WL is merely a first-order approximation of W. To clarify these points, we conduct tests of lognormality assumptions: Skewness-Kurtosis tests; Shapiro-Wilk tests (Shapiro and Wilk (1965), Royston (1982)); Shapiro-Francia tests (Shapiro and Francia (1972)); and Kolmogorov-Smirnov tests. These tests are implemented for quarterly and annual per capita consumption distributions, and for quarterly and annual price indices distributions. The P-values are shown in table 2.

Skewness-Kurtosis tests are known to be sometimes unsatisfactory, though rather against multimodal distributions, which does not seem to be the case here. Despite the rough definition of the null hypothesis associated with this type of test, their results are not too distant from the results of other tests, except for living standards in period A or annually, for which they would lead to wrong inferences at 10 percent level.

The Kolmogorov-Smirnov test has generally low power, and it cannot reject the lognormality of price indices at 5 percent level for quarters A and D, nor the lognormality of living standards for quarters A, B, C.

The Shapiro-Wilk W and Shapiro-Francia W' tests yield generally close results, though the W' approximation is probably more accurate with a sample size above 40. Observed differences between estimated P-values of W and W' invites to caution. However, at 5 percent and 10 percent levels, these two tests are always in agreement[11].

[11] The Shapiro-Wilk test for the 3-parameters lognormal hypothesis occupies clearly a distinctive position, since the null hypothesis is different. The "3-parameters lognormality" is nonetheless always rejected at usual levels for price indices, although that is never the case for nominal living standards. To extend the model to this type of distribution will therefore not eliminate the rejection of the lognormality of price indices.

One might believe that the rejection of the lognormality of price indice distributions comes from the clustering of observations. At the cluster level the lognormality of price indices is not rejected by the W' test in periods A and B at 10 percent level, although these features are obtained mostly because considering small sample sizes. For other quarters, the lognormal approximation for prices is always rejected, even with the cluster sample.

We give greater importance to the W' test with complete sample for its better statistical properties in this context. Then, the lognormality of the price index distributions is rejected in this data set. By contrast, the lognormality of the living standard distribution is never rejected by this test at 5 percent level, neither in quarters A and C, nor for the year.

Since we want to compare further on several estimators of the Watts' index, some relying on the lognormality, it is interesting to dispose of a benchmark data that does not overly determine the results of comparisons with a too good adequation to lognormality. The present data seems to be suitable to this aim[12].

5.2 Tests of independence

Table 3 shows the correlation coe¢cients between price indices and nominal living standards, and the correlation coe¢cients between the same variables in logarithms, at several periods and for several equivalence scales. Most of the correlation coe¢cients are not signi…cant even at 10 percent level. However, the results may be partly driven by the linear link between variables, which is implicitly assumed when considering these coe¢cients.

Table 4 shows the results of tests of independence between nominal living standards and price indices, based on deciles of these variables[13]. Â^2, ° (di¤erence between conditional probabilities of like and unlike order) and Kendall's ¿$_b$ test statistics have been calculated, as well as the Cramer's V association measure. Goodman and Kruskal (1954, 1959, 1963, 1972) discuss

[12] Using di¤erent equivalent scales do not change the results of these tests.

[13] Of course, deciles of variables in levels and in logarithms are identical.

measures of association for cross classi…cation[14].

The results of $Â^2$; ° and $¿_b$ tests indicate that there is independence between price indices and nominal living standards. The measure of association V is between 0.17 and 0.20, implying that the non-rejection of the independence hypothesis might be attributed to the small sample size.

5.3 Estimation of Distributions

Table 5 shows the maximum likelihood estimates of the parameters of log-normal distributions for price indices and per capita consumption, in each of the four quarters and the whole year. The estimated quarterly distributions of price indices are very close whether they are calculated from the sample of households or the sample of clusters. By contrast, there exist strong di¤erences between the characteristics of distributions for di¤erent quarters, more in the case of prices than for nominal living standards.

The distributions of living standards show substantial di¤erences across quarters. The estimated means of the l.n.l.s. are high in quarters A and in C, and are not signi…cantly di¤erent. They are signi…cantly lower in quarter D during a general poverty crisis. The estimated variabilities of l.n.l.s. are low in quarter B, high in quarter D, and intermediate in quarters A and C that are not signi…cantly di¤erent.

The characteristics of price indices distributions vary also with seasons. The estimated means of the l.p.i. are low in quarter B, moderate in C, high during the poverty crisis in D and in quarter A that are not signi…cantly di¤erent. The estimated variabilities of l.p.i. are low in quarter D, moderate in quarters A and B (not statistically di¤erent), and high in quarter C.

In all periods the estimates correspond to the case $¾_1$=$¾_2$ greater than 1=½ and ½; since ½ is negative in all periods, although it is not signi…cantly di¤erent from zero at the 5 percent level. Then, poverty increases with both

[14] Let be P, the number of concordances of the two classi…cation variables, and Q, the number of discordances, then

De…nition 10 ° = (P-Q)/(P+Q);

$¿_b$ = (P-Q)/((n^2 - $n_{i:}$)(n^2 - $n_{:j}$))$^{\frac{1}{2}}$
and Cramer's V = ($Â^2$/(n.Min(I-1,J-1)))$^{\frac{1}{2}}$.

variances of logarithms[15].

The estimates of the parameters describing the mean and the variance of lognormal distributions, denoted respectively ® and ^, show an ordering of quarters that is consistent with what has been found for means and variances of logarithms in the case of the prices, whereas this is not totally true for nominal living standards. The estimated mean of nominal living standards are lower in quarters B and D and are not signi…cantly di¤erent. They are higher in quarters A and C that are not signi…cantly di¤erent. The estimated variance of the nominal living standards is smaller in period B, while it is not signi…cantly di¤erent in other quarters.

The estimated mean of price indices is lower in quarter B and higher in periods A and D that are not signi…cantly di¤erent. The estimated variance of price indices is smaller in periods B and D that are not signi…cantly di¤erent in comparison with estimates in A and in C, that are as well statistically identical.

6 Estimation of Watts Indices

6.1 Basic Estimates and Comparative Statics

Six poverty lines are used and expressed in terms of Rwandan Francs (Frw).

z_3^0 is the …rst quintile of the annual living standards;

z_2^0 is the sum of the …rst quintiles of the quarterly living standards;

z_1^0 is four times the minimum of the …rst quintiles of the quarterly living standards. Three remaining poverty lines are calculated similarly from the second quintiles of the living standard distributions, and respectively denoted z_6^0 , z_5^0 , z_4^0.[16]

We …rst estimate the Watts poverty indices at period t, directly using ratios of Horwitz-Thompson estimators (see Gouriéroux (1981)). These estimates are denoted WD_t (for "direct") at quarter t:

[15] However, Wald tests show that for all periods, $¾_1$=$¾_2$ is always signi…cantly greater than 1/½, but never signi…cantly greater than ½.

[16] These poverty lines have been calculated from the price corrected living standard distributions. However, our concern in this paper is not to enter in the possible endogeneity in the de…nition of poverty lines. Their values should therefore be considered as …xed once for all , and as a mere benchmark for a convenient analysis of the distributions.

$WD_t = \frac{\sum_{s=1}^{n} \frac{\ln(y_{st}/z)1[y_{st}<z]}{\pi_{st}}}{\sum_{s=1}^{n} \frac{1}{\pi_{st}}}$ where π_{st} is the inclusion probability (in the sample) of household s at date t (s = 1,...,n).

The estimation of sampling standard errors of the poverty indicators is delicate because of the complexity of the actual sampling scheme[17]. Indeed, only one sector was drawn at the second stage of the sampling plan in every primary unit, which does not allow the direct calculus of the inter-strata variance. Another difficulty is the small sample size at several stages of the sampling scheme, which hampers a robust use of classical sampling variance formulae that are based on usual asymptotic properties. We use an estimator for sampling standard errors inspired from the method of balanced repeated replications[18], that is adapted to the actual survey (see appendix). Note that because of the sophisticated stratification involved in the sampling scheme, one expects relatively accurate estimates despite the small sample size, which besides can be verified with the size of the sampling errors in the tables. In fact, a survey of several thousands households based on simple random draws might well yield less precise estimates.

Table 6 shows estimates WD, WL, WM, together with sampling errors of WD and standard errors of WL and WM, for all quarters and the whole year. The poverty measured with WD is unambiguously higher in quarter D (after the dry season), and lower in period B (after bean harvests). Of course, measured poverty increases with the poverty line. The comparison with WD indicators without correction by the price index (Muller (1998)) shows that the correction for price variability entails a substantial increase in poverty such as estimated with WD, in periods A (from 40.4 percent to 51.8 percent depending on the poverty line), C (14.7 percent through 24.8 percent) and D (19.6 through 20.4 percent), and a notable reduction in period B (-11.2 through -12.0 percent). Over the whole year, for which solely the effect of the variability of prices mostly remains, the correction for prices augments considerably the poverty measure (12.4 through 19.4 percent).

We examine now the sensitivity of WL to values of parameters. Table 7 presents the ratios of WL estimated under constraints (respectively: $m_2 = 0$; $\sigma_2 = 0$; $m_2 = 0$ and $\sigma_2 = 0$) on the price distribution, over WL estimated

[17] Gouriéroux (1981) discusses usual sampling estimators. Kakwani (1993) provides an estimator for sampling standard errors of poverty indices, although only valid for simple random frame, which is not the case here.

[18] See Krewski and Rao (1981), Roy (1984) for discussions of the properties of this type of estimator.

without constraints. These ratios are respectively denoted r_1, r_2, r_3, and are calculated for every quarter and for the year, and using each poverty line. In the present data set, …xing the level of l.p.i. at 0 has a much stronger in‡uence (r_1 is between 0.731 and 1.219 for the di¤erent quarters and the di¤erent poverty lines) on WL than …xing the variability of l.p.i. at 0 (r_2 is between 0.909 and 0.989). Both restrictions lead to biased estimates of W, although if the e¤ect of l.p.i. variability at constant level is important, the e¤ect of the l.p.i. level at di¤erent seasons clearly dominates the impact of prices on poverty. There are slight di¤erences in the value of the r_i (i=1,2,3) following the poverty line, although these are much smaller than the di¤erences caused by a change in the quarter. At the annual level, both e¤ects of variability and level of logarithms of price indices are reduced by averaging living standards over four quarters. The complete omission of price e¤ects (m_2 = 0 and $¾_2$ = 0) shown with r_3, is associated with a strong underestimation of poverty in quarters: A (only 66 to 76 percent of poverty is retained), C (80 to 87 percent), D (80 to 85 percent), and for the year (75 to 84 percent); and with a notable overestimation in quarter B (111 to 130 percent). Clearly, accounting for geographical and seasonal price di¤erences is of considerable importance when estimating the Watts' index, with and without using the distribution model.

We have also calculated elasticities and relative variations of WL with respect to the parameters of the model. The parameters of the marginal distribution n.l.s. are the most in‡uential. The impact on WL of a marginal change in the mean of the l.n.l.s. (m_1) is always very strongly negative (elasticity e_1 from -28.90 through -13.95 for the di¤erent quarters and poverty lines). The elasticity of WL with respect to the variability of the l.n.l.s. ($¾_1$) is generally substantial (e_3 = 0.967 through 6.478).

Nonetheless, the parameters of the marginal distribution of the price index still play important roles. The elasticities of WL with respect to the mean of the l.p.i. or the poverty line are generally non negligible (respectively, e_2 = -0.202 to 0.290, and e_6 = -0.146 to 2.124), as well as the elasticity of WL with respect to the variability of the l.p.i. (e_4 = 0.226 to 0.182). Finally, the elasticity of WL with respect to the correlation between the l.p.i. and l.n.l.s. is almost null (e_5 = 0.0054 to 0.0623) in the observed sample.

The orders of magnitude of e_1, e_2, e_4 and e_5 change little when one considers di¤erent poverty lines in the same period, in contrast with the elasticity with respect to variability parameters, e_3 and e_6 (respectively elasticities with respect to $¾_1$ and z). Similarly, the orders of magnitude of e_1, e_3, e_4

and e_5 show little variation in different periods with the same poverty line, in contrast with e_2 and e_6 (respectively elasticities with respect to m_2 and z). On the whole, even if the strong effects of the nominal living standard distribution dominate other marginal variations, the elasticities with respect to price characteristics are clearly not negligible.

The decomposition of elasticities in two additive terms shows that both elasticities of the global variability and elasticities of the cumulating incidence of poverty play generally important roles with occasionally different signs. The role of the global variability is sometimes dominant.

Figures 1 and 2 show graphics of univariate variations in W with respect to distribution parameters, at values of parameters about annual estimates and using line z_3^0. The directions of variation are consistent with the theoretical signs derived in proposition 2. The main nonlinearities occur for the variation in W with m_1, and more moderately with σ_2 and ρ. Curves $W(m_1)$ and $W(\sigma_2)$ are convex, while the curve $W(\rho)$ is concave. Figure 3 illustrates the effects of bivariate variations in parameters of the global variability (σ_1, σ_2, ρ). The variations of ρ and especially σ_1 have more impact on W than the variations of the l.p.i. variability, σ_2:

6.2 Comparison of Estimators

For each line and each period, WD and WL are generally very close and never significantly different at the 5 percent level when sampling errors of WD are considered in the test[19]. This is as well the case when standard errors of WL, $\hat{\sigma}_{WL}$, are used in the comparison[20]. The differences between WD and WL are larger in absolute value in the quarter D during the annual poverty crisis (-12.0 percent through -5.4 percent), although they are still non significant. In other quarters they are depending on the poverty line: -5.8

[19] Of course, and this is also true for comparisons of WD and WM, or of WL and WM, it would be possible to combine standard errors associated to both estimators in the comparison. In that case, all estimators appear clearly not significantly different, although such approach is inaccurate in the sense that the covariance between estimators should be also accounted for. We prefer to consider fixed the value of one estimate and check if the other estimate is significantly different.

[20] Interestingly enough, the corresponding sampling standard errors are always larger than the standard errors of WL, derived from the model. Of course, this must not be interpreted as an argument in favour of WL instead of WD, since when the lognormality assumptions are not satisfied, WL and $\hat{\sigma}_{WL}$ are generally non consitent.

percent through -3.5 percent in A, -5.4 percent through -6.1 percent in B, -8.6 percen t through -5.4 percent in C. WL slightly overestimates poverty when compared with WM, even if the di¤erence is not signi…cant. This may be caused by a too thick left tail of the distribution of real living standards when lognormality is imposed. Note that the relative absolute deviation between WD and WL is not a monotonous function of the poverty line.

The distribution model therefore provides a good approximation of poverty in our context, when the MLE of the parameters of distributions are available, and even if the lognormality of distributions is rejected in several periods. These results justify the use of the model as an analytical tool and simulation device.

Let us turn now to the last estimator of the Watts' index. Using WM underestimates poverty in periods A, C, D and year, and overestimates poverty in period B. These underestimations and overestimations may be substantial (-23.4 through -11.4 percent in quarter D following the poverty line when compared with WL; -4.2 through -8.6 in quarter A; 9.8 through 21.0 in B; -3.6 through -7.3 in C; 13.31 through 34.1 percent for the year). The lower the poverty line, the greater the relative absolute deviation. However for all quarters, deviations of WM with respect to WD or WL are never signi…cant at the 5 percent level. At the annual level, WM and WL are never signi…cantly di¤erent, although WM and WD appear to be signi…cantly di¤erent at the 5 percent level, when using the sampling error of WD in the comparison, and for two lines out of six when using the standard error of WM. The di¤erences between WM or WL arise from the gap between MLE and MME of the parameters of distributions and from the fact that a null correlation has been assumed for WM. The fact that the di¤erences between WM and WD are often not signi…cant indicates that the model can be used for predictions of poverty in situations where only empirical means and standard deviations of price indices and nominal living standards are known, and with small sample sizes of magnitude common in LDC household surveys.

The relative variations between WD or WM, and WL, provide indications about the extent of the approximation involved in the model. Clearly, the differences caused by the approximation slightly change the estimated poverty, but the estimates remain close enough to provide useful and meaningful information, especially when direct sampling estimates are not available.

The distance of WD and WM from WL is generally larger for quarters in which lognormality of n.l.s. has been rejected (B and D). This is consistent with the restrictions imposed by the model, although the result was not obvious a priori since the lognormality of prices has been rejected at any quarters.

Finally, since WL is generally closer to WD than WM (except in quarter C), the use of WM is justified only in presence of sparse information. The available information is one major criterion for choosing between estimators WD, WL and WM. WD requires the observation of the survey sample and the accurate knowledge of the sampling scheme (not only the weights). WL requires the observation of the survey sample, without knowledge of the sampling scheme. WM requires only the knowledge of mean and standard deviation of n.l.s. and p.i.

7 Simulations of shocks in distribution of price indices

We examine now the consequences on poverty of non marginal shocks on the distribution of price indices. For all these simulations we do not incorporate the responses of households to changes in prices that they face, nor the change of the equilibrium of the economy that is possibly caused by price shocks. An approach followed by Ravallion and van de Valle (1991) is to estimate equivalent income functions using a demand model and simulate the new value of each household's equivalent income after the specific price changes. Here, we focus on the very short term effects neglecting all these responses. From table 5, showing the estimated mean and standard deviation of p.i. and l.p.i. at every quarter, we have calculated the larger absolute deviation between two successive quarters. These variations are used as a benchmark for the simulation of price variation. The calculus yields approximately 15 percent of variation for α_2; 61 percent for γ_2 ; 17 percent for σ_2 ; 300 percent for m_2 .

Four different simulation hypothesis have been examined: m_2 changed into 4 m_2; σ_2 into 1.2 σ_2; α_2 into 1.15 α_2; γ_2 into 1.6 γ_2. The simulated poverty indices are shown in table 8, along with the relative deviations with respect to WL, and their standard errors..

The absolute magnitude of the poverty change is a decreasing function of the poverty line. This is consistent with the gradient derived in section 2 and calculated with observed and simulated distribution parameters.

We first examine the results of the simulation of WL, with m_2 replaced by 4 m_2 (increase in the mean of l.p.i.). Both level and variability of prices augment in this scenario. The examination of standard errors of WS shows that all variations of W are significant at 5 percent level. Relative changes in poverty vary from a decrease of 47 percent to an increase of 70 percent depending on the period and the poverty line and are always considerable. For each simulation, the quarter considered corresponds to specific distribution parameters, which explains the massive differences in relative variation of poverty for different quarters. Quarter B is characterised by a large decrease in poverty (-47 through -37 percent, following the poverty line), while considerable increases in poverty occur at other quarters (82 through 129 percent in A; 28 through 40 percent in C; 48 through 70 percent in D).

The second series of simulations corresponds to a 20 percent increase in σ_2 (increase in the variance of l.p.i.). The change in σ_2 causes only very small increases in poverty at all quarters (1.4 through 3.0 percent in A; 1.4 through 3.5 percent in B; 1.6 through 3.4 percent in C; 0.4 through 1.0 percent in D).

The third series of simulations corresponds to 15 percent increase in α_2 (increase in the mean of p.i.). Substantial and always significant increase in poverty occur in quarters A (35 through 49 percent); B (41 through 63 percent); C (35 through 49 percent); D (28 through 40 percent). Quarters A and C show particularly similar evolutions whatever the chosen poverty line.

Finally, the third series of simulations describes the effect of 60 percent increase in γ_2 (increase in the variance of p.i.). The effects on poverty are very moderate and non significant in all quarters (-0.2 through +0.3 percent in A; 0.8 through 2.7 in B; 1.1 through 3.1 in C; 0.7 through 1.9 in D).

The fact that the choice of the poverty line can affect substantially the result of the relative variation in poverty shows the importance of considering a broad range of lines in this type of analysis.

If we are interested in shocks on price distributions of magnitude similar to changes in price distributions from one season to another, the simulations show that the change in variances of p.i. or l.p.i. can be neglected in a first order approximation since they have very small impact on poverty measurement. This implies that very large shocks in variances of p.i. and l.p.i. are necessary to perturb poverty measured with the Watts' index in Rwanda. By contrast, variations in means of p.i. and l.p.i. always entail strong variations in W.

8 Conclusion

Large geographical and temporal differences or changes in prices are likely to exist in agricultural developing countries due to market imperfections and seasonality. They may also occur during structural adjustment periods, or due to weather, economic or political shocks that are frequent in LDCs. The knowledge of their impact on living standards and poverty is therefore of the outmost importance.

We show in this article that using bivariate lognormal models of the distributions of price indices and nominal living standards, leads to an explicit formula of the Watts' poverty index, in terms of five parameters to estimate.

Using data from Rwanda for four quarters, we test and estimate the distribution model and we deduce a MLE of the poverty index. The comparison, based on sampling and standard errors, of this indirect estimator with direct estimates based on the sampling scheme, reveals that even when lognormality models are rejected, the MLE are often not statistically different from direct estimates.

Finally, estimates of the Watts' index, based on MME of parameters of the distribution, exploiting only the knowledge of empirical mean and variance of price indices and nominal living standards are often not significantly different from direct estimates. This implies that it is possible to generate credible and axiomatically valid estimates of poverty, from the sparse information usually available in official publications.

Finally, simulation using the model of changes in levels and variability of the prices and nominal living standards, show that if considerable changes in poverty may occur caused by changes in levels of prices (or logarithms of prices) similar to what is observed from one season to another, it is not the case for similarly common increases in the variance of price indices (or their logarithms).

The methods developed in this paper are associated with functional forms of distributions that are deliberately simple so as to be easy to implement in any organisation. However, they could be generalised with hypotheses relying on formulae expressed in terms of multiple integrals and estimation based on simulation methods.

Table 1: Mean and standard deviation of the main variables

	Annual	A	B	C	D
Total Consumption (corrected)	51176:15 (24985:80)	13521:52 (9527:40)	13232:20 (8192:52)	13452:85 (8249:68)	10969:57 (6092:44)
Total Production (corrected)	57158:02 (24985:80)	13240:50 (12178:27)	15548:30 (16610:28)	15416:63 (18171:03)	12952:59 (10662:06)
Per Capita Consumption (corrected)	10613:27 (5428:08)	2750:173 (1701:169)	2702:944 (1620:898)	2850:082 (1968:637)	2310:075 (1511:553)
Price Index	1:0487 (0:0634)	1:1087 (0:1294)	0:9534 (0:1015)	1:0476 (0:1316)	1:0847 (0:0978)
Per Capita Consumption (non corrected)	10905:18 (5355:731)	2995:399 (1826:006)	2539:347 (1475:742)	2902:023 (1834:125)	2468:417 (1524:948)

Standard deviations in parentheses.

Table 2: P-values of lognormality tests

Variable	1	2	3	4
price index in A	0.0030	0.00008	0.00062	0.00015
price index in B	0.0001	0.00000	0.00001	0.00000
price index in C	0.0000	0.00000	0.00001	0.00001
price index in D	0.0064	0.00016	0.00051	0.00162
annual per capita consumption	0.0916	0.32117	0.20879	0.90100
per capita consumption in A	0.0861	0.21209	0.11655	0.90100
per capita consumption in B	0.0431	0.01719	0.00868	0.90095
per capita consumption in C	0.5249	0.84615	0.52032	0.90100
per capita consumption in D	0.0000	0.0000	0.00001	0.90099

Variable	5	6	7	8	9
price index in A	0.364	0.0644	0.06862	0.10313	0.09022
price index in B	0.000	0.0207	0.00029	0.00113	0.00060
price index in C	0.011	0.0129	0.00073	0.00768	0.00940
price index in D	0.155	0.1431	0.06970	0.12212	0.12331
per capita consumption in A	0.816				
per capita consumption in B	0.163				
per capita consumption in C	0.934				
per capita consumption in D	0.028				

Tests:

1: 256 households. Skewness-Kurtosis of logarithm
2: 256 households. Shapiro-Wilk W of logarithm
3: 256 households. Shapiro-Francia W' of logarithm
4: 256 households. Shapiro-Wilk W for 3-parameters lognormal
5: 256 households. Kolmogorov-Smirnov of logarithm for $N(\hat{m},\hat{\sigma}^2)$
6: 90 clusters. Skewness-Kurtosis of logarithm
7: 90 clusters. Shapiro-Wilk W of logarithm
8: 90 clusters. Shapiro-Francia W' of logarithm
9: 90 clusters. Shapiro-Wilk W for 3-parameters lognormal

Table 3: Correlation coe¢cients between w and l, and between Lnw and lnl

Quarter	levels	Quarter	Logarithms
A	¡ 0:0448 (0:48)	A	¡ 0:1170 (0:0617)
B	¡ 0:0442 (0:48)	B	¡ 0:0371 (0:5547)
C	¡ 0:1103 (0:0782)	C	¡ 0:0945 (0:1315)
D	¡ 0:1124 (0:0726)	D	¡ 0:0471 (0:4529)

Table 4: Independence tests

Quarternscale				
A	0:340	0:1928	A	A
B	0:701	0:1784	A	A
C	0:304	0:1943	A	A
D	0:287	0:1951	A	A

In each line, are shown successively: P-value of $Â^2$ test; Cramer's V association measure; Result of ° test at 5 percent level (A = not rejected, R= rejected); Result of $¿_b$ test at 5 percent level (A = not rejected, R= rejected).

Table 5 : MLE of distribution parameters
(standard errors in parentheses)
Living standards (256 obs.)

Parameter	annual	A	B	C	D
$\hat{m}_1$	9:19964	7:84461	7:70622	7:80331	7:64110
	(0:0270)	(0:0356)	(0:0318)	(0:03659)	(0:03998)
$\hat{\sigma}_1^2$	0:43235	0:57239	0:50957	0:58779	0:64130
	(0:0191)	(0:0253)	(0:0225)	(0:02598)	(0:02830)
$\hat{\alpha}_1$	10862:79	3006:19	2530:20	2910:42	2557:37
	(306:05)	(115:44)	(85:402)	(115:29)	(112:17)
$\hat{\lambda}_1$	24253352:50	3503484:78	1898086:20	3495744:18	3327186:65
	(3032089)	(525243)	(261748)	(534119)	(544500)
$\hat{\rho}_1$	0.74799	0.87526	0.82491	0.88607	0.91940

Prices (256 obs.)

Parameter	annual	A	B	C	D
$\hat{m}_2$	0:045651	0:096362	¡ 0:053677	0:038192	0:077123
	(0:00565)	(0:01096)	(0:0104)	(0:0123)	(0:00863)
$\hat{\sigma}_2^2$	0:060308	0:11814	0:11104	0:13135	0:091994
	(0:00415)	(0:00817)	(0:00737)	(0:00892)	(0:00617)
$\hat{\alpha}_2$	1:04861	1:10887	0:95360	1:04793	1:08476
	(0:00737)	(0:01529)	(0:0122)	(0:01608)	(0:01157)
$\hat{\lambda}_2$	:004006	0:017283	0:011282	0:019110	0:010000
	(0:000850)	(0:00392)	(0:00244)	(0:00439)	(0:00214)
$\hat{\rho}_2$	0.10501	0.20602	0.18927	0.22656	0.1605

Prices (90 clusters)

Parameter	annual	A	B	C	D
$\hat{m}_2$	0.046366	0.097465	-0.052346	0.039100	0.076197
$\hat{\sigma}_2^2$	0.060084	0.11890	0.11255	0.13308	0.093792
$\hat{\alpha}_2$	1.04935	1.11019	0.95503	1.04912	1.08393
$\hat{\lambda}_2$	.0039823	0.017547	0.011628	0.019666	0.010381
$\hat{\rho}_2$	0.10463	0.20735	0.19185	0.22952	0.16358

The $\hat{\rho}_i$ are the correlations between estimators $\hat{\alpha}_i$ and $\hat{\lambda}_i$, i = 1,2.

$\hat{\rho}$: correlation coeﬃcient of the bivariate lognormal law.(256 observations)

	Annual	A	B	C	D
$\hat{\rho}$	¡ 0:05222	¡ 0:1179	¡ 0:03964	¡ 0:09437	¡ 0:04531
	(0:0649)	(0:0656)	(0:06288)	(0:06397)	(0:06321)

Table 6 : Watts Poverty indices
(with correction for price variability)

	z_6^0 >	z_5^0 >	z_4^0 >	z_3^0 >	z_2^0 >	z_1^0
	0:1804	0:1502	0:1118	0:1025	0:0657	0:05288
	(0:0228)	(0:0200)	(0:0161)	(0:0154)	(0:0116)	(0:0101)
	0:1870	0:1572	0:1186	0:1088	0:0691	0:05519
	(0:0191)	(0:0174)	(0:0149)	(0:0142)	(0:0108)	(0:00937)
	[¡ 0:0353]	[¡ 0:0445]	[¡ 0:0573]	[¡ 0:0579]	[¡ 0:0492]	[¡ 0:0419]
A	0:1790	0:1500	0:112	0:1022	0:0638	0:0504
	[¡ 0:0416]	[¡ 0:0474]	[¡ 0:0572]	[¡ 0:0603]	[¡ 0:0772]	[¡ 0:0859]
	(0:0209)	(0:0185)	(0:0150)	(0:0141)	(0:0099)	(0:0082)
	0:1446	0:1182	0:08585	0:07783	0:04499	0:03369
	(0:0227)	(0:0198)	(0:0155)	(0:0143)	(0:0101)	(:00909)
	0:1556	0:1274	0:09177	0:0829	0:0486	0:0372
	(0:0160)	(0:0144)	(0:0119)	(0:0112)	(0:00803)	(0:0067)
	[¡ 0:0707]	[¡ 0:0722]	[¡ 0:0645]	[¡ 0:0612]	[¡ 0:0743]	[¡ 0:0944]
B	0:1708	0:1416	0:1042	0:0948	0:0577	0:0450
	[0:0978]	[0:1116]	[0:1356]	[0:1434]	[0:1870]	[0:2104]
	(0:0198)	(0:0175)	(0:0141)	(0:0131)	(0:00901)	(0:0074)
	0:1764	0:1472	0:1118	0:1027	0:06482	0:05171
	(0:0179)	(0:0152)	(0:0129)	(0:0125)	(:00947)	(:00779)
	0:1864	0:1573	0:1194	0:1098	0:0706	0:0566
	(0:0191)	(0:0174)	(0:0149)	(0:0142)	(0:0109)	(0:0094)
	[¡ 0:0536]	[¡ 0:0642]	[¡ 0:0637]	[¡ 0:0647]	[¡ 0:0819]	[¡ 0:0864]
C	0:1798	0:1509	0:1136	0:1041	0:0659	0:0525
	[¡ 0:0357]	[¡ 0:0406]	[¡ 0:0488]	[¡ 0:0515]	[¡ 0:0656]	[¡ 0:0729]
	(0:0259)	(0:0230)	(0:0188)	(0:0176)	(0:0125)	(0:0105)

	0:2846	0:2427	0:1898	0:1764	0:1208	0:1023
	(0:0447)	(0:0413)	(0:0364)	(0:0352)	(0:0299)	(0:0278)
	0:3008	0:2622	0:2099	0:1960	0:1372	0:1150
	(0:0250)	(0:0234)	(0:0209)	(0:0202)	(0:0166)	(0:0150)
	[¡ 0:0539]	[¡ 0:0744]	[¡ 0:0958]	[¡ 0:100]	[¡ 0:1195]	[¡ 0:1104]
D	0:2665	0:2282	0:1760	0:1636	0:1080	0:0881
	[¡ 0:114]	[¡ 0:129]	[¡ 0:157]	[¡ 0:165]	[¡ 0:211]	[¡ 0:234]
	(0:0311)	(0:0283)	(0:0239)	(0:0227)	(0:0169)	(0:0145)
	0:11300	0:08599	0:05417	0:04696	0:02201	0:0153
	(:007923)	(:00675)	(:00568)	(:00549)	(:00402)	(:00341)
	0:1196	0:0934	0:0620	0:05454	0:02755	0:01947
	(0:0129)	(0:0113)	(0:0088)	(0:0082)	(0:0052)	(0:0041)
	[¡ 0:0552]	[¡ 0:0793]	[¡ 0:1263]	[¡ 0:1390]	[¡ 0:2011]	[¡ 0:2142]
Y	0:1355	0:1081	0:07434	0:06619	0:03569	0:02612
	[0:1331]	[0:1568]	[0:1995]	[0:2137]	[0:2957]	[0:3414]
	(0:0189)	(0:0161)	(0:0123)	(0:0112)	(0:0069)	(0:0054)

The lines in each cell for the quarters correspond respectively to WD, the sampling error of WD (in parentheses), WL, $\hat{\sigma}_{WL}$ (in parentheses), (WD-WL)/WL (in brackets),WM,(WM-WL)/WL (in brackets), $\hat{\sigma}_{WM}$ (in parentheses).

Table 7: Sentivity analysis of the Watts' index

Lines : z_3^0 and z_2^0

	A	B	C	D	A	B	C	D
r_1	0.765	1.181	0.903	0.845	0.742	1.207	0.893	0.830
r_2	0.941	0.945	0.937	0.983	0.924	0.927	0.918	0.977
r_3	0.712	1.123	0.842	0.829	0.676	1.127	0.816	0.809

line z_1^0:

	A	B	C	D
r_1	0.731	1.219	0.888	0.822
r_2	0.915	0.917	0.909	0.975
r_3	0.659	1.127	0.803	0.799

Lines $z_6^0$and z_5^0

	A	B	C	D	A	B	C	D
r_1	0.795	1.149	0.916	0.865	0.785	1.160	0.912	0.859
r_2	0.961	0.965	0.957	0.989	0.955	0.959	0.951	0.987
r_3	0.757	1.114	0.874	0.854	0.742	1.117	0.864	0.846

Line z_4^0

	A	B	C	D
r_1	0.769	1.176	0.905	0.848
r_2	0.945	0.949	0.940	0.984
r_3	0.719	1.122	0.847	0.833

Year

	z_3^0	z_2^0	z_1^0	z_6^0	z_5^0	z_4^0
r_1	0.830	0.807	0.797	0.859	0.849	0.834
r_2	0.969	0.957	0.951	0.982	0.978	0.971
r_3	0.802	0.769	0.754	0.841	0.828	0.808

r_1 is the ratio of the Watts' index under ($m_2 = 0$), over the Watts index without restriction; r_2 is the ratio of the Watts' index under (¾$_2$ = 0), over the Watts index without restriction; r_3 is the ratio of the Watts' index under ($m_2 = 0$ and ¾$_2$ = 0), over the Watts index without restriction.

Table 8: Simulations

$4m_2$:

	z_6^0 >	z_5^0 >	z_4^0 >	z_3^0 >	z_2^0 >	z_1^0
	0:339	0:295	0:2355	0:2196	0:1520	0:1264
A	[0:815]	[0:879]	[0:986]	[1:019]	[1:198]	[0:1290]
	(0:0282)	(0:0266)	(0:0240)	(0:0232)	(0:0193)	(0:0174)
	0:0985	0:0784	0:0541	0:0482	0:0264	0:0196
B	[¡ 0:367]	[¡ 0:384]	[¡ 0:411]	[¡ 0:419]	[¡ 0:456]	[¡ 0:474]
	(0:0151)	(0:0133)	(0:0106)	(0:0098)	(0:0066)	(0:0054)
	0:2388	0:2043	0:1585	0:1466	0:0974	0:0795
C	[0:281]	[0:299]	[0:327]	[0:335]	[0:380]	[0:402]
	(0:0266)	(0:0247)	(0:0217)	(0:0208)	(0:0165)	(0:0147)
	0:4436	0:3950	0:3271	0:3086	0:2276	0:1956
D	[0:475]	[0:507]	[0:558]	[0:574]	[0:658]	[0:701]
	(0:0237)	(0:0226)	(0:0209)	(0:0203)	(0:0176)	(0:0163)

1.2 ¾$_2$:

	z_6^0 >	z_5^0 >	z_4^0 >	z_3^0 >	z_2^0 >	z_1^0
	0:1894	0:1596	0:1209	0:1110	0:0710	0:0568
A	[0:014]	[0:015]	[0:019]	[0:020]	[0:0267]	[0:0301]
	(0:0065)	(0:0059)	(0:0051)	(0:0049)	(0:0038)	(0:0033)
	0:1578	0:1295	0:0936	0:0847	0:0500	0:0384
B	[0:0138]	[0:0162]	[0:0204]	[0:0218]	[0:0293]	[0:0333]
	(0:0043)	(0:0038)	(0:0032)	(0:00303)	(0:0022)	(0:00185)
	0:1893	0:1601	0:1220	0:1123	0:0727	0:0586
C	[0:0155]	[0:0178]	[0:0218]	[0:0230]	[0:0300]	[0:0336]
	(0:0058)	(0:0053)	(0:0046)	(0:0044)	(0:0034)	(0:0030)
	0:3021	0:2635	0:2112	0:1973	0:1384	0:1161
D	[0:0043]	[0:0050]	[0:0061]	[0:0065]	[0:0086]	[0:0097]
	(0:0045)	(0:0043)	(0:0038)	(0:0037)	(0:0031)	(0:0028)

1.15 ®$_2$:

	z'_6 >	z'_5 >	z'_4 >	z'_3 >	z'_2 >	z'_1
	0:2515	0:2149	0:1661	0:1535	0:1012	0:0822
A	[0:346]	[0:367]	[0:401]	[0:411]	[0:464]	[0:489]
	(0:0233)	(0:0211)	(0:0178)	(0:0169)	(0:0126)	(0:0108)
	0:2201	0:1841	0:1372	0:1252	0:0773	0:0606
B	[0:414]	[0:445]	[0:495]	[0:510]	[0:590]	[0:630]
	(0:0210)	(0:0187)	(0:0153)	(0:0143)	(0:0100)	(0:0083)
	0:2512	0:2154	0:1676	0:1552	0:1036	0:0847
C	[0:348]	[0:369]	[0:403]	[0:414]	[0:468]	[0:494]
	(0:0234)	(0:0212)	(0:0179)	(0:0170)	(0:0127)	(0:0110)
	0:3844	0:3397	0:2777	0:2611	0:1889	0:1608
D	[0:278]	[0:295]	[0:323]	[0:332]	[0:377]	[0:399]
	(0:0302)	(0:0281)	(0:0248)	(0:0239)	(0:0192)	(0:0172)

1.60°$_2$:

	z'_6 >	z'_5 >	z'_4 >	z'_3 >	z'_2 >	z'_1
	0:1865	0:1569	0:1185	0:1088	0:0693	0:0553
A	[¡ 0:0021]	[¡ 0:0016]	[¡ 0:00058]	[¡ 0:0002]	[0:00167]	[0:0027]
	(0:0162)	(0:0145)	(0:0120)	(0:0113)	(0:0082)	(0:0070)
	0:1569	0:1287	0:0931	0:0842	0:0497	0:0382
B	[0:0080]	[0:0103]	[0:0143]	[0:0156]	[0:0229]	[0:0269]
	(0:0137)	(0:0120)	(0:0096)	(0:0089)	(0:0060)	(0:0049)
	0:1884	0:1594	0:1215	0:1118	0:0724	0:0584
C	[0:0107]	[0:0132]	[0:017]	[0:0118]	[0:0265]	[0:0306]
	(0:0163)	(0:0146)	(0:0121)	(0:0114)	(0:0083)	(0:0071)
	0:3028	0:2644	0:2122	0:1983	0:1395	0:1171
D	[0:0068]	[0:0083]	[0:0109]	[0:0117]	[0:0164]	[0:0189]
	(0:0223)	(0:0205)	(0:0178)	(0:0170)	(0:0133)	(0:0117)

The …rst line of each cell is W simulated with the model. The second line in brackets is the proportion of variation compared with WL without change in parameters. The third line, in parentheses, is the standard error of the estimates, ¾$_{WS}$ or ¾$_{WMS}$: The tables of simulations correspond successively to the following changes in parameters : 4m$_2$ instead of m$_2$; 1:2¾$_2$ instead of ¾$_2$; 1:15®$_2$ instead of ®$_2$; 1:60°$_2$ instead of °$_2$.

Proof of proposition 1:

ln(y) = ln(w) - ln(I) is the sum of two normal random variables, of law $N(m_1 - m_2, \sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2)$, whose c.d.f. is denoted H. The Watts' index can be decomposed as follows

$$W(z) = \int_0^z \left[-\ln(y) + \ln(z) \right] d\mu(y) \qquad (36)$$

which yields, using the transfer theorem (see Monfort (1980)) with u = ln(y))

$$W(z) = \ln(z).H(\ln(z)) - \int_{-\infty}^{\ln z} u\, dH(u) \qquad (37)$$

and again with normalization of u with $t = \frac{u - m_1 + m_2}{\sqrt{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2}}$

$$W(z) = \ln(z).\Phi\left[\frac{\ln(z) - m_1 + m_2}{\sqrt{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2}}\right] - \int_{-\infty}^{\frac{\ln z - m_1 + m_2}{\sqrt{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2}}} \left[\sqrt{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2}\, t + m_1 - m_2\right] d\Phi(t) \qquad (38)$$

where Φ is the cumulative distribution function of the standard normal law, N(0,1). Then,

$$W(z) = (\ln(z) - m_1 + m_2).\Phi\left[\frac{\ln(z) - m_1 + m_2}{\sqrt{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2}}\right] - \sqrt{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2}\,.J(z) \qquad (39)$$

where

$$J(z) = \int_{-\infty}^{\frac{\ln z - m_1 + m_2}{\sqrt{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2}}} t\, d\Phi(t) \qquad (40)$$

Integration of eq. 40 yields

$$J(z) = -\frac{1}{\sqrt{2\pi}} e^{-\left(\frac{\ln(z) - m_1 + m_2}{\sqrt{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2}}\right)^2/2} \tag{41}$$

$$W = (\ln(z) - m_1 + m_2).\Phi\left(\frac{\ln z - m_1 + m_2}{\sqrt{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2}}\right) + \frac{1}{\sqrt{2\pi}}\sqrt{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2}\, e^{-\left(\frac{\ln(z) - m_1 + m_2}{\sqrt{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2}}\right)^2/2} \tag{42}$$

QED.

proof of proposition 3

The density of the couple $(\ln(w), \ln(l)) = (x_1, x_2)$ with respect to λ_2 is (when $\rho \neq 1$) :

$$f = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}} \exp\left\{\frac{-1}{2(1-\rho^2)}\left[\frac{(x_1 - m_1)^2}{\sigma_1^2} + \frac{(x_2 - m_2)^2}{\sigma_2^2} - \frac{2\rho(x_1 - m_1)(x_2 - m_2)}{\sigma_1\sigma_2}\right]\right\} \tag{43}$$

Using the theorem of change in variables, we obtain the density of our variables of interest (w,l) and we derive the log-likelihood of the sample:

$$LL = \sum_{i=1} -\ln(2\pi) - \ln(\sigma_1) - \ln(\sigma_2) - \frac{1}{2}\ln(1-\rho^2) - \ln(w) - \ln(l) - \frac{1}{2(1-\rho^2)}\left[\frac{(x_1 - m_1)^2}{\sigma_1^2} + \frac{(x_2 - m_2)^2}{\sigma_2^2} - \frac{2\rho(x_1 - m_1)(x_2 - m_2)}{\sigma_1\sigma_2}\right] \tag{44}$$

Under the usual regularity conditions (e.g. Gouriéroux and Monfort (1989)), the MLE exist are unique, consistent and efficient.

Then, the components of the score vector are:

$$\frac{\partial LL}{\partial m_1} = \sum_{i=1} \frac{(x_1 - m_1)}{(1-\rho^2)\sigma_1^2} - \frac{\rho(x_2 - m_2)}{(1-\rho^2)\sigma_1\sigma_2} \tag{45}$$

$$\frac{\partial LL}{\partial m_2} = \sum_{i=1} \frac{(x_2 - m_2)}{(1-\rho^2)\sigma_2^2} - \frac{\rho(x_1 - m_1)}{(1-\rho^2)\sigma_1\sigma_2} \tag{46}$$

$$\frac{\partial LL}{\partial \sigma_1} = -\frac{n}{\sigma_1} + \sum_{i=1} \frac{(x_1 - m_1)^2}{\sigma_1^3(1-\rho^2)} - \frac{\rho}{1-\rho^2}\frac{(x_1 - m_1)(x_2 - m_2)}{\sigma_1^2\sigma_2} \tag{47}$$

$$\frac{\partial LL}{\partial \sigma_2} = -\frac{n}{\sigma_2} + \sum_{i=1} \frac{(x_2 - m_2)^2}{\sigma_2^3(1-\rho^2)} - \frac{\rho}{1-\rho^2}\frac{(x_1 - m_1)(x_2 - m_2)}{\sigma_2^2\sigma_1} \tag{48}$$

$$\frac{\partial LL}{\partial \rho} = \frac{n.\rho}{1-\rho^2} + \frac{1}{1-\rho^2}\sum_{i=1} \frac{(x_1 - m_1)(x_2 - m_2)}{\sigma_2\sigma_1} \tag{49}$$

$$- \frac{\rho}{(1-\rho^2)^2} \sum_{i=1} \left[\frac{(x_1 - m_1)^2}{\sigma_1^2} + \frac{(x_2 - m_2)^2}{\sigma_2^2} - 2\rho\frac{(x_1 - m_1)(x_2 - m_2)}{\sigma_2\sigma_1} \right] \tag{50}$$

The MLE are obtained by cancelling eqs. 45 to 49 and solving.

$$\hat{m}_1 = \frac{1}{n}\sum_{i=1} \ln w_i \tag{51}$$

$$\hat{m}_2 = \frac{1}{n}\sum_{i=1} \ln l_i \tag{52}$$

$$\hat{\sigma}_1 = \sqrt{\frac{1}{n}\sum_{i=1} (\ln x_1 - \hat{m}_1)^2} \tag{53}$$

$$\hat{\sigma}_2 = \sqrt{\frac{1}{n}\sum_{i=1} (\ln x_2 - \hat{m}_2)^2} \tag{54}$$

$$\hat{\rho} = \frac{\frac{1}{n}\sum_{i=1}^{n}(\ln w - \hat{m}_1)(\ln l - \hat{m}_2)}{\hat{\sigma}_1\hat{\sigma}_2} \tag{55}$$

The Fisher information matrix of one observation on $(m_1, m_2, \sigma_1; \sigma_2; \rho)$ is

$$IF = \begin{bmatrix} -E\left(\frac{\partial^2 \ln f}{\partial m_1^2}\right) & -E\left(\frac{\partial^2 \ln f}{\partial m_1 \partial m_2}\right) & -E\left(\frac{\partial^2 \ln f}{\partial m_1 \partial \sigma_1}\right) & -E\left(\frac{\partial^2 \ln f}{\partial m_1 \partial \sigma_2}\right) & -E\left(\frac{\partial^2 \ln f}{\partial m_1 \partial \rho}\right) \\ -E\left(\frac{\partial^2 \ln f}{\partial m_2 \partial m_1}\right) & -E\left(\frac{\partial^2 \ln f}{\partial m_2^2}\right) & -E\left(\frac{\partial^2 \ln f}{\partial m_2 \partial \sigma_1}\right) & -E\left(\frac{\partial^2 \ln f}{\partial m_2 \partial \sigma_2}\right) & -E\left(\frac{\partial^2 \ln f}{\partial m_2 \partial \rho}\right) \\ -E\left(\frac{\partial^2 \ln f}{\partial \sigma_1 \partial m_1}\right) & -E\left(\frac{\partial^2 \ln f}{\partial \sigma_1 \partial m_2}\right) & -E\left(\frac{\partial^2 \ln f}{\partial \sigma_1^2}\right) & -E\left(\frac{\partial^2 \ln f}{\partial \sigma_1 \partial \sigma_2}\right) & -E\left(\frac{\partial^2 \ln f}{\partial \sigma_1 \partial \rho}\right) \\ -E\left(\frac{\partial^2 \ln f}{\partial \sigma_2 \partial m_1}\right) & -E\left(\frac{\partial^2 \ln f}{\partial \sigma_2 \partial m_2}\right) & -E\left(\frac{\partial^2 \ln f}{\partial \sigma_2 \partial \sigma_1}\right) & -E\left(\frac{\partial^2 \ln f}{\partial \sigma_2^2}\right) & -E\left(\frac{\partial^2 \ln f}{\partial \sigma_2 \partial \rho}\right) \\ -E\left(\frac{\partial^2 \ln f}{\partial \rho \partial m_1}\right) & -E\left(\frac{\partial^2 \ln f}{\partial \rho \partial m_2}\right) & -E\left(\frac{\partial^2 \ln f}{\partial \rho \partial \sigma_1}\right) & -E\left(\frac{\partial^2 \ln f}{\partial \rho \partial \sigma_2}\right) & -E\left(\frac{\partial^2 \ln f}{\partial \rho^2}\right) \end{bmatrix}$$

We derive the coordinates of the gradient of ln(f) to obtain

$$\frac{\partial^2 \ln f}{\partial (m_1)^2} = -\frac{1}{\sigma_1^2(1-\rho^2)} \tag{57}$$

$$\frac{\partial^2 \ln f}{\partial (m_2)^2} = -\frac{1}{\sigma_2^2(1-\rho^2)} \tag{58}$$

$$\frac{\partial^2 \ln f}{\partial (\sigma_1)^2} = \frac{1}{\sigma_1^2} - \frac{1}{1-\rho^2}\left(3\frac{(\ln w - m_1)^2}{\sigma_1^4} - 2\rho\frac{(\ln w - m_1)(\ln l - m_2)}{\sigma_1^3\sigma_2}\right) \tag{59}$$

$$\frac{\partial^2 \ln f}{\partial(\sigma_2)^2} = \frac{1}{\sigma_2^2} - \frac{1}{1-\rho^2}\left[3\frac{(\ln l - m_2)^2}{\sigma_2^4} - 2\rho\frac{(\ln w - m_1)(\ln l - m_2)}{\sigma_2^3\sigma_1}\right] \quad (60)$$

$$\frac{\partial^2 \ln f}{\partial(\rho)^2} = \frac{1}{(1-\rho^2)^2}\left[\begin{array}{c} 1+\rho^2+6\rho\frac{(\ln w - m_1)(\ln l - m_2)}{\sigma_2\sigma_1} \\ -\frac{(\ln w - m_1)^2}{\sigma_1^2} - \frac{(\ln l - m_2)^2}{\sigma_2^2}\end{array}\right] - \frac{4\rho^2}{(1-\rho^2)^3}\left[\begin{array}{c} -2\rho\frac{(\ln w - m_1)(\ln l - m_2)}{\sigma_2\sigma_1} \\ +\frac{(\ln w - m_1)^2}{\sigma_1^2} + \frac{(\ln l - m_2)^2}{\sigma_2^2}\end{array}\right] \quad (61)$$

$$\frac{\partial^2 \ln f}{\partial m_1 \partial m_2} = \frac{\partial^2 \ln f}{\partial m_2 \partial m_1} = \frac{\rho}{\sigma_1\sigma_2(1-\rho^2)} \quad (62)$$

$$\frac{\partial^2 \ln f}{\partial m_1 \partial \sigma_1} = \frac{\partial^2 \ln f}{\partial \sigma_1 \partial m_1} = \frac{1}{(1-\rho^2)\sigma_1^2}\left[-2\frac{(\ln w - m_1)}{\sigma_1} + \rho\frac{(\ln l - m_2)}{\sigma_2}\right] \quad (63)$$

$$\frac{\partial^2 \ln f}{\partial m_2 \partial \sigma_2} = \frac{\partial^2 \ln f}{\partial \sigma_2 \partial m_2} = \frac{1}{(1-\rho^2)\sigma_2^2}\left[-2\frac{(\ln l - m_2)}{\sigma_2} + \rho\frac{(\ln w - m_1)}{\sigma_1}\right] \quad (64)$$

$$\frac{\partial^2 \ln f}{\partial m_1 \partial \sigma_2} = \frac{\partial^2 \ln f}{\partial \sigma_2 \partial m_1} = \frac{\rho}{(1-\rho^2)}\frac{(\ln l - m_2)}{\sigma_1\sigma_2^2} \quad (65)$$

$$\frac{\partial^2 \ln f}{\partial m_2 \partial \sigma_1} = \frac{\partial^2 \ln f}{\partial \sigma_1 \partial m_2} = \frac{\rho}{(1-\rho^2)}\frac{(\ln w - m_1)}{\sigma_2\sigma_1^2} \quad (66)$$

$$\frac{\partial^2 \ln f}{\partial m_1 \partial \rho} = \frac{\partial^2 \ln f}{\partial \rho \partial m_1} = \frac{1}{(1-\rho^2)^2}\left[-(1+\rho^2)\frac{(\ln l - m_2)}{\sigma_1\sigma_2} + 2\rho\frac{(\ln w - m_1)}{\sigma_1^2}\right] \quad (67)$$

$$\frac{\partial^2 \ln f}{\partial m_2 \partial \rho} = \frac{\partial^2 \ln f}{\partial \rho \partial m_2} = \frac{1}{(1-\rho^2)^2}\left[-(1+\rho^2)\frac{(\ln w - m_1)}{\sigma_1\sigma_2} + 2\rho\frac{(\ln l - m_2)}{\sigma_2^2}\right] \quad (68)$$

$$\frac{\partial^2 \ln f}{\partial\sigma_1\partial\sigma_2} = \frac{\partial^2 \ln f}{\partial\sigma_2\partial\sigma_1} = \frac{\rho}{1-\rho^2}\frac{(\ln w - m_1)(\ln l - m_2)}{\sigma_2^2\sigma_1^2} \tag{69}$$

$$\frac{\partial^2 \ln f}{\partial\sigma_1\partial\rho} = \frac{\partial^2 \ln f}{\partial\rho\partial\sigma_1} = \frac{1}{(1-\rho^2)^2\sigma_1}\left[2\rho\frac{(\ln w - m_1)^2}{\sigma_1^2} - (1+\rho^2)\frac{(\ln w - m_1)(\ln l - m_2)}{\sigma_1\sigma_2}\right] \tag{70}$$

$$\frac{\partial^2 \ln f}{\partial\sigma_2\partial\rho} = \frac{\partial^2 \ln f}{\partial\rho\partial\sigma_2} = \frac{1}{(1-\rho^2)^2\sigma_2}\left[2\rho\frac{(\ln l - m_2)^2}{\sigma_2^2} - (1+\rho^2)\frac{(\ln w - m_1)(\ln l - m_2)}{\sigma_1\sigma_2}\right] \tag{71}$$

Then, taking minus expectations and averaging on the observations, the average Fisher information matrix is

$$I_F^1 = \begin{bmatrix} \frac{1}{\sigma_1^2(1-\rho^2)} & \frac{-\rho}{\sigma_1\sigma_2(1-\rho^2)} & 0 & 0 & 0 \\ \frac{-\rho}{\sigma_1\sigma_2(1-\rho^2)} & \frac{1}{\sigma_2^2(1-\rho^2)} & 0 & 0 & 0 \\ 0 & 0 & \frac{(2-\rho^2)}{\sigma_1^2(1-\rho^2)} & \frac{-\rho^2}{\sigma_1\sigma_2(1-\rho^2)} & \frac{\rho}{\sigma_1^2(1-\rho^2)} \\ 0 & 0 & \frac{-\rho^2}{\sigma_1\sigma_2(1-\rho^2)} & \frac{(2-\rho^2)}{\sigma_2^2(1-\rho^2)} & \frac{\rho}{\sigma_2^2(1-\rho^2)} \\ 0 & 0 & \frac{\rho}{\sigma_1^2(1-\rho^2)} & \frac{\rho}{\sigma_2^2(1-\rho^2)} & \frac{(1+\rho^2)}{(1-\rho^2)^2} \end{bmatrix} \tag{72}$$

From eq. 51, we have $E(\hat{m}_i) = m_i$ and $\hat{m}_i$ is unbiased, $i = 1, 2$. The MLE are convergent and asymptotically normal with asymptotic variance deduced from the inverse of the Fisher information matrix.

Then, under usual regularity conditions for the MLE, using central limit theorems, we can derive the asymptotic variance-covariance matrices by inverting I_F.

This yields

$$\sqrt{n}\begin{pmatrix} \hat{m}_1 - m_1 \\ \hat{m}_2 - m_2 \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}; \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix}\right) \tag{73}$$

and independently

$$\sqrt{n}\begin{pmatrix} \hat{\sigma}_1 - \sigma_1 \\ \hat{\sigma}_2 - \sigma_2 \\ \hat{\rho} - \rho \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}; \begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{12} & B_{22} & B_{23} \\ B_{13} & B_{23} & B_{33} \end{bmatrix}\right) \tag{74}$$

where the B_{ij} are given in the proposition.

Proof of proposition 4:

The characteristic function of the binomial normale law μ, (i.e. $N(m, \Sigma)$); is the following (Johnson and Kotz (1973)).

$$\begin{aligned} G(t1; t2) &= \int e^{i\cdot(u_1 t_1 + u_2 t_2)} d\mu(u_1; u_2) \\ &= e^{i\cdot(t_1 m_1 + t_2 m_2) - \frac{1}{2}(t_1^2\sigma_1^2 + t_2^2\sigma_2^2 - 2t_1 t_2 \rho\sigma_1\sigma_2)} \end{aligned} \tag{75}$$

Eq. 75 gives for $t_1 = -i\cdot r_1$ and $t_2 = -i\cdot r_2$, $u_1 = \ln(v_1)$ and $u_2 = \ln(v_2)$:

$$\int v_1^{r_1} v_2^{r_2} \cdot dLN(v_1; v_2) = e^{r_1 m_1 + r_2 m_2 + \frac{1}{2}(r_1^2\sigma_1^2 + r_2^2\sigma_2^2 - 2r_1 r_2 \rho\sigma_1\sigma_2)} \tag{76}$$

In particular, we have for $(X_1, X_2) = (w, I)$ following a joint lognormal distribution

$$EX_i = e^{m_i + \sigma_i^2/2}; \ i = 1; 2 \tag{77}$$

$$E(X_i^2) = e^{2m_i + 2\sigma_i^2}; \ i = 1; 2 \tag{78}$$

$$V X_i = e^{2m_i + \sigma_i^2} \cdot (e^{\sigma_i^2} - 1); \ i = 1; 2 \tag{79}$$

$$Cov(X_1; X_2) = \{e^{\rho\sigma_1\sigma_2} - 1\} \cdot e^{m_1 + m_2 + \frac{\sigma_1^2 + \sigma_2^2}{2}} \tag{80}$$

$$R = \rho(X_1; X_2) = \frac{e^{\rho\sigma_1\sigma_2} - 1}{\sqrt{(e^{\sigma_1^2} - 1) \cdot (e^{\sigma_2^2} - 1)}} \tag{81}$$

Eqs. 77 and 79 enable us to define the MME $(\hat{m}_1, \hat{m}_2, \hat{\sigma}_1; \hat{\sigma}_2)$' that does not depend on the value of ρ.

A MME for ρ can as well be defined as a solution of eq. 81 :
$\hat{\rho} = \frac{\ln\left(1 + R \cdot \sqrt{(e^{\sigma_1^2} - 1) \cdot (e^{\sigma_2^2} - 1)}\right)}{\sigma_1 \cdot \sigma_2}$ and replacing R, $\sigma_1; \sigma_2$ with their empirical equivalent.

Sampling errors:

The poverty indicator of a sub-population is estimated by a ratio of the type $\bar{y}_x' = z'/x'$ where ' denotes the Horwitz-Thompson estimator for a total (sum of values for the variable of interest weighted by the inverse of inclusion probability). z is the sum of the poverty in the sub-population and x is the size of the sub-population. The variance associated with the sampling error is then approximated by:

$$V(\bar{y}_x') = \left[V(z') - 2\bar{y}_x' \mathrm{Cov}(z'; x') + (\bar{y}_x')^2 V(x')\right] / (x')^2 \tag{82}$$

which can be obtained from a Taylor expansion at the first order from function $Y = f(Z/X)$ around (E y', Ex') and because E z' $\neq$ 0 and x' does not cancel, where the appropriate expectancies are estimated by x' and $\bar{y}_x'$.

We divide the sample of communes (first actual stage of the sampling since all the prefectures are drawn) in five super-strata (α = 1 to 5) so as to group together the communes sharing similar characteristics. Several sectors are assumed to have been drawn in each strata. This allows the estimation of the variance intra-strata, while the calculation of the variance intra-commune was impossible since in fact only one sector had been drawn in each commune. Then, the Horwitz-Thompson formula for superstrata α is:

$$z_\alpha' = \sum_h \frac{M_h}{m_{h\alpha}} \sum_{i=1}^{m_{h\alpha}} \frac{N_{hi}}{n_{hi}} \sum_{j=1}^{n_{hi}} \frac{Q_{hij}}{q_{hij}} \sum_{k=1}^{q_{hij}} z_{hijk} \tag{83}$$

and

$$x_\alpha' = \sum_h \frac{M_h}{m_{h\alpha}} \sum_{i=1}^{m_{h\alpha}} \frac{N_{hi}}{n_{hi}} \sum_{j=1}^{n_{hi}} \frac{Q_{hij}}{q_{hij}} \sum_{k=1}^{q_{hij}} x_{hijk} \tag{84}$$

where M_h is the number of communes in prefecture h; $m_{h\alpha}$ is the number of communes in prefecture h and drawn in superstrata α; N_{hi} is the number of sectors in commune i of prefecture h and superstrata α; n_{hi} is the number of sectors drawn in commune i of prefecture h and superstrata α; Q_{hij} is the number of households in sector j of commune i of prefecture h; q_{hij} is the number of households drawn in sector j of commune i of prefecture h and superstrata α. A similar formula can also be used to account for the intermediary drawing of several districts in every sector.

Cov(z',x') is estimated by:

$$\widehat{Cov}(z';x') = \frac{1}{20}\sum_{\alpha=1}(z'_{\alpha} - z').(x'_{\alpha} - x')$$

and similar formula for V(x) and V(z) are obtained by making x = z.

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