Protecting Buyers from Fine Print: Online Appendix
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In this Appendix, we substantiate various claims in Section 4. We use bullet points to distinguish between these claims.

- We cannot use our previous notion of an equilibrium when analyzing unregulated games with naive buyers because this relies on the supremum of profits across BPEs of G being realized in a BPE of G. This property does not necessarily hold in games with both sorts of buyer. In particular, A3 implies that there are BPEs in which the monopolist offers an unfavorable contract priced at q, and mixes between complex contracts, priced at p. Naive buyers accept the unfavorable contract, provided that $q < p$, while sophisticated buyers never select the unfavorable contract. Profit increases as q approaches p from below, but the monopolist can only discriminate between buyers if $q \neq p$. In other words, the supremum of BPE profits cannot be realized in any BPE of G. We will also require the menu offered by the ε-commitment type and the menu required to realize the supremum of BPE profits to be disjoint. (This property is automatically satisfied when all buyers are sophisticated.)

- A3b implies that $b_f > 4k > 0$. If n were close to 1 then A3c would imply that $b_f < 2k < 4k$, contrary to A3b.

- A1, A3b and A3c imply that the right-hand side of the inequality in A3d is positive because $nb_f < 2k - (1-n)(b_d - c_d + c_f)$ implies that

$$\widehat{p} - nb_f - (1-n)(b_d - c_d + c_f) > \widehat{p} - 2k = \frac{2k}{1-Y} - 2k > 0.$$

- **Claim** *If A3 then, in every ε-equilibrium, an unregulated monopolist:*

  a *Offers complex contracts priced at p and at q, where $0 < q < p < \widehat{p}$. The cheaper contract is unfavorable, and is accepted by naive buyers; the other contract is unfavorable with probability k/p, and is otherwise favorable, and is only accepted by sophisticated buyers;*

  b *Earns less than* $\min\{\widehat{p} - nc_u - (1-n)c_f, b_f - c_f\}$.

Proof

a We prove the result in a number of steps. We first argue that G has a BPE outcome in which sophisticated buyers accept a simple contract. We then characterize BPEs in which sophisticated buyers select a complex contract, arguing that the menu must satisfy the conditions in part a in every such BPE, and that A3 implies existence of such BPEs. We use these observations to construct ε-commitment types in perturbed games, and apply our refinement to demonstrate that all buyers trade complex contracts in every ε-equilibrium.

Step 1 *Simple menu*

As default terms are second best, there is a BPE in which the monopolist offers a simple contract priced at b_d and an unfavorable contract priced at b_f; sophisticated buyers accept the former contract and would reject any complex contract at a non-negative price; naive buyers accept the unfavorable contract. Call this the *simple menu.* The monopolist then earns

$$\pi^s \equiv n(b_f - c_u) + (1-n)(b_d - c_d),$$

which is positive (by A1, A2 and A3a). Arguments at the beginning of Section 4 imply that the monopolist offers the simple menu at every BPE in which sophisticated buyers accept a simple contract.

Step 2 *Complex menus*

We now consider putative BPEs in which sophisticated buyers select complex contracts. An argument in Section 4 implies that naive buyers must then also accept a complex contract.

There are two cases to consider: in the first case, the monopolist offers a single complex contract, priced at P. Naive buyers accept, while sophisticated buyers mix between accepting and reading. Lemma 2 implies that the contract must be favorable with probability $1 - \frac{2k}{P}$ and otherwise unfavorable; so $P \in (0, b_f)$. The monopolist then earns $P - c_f$. If sophisticated buyers believe that every complex contract in every other menu is unfavorable then $P > 0$ implies that the monopolist cannot profitably deviate if and only if $P - c_f \geq \pi^s$: that is, when deviation to offering the simple menu is unprofitable.

In the second case, naive and sophisticated buyers accept complex contracts at different prices. Naive buyers must then pay less than sophisticated buyers, but must accept an unfavorable contract. Accordingly, consider a putative BPE in which the monopolist offers $\{q, u\}$, and mixes between offering $\{p, f\}$ and $\{p, u\}$, where $p > q$. Call this a *complex menu.*

Suppose that the monopolist offers a single complex contract priced at P in a BPE, and consider a putative BPE in which the monopolist offers a complex menu with $p = P$. Sophisticated buyers cannot profitably deviate; and, as $q < P < b_f$, naive buyers would accept the unfavorable contract priced at q. The monopolist then earns $n(q - c_u) + (1-n)(P - c_f)$, which exceeds $P - c_f$ whenever q is close enough to P (by A1). The premise then implies that the monopolist cannot profitably deviate from offering some complex menus. In sum, a BPE with a single offered contract can only exist if the monopolist offers some complex menu in another BPE; and expected profit is higher in the BPE with a complex menu. We will now explain why A3 implies existence of BPEs in which the monopolist offers a complex menu.

Define π^* as $\widehat{p} - nc_u - (1-n)c_f$. We claim that, for every ε which satisfies A3d, there is a BPE in which the monopolist offers a complex menu, charging sophisticated [resp. naive] buyers $p \equiv \widehat{p} - \frac{\varepsilon}{2(1-n)}$ [resp. $q \equiv \widehat{p} - \frac{\varepsilon}{2n}$] and earning $\pi^* - \varepsilon$, where $q > 0$ because $\varepsilon < 2n\widehat{p}$. (A3c implies that $n < 1/2$; so $q < p$.)

If the monopolist offers a complex menu in which $q > 0$ then sophisticated buyers cannot profitably deviate to selecting the cheaper contract (which is unfavorable). In addition, arguments used in the proof of Proposition 3.1 imply that they cannot profitably deviate from sometimes reading (and not rejecting) the more expensive contract because $2(b_d - c_d + c_f) < 4k < b_f$ (in A3b). Furthermore, if sophisticated buyers believe that every complex contract in every other menu is unfavorable then they cannot profitably deviate from not accepting any positively priced complex contract in any menu offered off the path of the putative BPE.

By construction, the monopolist earns $\pi^* - \varepsilon$ in the putative BPE, and can therefore not profitably deviate to another menu if $\pi^* - \varepsilon > \pi^s$: the profit earned by offering the simple menu. Rearranging expressions, this is exactly equivalent to

$$\varepsilon < \widehat{p} - nb_f - (1-n)(b_d - c_d + c_f)$$

in A3d.

These arguments therefore imply that, for every ε which satisfies A3d, G has a BPE in which the monopolist offers a complex menu and earns $\pi^* - \varepsilon$.

Step 3 *Equilibria*

Step 1 implies that G has a BPE in which the monopolist offers the simple menu, and earns π^s. Step 2 implies that $\pi^* > \pi^s$; so, for every ε which satisfies A3d, she earns $\pi^* - \varepsilon > \pi^s$ in some BPE of G. Step 2 also implies that the monopolist earns no more than $\widehat{p} - c_f$ in any BPE at which she offers a single complex contract; so $\varepsilon < n(c_f - c_u)$ precludes an ε-commitment type offering a single complex contract. Every ε-commitment type must therefore offer a complex menu. We first argue that the monopolist can therefore not offer the simple menu in any ε-equilibrium of G:

Consider any BPE of a perturbed game $G(e, \varepsilon)$. Define $\xi(e, \varepsilon)$ as the probability with which the normal type is prescribed to offer the simple menu. If $\xi(e, \varepsilon) = 1$ then the normal type could profitably deviate to mimicking the ε-commitment type, as it would then earn $\pi^* - \varepsilon > \pi^s$ because sophisticated buyers would accept. The same argument precludes the normal type mixing between offering the simple menu and the complex menu offered by the the ε-commitment type. There can therefore be no sequence of BPEs which satisfies $\lim_{e\to 0} \xi(e, \varepsilon) = 1$; so G cannot have an ε-equilibrium in which the monopolist offers the simple menu.

We now argue that such an ε-equilibrium exists. Consider any perturbed game $G(e, \varepsilon)$ in which the ε-commitment type offers a complex menu consisting of an unfavorable contract priced at $q = \widehat{p} - \frac{\varepsilon}{2n}$, and a contract priced at $p = \widehat{p} - \frac{\varepsilon}{2(1-n)}$, which is unfavorable with probability k/p, and is otherwise favorable. The ε-commitment type's strategy is therefore played by the monopolist in a BPE of G where she earns $\pi^* - \varepsilon$. If sophisticated buyers believe that every complex contract in every other menu is unfavorable then $G(e, \varepsilon)$ has a BPE in which the normal type pools with the ε-commitment type whenever ε satisfies A3d. Taking limits as $e \to 0$, G has an ε-equilibrium in which the monopolist offers $\{q, u\}$, and mixes between offering $\{p, f\}$ and $\{p, u\}$.

b Part a implies that naive buyers pay less than sophisticated buyers, who in turn pay less than $\widehat{p}$. Hence, the monopolist earns less than π^* in every ε-equilibrium. We prove this part by arguing that A3 implies that $\pi^* < b_f - c_f$:

an inequality which holds if and only if $n(c_f - c_u) < b_f - \widehat{p}$.

A3a and A3b imply that $nb_f > 2n(c_f - c_u)$; so A3c implies that

$$2n(c_f - c_u) < 2k - (1-n)(c_f - c_u),$$

and therefore that

$$n(c_f - c_u) < \frac{n}{1+n}2k.$$

Hence, $\pi^* < b_f - c_f$ if

$$\frac{n}{1+n}2k < b_f - \widehat{p}$$

which, substituting for $\widehat{p}$ and rearranging, is equivalent to

$$\frac{2k}{1-Y} < b_f - \frac{n}{1+n}2k \text{ or } Y < 1 - \frac{2k}{b_f - \frac{n}{1+n}2k}.$$

The right-hand side of this inequality is positive because $b_f > 4k$, so the condition is equivalent to the square of the right hand side exceeding Y^2. Expanding these expressions and rearranging: $n < 1$ implies that

$$Y^2 < (1 - \frac{2k}{b_f - \frac{n}{1+n}2k})^2,$$

and therefore that $\pi^* < b_f - c_f$.

The monopolist cannot earn as much as $\widehat{p} - nc_u - (1-n)c_f = \pi^*$ because it cannot offer favorable contracts exclusively to sophisticated buyers.■

- A3b implies that $b_d - b_f < -\widehat{p}$. To see this, substitute for $\widehat{p}$ and rearrange; so that the inequality becomes

  $$Y < 1 - \frac{2k}{b_f - b_d}.$$

  The right-hand side is positive because $b_f - b_d > b_f - k > 2k$; so the inequality is satisfied if and only if

  $$Y^2 < (1 - \frac{2k}{b_f - b_d})^2:$$

  viz.

  $$1 - \frac{4k}{b_f} < 1 - \frac{4k}{b_f - b_d} + \frac{4k^2}{(b_f - b_d)^2}.$$

  It is easy to confirm that this is equivalent to $(k - b_d)b_f + b_d^2 > 0$, which holds because A3b implies that $b_d < k$.