Marriage Market and Labor Market Sorting

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Abstract

We build a novel equilibrium model in which households' labor supply choices form the link between sorting on the marriage market and sorting on the labor market. We first show that in theory, the nature of home production – whether partners' hours are complements or substitutes – shapes marriage market sorting, labor market sorting and labor supply choices in equilibrium. We then estimate our model on German data to assess the nature of home production in the data, and find that spouses' home hours are *complements*. We investigate to what extent complementarity in home hours drives sorting and inequality. We find that the home production complementarity – by strengthening positive marriage sorting and reducing the gender gap in hours and labor sorting – puts significant downward pressure on the gender wage gap and within-household income inequality, but it fuels between-household inequality. Our estimated model sheds new light on the sources of inequality in today's Germany and – by identifying important shifts in home production technology towards more complementarity over time – on the evolution of inequality.

Keywords. Sorting, Marriage Market, Labor Market, Hours, Household Income Inequality, Gender Wage Gap, Home Production, Technological Change.

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1 Introduction

Positive assortative matching is a defining feature of both the labor market and the marriage market and has important implications for inequality. On the marriage market, the matching of partners with similar education impacts both within- and between-household income inequality. At the same time, positive sorting in the labor market between workers and firms or jobs reinforces wage inequality across skills. However, even though inequality in economic outcomes results from agents interacting in *both* the marriage and the labor market, we know very little about the interplay between sorting on both markets; and about how the two markets *jointly* shape inequality.

This paper shows that there is a natural link between sorting in the marriage and the labor market: households' choices of how many hours to allocate to market work versus home production. We build a new model with rich heterogeneity and sorting on both markets and show that in theory the nature of home production shapes equilibrium. If spouses' home hours are complementary, then the equilibrium is a 'progressive' one where spouses share home production tasks, supply similar amounts of labor hours, and there is positive sorting on both marriage and labor market. We then estimate our model to investigate the nature of home production in the data. We find that partners' home production time is indeed complementary in today's Germany and this complementarity has become stronger over time. Our estimated model suggests that this technological change in home production was the main driver behind reduced gender disparities between 1990 and 2016, and had opposite effects as technological change in the labor market. Importantly, increases in *both* positive marriage and labor market sorting further mitigated gender disparities in Germany over the last decades.

In studying labor and marriage markets jointly, we are motivated by three sets of facts, based on the German Socioeconomic Panel (henceforth, GSOEP), showing a striking relationship between the two markets. First, as is well documented in the literature, there is positive sorting in both markets: on spouses' education in the marriage market and between workers' education and jobs' skill requirements in the labor market. Importantly, men are better matched to jobs in the labor market, in the sense that for each given level of education they are matched to more demanding jobs. Second, men and women who are more strongly sorted in the marriage market (i.e. those whose education is more similar to their partner's education) are also more strongly sorted in the labor market (i.e. they tend to have the 'right' education level for the jobs they perform). Third, households' labor supply choices, and particularly the split between hours worked in the labor market vs. hours spent in home production, form an important link between the two markets: Time allocation choices are not only impacted by marriage market sorting (spouses with more similar education have a higher correlation of their hours worked; and similarly for hours in home production), but also have an impact on labor market sorting (controlling for hours worked eliminates most of the gender gap in sorting on the labor market).

We aim to capture these features of the data with a novel equilibrium model, in which households' endogenous labor supply choices form the link between the marriage and the labor market. The model is static, and individuals differ in skills and face three decisions. First, in the marriage market, men and women choose whether and whom to marry. Second, there is a (collective) household problem, where each household formed in the marriage stage decides how to allocate time to labor market vs. home production. This choice determines the consumption of a private and public good. Last, in the labor market, individuals match with firms/jobs of different productivity, which determines their wages.

The crucial twist of our model is that for matching on the labor market, not only the individuals' skills matter but also their endogenous hours worked. The optimal job match in the labor market depends on workers' *effective* skills, which are an increasing function of both skills and hours. Thus, hours worked increase individuals' productivity.[1] Since the time allocation choice of a couple depends on the characteristics of both partners (and thus on marriage sorting) and at the same time impacts the allocation of individuals to jobs (and thus labor market sorting), marriage market sorting affects labor market sorting. At the same time, when making their marital and household labor supply choices, individuals internalize that an increase in labor hours improves job quality and wages. Therefore, labor market sorting also affects marriage market sorting. This interrelation between the two markets is the unique feature of our model but also makes the problem complex.

We focus on a tractable transferable utility (TU) representation of our model and characterize two benchmark equilibria depending on the model's primitives. Both feature positive assortative matching between workers and jobs in the labor market driven by productive complementarities. However, the two equilibria differ in household and marriage outcomes depending on properties of the home production function. On the one hand, if home production exhibits complementarity in partners' time inputs, a *monotone equilibrium* arises, characterized by positive sorting in the marriage market and labor hours that are increasing in both own and partner's skills. This equilibrium reflects a 'progressive' economy with a high frequency of two-earner households and where spouses are similar in terms of skills and their split between work and home production. The complementarity in home hours is therefore a force towards positive marriage sorting as well as balanced labor supply, labor market sorting and pay across gender. This leads to a narrow gender wage gap, low within-household income inequality, but high inequality between households. On the other hand, if partners' time inputs are substitutable in home production, a *non-monotone* equilibrium arises, featuring negative assortative matching in the marriage market and labor hours that are increasing in own but *decreasing* in partner's skill. This equilibrium reflects a 'traditional' economy with a high degree of household specialization and disparity in partners' skills – features that widen the gender wage gap and within-household income inequality but put downward pressure on between-household inequality.

The key insight from our model is that marriage and labor market sorting are linked in an intuitive way by households' labor supply choices. The nature of this link depends on whether spouses' hours in

[1] We base this assumption on our own evidence of a positive impact of hours worked in the labor market on *hourly* wages in the GSOEP, both in Figure 4 and Table 9, column (3); and also on empirical evidence from the literature arguing that more hours worked lead to higher productivity, especially if it is costly to hand off clients, patients or customers to the next worker on the shift, for instance due to increased coordination costs. It has been shown that hours worked are of special importance in occupations in business and finance as well as legal occupations, which have high requirements for particular hours or considerable client contact (e.g. Goldin (2014)).

home production are complementary or substitutable, a feature that needs to be investigated empirically.

To do so, we first develop a quantitative version of our model that is amenable to estimation. We do so by imposing minimal changes to our baseline model in order to preserve its parsimony and key mechanism. We introduce marriage taste shocks to allow for mismatch in the marriage market. We allow for labor supply shocks to generate variation in hours choices even within the same couple-type. Last, we add a random productivity component to workers' skills to account for imperfect matching in the labor market. We show that this model is identified. Importantly, after parameterizing our model, we allow for two asymmetries between men and women that will be disciplined by the data: gender differences in home productivity and in labor market productivity (where the latter could also be interpreted as discrimination). Our objective is to determine the nature of home production in the data; and to use the estimated model to assess the role of home production technology (and that of other model primitives, like the labor market technology) in inequality, both in the cross-section and over time.

We first focus on MODERN GERMANY – our benchmark estimation. We estimate the model on GSOEP data for West Germany from 2010 to 2016 combined with data on job characteristics. Our model matches key features of the marriage market equilibrium (such as the degree of marital sorting and the correlation of home hours within couples) and the labor market equilibrium (such as moments of the wage distributions). To further validate the model, we show that it reproduces the three stylized facts outlined above, even though they are not targeted in the estimation, as well as several untargeted moments of inequality. Importantly, our estimates indicate that spouses' home production time is complementary, informed by a strong positive correlation of their home production hours.

In order to showcase our model's mechanism, we conduct comparative statics of the gender wage gap and within/between household wage inequality with respect to three parameters that significantly impact inequality: (i) complementarity of partners' home production time, (ii) women's relative productivity in the labor market, (iii) women's relative productivity in home production. Our insights are the following: First, eliminating asymmetries in productivity across gender (whether at home or at work) naturally reduces the gender wage gap. But, interestingly, this is not the only way to reduce gender disparities: An increase in complementarity of partners' home production hours has qualitatively similar effects. Second, a decline in the gender wage gap tends to go hand in hand with a decline of gender gaps in labor hours and labor market sorting, and with an increase in marriage market sorting. Third, while the effect of these parameter changes on overall household income inequality depends on the specific exercise, in all cases, the gender wage gap moves hand in hand with *within*-household inequality but in opposite direction as *between*-household inequality.

Having well understood our model mechanism, we then focus in our main quantitative exercise on GERMANY OVER TIME. The last 25 years have shown a large decline in gender disparities (gender wage gap and within-household income inequality) and increases in overall household income inequality and its between-component. We ask whether and how our model can rationalize these trends. To this end, we also estimate our model on data from the beginning of the 1990s and then compare it to our baseline estimation. The model estimates reveal significant changes in home production over time with today's Germany being characterized by increased relative productivity of men at home and stronger complementarity in spouses' home hours, indicating a switch towards a more 'progressive' economy (the monotone equilibrium of our model). Changes in home production technology account for around 70% of the observed decline in the gender gap and for the entire drop in within-household inequality. In contrast, changes in labor market technology fueled inequality across the board, significantly pushing up household income inequality and its between component, and preventing gender inequality from falling even further.

We then assess the quantitative role of marriage and labor market sorting in these inequality shifts. Between 1990 and 2016, positive marriage sorting has increased by around 10% and positive labor sorting by 8%. How would inequality have evolved when fixing sorting patterns at their 1990-level? In these counterfactuals, we find that changes in *both* marriage and labor sorting have had a large mitigating impact on gender inequalities (wage gap and within-household inequality) and have amplified overall inequality and between-household inequality. Stronger marriage market sorting over time generated more balanced labor market outcomes – in hours, sorting, and pay – across gender. In turn, the increase in labor sorting over the past decades also significantly narrowed gender disparities as it was predominantly driven by women's improved labor sorting, helping them to catch up with men's pay.

Our main takeaway from this exercise is that technological change in home production – with stronger home production complementarities being an important component – mitigated gender inequality in Germany over the last three decades. The estimated changes in home production directly lead to more balance in spouses' labor supply choices; but they also lead to more marriage sorting and a reduced gender gap in labor sorting, further spurring convergence in labor market outcomes across gender.

2 The Literature

This paper relates to four strands of literature: the literature on gender gaps in the labor market highlighting the importance of the gap in labor supply; sorting on the marriage market; sorting on the labor market; and the interaction between labor and marriage markets.

Gender Gaps in Labor Supply and Pay. A growing literature studies the link between the gender gap in labor supply and the gender gap in pay. The standard channel works through earnings, where family and fertility choices have a permanent effect on the gender earnings gap (Dias, Joyce, and Parodi, 2018; Angelov, Johansson, and Lindahl, 2016; Kleven, Landais, and Søgaard, 2019). Because the wage rate is kept fixed in these papers, any gender gap in pay can only be attributed to *earnings* not to *hourly wages* (which is what we focus on). In assuming that hours worked affect workers' productivity in the market, we follow more closely the literature documenting significant labor market returns to hours (Aaronson and French, 2004; Gicheva, 2013; Goldin, 2014; Cortés and Pan, 2019; Bick, Blandin, and Rogerson, 2020).[2] Other work links gender pay gaps to gender differences in preferences for work

[2] Bick, Blandin, and Rogerson (2020) find that hourly wages of elderly U.S. men are non-monotone, increasing until 50

flexibility (Mas and Pallais, 2017 and Bertrand, Goldin, and Katz, 2010) and to sorting into occupations that require different hourly inputs (Erosa, Fuster, Kambourov, and Rogerson, 2017). Finally, there is work highlighting the importance of information frictions for gender pay gaps (without considering the marriage market): If employers believe that women have less market attachment relative to men, they get paid less (Albanesi and Olivetti, 2009, Gayle and Golan, 2011).

Our paper builds on this work in that we also propose the gender gap in hours as a key factor behind the gender pay gap. However, in contrast to both the purely empirical and the structural papers we cited, our work takes into account an endogenous marriage market which shapes labor supply choices.

Marriage Market Sorting. A large literature measures marriage sorting in the data and finds evidence of positive assortative matching in education in different countries and increases in marriage sorting over time (Browning, Chiappori, and Weiss, 2014; Eika, Mogstad, and Zafar, 2019; Greenwood, Guner, Kocharkov, and Santos, 2016; Greenwood, Guner, and Vandenbroucke, 2017). We confirm these findings on positive marriage sorting on education for Germany.

Another approach has been to study marriage market sorting using structural models. Several papers have investigated how pre-marital investments in education interact with marriage patterns in a static framework (Chiappori, Iyigun, and Weiss, 2009 and Fernández, Guner, and Knowles, 2005) or over the life cycle (Chiappori, Costa-Dias, and Meghir, 2018) and how post-marital investments in a partner's career interact with family formation and dissolution (Reynoso, 2019). Further, there is structural work analyzing how exogenous changes in wages, education and family values (Goussé, Jacquemet, and Robin, 2017a), exogenous wage inequality shifts (Goussé, Jacquemet, and Robin, 2017b), the adoption of unilateral divorce (Reynoso, 2019), or different tax systems (Gayle and Shephard, 2019) affect household behavior and marriage sorting.[3] Finally, in a household model with *exogenous* marriage sorting (and exogenous labor market), Lise and Seitz (2011) analyze the effect of an increase in marriage sorting on between and within household consumption inequality.

Like in these papers, marriage market sorting is an important margin also in our model. While education is exogenous in our setting, we could think of the choice of how many hours to work as an 'investment' in individuals' effective skills. But this investment happens *post-marriage* market and *pre-labor* market, and therefore is impacted by marriage sorting while impacting labor market sorting, so the timing is different than in existing work. Crucially none of these papers endogenizes the labor market or features labor market sorting, which is the key addition of our work.[4]

Labor Market Sorting. A body of literature investigates sorting on the labor market, documenting positive assortative matching between workers and firms (Card, Heining, and Kline, 2013; Hagedorn, Law, and Manovskii, 2017; Bagger and Lentz (2018); Bonhomme, Lamadon, and Manresa,

hours per week, and then decreasing. Note that in our sample, hardly anyone (<0.3%) works more than 50 hours per week, justifying that we do not allow for non-monotone effects of hours on wages in our model (we allow for non-linear effects).

[3]In an influential paper, Voena, 2015 also focuses on the adoption of unilateral divorce and its effects on household behavior, especially asset accumulation. In her paper, the marriage market is exogenous.

[4]The exception is Fernández, Guner, and Knowles (2005) who endogenize the wages of their two worker types, low and high skill, but as in the other papers, their model does not feature any labor market sorting.

2019); or workers and jobs (Lindenlaub, 2017; Lise and Postel-Vinay, Forthcoming; Lindenlaub and Postel-Vinay, 2020) without taking the marriage market into account. In this strand, our paper is perhaps closest to Pilossoph and Wee (2019b) who consider spousal joint search on the labor market to explain the marital premium, but taking marriage market sorting as given. Our contribution is to explore how the forces that determine who marries whom shape labor market sorting and pay.

Interplay between Marriage and Labor Market. Our work is most closely related to a nascent literature on the interplay between marriage and labor markets. These papers have focused on the effects of spouses' joint labor search (Pilossoph and Wee, 2019a and Flabbi, Flinn, and Salazar-Saenz, 2020), changes in wage structure and home technology (Greenwood, Guner, Kocharkov, and Santos, 2016), and changes in the skill premium (Fernández, Guner, and Knowles, 2005) on marital sorting and household inequality keeping the labor market in *partial* equilibrium.

To the best of our knowledge, this is the first paper that features both the marriage market and the labor market in *equilibrium* with market clearing, price determination and sorting in both markets. Jointly considering marriage and labor market *sorting* is novel and so is our mechanism of how the two markets and sorting margins are linked (i.e. through endogenous labor supply).

3 Descriptive Evidence

3.1 Data

We use two different data sources: the German Socioeconomic Panel (GSOEP) and the Employment Survey of 2012 (to which we refer as the BIBB Survey).

GSOEP: The GSOEP is a household survey conducted by the German Institute of Economic Research (in German: DIW Berlin) starting in 1984. In 1990, it was extended to include states from the former German Democratic Republic. The core study of the GSOEP surveys about 25,000 individuals living in 15,000 households each year. All individuals aged 16 and older respond to the individual questionnaire, while the head of household additionally answers a household questionnaire. This survey is suitable for our project not only because of its longitudinal nature, but also because it collects very rich information: it includes comprehensive information on demographics (such as marital status, education level, nationality, and family background), labor market variables (including actual and contractual hours worked, wages and occupation), and home production (such as detailed time use variables). Important for our study, it contains the same information for the heads of household and their partner (whether married or cohabiting). Additionally, it is possible to link all this information to marital and birth histories, which allows us to identify whether there are children in the household, and their ages. The GSOEP is our main dataset both for empirical analysis and estimation. In our baseline analysis, we restrict our attention to West Germany, and pool observations from the period 2010-2016.

BIBB. The BIBB is our supplementary data source, collected in 2012 by the German Federal Institute of Vocational Training (in German: BIBB) and the German Federal Institute for Occupational

Safety and Health (in German: BAuA). This survey collects information on 20,000 individuals and is representative of the employed population working at least 10 hours per week. The BIBB survey contains information on the characteristics of employees (such as job satisfaction, tenure in their jobs, etc.), and the relationship between education and employment. Most important for us, it contains information about the characteristics of occupations, reported by each individual. Respondents are asked about their job tasks and the knowledge required to perform them. We use this information to construct a measure of *task complexity* of each occupation, as a proxy for job productivity. We merge this measure into the GSOEP, using four-digit occupational codes from the German Classification of Occupations 1992.

3.2 Empirical Evidence

We first present evidence related to sorting in the marriage market, sorting in the labor market, and the interaction between both sorting margins. We then highlight that the allocation of hours between the labor market and home production is an important link between both markets. A description of the sample restrictions and the main variables can be found in Appendix D.1.

Marriage Market Sorting. We find evidence of positive assortative matching (PAM) in education in the German marriage market in line with the existing evidence (Eika, Mogstad, and Zafar (2019) for US and Germany and Greenwood, Guner, Kocharkov, and Santos (2016), Greenwood, Guner, and Vandenbroucke (2017) for the US). Table 1 reports the matching frequencies by education for the period 2010-2016, suggesting that almost 60% of individuals marry someone of the same level of education, with a correlation between the education level of spouses equal to 0.47. The correlation of spouses' education is our main measure of marriage market sorting.[5] Furthermore, marriage market sorting increased over time. For the period 1990-1996, the correlation between education of partners was 0.44.[6]

Table 1: Marriage Matching Frequencies by Education

	Low Education Men	Medium Education Men	High Education Men
Low Education Women	**0.16**	0.08	0.03
Medium Education Women	0.13	**0.25**	0.11
High Education Women	0.03	0.05	**0.16**

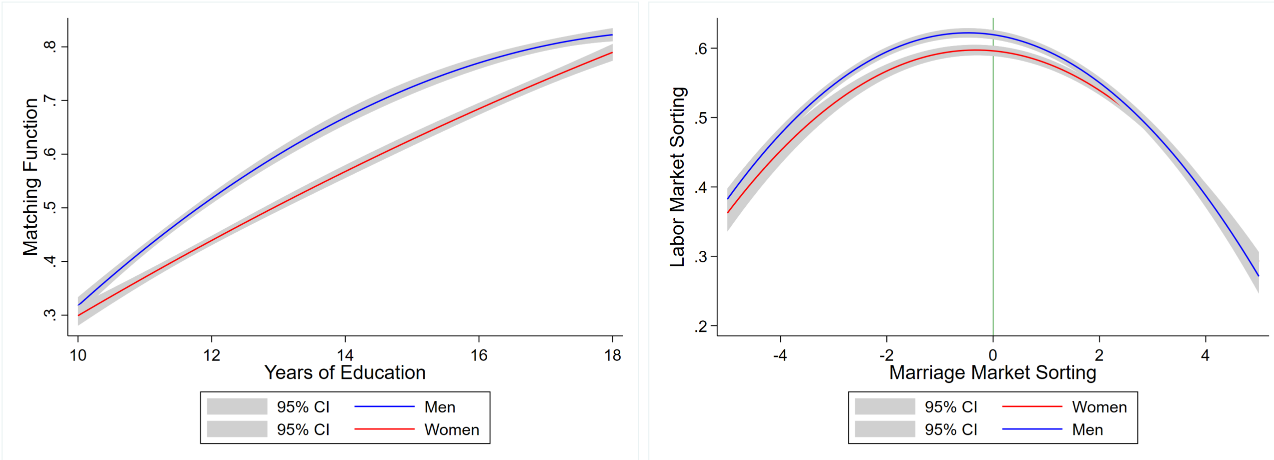
Notes: Low education includes high school and vocational education with less than 11 years of schooling. Medium Education is vocational education with more than 11 years of schooling. High Education is defined as college and more. We consider the maximum level of education attained by each individual and keep only one observation per couple.

Labor Market Sorting. We also document positive assortative matching in the labor market. We do not have firm identifiers in the GSOEP, so we measure labor sorting based on the relationship between worker and *job* attributes, where a job is defined by the occupation of the individual. The

[5] As discussed in Chiappori, Dias, and Meghir (2020), *correlation* is one of the measures that complies with two desirable properties – a 'monotonicity' condition and whether it captures the case of 'perfectly assortative matching' – that a measure of sorting (and sorting changes) should have. Eika, Mogstad, and Zafar (2019) propose an alternative measure of marriage sorting based on the frequency of couples' education relative to random matching. This measure equals 1.73 in West Germany: individuals are 73% more likely to marry someone of the same education, relative to random matching.

[6] For years 1990-1996, the sorting measure from Eika et al. (2019) is 1.59, also suggesting increased sorting over time.

Figure 1: Labor Market Matching Function (left); Labor Market and Marriage Market Sorting (right)



match-relevant attribute of workers in the labor market is 'years of education'. In turn, for jobs we use information on the task requirement of each occupation to construct a measure of its *task complexity* (see Appendix E.3). The correlation between years of education of workers and task complexity of jobs is 0.62, indicating positive assortative matching between workers and jobs on the labor market.

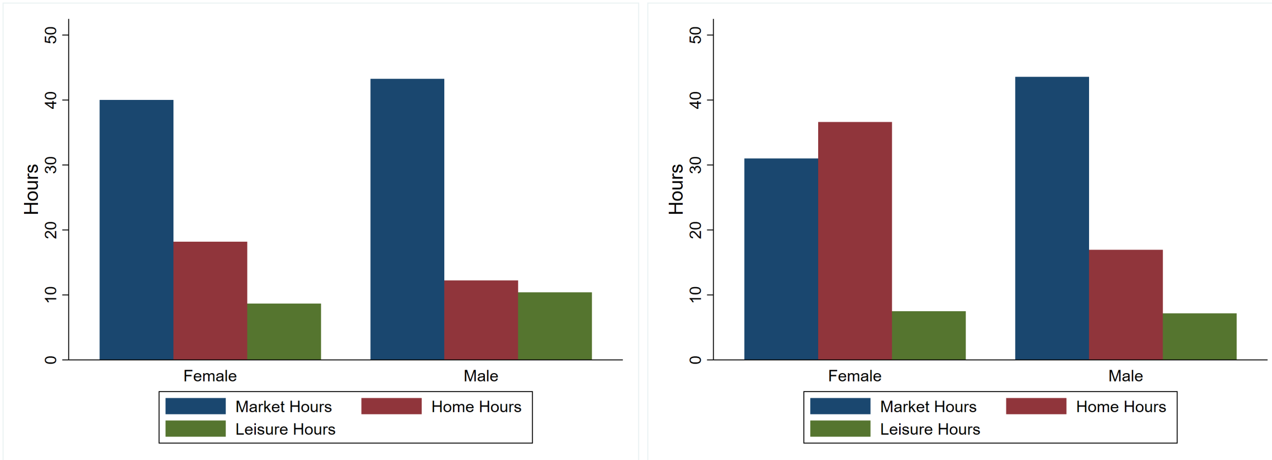
Figure 1 (left) plots the fitted matching function (job attribute as a function of worker characteristic) by gender, conditional on employment. Both men and women are positively sorted in the labor market, indicated by a positive slope of the matching function. However, men are 'better' matched: for a given education level, men are on average matched to better jobs than women. This pattern is also reflected in the correlation of worker and job attributes by gender, which is 0.64 for men and 0.62 for women.[7]

Labor Market Sorting and Marriage Market Sorting. Next we assess the relationship between labor market and marriage market sorting. To do so graphically, we measure marriage market sorting by the difference between the years of education of the individual under consideration and the years of education of their partner, with 'zero' indicating the maximum amount of sorting. We measure labor market sorting as before as the correlation between years of education (worker characteristic) and the task complexity of the occupation (job characteristic). We then plot the relation between labor market and marriage market sorting by gender in Figure 1 (right), where the green vertical line indicates maximum marriage market sorting. The striking – and we believe novel – feature is that labor market sorting is maximized when marriage market sorting is maximized, both for men and for women. In Appendix A.1, we substantiate this finding using regressions that control for important covariates.

The Role of Hours. We now provide evidence on a salient link between the two markets: hours worked on the labor market vs. hours spent in home production. First, we document that the time allocation choice is 'impacted' by the partnership status as well as marriage market sorting. Second, we

[7]Differences in labor market sorting across gender are even larger (with a correlation 0.58 for men vs. 0.48 for women) if we do not condition on participation and we treat unmatched individuals as matched to a job with attribute zero. This suggests that non-participation is one of the dimensions through which women are worse matched in the labor market.

Figure 2: Time allocation for Singles (left) and Couples (right)



document that at the same time, the time allocation choice 'impacts' labor market sorting.[8]

As is well documented (Gayle and Shephard, 2019; Goussé, Jacquemet, and Robin, 2017b), an individual's time allocation between the activities 'work', 'home production' and 'leisure' is related to their partnership status. While among singles (left panel in Figure 2) gender differences in time allocation across activities are minor, gender differences are much more pronounced for couples (right panel). Indeed, in couples, women spend about 10 hours less per week working on the labor market but about 10 hours more in home production compared to their male partners. Neither for couples nor for singles are there significant gender differences in leisure, justifying that we abstract from it in our model below.

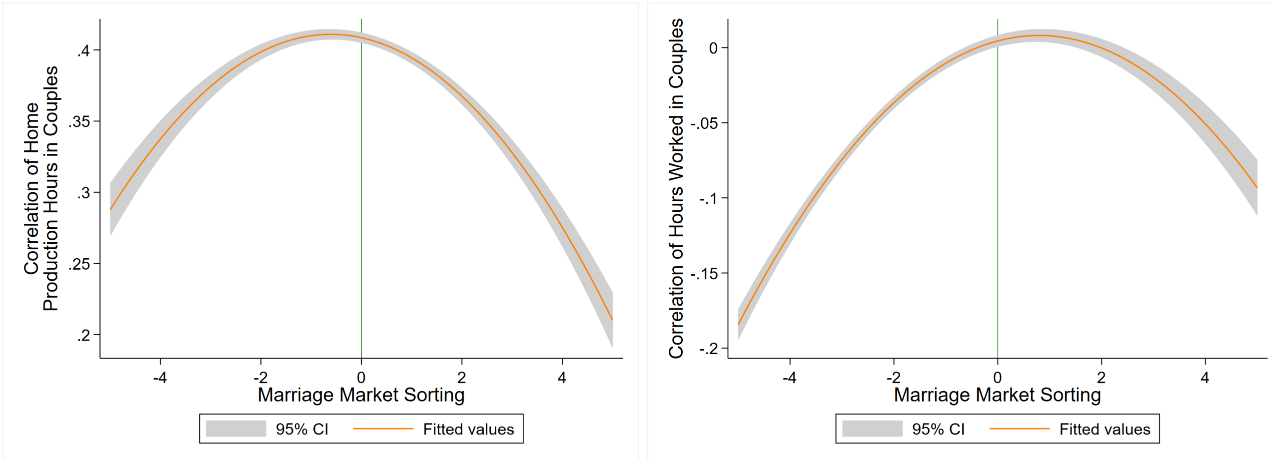
We also document the relationship between hours and marriage market sorting. Figure 3 focuses on labor and home production hours of partners and how they relate to their marriage sorting. The left panel plots the correlation between home production hours within couples against our summary measure of marriage market sorting (difference in partners' years of education); the right panel has the same structure but with the correlation of labor hours within couples on the vertical axis. Interestingly, both home production hours (left panel) and labor market hours (right panel) are more complementary among those partners that are well sorted on education (whose difference in education is zero), as indicated by the inverse U-shape of the hours' correlation function. Note that the pattern for home production is even more pronounced than for market hours, with a strong positive correlation of home hours among partners with the same education compared to partners with differences in education.[9]

We further explore what drives the complementarity in home production time of spouses by looking at different components of home production, especially since most of the family economics literature emphasizes household specialization. We find that complementarities are strongest in childcare and least pronounced when it comes to housework (see Figure 13 in Appendix A.2).

[8] We use self-reported actual hours for labor hours but our facts are robust when considering contractual hours.

[9] For consistency between the right and the left panels of Figure 3, we condition on both partners participating in the labor market. However, the pattern in the left panel holds if we do not condition on labor market participation.

Figure 3: Time Allocation and Marriage Sorting



One concern is that the relationships between marital sorting and complementarities in hours in Figure 3 are based on marriage market sorting bins that pool individuals from different education groups. Not controlling for education allows for the possibility that hours only depend on own education but do not vary with partner's education if, e.g., low (high) educated workers always put low (high) hours independently of the partner's type. Also, the relationships in Figure 3 cannot be interpreted as causal, since there might be other confounding factors or unobserved heterogeneity driving both partners choice of hours. We discuss and address these concerns in detail in Appendix A.2. There, we control for education and other covariates that might be correlated with hours' choices (Table 6). Moreover, we show that complementarities in market hours are stronger when we exploit exogenous variation in childcare availability across states and time, instrumenting for female labor market hours (Table 7). Finally, we also show in these regressions that the correlation between partners' hours is larger for those who are better sorted in the marriage market, in line with the descriptive evidence of Figure 3. We provide the details of our identification strategy in Appendix A.2.

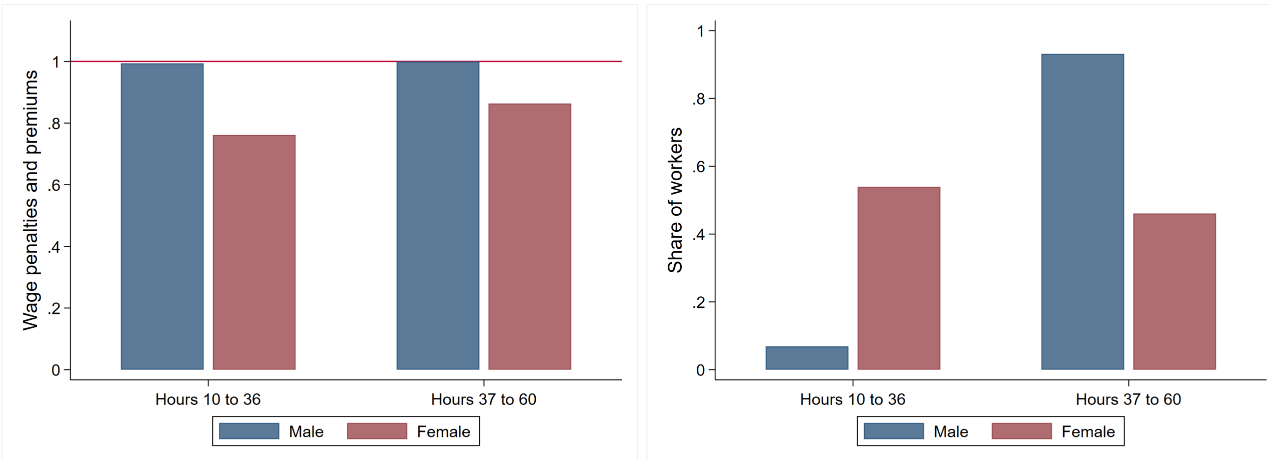
The second point we stress is that the time split between labor market and home production also relates to labor market outcomes: We first show that in Germany there is a large hourly wage penalty for working part time, suggesting that hours are a productive input in the labor market. This is in line with evidence by Aaronson and French (2004), Goldin (2014) and Bick, Blandin, and Rogerson (2020) for the US. Figure 4 (left) shows a sizable part time penalty, especially for women. The left panel shows the wage penalties of various groups *relative* to men working at least full-time (blue bar that equals one).[10] While full-time women have a wage penalty of 14.7 percentage points relative to full-time men, when they work part time the wage penalty increases to 26.6 percentage points.[11] Figure 4 (right) shows that while few men work less than full time (less than 10% of employed men), more than 50%

[10] Full-time is defined as working between 37 and 60 hours per week.

[11] The bars in Figure 4 (left) reflect the estimates of the regression of the logarithm of hourly wages on an indicator for part time work, a gender indicator and the interaction between both. We control for age, marital status, birth place, education.

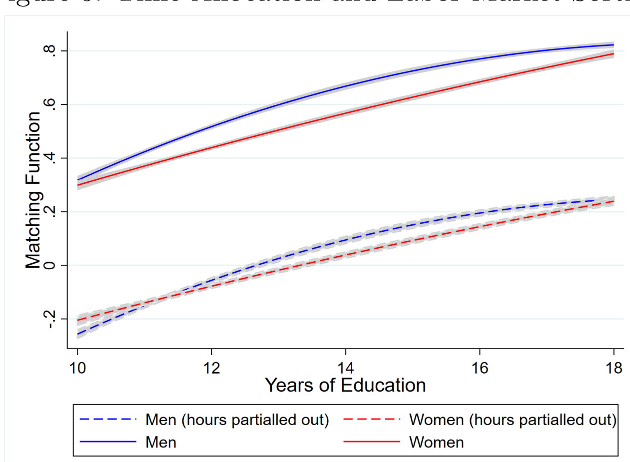
of employed women do so, and are thus particularly affected by the documented wage penalties. The here reported effect of hours on wages cannot be interpreted as causal though. To address selection, we identify the effect of hours on the hourly wage in a panel regression with individual fixed effects below, where we instrument for hours worked (see Section 7.3.1). We again find a significant wage penalty for not working full time: And increase from 30 to 40 hours per week raises the hourly wage by around 4%.

Figure 4: Part-Time Wage Penalties (left) and Share of Workers in Full/Part Time Work (right)



Finally, we highlight that the number of hours worked is associated with *sorting* on the labor market. Indeed, when accounting for differences in hours worked across gender, the discrepancy in their matching functions from Figure 1 shrinks considerably. This is documented in Figure 5, where the solid lines replicate the the matching functions by gender from Figure 1. In turn, the dashed lines plot the residualized matching function for both genders, after partialling out hours worked. While much smaller, we see that even when controlling for the number of hours worked some differences in labor market sorting across gender persist, which must be accounted by other factors.

Figure 5: Time Allocation and Labor Market Sorting



In sum, we highlight three sets of facts. First, there is evidence of PAM both in the labor and the marriage market. But on the labor market women are more mismatched than men, indicated by a lower matching function. Second, there is a strong relation between labor market and marriage market sorting with labor market sorting (for both men and women) being maximized when marriage market sorting is. Third, the split between hours worked in the labor market vs. hours spent in home production is a potentially important link between the two markets. We not only show that time allocation choices depend on marriage market sorting but also they are themselves associated with labor market sorting. Motivated by these facts we now build a model with endogenous labor *and* marriage market. We also use these facts to justify several assumptions and to guide our modeling choices regarding the link between labor and marriage markets, where we focus on the hours margin. Finally, we come back to these facts when validating our estimated model below.

4 The Model

We start with an overview: Men and women first decide whom to marry in a competitive market based on their education/skills. Each matched household then optimally chooses private consumption and the time allocation between home production and labor market work, which also pins down the public good consumption. Finally, individuals match with heterogenous firms in a competitive labor market.

4.1 Environment

There are two types of agents, individuals and firms. There is a measure one of firms. Firms are characterized by productivity $y \in \mathcal{Y} = [\underline{y}, \overline{y}]$, distributed according to a continuously differentiable cdf G, with positive density g. Among the individuals, there is an equal measure of men (denoted by subscript m) and women (denoted by subscript f). The overall measure of individuals is one. Both men and women have exogenously given skills: Denote women's skills by $x_f \in \mathcal{X}_f = [0, \overline{x}_f]$, where x_f is distributed with a continuously differentiable cdf N_f, with density $n_f > 0$. Analogously, men have skills $x_m \in \mathcal{X}_m = [0, \overline{x}_m]$, distributed according to the continuously differentiable cdf N_m with density $n_m > 0$.

In the marriage market, men and women match on skills, so the relevant distributions for marriage matching are N_m and N_f. In the labor market, however, what matters for output is not only skills but also hours worked, which will be chosen optimally in each couple. Each individual is endowed with one unit of time that can be allocated to paid work in the labor market, denoted by $h_i, i \in \{f, m\}$, or non-paid work at home towards the production of a public good, $1 - h_i$ (based on Figure 2, which shows no large differences in leisure across gender, we abstract from it). Note that $h_i = 0$ captures non-participation. By increasing hours worked in the labor market, each individual 'invests' in his/her *effective skill* $\tilde{x} := e(x, h)$, $\tilde{x} \in \tilde{\mathcal{X}}$, with endogenous cdf $\tilde{N}(X) := \mathbb{P}[\tilde{x} \leq X] = \frac{1}{2}\mathbb{P}[\tilde{x}_f \leq X] + \frac{1}{2}\mathbb{P}[\tilde{x}_m \leq X]$. We assume that e is twice differentiable, strictly increasing in each argument, supermodular, and $e_h(x, \cdot)$ is strictly positive and finite for all x.[12] Thus, putting more labor hours is *as if* the worker is more

[12] We use subscripts of functions to denote derivatives throughout.

skilled. The effective skill or index $\tilde{x}$ is the output-relevant worker characteristic on the labor market. We base this assumption on evidence that more hours worked lead to higher productivity (see (Gicheva, 2013), (Goldin, 2014) and (Cortés and Pan, 2019), and our own evidence). One reason may be that costs of handing off clients or projects to the next worker on the shift are high especially in skilled jobs. This assumption that not only skills but also hours worked matter for labor market matching means that multiple attributes are matching-relevant even if the actual assignment is simplified and based on the index $\tilde{x}$.

Denote by $z(\tilde{x}, y)$ the output generated by an individual of type $\tilde{x}$ matched to a firm of type y. We assume that production function z is twice differentiable, increasing in each argument, and $z_{\tilde{x}}(\cdot, y)$ is strictly positive and finite for all y. Individuals and firms split the output they generate into wages and profits, where workers use their wages to finance the private consumption good $c_i, i \in \{f, m\}$.

The public good production function is given by p, which takes as inputs each couple's hours at home, so that $p(1-h_m, 1-h_f)$ is the public good produced by a couple spending $(1-h_m, 1-h_f)$ in home production (recall hours at home equal the hours *not* spent in the labor market). We assume that p is twice differentiable with $p_1 > 0, p_2 > 0, p_{11} < 0$ and $p_{22} < 0$ and with the Inada conditions $\lim_{h_f \to 0} p_2(1-h_m, 1-h_f) = 0$ and $\lim_{h_f \to 1} p_2(1-h_m, 1-h_f) = \infty$, and similarly for p_1.

Denote the utility function of an individual by u, where $u(c_i, p)$ is the utility from consuming private good c_i and public good p. We assume that u is twice differentiable with $u_1 > 0, u_2 > 0, u_{11} \leq 0, u_{22} \leq 0$, and we further restrict the class of utility functions below.

The model is static and agents make three decisions. In the *marriage market stage*, men and women choose their partner to maximize their value of being married. The outcome is a marriage market matching function, matching each woman x_f to some man x_m (or single-hood), and a market clearing price. In the second stage, the *household decision problem*, each matched couple chooses private consumption and allocates hours to the various activities – work in the labor market and at home – under anticipation of the labor market outcomes (matching function and wage function). This stage renders both private consumption and public consumption (and thus the time allocation), pinning down individuals' effective types. In the third stage, the *labor market stage*, agents take marriage market and household choices as given and match with firms based on their effective skills so that their wage income is maximized (or equivalently, each firm chooses an effective worker type to maximize profits). This problem pins down a labor market matching function and a market-clearing wage function.

Both matching markets, the labor and marriage market, are competitive (full information and no search frictions) and there is no risk. The two markets and sorting choices therein are linked through the labor supply choice, which can be interpreted as a pre-labor market and post-marriage market continuous investment in 'effective' skills. This link is the key element of our model.

4.2 Decisions

In terms of exposition, we will proceed backwards, so we describe the decision stages in reverse order.

Labor Market. Taking marriage/household choices (in particular, the associated hours choices) as given, firms choose the *effective* worker type that maximizes their profits:

$$\max_{\tilde{x}} z(\tilde{x}, y) - w(\tilde{x}) \tag{1}$$

where $w : \tilde{\mathcal{X}} \to \mathbb{R}_+$ is the endogenous wage function taken as given. Market clearing pins down the labor market matching function $\mu : \tilde{\mathcal{X}} \to \mathcal{Y}$, mapping workers' effective skills to firm types. And if $\tilde{\mathcal{X}}$ is an interval (as it will be the case below) then the first order condition, which gives a differential equation in w, pins down the wage function as

$$w(\tilde{x}) = w_0 + \int_0^{\tilde{x}} z_{\tilde{x}}(t, \mu(t))dt, \tag{2}$$

where w_0 is the constant of integration. It is important to note that $w(\tilde{x})$ is the wage of worker $\tilde{x}$ *per unit of time* (recall our time endowment is normalized to one unit and we will give a specific interpretation to what that time unit is when going to the data below), *not* the worker's earnings.

The optimal matching μ, which features PAM if $z_{\tilde{x}y} > 0$, will match up the exogenous firm distribution, G, and the *endogenous* worker distribution of effective types, $\tilde{N}$, in a measure-preserving way. Importantly, the labor market matching function μ depends on the hours choice (through $\tilde{N}$), which in turn will depend on the marriage partner. Thus, sorting on the two markets is connected.

Household Problem. Each couple (x_f, x_m), taking the partner choice from the marriage market stage as given and anticipating the wage function and labor market matching function (w, μ), solves the following *cooperative household problem*. One partner (here wlog the male partner) maximizes his utility subject to the household budget constraint and a constraint that ensures a certain level of utility for the female partner, by choosing the couple's private consumption and the hours allocation:

$$\begin{aligned} \max_{c_m, c_f, h_m, h_f} \quad & u(c_m, p(1-h_m, 1-h_f)) \\ s.t. \quad & c_m + c_f - w(\tilde{x}_m) - w(\tilde{x}_f) = 0 \\ & u(c_f, p(1-h_m, 1-h_f)) \geq \overline{v}, \end{aligned} \tag{3}$$

where at this stage $\overline{v}$ is taken as a parameter by each household (it will be a function of female skills and endogenously determined in the next stage, the marriage market stage). When solved for all feasible $\overline{v} \in [0, \overline{v}_{max}(x_f, x_m)]$ (where $\overline{v}_{max}(x_f, x_m)$ is the maximum that x_f can obtain when matched with x_m), problem (3) traces out the household's *pareto-frontier*. The solution to this problem yields the hours functions $h_i : \mathcal{X}_m \times \mathcal{X}_f \times [0, \overline{v}_{max}(x_f, x_m)] \to [0, 1]$ and consumption functions $c_i : \mathcal{X}_m \times \mathcal{X}_f \times [0, \overline{v}_{max}(x_f, x_m)] \to \mathbb{R}_+$. That is, for each partner in any matched couple (x_m, x_f) and for a given utility split $\overline{v}$, the problem pins down private consumption $c_i(x_m, x_f, \overline{v})$ and labor hours $h_i(x_m, x_f, \overline{v})$ (and therefore home production $p(1-h_m, 1-h_f)$). Because the household problem is set up in a cooperative way, these allocations are pareto-efficient for any given wage function.

Marriage Market. Anticipating for each *potential* couple the solution to the household problem (h_f, h_m, c_f, c_m) (and implicitly the labor market outcomes (μ, w)), the value of marriage of husband x_m marrying wife x_f is given by the value of household problem (3) and thus:

$$\Phi(x_m, x_f, v(x_f)) := u(c_m(x_m, x_f, v(x_f)), p(1 - h_m(x_m, x_f, v(x_f)), 1 - h_f(x_m, x_f, v(x_f)))),$$

where we now make explicit that v, the marriage market clearing price, is an *endogenous* function of x_f and pinned down in the equilibrium of the marriage market. The marriage market problem for any man x_m is then to choose the optimal female partner x_f by maximizing this value:

$$\max_{x_f} \Phi(x_m, x_f, v(x_f)). \tag{4}$$

The FOC of this problem (together with marriage market clearing) determines the marriage market matching function $\eta : \mathcal{X}_f \to \mathcal{X}_m$, matching each woman x_f to man $\eta(x_f)$ and a transfer function $v : \mathcal{X}_f \to \mathbb{R}_+$, where $v(x_f)$ is the payoff of wife x_f. In particular, the FOC of (4) gives a differential equation in v, which can be solved given the marriage matching function η. In turn, the marriage matching function depends on the complementarities between men's and women's skills in Φ. Particularly tractable matching patterns result if the differential version of the Legros and Newman (2007) condition for positive sorting is satisfied. There is PAM on the marriage market in partners' skills if and only if

$$\Phi_{x_m x_f} \geq \frac{\Phi_{x_f}}{\Phi_v} \Phi_{x_m v}.$$

The marriage market matching function is then determined by market clearing, mapping N_f to N_m in a measure-preserving and – in the case of PAM – increasing way. Note that in principle, individuals can decide to remain single, which – given that there is an equal mass of men and women – will not happen in our baseline model if the value of marriage Φ is positive for all potential couples.

Figure 6 summarizes these decision stages and the endogenous variables they pin down.

4.3 Equilibrium

We now formally define equilibrium.

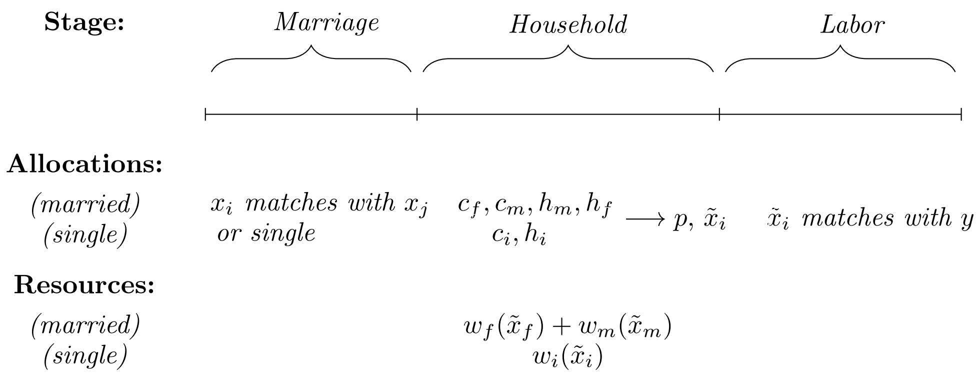
Definition 1 (Equilibrium)**.** *An equilibrium is given by a tuple* $(\eta, v, h_m, h_f, c_f, c_m, \tilde{N}, \mu, w)$ *such that*

1. *given* (η, v) *and* (h_m, h_f)*, the pair* (μ, w) *is a competitive equilibrium of the labor market;*
2. *given* (η, v) *and* (μ, w)*, the tuple* (h_f, h_m, c_f, c_m) *solves the household problem, pinning down* $\tilde{N}$*;*
3. *given* (μ, w) *and* (h_m, h_f, c_f, c_m)*, the pair* (η, v) *is a competitive equilibrium of the marriage market.*

We next define a *monotone* equilibrium, which will be our main benchmark below.

Definition 2 (Monotone Equilibrium)**.** *An equilibrium is monotone if it satisfies Definition 1 and:*

Figure 6: The Decision Stages of Individual $i \in \{f, m\}$ of Skill Type x_i



1. *labor market matching μ satisfies PAM, $\mu(\tilde{x}) = G^{-1}(\tilde{N}(\tilde{x}))$;*
2. *labor hours h_i are increasing in own type x_i and in partner's type x_j, $i \neq j, i, j \in \{f, m\}$, as well as in transfer v;*
3. *marriage market matching η satisfies PAM, $\eta(x_f) = N_m^{-1}(N_f(x_f))$, and v is increasing in x_f.*

Thus, in a monotone equilibrium, there are three additional requirements relating to the three different stages of this model. Importantly, 1. matching on the labor market is PAM; 2. hours worked in the labor market are increasing in own and in the partner's type; 3. matching on the marriage market is PAM and the transfer to the wife is increasing in her type. Under 2. and 3., we obtain that a woman's effective type as a function of x_f, $\gamma_f(x_f) := e(x_f, h_f(x_f, \eta(x_f), v(x_f)))$, is strictly increasing in x_f since then $d\gamma_f/dx_f = e_{x_f} + e_{h_f}[\partial h_f/\partial x_f + (\partial h_f/\partial x_m)(\partial \eta(x_f)/\partial x_f) + (\partial h_f/\partial v)(\partial v/\partial x_f)] > 0$, implying that γ_f can be inverted (and similarly for γ_m). As a result, in a monotone equilibrium, the cdf of effective types has a closed form, where the probability of an effective type $\tilde{x} \leq X$ is given by:[13]

$$\tilde{N}(X) = \frac{1}{2} N_f(\gamma_f^{-1}(X)) + \frac{1}{2} N_m(\gamma_m^{-1}(X)). \tag{5}$$

This discussion highlights an important point: The equilibrium hours function, h_f, not only depends on her own skill type x_f but also on marriage market outcomes: the skill type of her partner, $\eta(x_f)$, as well as the transfer guaranteed to her in the marriage $v(x_f)$; and similarly for men's hours function h_m. As a result, labor supply choices form the link between the marriage market (they are determined by the household and depend on who is matched to whom on the marriage market η) and the labor market (they affect the effective skill cdf $\tilde{N}$ and thus labor market matching μ and wages w).

This interdependence of marriage and labor market sorting is the crucial feature of our model. But

[13] To see this, observe that

$$\tilde{N}(X) = \frac{1}{2}\mathbb{P}[\gamma_f(x_f) \leq X] + \frac{1}{2}\mathbb{P}[\gamma_m(x_m) \leq X] = \frac{1}{2}\mathbb{P}[x_f \leq \gamma_f^{-1}(X)] + \frac{1}{2}\mathbb{P}[x_m \leq \gamma_m^{-1}(X)] = \frac{1}{2}N_f(\gamma_f^{-1}(X)) + \frac{1}{2}N_m(\gamma_m^{-1}(X)).$$

it also makes the problem challenging from a theoretical point of view since we seek the simultaneous equilibrium of two intertwined matching markets, which are related through the time allocation choice. Equilibrium existence requires to show that there is a fixed point in the space of continuous and bounded functions (the hours functions). Even aside from existence, the analysis of equilibrium properties and comparative statics is difficult in light of the equilibrium links; and it is further complicated by the possible feature of *imperfectly transferable utility* (ITU), where the hours functions depend on transfer v.

To gain tractability and intuition into the main mechanisms of the model, we focus on a certain class of models (quasi-linear ones) that yields the *transferable utility* (TU) property.

4.4 Transferable Utility Representation

Under which conditions on primitives will this model with ITU be TU representable? We discuss the two stages where ITU will generally be present: the household problem and the marriage matching problem.

Household Stage. In the collective approach laid out above, the household maximization problem generally features ITU. This implies that the household demand for private and public consumption depend on distribution factors (e.g. the income distribution) and are determined simultaneously: The aggregate split into public and private consumption *cannot* be solved for independently of how private consumption is shared. In turn, if the household problem has the TU property, then the household agrees to maximize the sum of utilities (since in this case the pareto weight on each partner's utility or, equivalently, the Lagrange multiplier of the problem equals one) and aggregate demand for private consumption c and public consumption p can be determined independently of the couple's sharing rule for private consumption and thus independently of $\overline{v}$. As a consequence, the hours functions (h_f, h_m) are independent of $\bar{v}$. In this case, the household acts as a single decision-maker so the collective model collapses to the unitary model.

Marriage Market Stage. Generally, our model features ITU at the marriage market stage as well. In particular, the value of the marriage problem $\Phi(x_m, x_f, v(x_f))$ may not be additively separable in v. However, if the TU property ensues, and – with some abuse of notation – we obtain $\Phi(x_m, x_f) - v(x_f)$. As a result, supermodularity of the value of marriage in (x_f, x_m) is independent of properties of v (meaning the matching problem can be solved by maximizing the total value of marriage, independently of how this value is shared which is captured by v).[14]

The TU representation obtains if the utility function falls into a known class, the *Gorman form* (see Browning, Chiappori, and Weiss (2014) for a detailed discussion).[15] We give two examples of

[14]Note that depending on the precise utility function, we could also have $\Phi(x_f, x_m) - m(v(x_f))$ with some function m. The important point is that v is separable from the remaining part that depends on x_m.

[15]More generally, with more than one private consumption good, the Gorman form of i's utility is given by $u^i(p, c_1, ..., c_n) = z^i(c_2, ..., c_n) + k(p)c_1$, which is *linear in at least one private consumption good*, here c_1, with common coefficient $k(p)$, meaning that the marginal utility w.r.t. c_1 is equalized across partners. Thus, utility can be costlessly transferred between partners at a constant rate using c_1. This finding on the TU representation under the Gorman form mimics a known result from consumer theory that aggregate demand is independent of the income distribution if and only if indirect utility can be represented by functions of the Gorman polar form.

utility functions that can be represented as the Gorman form: (i) The utility function is linear in money/consumption (standard TU case), e.g., the quasi-linear utility $u(c,p) = c + k(p)$, for some function k. (ii) The utility is linear in money after a monotone transformation. Examples of this second case include $u(c,p) = m(c)k(p)$, $u(c,p) = log(cp)$ or $u(c,p) = F(c+p)$ with F strictly increasing. See Appendix B.1 for the details of the TU representation of our model for these examples.

5 Analysis

5.1 The Quasi-Linear Class

We consider the quasi-linear class as our baseline as in this case, we can derive analytical properties of the model, providing insights into the model's mechanism. Specifically, we assume the utility function:[16]

$$u(c_i, p) = c_i + p. \tag{6}$$

5.2 Monotone Equilibrium

Our first objective is to derive conditions under which any stable equilibrium is *monotone* in the sense of Definition 2. The monotone equilibrium will be our benchmark. We call an equilibrium stable if it is robust to small perturbations (see Appendix B.2.1 for a precise definition). We then show in that Appendix that if there exists an equilibrium there is at least one stable one.

Labor Market Stage. As is well-known, if technology z is supermodular, then the worker-firm assignment in the labor market will satisfy PAM, that is matching function μ is increasing. Workers with higher effective skills match up with more productive firms, where $\mu(\tilde{x}) = G^{-1}(\tilde{N}(\tilde{x}))$ is the firm matched to worker $\tilde{x}$. We reiterate that $\tilde{N}$ is endogenous. In turn, the equilibrium wage function is given by (2).

Household Stage. With quasi-linear utility (6), the household problem (3) takes the form:

$$\max_{h_m, h_f} w(\tilde{x}_m) + w(\tilde{x}_f) - \overline{v} + 2p(1-h_m, 1-h_f), \tag{7}$$

where we substituted both the household's budget constraint and the wife's constraint to receive at least utility $\overline{v}$ into the objective function. Clearly, as a consequence of TU, the overall split between public and private consumption (and thus the time allocation choice) can be made independently of how utility is shared, captured by $\overline{v}$. As a consequence, the hours functions (h_f, h_m) (and thus the public good) will only depend on types (x_f, x_m) but no longer on $\overline{v}$.

[16] We here choose the simplest functional form in the Gorman class to reduce notation that obscures the main mechanism and intuition. Generalizations to other utilities in the Gorman class are possible, e.g. $u = F(c+p)$ with $F' > 0$.

The FOCs for h_f and h_m are given by

$$w_{\tilde{x}_f} e_{h_f} - 2p_2 = 0 \tag{8}$$

$$w_{\tilde{x}_m} e_{h_m} - 2p_1 = 0, \tag{9}$$

where p_k indicates the partial derivative of p with respect to its $k's$ argument, $k \in \{1, 2\}$. In any interior solution for the hours choices of partners, each of these FOCs equalizes the marginal benefit of an additional hour in the labor market, captured by the wage gain, with its marginal cost stemming from a reduction in home production that affects both partners (hence the multiplication by 2).

FOCs (8) and (9) give rise to two 'best-response' functions, one of wife's to husbands labor hours and one of husband's to wife's hours. We will focus on equilibria that are stable, i.e. where the husband's best-response function crosses the wife's best-response function from below in the (h_m, h_f)-space.

To characterize under which conditions an equilibrium is monotone, we need to pursue equilibrium comparative statics of the hours functions with respect to skill types, derived from the system of FOCs, evaluated at the equilibrium marriage market matching function η:

$$\frac{\partial h_f}{\partial x_f} = \frac{-\frac{\partial^2 w(\tilde{x}_f)}{\partial h_f \partial x_f}\frac{\partial^2 u}{\partial h_m^2} + \eta'(x_f)\frac{\partial^2 w(\tilde{x}_m)}{\partial h_m \partial x_m} p_{12}}{|H|} \tag{10}$$

$$\frac{\partial h_m}{\partial x_f} = \frac{-\eta'(x_f)\frac{\partial^2 u}{\partial h_f^2}\frac{\partial^2 w(\tilde{x}_m)}{\partial h_m \partial x_m} + 2\frac{\partial^2 w(\tilde{x}_f)}{\partial h_f \partial x_f} p_{12}}{|H|} \tag{11}$$

$$\frac{\partial h_m}{\partial x_m} = \frac{-\frac{\partial^2 w(\tilde{x}_m)}{\partial h_m \partial x_m}\frac{\partial^2 u}{\partial h_f^2} + (\eta^{-1}(x_m))'\frac{\partial^2 w(\tilde{x}_f)}{\partial h_f \partial x_f} p_{12}}{|H|} \tag{12}$$

$$\frac{\partial h_f}{\partial x_m} = \frac{-(\eta^{-1}(x_m))'\frac{\partial^2 w(\tilde{x}_f)}{\partial h_f \partial x_f}\frac{\partial^2 u}{\partial h_m^2} + 2\frac{\partial^2 w(\tilde{x}_m)}{\partial h_m \partial x_m} p_{12}}{|H|}. \tag{13}$$

The denominator in these expressions is given by $|H| = \frac{\partial^2 u}{\partial h_f^2}\frac{\partial^2 u}{\partial h_m^2} - 4p_{12}$ (where the notation indicates this is the determinant of the Hessian of the household problem). By our definition of stability (Definition 3 in the Appendix B.2.1), $|H|$ is positive and moreover $\partial^2 u/\partial h_f^2 \leq 0$ and $\partial^2 u/\partial h_m^2 \leq 0$. Further, we show in Lemma 1 (Appendix B.2.1) that if an equilibrium exists, then there is at least one stable one.

In any stable equilibrium, (10)-(13) are positive if home hours are complementary (p is supermodular), wages are supermodular in hours and skills, $\partial^2 w(\tilde{x}_f)/\partial h_f \partial x_f > 0$ and $\partial^2 w(\tilde{x}_m)/\partial h_m \partial x_m > 0$, and if marriage market matching is PAM, $\eta' > 0$. We will specify these conditions in terms of primitives below and give an interpretation. If (10)-(13) are positive, then the distribution of effective types, $\tilde{N}$, can be pinned down in closed form and is given by (5).

Marriage Market Problem. Given the equilibrium hours functions (h_f, h_m), we obtain the value of marriage Φ as the value of the household problem (7):

$$\Phi(x_m, x_f, v(x_f)) = w(e(x_m, h_m(x_m, x_f))) + w(e(x_f, h_f(x_f, x_m))) - v(x_f) + 2p(1 - h_m(x_m, x_f), 1 - h_f(x_m, x_f)). \tag{14}$$

Complementarities among partners' types in Φ determine marriage market matching patterns. Under TU, $\partial^2\Phi(x_m, x_f, v(x_f))/\partial x_m \partial x_f = \Phi_{x_m x_f}$. If $\Phi_{x_m x_f} \geq 0$, then marriage matching is PAM, $\eta' > 0$.[17]

For consistency, we again adopt the male partner's perspective. Maximizing (14) with respect to x_f, while taking into account that $h_m(x_m, x_f)$ and $h_f(x_m, x_f)$ are already optimized so that they do not respond to further changes in x_f (by the Envelope Theorem), yields:

$$\Phi_{x_f} = 0 \qquad \Leftrightarrow \qquad w_{\tilde{x}_f} e_{x_f} - v_{x_f} = 0 \tag{15}$$

The transfer to the female partner, v, reflects the marginal impact of her type on her wage: When a man chooses a woman, he trades off the marginal benefits of choosing a higher type (which equals the marginal impact on her wage, $\partial w/\partial x_f = w_{\tilde{x}_f} e_{x_f}$) with the marginal costs (which equals the marginal increase in transfer to her, v_{x_f}). The higher the marginal wage return from a more productive female type, the larger is the increase in her compensation within the marriage. The reason why transfer v does not depend on the woman type's contribution to the public good p is that types only indirectly affect the public good production through the hours choice (and since hours were already optimized, the change in x_f has no impact on home production by the Envelope Theorem).

Then, the cross-partial of Φ can be computed from (15) as:

$$\begin{aligned} \Phi_{x_f x_m} &= \frac{\partial^2 w}{\partial x_f \partial x_m} \\ &= \frac{\partial^2 w}{\partial x_f \partial h_f} \frac{\partial \widehat{h}_f}{\partial x_m} \end{aligned} \tag{16}$$

which is positive if wages are complementary in partners' skill types or, zooming in, when wages are supermodular in type and hours and when female labor hours are increasing in her partner's type, $\partial \widehat{h}_f / \partial x_m > 0$. Note that here, the comparative static of female hours with respect to male type is computed for *any* potential couple (x_f, x_m), not just for the ones that form in equilibrium $(x_f, \eta(x_f))$ (we still need to determine η at this stage), and we use the *hat*-notation in (16) to make this distinction from the equilibrium comparative static (13) clear. In turn, wages are supermodular in type and hours if they are convex in effective types since $\partial^2 w/\partial x_f \partial h_f = w_{\tilde{x}_f \tilde{x}_f} e_{h_f} e_{x_f} + w_{\tilde{x}_f} e_{x_f h_f}$, meaning that the marginal wage return to skill increases when putting in more labor hours. The sorting conditions are

[17] Under ITU, $\partial^2\Phi(x_m, x_f, v(x_f))/\partial x_m \partial x_f \geq 0$ is equivalent to $\Phi_{x_m x_f} \geq \frac{\Phi_{x_f}}{\Phi_v} \Phi_{x_m v}$ (Legros and Newman, 2007). To see this, the FOC of problem (4) is given by $\Phi_{x_f} + \Phi_v v_{x_f} = 0$, while $\partial^2\Phi(x_m, x_f, v(x_f))/\partial x_m \partial x_f = \Phi_{x_m x_f} + \Phi_{x_m v} v_{x_f}$. Plugging the FOC into the latter condition gives the known condition for PAM in ITU problems $\partial^2\Phi(x_m, x_f, v(x_f))/\partial x_m \partial x_f \geq 0 \Leftrightarrow \Phi_{x_m x_f} \geq \frac{\Phi_{x_f}}{\Phi_v} \Phi_{x_m v}$. In the quasi-linear class, this becomes $\Phi_{x_m x_f} \geq 0$ since $\Phi_{x_m v} = 0$.

intuitive: There is PAM in the marriage market, so that x_f is matched to $\eta(x_f) = N_m^{-1}(N_f(x_f))$, if labor hours of spouses are complementary in the sense that an individual's labor hours are increasing in partner's type, and if at the same time working more hours boosts the marginal wage return to skill.

Using the formula for $\partial\widehat{h}_f/\partial x_m$ (see Appendix B.2.3), we can re-express $\Phi_{x_f x_m}$ in a more symmetric way, which highlights the importance of the home production function also at the marriage stage:

$$\Phi_{x_f x_m} = 2p_{12}\frac{\frac{\partial^2 w(\tilde{x}_f)}{\partial h_f \partial x_f}\frac{\partial^2 w(\tilde{x}_m)}{\partial h_m \partial x_m}}{|H|}. \tag{17}$$

Complementarity in home production (supermodular p) along with wages that are supermodular in skill and hours induce $\Phi_{x_f x_m} > 0$ and thus PAM in the marriage market.

We can now state our result on monotone equilibrium, expressing the discussed conditions in all three stages of the model in terms of primitives.

Proposition 1 (Monotone Equilibrium)**.** *If p is strictly supermodular and z is weakly convex in effective types $\tilde{x}$ and supermodular, then any stable equilibrium is monotone.*

The proof is in Appendix B.2.3. The first requirement, strict supermodularity of p, ensures that hours in home production (and thus also labor hours) of any two partners are complements. The second requirement, convexity of z in effective types, ensures that wages are convex in effective types, translating into the property that wages are supermodular in skill and hours, $\partial^2 w(\tilde{x}_i)/\partial h_i \partial x_i > 0, i \in \{f, m\}$. It is clear from (10)-(13) and (17) that any stable equilibrium that satisfies these two conditions will feature PAM on the marriage market and labor hours that are increasing in own and partner's type. Together, with the assumption that z is supermodular (which was imposed throughout), inducing PAM in the labor market, all three requirements for monotone equilibrium are satisfied, see Definition 2.

We provide details on stability of equilibrium in Appendix B.2.1, where we show that if an equilibrium exists there is at least one stable one.

5.3 Properties of Monotone Equilibrium

We now give more intuition for the properties of monotone equilibrium in Proposition 1. We also qualitatively link these properties to our stylized facts, before going to our quantitative analysis below.

Comparative Statics in Cross-Section of Couples. We first compute comparative statics in the cross-section of couples, taking equilibrium wage and matching functions as given. That is, how do outcomes change if we move up the type distribution of men or women? For concreteness, we compare two women with $x_f' > x_f$ and analyze the difference in their marriage sorting, household allocation of consumption and hours worked, and labor sorting, as well as the differences in their partners' outcomes.

Corollary 1 (Comparative Statics in the Cross-Section)**.** *Assume a monotone equilibrium. Compare two women with $x_f' > x_f$. Then:*

1. *Marriage Market: woman x'_f has a 'better' partner, $\eta(x'_f) > \eta(x_f)$ (PAM);*
2. *Household Allocation: Both partners in couple $(x'_f, \eta(x'_f))$ work more labor hours and less at home compared to couple $(x_f, \eta(x_f))$ and have thus less public consumption; in turn, both partners $(x'_f, \eta(x'_f))$ have more private consumption and also a higher utility than $(x_f, \eta(x_f))$;*
3. *Labor Market: Both partners $(x'_f, \eta(x'_f))$ have higher effective types and therefore better firm matches compared to $(x_f, \eta(x_f))$ (PAM in $(y, \tilde{x}_i)$ and also in (y, x_i)); and they have higher wages.*

Despite its stylized-ness, the monotone equilibrium of our model can account for several salient features of the data in a qualitative way.

First, there is PAM in the marriage market (in line with Table 1). At the heart of why couples match positively is the supermodular home production function, inducing supermodularity of marriage value Φ in skill types. Based on (17), increasing, say, the female skill x_f increases her wage and thus the marriage value $\Phi(x_f, x_m)$. Additionally augmenting her partner's skill x_m increases the marriage value even further if this change boosts her marginal wage return. Since there is no direct effect of male type on female wage, this effect must work through a change in female labor hours. An increase in the partner's skill increases his labor hours if wages are complementary in skill and hours. This increase in his labor hours implies a decrease in his home hours. Because home hours are complementary among spouses if $p_{12} > 0$, she also reduces her home hours and increases her labor hours. This has a positive effect on Φ_{x_f} if *her* wage is complementary in her skill and labor hours, which is true if z is convex.

Second, labor hours are complementary within couples, meaning that labor hours are increasing in own skill and partner's skills: Increasing, say, the female type, not only makes her own labor hours go up but also induces her partner to work more. As a result, partners' hours co-move. There are two reasons behind this result. First, for a *given male partner type* x_m (for exogenous marriage matching), an increase in the female skill increases her labor hours if z is convex. But this reduces her home hours, inducing her partner to also work less at home and more in the market due to $p_{12} > 0$. As a result, *both* partners increase their labor hours as the female skill improves. Second, this complementarity gets an extra kick under endogenous marriage market sorting: If an increase in her skill x_f leads to a better partner x_m – which is the case under PAM in the marriage market – then this better partner puts more labor hours and less home hours (given wages are supermodular in (x_m, h_m) or given that z is convex). And since $p_{12} > 0$, the wife adjusts hours in the same direction (less home hours and more labor hours), reinforcing the co-movement of hours within a couple. Thus, PAM on the marriage market fuels the complementarity of hours within couples – a feature we saw in the data (Figure 3).

Third, more skilled couples have higher effective types since they work more, and therefore obtain a more productive labor market match. This implies there is PAM not only between firm productivity and effective types $(\tilde{x}, y)$ – which happens here by assumption of supermodular z – but also between firm productivity and raw education (x, y), another feature that is in line with the data (Figure 1 (left)).

Relation between Labor Market and Marriage Market Sorting. The unique feature of our model is the link between labor and marriage market equilibrium and, in particular, labor and marriage sorting. This link becomes most transparent when highlighting how the labor market matching function depends on the marriage market matching function and vice versa.

Start with μ. Consider the total derivative $d\mu(\tilde{x}_i)/dx_i$, which – when positive – indicates PAM on the labor market in (x, y) (where we consider the matching between firms' y and *skills*, not effective skills):

$$\frac{d\mu(\tilde{x}_i)}{dx_i} = \mu' \left(e_x + e_h \left(\frac{\partial h_i}{\partial x_i} + \frac{\partial h_i}{\partial x_j} \eta' \right) \right) \tag{18}$$

Equation (18) illustrates how labor market sorting $d\mu(\tilde{x}_i)/dx_i$ depends on marriage market sorting η'. When marriage market matching is PAM, $\eta' > 0$, then $d\mu(\tilde{x}_i)/dx_i > 0$ (given that hours of spouses are complementary $\partial h_i/\partial x_j > 0$): the matching between (x, y) is given by an increasing function. The intuition is straightforward. PAM on the marriage market induces individuals with higher x_i to have a better partner x_j and therefore to work more hours, which translates into a higher effective type $\tilde{x}_i$ and thus a better labor market match $\mu(\tilde{x}_i)$, compared to when marriage market sorting is not PAM.

In a stylized way, this property of the monotone equilibrium is related to our empirical fact that labor market sorting is stronger for positively sorted couples (Figure 1, right).

Next consider η, highlighting that the link between marriage and labor market sorting also goes in the other direction: the cross partial $\Phi_{x_f x_m}$ (which determines the sign of marriage sorting) depends on labor market matching μ through the wage function. In (17), we can express the complementarities of wages in (h_f, x_f) as $\partial^2 w/\partial h_f \partial x_f = w_{\tilde{x}_f \tilde{x}_f} e_{h_f} e_{x_f} + w_{\tilde{x}_f} e_{x_f h_f} = (z_{\tilde{x}_f \tilde{x}_f} + z_{\tilde{x}_f y} \mu') e_{h_f} e_{x_f} + z_{\tilde{x}_f} e_{x_f h_f}$. Positive labor market sorting $\mu' > 0$ amplifies the convexity of the wage function in effective skills, strengthening the complementarity of wages in skills and hours and thus the complementarity of the wage in partners' types (x_f, x_m), resulting in $\Phi_{x_f, x_m} > 0$. Thus, labor market sorting affects marriage market sorting.

Labor Market Sorting across Gender. An interesting feature of our model is that it can generate a gender gap in labor market sorting: If the home production function is such that women spend relatively more time at home (e.g. if they are relatively more productive at home), then men will be 'better' matched on the labor market compared to women of the same skill. Thus, our competitive model can generate a gender gap in sorting and wages even in the absence of discrimination or differential frictions.

To see this, consider labor market sorting in terms of firm productivity and skills (y, x_i), and how it varies across gender $i \in \{f, m\}$. Consider a man and a woman with $x_f = x_m$. We say that x_m is 'better sorted' than x_f if $\mu(e(x_m, h_m)) > \mu(e(x_f, h_f))$. For each man and woman of equal skills, $x_f = x_m$, men x_m is better sorted if he works more hours on the labor market, $h_m(x_m, \eta^{-1}(x_m)) > h_f(\eta(x_f), x_f)$, which helps rationalizing our finding in the data on the differential sorting of men and women in the labor market (Figure 1, left). Further controlling for hours worked, $h_m(x_m, \eta^{-1}(x_m)) = h_f(\eta(x_f), x_f)$, closes the sorting gap in the model and considerably shrinks it in the data (Figure 5).

5.4 Non-Monotone Equilibrium

The monotone equilibrium captures – albeit in a stylized way – several salient features of the data. Some features of the monotone equilibrium, in particular the complementarity of spouses' hours, may be in contrast to the traditional and more standard view of the household, which relies on specialization. Historically, it is plausible that a different equilibrium was in place, in which partners' hours in home production were substitutable and where positive sorting on the marriage market was less pronounced or sorting was even negative, giving rise to specialization of household members. We capture this different regime in a stylized way by an equilibrium that – with some abuse – we call *non-monotone equilibrium* and we highlight the role played by properties of the home production function. We define a non-monotone equilibrium as the monotone one with two differences. First, there is NAM in the marriage market. And second, labor hours are *decreasing* in partner's type.

Proposition 2 (Non-Monotone Equilibrium)**.** *If p is strictly submodular and z weakly convex in effective types $\tilde{x}$ and supermodular, then any stable equilibrium is non-monotone.*

The proof is in Appendix B.2.5. This result highlights the key role of home production complementarities/substitutabilities in shaping equilibrium. Making hours at home substitutable, $p_{12} < 0$, gives rise to an equilibrium that relies on 'specialization', where a more skilled partner puts more labor hours while own labor hours go down in response. At the same time, the partner spends less time in home production while own home production increases. This specialization within the household is clearly a force towards NAM in the marriage market, which indeed materializes. The reason is that increasing the partner's type pushes own labor hours *down*, hurting own labor market prospects especially for skilled individuals. Skilled individuals then prefer to match with less skilled partners.

The only feature that both equilibria have in common is PAM on the labor market not only in $(y, \tilde{x})$ but, importantly, also in (y, x). This follows from the positive slope of the labor matching function (18).[18]

Thus, complementarity vs. substitutability of home hours shapes equilibrium. In particular, $p_{12} \lesseqgtr 0$ determines whether marriage partners match positively and whether their hours – both at home and at work – are complementary. The monotone equilibrium captures 'progressive' times while the non-monotone one reflects a 'traditional' division of labor. To our knowledge, this mechanism in which home production complementarities are the key determinant of both marriage and labor market outcomes is new in the literature. We now investigate this property of home production and mechanism in the data.

6 Quantitative Model

One advantage of our parsimonious model is that we obtain clean analytical properties that illuminate the model mechanism. To evaluate its quantitative importance, we now augment our model so that it

[18]Further note that the distribution of effective types $\tilde{N}(X) = \frac{1}{2}N_f(\gamma_f^{-1}(X)) + \frac{1}{2}N_m(\gamma_m^{-1}(X))$ will be pinned down just like in the monotone equilibrium since own labor hours are still increasing in own type so that γ_i is still monotone.

can match the data. We do so by implementing some minimal departures from our baseline model in order to preserve its mechanism and underlying intuition.

6.1 Set-Up and Decisions

Our objective is to build a quantitative version of our baseline model that can match key facts of the data while minimally departing from our original set-up. To this end, we augment the model by including shocks in each of the three stages – marriage market, household decision stage, and labor market – so that we capture the following: imperfect sorting and non-participation on both marriage and labor markets as well as heterogeneity in hours choices across couples of the same type. We make three changes:

First, in order to capture some mismatch in both the marriage and the labor market, we augment individuals' education/skill x by an idiosyncratic i.i.d. productivity component ν. We assume that individuals are characterized by *discrete* human capital $s := k(x, \nu) \in \mathcal{S}$, distributed according to cdf N_s, where s takes the role of x from the baseline model. In the labor market, the match relevant attribute of a worker is her effective human capital $\tilde{s} := e(h, s)$ (instead of $\tilde{x}$), whose distribution we denote by $\tilde{N}_s$. So the firm's problem becomes

$$\max_{\tilde{s}} \quad z(\tilde{s}, y) - w(\tilde{s}).$$

Second, we account for heterogeneity in labor supply *within* (s_f, s_m)-type couples and *within* s_i-type singles (and for non-participation) by introducing idiosyncratic labor supply shocks. We denote by $\delta_m^{h_m}$ and $\delta_f^{h_f}$ the idiosyncratic preference of man m and woman f for hours alternative $h_i, i \in \{f, m\}$. In this quantitative version of our model, hours are discrete elements of the choice set $\mathcal{H}$, $h_i \in \mathcal{H}$. Each decision-maker (single or couple) draws a vector of labor supply shocks, one for each discrete alternative h_i. These shocks realize after marriage.

In the household decision stage, partners now maximize utility *plus* labor supply shock:

$$\begin{aligned} & \max_{c_m, c_f, h_m, h_f} && u(c_m, p^M(1 - h_m, 1 - h_f)) + \delta_m^{h_m} \\ & s.t. && c_m + c_f - w(\tilde{s}_m) - w(\tilde{s}_f) = 0 \\ & && u(c_f, p^M) + \delta_f^{h_f} \geq \overline{v}. \end{aligned} \tag{19}$$

where we introduce the notation p^M for the home production technology of couples (Married).

Similarly, the consumption-time allocation problem of singles is given by

$$\begin{aligned} & \max_{c_i, h_i} && u(c_i, p^U(1 - h_i)) + \delta_i^{h_i} \\ & s.t. && c_m - w(\tilde{s}_m) = 0 \end{aligned} \tag{20}$$

where we denote by p^U the home production function of singles (Unmarried).

Third, to accommodate the fact that marriage market matching on *human capital* s may not be perfectly assortative, we introduce an idiosyncratic taste shock for partners' s-types. We denote by β^s_m and β^s_f the idiosyncratic taste of man m and woman f for a partner with human capital $s \in \{\mathcal{S} \cup \emptyset\}$ where $s = \emptyset$ indicates the choice of remaining single. Each individual draws a vector of taste shocks, one for each discrete alternative s. So, individuals in the marriage market value potential partners not only for their impact on the economic joint surplus (as before) but also for their impact on the non-economic surplus (which depends on preference shocks β^U_f or β^U_m). The marriage problem of a man with s_m reads:

$$\max_{s} \quad \Phi(s, s_m, v(s)) + \beta^s_m$$

where the choice of marrying a woman with human capital type $s = s_f$ needs to be weighed against the choice of remaining single $s = \emptyset$ ($\Phi(\emptyset, s_m, v(\emptyset))$ is the economic value of remaining single).

Similar to the baseline model under quasi-linear utility, Φ is the value of the household problem, which captures the *economic* surplus from marriage. Different from the baseline model, due to the introduction of labor supply shocks that have not yet realized at the time of marriage, Φ is the *expected* economic surplus from marriage. The expectation is taken over the different hours alternatives of the couple whose choice probabilities are pinned down at the household stage. See Appendix C for details. This change (introduction of β^U) helps us account for imperfect marriage market sorting between human capital types (s_f, s_m) and for non-participation/single-hood. Since marriage market matching is no longer pure (due to both the discreteness of the match attribute s and the idiosyncratic shocks β^U) $\eta : \{\mathcal{S} \cup \emptyset\}^2 \to [0, 1]$ here denotes the matching *distribution* (as opposed to the matching function).

6.2 Functional Forms

We parameterize our model as follows.[19] The production function on the labor market is given by $z(\tilde{s}, y) = A_z \tilde{s}^{\gamma_1} y^{\gamma_2} + K$ where A_z is a TFP term, (γ_1, γ_2) are the curvature parameters reflecting the elasticity of output with respect to skill and firm productivity, and K is a constant.

For couples, the public good production function is assumed to be CES

$$p^M(1 - h_m, 1 - h_f) = A_p \Big[\theta\big(1 - h_f\big)^\rho + (1 - \theta)\big(1 - h_m\big)^\rho\Big]^{\frac{1}{\rho}}$$

where A_p is the TFP in home production, θ is the relative productivity of a woman, and ρ is the complementarity parameter that determines the elasticity of substitution, $\sigma := 1/(1 - \rho)$ (where $\sigma <$ $(>)1$ indicates that spouses' home hours are strategic complements (substitutes)). In turn, we assume that home production for single women and men are respectively given by $p^U(1 - h_f) = A_p\theta\big(1 - h_f\big)$ and $p^U(1 - h_m) = A_p(1 - \theta)\big(1 - h_m\big)$.

The utility function of individual i is given by $u(c_i, p) = c_i + p$ where $p \in \{p^M, p^U\}$ for spouses

[19] The functional forms for the production function and home production function below will not comply with all the assumptions we made in the analytical model to show that whenever an equilibrium exists there is at least one stable one. However, this is not an issue as the conditions we derived there were *sufficient*, not necessary. And in the quantitative model, we never ran into issues related to existence or stability.

and singles and where we assume that both men and women have the same preferences. We adjust the private consumption of singles by the McClemens factor (singles consume .61 of their wage income relative to couples (Anyaegbu, 2010)).

Human capital as a function of skill and productivity shock is given by $s \propto x + \nu$, where we assume that s is proportional to the sum of observed skill and unobserved productivity.

We allow workers' effective human capital to depend on gender:

$$\begin{aligned} \tilde{s}_f &= \psi s_f h_f \\ \tilde{s}_m &= s_m h_m \end{aligned} \tag{21}$$

where, if a man and a woman have the same (s, h)-combination, $\tilde{s}_f \leq \tilde{s}_m$ if $\psi \leq 1$. We thus allows for a labor market penalty for women that could reflect gender discrimination or productivity differences.

In turn, we measure job productivity y directly in the data, so there is no need for parameterization.

Finally, both marriage taste shocks and labor supply shocks follow extreme-value type-I distributions:

$$\begin{aligned} \beta^s &\sim \text{Type I}(\bar{\beta}^t, \sigma_\beta^t) \qquad \text{for } t \in \{M, U\} \text{ and } s \in \ \{\mathcal{S} \cup \emptyset\} \\ \delta^{h^t} &\sim \text{Type I}(\bar{\delta}, \sigma_\delta) \qquad \text{for } t \in \{M, U\} \text{ and } h^t \in \ \mathcal{H} \end{aligned}$$

where we allow for different preference shock distributions for marriage partners and singles.[20] We normalize the location parameter of both labor supply and marriage market preference shocks to zero, $\bar{\delta} = \bar{\beta}^M = \bar{\beta}^U = 0$. The index t indicates the household type, where we specify the labor supply shocks:

$$\delta^{h^t} = \begin{cases} \delta^{h_i}, i \in \{f, m\} & \text{if } t = U \\ \delta^{h_f} + \delta^{h_m} & \text{if } t = M. \end{cases}$$

That is, when making hours choices, a decision-making unit draws *a single* labor supply shock, δ^{h^t}, that is extreme value distributed. In the case of singles, the decision-making unit is just one person and hence, as is standard, this agent draws a shock for each hours alternative. In the case of spouses however, the decision-making unit is the couple. Therefore, that household draws a single shock for each *joint* time allocation of the spouses (equivalently, the sum of the spouses' shocks is assumed to be extreme-value distributed). We make this adjustment to the standard setting, where *each* individual agent draws an extreme-value shock when making a discrete choice, in order to obtain tractable choice probabilities that help with computation and identification of the model.[21]

[20] Note that without different scales for partner and single choices our parsimonious model (featuring no couple/single-specific parameters) would have difficulty to generate enough singles. Allowing for different scales, however, means that our marriage market resembles a nested logit problem with degenerate (single) nest, associated with known identification issues for the scale of the degenerate nest Hunt (2000). This is why we fix σ_β^U outside of the main estimation below.

[21] Our approach is related to the approach taken by Gayle and Shephard (2019) who use this logic of households drawing one shock for each of the spouses' hours combinations for their numerical solution. Here we make this assumption upfront.

6.3 Model Solution

Appendix C describes the solution of the quantitative model in detail. Our numerical solution consists of solving a fixed point problem in the wage function w (or, equivalently, in the hours functions (h_f, h_m)). For any given wage function, agents make optimal marriage and household choices as well as labor market choices. Labor market choices then give rise to a new wage function that, in equilibrium, needs to coincide with the initially postulated wage function. We implement a search algorithm that iterates between the problem of households and firms, producing a new wage function at each round, and that halts when the wage function satisfies a strict convergence criterion. Our procedure ensures that at convergence, both the labor *and* the marriage market are in equilibrium and households act optimally.

A challenge in our fixed point algorithm is that when partners determine whether a particular hours choice is optimal (which – as discussed – can be understood as an 'investment' in effective skills), they must compare the payoff of this investment with alternative investments.[22] But the competitive wage only determines the price for *equilibrium* investment. In order to obtain the off-equilibrium wages without significantly perturbing the equilibrium wages, we use a *tremble strategy*. Trembling is used in game theory to pin down off-equilibrium choices. Here, we apply this idea to our context. We postulate that a small fraction of agents are tremblers who make a mistake by choosing off-equilibrium hours. This ensures that also off-equilibrium choices will be priced and individuals can compare *all* investment choices when solving the household problem. While trembling is a widely used concept in game theory, we believe the application to matching markets with investment is new.

7 Estimation

We estimate our model in order to assess whether partners' home production time is complementary or substitutable in the data; and how this home production property (and how it evolved over time) shapes empirical sorting patterns on both marriage and labor market, and ultimately household income inequality as well as gender disparities in labor market outcomes in the cross-section and over time.

7.1 The Data

We again use data from the German SOEP combined with information from the dataset of occupational characteristics (BIBB). The challenge is to bridge our static model with the panel data which is intrinsically dynamic and contains life-cycle features. We deal with it as follows. For the estimation of

The reason why it proves useful is that under TU, household problem (19) then becomes:

$$\max_{c_m, c_f, h_m, h_f} u(c_m, p(1-h_f, 1-h_m)) + u(c_f, p(1-h_f, 1-h_m)) + \delta^{h_f} + \delta^{h_m}$$

where we obtain the standard conditional logit choice probabilities for *joint* hours allocations given that the *sum* of shocks $\delta^{h_f} + \delta^{h_m}$ is extreme-value distributed.

[22] This issue is similar to the one in Cole, Mailath, and Postlewaite (2001) who study bargaining in a matching problem with pre-match investment.

worker unobserved heterogeneity (which will be done outside of the model), we exploit the full panel structure in order to make use of techniques that control for unobserved time-invariant characteristics. In turn, for the structural estimation of the model we construct a dataset that features each individual only once while accounting for his/her 'typical' outcomes. To be able to assess the typical outcomes, we focus in our baseline analysis on a restricted time period (2010-2016) so that each individual is captured in only one life-cycle stage and we focus on observations that are not too different in age (25-50). We consider each individual as one observation and generate summary measures (or 'typical' outcomes) of the life-cycle stage we see them in. We then define for each individual the typical occupation (based on a combination of tenure and job ladder features), typical labor hours and typical wage in that occupation, and typical home hours while holding that occupation, as well as the typical marital status. In line with our model, we only consider those individuals who are either married/cohabiting or have never been married and are thus single. We drop divorced and widowed people because they likely behave differently than the singles in our model. Our final sample contains 5,153 individuals, 50% of which are men. See Appendix D.2 for the details of the sample construction.

7.2 Identification

We need to identify 10 parameters and two distributions. We group the parameters into 5 categories and discuss the identification group-wise. We have parameters pertaining to the home production function (θ, ρ, A_p), the production function $(\gamma_1, \gamma_2, A_z, K)$, labor supply shock and marriage preference shock distributions $(\sigma_\delta, \sigma_\beta^M)$ and a wedge (ψ), which could be interpreted as a productivity wedge (reducing women's productivity relative to men) or a discrimination wedge (firms/jobs discriminate against women). Finally, we have the distributions of worker human capital and job productivity (N_s, G). We provide formal identification arguments in Appendix E.1 and summarize the logic here. Our estimation will mostly be parametric. Nevertheless, we consider it useful to lay out non-/semi-parametric arguments in order to understand the source of data variation that pins down our parameter estimates. We will also clarify which parametric restrictions (mainly pertaining to the shock distributions) are important.

The home production function, and thus (θ, ρ, A_p), is identified from choice probabilities for home hours by households of different s-types. The formal identification uses the assumption that the labor supply shock for different hours choices of husband and wife follows a type-I extreme-value distribution.

The production function, and thus $(\gamma_1, \gamma_2, A_z, K)$, is identified from wage data. In our competitive environment, there is a tight link between wages and the marginal product (and thus technology), which allows us to do so. The curvature and TFP parameters, $(\gamma_1, \gamma_2, A_z)$, can be identified from the first derivative of the wage function (the marginal product), following arguments from the literature on the identification of hedonic models (Ekeland, Heckman, and Nesheim, 2004). In turn, the constant in the production function (K) can be identified from the minimum observed hourly wage.

The pair $(\sigma_\delta, \sigma_\beta^M)$ associated with our shock distributions is identified as follows (note that in each distribution we make one normalization choice). In the absence of labor supply shocks, any two couples of the same type (s_f, s_m) would choose the same combination of hours. Hence, the variation in hours choices *by couple type* pins down the scale parameter of the labor supply shock distribution σ_δ. Similarly, in the absence of any preference shocks for marriage partners ($\sigma_\beta^M = 0$), the model would produce perfect assortative matching on the marriage market with $corr(s_f, s_m) = 1$. The extent of marriage market sorting and mismatch identifies the scale parameter of preference shocks for partners, σ_β^M. Note that the standard result in the literature that the scale parameter is not identified separately from the utility associated with the discrete choices (e.g. Keane, Todd, and Wolpin (2011)) does not apply in our context. The reason is that we are able to identify utility in a prior step from household labor supply choices. Importantly, we do not exploit variation in partner choices to identify the utility and therefore, this variation can be used to identify the scale of the marriage shock distribution. The extreme-value assumption of the shock distributions yield tractable choice probabilities that we use in our identification argument.

The productivity or discrimination wedge of women, ψ, is identified by the hourly gender wage gap *conditional on hours and s-type*. If there was no wedge, $\psi = 1$, women and men with the same (s, h)-bundle should receive the exact same wage. A gap can only be rationalized by $\psi \neq 1$.

Finally, the worker and job heterogeneity will be identified directly from the data. We use the empirical distributions of workers' human capital and occupations' productivity for (N_s, G). In sum:

Proposition 3 (Identification)**.** *The production function and women's productivity wedge are identified. If $\delta^{h^t} \sim$ Type I$(0, \sigma_\delta), t \in \{M, U\}$, then the home production function and the scale of labor supply shocks, σ_δ, are identified. If $\beta^s \sim$ Type I$(0, \sigma_\beta^M)$ and Assumption D1 (Appendix) holds, then the scale parameter of the marriage shock distribution, σ_β^M, is identified.*

Our identification result informs the moments we choose to pin down our parameters. The first set of moments (5) relates to the division of labor and to the complementarity of hours within households (labor force participation ratio of women and men, labor force participation of married to single individuals by gender, full time work ratio of women and men, correlation of spouses' home production hours) and is supposed to identify the home production function. The second set of moments (4) relates to the hourly wage distribution (mean, variance, 90-10 and 90-50 inequality) and is meant to identify the production function. The third set of moments (2) concerns the marriage market (correlation of spouses' human capital types and fraction of single men) and is used to identify the marriage shock parameter. The fourth set of moments (4) relates to the hours variation across households of given human capital (female labor force participation rate by couple type and single type, where we select 2 types), which identifies the scale of the labor supply shock. The last set of moments (2) relates to the gender wage gap conditional on the same human capital-work hours of men and women and identifies the female labor wedge. In total we have 17 moments, see Table 14 in Appendix E.1 for the details.

7.3 Two-Step Estimation

We propose a two-step estimation procedure. The first step estimates worker and job heterogeneity as well as the constant in the production function *outside* of the model. In a second step, given the worker and job distributions, we estimate the structural parameters of the primitives *within* the model.

7.3.1 First Step: Estimation Outside the Model

The first step of our estimation concerns the estimation of worker types s (or (x, ν)) and job types y. Except for x (education), these types are not directly observed. Moreover, even though we observe the educational group of a worker we need to translate it into productivity units.

Estimation of Worker Types. Let $ed \in \{hs, voc, c\}$ be the education level of an individual (standing for *h*igh *s*chool or less, *voc*ational training and *c*ollege) and ν be their ability. Based on our theory, we specify an empirical model for hourly wages, namely as a function of effective types (which in turn are a function of education, ability and hours worked). That is, we assume the empirical log hourly wage of individual i at time t is given by

$$\ln w_{it} = \nu_i + \sum_{ed \in \{voc,c\}} \alpha^{ed} x_{it}^{ed} + \beta_1 h_{it} + \beta_2 h_{it}^2 + \beta_z' Z_{it} + \kappa_s + \rho_t + \epsilon_{it} \tag{22}$$

where x_{it}^{ed} are indicators for the education group of an individual (meant to capture x in our model). This indicator is equal to one if individual i belongs to education group ed at time t. Coefficient α^{ed} identifies the 'value' of education ed in terms of log wage units where positive returns to education will be indicated by $0 < \alpha^{voc} < \alpha^c$. While these coefficients indicate the average return to education for all individuals in a certain category, ν_i is a person fixed effect capturing unobserved time-invariant ability, with model counterpart ν. So ν_i is a person specific deviation from the mean wage of their education group. In turn, h_{it} denote the hours worked in the market in a typical week (capturing the time 'investment' in labor productivity in our model). Finally, Z_{it} are additional time-varying controls for the individual, κ_s and ρ_t are state and time fixed effects, and ϵ_{it} is a mean-zero error term.[23]

We thus make use of the dynamic features of the (panel) data to estimate individual unobserved heterogeneity ν. For computational tractability, we divide individuals in each education bin into two groups depending on their ν_i (above and below the median). We compute the level of the low and the high ability based on the average fixed effect in their group, so $\nu_i \in \{\nu_L, \nu_H\}$. Hence, individuals belong to one of six human capital bins (three education types times two ability types). We then order individuals by their human capital $s_i = \alpha^{ed} x_i^{ed} + \nu_i$, giving us a global ranking of worker types. We use the empirical cdf over s_i as our estimate for workers' human capital distribution N_s.

[23] We do not include occupation fixed-effect since in our model, conditional on $\tilde{s}$ (which we control for here by controlling for (x, ν, h)), the wage does not depend on occupation in our competitive equilibrium. But even doing so – which we have done for robustness – does not significantly change the impact of x or ν on the hourly wage.

There are three challenges in implementing (22): First, there may be confounding factors impacting both hours and wages. While we deal with time-invariant unobserved heterogeneity using the panel regression with individual fixed effects, time-varying unobserved heterogeneity, such as productivity shocks or health shocks, could still be problematic. To address this concern, we use an instrumental variable (IV). In our model, there is a systematic relationship between the hours worked of an individual and the hours worked by their partner, so we use the partner's hours as an instrument for own hours. Identification relies on changes in spousal labor hours over time. The identifying assumption is that conditional on the individual fixed effect and education, partner's hours are exogenous in the wage regression and that partner's labor hours impact own wage *only* through own labor hours, which is satisfied in our model.Second, we only observe wages for those who work and labor market participation is not random. To account for selection, we apply a Heckman selection correction. Third, even when we control for selection, using (22) we can only estimate types for those individuals who are employed for at least two periods in our panel. We therefore impute the fixed effects for those who we never observe participating using the *multiple imputation* method.

We provide the details on the sample as well as on the IV, selection and imputation in Appendix E.2. The estimation results of (22) are in Table 9 and the estimated skill distribution in Table 10. And note that regression (22) delivers a causal effect of hours on hourly wages, where we find that increasing weekly hours worked from 30 to 40 increases hourly wages by around 4%.[24]

Estimation of Occupation Types. The empirical counterpart of our model's firms are occupations (we do not observe firms in the SOEP). We here describe how we measure the occupational types y in the data. Our objective is to obtain a one-dimensional ranking of occupations in terms of their task complexity, as a proxy for their productivity. Ideally, we want to use information that does not heavily rely on wages since wages are a function of the match, not just of the occupational type.

Our main dataset is the BIBB (comparable to the O*NET in the US), giving extensive information on task use in each occupation, were we focus on 16 tasks measured on a comparable scale (e.g. prevalence of problem solving, difficult decisions, responsibility for others etc.). We measure the occupations' types in two steps. First, we use a Lasso wage regression to select the important/pay-off relevant tasks. In a second step, we run a principal component analysis (PCA) to reduce the task dimensions further to a single one, where we use the (normalized) first principal component as our one-dimensional occupation characteristic y. It is important to note that we use the wage regression only to select the relevant tasks but we do not use the estimated coefficients. See Appendix E.3 for the details of this approach and for the alternative approaches that we pursued for robustness and which have led to similar results.

Estimation of Constant in Production Function. We assume that the constant in the production function is not shared between workers and firms but accrues to the worker in form of a

[24]Our estimated effect is smaller but comparable to Aaronson and French (2004) (who also use a panel regression with fixed effects with an IV for hours and find that increasing hours from 20 to 40 per week increases the hourly wage by 25%); and to Bick, Blandin, and Rogerson (2020) who find that the effect of increasing hours from 30 to 40 per week increases hourly wages by 11% for elderly men in the U.S.

minimum hourly wage (the wage of someone with the lowest human capital who will be matched to the lowest productive occupation $y = 0$). This way, we obtain $K = 6.32$.

7.3.2 Second Step: Internal Estimation

There are nine remaining parameters of the model, $\Lambda \equiv (\theta, \rho, A_p, \gamma_1, \gamma_2, A_z, \psi, \sigma_\delta, \sigma_\beta^M)$. They are disciplined by the moments described above, which are chosen based on our identification arguments discussed in Section 7.2. To estimate those parameters, we apply the method of simulated moments ((McFadden, 1989); (Pakes and Pollard, 1989)). For any vector Λ of structural parameters the model produces the 17 moments outlined above, $mom_{sim}(\Lambda)$, that will also be computed in the data, mom_{data}. We then use a global search algorithm to find the values of parameters that minimize the distance between simulated and observed moments. Formally, the vector $\hat{\Lambda}$ of ten estimates solves

$$\hat{\Lambda} = \arg\min_{\Lambda} \quad [mom_{sim}(\Lambda) - mom_{data}]'\mathcal{V}[mom_{sim}(\Lambda) - mom_{data}]$$

where $\mathcal{V}$ is specified as the inverse of the diagonal of the covariance matrix of the data.

7.4 Results and Fit

We report the parameters that we fixed outside of the structural estimation in Table 13 (Appendix E.4.1): the minimum hourly wage of 6.32 Euros (K), and several normalizations pertaining to the shock distributions, $(\bar{\delta}, \bar{\beta}^M, \bar{\beta}^U, \sigma^U)$. The estimated parameters are in Table 2. The home production function indicates that women are significantly more productive at home than men ($\theta = 0.78$). The (large) differences in labor force participation and full time work across gender call for this relatively high female productivity at home. Importantly, our estimates indicate that spouses' hours at home (and therefore also in the labor market) are complements with $\rho = -0.54$, pushing the model towards the *monotone equilibrium* of the baseline model. The moment indicating a positive correlation of spouses' home hours informed this complementarity. In terms of labor market production function, our estimates indicate that it is concave in both the workers' effective skill as well as the jobs' productivity ($\gamma_1 < 1, \gamma_2 < 1$). Labor market TFP A_z is estimated to be higher than home production TFP A_p. The empirical gender wage gap *conditional* on hours and human capital calls for a female productivity/discrimination wedge, which we estimate as $\psi = 0.84$. This implies that women are 16% less productive than men for a given type and choice of hours.

Finally, regarding the marriage preference and labor supply shocks, our estimated scale parameters ensure that we match the fraction of singles, the extent of mismatch in the marriage market and the heterogeneity in hours choices by households of the same human capital type. We also report the standard errors of the estimates.[25] The last column presents our sensitivity analysis (Andrews,

[25] The covariance matrix of the estimator is computed as the sandwich matrix $Var = [D_m'\mathcal{V}D_m]^{-1}D_m'\mathcal{V}C\mathcal{V}'D_m[D_m'\mathcal{V}D_m]^{-1}$, where D_m is the 10×17 matrix of the partial derivative of moment con-

Gentzkow, and Shapiro (2017) and Gayle and Shephard (2019)) where we report the most important moments that explain 50% of the impact on each parameter in estimation.[26] Our sensitivity analysis is in line with our identification arguments. For example, the correlation of spouses' hours M_5 is an important moment disciplining the home production complementarities, ρ; or, the female productivity wedge, ψ, is most related to the within-type gender wage gaps M_{12} and M_{13}.

Table 2: Estimated Parameters

Parameter	**Estimate**	**s.e.**	**Top Sensitivity Moments**
Female Relative Productivity in Home Production, θ	0.78	0.02	M11, M2, M9
Complementarity Parameter in Home Production, ρ	-0.54	0.22	M5, M3, M13
Home Production TFP, A_p	41.38	0.98	M11
Elasticity of Output w.r.t. $\tilde{s}$, γ_1	0.59	0.05	M2, M11, M8, M9
Elasticity of Output w.r.t. y, γ_2	0.16	0.07	M2, M8, M11
Production Function TFP, A_z	42.33	2.28	M2, M11, M8
Female Productivity Wedge, ψ	0.84	0.03	M13, M12
Labor Supply Shock (scale), σ_δ	7.51	0.40	M2, M11
Preference Shock for Partners (scale), σ_β^M	0.19	0.02	M11, M2, M13, M10

Notes: s.e. stands for Standard Errors. *Top Sensitivity Moments* reports the most important moments explaining 50% of the total impact on each parameter in estimation, based on our sensitivity measure (details in footnote 26). $M1$, ..., $M17$ denote the 17 moments targeted in estimation, see Table 15, Appendix E.4.

Figure 7 summarizes the fit between model and data moments, where we plot all 17 moments (red dots indicate the level of these moments in the model) as well as their blue confidence interval of the corresponding data moment (computed from a bootstrap sample). We re-scaled some moments ($M_6 - M_9$) to be able to plot them all in the same graph. Table 15 in Appendix E.4 reports the fit in detail and indicates the moments corresponding to numbers 1-17. Our model achieves a good fit with the data, with nearly all model moments lying in the confidence interval of their data moments.

7.5 Model Validation

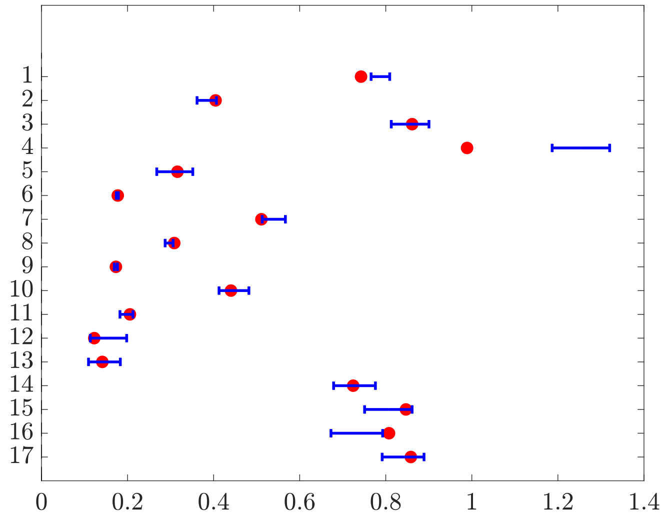
Our model achieves a good fit with the data along the moments we target. In this section, we investigate the model fit regarding un-targeted features of the marriage and labor market. Our identification strategy mostly relied on targeting *average* moments instead of moments by detailed heterogeneity. We now assess whether our model can reproduce the rich patterns of marriage and labor market sorting and the link (hours) between them that we documented in Section 3.2 – none of them being targeted.

Marriage Market Sorting. Table 16, Appendix E.4, displays the matching frequency of marriages by three education types (low, medium and high) in data and model. The main panel indicates the frequencies of different types of couples while the bottom row (right column) indicates the frequencies of

ditions with respect to each parameter at $\Lambda = \hat{\Lambda}$ and C is the covariance matrix of the data moments.

[26] We compute the sensitivity of each parameter to the moments as $|Sensitivity| = |-\left[D_m' \mathcal{V} D_m\right]^{-1} D_m' \mathcal{V}|$, defined by Andrews, Gentzkow, and Shapiro (2017), see footnote 25 for notation.

Figure 7: Model Fit: Model Moments (red) with Data Confidence Intervals (blue)



single men (women) by education. Data frequencies are in parentheses. Note that in our estimation, we only targeted the *average* correlation of couples' *human capital* types (i.e. s-types), as s is the relevant matching characteristic on the marriage market in our model. We did not target marital matching on *education*, x, especially not the detailed matching frequencies. Nevertheless, the model matches well the observed marriage frequencies by education type: A considerable fraction of couples matches along the diagonal, while the off-diagonal cells indicate that mixed couples (especially high-low couples) are rare – a sign of positive assortative matching in education. Our model also captures that medium educated men and women are most likely to be single, where we only targeted the average fraction of male singles.

Labor Market Market Sorting. Despite the fact that we do not target labor market sorting in our estimation, our model matches the data reasonably well. We report in Figure 8, left panel, the labor market matching function for men (blue) and women (red) in the model (solid) and data (dashed). It is given by job productivity y as a function of individuals' human capital s. Our model captures that labor market sorting is PAM and that men are better matched for any given level of human capital.

Relationship between Labor Market Sorting and Marriage Market Sorting. We documented in Section 3.2 a strong link between labor market and marriage market sorting in the data, where labor market sorting is maximized for individuals who are well matched in the marriage market. Figure 9 (left panel), which compares data and model, indicates that our model reproduces this pattern. Note that consistent with our quantitative model (and in contrast to Section 3.2), we here proxy marriage market sorting by spouses' differences in *human capital* s-types (as opposed to differences in education), also in the data. Similarly, labor market sorting is measured by the correlation of (s, y) (instead of (x, y)).

Hours as the Link between Marriage Market Sorting and Labor Market Sorting. The key feature of our model is that marriage and labor markets are linked in equilibrium, namely through the household's time allocation choice. Here we show that the model replicates salient features of the data according to which hours are associated with both marriage and labor market outcomes.

Figure 9, right panel, shows that both in data (dashed) and model (solid), the correlation of spouses' home production hours is highest when marriage market sorting is strongest (i.e. when partners' human capital is equalized $s_f \approx s_m$, around the vertical line at 'zero'). This is a natural prediction of our model: Spouses of similar human capital can act more on the hours complementarity in home production and better align their hours relative to couples with large human capital differences who tend to specialize.

Finally, households' time allocation choices in our model are also related to labor market sorting. When re-plotting the labor market matching function from Figure 8 (left) but *controlling* for hours worked, then the difference in sorting across gender nearly vanishes in the model (solid) and also in the data (dashed), Figure 8, right. This is indicated by similar labor market matching functions for both genders relative to the left panel. In sum, the monotone equilibrium of our model – driven by home hour complementarity – very much fits the rich empirical patterns of marriage sorting, labor sorting, hour allocations and their interconnections.

Figure 8: Labor Market Matching Function, Original (left) and with Hours Partialled Out (right)

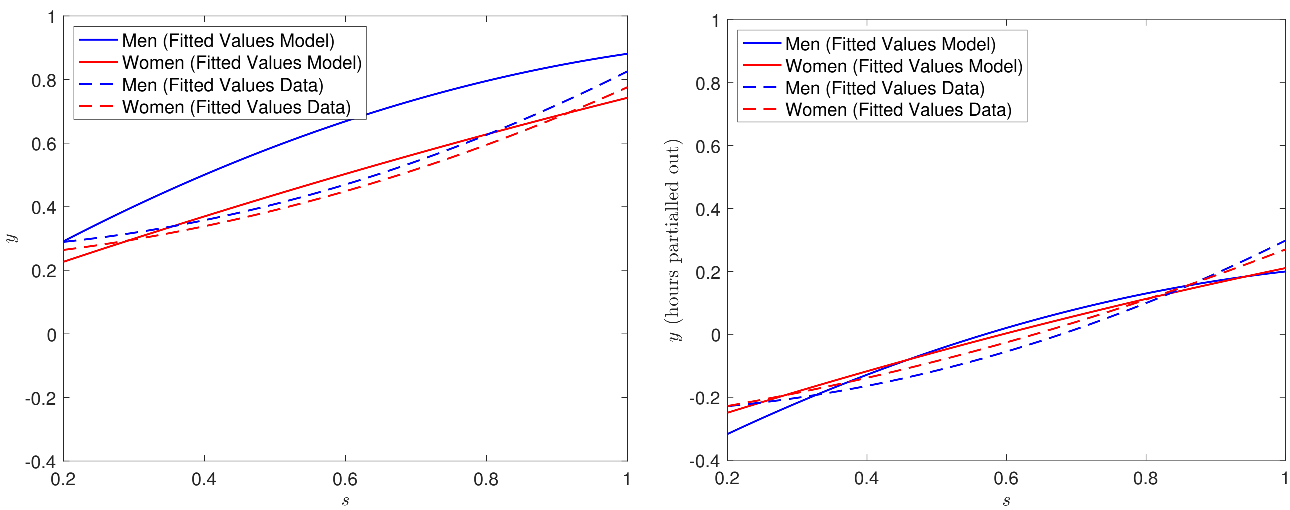
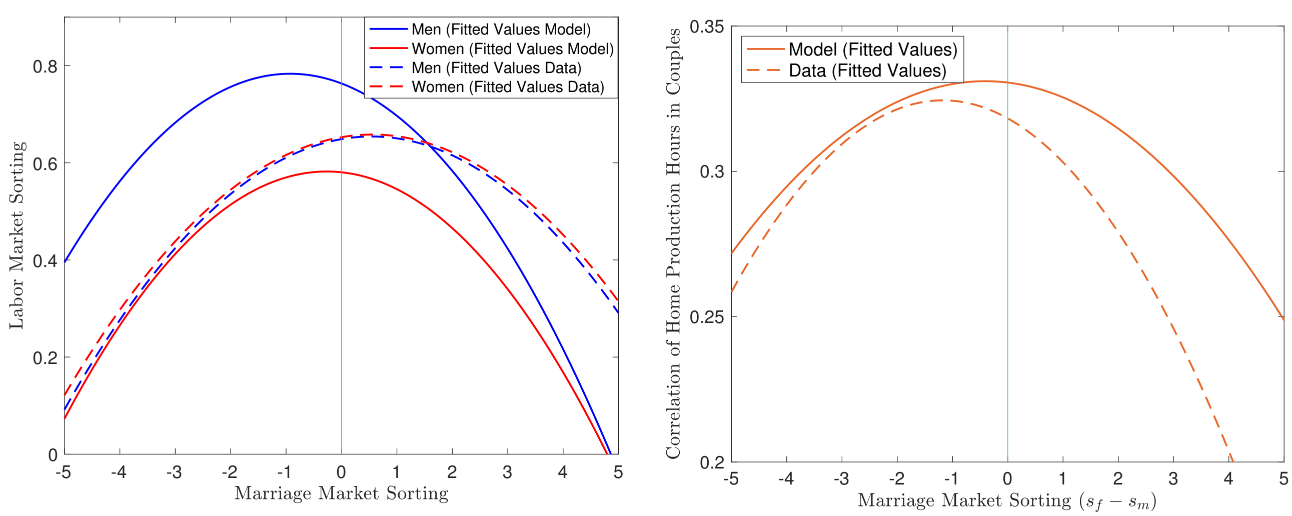


Figure 9: Labor Market Sorting and Marriage Market Sorting (left); Home Production Hours and Marriage Market Sorting (right)



8 Application: The Drivers of Inequality

In our main quantitative exercise, we use our model to shed new light on the sources of gender disparities in the labor market and household income inequality. Our analysis focusses on two different contexts: Today's Germany (Section 8.1) and Germany over time (Section 8.2).

8.1 Inequality Through the Lens of our Model

We first focus on a recent period, 2010-2016. Throughout we concentrate on West Germany to facilitate the over-time analysis below. We analyze the gender wage gap and income inequality within and between households through the lens of our model. We start with investigating the performance of our model in reproducing the observed inequality. We then analyze comparative statics with respect to the model's key determinants of inequality. Finally, to highlight how the interplay between marriage and labor market affects inequality, we shut down sorting in one market and replace it by random matching. Understanding how sorting in the two markets mitigates or amplifies inequality will help us better understand the subsequent exercises on the evolution of inequality in Germany across time.

8.1.1 Inequality in Data and Model

To assess the extent of inequality in data and model, we focus on two measures: the gender wage gap and household income variance, including its decomposition into between and within household components. These statistics are reported in Table 3.[27] While our model underestimates the level of the income variance (83 in the model versus 98 in the data), we capture the split of within- and between-household inequality well (54-46 split in the model vs. 50-50 in the data).[28] Moreover, regarding inequality within households, the model captures the gap between men and women accurately: It predicts that the share of female wages in overall household wage income is 31% (in the data it is 33%). Last, our model produces a sizable unconditional gender wage gap (23%), slightly overestimating the observed gap (20%).

Table 3: Gender and Household Inequality

	Model	**Data**
Total Household Wage Variance	83.4822	97.7822
Within Household Wage Variance	38.0682	49.1806
... share in total variance	0.4560	0.5030
Between Household Wage Variance	45.4140	48.6017
... share in total variance	0.5440	0.4970
Share of Female Wage in Overall HH Wage Income	0.3144	0.3285
Gender Wage Gap	0.2314	0.1973

[27] Our measure of the gender wage gap includes all individuals in the sample, singles and in couples, conditional on emplyment. In turn, both the female's share in household income and the total income variance and its decomposition are computed based on the sample of couples. All couples are included, independent of employment status.

[28] The within-component is measured by the variance of wages within a couple, averaged across all couples. The between-component is measured as the variance of the average income of each couple.

Our model is thus able to reproduce key features of observed inequality that were not targeted in estimation. This validation suggests that our model is an adequate tool through which we can investigate the main drivers of inequality, and understand the sources of changing inequality in Germany over time.

8.1.2 Comparative Statics

To highlight the key forces behind inequality we begin with comparative statics exercises in our estimated model. This will help us understand changes in inequality over time through the lens of our model below. The gender wage gap in our model is driven by (endogenous) gender differences in hours worked, and (exogenous) differences in human capital and labor productivity. In turn, differences in hours worked across gender are mostly impacted by the relative productivity of women at home, θ, the home production complementarity, ρ, and the labor market productivity wedge ψ. Clearly, if both $\theta = 0.5$ (men and women are equally productive at home) and $\psi = 1$ (men and women are equally productive in the labor market/women are not discriminated against), this would eliminate the gender wage gap entirely. But given that $\theta \neq 0.5, \psi \neq 1$, the level of complementarities in home hours, ρ, is a third key determinant of gender inequality in our model. We are interested in the comparative statics effects of these parameters on the gender wage gap, and and also on intra and inter-household income inequality.

One can think of several policies and technological changes that impact these parameters. Anti-discrimination policies (such as gender quota or equal pay policies) can affect ψ. Childcare availability and parental leave policies (such as "daddy months") might affect θ. As the child-related tasks women were expected to perform at home are performed by someone else, gender differences in productivity of home hours are likely to decline. Further, changes in home production technologies that facilitate the house chores women traditionally specialized in (Greenwood, Guner, Kocharkov, and Santos (2016), Greenwood (2019)), affect θ and ρ. For instance, if partners' time is inherently more complementary in childcare relative to other home production activities (as we show in Appendix A.2), the outsourcing and reduction of time dedicated to non-childcare activities shift the composition of home production towards more complementary tasks, affecting ρ. Finally, an increase in the returns to investment in children (Lundberg, Pollak, and Stearns (2016)) could also impact ρ, as parents make more *joint* investments.

The Effect of ρ. We first investigate a change in home production technology that increases the complementarities in spouses' home hours. Recall that our estimate $\rho = -0.54$ indicates home hours are strategic complements. We are interested in the effects on inequality when ρ becomes even more negative, and in the underlying mechanism (marriage sorting, hours, labor sorting).

Figure 10, first row, plots the effect of ρ on different inequality measures: the gender wage gap (panel a), within and between household income inequality (panel b) and overall household income inequality (panel c). It shows that a decline in ρ (moving from the right to the left on the x-axis) *decreases* the gender gap significantly. Starting from our estimate $\rho = -0.54$ and decreasing this parameter to -2 decreases the gender wage gap by almost 13%. This is due to a direct of complementarities on hours and several indirect effects through sorting: First, because complementarity in home production (and thus also in labor hours) among partners increases, the desire for positive sorting on the marriage market is stronger, resulting in more positive assortative matching, panel d. Both, increased marriage sorting (indirectly) and stronger complementarities in home production (directly) induce spouses to better align their hours. Women increase their labor hours while men decrease theirs, leading to a smaller gender gap in labor hours, panel e, which puts downward pressure on the wage gap. Moreover, because women 'improve' a sorting-relevant attribute (work hours) relative to men, the gender gap in labor market sorting declines, panel f, reducing the gender wage gap even further.

How does this change in home production complementarities affect household incom inequality? Figure 10c shows that overall income inequality *declines* with stronger complementarities. This decline is driven by the decrease in within-household inequality (mirroring the decline in the gender wage gap), which dominates the increase in between inequality that stems from stronger marriage sorting.

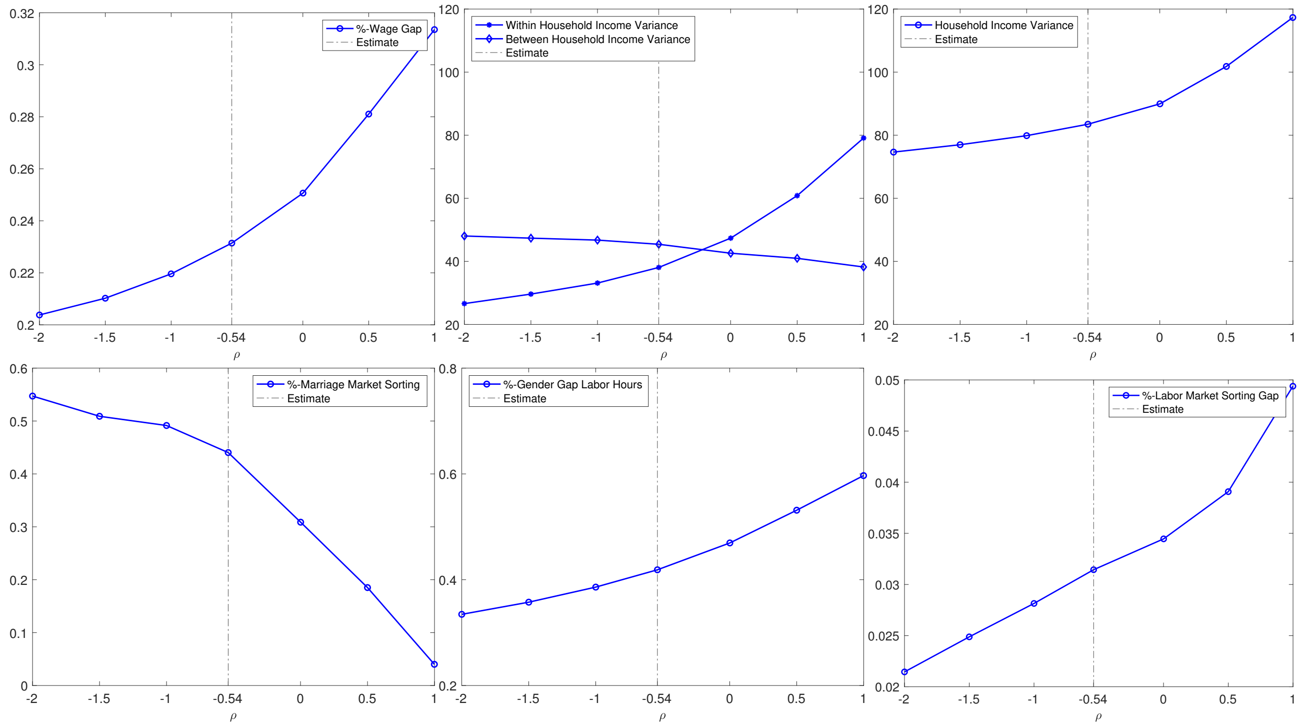
This comparative statics exercise also highlights what features of the data call for $\rho < 0$, pushing the economy towards the monotone equilibrium: If it was the case that $\rho > 0$ (i.e. if hours were strategic substitutes), then marriage market sorting would be random, see Figure 10d where marriage sorting becomes drops significantly as ρ becomes positive. And the correlation of spouses' home hours would tend to zero (not plotted here) even though in the data it is positive (equal to 0.32, M_4, Table 15).

THE EFFECT OF θ. Next we are interested in the effect of women's relative productivity at home on our inequality measures. Clearly, the gender wage gap is increasing in θ, Figure 15a in Appendix F.1. Indeed, eliminating the gap in home productivity (from estimate $\theta = 0.78$ to $\theta = 0.5$) would cut the gender wage gap by almost half. The mechanism is as follows: increasing female home productivity decreases the incentive for positive marriage sorting (panel d) and pushes toward household specialization with a large gender gap in labor hours (panel e). This negatively affects women's wages directly, as well as indirectly through a larger labor market sorting gap (panel f). Interestingly, overall household income inequality increases as women become more productive at home, Figure 15c. Here, this is driven by an increase in within-household inequality (mimicking the evolution of the gender wage gap), which dominates the decline in between-household inequality driven by a drop in marriage market sorting.

THE EFFECT OF ψ. Last, we analyze the comparative statics of the female labor market wedge. Figure 16a in Appendix F.1 shows that eliminating the wedge (increasing ψ from our estimate $\psi = 0.84$ to $\psi = 1$) would reduce the gender gap by about 25%. There is a direct positive effect of ψ on female productivity and thus wages but also several indirect effects: First, wife's labor hours increase in productivity ψ relative to the husband's, reducing the gender hours gap, panel e, and thus gender wage gap. Second, the reduction in the gender hours gap leads to a decline in the labor market sorting gap, panel f, further curbing the gender wage gap; third, smaller gender disparities on the labor market are associated with an increase in marriage market sorting, panel d, since in a world where men and women are more equal the motive for positive sorting strengthens. The increase in marriage sorting reinforces the drop in both hours and labor sorting gaps, further dampening the gender wage gap.

In Figure 16b and c, we study the effects of ψ on the variance of income, both within and across

Figure 10: a. Gender Wage Gap, b. Household Income Variance, c. Income Variance Decomposition, d. Marriage Market Sorting, e. Gender Gap Labor Hours, f. Gender Gap Labor Market Sorting.



households. An increase in female productivity ψ leads to lower within-household inequality (moving in the same direction as the gender wage gap), but higher between-household inequality, driven by the increase in marriage sorting. These countervailing forces make the impact on overall inequality ambiguous.

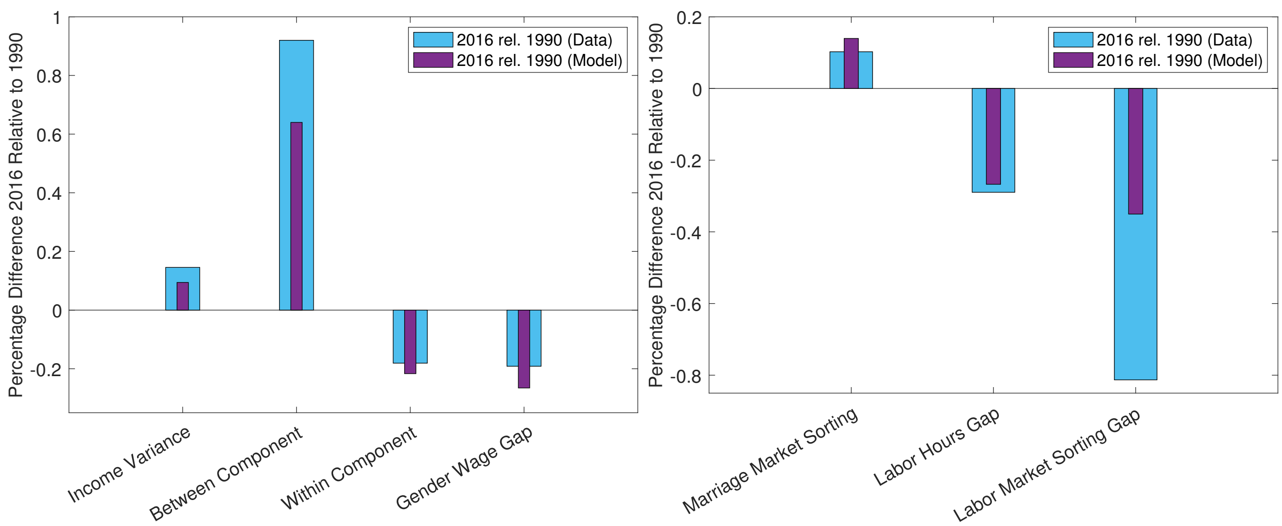
We derived several insights: First, eliminating asymmetries in productivity across gender (whether at home through $\theta \to 0.5$ or at work through $\psi \to 1$) reduces the gender wage gap. But this is not the only way to reduce gender disparities: an increase in home production complementarity (decrease in ρ, the key parameter of our model in shaping equilibrium) has qualitatively similar effects. Second, a decline in the gender gap tends to go hand in hand with a decline in the labor hours gap and in the labor market sorting gap and with an increase in marriage market sorting. Third, while the effect of these parameters on overall income inequality depends on the specific exercise, in all cases, the gender gap co-moves positively with within-household inequality but negatively with between-household inequality.

8.2 Inequality Over Time

Over the last few decades, inequality in Germany has changed significantly. Household income variance is 15% higher today than 30 years ago, which masks diverging trends of within-household inequality (which declined by 18%) and between-household inequality (which increased by 92%). In turn, the gender wage gap declined by almost 20% over this period, see the turquoise bars in Figure 11, left panel. At the same time, both the marriage and the labor market have undergone notable changes. Marriage market PAM increased by 10%, while gender disparities in labor outcomes declined. The gender gap in labor hours fell by almost 30% and the gender gap in labor market sorting by almost 60% (turquoise bars, right panel).

We are interested in how our model rationalizes these trends in a unified way. We first investigate how the model primitives have changed over time and how these changes affected inequality. We then ask whether the shifts in labor and marriage sorting amplified or mitigated inequality.

Figure 11: Inequality Changes Over Time (left); Sorting and Hours Changes Over Time (right).



To assess over-time changes in inequality with our model, we compare our estimation from 2010-2016 with the re-estimated model in an earlier period, 1990-1996.[29] For re-estimation on the 1990-1996 sample, we re-assess the skill and job distributions for the earlier period, and re-estimate all parameters except those pertaining to the labor supply preference shock, which we set to the level of our current period benchmark (Section 7.4). This is to tie our hands and force the model mechanism to explain the data, as opposed to giving changes in shock distributions a too prominent role.[30] The model fit along targeted moments is in Table 17 in Appendix F.2, which also indicates that both labor and marriage market underwent statistically significant changes over time (column 5). Regarding the un-targeted inequality moments of the data (Figure 11, left panel), the model replicates the over-time changes well, where the turquoise bars indicate changes in the data and the purple bars changes in the model.

To understand the driving forces behind the inequality changes, we now zoom further into the model. We compare the parameter estimates for both periods in Table 18, Appendix F.2. There have been significant changes in home production with today's Germany being characterized by a lower θ (drop from 0.88 to 0.78, meaning men became relatively more productive at home over time) and lower ρ (drop from -0.16 to -0.54, indicating increased complementarity in spouses' home hours); and a narrowing labor productivity wedge ψ (increase from 0.76 to 0.84, raising relative female productivity). These changes indicate that Germany has become a more 'progressive' society and economy over the

[29] In the GSOEP, 1990 is the first year that features the time use variables used in our analysis of the later period.
[30] We did have to free up the scale of marriage shocks in 1990-1996 in order to give the model a chance to match the data.

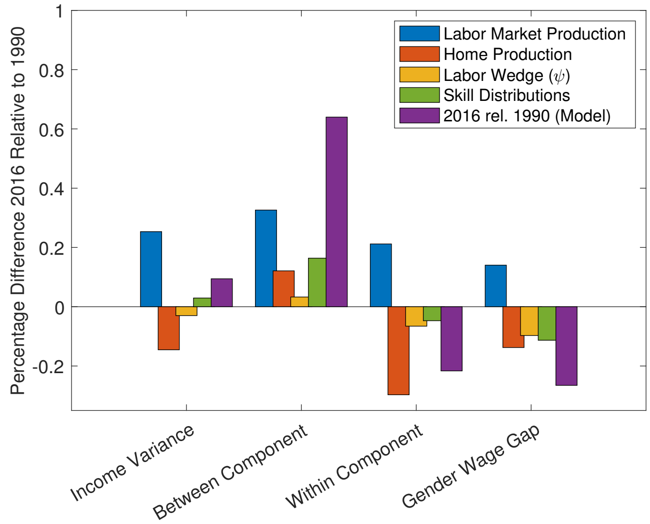
last three decades, with more gender equality at home and work. In turn, the labor market technology has become more convex in effective skills, resembling skill-biased technological change, and has a higher TFP than before.

How much of the documented changes in inequality can be explained by these changes of model parameters? Figure 12 provides a detailed decomposition and also re-displays, for comparison, the purple bars (from Figure 11, left panel) of shifts in the inequality outcomes when *all* parameters change from the early to the later level. The remaining bars give the percentage change in inequality outcomes relative to the early period (1990-1996) if one parameter group changes *in isolation* while the others remain fixed at the 1990-1996 level: we consider changes in the labor market production function, blue, home production, orange, labor productivity wedge, yellow, and human capital distribution, green.

In line with our comparative statics exercises, the documented changes in home production technology reduced gender disparities (gender wage gap and within-household inequality) as well as overall household inequality, while they fueled between-household inequality. Figure 12 (orange bars) shows that these effects are also quantitatively sizable. If only home production had changed over time, within-household inequality would have declined by 30% (accounting for more than the observed change) and the gender wage gap by 14% (accounting for more than 70% of the observed drop). Home production changes were thus the biggest driver behind the decline in gender inequality. In turn, home production shifts put upward pressure on between-household inequality, accounting for almost 20% of the observed increase. But since this effect was dominated by the downward pressure on within inequality, the net effect of technological change in home production on overall household income inequality was negative. Splitting home production further into the contributions of our model's key parameters θ and ρ (Figure 17 in Appendix F.2) reveals that changes in relative productivity parameter θ were the main driver behind the inequality shifts (accounting for around 2/3 of the total home production effects), while the impact of complementarity parameter ρ was smaller but still sizable (around 1/3 of the effects). The effects of changes in the labor market wedge ψ on inequality (yellow bars, Figure 12) – while qualitatively similar to those of home production technology – were quantitatively smaller. Finally, changes in labor market technology (blue bars) fueled inequality across the board, significantly pushing up household income variance (through both between- and within-components) and preventing gender inequality from falling even further. Thus, technological change in home production and in labor market production have pushed inequality, and especially gender disparities, in opposite directions.

Our comparative statics in Section 8.1.2 clarify the mechanism of why the estimated changes of home production technology and the labor wedge push towards more gender equality. Both changes induced women to work more (leading to a decline in the gender gap of labor hours), which in turn made women sort relatively better on the labor market (reducing the gender gap in labor sorting). More gender parity in labor market outcomes in turn strengthened the desire for positive sorting in marriage, reinforcing the push towards more equal labor (and home) hours across gender. Figure 11, right panel, demonstrates that these shifts were not only present in the model (purple bars) but also in the data

Figure 12: Mechanism Behind Inequality Changes



(turquoise bars). Our evidence and estimates suggest that Germany underwent significant changes over the last decades towards an equilibrium that resembles the monotone equilibrium from our theory, with stronger home production complementarities and, consequently, increased marriage sorting as well as stronger co-movements of spouses' hours, labor market sorting and wages.

We end by returning to the key feature of our model: equilibrium sorting in both labor and marriage markets. We assess the quantitative role of changes in marriage and labor market sorting for inequality shifts. Between 1990-96 and 2010-16, positive marriage sorting has increased by around 10% and positive labor sorting by 8%. We compute the elasticity of each inequality outcome with respect to sorting in each market as (%ΔInequality)/(%ΔLabor Sorting) and (%ΔInequality)/(%ΔMarriage Sorting). *Inequality* refers to one of our four inequality outcomes (gender wage gap, household income variance, within/between component) and the percentage change is computed between the baseline model in 2010-2016 and the counterfactual model. This counterfactual inputs the estimated parameters from 2010-2016 but keeps either labor market or marriage market sorting constant at the past period's (1990-1996) level.[31] This way we isolate the role of the observed changes in sorting for inequality shifts.

Table 4: Elasticity of Inequality with Respect to Sorting

	Gender Wage Gap	**Income Variance**	**Within Variance**	**Between Variance**
Marriage Market Sorting	-0.0036	0.0117	-0.1173	0.1234
Labor Market Sorting	-1.2349	0.0227	-0.7356	0.7351

Table 4 reports the elasticities. We find that *both* marriage and labor sorting have had mitigating impact on gender inequalities (wage gap and within household inequality) and have amplified over-

[31]To implement the past marriage sorting in the counterfactual model, we adjust σ_β^M. In turn, to implement the past labor sorting, we take into account labor market matching $\mu(\tilde{s})$ from the past period when computing the wage function.

all inequality and between-household inequality. For instance, a 1% increase in marriage sorting has decreased within-household inequality by 0.117%, while it increased between-household inequality by 0.123%. The elasticity of the gender wage gap is also negative, albeit smaller. Stronger marriage market sorting generated more balanced labor market outcomes – in hours, sorting, and pay – across gender.

The effects of changes in labor sorting on inequality are even larger. A 1% rise in labor market sorting has increased the overall income variance by 0.023% and the between-household income variance by 0.735%. In turn, a 1 % increase in labor sorting has reduced the gender wage gap by 1.235% and within inequality by 0.736%. Surprisingly at first sight, the increase in labor sorting over the past decades significantly *narrowed* gender disparities. The reason is that this increase was predominantly driven by women's improved labor sorting (the gender gap in labor sorting has declined over time, Figure 11, right panel), helping them to catch up with men's pay. Stronger positive sorting between workers and jobs – when over-proportionally benefitting women – can spur gender convergence in labor market outcomes.

9 Conclusion

Employers value workers not only for their skills but also for their time input. In such a setting, if labor supply decisions are made at the household level so that they depend on the characteristics of *both* spouses, then marriage market sorting affects labor market sorting. In turn, if individuals anticipate their hours choices as well as their sorting and pay in the labor market when deciding whom to marry, then labor market sorting affects marriage market sorting. The interaction of *both* the marriage and the labor market crucially impacts inequality across gender and within/between households. And policies affecting who marries whom (such as tax policies) or home production technology (such as parental leave or universal childcare) can therefore mitigate or amplify inequality, calling for a better understanding of these spillovers across markets.

The interplay between labor market and marriage market and its effect on inequality are at the center of this paper. We build a novel equilibrium model in which households' labor supply choices form the natural link between the two markets and their sorting margins. We first show that in theory, the nature of home production – whether partners' hours are complements or substitutes – shapes marriage market sorting, labor supply choices and labor market sorting in equilibrium.

We then ask what is the nature of home production in the data. To this end, we estimate our model on data from today's Germany and find that spouses' home hours are strategic *complements*, pushing towards positive sorting in both markets and co-movement of labor hours of spouses. This is in contrast to what would happen in a 'traditional' economy based on substitution in home production and specialization of spouses. Investigating the key drivers behind inequality based on primitives, we find that the gender wage gap and within household income inequality would decrease not only if gender productivity differences at home or in the labor market were reduced, but also if home production hours were even more complementary among partners. Home production complementarities induce spouses to split their time similarly between work in the market and at home. And they also increase marriage sorting and reduce the gender gap in labor sorting, both mitigating gender disparities further.

Our main quantitative exercise analyzes how our model can rationalize changes in inequality over time. We find that home production time of spouses has become more complementary over time and that this technological change in home production can account for a significant part of the decline in gender inequality in Germany. In contrast, technological change in the labor market has fueled inequality across the board, including gender gaps. Highlighting the unique feature of our model, we show that sorting on both markets has significant quantitative effects on inequality: We find that *both* stronger marriage market sorting and labor market sorting over time have amplified overall inequality and between-household inequality, but have had a mitigating impact on gender inequalities (wage gap and within-household inequality) – highlighting a new role of sorting for gender convergence in pay.

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Appendix

A Empirical Evidence

A.1 Marriage Market and Labor Market Sorting

We provide additional evidence of the relationship between marriage market sorting and labor market sorting presented in Figure 1, using regressions. We run the following regression at the individual level:

$$\text{Occup. Rank}_{its} = \beta_0 + \beta_1 \text{Educ}_{its} + \beta_2 \text{Educ}_{its} \times \text{PAM}_{its} + \beta_3 \text{PAM}_{its} + X_{its}\mathbf{\Gamma} + \delta_t + \delta_s + \epsilon_{cts}$$

where Occupation Rank is the percentile rank of the occupation as defined in Appendix E.3. Educ is defined as the highest education bin individuals attain during the time we observe them in the sample, where the education bins are defined as in Section 3.2. PAM is an indicator variable that takes value 1 when both spouses are in the same education bin. We also control for the age of the individual, the income of the household and the presence of children. δ_t and δ_s capture year and state fixed effects.

We pool data from the period 2010-2016 and restrict our attention to individuals in couples (married or cohabiting). We run the regressions separately for women, and men, and also for the pooled sample. The results in Table 5 suggest a positive correlation between own education and productivity attribute of the job (as indicated by a positive and significant β_1). This correlation becomes *larger* when spouses are well matched on education, as suggested by a positive and significant β_3. This in line with the evidence in Figure 1 on the inverse u-shaped relation between labor market sorting and marriage market sorting.

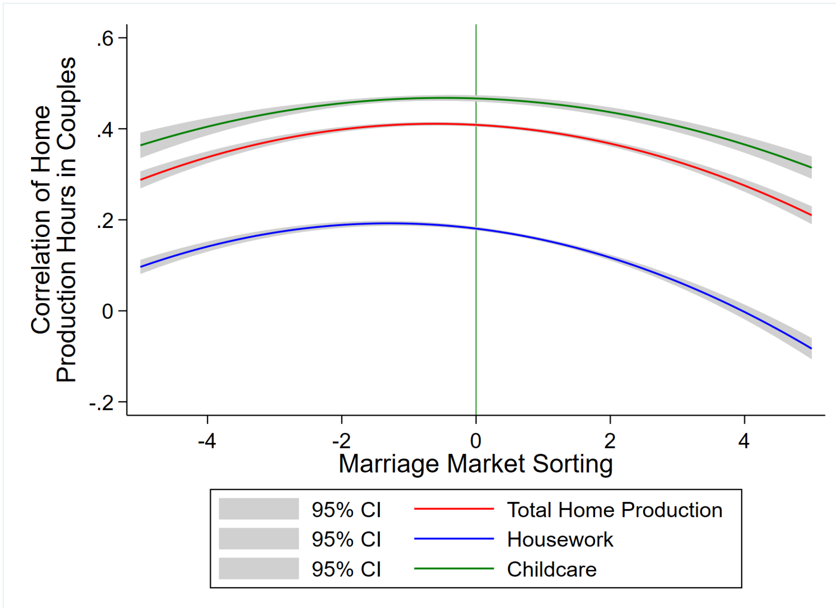
Table 5: Labor Market Sorting and Marriage Market Sorting

	(1) Occupation Rank	(2) Occupation Rank	(3) Occupation Rank
Educ	0.162***	0.179***	0.174***
	(0.008)	(0.005)	(0.004)
Educ × PAM	0.081***	0.044***	0.060***
	(0.010)	(0.007)	(0.006)
PAM	-0.144***	-0.107***	-0.121***
	(0.021)	(0.015)	(0.012)
Observations	5,079	5,948	11,027
R-squared	0.380	0.450	0.403
Demographic Controls	Yes	Yes	Yes
State and Year FE	Yes	Yes	Yes
Sample	Women	Men	All
Period	2010-2016	2010-2016	2010-2016

Notes: Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1. Occupation Rank is the job attribute percentile of the individual's occupation. Educ is measured as the individual's highest education bin we observe during the sample period. PAM is an indicator variable that takes value 1 when spouses are in the same education bin. We restrict the sample to individuals in couples, both married and cohabiting. Demographic controls: age, household income and presence of a child in the household.

A.2 Complementarities in Home Production and Market Hours

Figure 13: Hours Correlation and Marriage Sorting by Home Production Components



Note: We split total home production between time allocated to childcare-related activities and housework (including household chores, such as cooking and cleaning, errands, and repairs around the house and the garden).

In Tables 6 and 7 we further explore the correlation between male and female hours, and how it is related to marriage market sorting. This is to rule out some of the concerns discussed in section 3.2.

We run the following regression at the couple level:

$$hours^m_{cts} = \alpha_0 + \alpha_1 hours^f_{cts} + X_{cts}\mathbf{\Gamma} + \delta_t + \delta_s + \epsilon_{cts} \tag{23}$$

where $hours^f_c$ and $hours^m_c$ are either the home or the market hours of the female and the male partner in couple c, respectively. In all regressions, we control for the years of education of the male and female partners, the age of both partners, the income of the household and the presence of children. Finally, δ_t and δ_s capture year and state fixed effects.

Our results in Table 6 suggest strong complementarities in *home production hours* between spouses (column 1). In line with our findings in Figure 3, complementarities are stronger when spouses are better sorted in the marriage market (column 2).[32]

We also provide additional evidence on the source of complementarities. In columns 3 and 4, Table 6, we restrict attention to couples with children younger than 18 years old in the household. We find that complementarities in childcare are larger than complementarities in housework (including household chores, errands, and repairs around the house), in line with Figure 13.

[32] For consistency with our regressions in Table 7 we condition on the sample of couples in which both partners work in the labor market. Our results are qualitatively unchanged (but smaller) when we do not impose this restriction.

One concern is that there is unobserved heterogeneity driving both male and female hours. To control for time-invariant unobserved characteristics, we exploit the panel structure of the data and include individual fixed effects in the regressions of Table 6. Our main results are robust (available on request).

We show our results for *labor market hours* in Table 7.[33] We first replicate our results from Column 1 and 2 of Table 6 but using market hours instead. Our coefficients are close to zero and not significant. Several factors could be driving the results: First, market hours are more rigid than home hours, which makes it more difficult to adjust them in the short run. Second, gender norms in Germany are such that men are less likely to adjust their labor market hours in response to life-events, such as the arrival of a child, since they are less involved in childcare. Third, confounding factors could affect both male and female hours. For instance, as in our model in Section 6, unobserved ability could impact both own hours and partner's hours; also, as in our model, there could be household labor supply shocks affecting both partners' market hours.

In order to address these concerns, we take advantage of the exogenous change in childcare availability in Germany to instrument for female hours in regression (23). In particular, we exploit the variation across time and across states in the share of the childcare slots available for children aged 1 to 3 years old, in response to a law passed in December 2008. This law ("Das Kinderförderungsgesetz") aimed to guarantee universal and subsidized childcare for children aged 1 to 3 years old by August 1st, 2013.

Specifically, we use as instrument for female market hours the share of children aged 1 to 3 years old enrolled in childcare in a state and year, and its interaction with the presence of a child of that age range in the household.[34] Our identification assumption is that the exogenous increase in childcare availability affects female labor force participation and hours, but it only affects male hours through changes in his partner's hours. We present different pieces of evidence to support our identifying assumption: first, gender norms in Germany are such that women take most of the burden of childcare (almost 80% of childcare is done by women) independently of the education levels of the partners. Second, women with small children are likely to drop out of the labor force or to work part-time, while most men (even with small children) in Germany always work full-time. In particular, we find that the presence of a small child in the household has a strong negative impact on female market hours, but a non-significant impact on male market hours. Therefore, we would expect that policies that allow households to outsource childcare would mainly have an impact on the allocation of maternal time, but not on paternal time. Finally, previous literature looking at the impact of childcare expansions in Germany on labor market outcomes focus on women's outcomes, with positive impacts on maternal employment ((Boll

[33] As mentioned above, we restrict our attention to couples in which both partners participate in the labor market. We winsorize market hours to 10 hours at the bottom and 60 hours at the top. For the IV regressions (see below) we extend our sample period from 2010-2016 to 2006-2016, in line with the timing of our instrument. We also include in all regressions a control for the presence of children between 1 and 3 years old in the household.

[34] Both the number of children enrolled in childcare and the total population by age group, state and year were obtained from the Federal Statistical Office (Statistisches Bundesamt). We combined them to construct the share of children enrolled in childcare by state and year. Under the assumption of excess demand for childcare slots and full take-up of available slots, this corresponds to the share of slots offered at each year and state (Müller and Wrohlich, 2020).

and Lagemann, 2017), (Müller and Wrohlich, 2020), (Bauernschuster and Schlotter, 2015)).[35]

We report the results of the IV regressions in columns 3 and 4 in Table 7. Once we instrument for female market hours we find a sizable positive impact of female market hours on male market hours (column 3). The effect is larger for well sorted couples (column 4).[36]

In column 5 we report the first stage of the regression in column 3 of Table 7. Results suggest that our instruments are indeed relevant in the regression of female hours. The F-stat in the first stage reported in column 5 is 1,130.

Table 6: Complementarity in Home Production Hours

	(1) Home Hours Male	(2) Home Hours Male	(3) Childcare Hours Male	(4) Housework Hours Male
Home Hours Female	0.174***	0.184***		
	(0.017)	(0.021)		
Childcare Hours Female			0.176***	
			(0.032)	
Housework Hours Female				0.136***
				(0.027)
Demographic Controls	Yes	Yes	Yes	Yes
State and Year FE	Yes	Yes	Yes	Yes
Sample	All	Same Education	Children Present	Children Present
Period	2010-2016	2010-2016	2010-2016	2010-2016
Observations	9,422	5,475	3,791	3,649
R-squared	0.173	0.186	0.195	0.095

Notes: Standard errors clustered at the state level in parentheses. *** p<0.01, ** p<0.05, * p<0.1. All regressions control for years of education and age of the spouses, household income, and presence of children. Missing data on home hours are imputed using market hours and considering 70 hours of available time to work (home + market) per week. We pooled observations from West and East Germany for consistency with regression on market hours in Table 7.

[35]In particular, Müller and Wrohlich (2020) use the same policy as we do to estimate the impact of childcare availability on maternal time allocation, in line with our first stage. However, different from us, they use data from the German Micro-census and their empirical strategy is different from ours.

[36]Unfortunately, we cannot use the same instrument for home hours in Table 6, since the presence of a small child has a significant direct impact on the hours allocated to home production by *both* parents, violating the exclusion restriction.

Table 7: Complementarity in Labor Market Hours

	OLS Market Hours Male	OLS Market Hours Male	IV Market Hours Male	IV Market Hours Male	First Stage Market Hours Female
Market Hours Female	-0.004	0.008	0.334*	0.444*	
	(0.013)	(0.020)	(0.176)	(0.230)	
Share of Childcare Slots (1-3 yo)					7.893*
					(4.037)
Share of Childcare Slots (1-3 yo) ×					6.870***
Presence of a Child in the HH (1-3 yo)					(1.378)
Demographic Controls	Yes	Yes	Yes	Yes	Yes
State and Year FE	Yes	Yes	Yes	Yes	Yes
Sample	All	Same Education	All	Same Education	All
Period	2010-2016	2010-2016	2006-2016	2006-2016	2006-2016
Observations	9,316	5,414	15,047	8,746	17,816
R-squared	0.035	0.038			0.237

Notes: Standard errors clustered at the state level in parentheses. *** $p<0.01$, ** $p<0.05$, * $p<0.1$. All regressions control for years of education and age of the spouses, household income, and presence of children. Columns (3) and (4) instrument female market hours with the share of available slots for daycare for children 1 to 3 years old, and its interaction with the presence of a child of that age group in the household. Columns (2) and (4) restrict the sample to couples in which both spouses have the same level of education. We pooled observations from West and East Germany since a large part of the initial regional variation in childcare availability comes from differences between the Eastern and Western States.

B Theory

B.1 TU Representation: Examples

We give two examples of utility functions under which our problem features TU. First, consider a utility function that is multiplicatively separable in private and public consumption, $u(c,p) = m(c)k(p)$, where, as standard, m and k are assumed to be strictly increasing. Then, the wife's constraint in (3) reads $m(c)k(p) = \overline{v} \Leftrightarrow c = m^{-1}(\overline{v}/k(p))$, and the constrained household maximization problem (3) can be simplified as follows, when substituting in both the budget constraint and the constraint on the wife's utility:

$$
\begin{aligned}
& \max_{h_m,h_f} m(w(\tilde{x}_m) + w(\tilde{x}_f) - m^{-1}(\overline{v}/k(p)))k(p) \\
\Leftrightarrow \quad & \max_{h_m,h_f} m([w(\tilde{x}_m) + w(\tilde{x}_f)][m^{-1}(k(p))] - m^{-1}(\overline{v})) \\
\Leftrightarrow \quad & \max_{h_m,h_f} [w(\tilde{x}_m) + w(\tilde{x}_f)][m^{-1}(k(p))]
\end{aligned}
$$

These manipulations yield an objective function, which is linear in money, complying with the Gorman form. A second example is the log-utility function $u(c,p) = \log(c) + \log(p)$. We then obtain from (3):

$$
\begin{aligned}
& \max_{h_m,h_f} \log\left(w(\tilde{x}_m) + w(\tilde{x}_f) - \frac{\exp(\overline{v})}{p}\right) + \log(p) \\
\Leftrightarrow \quad & \max_{h_m,h_f} \log\left((w(\tilde{x}_m) + w(\tilde{x}_f))p - \exp(\overline{v})\right) \\
\Leftrightarrow \quad & \max_{h_m,h_f} (w(\tilde{x}_m) + w(\tilde{x}_f))p
\end{aligned}
$$

which is again linear in money and independent of $\overline{v}$. Here the arguments in the log are linear but they can be specified in a flexible way, e.g. CRRA utility for each component, as long as the overall utility function is log-additive in consumption of the private and the public good.

B.2 Monotone Equilibrium: Derivations and Proofs

B.2.1 Stability

We begin by analyzing stability of equilibrium. In particular, since our comparative statics apply to any stable equilibrium, where we refer to stability of the household problem. (In turn, stability in the marriage and labor market is trivially satisfied in the competitive equilibrium.) We therefore first define stability of the household problem.

Definition 3. *The equilibrium in the household stage, given by (h_f, h_m), is stable for a given wage* *function w if*

$$w_{\tilde{x}_m\tilde{x}_m}e^2_{h_m} + w_{\tilde{x}_m}e_{h_mh_m} + 2p_{11} < 0 \tag{24}$$

$$w_{\tilde{x}_f\tilde{x}_f}e^2_{h_f} + w_{\tilde{x}_f}e_{h_fh_f} + 2p_{22} < 0 \tag{25}$$

$$\left(w_{\tilde{x}_m\tilde{x}_m}e^2_{h_m} + w_{\tilde{x}_m}e_{h_mh_m} + 2p_{11}\right)\left(w_{\tilde{x}_f\tilde{x}_f}e^2_{h_f} + w_{\tilde{x}_f}e_{h_fh_f} + 2p_{22}\right) - 4p_{12}p_{21} > 0. \tag{26}$$

We now explain what is behind this definition of stability. Stability is reflected in the properties of the spouses' 'best response' functions, which are implicitly given by the household's FOCs (8)-(9). The first FOC gives rise to a 'best response' function r_f for the woman's hours to man's hours, given by $h^*_f = r_f(h^*_m)$ for all $h^*_m \in [0,1]$. The second FOC gives rise to a 'best response' function r_m for the man's hours to the woman's hours, given by $h^*_m = r_m(h^*_f)$ for all $h^*_f \in [0,1]$. An equilibrium in the household stage is any solution (h^*_m, h^*_f) to (8)–(9), an intersection of the best response functions. It is stable if small perturbations in the hours choices induce the agents to converge back to equilibrium. This is true iff (24)–(26) hold:

Inequalities (24)–(25) require that, given the labor hour choice of the partner, own hours adjust properly if the marginal benefit from 'investing' (i.e. the marginal wage benefit from more labor hours) is higher/lower than the marginal cost (forgone home production), i.e., if higher then hours go up; if lower then hours go down.

In turn, inequality (26) requires that, at the crossing point of the two best response functions, $|\partial r_f/\partial h^*_m| < |\partial r^{-1}_m/\partial h^*_m|$. That is, the slope of r_f is smaller than the slope of the inverse of r_m when they cross in the (h^*_m, h^*_f)-space, or r_m crosses r_f from below (above) when the BR-functions are upward (downward) sloping at the crossing. To see that inequality (26) requires that, at the crossing of the two best response functions, $|\partial r_f/\partial h^*_m| \leq |\partial r^{-1}_m/\partial h^*_m|$, note that $|\partial r^{-1}_m/\partial h^*_m| > |\partial r_f/\partial h^*_m|$ is equivalent to

$$\begin{aligned}
\Leftrightarrow \quad & \left| -\frac{w_{\tilde{x}_f\tilde{x}_f}e^2_{h_f} + w_{\tilde{x}_f}e_{h_fh_f} + 2p_{22}}{2p_{12}}\right|\left| -\frac{w_{\tilde{x}_m\tilde{x}_m}e^2_{h_m} + w_{\tilde{x}_m}e_{h_mh_m} + 2p_{11}}{2p_{12}}\right| && > 1 \\
\Leftrightarrow \quad & \left| -\left(w_{\tilde{x}_f\tilde{x}_f}e^2_{h_f} + w_{\tilde{x}_f}e_{h_fh_f} + 2p_{22}\right)\right|\left| -\left(w_{\tilde{x}_m\tilde{x}_m}e^2_{h_m} + w_{\tilde{x}_m}e_{h_mh_m} + 2p_{11}\right)\right| && > (2p_{12})^2 \\
\Leftrightarrow \quad & \left(w_{\tilde{x}_f\tilde{x}_f}e^2_{h_f} + w_{\tilde{x}_f}e_{h_fh_f} + 2p_{22}\right)\left(w_{\tilde{x}_m\tilde{x}_m}e^2_{h_m} + w_{\tilde{x}_m}e_{h_mh_m} + 2p_{11}\right) && > (2p_{12})^2
\end{aligned}$$

where the last inequality follows from conditions (24)–(25), ensuring that in a stable equilibrium, both terms in brackets on the LHS are negative. The crossing of the best response functions described by (26) guarantees that small perturbations away from the equilibrium hours induce dynamics so that the resulting hours adjustments make the household converge back to the equilibrium hours.

With Definition 3 at hand, we can now show the following.

Lemma 1. *For any wage function w with the property that $\partial w/\partial h_i = w_{\tilde{x}_i}e_{h_i}$ is strictly positive and finite for all $h_i \in [0,1]$, a stable equilibrium of the household problem exists.*

Proof. We first show that, given $h_m^* \in [0,1]$, there exists a solution to (8), $w_{\tilde{x}_f} e_{h_f} = 2p_2$ that satisfies also (25). To see this, notice that, for any h_m^*, the LHS, $\partial w/\partial h_f = w_{\tilde{x}_f} e_{h_f}$, is always strictly positive and finite under the premise, while the RHS goes to infinity as h_f^* goes to one (i.e. when $1 - h_f$ goes to zero), and to zero as h_f^* goes to zero (and $1 - h_f$ to one) by the assumed Inada conditions on p. Hence, by the Intermediate Value Theorem, there is at least one solution to (8), and the first solution satisfies (24) (the RHS crosses the LHS from below when plotted against h_f). Denote this solution by $h_f^* = r_f(h_m^*)$ for each h_m^*, and notice that it is continuously differentiable in h_m^*.

Similarly, given $h_f^* \in [0,1]$, there exists a solution to (9) that satisfies also (24). We denote this solution by $h_m^* = r_m(h_f^*)$ for each h_f^*, which is continuously differentiable in h_f^*.

Next, we note that $r_f(0) > 0$ and $r_f(1) < 1$, which follows from the strictly positive and finite value of the LHS in (8), $w_{\tilde{x}_f} e_{h_f} = 2p_2$, and the boundary properties of p_2. Similarly, $r_m(0) > 0$ and $r_m(1) < 1$. Since r_m and r_f are continuous functions, there exists a pair (h_f^*, h_m^*) such that $h_f^* = r_f(h_m^*)$ and $h_m^* = r_m(h_f^*)$ and that also satisfies (26), i.e. $|\partial r_f/\partial h_m^*| \leq |\partial r_m^{-1}/\partial h_m^*|$.

Thus, a stable equilibrium of the household problem exists.[37] □

Lemma 1 shows that for *any* given wage function that satisfies the stated assumption on $\partial w/\partial h_i$, there exists at least one stable equilibrium. We now argue that, as a result, also for the *equilibrium* wage function, there exists at least one stable equilibrium. To show this, the equilibrium wage function needs to satisfy the property that $\partial w/\partial h_i$ is strictly positive and finite. It is given by (2), and so $\partial w/\partial h_i = z_{\tilde{x}}(\tilde{x}_i, \mu(\tilde{x}_i)) e_{h_i}$, which under the assumption of the model is strictly positive and finite.

Thus, whenever an equilibrium exists, we know that there exists at least one stable one. And any stable equilibrium satisfies the conditions of Definition 3, which we use to sign our comparative statics of the household problem.

B.2.2 Spouses' Hours as Strategic Complements or Substitutes

Differentiating each FOC w.r.t. to (h_f, h_m), we obtain the slopes of the 'best response' functions for women (men) to men's (women's) hours, respectively:

$$
\begin{aligned}
0 &= (w_{\tilde{x}_f \tilde{x}_f} e_{h_f}^2 + w_{\tilde{x}_f} e_{h_f h_f}) dh_f + 2p_{12} dh_m + 2p_{22} dh_f \\
\Leftrightarrow \quad \frac{dh_f}{dh_m} &= -\frac{2p_{12}}{w_{\tilde{x}_f \tilde{x}_f} e_{h_f}^2 + w_{\tilde{x}_f} e_{h_f h_f} + 2p_{22}} \qquad (27) \\
0 &= (w_{\tilde{x}_m \tilde{x}_m} e_{h_m}^2 + w_{\tilde{x}_m} e_{h_m h_m}) dh_m + 2p_{12} dh_f + 2p_{11} dh_m \\
\Leftrightarrow \quad \frac{dh_m}{dh_f} &= -\frac{2p_{12}}{w_{\tilde{x}_m \tilde{x}_m} e_{h_m}^2 + w_{\tilde{x}_m} e_{h_m h_m} + 2p_{11}} \qquad (28)
\end{aligned}
$$

[37] To see this graphically in the (h_m^*, h_f^*) space, notice that r_f starts above the inverse of r_h and ends below. Continuity implies there is a crossing between the two, and the first one is such that the inverse of r_m crosses the r_f from below. This implies that at the crossing point $|\partial r_f/\partial h_m^*||\partial r_m/\partial h_f^*| \leq 1$.

In any stable equilibrium (since the denominators of these expressions are negative), the best response functions are upward (downward) sloping, and thus hours are strategic complements (substitutes), if p is supermodular (submodular).

B.2.3 Proof of Proposition 1

We now check that under the specified conditions the equilibrium is monotone as specified in Definition 2.

1. PAM in the labor market in $(y, \tilde{x}_i)$ materializes due to $z_{\tilde{x}y} > 0$.

2. The properties of how own labor hours depend on own type and partner type follow from the equilibrium comparative statics that can be derived from the system of FOCs of the household problem. We differentiate system (8) - (9) w.r.t. x_f, taking as given the equilibrium wage function and the equilibrium marriage market matching function (meaning that hours do not only depend on own type but also on the partner's type along the equilibrium assignment η):

$$A\frac{\partial h_f}{\partial x_f} + B\frac{\partial h_m}{\partial x_f} = -(w_{\tilde{x}_f\tilde{x}_f}e_{h_f}e_{x_f} + w_{\tilde{x}_f}e_{x_fh_f})$$
$$C\frac{\partial h_f}{\partial x_f} + D\frac{\partial h_m}{\partial x_f} = -\eta'(x_f)(w_{\tilde{x}_m\tilde{x}_m}e_{h_m}e_{x_m} + w_{\tilde{x}_m}e_{x_mh_m})$$

where

$$A := w_{\tilde{x}_f\tilde{x}_f}e_{h_f}^2 + w_{\tilde{x}_f}e_{h_fh_f} + 2p_{22}$$
$$B := 2p_{12}$$
$$C := 2p_{12}$$
$$D := w_{\tilde{x}_m\tilde{x}_m}e_{h_m}^2 + w_{\tilde{x}_m}e_{h_mh_m} + 2p_{11}$$

Denote $|H| := AD - BC$, which is the determinant of the Hessian of the household problem and positive in any *stable* equilibrium, see stability condition (26) of Definition 3. Solving the system yields

$$\frac{\partial h_f}{\partial x_f} = \frac{-(w_{\tilde{x}_f\tilde{x}_f}e_{h_f}e_{x_f} + w_{\tilde{x}_f}e_{x_fh_f})\left(w_{\tilde{x}_m\tilde{x}_m}e_{h_m}^2 + w_{\tilde{x}_m}e_{h_mh_m} + 2p_{11}\right) + 2\eta'(x_f)((w_{\tilde{x}_m\tilde{x}_m}e_{h_m}e_{x_m} + w_{\tilde{x}_m}e_{x_mh_m}))p_{12}}{|H|}$$
$$= \frac{-\frac{\partial^2 w(\tilde{x}_f)}{\partial h_f \partial x_f}\frac{\partial^2 u}{\partial h_m^2} + 2\eta'(x_f)\frac{\partial^2 w(\tilde{x}_m)}{\partial h_m \partial x_m}p_{12}}{|H|}$$
$$\frac{\partial h_m}{\partial x_f} = \frac{-\eta'(x_f)\left(w_{\tilde{x}_f\tilde{x}_f}e_{h_f}^2 + w_{\tilde{x}_f}e_{h_fh_f} + 2p_{22}\right)(w_{\tilde{x}_m\tilde{x}_m}e_{h_m}e_{x_m} + w_{\tilde{x}_m}e_{x_mh_m}) + (w_{\tilde{x}_f\tilde{x}_f}e_{h_f}e_{x_f} + w_{\tilde{x}_f}e_{x_fh_f})2p_{12}}{|H|}$$
$$= \frac{-\eta'(x_f)\frac{\partial^2 u}{\partial h_f^2}\frac{\partial^2 w(\tilde{x}_m)}{\partial h_m \partial x_m} + 2\frac{\partial^2 w(\tilde{x}_f)}{\partial h_f \partial x_f}p_{12}}{|H|}.$$

These expressions are equivalent to (10) and (11). They are positive in any stable equilibrium (where conditions (24)-(26) from Definition 3 are satisfied), meaning hours are increasing in own types and in partner's types, given that (i) hours are complementary in home production (p supermodular so that

$p_{12} > 0$), (ii) matching on the marriage market is PAM ($\eta' > 0$), for which we will provide conditions below, and (iii) wages are convex in effective types (ensuring that $\frac{\partial^2 w(\tilde{x}_f)}{\partial h_f \partial x_f} > 0$ and $\frac{\partial^2 w(\tilde{x}_m)}{\partial h_m \partial x_m} > 0$), which can be ensured from primitives if z is weakly convex. To see this note that $\frac{\partial^2 w(\tilde{x}_f)}{\partial h_f \partial x_f} = z_{\tilde{x}_f} e_{x_f h_f} + (z_{\tilde{x}_f \tilde{x}_f} + z_{\tilde{x}_f y} \mu'(\tilde{x}_f)) e_{x_f} e_{h_f}$. Analogously, we can compute how hours respond to changes in male types (12) and (13) (omitted here for brevity).

3. In a stable equilibrium, PAM in the marriage market results if (as indicated in the text)

$$\begin{aligned}\Phi_{x_f x_m} &= \frac{\partial^2 w}{\partial x_f \partial h_f} \frac{\partial \hat{h}_f}{\partial x_m} \\ &= 2\frac{p_{12} \frac{\partial^2 w(\tilde{x}_f)}{\partial h_f \partial x_f} \frac{\partial^2 w(\tilde{x}_m)}{\partial h_m \partial x_m}}{|H|} > 0\end{aligned}$$

where we substituted in the first line the 'partial' equilibrium comparative static $\partial \hat{h}_f / \partial x_m$ (see below) to obtain to the second line. Thus, $\Phi_{x_f x_m} > 0$ if $z_{\tilde{x}\tilde{x}} \geq 0$ (which renders $\partial^2 w / \partial x_i \partial h_i > 0$, see part 2. above) and $p_{12} > 0$.

To show that $\partial \hat{h}_f(x_m, h_f)/\partial x_m > 0$ (i.e. hours are increasing in partner's type for *any* couple (x_f, x_m), not only along the equilibrium assignment, which is to be solved for in this step), we differentiate system (8) - (9) w.r.t. x_m for any given x_f:

$$\begin{aligned}\frac{\partial \hat{h}_m}{\partial x_m} &= \frac{-\frac{\partial^2 w(\tilde{x}_m)}{\partial h_m \partial x_m} \frac{\partial^2 u}{\partial h_f^2}}{|H|} \\ \frac{\partial \hat{h}_f}{\partial x_m} &= \frac{2\frac{\partial^2 w(\tilde{x}_m)}{\partial h_m \partial x_m} p_{12}}{|H|}\end{aligned}$$

where we used the second expression to sign Φ_{x_f, x_m} above.

B.2.4 Proof of Corollary 1

The proof of Corollary 1 follows immediately from the properties of monotone equilibrium:

1. Follows from PAM on the marriage market.
2. That both $(x'_f, \eta(x'_f))$ work more labor hours and less at home compared to couple $(x_f, \eta(x_f))$ follows from the monotone equilibrium property that labor market hours are increasing in own and partner's types. That both partners of couple $(x'_f, \eta(x'_f))$ have more private consumption and less public consumption is due to the following: Woman x'_f has more private consumption than x_f since $c_f = v - p$ and $\partial v / \partial x_f = w_{\tilde{x}_f} e_{x_f} > 0$ and $dp/dx_f = -p_2(\partial h_f / \partial x_f + (\partial h_f / \partial x_m)\eta') - p_1(\partial h_m / \partial x_f + (\partial h_m / \partial x_m)\eta') < 0$. That the private consumption of partner $\eta(x'_f)$ is higher

compared to the other couple's partner follows from $c_m = w_f + w_m - c_f$ and thus

$$\begin{aligned}\frac{dc_m}{dx_f} &= \frac{dw_f}{dx_f} + \frac{dw_m}{dx_f} - \frac{dc_f}{dx_f} \\ &= w_{\tilde{x}_f} e_{h_f}\left(\frac{\partial h_f}{\partial x_f} + \frac{\partial h_f}{\partial x_m}\eta'\right) + w_{\tilde{x}_f} e_{x_f} + w_{\tilde{x}_m} e_{h_m}\left(\frac{\partial h_m}{\partial x_f} + \frac{h_m}{\partial x_m}\eta'\right) + w_{\tilde{x}_m} e_{x_m}\eta' - \frac{\partial v}{\partial x_f} + \frac{dp}{dx_f} \\ &= (w_{\tilde{x}_f} e_{h_f} - p_1)\left(\frac{\partial h_f}{\partial x_f} + \frac{\partial h_f}{\partial x_m}\eta'\right) + (w_{\tilde{x}_m} e_{h_m} - p_2)\left(\frac{\partial h_m}{\partial x_f} + \frac{\partial h_m}{\partial x_m}\eta'\right) + w_{\tilde{x}_m} e_{x_m}\eta' > 0\end{aligned}$$

where the positive sign follows from the FOCs of the household problem w.r.t. hours (the first two terms are positive if the FOCs hold). Finally, her utility increases in her type which follows from $\partial v/\partial x_f = w_{\tilde{x}_f} e_{x_f} > 0$. That her partner's utility increases in her type follows from $u = c_m + p$ and thus, $du/dx_f = dc_m/dx_f + dp/dx_f = w_{\tilde{x}_m} e_{x_m}\eta' > 0$ (where we applied the Envelope Theorem).

3. The first statement on effective types follows from 2. and the premise. The statement on PAM on the labor market follows from the comparison of effective types of the two couples and $z_{\tilde{x}y} > 0$. The last statement on wages follows since wages are increasing in effective types.

B.2.5 Proof of Proposition 2

We check that the properties of non-monotone equilibrium are satisfied under the specified conditions.

1. PAM in the labor market in $(y, \tilde{x}_i)$ materializes due to $z_{\tilde{x}y} > 0$.

2. The properties of how labor hours depend on own type and partner type follow straight from comparative statics expressions (10)–(13): $\partial h_i/\partial x_i > 0$ in a stable equilibrium if $p_{12} < 0$ and $\eta' < 0$ as well as $z_{\tilde{x}\tilde{x}} > 0$ (which renders $\partial^2 w/\partial x_i \partial h_i > 0$). Further, $\partial h_i/\partial x_j < 0$ under the same conditions. It remains to verify that $\eta' < 0$, see 3.

3. NAM in the marriage market results if

$$\Phi_{x_f x_m} = 2\frac{p_{12}\frac{\partial^2 w(\tilde{x}_f)}{\partial h_f \partial x_f}\frac{\partial^2 w(\tilde{x}_m)}{\partial h_m \partial x_m}}{|H|} < 0$$

Thus, in a stable eq., $\Phi_{x_f x_m} < 0$ if $z_{\tilde{x}\tilde{x}} > 0$ (which renders $\partial^2 w/\partial x_i \partial h_i > 0$ for $i \in \{f, m\}$) and $p_{12} < 0$.

C Solution of the Quantitative Model

The solution of our quantitative model consists of solving for a fixed point in the wage function (as a function of effective types) such that under this wage function, marital choices, household labor supply, and labor market sorting are all consistent. That is, we find the market-clearing wage function that induces households that form in the marriage market to optimally supply labor (pinning down their effective types) such that, when optimally sorting into firms on the labor market, this gives rise to that exact same wage function.

We first solve for the optimal matching in the marriage market and households' labor supply choices *given* a wage function. Given the induced labor supply decisions, individuals move to the labor market where they optimally match with firms. Sorting in the labor market endogenously determines a new wage function (again as a function of effective types) that supports this particular matching. Given this new wage function, new marriage and labor supply decisions are made that, in turn, again affect wages in the labor market. We iterate between the problem of households on the one hand and of workers and firms on the other until the wage function converges (until a fixed point in the wage function is found).

We next describe the solution in each decision stage, starting backwards from the labor market and then going to household and marriage problems. Finally, we outline the algorithm to find the fixed point.

C.1 Partial Equilibrium in the Labor Market ([lpe])

First, we show how we solve for the matching and wage functions in the labor market, (μ, w). Consider our *exogenous* distribution of firms, $y \sim G$, and *any given* distribution of effective types, $\tilde{s} \sim \tilde{N}_s$. Note that even though $\tilde{N}_s$ is an endogenous object in our model, from a partial equilibrium perspective where marital and household choices are taken as given, firms take the distribution $\tilde{N}_s$ as fixed.

To solve for the optimal matching between firms and workers note that the production function $z(\tilde{s}, y)$ is assumed to be supermodular. By the well known Becker-Shapley-Shubik result (**?** and **?**) the optimal matching in the labor market is positive assortative between y and $\tilde{s}$. Hence, a worker with effective skill $\tilde{s}$ matches to firm $\mu(\tilde{s})$, where

$$\mu(\tilde{s}) = G^{-1}[\tilde{N}_s(\tilde{s})].$$

Moreover, the wage function w is derived from the firms' first order conditions (2), evaluated at the optimal matching μ. In the quantitative model where G and $\tilde{N}_s$ are discrete, we approximate the integral in (2) numerically using trapezoidal integration.

The output from solving the equilibrium in the labor market given marital and household choices is the tuple (μ, w) as defined above.

C.2 Optimal Household Choices ([hh])

Second, we derive the solution of the household problem that yields spouses' optimal consumption, (c_f, c_m), their optimal labor supply (h_f, h_m), and the distribution of effective types $\tilde{N}_s$.

Individuals arrive at the household stage either as singles with human capital s_i or in a couple with human capital bundle (s_f, s_m). We denote the household human capital type by two-dimensional vector $\mathbf{s} = (s_f, s_m) \in \{\mathcal{S} \cup \emptyset\}^2$ where, for example, $(s_f, \emptyset)$ denotes the household of single woman of type s_f.

When solving their household problem agents *take as given* wage function w, the marriage market matching distribution η, and the marriage market clearing price v.

Given prices and marriage outcomes, couples solve problem (19) and singles solve problem (20). Replacing the budget constraint into the objective function and noting the transferable utility structure of the problem given the quasilinear utility function, the collective problem of couple (s_f, s_m) after labor supply preference shocks realize is given by:

$$\max_{h_m,h_f} w(\tilde{s}_m) + w(\tilde{s}_f) + 2p^M(1-h_m, 1-h_f) + \delta_m^{h_m} + \delta_f^{h_f} \tag{29}$$

where $w(\tilde{s}_m)$ and $w(\tilde{s}_f)$ depend on hours through the effective human capital types (21).

Similarly, the problem of a single woman of type s_f after realization of her labor supply preference shock is

$$\max_{h_f} w(\tilde{s}_f) + p^U(1-h_f) + \delta_f^{h_f} \tag{30}$$

and the problem of a single man s_m is given by

$$\max_{h_m} w(\tilde{s}_f) + p^U(1-h_m) + \delta_m^{h_m}. \tag{31}$$

To derive aggregate labor supply and the distribution of effective types $\tilde{N}_s$, we need to introduce some notation.

We denote the *alternative* of hours that a decision maker chooses by $\mathbf{h} \in \{\mathcal{H} \cup \emptyset\}^2 := \{\{0, ..., 1\} \cup \emptyset\}^2$ (where $\emptyset$ indicates the hours of the non-existing partner when the individual is single). We denote by $\mathbf{h}^t$ the hours alternative chosen by a decision maker of type $t \in \{M, U\}$:

$$\mathbf{h}^t = \begin{cases} (h_i, \emptyset), i \in \{f, m\} & \text{if } t = U \\ (h_f, h_m) & \text{if } t = M. \end{cases}$$

where type $t = U$ indicates *single* and type $t = M$ indicates *couple*.

Also, we denote the economic utility associated with hours alternative $\mathbf{h}^t$ of household type $t \in \{M, U\}$ with human capital type $\mathbf{s} \in \{\mathcal{S} \cup \emptyset\}^2$ by $\overline{u}_{\mathbf{s}}^t(\mathbf{h}^t)$, where

$$\overline{u}_{\mathbf{s}}^t(\mathbf{h}^t) = \begin{cases} w(\tilde{s}_i) + p^U(1-h^t) & \text{if } t = U \\ w(\tilde{s}_m) + w(\tilde{s}_f) + 2p^M(1-h_m, 1-h_f) & \text{if } t = M. \end{cases} \tag{32}$$

We obtain the optimal labor supply and private consumption (c_m, c_f, h_m, h_f) for each household by solving problems (29)-(31). Given our assumption that the labor supply shock distribution is Type-I extreme value, we then obtain the fraction of agents that optimally chooses each hours alternative. The probability that household type $t \in \{M, U\}$ with human capital type $\mathbf{s} \in \{\mathcal{S} \cup \emptyset\}^2$ chooses hours alternative $\mathbf{h} \in \{\mathcal{H} \cup \emptyset\}^2$ is

$$\pi_{\mathbf{s}}^t(\mathbf{h}^t) = \frac{\exp(\overline{u}_{\mathbf{s}}^t(\mathbf{h}^t)/\sigma_\delta)}{\sum_{\tilde{\mathbf{h}} \in \{\mathcal{H} \cup \emptyset\}^2} \exp(\overline{u}_{\mathbf{s}}^t(\tilde{\mathbf{h}}^t)/\sigma_\delta)} \tag{33}$$

Denoting the fraction of *households* who are type $\mathbf{s}$ by $\eta_{\mathbf{s}}$, the fraction of households who are of type $\mathbf{s}$ *and* choose hours alternative $\mathbf{h}$ is given by

$$\eta_{\mathbf{s}} \times \pi^t_{\mathbf{s}}(\mathbf{h}^t).$$

From this distribution of *household* labor supply we back out the distribution of *individual* labor supply. To do so, we compute the fraction of men and women of each *individual* human capital type, s_i in household $\mathbf{s}$, optimally choosing each *individual* hours alternative h_i associated with household labor supply $\mathbf{h}$. Given the distribution of individual labor supply, we can compute the distribution of effective human capital types, $\tilde{N}_s$. First, note that the support of the distribution is obtained by applying functional forms (21) for any combination of *individual* hours and skill types. Second, to each point in the support of $\tilde{s}(s,h)$ we attach the corresponding individual frequencies from the individual labor supply distribution backed out as explained above.

The output from solving the household problem given (w, μ, η) is the tuple $(h_f, h_m, c_f, c_m, \tilde{N}_s)$.

C.3 Partial Equilibrium in the Marriage Market ([mpe])

In the marriage stage, individuals draw idiosyncratic taste shocks for partners and single-hood, β^s_i, with $i \in \{f, m\}$ and $s \in \{\mathcal{S} \cup \emptyset\}$. At this stage, labor supply shocks are not yet realized. As a result, the *ex ante* economic value from marriage of type (s_f, s_m) is the expected value of (29); and the ex-ante economic value from female and male singlehood is the expected value of (30) and (31). In both cases, the expectation is taken over the distribution of δ-shocks. Denoting the utility transfer to a female spouse of type s_f by $v(s_f)$, the values of being married (economic plus non-economic) for a woman type s_f and a man type s_m in couple (s_f, s_m) are given by

$$\begin{aligned}
\Phi_f(s_m, s_f, v(s_f)) + \beta^{s_m}_f &:= v(s_f) + \beta^{s_m}_f \\
\Phi_m(s_m, s_f, v(s_f)) + \beta^{s_f}_m &:= \mathbb{E}_\delta\Big\{\max_{h_m, h_f} w(\tilde{s}_m) + w(\tilde{s}_f) + 2p^M(1-h_m, 1-h_f) + \delta^{h_m}_m + \delta^{h_f}_f\Big\} + \beta^{s_f}_m - v(s_f) \\
&= \sigma_\delta\Big[\kappa + \log\Big(\sum_{\mathbf{h}^M \in \mathcal{H}^2} \exp\{\overline{u}^M_{\mathbf{s}}(\mathbf{h}^M)/\sigma_\delta\}\Big)\Big] + \beta^{s_f}_m - v(s_f)
\end{aligned}$$

where $\kappa = 0.57722$ is the Euler constant, $\overline{u}$ is defined in (32) and $\mathbb{E}_\delta$ indicates the expectation is taken over the distribution of δ-shocks.

In turn, the value of being single for woman s_f is

$$\Phi_f(\emptyset, s_f) + \beta^\emptyset_f := \sigma_\delta\Big[\kappa + \ln\Big(\sum_{\mathbf{h}^U \in \mathcal{H}} \exp\{\overline{u}^U_{(s_f,\emptyset)}(\mathbf{h}^U)/\sigma_\delta\}\Big)\Big] + \beta^\emptyset_f$$

and for man s_m it is

$$\Phi_m(s_m, \emptyset) + \beta^\emptyset_m := \sigma_\delta\Big[\kappa + \ln\Big(\sum_{\mathbf{h}^U \in \mathcal{H}} \exp\{\overline{u}^U_{(\emptyset,s_m)}(\mathbf{h}^U)/\sigma_\delta\}\Big)\Big] + \beta^\emptyset_m$$

Every man s_m and every woman s_f chooses the skill type of their partner or to remain single to maximize their value on the marriage market:

$$\max \quad \{\max_{s_f \in \mathcal{S}} \Phi_m(s_m, s_f, v(s_f)) + \beta_m^{s_f}, \Phi_m(s_m, \emptyset) + \beta_m^{\emptyset}\}$$
$$\max \quad \{\max_{s_m \in \mathcal{S}} \Phi_f(s_m, s_f, v(s_f)) + \beta_f^{s_m}, \Phi_f(\emptyset, s_f) + \beta_f^{\emptyset}\}$$

These problems thus also capture the marriage market participation margin.

In practice, using the transferable utility property of our model, we solve for the *optimal* marriage matching by maximizing the total sum of marital values across all individuals in the economy, using a linear program. We denote the matching distribution by η, which solves

$$\begin{aligned} &\max_{\eta_{(s,s')} \in [0,1]} \sum_{(s,s') \in \{\mathcal{S} \cup \emptyset\}^2} \eta_{(s,s')} \times \left(\Phi(s,s') + \tilde{\beta}\right) \\ s.t. \quad &\sum_{s \in \mathcal{S}} \eta_s = 1/2 \\ &\sum_{s' \in \mathcal{S}} \eta_{s'} = 1/2 \end{aligned}$$

where $\eta_{(s,s')}$ denotes the mass of household type $(s,s') \in \{\mathcal{S} \cup \emptyset\}^2$ under matching η; η_s denotes the marginal distribution of η with respect to the first dimension; $\eta_{s'}$ denotes the marginal distribution of η with respect to the second dimension; $\Phi(s,s')$ denotes the economic value from marriage for the different types of households, $\Phi(s,s') = \{\Phi_m(s,s',v(s')) + \Phi_f(s,s',v(s')), \Phi_f(\emptyset,s'), \Phi_m(s,\emptyset)\}$; and $\tilde{\beta}$ denotes $\beta_f^{s_m} + \beta_m^{s_f}$ for couples and $\beta_i^{\emptyset}$ $(i = \{f,m\})$ for singles. Note that the restrictions of this linear program impose that the mass of women and men in all households (couples or singles) must be equal to the total mass of women and men in the economy (which is 1/2 for both sexes).

We obtain the equilibrium matching in the marriage market, η, by solving this linear program, taking prices and allocations in households and the labor market, (w, μ, h_f, h_m), as given.

C.4 General Equilibrium of the Model

Once we have derived the solution of each of the stages taking the output from the other stages as given, we solve for the *general equilibrium* of the model by searching for the prices, allocations, and assignments such that all markets are simultaneously in (partial) equilibrium. To preview, we start "backwards" from the output of the labor market stage with an arbitrary initial wage function indicating a wage offered to each effective type. In the household stage, each *potential* household takes those wages as given and makes their labor supply choices. These optimal labor supply choices (in each potential household) are then used by each individual in the marriage market to compute the value of single-hood and marriage with different partners, leading to marriage choices. The hours choices of *formed* households give rise to a distribution of *effective* types. With this endogenous distribution of effective types we go back to the labor market stage, where we match workers' effective types with firms' productivities optimally. This labor market matching gives rise to a new wage function supporting this allocation. With the new wage function at hand, we solve and update the household and marriage problems and iterate until we obtain a wage function from the labor market stage that when taken as given by households gives rise to an aggregate labor supply that, in turn, produces those same wages, i.e. until we have found the fixed point in the wage function.

C.4.1 Trembling Effective Types

A challenge in the search for the equilibrium is that each household type needs to face a wage for *any* hours choice in order to make its optimal labor supply choices. However, it may be the case that at a given iteration of our fixed point algorithm, the wage function is such that certain levels of hours are *not* chosen by some household types. Therefore, in the next iteration, agents would face a wage function that only maps *realized* effective types to a wage (i.e. a wage function 'with gaps in the support'), see subsection C.1. The problem then is that agents do not know the payoff from all potential hours choices when they try to make their optimal choice.

To fill in the gaps so that households of each type observe wages for *any* hours alternatives, we develop a trembling strategy. The trembling strategy consists of drawing a random sample of women and men and force them to supply a suboptimal amount of hours from the set of unchosen hours in each iteration. In practice, for each group of women with skill type s_f and each group of men with skill type s_m, we track their optimal choices for a given wage function and determine the hours that were *not* chosen with positive probability. We then draw a 1% random sample of women and men within each of those skill types and assign them uniformly to the unchosen hours. So we force maximally 1% of each skill type (the 'tremblers') to choose sub-optimal hours, or in other words, to tremble. Finally, we construct the distribution of effective types $\tilde{N}_s$ by taking into account both 'trembling' effective types and 'optimal' effective types.

C.4.2 Fixed Point Algorithm

To solve for the general equilibrium we denote by $\tilde{N}_s^*$ the distribution of *realized* effective types (based on *optimal* hours choices, not *trembling* hours choices). Similarly, we denote by w^* the wage as a function of *realized* effective types only, where recall that the *full support* wage function is denoted by w. The fixed point algorithm we designed to solve for the equilibrium is as follows:

0. Initiate a round-zero wage function for the full support of effective types, w^0.

At any round $r \geq 1$

1. Input w^{r-1} and solve [hh] and [mpe]. Update $\tilde{N}_s^{*r}$.
2. Input $\tilde{N}_s^{*r}$ and solve [lpe]. Update w^{*r}.
3. Update w^r:
   (a) We determine w^{*r} from step 2. above.

(b) Simultaneously, we fill in the wage for effective types that did not realize at round r by solving step 2. for *trembling types*, yielding w^r.

4. Move to round $r+1$ by going back to step 1. above and continue iterating until the wage function converge, that is, $w^{r+1}(\tilde{s}) - w^r(\tilde{s}) < \epsilon$ for $\epsilon > 0$ and small, element-by-element (for each $\tilde{s}$).
5. (OUTPUT) Compute the general equilibrium as the tuple of outputs from [hh], [mpe], and [lpe] at the round where the wage function w^r converged.

D Sample Construction

D.1 Sample for Empirical Facts

We now describe the sample restrictions and the variables used for our empirical facts in Section 3.

D.1.1 Sample restrictions

To construct our *Main sample*, we pool observations from the period 2010-2016, for the original GSOEP samples and their refreshments.[38] We restrict our attention to West Germany. Following other papers in the literature, such as Heise and Porzio (2019), we drop Berlin from the sample since it cannot be unambiguously assigned to East or West Germany.

We impose the following demographic restrictions: we keep all individuals in private households, either single or in heterosexual couples (married or cohabiting). We restrict our analysis to the first marital spell of the life of an individual, which could be either never married or the individual's first marriage.[39] We restrict our sample to individuals in their prime working age, 22-55 years old. We apply these restrictions at the individual level, but not at the couple level, which implies that in some cases, one of the spouses could be part of the sample, even if the other spouse is not.

Regarding the labor market, we exclude from our sample those individuals who are self-employed or those who are still in school, those working in odd occupations (`kldb92`$\geq$ 9711) and the employed individuals with missing data on the occupational code.

D.1.2 Variable description

We now describe the variables we use in the empirical analysis.

1. **Education Variables:** We classify individuals into three education bins: low education includes those with a high school degree or with a vocational degree, with less than 11 years of schooling. Medium education includes those individuals with a vocational degree and more than 11 years of schooling. High education includes those with a college degree or higher. Education levels

[38] We exclude from our analysis the migrants and refugees samples, the oversampling of low income individuals and single parents, and the oversampling of high income earners.

[39] Since cohabitants are defined as never married, we cannot rule out that they are in a cohabiting relationship that is not the first one.

are defined based on the ISCED-97 classification. Alternatively, we use years of education as our education measure, and we left-truncate this variable at 10 years of education.

2. **Marriage Market Sorting:** For the figures, we define marriage market sorting as the difference between years of education of the partners in couples (own years of education minus partner's years of education). When the analysis is at the household level, we compute marriage market sorting as the difference between the years of education of the male partner and the years of education of the female partner, for consistency across couples.[40]
3. **Matching Function:** Our matching function is given by the task complexity of the occupation in which the individual is employed, defined at the 4-digit level of the `kldb92` classification. For a detailed description of how this measure is constructed, refer to Appendix E.3.
4. **Labor Market Sorting:** We define labor market sorting as the correlation between the individual's years of education and their matched job characteristic.
5. **Hours:**
   (a) **Market hours**: We define market hours as the number of self-reported hours of an individual in the labor market in a given week (including overtime). We winsorize market hours to 10 and 60 hours, at the bottom and the top, respectively.
   (b) **Home Hours:** We measure home hours as the weekly time an individual allocates to the following activities: childcare, housework (which includes household chores such as cooking, cleaning, etc.), running errands, repairs around the house or the car, and garden work. Since home hours are measured on a typical week day, we multiply these hours by 5, for consistency with market hours. When home hours are not available, we use information on market hours to impute them, assuming a total of 70 hours available per week to allocate to home production and market work.
   (c) **Leisure Hours:** We measure leisure hours as the weekly hours allocated to hobbies and other leisure activities.

D.2 Estimation Sample

In this section we describe the sample restrictions and variable definitions of our estimation sample.

D.2.1 Sample Restrictions

In order to construct our *Baseline Estimation Sample*, we use data from West Germany for the period 2010-2016 of the original GSOEP panel and its refreshments (as discussed above).[41] For our *Past Estimation Sample* we use data for the period 1990-1996. We drop those individuals that appear in

[40] We exclude couples for which the difference in years of education between the partners is 6.5 years or more. We pool together couples in which the difference in years of education between the partners is either 5 or 6 years, in order to increase the sample size in this bin.

[41] We drop from our sample individuals who are observed both in the West and in the East during the time they are in their typical occupation, as defined below.

both periods. We then apply the following restrictions:

1. **Age restrictions**: We restrict our attention to individuals between 22 and 55 years. In the case of couples, we keep them in our sample if both partners are within this age range.[42]
2. **Marital Status Restrictions**: We focus on individuals who are either single or in heterosexual couples (married or cohabiting). We restrict our analysis to the first marital spell of the life of an individual, as discussed above. We drop observations corresponding to periods after the first marriage ended, or for which the end date of the first marriage cannot be identified. We drop observations for which we can identify more than one spouse/partner during the sample period.
3. **Labor Market Restrictions:**
   (a) We exclude from our sample observations corresponding to individuals working in odd occupations (`kldb92`$\geq$ 9711) or employed but with missing data on the occupation code.
   (b) We exclude observations corresponding to employed individuals for whom information on hourly wages is missing.
4. **Additional restrictions:**
   (a) We exclude observations from individuals to whom we cannot assign a 'type', based on our estimation of individual types described in Appendix E.2.
   (b) We exclude observations corresponding to individuals in couples but for whom information on the spouse/partner is not available in any of the years in which they appear in the sample.

D.2.2 Definition of Typical Occupation, Hours, Wages and Marital Status

In this section we explain the methodology to create summary variables for the individuals in the GSOEP panel (to which we refer to as 'typical variables'), in line with the static nature of our model. We define the concepts of typical occupation, typical working and home hours, and typical wages. For each individual we also define the typical marital status and the corresponding variables for their partner.

Typical occupation: we apply the following rules to determine the typical occupation of an individual:

1. If the individual appears in the sample only once, we assign them the occupation corresponding to that particular year.
2. If the individual appears more than once in the sample, but only reports one occupation, we assign them the unique occupation that we observe over the sample period.
3. If an individual appears more than once in the sample, and at some point was 'self-employed', 'studying' or 'not-employed', we check the duration of these states.
   (a) If the individual was either 'not-employed', 'self-employed' or 'studying' for strictly more than half of the time they appear in the sample, we consider that status as the typical occupation.
   (b) If an individual was in one of these states exactly half of the time they appear in the sample,

[42] Even if we impose the age restriction at the individual level, we would drop observations for married individuals for which information of their spouses is not available in any of the years they appear in the sample.

but they spent the other half on the other two status, we still classify them to that status.

(c) If the conditions above do not hold but the individual spent more than 75% of her time between 'not-employed', 'self-employed' and 'studying', even if there is another occupation, we assign them to 'not-employed', 'self-employed' and 'studying' depending on which one has the longest duration.

4. If we observe an individual more than once, and only in one occupation other than 'not employed', 'self-employment' and 'studying', and they spent less than 75% of the time in these three states, we assign the individual to that unique occupation.
5. If we observe an individual having more than one occupation during the period they appear in the sample, we follow these rules:
   (a) We construct the difference in percentiles between the highest ranked occupation and the lowest ranked occupation the individual held during the sample period, where we rank occupations as described in Appendix E.3.
   (b) When the difference is larger than 0.1, we assign the individual to the highest ranked occupation he held.
   (c) If this difference is lower than 0.1, we assign the individual to the occupation with the longest tenure.
   (d) If the difference is lower than 0.1, but we observe the individual in more than one occupation with the same tenure, we assign the individual to the better ranked occupation (between those with the longest tenure).
6. After applying all these rules, we exclude from our sample those individuals whose typical occupation is 'self-employment' or 'studying'.

Typical market hours: For each individual, we define typical market hours as the mean of the self-reported total worked hours (including overtime) during the years they were working in the typical occupation, defined as above. We winsorize market hours to 10 and 60 hours, at the bottom and the top, respectively. For those individuals whose typical occupation is 'not employed', typical market hours take value zero.

Typical wages: For each individual, we define typical wages as the mean of hourly wages earned in years the individual worked in the typical occupation.[43]

Typical home hours: For each individual, we construct typical home hours (defined as in Appendix D.1) as the mean of hours the individual works at home while employed in the typical occupation. We impute missing home hours using data on labor market hours, as discussed in Appendix D.1.

Typical marital status: We define the typical marital status following these rules:

1. If the individual had only one marital status during the sample period, we consider that marital status as the typical one.

[43] Hourly wages are constructed based on inflation adjusted monthly earnings, divided by monthly hours (constructed as weekly hours times 4.3). Hourly wages are trimmed at the bottom and top 1% percentile. Data to deflate wages comes from the OECD: https://data.oecd.org/price/inflation-cpi.htm

2. If the individual switched from being single to being married during the time they appear in the sample, we assign the marital status observed when employed in the typical occupation. If they were observed in both marital status while working in the typical occupation, we assign them to marriage.
3. We exclude from the sample those individuals that, even if assigned to marriage based on the previous rules, report more than one spouse during the time they appear in the sample. We also exclude their corresponding partners.

Corresponding variables for spouses: For every variable defined above (typical occupation, typical market and home hours, and typical wages) we define the analogous variable for the spouse.

Following the rules above, our *Baseline Estimation Sample* (West Germany, 2010-2016) has 3,857 individuals living in 2,326 households. Of these households, 1,531 are couples and 795 are single individuals (418 are single women and 377 are single men).

Our *Past Estimation Sample* (West Germany, 1990-1996) consists of 2,336 individuals, living in 1,294 households, of which 1,042 are couples and 252 are singles (117 single women and 135 single men).

E Estimation

E.1 Identification

Identification of the Worker and Job Distribution. We identify the distributions (G, N_s) directly from the data. We treat the distribution of occupational attributes G as observable. We identify the workers' human capital distribution N_s from workers' education and fixed effect in a panel wage regression. See Section 7.3 for the details on estimation.

E.1.1 Proof of Proposition 3

Identification of the Production Function. We follow arguments on the estimation of hedonic models to show identification of the production function z. In principle, this argument is non-parametric, but in line with our parametric estimation, we focus here illustrate the parametric approach. We mainly follow Ekeland, Heckman, and Nesheim (2004), Section IV.D, and also make use of their discussion of the identification strategy proposed by Rosen (1974) and criticized by Brown and Rosen (1982). The identification is based on the firm's FOC and exploits the non-linearity of our matching model, which is an important source of identification just as in Ekeland, Heckman, and Nesheim (2004). Recall the firm's FOC in the model is given by:

$$w_{\tilde{s}}(\tilde{s}) = z_{\tilde{s}}(\tilde{s}, \mu(\tilde{s})) \tag{34}$$

This equation can be used to identify the parameters of interest. There are two steps:

1. Estimate the marginal return $w_{\tilde{s}}$ as the derivative of the kernel regression of w (observed) on $\tilde{s}$. Denote this estimate by $\widehat{w_{\tilde{s}}}$. We treat the derivative of the wage as observable. Also note that we only observe $\tilde{s}$ for men in the data (for women, there is – at this stage – an unknown productivity wedge ψ), and so for this argument we focus on the subsample of men.

2. Estimate FOC (34) after applying a log transformation and taking into account measurement error:

$$\log(\widehat{w}_{\tilde{s}}(\tilde{s})) = \log(z_{\tilde{s}}(\tilde{s}, \mu(\tilde{s}))) + \epsilon \tag{35}$$

   where, for concreteness, we assume the functional form for z, $z(\tilde{s}, y) = A_z \tilde{s}^{\gamma_1} y^{\gamma_2} + K$ (see main text), and where we treat $\tilde{s}$ and the matching μ as observed. Note that this functional form of z circumvents the identification problem of Rosen (1974), discussed in Brown and Rosen (1982) and Ekeland, Heckman, and Nesheim (2004), since the slope of the wage gradient in $\tilde{s}$ is *not* equal to the slope of the marginal product in $\tilde{s}$. We assume that ϵ is the measurement error of the marginal wage, with mean zero and uncorrelated with the rhs variables. Regression (35) identifies $(A_z, \gamma_1, \gamma_2)$.

In turn, the constant in the production function K is identified from the wage of the lowest productive type $s = 0$ (and thus $\tilde{s} = 0$), with $w(0) = \int_0^0 z_{\tilde{s}}(t, \mu(t))dt + K = K$.

Identification of the Female Productivity Wedge. We can identify ψ from the within sh-type (agents with the same s and same work hours h) wage gap across gender. Denote the gender wage gap within individuals of hours-human-capital type $sh = \hat{s}\hat{h}$ in the data by $gap(\hat{s}\hat{h})$, which we treat as observed for any $\hat{s}\hat{h}$. We here focus on any 'interior' type with $\hat{h} > 0$. Moreover, to ease exposition, we focus on identifying $\psi \in [0, 1]$, as this is the empirically relevant case (but the argument can be extended to $\psi > 1$).

Then, given the wage function and our assumption that effective skill types of women and men are given by $\tilde{s}_f = \psi s_f h_f$ and $\tilde{s}_m = s_m h_m$, the observed gender wage gap at $\hat{s}\hat{h}$ can be expressed as:

$$gap(\hat{s}\hat{h}) = \frac{w(\hat{s}\hat{h}) - w\left(\psi\hat{s}\hat{h}\right)}{w(\hat{s}\hat{h})},$$

where we made the dependence of the female wage on ψ explicit. Note that (G, N_s) were identified directly from the data and so we observe which worker matches to which firm. Thus, we consider the matching μ as known at this stage.

Then for any observed $gap(\hat{s}\hat{h})$ with $0 \leq gap(\hat{s}\hat{h}) \leq 1 - K/w(\hat{s}\hat{h})$, the female wage is given by:

$$w\left(\psi\hat{s}\hat{h}\right) = w(\hat{s}\hat{h})(1 - gap(\hat{s}\hat{h})) \tag{36}$$

For a given (observed) μ, the RHS is independent of ψ, positive and finite. In turn, the LHS is positive and finite; and it is a continuous and strictly increasing function in ψ with $w\left(\psi\hat{s}\hat{h}\right) = K$ for $\psi = 0$ and

$w\left(\psi \hat{s}\hat{h}\right) = w(\hat{s}\hat{h})$ for $\psi = 1$.

Hence, one of the following is true: either there is an interior gap, $0 < gap(\hat{s}\hat{h}) < 1 - K/w(\hat{s}\hat{h})$, and so by the Intermediate Value Theorem there exists a unique $\psi \in (0,1)$ for which (36) holds; or, a minimal gap $gap(\hat{s}\hat{h}) = 0$ pins down $\psi = 1$ or a maximal gap $gap(\hat{s}\hat{h}) = 1 - K/w(\hat{s}\hat{h})$ pins down $\psi = 0$. Thus, ψ is identified from gender wage gaps of agents with the same hours-human-capital combination.

Identification of the Scale of the Labor Supply Shock. Recall that the choice set of singles differs from that of couples. In Appendix C, we introduced the notation where we denote the *alternative* of hours that a decision maker $t \in \{M, U\}$ chooses by $\mathbf{h}^t \in \{\mathcal{H} \cup \emptyset\}^2 := \{\{0, ..., 1\} \cup \emptyset\}^2$ with:

$$\mathbf{h}^t = \begin{cases} (h_i, \emptyset), i \in \{f, m\} & \text{if } t = U \\ (h_f, h_m) & \text{if } t = M. \end{cases}$$

where type $t = U$ indicates *unmarried* and type $t = M$ indicates *married.*

Also, we denote the sum of economic utility and utility derived from preference shocks of decision-maker t with human capital type $\mathbf{s} \in \{\mathcal{S} \cup \emptyset\}^2$ by $\overline{u}^t_{\mathbf{s}}(\mathbf{h}^t) + \delta^{\mathbf{h}^t}$, where

$$\overline{u}^t_{\mathbf{s}}(\mathbf{h}^t) + \delta^{\mathbf{h}^t} = \begin{cases} u(c_i, p^U(1 - h_i)) + \delta^{h_i}, i \in \{f, m\} & \text{if } t = U \\ u(c_f, p^M(1 - h_m, 1 - h_f)) + u(c_m, p^M(1 - h_m, 1 - h_f)) + \delta^{h_f} + \delta^{h_m} & \text{if } t = M. \end{cases}$$

The probability that household type t with human capital $\mathbf{s}$ chooses hours alternative $\mathbf{h}$ is

$$\pi^t_{\mathbf{s}}(\mathbf{h}^t) \quad = \quad \frac{\exp(\overline{u}^t_{\mathbf{s}}(\mathbf{h}^t)/\sigma_\delta)}{\sum_{\tilde{\mathbf{h}} \in \{\mathcal{H} \cup \emptyset\}^2} \exp(\overline{u}^t_{\mathbf{s}}(\tilde{\mathbf{h}}^t)/\sigma_\delta)} \tag{37}$$

which follows from our assumption on the preference shock distribution (Type-I extreme value).

Let $\mathbf{h}^U = \mathbf{0} := (0, \emptyset)$ denote the hours for a single who puts all available time into home production and works zero hours in the labor market. We consider alternative $\mathbf{h}^U = \mathbf{0}$ as our normalization choice and obtain for a single male of human capital type $\mathbf{s} = (s_m, \emptyset))$ the relative choice probabilities:

$$\begin{aligned} \frac{\pi^U_{\mathbf{s}}(\mathbf{h}^U)}{\pi^U_{\mathbf{s}}(\mathbf{0})} &= \frac{\exp(\overline{u}^U_{\mathbf{s}}(\mathbf{h}^U)/\sigma_\delta)}{\exp(\overline{u}^U_{\mathbf{s}}(\mathbf{0})/\sigma_\delta)} \\ \log\left(\frac{\pi^U_{\mathbf{s}}(\mathbf{h}^U)}{\pi^U_{\mathbf{s}}(\mathbf{0})}\right) &= \frac{\overline{u}^U_{\mathbf{s}}(\mathbf{h}^U) - \overline{u}^U_{\mathbf{s}}(\mathbf{0})}{\sigma_\delta} \\ &= \frac{w(s_m h_m) - w(s_m 0) + p^U(1 - h_m) - p^U(1 - 0)}{\sigma_\delta} \\ &= \frac{w(s_m h_m) + p^U(1 - h_m) - p^U(1)}{\sigma_\delta} \end{aligned} \tag{38}$$

where the wage from not working is set to zero and where h_m is the male hours associated to this single household's hours choice, $\mathbf{h}^U = (h_m, \emptyset)$. We treat human capital types as observed at this stage and consider two single types $\mathbf{s}' = (s'_m, \emptyset)$ and $\mathbf{s}'' = (s''_m, \emptyset)$. Then we can consider the difference in relative choices of these two single men:

$$\log\left(\frac{\pi^U_{\mathbf{s}'}(\mathbf{h}^U)}{\pi^U_{\mathbf{s}'}(\mathbf{0})}\right) - \log\left(\frac{\pi^U_{\mathbf{s}''}(\mathbf{h}^U)}{\pi^U_{\mathbf{s}''}(\mathbf{0})}\right) = \frac{1}{\sigma_\delta}\left(w(s'_m h_m) - w(s''_m h_m)\right).$$

The LHS is observed in the data (how does the relative choice probability for hours alternative $\mathbf{h}^U \neq \mathbf{0}$ change in the population of male singles as one varies human capital s_m), and on the RHS, the wage difference (it is the effect of men's human capital on wages given the hours choice $\mathbf{h}^U \neq \mathbf{0}$) is also observed and different from zero as the wage strictly increases in human capital. Thus, σ_δ is identified.

Identification of the Home Production Function. Let $\mathbf{h}^M = \mathbf{1} := (1, 1)$ denote the vector of hours for couples in which both spouses put zero hours into home production and thus work full time in the labor market. Alternative $\mathbf{h}^M = \mathbf{1}$ is our normalization choice and we obtain the relative choice probabilities of married couple $\mathbf{s}$ of choosing hours $\mathbf{h}^M \neq \mathbf{1}$ versus $\mathbf{h}^M = \mathbf{1}$ as:

$$\begin{aligned}\frac{\pi^M_{\mathbf{s}}(\mathbf{h}^M)}{\pi^M_{\mathbf{s}}(\mathbf{1})} &= \frac{\exp(\overline{u}^M_{\mathbf{s}}(\mathbf{h}^M)/\sigma_\delta)}{\exp(\overline{u}^M_{\mathbf{s}}(\mathbf{1})/\sigma_\delta)} \\ \log\left(\frac{\pi^M_{\mathbf{s}}(\mathbf{h}^M)}{\pi^M_{\mathbf{s}}(\mathbf{1})}\right) &= \frac{\overline{u}^M_{\mathbf{s}}(\mathbf{h}^M) - \overline{u}^M_{\mathbf{s}}(\mathbf{1})}{\sigma_\delta} \\ &= \frac{w_f(\psi s_f h_f) - w_f(\psi s_f) + w_m(s_m h_m) - w_m(s_m) + 2p^M(1-h_m, 1-h_f)}{\sigma_\delta} \end{aligned} \tag{39}$$

where we used that $2p^M(0,0) = 0$ by assumption in our quantitative model. Note that the LHS of (39) (relative choice probabilities) is observed, and on the RHS, wages of men and women with types (s_f, s_m) conditional on hours are also observed in the data, and σ_δ is known at this stage. Thus, home production function p^M is non-parametrically identified since we can specify (39) for all hours alternatives $\mathbf{h}^M$ chosen in the data. Note that we can identify p^M from a couple of any type $\mathbf{s} = (s_f, s_m)$.

By a similar argument the home production function of singles, p^U, is identified.

Identification of the Scale of the Marriage Taste Shock. In this section, we show that σ^M_β is identified once the parameters of the utilities are identified, where we will impose the following (mild) assumption.

Assumption D1 (Identification)**.**
(Marriage Sorting.) There exists male types $(s'_m, s''_m) \in \mathcal{S}^2$, $s''_m > s'_m$, *and female types* $(s'_f, s''_f) \in \mathcal{S}^2$, $s''_f > s'_f$, *such that* $\frac{\eta_{(s''_f, s''_m)}}{\eta_{(s'_f, s''_m)}} \neq \frac{\eta_{(s''_f, s'_m)}}{\eta_{(s'_f, s'_m)}}$.

The assumption states that there is marriage market sorting, at least somewhere in the support of (s_f, s_m). To see this, consider the following case for illustration: Suppose that for all $s''_m > s'_m$, and $s''_f > s'_f$, $\frac{\eta_{(s''_f, s''_m)}}{\eta_{(s'_f, \hat{s}''_m)}} > \frac{\eta_{(s''_f, s'_m)}}{\eta_{(s'_f, \hat{s}'_m)}}$. In words, $\eta_{(s_f, s_m)}$ has the monotone likelihood ratio property, or equivalently, is log-supermodular. Then, higher male types s_m are matched with higher female types s_f in the first-order stochastic dominance sense, i.e. there is positive sorting in s-types.

Let $\eta_{(s_f,s_m)}$ be the probability that a man s_m chooses woman s_f on the marriage market, conditional on marrying. Under the assumption that the taste shock is extreme-value distributed (and following the same derivations as for the choice probabilities of hours), $\eta_{(s_f,s_m)}$ is given by:

$$\eta_{(s_f,s_m)} = \frac{\exp(\Phi(s_m, s_f, v(s_f)))/\sigma_\beta^M)}{\sum_{s'_f} \exp(\Phi(s_m, s'_f, v(s'_f)))/\sigma_\beta^M)}$$

where, as before, we denote by $\Phi(s_m, s_f, v(s_f))$ the expected value of man s_m from being married to woman s_f and paying her the transfer $v(s_f)$. This value is given by:

$$\begin{aligned}\Phi(s_m, s_f, v(s_f)) &:= \sigma_\delta\Big[\kappa + \log\Big(\sum_{\mathbf{h}^M\in\mathcal{H}^2} \exp\Big\{\overline{u}_{\mathbf{s}}^M(\mathbf{h}^M)/\sigma_\delta\Big\}\Big)\Big] - v(s_f) \\ &= \sigma_\delta\Big[\kappa + \log\Big(\sum_{\mathbf{h}^M\in\mathcal{H}^2} \exp\Big\{\frac{w(s_m h_m) + w\left(\psi s_f h_f\right) + 2p^M(1-h_m, 1-h_f)}{\sigma_\delta}\Big\}\Big)\Big] - v(s_f)\end{aligned}$$

Using the ratio of probabilities of choosing two different women s''_f and s'_f, we obtain:

$$\log\left(\frac{\eta_{(s''_f,s_m)}}{\eta_{(s'_f,s_m)}}\right) = \frac{\Phi(s''_f, s_m, v(s''_f)) - \Phi(s'_f, s_m, v(s'_f))}{\sigma_\beta^M}.$$

It follows from Assumption D1, that there exist types $(s'_f, s''_f, s'_m, s''_m)$ such that

$$\log\left(\frac{\eta_{(s''_f,s''_m)}}{\eta_{(s'_f,s''_m)}}\right) - \log\left(\frac{\eta_{(s''_f,s'_m)}}{\eta_{(s'_f,s'_m)}}\right) = \frac{\Phi(s''_f, s''_m, v(s''_f)) - \Phi(s'_f, s''_m, v(s'_f))}{\sigma_\beta^M} - \frac{\Phi(s''_f, s'_m, v(s''_f)) - \Phi(s'_f, s'_m, v(s'_f))}{\sigma_\beta^M} \neq 0.$$

which, using the expression for $\Phi(s_m, s_f, v(s_f))$ from above, we can spell out as:

$$\begin{aligned}\log\left(\frac{\eta_{(s''_f,s''_m)}}{\eta_{(s'_f,s''_m)}}\right) - \log\left(\frac{\eta_{(s''_f,s'_m)}}{\eta_{(s'_f,s'_m)}}\right) = \frac{\sigma_\delta}{\sigma_\beta^M}\Bigg(&\log\Big(\sum_{\mathbf{h}^M\in\mathcal{H}^2} \exp\Big\{\frac{w(s''_m h_m) + w\left(\psi s''_f h_f\right) + 2p^M(1-h_m, 1-h_f)}{\sigma_\delta}\Big\}\Big) \\ &- \log\Big(\sum_{\mathbf{h}^M\in\mathcal{H}^2} \exp\Big\{\frac{w(s''_m h_m) + w\left(\psi s'_f h_f\right) + 2p^M(1-h_m, 1-h_f)}{\sigma_\delta}\Big\}\Big) \\ &- \log\Big(\sum_{\mathbf{h}^M\in\mathcal{H}^2} \exp\Big\{\frac{w(s'_m h_m) + w\left(\psi s''_f h_f\right) + 2p^M(1-h_m, 1-h_f)}{\sigma_\delta}\Big\}\Big) \\ &+ \log\Big(\sum_{\mathbf{h}^M\in\mathcal{H}^2} \exp\Big\{\frac{w(s'_m h_m) + w\left(\psi s'_f h_f\right) + 2p^M(1-h_m, 1-h_f)}{\sigma_\delta}\Big\}\Big)\Bigg) \\ \neq 0\end{aligned}$$

Since the LHS is not zero by assumption, the RHS is also not zero. Moreover, all objects on the RHS are either observed (wages) or identified at this stage (home production function and σ_δ, except σ_β^M). We can solve this equation for σ_β^M, giving a unique solution. Thus, σ_β^M is identified.

E.2 Estimation of Worker Types

Sample Selection. Our sample consists of individuals in the GSOEP from 1984-2018 who are between 20 and 60 years old and are either married/cohabiting or single. We exclude individual-year observations when the individual indicated self-employment and when they worked in poorly defined occupations (`kldb92`$\geq$ 9711). We also exclude observations with missing information on education or with missing (not zero) labor force experience. Our panel consists of around 212,000 person-year observations.

Key Variables. For *weekly hours*, we use reported actual hours which, when positive, we winsorized by 10 hours from below and 60 hours from above to deal with outliers. For *labor force experience* we use the reported labor force experience, and we impute it by potential experience if this information is missing.[44] For *education*, we use three categories: In the group of *low education*, there are those whose highest degree is lower secondary, high school or vocational with weakly less than 11 years of education (around 35%). In the group of *medium education*, there are those with vocational degree and above 11 years of education (around 44%). In the group of *high education*, there are those with college degree or more (around 20%). For *occupation codes*, we use the variable `kldb92_current`, which consistently codes occupations across the entire panel. Our wage variable are *log hourly wages*, inflation-adjusted in terms of 2016 Euros. For the definition of 'demographical cells' in the selection stage below, we additionally use a variable that indicates whether *children below 3 years old* are in the household, *age bins* (≤ 25, > 25 and ≤ 40, > 40 and ≤ 50, > 50) and the *state* of residence.

Selection Equation. To account for selection into labor force participation in the wage regression, we first run a selection regression. To do so, we need an instrument that affects participation but is excluded from wage regression (22). Since the variation in participation in our sample is mainly due to women, we use the 'progressiveness' in an individual's narrowly defined demographic cell. We proxy by the share of females working in a narrowly defined demographic cell. Our cells are defined by a combination of state, year, age and an indicator whether a child below 3 years is in the household.[45] When defining this variable for a particular individual, we employ the 'leave-one-out' method and do not count the individual's labor force participation when computing this statistic. We further drop cells with less than five observations. We end up with around 2,500 cells with more than 5 observations. Note that we experimented with additional cell characteristics (education and country of origin) but there is a trade-off between number of observations by cell and making the cells more specific to the demographic groups. Defining the cells by these additional variables would imply to drop more than

[44] For men, potential and actual experience are almost perfectly correlated, which is why this imputation should work well for them. For women, the correlation is much lower, which is why we do not impute here.

[45] German children start kinder-garden when they are 3 years old. Before age 3, children are predominantly at home (in 2013, only 29% of children aged 0-2 were in daycare, Source: OECD (2016)), so age 3 is an important threshold when it comes to mothers' labor force participation.

twice as many observations due to small cell size.

Our assumption on this IV is that the following exclusion restriction holds: 'progressiveness of an individual's demographic cell' only affects wages through labor force participation but not in other ways.

We run the following probit selection regression:

$$emp_{it} = \alpha\, share_{j(i)t} + \sum_{ed \in \{voc,c\}} \alpha^{ed} x_{it}^{ed} + \beta_z' Z_{it} + \kappa_s + \rho_t + \epsilon_{it} \tag{40}$$

where the dependent variable is an indicator of whether individual i is employed at time t, $share_{j(i)t}$ is the progressiveness measure in the demographic cell j of individual i at time t (given by the share of women working in the cell, see description of this IV above), x_{it}^{ed} captures education indicators (indicator variables for medium and high education, so that low education is the reference group), Z_{it} is a vector of demographic individual controls (linear and quadratic labor force experience in years, household size) and κ_s and ρ_t are state and year fixed effects and ϵ_{it} is a mean-zero error term. We cluster standard errors on the cell level.

The results are in Table 8, where we label our IV $share_{j(i)t}$ by *Share of Working Women in Cell*. There is a strong positive effect of the share of women working in the demographic cell on labor force participation of an individual in that cell (coefficient of 1.355 with standard error 0.0284).

Table 8: Selection Regression

	(1)
	Employed
Share of Working Women in Cell	1.355***
	(0.0284)
Experience	0.0919***
	(0.00216)
Experience^2	-0.00164***
	(0.0000599)
Medium Educ	0.453***
	(0.00932)
High Educ	0.851***
	(0.0115)
HH Size	-0.0462***
	(0.00539)
Constant	-0.918***
	(0.0358)
Observations	212894
Pseudo R^2	0.144

Standard errors in parentheses

Controls that are included but not reported: state fixed effects, year fixed effects

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Wage Regression. We further restrict the sample to those that are employed and have non-missing hourly wage. Since we instrument hours worked and hours worked squared by (i) the hours worked by the partner, (ii) the hours worked by the partner squared and (iii) whether partner is present (where we set partner's hours to zero in both cases, if partner is present but not working and if partner is not present). We therefore drop observations whose partner reports to be employed but has zero reported labor hours and observations whose partner has missing employment and hours information. Since we include individual fixed effects we also drop singleton observations (those who only show up in a single year of the panel). This leads to a sample of 133,214 person-year observations. Based on our model wage function, we choose the following regression specification:

$$\ln w_{it} = \nu_i + \beta_0 IMR_{it} + \beta_1 h_{it} + \beta_2 h_{it}^2 + \sum_{ed \in \{voc,c\}} \alpha^{ed} x_{it}^{ed} + \beta_z' Z_{it} + \kappa_s + \rho_t + \epsilon_{it} \tag{41}$$

where ν_i is a person-fixed effect, IMR_{it} is the inverse mills ratio of individual i in year t from the selection probit regression, h_{it} are weekly hours worked, and x_{it}^{ed}, Z_{it}, κ_s and ρ_t are as in the selection regression (40). Note that we could not include h_{it} into the selection equation since $h_{it} = 0$ versus $h_{it} > 0$ is a perfect predictor of employment. Nevertheless, based on our model, it is important to control for hours worked in the hourly wage regression. We again cluster standard errors on the cell level.

Table 9 contains the results. In column (1) an (2) we report the first stage regressions (for two variables to be instrumented: weekly hours and weekly hours squared) and column (3) contains the second stage regression. The three IV's for the hours worked variables (partner's hours, partner's hours squared and partner present) are not subject to the weak instrument problem according to the F-statistics. Regarding the second stage, we note that the inverse mills ratio is positive and significant, indicating that individuals are positively selected into working and not controlling for selection here would have biased the coefficients upward. Moreover, we note that weekly hours worked have a strong positive effect on wages, justifying our model assumption that hours affect productivity and thus hourly wages. In particular, increasing hours from 30 to 40 hours per week yields an hourly wage return of $(40 \times 0.119 - 40^2 \times 0.00164) - (30 \times 0.119 - 30^2 \times 0.00164) = 0.042$, so of around 4%.

Imputation. Based on these results, we are able to obtain x-types and ν-types (and thus s-types) for around 17,000 individuals. We impute fixed effects of the remaining ones (around 11,600 individuals) based on the *multiple imputation* approach. As auxiliary variables in this imputation we choose covariate in our data set that are most correlated with the individual fixed effects (such as *education*, *gender* and *full time labor force participation*). After imputation, we use the subset of individuals for structural estimation the comply with our final sample restrictions, Appendix D.2. In our final estimation sample (for baseline period, 2010-2016), we have 3,857 unique individuals. For 24% of them we have imputed ν_i.

We then divide individuals into the three education groups and assess within each group whether an individual has a low (below median) or high (above median) fixed effect, so there are two subgroups in each education bin. We compute the subgroup fixed effect, $\overline{\nu}_j$, as the mean of the individuals fixed effects belonging to subgroup j. This way we obtain six subgroup fixed effects (two for each education group). Finally, we compute the human capital type for each individual as $s_i = \alpha^{ed} x_i^{ed} + \overline{\nu}_j(i)$ where $\overline{\nu}_j(i)$ is the fixed effect of individual i's subgroup j. We obtain six s-types. The resulting human capital distribution (s-types by education group) is displayed in Table 10.

Table 10: Worker Distribution of s-Types by Education

education \ s-type	.4416637	.4764665	.5525224	.6088882	.7007952	.8341857	Total
1 (low)	523	0	601	0	0	0	1,124
2 (medium)	0	617	0	1,046	0	0	1,663
3 (high)	0	0	0	0	447	623	1,070
Total	523	617	601	1,046	447	623	3,857

E.3 Estimation of Occupational Types

Sample Selection. Our main data source for measuring occupation types is the BIBB (see Section 3.1 for a detailed description). It contains data on task usage in 1,235 occupations defined by the 4-digit code `kldb92`, which we also use for our analysis in the GSOEP. This data is reported by individuals who work in these occupations. In order to reduce the problem of noisy reporting, we drop occupations in which the task information is based on less than 5 individuals. We are left with task data for 613 occupations. These are the most common occupations and we will base our structural estimation exercise on the subset (608 occupations) that can be merged to the occupations held by individuals in our GSOEP sample.

Task Data. The BIBB contains data on how intensely different 4-digit occupations use different types of tasks.[46] These intensity measures are continuous and we normalize them to be on the unit interval. The reported tasks are comprised of: Detailed Work, Same Cycle, New Tasks, Improve Process, Produce Items, Tasks not Learned, Simultaneous Tasks, Consequence of Mistakes, Reach Limits, Work Quickly, Problem Solving, Difficult Decisions, Close Gaps of Knowledge, Responsibility for Others, Negotiate, Communicate.

Model Selection Stage. We merge the task data from the BIBB into occupations held by individuals in the GSOEP. As with the worker types, we here use the entire SOEP panel (here: pooled). We run a Lasso regression of log hourly wages on the task descriptors listed in the last paragraph in order to systematically select the tasks that matter for pay. This procedure selects 13 tasks (all tasks from the dataset except: Improve Process, Consequence of Mistakes and Difficult Decisions).

Principal Component Analysis. Because we want to reduce the occupational type to a single dimension, we collapse the information of the 13 selected tasks into a single measure using a standard

[46] For confidentiality reasons, the BIBB Public Use Files do not contain information on the 4-digit kldb 1992 classification of occupations of each individual (it contains 3-digit levels). However, we were able to obtain summary measures of occupational task content at the 4-digit level, kldb 1992, without reference to individual identifiers.

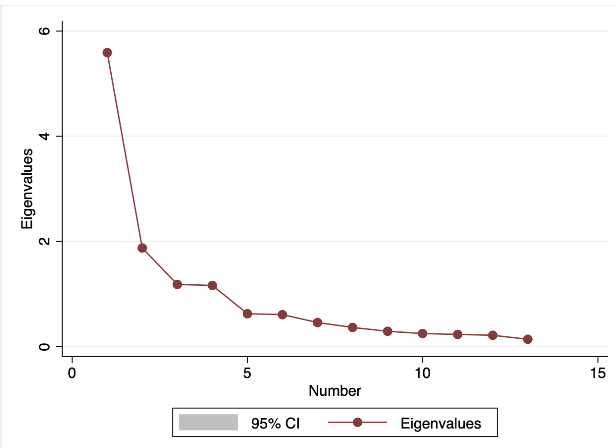
Table 9: Wage Regression

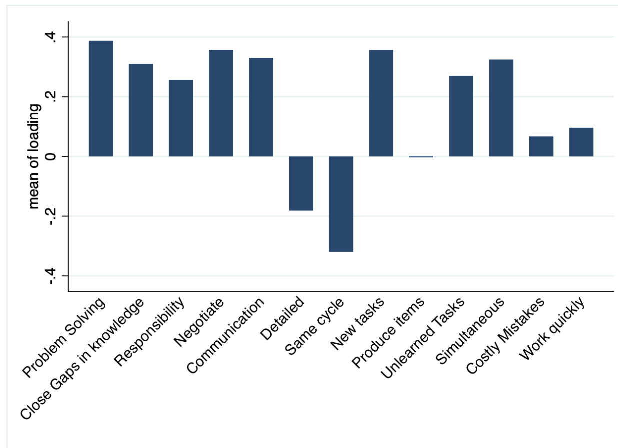
	(1) Weekly Hours Worked	(2) Weekly Hours Worked^2	(3) Log Hourly Wage
Partner's Weekly Hours Worked	-0.0574***	-4.607***	
	(0.00480)	(0.389)	
Partner's Weekly Hours Worked^2	0.000895***	0.0796***	
	(0.000105)	(0.00857)	
Partner Present	1.133***	60.85***	
	(0.164)	(12.57)	
Experience	0.377***	28.53***	0.0332***
	(0.0251)	(1.718)	(0.00182)
Experience^2	-0.00460***	-0.362***	-0.000555***
	(0.000399)	(0.0298)	(0.0000286)
Medium Educ	0.242	27.28*	0.0264**
	(0.216)	(15.89)	(0.0103)
High Educ	4.289***	299.8***	0.195***
	(0.458)	(30.93)	(0.0244)
HH Size	-1.280***	-80.32***	0.0218***
	(0.0510)	(3.495)	(0.00467)
Inverse Mills Ratio	-3.519***	-215.2***	0.133***
	(0.255)	(18.38)	(0.0197)
Weekly Hours Worked			0.119***
			(0.0156)
Weekly Hours Worked^2			-0.00164***
			(0.000211)
Observations	133214	133214	133214
F	69.92	59.52	162.902
R^2			-0.386

Standard errors in parentheses
Controls that are included but not reported: state fixed effects, year fixed effects.
Columns (1) and (2) are the first stage regressions with weekly hours worked and weekly hours worked squared as dependent variables. Column (3) is the second stage regression. Weekly hours and weekly hours squared are instrumented in the second stage.
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

dimension reduction technique (principal component analysis). We then select the first principal component, which captures the most variation of the underlying task variables in the sample of employed workers in the SOEP: It captures around 43% of the underlying variation and – based on the loadings on the underlying task descriptors – our interpretation of this component is *task complexity* or *high skill requirement*. This interpretation is based on positive loadings on all task variables except *detailed work* and *same cycle*, which arguably are the only tasks in the dataset that indicate routine work. We report a scree-plot with eigenvalues of the different principal components and a plot with loadings of the first PC in Figure 14, left and right panel, respectively.

Figure 14: Principal Component Analysis





We then compute the mean of this measure by occupation and denote it by $\hat{y}$. Once matched to our main sample (see Appendix D.2), we define our final measure of occupational type as the ranking of occupations in the task complexity distribution, i.e. $y = \widehat{G}(\hat{y})$, where $\widehat{G}$ is the cdf of $\hat{y}$. So $y \sim G$ where $G = U[0, 1]$. The reason for this transformation is that the occupational task data only has an ordinal interpretation. Note that our production function is flexible enough to capture the true output as a function of non-transformed types $\widehat{y}$. Examples of occupations in the top 5% of the G distribution include engineers and programmers. Examples of occupations in the lowest 5% include janitors and cleaners.

Alternative Approaches. We considered 2 alternative approaches for the estimation of occupational types. The first alternative is to not use wage data at all but instead, to directly reduce the multi-dimensional task data to a single measure by PCA. The second alternative is to rely more heavily on wages by first selecting important tasks via Lasso and then using the *predicted* wage based on these important tasks as our measure for occupation types. With all three approaches, we get very similar results. We show the summary statistics and correlations of all three measures in Tables 11 and 12.

We also considered determining the occupational types through an occupational fixed effect in the wage regression we used to recover worker types (equation (22)). This would have meant to run a two-way fixed effects regression. We chose our alternative approach that does not rely on occupational fixed effects for the following reasons: First, based on our model featuring a competitive labor market, the wage function does not depend on an occupational fixed effect/type when controlling for workers' effective types: all workers with the same effective type $\tilde{s}$ should be matched to the same occupation. Second, the two-way fixed effects approach is known to be problematic under limited worker mobility (limited mobility bias) and when one is interested in sorting (since the correlation between worker and firm/occupation fixed effects is not accurately capturing sorting).

Table 11: Summary Statistics of Alternative Measures

	mean	sd	min	max
y (baseline)	.573	.193	0	1
y(PCA)	.585	.188	0	1
y(Lasso)	.565	.196	0	1

Table 12: Correlation Across Alternative Measures

	y	y (PCA)	y (Lasso)
y (baseline)	1		
y (PCA)	0.983***	1	
y (Lasso)	0.946***	0.922***	1
Observations		608	

y is our baseline measure based on two steps: Lasso and PCA.
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

E.4 Internal Estimation

E.4.1 Parameters Set Outside the Model

Table 13: Exogenously Fixed Parameters

Parameter		**Value**
Hourly Minimum Wage	K	6.32
Labor Supply Shock (location)	$\bar{\delta}$	0.00
Preference Shock for Partners (location)	$\bar{\beta}^M$	0.00
Preference Shock for being Single (location)	$\bar{\beta}^U$	0.00
Preference Shock for being Single (scale)	σ_β^U	29.25

E.4.2 Construction of Moments

Our main estimation targets the 17 moments defined in Table 14. Below we provide details on how these moments are constructed both in the data and in the model.

For moment M1, we compute the female to male ratio in labor force participation on the whole sample (married individuals and single). We define labor force participation in the data and in the model as a dichotomous variable that takes value 1 when the individual work positive hours, and zero otherwise.

For moment M2 we compute the ratio of full time work by women to full time work by male, where full time in the data is defined as working more than 37.5 hours per week, which corresponds to working more than 46% of the available time.[47] In the model, we define full time as working more than 46% of total time in the market, corresponding to the 5th entry in our hours grid. We consider both individuals that are single and in couples. M2 takes value 0 when the individual either works part time or does not work, and takes value 1 when working full time. For M3 and M4, we compute the married to single ratio in labor force participation, for men and women separately. In M14-M15, we compute the female labor force participation rate for women in couples where both partners are of similar type. For M14, we pool couples where both partners are either of type 3 or type 4, while for M15 we pool couples in which both partners are either of type 5 or type 6. For M16 and M17, we focus only on the sample of single women. M16 computes the female labor force participation of single women of types 3 and 4, while M17 pools together single women of types 5 and 6.

In M5, we construct home hours $(1-h)$ as the share of home hours in total time (defined as hours worked at home plus hours worked in the market), in both data and model. To deal with the fact that different individuals report different number of total hours allocated to home production and market work, we first constructed a common denominator for all individuals, given by the 95th percentile of the sum of home and market hours (81.6 hours per week in our estimation sample). Then, we use the hours allocated to home production by each individual to construct their share of home hours in total time, using this common denominator.[48] M5 is computed based on the sample of individuals in couples.

Wage moments (moments M6-M9) use data on hourly wages for all individuals in our estimation sample (singles and in couples), conditional on employment. The model counterpart of these moments are constructed in the same way.

Moment M10 is constructed as the correlation of partners' s-types in both data and model. Moment M11 is the share of single men in the sample, both in data and model.

Finally, moments M12 and M13 measure the gender wage gap by (s, h)-types. To construct these moments in the data, we focus on two (s, h)-type combinations: all individuals (singles or in couples) of s-type 2 and s-type 4 that work full time in the labor market. We define full time work in the data and the model as described above.

[47] To determine the total available time, we take the percentile 95th of the distribution of total time spent in the labor market and in home production and add them, which is 81 hours in our data.

[48] For those individuals that report more than 81.6 hours of home production, we assign them the value of the 95th percentile, to avoid that the share of time spent in home production is larger than 1.

Table 14: Moments

Moment Description	**Definition**
Labor Force Participation Female to Male Ratio (M1)	$\frac{Pr(h_f>0)}{Pr(h_m>0)}$
Full Time Work Female to Male Ratio (M2)	$\frac{Pr(h_f=\hat{h})}{Pr(h_m=\hat{h})}, \hat{h} \geq 37.5$
Labor Force Participation Married to Single Ratio by Gender (M3-M4)	$\frac{Pr(h_i>0\|Married)}{Pr(h_i>0\|Single)}, i \in \{f, m\}$
Correlation of Spouses' Home Hours (M5)	$corr(1-h_f, 1-h_m)$
Mean Hourly Wage (M6)	$\mathbb{E}[w]$
Variance Hourly Wage (M7)	$Var[w]$
Upper Tail Wage Inequality (M8)	$\frac{w^{90}}{w^{50}}$
Overall Wage Inequality (M9)	$\frac{w^{90}}{w^{10}}$
Correlation between Spouses' Types (M10)	$corr(s_m, s_f)$
Fraction of Single Men (M11)	$\frac{\#SingleM}{\#M}$
Gender Wage Gap by Effective Type (M12-M13)	$\frac{\mathbb{E}[w(h_i s_i)\|i=m,h_i=\hat{h},s_i=\hat{s}]-\mathbb{E}[w(s_i h_i)\|i=f,h_i=\hat{h},s_i=\hat{s}]}{\mathbb{E}[w(h_i s_i)\|i=m,h_i=\hat{h},s_i=\hat{s}]}$
Female Labor Force Participation by Couple Type (M14-M15)	$Pr(h_f > 0\|s_f = s_m = \hat{s})$
Female Labor Force Participation of Single Women by Type (M16-M17)	$Pr(h_f > 0\|Single, s_f = \hat{s})$

E.4.3 Results

Table 15: Targeted Moments

	Model	Data
M1. Labor Force Participation Female to Male Ratio	0.7426	0.7864
M2. Full Time Work Female to Male Ratio	0.4046	0.3834
M3. Labor Force Participation Married to Single Ratio, Men	0.8612	0.8556
M4. Labor Force Participation Married to Single Ratio, Women	0.9890	1.2534
M5. Correlation of Spouses Home Hours	0.3159	0.3120
M6. Mean Hourly Wage	17.7271	17.6354
M7. Variance Hourly Wage	51.1067	53.9061
M8. Upper Tail (90-50) Wage Inequality	3.0852	2.9686
M9. Overall (90-10) Wage Inequality	1.7294	1.7271
M10. Correlation between Spouses Types	0.4403	0.4468
M11. Fraction of Single Men	0.2055	0.1976
M12. Gender Wage Gap by Effective Type 2	0.1227	0.1557
M13. Gender Wage Gap by Effective Type 4	0.1414	0.1464
M14. Female Labor Force Participation by Couple Types 3 and 4	0.7242	0.7308
M15. Female Labor Force Participation by Couple Types 5 and 6	0.8468	0.8071
M16. Female Labor Force Participation of Single Women Type 3 and 4	0.8076	0.7320
M17. Female Labor Force Participation of Single Women Type 5 and 6	0.8583	0.8429

Notes: Moments are computed as discussed in Table 14.

Table 16: Un-targeted Moments: Marriage Matching Frequencies - Model and (Data)

	Low Educ Men	**Medium Educ Men**	**High Educ Men**	**Single Women**
Low Educ Women	0.0820 (0.0747)	0.0721 (0.0449)	0.0230 (0.0126)	0.0361 (0.0365)
Medium Educ Women	0.1016 (0.0860)	0.1246 (0.2159)	0.0984 (0.0695)	0.0787 (0.0747)
High Educ Women	0.0361 (0.0149)	0.0459 (0.0485)	0.0754 (0.0986)	0.0557 (0.0562)
Single Men	0.0557 (0.0527)	0.0656 (0.0714)	0.0492 (0.0430)	0.0000

Notes: *Low Educ* includes high school and vocational education with less than 11 years of schooling. *Medium Educ* is defined as vocational education with more than 11 years of schooling. *High Educ* is defined as college and more. Data frequencies are shown in parenthesis.

F Quantitative Analysis

F.1 Comparative Statics

Figure 15: a. Gender Wage Gap, b. Household Income Variance, c. Income Variance Decomposition, d. Marriage Market Sorting, e. Gender Gap Labor Hours, f. Gender Gap Labor Market Sorting.

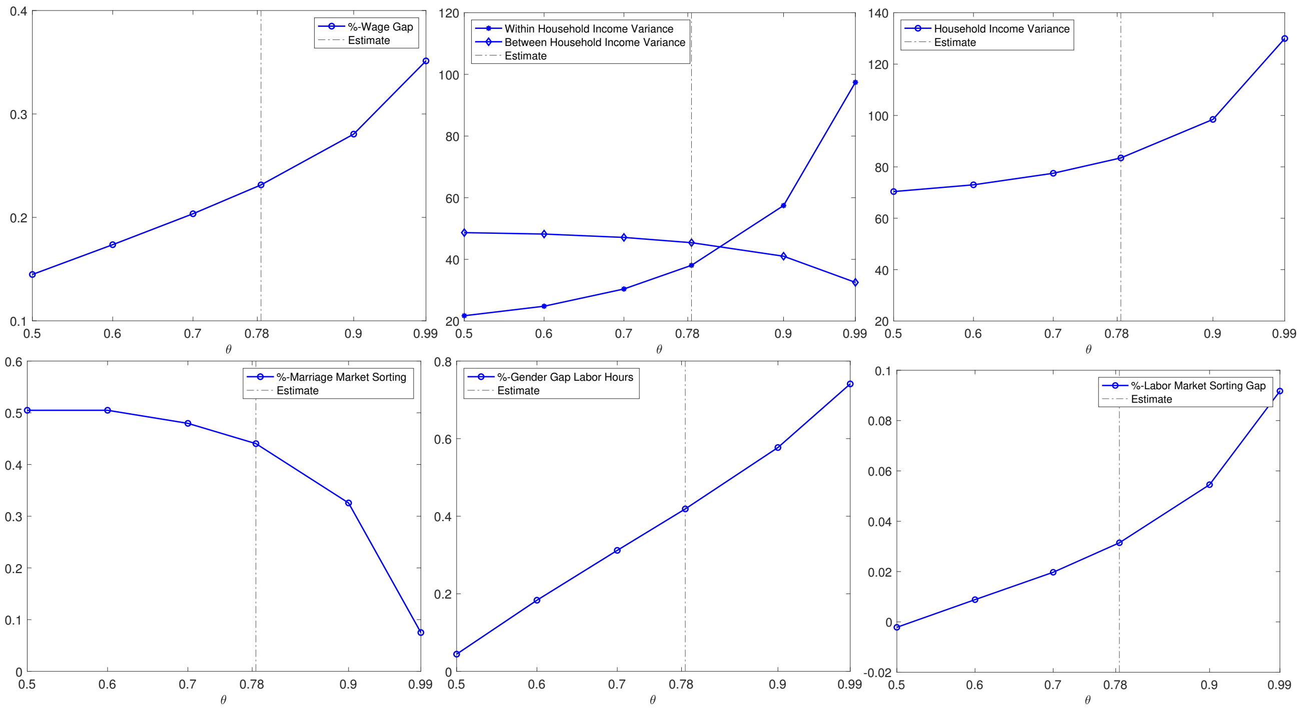
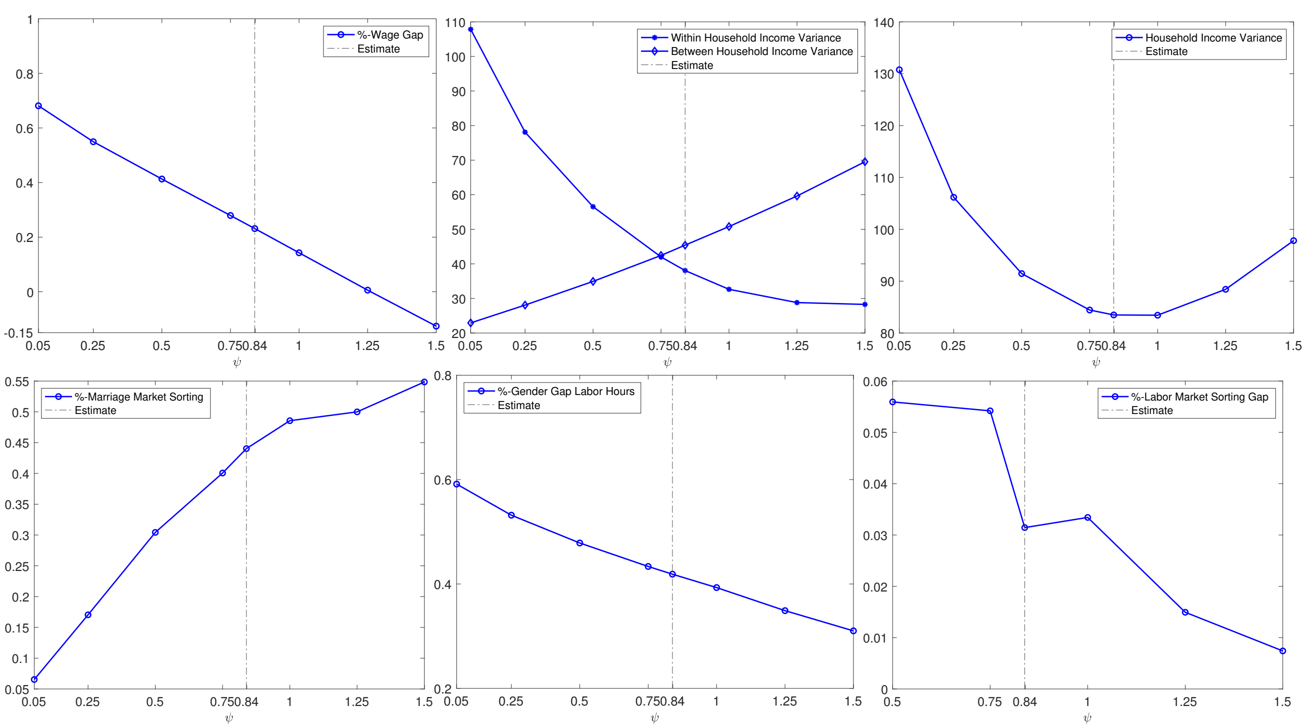


Figure 16: a. Gender Wage Gap, b. Household Income Variance, c. Income Variance Decomposition, d. Marriage Market Sorting, e. Gender Gap Labor Hours, f. Gender Gap Labor Market Sorting.



F.2 Inequality Over Time

Table 17: Data and Model moments: 1990-1996 versus 2010-2016

	Past Period		**Current Period**		**Data Diff.**
	Model	Data	Model	Data	p-value
M1. Labor Force Participation Female to Male Ratio	0.6266	0.5875	0.7426	0.7864	0.0000
M2. Full Time Work Female to Male Ratio	0.2374	0.3336	0.4046	0.3834	0.2324
M3. Labor Force Participation Married to Single Ratio, Men	0.7768	0.6343	0.8612	0.8556	0.0190
M4. Labor Force Participation Married to Single Ratio, Women	1.0057	1.0844	0.9890	1.2534	0.0000
M5. Correlation of Spouses Home Hours	0.1672	0.1518	0.3159	0.3120	0.0019
M6. Mean Hourly Wage	16.9807	17.0106	17.7271	17.6354	0.0000
M7. Variance Hourly Wage	35.3776	37.2476	51.1067	53.9061	0.0026
M8. Upper Tail (90-50) Wage Inequality	2.5262	2.3486	3.0852	2.9686	0.0000
M9. Overall (90-10) Wage Inequality	1.5604	1.5841	1.7294	1.7271	0.0000
M10. Correlation between Spouses Types	0.3864	0.4052	0.4403	0.4468	0.0000
M11. Fraction of Single Men	0.1186	0.1147	0.2055	0.1976	0.2069
M12. Gender Wage Gap by Effective Type 2	0.1682	0.1814	0.1227	0.1557	0.0000
M13. Gender Wage Gap by Effective Type 4	0.1719	0.1839	0.1414	0.1464	0.4885

Notes: Moments are computed as discussed above in Appendix E.4. The last column of the table reports the p-value of the hypothesis test of the differences between the data moments in the two samples being zero. We use a standard T-test for differences in means (M6), a standard Levene test for differences in variances (M7) and standard tests for differences in proportions (M11). We use a Fisher transformation to construct the test statistic for the differences in correlations between samples (M5 and M10). We use a two-sample Wald test for differences in ratios across samples (M1, M2, M3, M4, M8, M9, M12 and M13). To construct the statistic for the Wald tests for difference in ratios, we use bootstrap techniques for the variance estimation.

Table 18: Estimated Parameters: 1990-1996 versus 2010-2016

		Past Period		**Current Period**	
		Estimate	s.e.	Estimate	s.e.
Female Relative Productivity in Home Production	θ	0.88	0.01	0.78	0.05
Complementarity Parameter in Home Production	ρ	-0.16	0.19	-0.54	0.32
Home Production TFP	A_p	38.48	1.52	41.38	2.34
Elasticity of Output w.r.t. $\tilde{s}$	γ_1	0.41	0.07	0.59	0.08
Elasticity of Output w.r.t. y	γ_2	0.16	0.12	0.16	0.13
Production Function TFP	A_z	40.50	5.17	42.33	4.57
Female Productivity Wedge	ψ	0.76	0.03	0.84	0.03
Preference shock for Partners (scale)	σ_β^M	0.05	0.01	0.19	0.02

Notes: s.e. denotes standard errors. See Section 7.4 for a description of how these standard errors are computed.

Figure 17: Inequality Changes Over Time: Detailed Decomposition