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Problem solving - what have we learned since Polya's introspection

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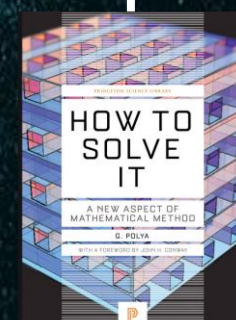
twitter: #PolyaPS



Solving the Problem of Powerful Instruction

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There is only one place to start, one person to start with, when you talk about problem solving:

George Pólya

When I finished my first paper on problem solving, I sent it to Pólya with this note:

"Isaac Newton wrote:

'If I have seen further, it is by standing on the shoulders of giants.'

Now that I work in problem solving, I truly understand the meaning of his words."

So, let us turn to the task:

Solving the problem of Powerful Instruction

- or -

What are the properties of classrooms that produce students who are powerful (mathematical) thinkers and problem solvers?

This is a very challenging problem, so we should approach it using one of Pólya’s strategies:

“If you cannot solve the given problem, try to decompose it into a series of subproblems.”

Here are 3 problems in sequence:

1. Can we understand and support individual students' problem solving?
2. Can we understand teachers' decision making?
3. Can we understand productive learning environments?

1. Can we understand and support individual students' problem solving? (1975-1985)

The challenge was: Pólya's descriptions felt right. But, how could we help students learn to *use* the heuristic strategies he described?

A Brief Overview

- What is Problem Solving?
- Aspects of Problem Solving, in Mathematics *and in Writing* (!)
- Examples and Evidence

What is Problem Solving?

A Working Definition:

You are engaged in Problem Solving when you are trying to achieve something, and you do not know a straightforward way to do so.

Examples:

Finding the product of two 37-digit numbers is NOT problem solving. (It's hard and you may goof, but you know how to do it.)

Writing an essay trying to convince someone of your perspective; and

Working a mathematics problem where you have to make sense of it and figure out what to do,

ARE acts of problem solving.

The Big Picture

The following four categories of knowledge determine the quality (and success) of problem solving attempts:

(i) the knowledge base

(ii) problem solving strategies (heuristics)

(iii) "control": monitoring and self-regulation, or metacognition

(iv) beliefs, and the practices that give rise to them.

Category 1:

The knowledge base

The Knowledge Base

What you know makes a difference.

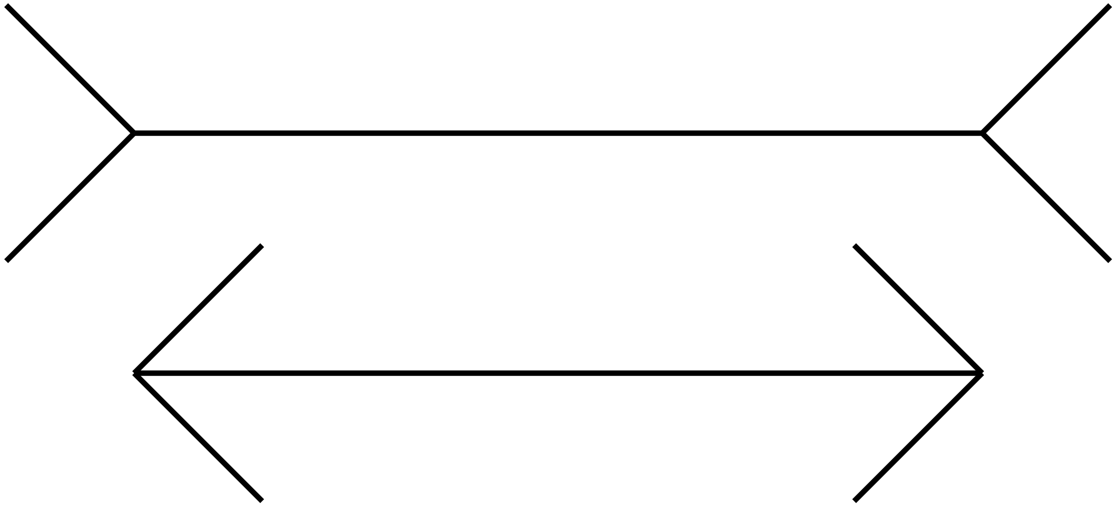
(No surprise there!)

But, the nature of "knowing" is more complex than you might think.

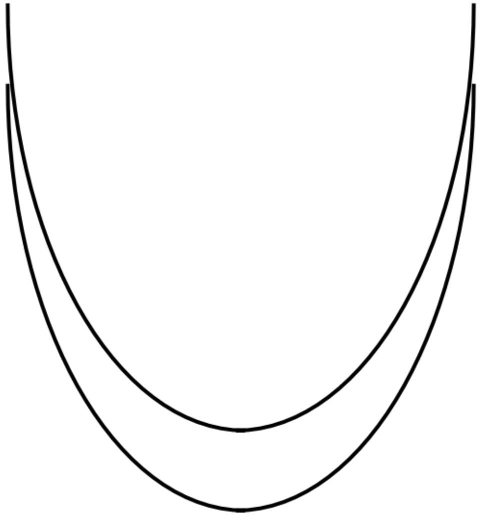
For example, we do *not* perceive reality directly!

If we did, optical illusions would be impossible…

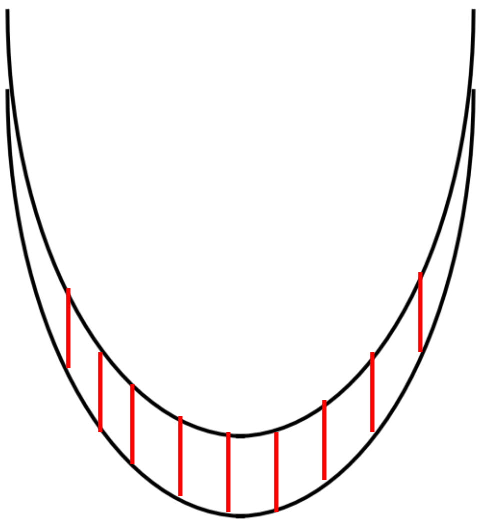
For example, which horizontal line segment is longer?



Here is another picture. What can you tell me about the two curves?



In fact, the curve on the top is a vertical translation of the curve on the bottom!



The point is that we construct "interpretive filters" that shape what we see…

And understand!

Here is another indication we know more about your thought processes than you might think…

Memorize the following numbers.

687 and 492

Then close your eyes and try to do the multiplication in your head:

$$\begin{array}{r} 687 \\ \times \ \underline{492} \end{array}$$

I’m waiting…

You just can’t do it, can you?

Category 2:

Problem Solving Strategies

Problem Solving Strategies
(Also Known as Heuristics)

Examples in Writing:

- Organize and outline the paper.
- Use Topic Sentences for paragraphs.
- Simple writing instructions:

  Tell them what you're going to tell them

  Tell them

  Tell them what you told them

In Mathematics:

Here are some of the problem solving strategies described in George Pólya’s book *How to Solve It:*

- draw a diagram
- look at cases
- solve an easier related problem…

The issue: Pólya's strategies may sound simple, but they' re not as easy to use as you might think.

For example, consider the strategy "Make sense of the problem by looking at examples."

How do you know which examples to look at?

Here are some problems…

- What is the sum of the first n odd #s?
- What is the sum of the numbers

$$\frac{1}{1\text{x}2} + \frac{1}{2\text{x}3} + \frac{1}{3\text{x}4} + \frac{1}{4\text{x}5} + \ldots + \frac{1}{(n)\ \text{x}\ (n+1)}\ ?$$

Try n = 1,2,3,4,5 and look for a pattern.

Let

$$P(x) = a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^n$$

and

$$Q(x) = a_n + a_{n-1} x + a_{n-2} x^2 + \ldots + a_0 x^n$$

What can you say about the relationship between the roots of P(x) and the roots of Q(X)?

Select *easily factorable* polynomials.

Given a_0 and a_1,

define $$a_{n+1} = \frac{1}{2}(a_{n-1} + a_n).$$

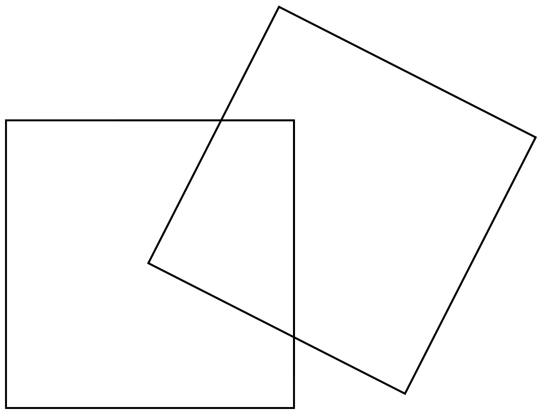
Does $\underset{n \to \infty}{Lim}(a_n)$ exist?

If so, what is it?

**Pick *nice* values such as 0 and 1. **

Take two squares the same size; put a corner of one on the center of the other.

What is the maximum intersection?
What is the minimum intersection?



Pick special orientations - e.g., at 0 or 45 degrees to the horizontal.

Of all the triangles with perimeter P, which one has the largest area?

A range of empirical values may give you a “feel” for the answer…

Steps in using a simple strategy like "Exploiting an easier related problem"

1. Think to use the "strategy".
2. Know which version of the strategy to use.
3. Generate appropriate and potentially useful easier related problems.
4. Select the right easier related problem.
5. Solve it.
6. Be able to exploit it….

   The Moral: The strategies are tough,and you need detailed training.

The Results

Students solved problems I couldn't.

Category 3:

"Control": Monitoring and Self-Regulation, or Metacognition

Monitoring and Self-Regulation, or Metacognition

What matters isn't simply what you know – it's how and when you use what you know!

Examples from writing

- Does your paper (or letter, or…) meander, because you’ve lost track of the argument?
- Is it incomprehensible because *you* know the reasons behind what you’re saying but you haven’t told the readers?
- Have you lost track of your audience?

A math example:

Determine

$$\int \frac{x}{x^2 - 9}\, dx.$$

Half the students used the substitution

$$u = x^2 - 9.$$

Half of the remaining students used the technique of partial fractions:

$$\frac{1}{x^2 - 9} = \frac{A}{x - 3} + \frac{B}{x + 3}.$$

And half of the rest used a trig substitution,

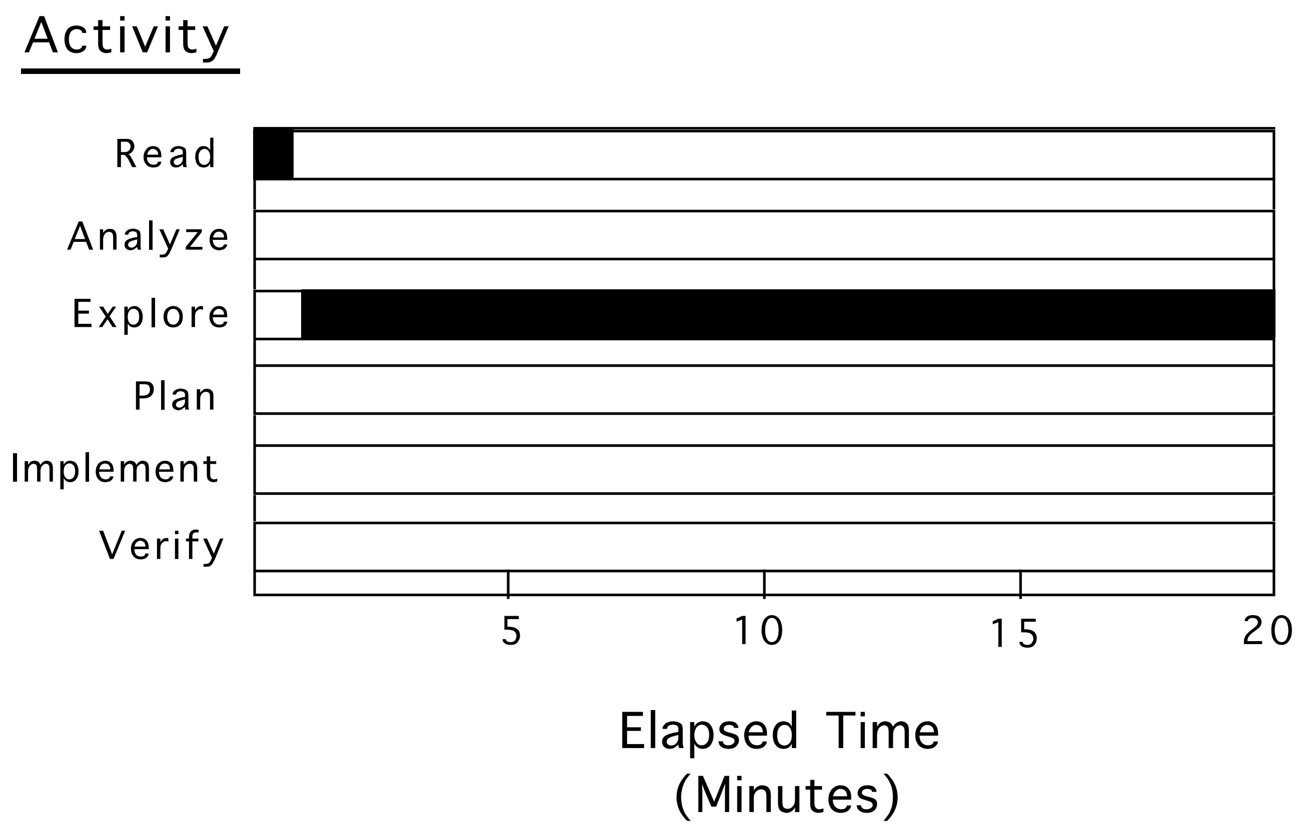
$$x = 3\sin\alpha.$$

They violated a fundamental rule of problem solving:

Never do anything difficult until you have made sure you need to!

I could show you students going off on a 20-minute wild goose chase…

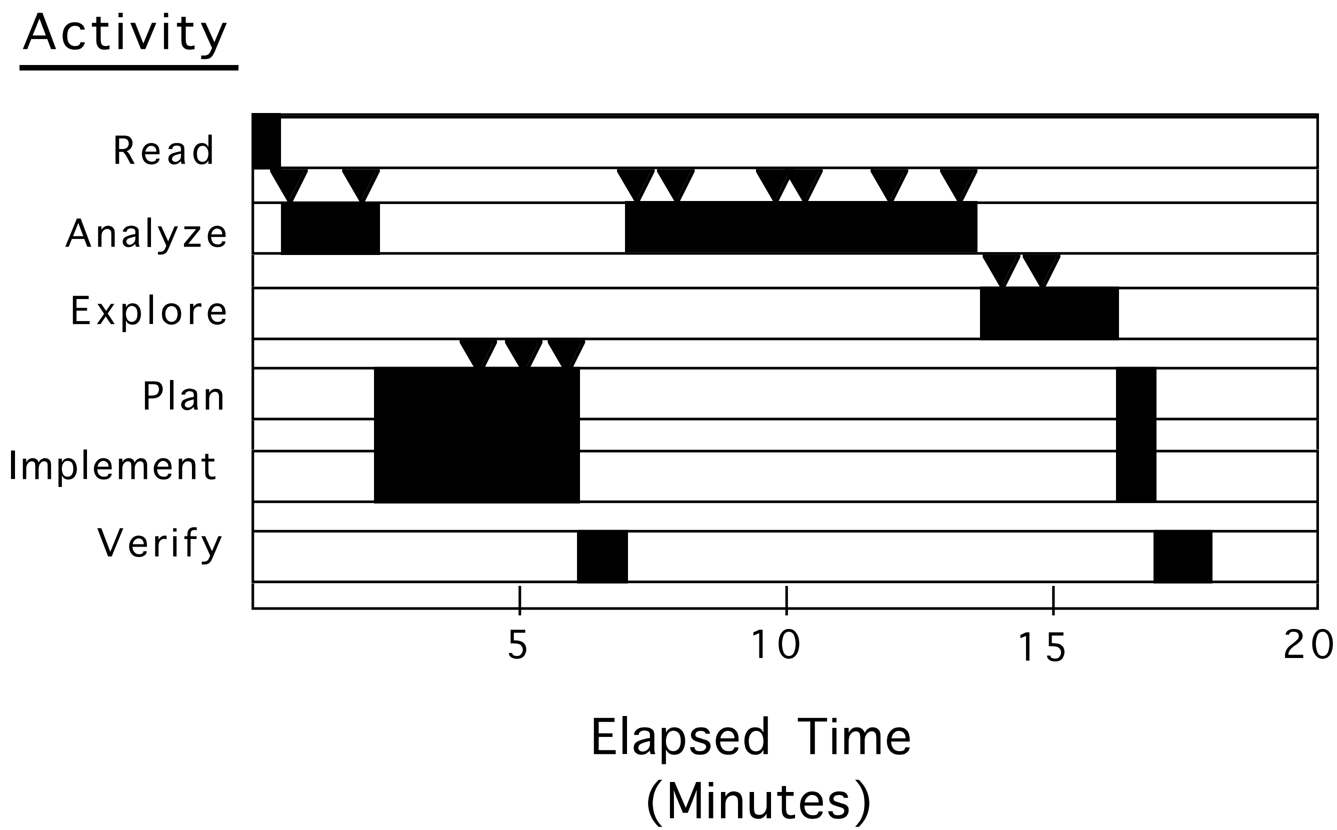
But I’ll save you the pain.



Time-line graph of a typical student attempt to solve a non-standard problem.

If there were time, I’d show a contrasting example,

A mathematician working a complex 2-part problem, and making very effective use of what he knows.



Time-line graph of a mathematician working a difficult problem

Methods for Inducing Good "Control"

1. Watching videotapes
2. Role-modeling solutions
3. Serving as "control" for class
4. Asking nasty questions during problem solving sessions….

What (exactly) are you doing?

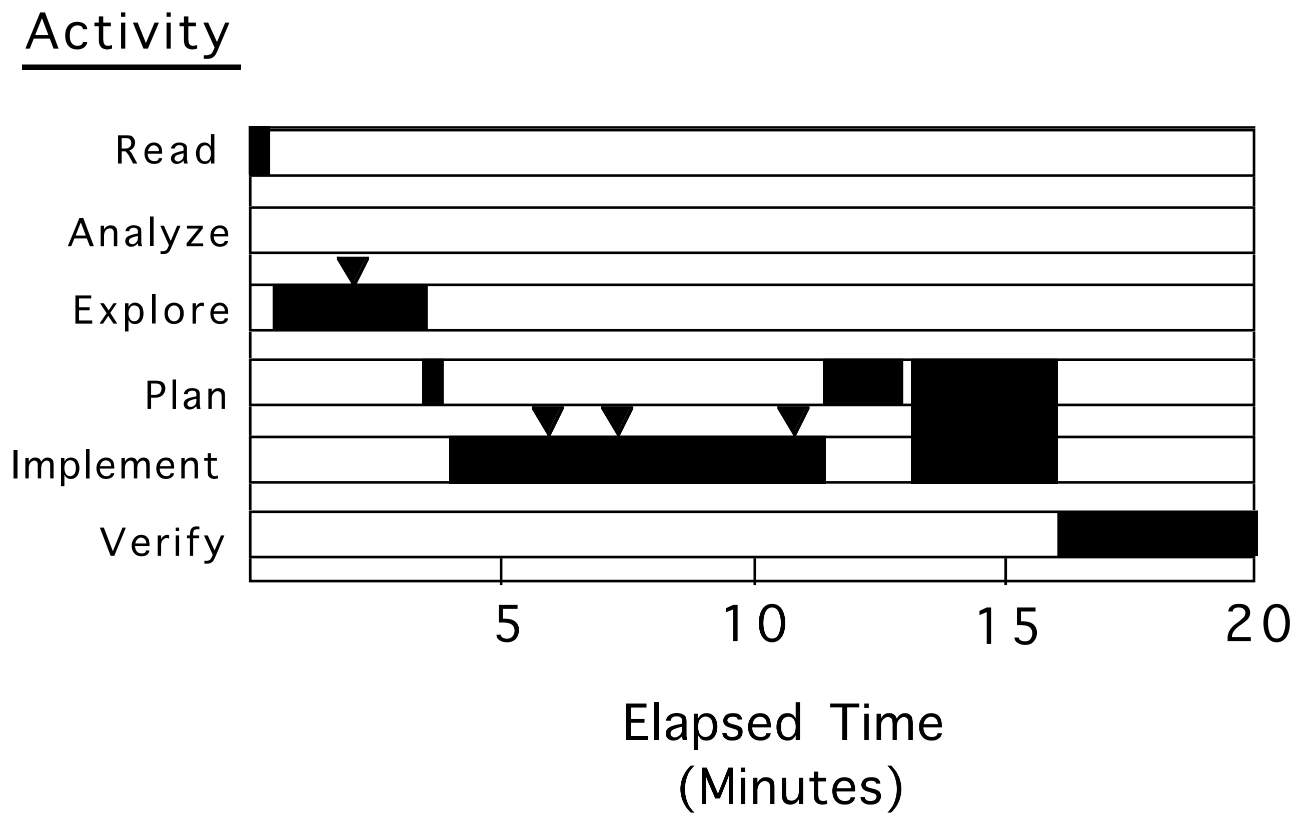
(Can you describe it precisely?)

Why are you doing it?

(How does it fit into the solution?)

How does it help you?

(What will you do with the outcome when you obtain it?)



Time-line graph of two students working a problem after the problem solving course.

Category 4:

Beliefs, and the Practices that Give Rise to Them.

Beliefs about writing

Writing is easy - you just write down what's in your head.

Writing is like telling a story. You start at the beginning and follow the narrative.

(Both of these beliefs cause problems. I spent about 5000 hours writing my problem solving book…)

U. S. National Assessment of Educational Progress

Carpenter, Lindquist, Matthews, & Silver, 1983

An army bus holds 36 soldiers. If 1128 soldiers are being bussed to their training site, how many buses are needed?

29%	31R12
18%	31
23%	32
30%	other

Kurt Reusser asks 97 1st and 2nd graders:

> There are 26 sheep and 10 goats on a ship.
> How old is the captain?

76 students "solve" it, using the numbers.

H. Radatz gives non-problems such as:

> Alan drove the 50 miles from Berkeley to Palo Alto at 8 a.m. On the way he picked up 3 friends.

No question is asked. Yet, from K-6, an increasing % of students "solve" the problem by combining the #'s and producing an "answer!

Some Typical Student Beliefs about Mathematics

1. There is <u>one</u> right way to solve any mathematics problem.
2. Mathematics is passed on from above for memorization.
3. Mathematics is a solitary activity.
4. All problems can be solved in 5 minutes or less.
5. Formal proof has nothing to do with discovery or invention.
6. School mathematics has little or nothing to do with the real world.

I could go on, with lots of examples and lots of data. But here are some bottom lines.

On Beliefs and Practices

Students develop a host of beliefs that can either enhance or impede their mathematical effectiveness.
They learn those beliefs as the result of the abstraction of typical practices in their mathematics classrooms.
Rich classroom practices can result in students' developing more productive beliefs and behaviors.

Summary and Conclusions:

The following four categories of knowledge determine the quality (and success) of problem solving attempts:

(i) the knowledge base

(ii) problem solving strategies (heuristics)

(iii) "control": monitoring and self-regulation, or metacognition

(iv) beliefs, and the practices that give rise to them.

Students develop their sense of mathematics (or any other subject matter) from their experience with it.

It is possible to create a culture of mathematical sense-making in the classroom, where students experience mathematics as a form of sense-making.
In such a context, they can develop the kinds of knowledge and beliefs that will enable them to be effective problem solvers.
That is the kind of environment one would hope to see in our mathematics classrooms. In fact…

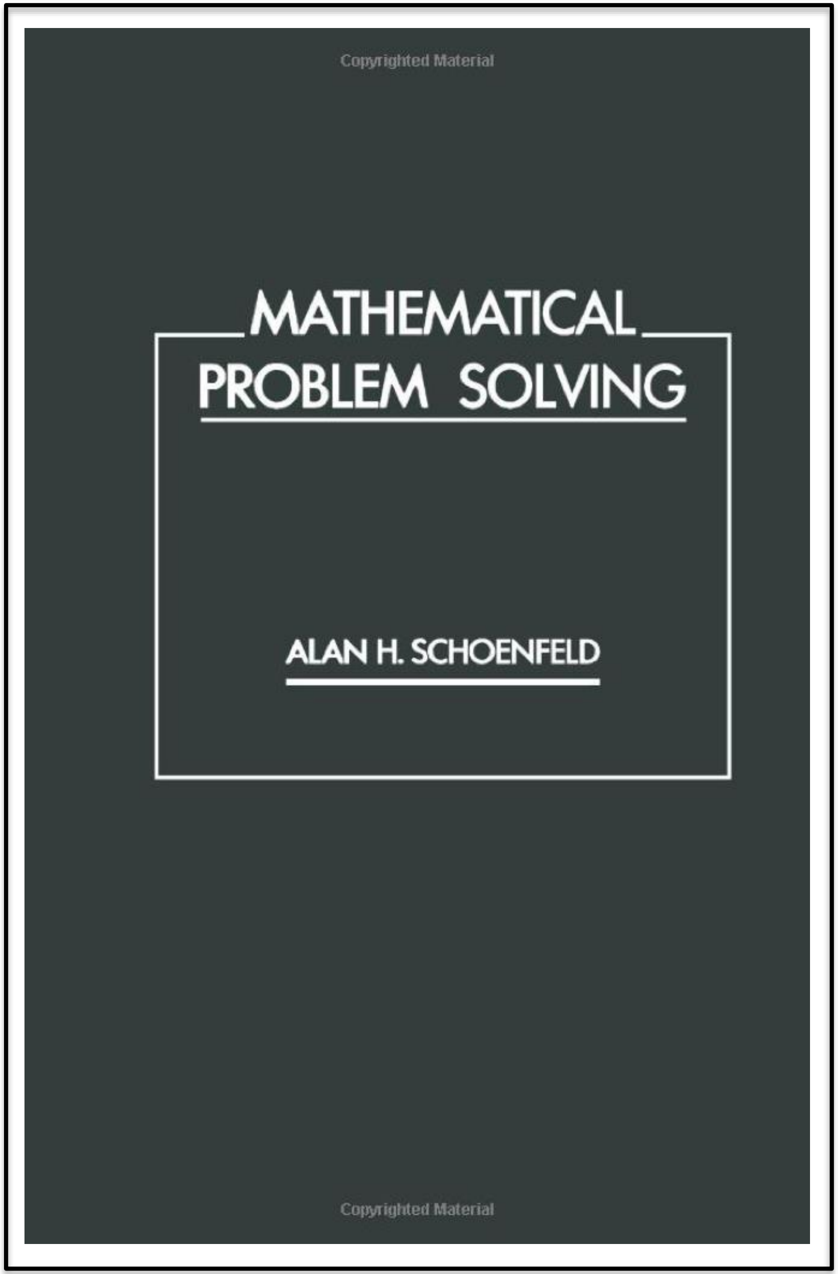
The issue is not just mathematics.

Properly conceived, writing or any other academic discipline is a sense-making, problem-solving activity.

They can *all* be taught as such – and there is evidence that teaching for problem solving “works.”

We owe this to our students.

For Detail, see:



2. Can we understand teachers' decision making? (1985-2010)

Can we understand how and why teachers make the decisions they do, in the midst of the classroom? More generally, what about decision making in any knowledge-intensive, socially complex domain?

YES.

The bottom line:

People's moment-by-moment decision making in teaching; in medicine; and … in *all* knowledge-rich domains, can be modeled as a function of their:

- Resources (esp. knowledge)
- Orientations (esp. beliefs)
- Goals

For detail, see…

How We Think

A Theory of Goal-Oriented Decision-Making and its Educational Applications

Alan H. Schoenfeld

So, let's turn to the original, most challenging problem:

What matters in classrooms?

(2005 – present)

For the rest of this talk, I'll focus on 2 key questions:

What *really* matters?

and,

How can we help make it happen??

Question 1:

What are 5 essential (i.e., necessary and sufficient) properties of classrooms from which students will emerge as knowledgeable and resourceful thinkers and problem solvers?

Why 5 (or fewer)?

It’s as many as most folks can keep in mind. (In fact, it may be too many to work on at one time.)

If you have 20, you might as well have none. People can’t keep that many things in their heads, and long check lists don’t help. What matters is what people can act on, in teaching and coaching.

What properties should those 5 things have?

They're all you need (there's nothing essential missing).

They each have a certain "integrity" and can be worked on in meaningful ways.

Their framing supports professional growth.

I’m going to spare you the details of the 6 years of R&D that resulted in TRU …

Say Thanks!*

… and try to give you a sense of the framework directly.

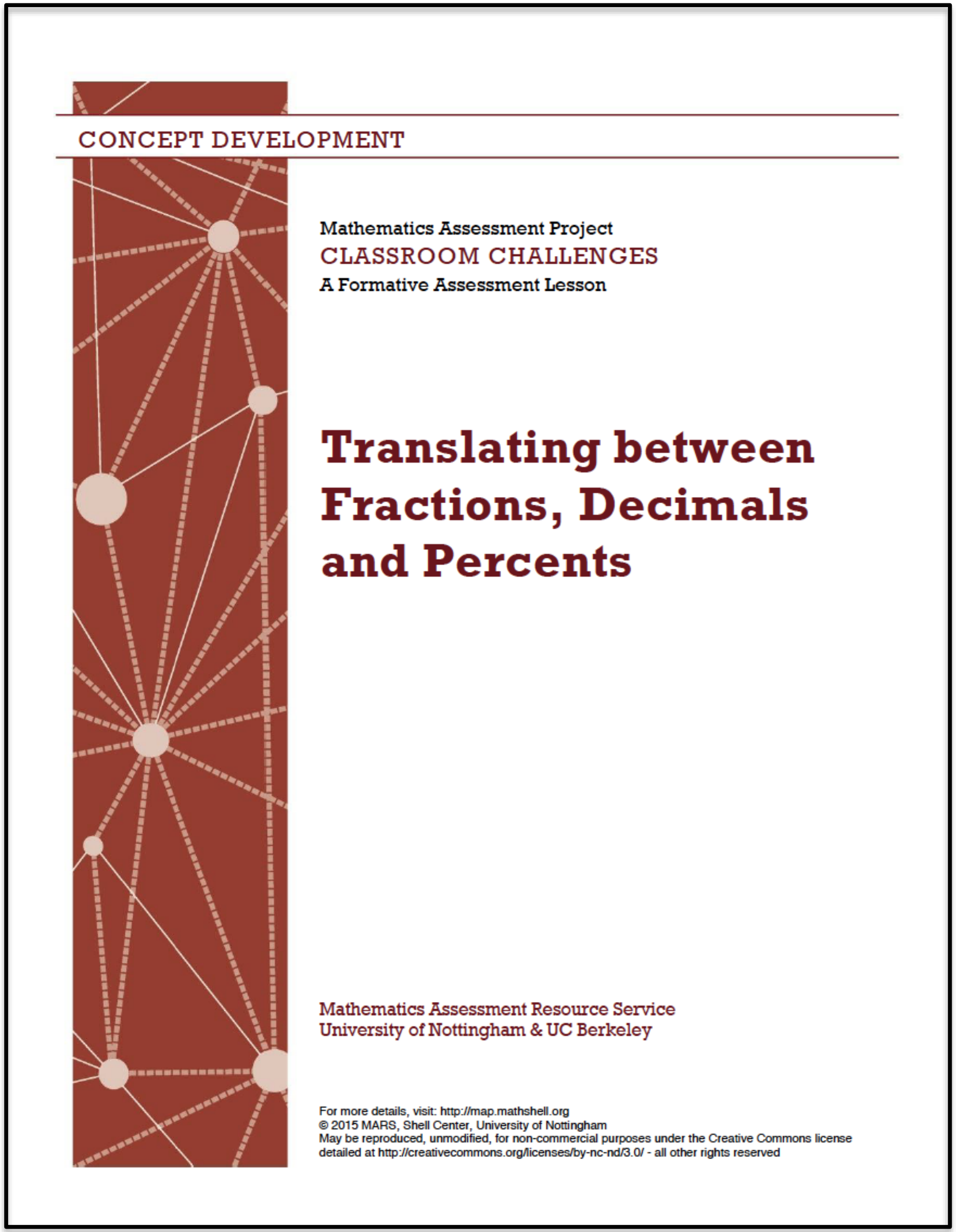
*See https://truframework.org/ for papers!

This is an experiment, I hope it works.

I am going to show you a video clip that shows students engaging powerfully with mathematics, and then I'll use it to illustrate the TRU Framework.

The video is of students from a
low SES inner-city school in Chicago.

They are working a
Formative Assessment Lesson
titled “Translating between Fractions,
Decimals, and Percents.”


CONCEPT DEVELOPMENT

Mathematics Assessment Project
CLASSROOM CHALLENGES
A Formative Assessment Lesson

Translating between Fractions, Decimals and Percents

Mathematics Assessment Resource Service
University of Nottingham & UC Berkeley

For more details, visit: http://map.mathshell.org
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This lesson is available for free, along with 99 other formative assessment lessons (a.k.a. "Classroom Challenges"). Just google "mathematics assessment" to find the Mathematics Assessment Project website.

To date we have over 7,500,000 lesson downloads. (More later.)

Fractions, decimals, percents

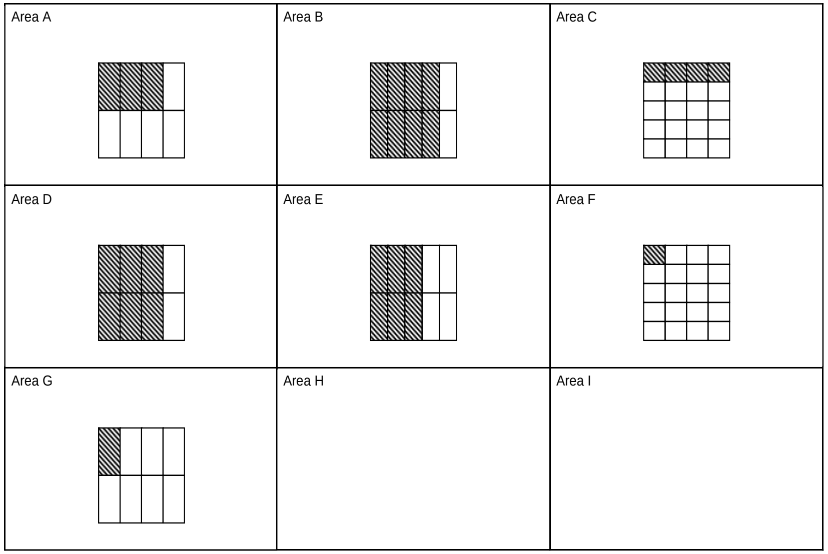
Take turns to:

1. Fill in the missing decimals and percents.
2. Place the cards in order of size.
3. Check that you agree.

0.2 ___%	0.05 ___%	_.___ 80%
0.375 ___%	_.___ 12.5%	0.75 ___%
1.25 ___%	_.___ 50%	_.___ ___%

Fractions, decimals, percents

0.2 ___%	0.05 ___%	_.___ 80%
0.375 ___%	_.___ 12.5%	0.75 ___%
1.25 ___%	_.___ 50%	_.___ ___%

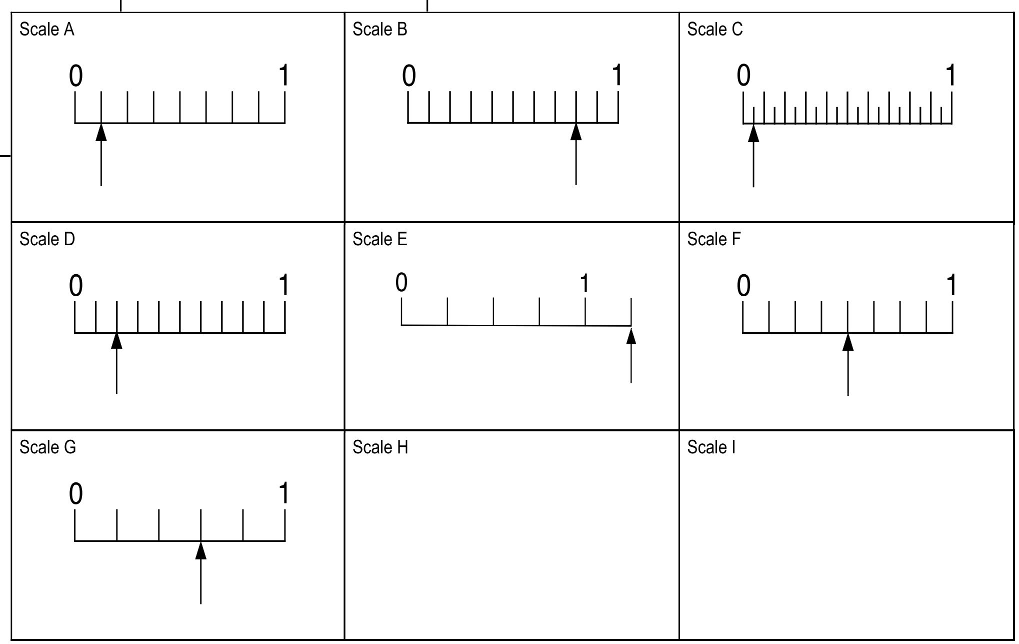


1. Match each area card to a decimals/percents card.
2. Create a new card or fill in spaces on cards until all the cards have a match.
3. Explain your thinking to your group.
4. Place your cards in order of size. Check that you all agree.

Fractions, decimals percents

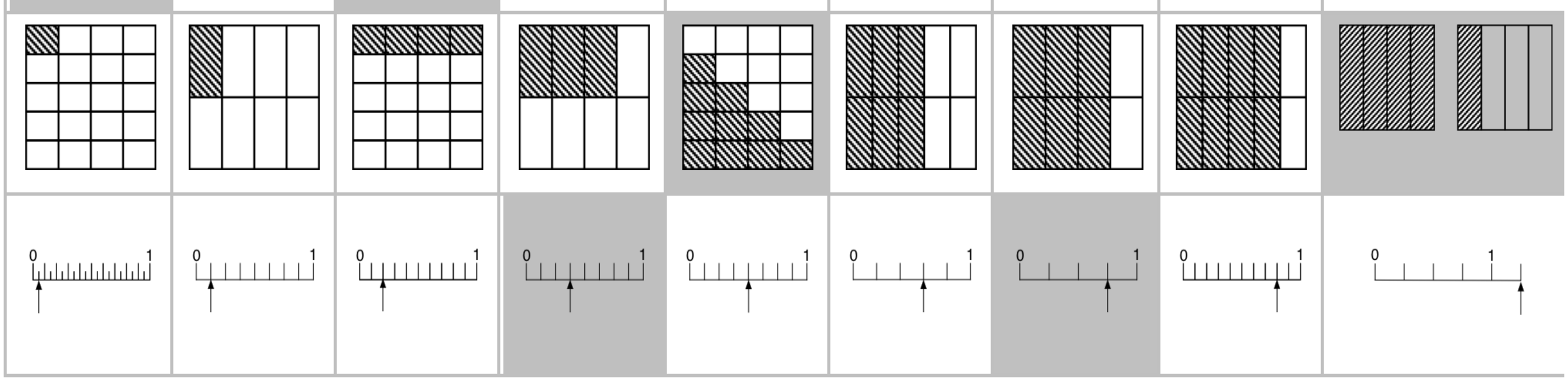
$\frac{3}{8}$	$\frac{4}{5}$	$\frac{1}{2}$
$\frac{3}{4}$	$\frac{6}{10}$	$\frac{5}{4}$
$\frac{1}{8}$		

- Add these cards.
- Place all cards in order of size.
- Check that you agree.



Here’s the full solution.

0.05 5%	0.125 12.5%	0.2 20%	0.375 37.5%	0.5 50%	0.6 60%	0.75 75%	0.8 80%	1.25 125%
$\frac{1}{20}$	$\frac{1}{8}$	$\frac{1}{5}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{6}{10}$	$\frac{3}{4}$	$\frac{4}{5}$	$\frac{5}{4}$



The gray cards are the ones that students had to create for themselves.

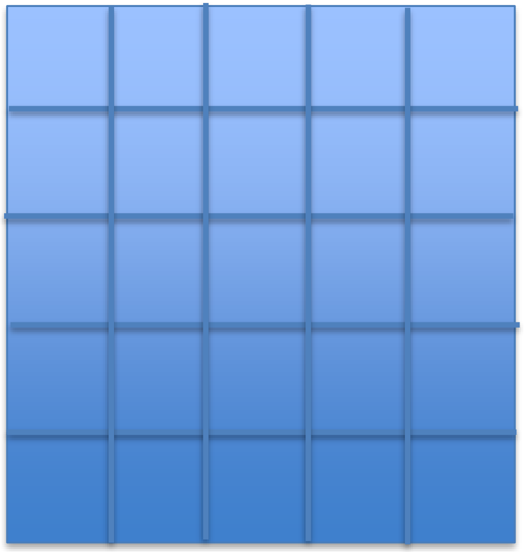
In this part of the lesson the students work in small groups.
The teacher circulates.

In the first part of the clip one student is explaining to another how to convert 50% to a decimal.

Note how all the groups are explaining to each other.

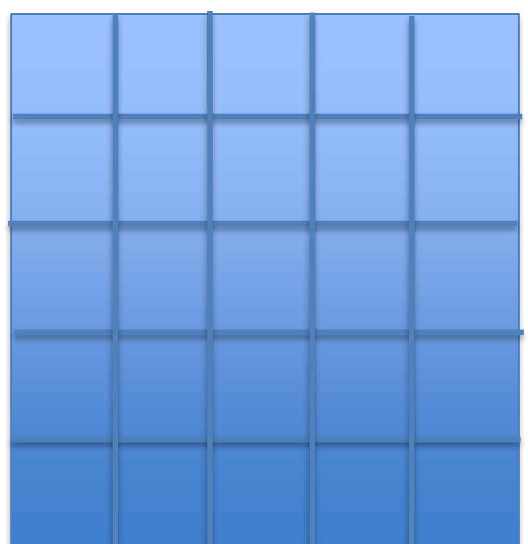
Later on, the students need to draw an area model and fraction for 1.25.

One student draws this:



A second student challenges him, saying he has shaded in 1, not 1.25.

The first student says no, each square is worth 5, so they add up to 125.



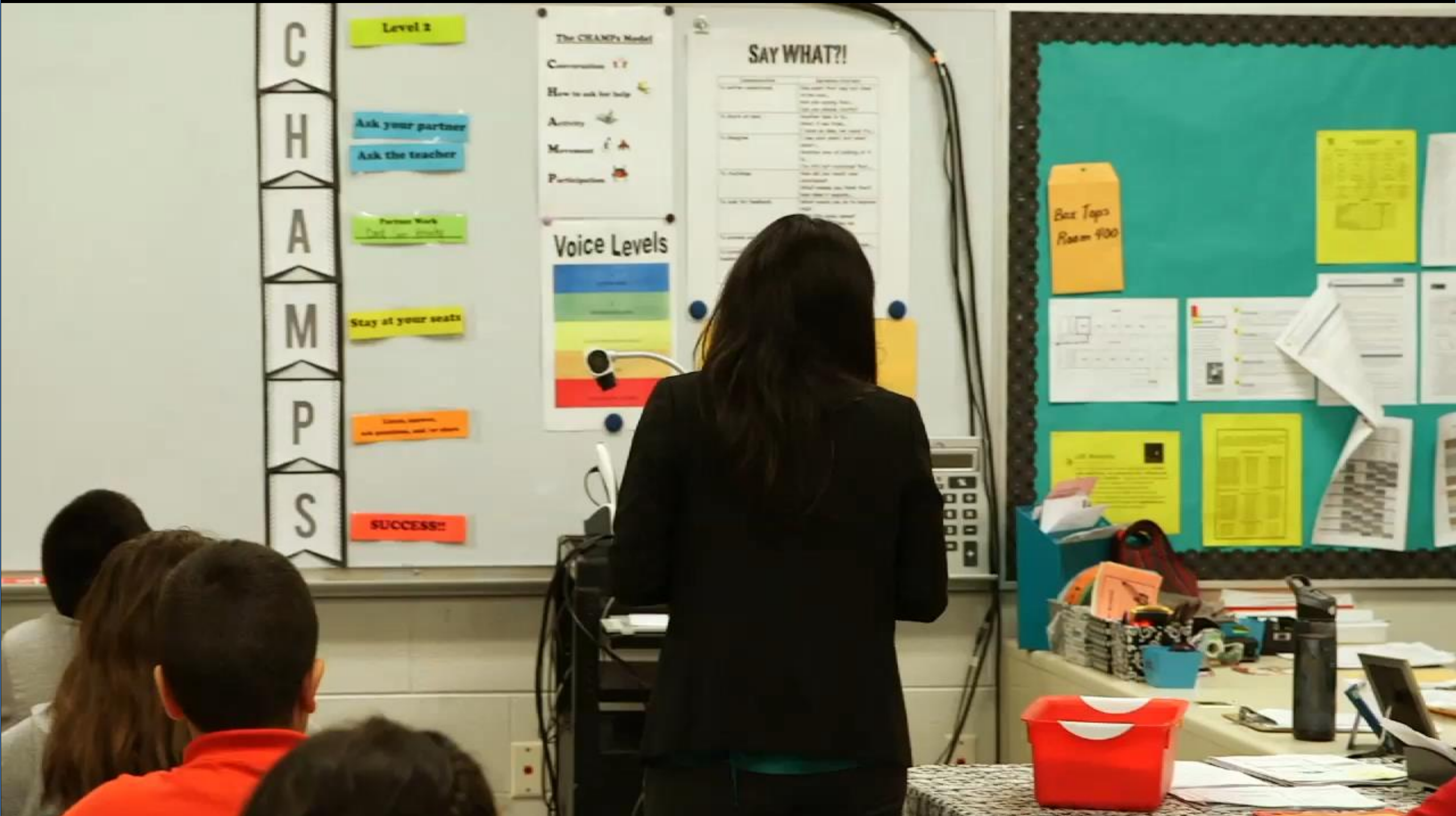
The second student leads the first through this argument:

You have a whole, which is 100%.

Which is bigger, 125% or 100%?

Doesn't that mean 125% should be bigger than a whole?

He leads the other student to see that 125% is 1 ¼.


C
H
A
M
P
S
Level 2
Ask your partner
Ask the teacher
Partner Work
Stay at your seats
SUCCESS!!
The CHAMPs Model
Conversation
How to ask for help
Activity
Movement
Participation
Voice Levels
SAY WHAT?!
Box Tops
Room 900

Here you see the students struggling meaningfully with the mathematics – and really learning.

(This is only part of the lesson, the teacher does have a role.)

We have a great deal of evidence that in (Western) classrooms using lessons and pedagogy like this, students learn much more than in classrooms when the teacher shows them what to do and they practice.

Now back to the first main question:

Question 1:

What are 5 essential (i.e., necessary and sufficient) properties of classrooms from which students will emerge as knowledgeable and resourceful thinkers and problem solvers?

Our distillation of the research, and a great deal of empirical work, suggests that the following five dimensions of classroom activity are essential.

I'll illustrate them by referring to the tape.

The Five Dimensions of Powerful Classrooms

The Content	Cognitive Demand	Equitable Access to Content	Agency, Ownership, and Identity	Formative Assessment
The extent to which the content students engage with represents our best current disciplinary understandings (as in CCSS, NGSS, etc.). Students should have opportunities to learn important content and practices, and to develop productive disciplinary habits of mind.	*The extent to which classroom interactions create and maintain an environment of productive intellectual challenge conducive to students' disciplinary development. Students should be able to engage in sense making and "productive struggle."*	*The extent to which classroom activity structures invite and support the active engagement of all of the students in the classroom with the core content being addressed by the class. No matter how rich the content being discussed, for example, a classroom in which a small number of students get most of the "air time" is not equitable.*	*The extent to which students have opportunities to "walk the walk and talk the talk," building on each other's ideas, in ways that contribute to their development of agency (the willingness to engage) and ownership over the content, resulting in positive identities as thinkers and learners.*	*The extent to which classroom activities elicit student thinking and subsequent instruction responds to those ideas, building on productive beginnings and addressing emerging misunderstandings. Powerful instruction "meets students where they are" and gives them opportunities to deepen their understandings.*

Note how this framework focuses on the student point of view.

Four of the five dimensions have to do with the ways in which the students experience the content.

What’s important about this framework?

Here are some central points.

Five central points about TRU:

1. **The TRU Dimensions are necessary and sufficient. That is,**

   If things go well along all 5 dimensions, students will emerge from the classroom as powerful thinkers.
   If things go badly along *any* of the dimensions, they will not. (See the papers at http://truframework.org/ for detail…)

Five central points about TRU:

2. **TRU involves a fundamental shift in perspective, from teacher-centered to student-centered.**

The key question is *not:*
"Do I like what the teacher is doing?"
It is:
"What does instruction feel like, from the point of view of the student?"

Observe the Lesson Through a Student's Eyes

The Content	• What's the big idea in this lesson? • How does it connect to what I already know?
Cognitive Demand	• How long am I given to think, and to make sense of things? • What happens when I get stuck? • Am I invited to explain things, or just give answers?
Equitable Access to Content	• Do I get to participate in meaningful math learning? • Can I hide or be ignored? In what ways am I kept engaged?
Agency, Ownership, and Identity	• What opportunities do I have to explain my ideas? In what ways are they built on? • How am I recognized as being capable and able to contribute?
Formative Assessment	• How is my thinking included in classroom discussions? • Does instruction respond to my ideas and help me think more deeply?

Five central points about TRU:

3. **TRU does not tell you how to teach, because there are many different ways to be an effective teacher.**

TRU describes the *principles* of powerful instruction, so it serves to *problematize* instruction. Asking, “how am I doing along this dimension; how can I improve?” can lead to richer instruction without imposing a particular style or norms on teachers.

This is critical when you think about Professional Development.

There is not one "right" way to implement professional development using TRU. What matters is that teachers have experiences that enable them to get better at the 5 dimensions.

There are many powerful ways to do this… as in Chicago, NY, Silicon Valley, Oakland, San Francisco… *See Schoenfeld et al, in press.*

Five central points about TRU:

4. **TRU is NOT a tool or set of tools.** TRU is a perspective regarding what counts in instruction, and TRU provides a language for talking about instruction in powerful ways. With this understanding, you can make use of any productive tools wisely.

But we have tools, of course…

See

http://TRUFramework.org

for the tools I'll show you when discussing Q2.

Five central points about TRU:

5. TRU doesn't compete with other initiatives; it works with them and makes them stronger.

You can use it to "problematize" the approaches you take.

Question 2:

How can we support teachers in developing the knowledge and skills that enable them to craft such learning environments?

Tools to Support
Powerful Classroom Instruction

The Mathematics Assessment Project
has produced 100
"Formative Assessment Lessons"
(FALs) to help teachers engage in
"diagnostic teaching."

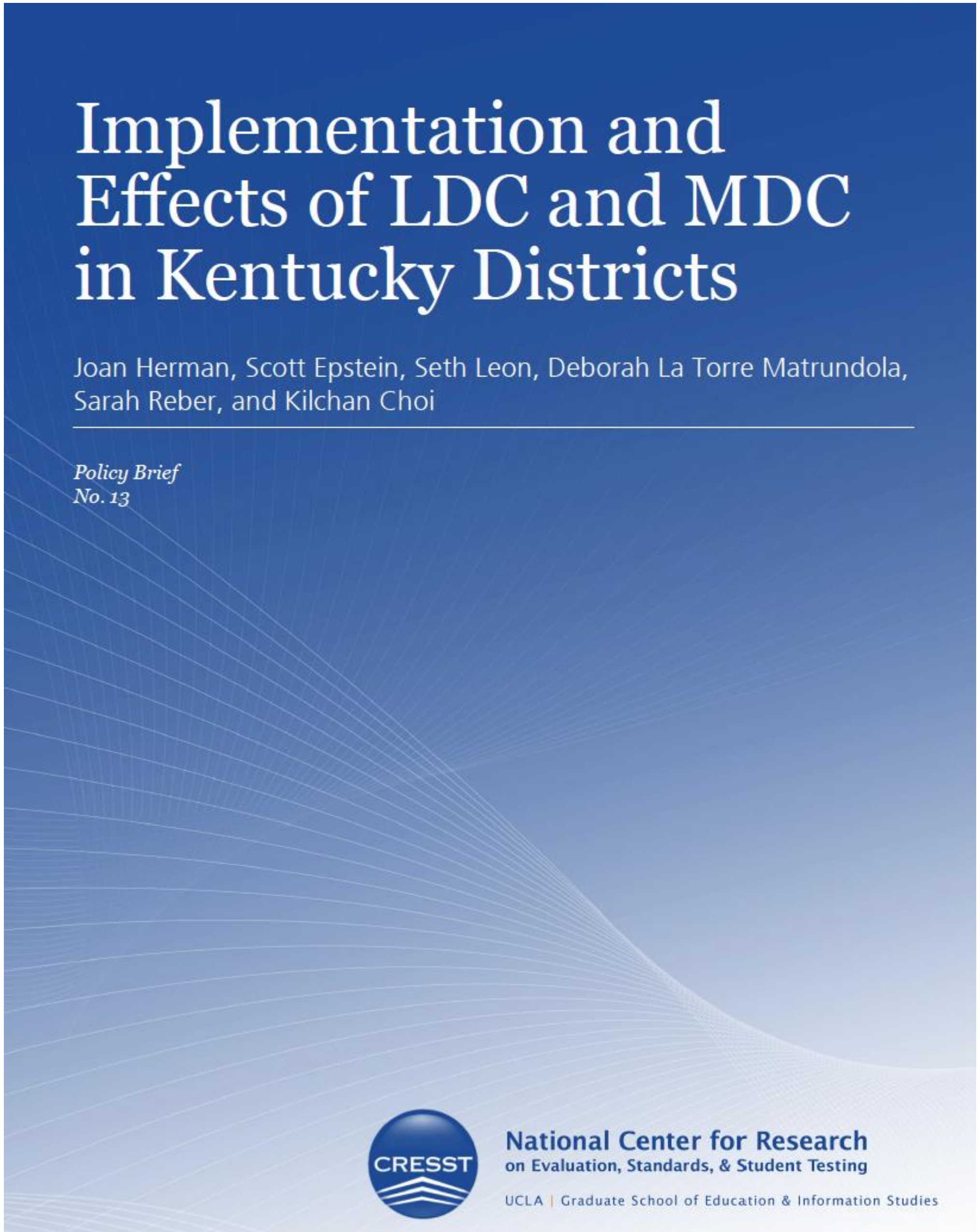
There have been more than 7,500,000 lesson downloads, from http://map.mathshell.org/.

You saw a bit of the Fractions, Decimals, and Percents Lesson

The FALs make a BIG difference.

I have TONS of evidence, which I'll spare you.

Well, maybe 2 slides…


Implementation and Effects of LDC and MDC in Kentucky Districts

Joan Herman, Scott Epstein, Seth Leon, Deborah La Torre Matrundola, Sarah Reber, and Kilchan Choi

Policy Brief
No. 13

CRESST
National Center for Research on Evaluation, Standards, & Student Testing
UCLA | Graduate School of Education & Information Studies

MDC = “Math design Collaborative,” which was designed to help implement the Formative Assessment Lessons.

The results:

Participating teachers were expected to implement between four and six Formative Assessment Lessons, meaning that students were engaged only 8-12 days of the school year.

Nonetheless, the studies found statistically significant learning effects of approximately **4.6 months** for the Formative Assessment Lessons.

A Tool for Planning for and Reflecting on Teaching

The ***TRU Conversation Guide*** is designed to foster reflective conversations about instruction.

Frame each dimension with questions:

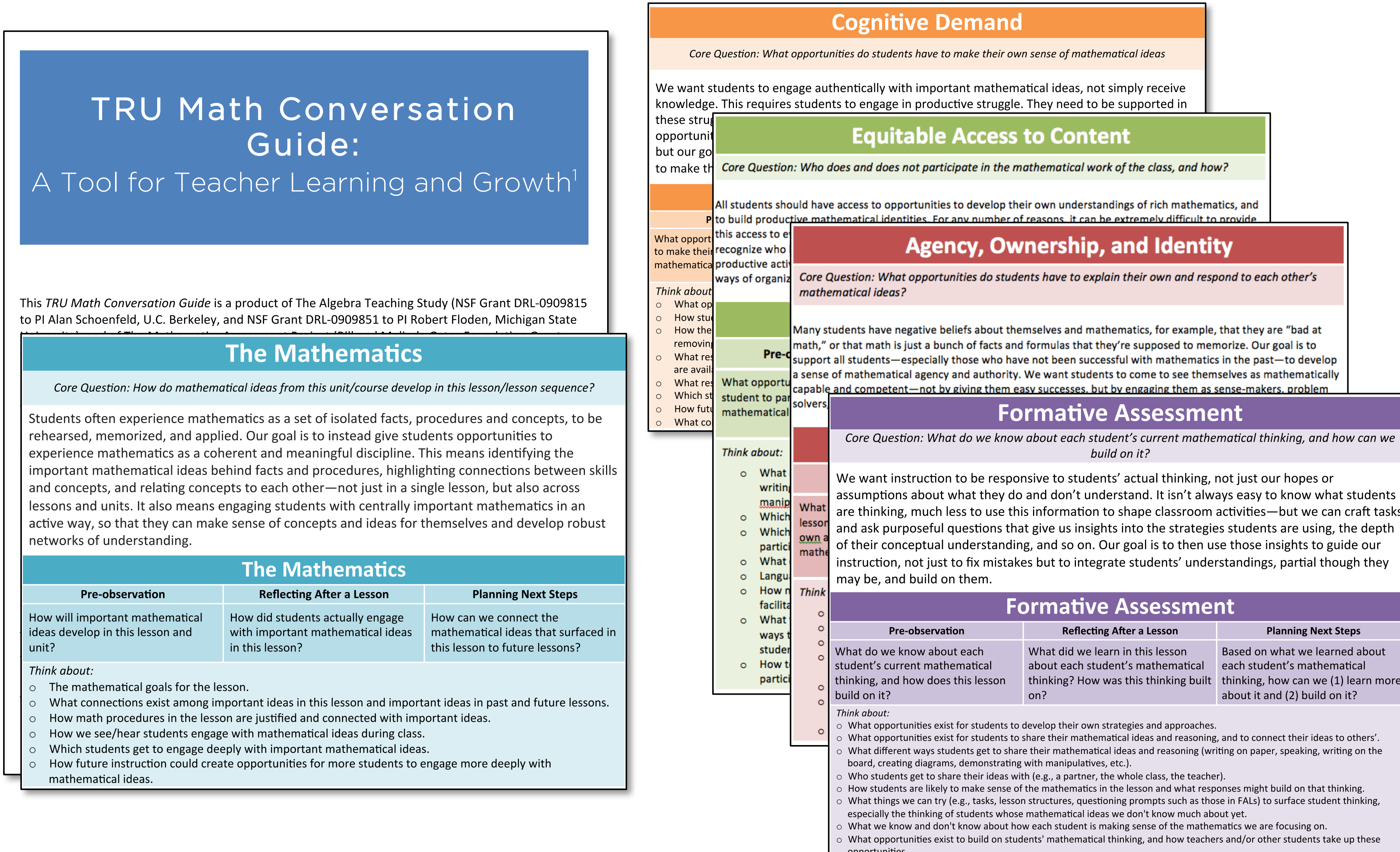
The Content
How do ideas from this unit/course develop in this lesson/lesson sequence?
Cognitive Demand
What opportunities do students have to make their own sense of important ideas?
Equitable Access to Content
Who does and does not participate in the meaningful work of the class, and how?
Agency, Ownership, and Identity
What opportunities do students have to explain their own and respond to each other's ideas?
Formative Assessment
What do we know about each student's current thinking, and how can we build on it?

. . . and expand the questions,

to *problematize* instruction.

That is: Ask a series of questions that help to plan for instruction that provides students with deeper opportunities along each of the five dimensions.

The TRU Conversation Guide



TRU Math Conversation Guide:
A Tool for Teacher Learning and Growth[1]

This *TRU Math Conversation Guide* is a product of The Algebra Teaching Study (NSF Grant DRL-0909815 to PI Alan Schoenfeld, U.C. Berkeley, and NSF Grant DRL-0909851 to PI Robert Floden, Michigan State

Cognitive Demand

Core Question: What opportunities do students have to make their own sense of mathematical ideas

We want students to engage authentically with important mathematical ideas, not simply receive knowledge. This requires students to engage in productive struggle. They need to be supported in

Equitable Access to Content

Core Question: Who does and does not participate in the mathematical work of the class, and how?

All students should have access to opportunities to develop their own understandings of rich mathematics, and

Agency, Ownership, and Identity

Core Question: What opportunities do students have to explain their own and respond to each other's mathematical ideas?

Many students have negative beliefs about themselves and mathematics, for example, that they are "bad at math," or that math is just a bunch of facts and formulas that they're supposed to memorize. Our goal is to support all students—especially those who have not been successful with mathematics in the past—to develop a sense of mathematical agency and authority. We want students to come to see themselves as mathematically

The Mathematics

Core Question: How do mathematical ideas from this unit/course develop in this lesson/lesson sequence?

Students often experience mathematics as a set of isolated facts, procedures and concepts, to be rehearsed, memorized, and applied. Our goal is to instead give students opportunities to experience mathematics as a coherent and meaningful discipline. This means identifying the important mathematical ideas behind facts and procedures, highlighting connections between skills and concepts, and relating concepts to each other—not just in a single lesson, but also across lessons and units. It also means engaging students with centrally important mathematics in an active way, so that they can make sense of concepts and ideas for themselves and develop robust networks of understanding.

The Mathematics

Pre-observation	Reflecting After a Lesson	Planning Next Steps
How will important mathematical ideas develop in this lesson and unit?	How did students actually engage with important mathematical ideas in this lesson?	How can we connect the mathematical ideas that surfaced in this lesson to future lessons?

Think about:

- The mathematical goals for the lesson.
- What connections exist among important ideas in this lesson and important ideas in past and future lessons.
- How math procedures in the lesson are justified and connected with important ideas.
- How we see/hear students engage with mathematical ideas during class.
- Which students get to engage deeply with important mathematical ideas.
- How future instruction could create opportunities for more students to engage more deeply with mathematical ideas.

Formative Assessment

Core Question: What do we know about each student's current mathematical thinking, and how can we build on it?

We want instruction to be responsive to students' actual thinking, not just our hopes or assumptions about what they do and don't understand. It isn't always easy to know what students are thinking, much less to use this information to shape classroom activities—but we can craft tasks and ask purposeful questions that give us insights into the strategies students are using, the depth of their conceptual understanding, and so on. Our goal is to then use those insights to guide our instruction, not just to fix mistakes but to integrate students' understandings, partial though they may be, and build on them.

Formative Assessment

Pre-observation	Reflecting After a Lesson	Planning Next Steps
What do we know about each student's current mathematical thinking, and how does this lesson build on it?	What did we learn in this lesson about each student's mathematical thinking? How was this thinking built on?	Based on what we learned about each student's mathematical thinking, how can we (1) learn more about it and (2) build on it?

Think about:

- What opportunities exist for students to develop their own strategies and approaches.
- What opportunities exist for students to share their mathematical ideas and reasoning, and to connect their ideas to others'.
- What different ways students get to share their mathematical ideas and reasoning (writing on paper, speaking, writing on the board, creating diagrams, demonstrating with manipulatives, etc.).
- Who students get to share their ideas with (e.g., a partner, the whole class, the teacher).
- How students are likely to make sense of the mathematics in the lesson and what responses might build on that thinking.
- What things we can try (e.g., tasks, lesson structures, questioning prompts such as those in FALs) to surface student thinking, especially the thinking of students whose mathematical ideas we don't know much about yet.
- What we know and don't know about how each student is making sense of the mathematics we are focusing on.
- What opportunities exist to build on students' mathematical thinking, and how teachers and/or other students take up these opportunities.

Agency, Ownership, and Identity

Core Questions: What opportunities do students have to see themselves and each other as powerful doers of mathematics? How can we create more of these opportunities?

Many students have negative beliefs about themselves and mathematics, for example, that they are "bad at math," or that math is just a bunch of facts and formulas that they're supposed to memorize. Our goal is to support all students—especially those who have not been successful with mathematics in the past—to develop a sense of mathematical agency and ownership over their own learning. We want students to come to see themselves as mathematically capable and competent—not by giving them easy successes, but by engaging them as sense-makers, problem solvers, and creators of mathematical ideas.

Planning

What opportunities might exist for students to generate and explain their own ideas? To respond to each other's ideas? How can we create more opportunities?

Reflecting

How have we seen students explain their own and respond to each other's ideas? What has that looked and sounded like in specific cases?

Things to think about

- Who generates the ideas that get discussed?
- What kinds of ideas do students have opportunities to generate and share (strategies, connections, partial understandings, prior knowledge, representations)?
- Who evaluates and/or responds to others' ideas?
- How deeply do students get to explain their ideas?
- How does (or how could) the teacher respond to student ideas (evaluating, questioning, probing, soliciting responses from other students, etc.)?
- How are norms about students' and teachers' roles in generating ideas developing?
- How are norms about what counts as mathematical activity (justifying, experimenting, connecting, practicing, memorizing, etc.) developing?
- Which students get to explain their own ideas? To respond to others' ideas in meaningful ways?
- Which students seem to see themselves as powerful mathematical thinkers right now?
- How might we create more opportunities for more students to see themselves and each other as powerful mathematical thinkers?

To support collegial observations,

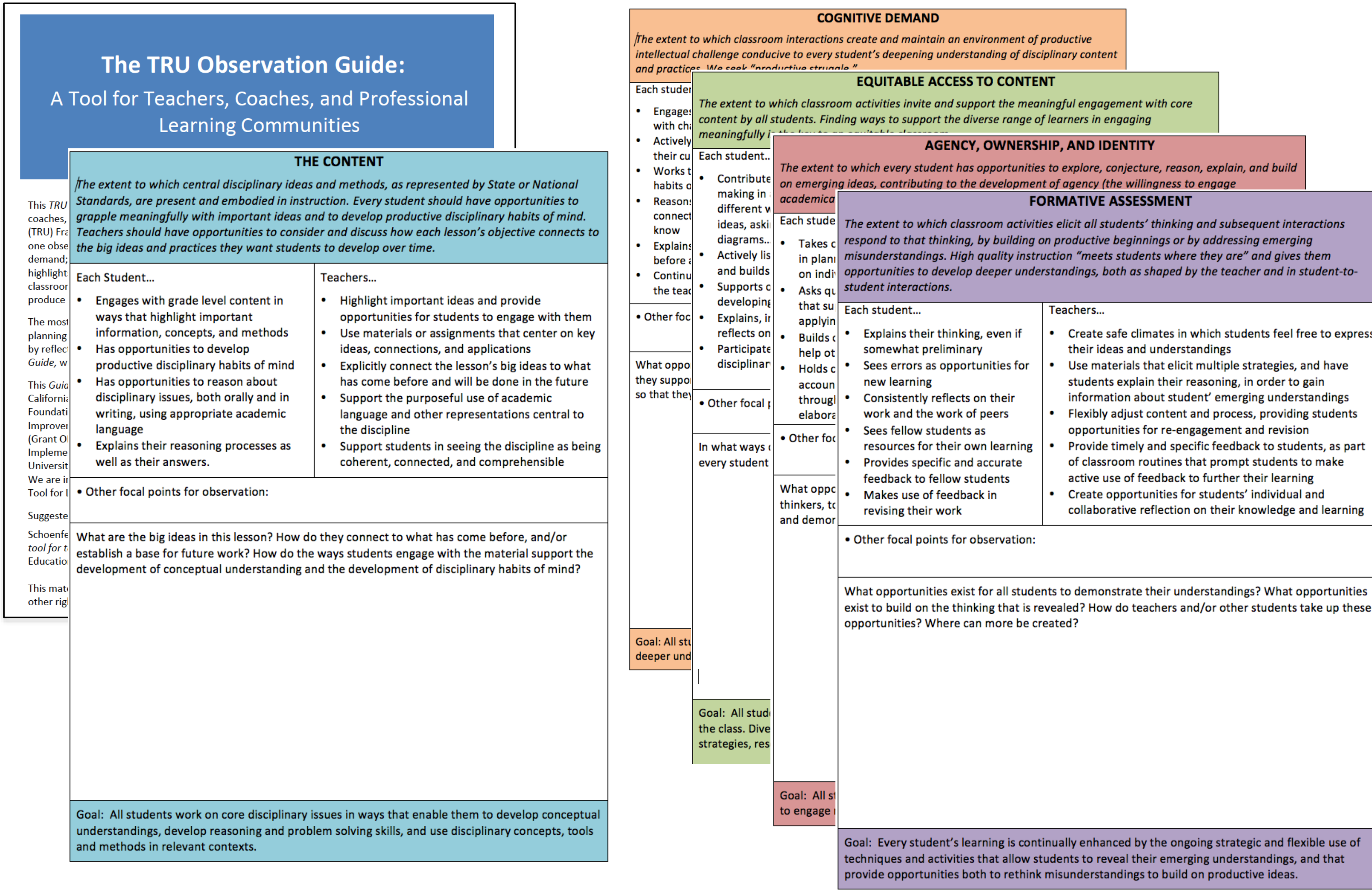
we offer the

TRU Observation Guide,

Which highlights things to look for is a lesson is going well.

The guide can be used by coaches or TLCs for planning and debriefing classroom observations…

The TRU Observation Guide



The TRU Observation Guide:

A Tool for Teachers, Coaches, and Professional Learning Communities

COGNITIVE DEMAND

The extent to which classroom interactions create and maintain an environment of productive intellectual challenge conducive to every student's deepening understanding of disciplinary content and practices. We seek "productive struggle."

EQUITABLE ACCESS TO CONTENT

The extent to which classroom activities invite and support the meaningful engagement with core content by all students. Finding ways to support the diverse range of learners in engaging meaningfully i...

AGENCY, OWNERSHIP, AND IDENTITY

The extent to which every student has opportunities to explore, conjecture, reason, explain, and build on emerging ideas, contributing to the development of agency (the willingness to engage academica...

THE CONTENT

The extent to which central disciplinary ideas and methods, as represented by State or National Standards, are present and embodied in instruction. Every student should have opportunities to grapple meaningfully with important ideas and to develop productive disciplinary habits of mind. Teachers should have opportunities to consider and discuss how each lesson's objective connects to the big ideas and practices they want students to develop over time.

Each Student...	Teachers...
• Engages with grade level content in ways that highlight important information, concepts, and methods • Has opportunities to develop productive disciplinary habits of mind • Has opportunities to reason about disciplinary issues, both orally and in writing, using appropriate academic language • Explains their reasoning processes as well as their answers.	• Highlight important ideas and provide opportunities for students to engage with them • Use materials or assignments that center on key ideas, connections, and applications • Explicitly connect the lesson's big ideas to what has come before and will be done in the future • Support the purposeful use of academic language and other representations central to the discipline • Support students in seeing the discipline as being coherent, connected, and comprehensible

• Other focal points for observation:

What are the big ideas in this lesson? How do they connect to what has come before, and/or establish a base for future work? How do the ways students engage with the material support the development of conceptual understanding and the development of disciplinary habits of mind?

Goal: All students work on core disciplinary issues in ways that enable them to develop conceptual understandings, develop reasoning and problem solving skills, and use disciplinary concepts, tools and methods in relevant contexts.

FORMATIVE ASSESSMENT

The extent to which classroom activities elicit all students' thinking and subsequent interactions respond to that thinking, by building on productive beginnings or by addressing emerging misunderstandings. High quality instruction "meets students where they are" and gives them opportunities to develop deeper understandings, both as shaped by the teacher and in student-to-student interactions.

Each student...	Teachers...
• Explains their thinking, even if somewhat preliminary • Sees errors as opportunities for new learning • Consistently reflects on their work and the work of peers • Sees fellow students as resources for their own learning • Provides specific and accurate feedback to fellow students • Makes use of feedback in revising their work	• Create safe climates in which students feel free to express their ideas and understandings • Use materials that elicit multiple strategies, and have students explain their reasoning, in order to gain information about student' emerging understandings • Flexibly adjust content and process, providing students opportunities for re-engagement and revision • Provide timely and specific feedback to students, as part of classroom routines that prompt students to make active use of feedback to further their learning • Create opportunities for students' individual and collaborative reflection on their knowledge and learning

• Other focal points for observation:

What opportunities exist for all students to demonstrate their understandings? What opportunities exist to build on the thinking that is revealed? How do teachers and/or other students take up these opportunities? Where can more be created?

Goal: Every student's learning is continually enhanced by the ongoing strategic and flexible use of techniques and activities that allow students to reveal their emerging understandings, and that provide opportunities both to rethink misunderstandings to build on productive ideas.

AGENCY, OWNERSHIP, AND IDENTITY

The extent to which every student has opportunities to explore, conjecture, reason, explain, and build on emerging ideas, contributing to the development of agency (the willingness to engage academically) and ownership over the content, resulting in positive mathematical identities.

Each student…	Teachers…
• Takes ownership of the learning process in planning, monitoring, and reflecting on individual and/or collective work • Asks questions and makes suggestions that support analyzing, evaluating, applying and synthesizing mathematical ideas • Builds on the contributions of others and help others see or make connections • Holds classmates and themselves accountable for justifying their positions, through the use of evidence and/or elaborating on their reasoning	• Provide time for students to develop and express mathematical ideas and reasoning • Work to make sure all students have opportunities to have their voices heard • Encourage student-to-student discussions and promote productive exchanges • Assign tasks and pose questions that call for mathematical justification, and for students to explain their reasoning • Employ a range of techniques that attribute ideas to students, to build student ownership and identity

• Other focal points for observation:

What opportunities do all students have to see themselves and others as proficient mathematical thinkers, to grapple with challenges and construct new understandings, to build on others' ideas, and demonstrate their understandings? How can more of these opportunities be created?

Goal: All students build productive mathematical identities through taking advantage of opportunities to engage meaningfully with the discipline and share and refine their developing ideas.

The first version of the Observation Guide was actually built by San Francisco Unified School District, and it’s being used in a number of school districts across the US.

So, (1) these ideas work at the “ground level.” They’re not just “academic.” And (2), it shows how people can make good use of *principles* and *ideas* when they’re not told what to do.

These and other tools are available at

https://truframework.org/

and

http://map.mathshell.org/...

In concluding, let me return to the titles of this talk...

Problem solving – What have we learned since Pólya's introspection?

and

Solving the problem of Powerful Instruction

We've learned a LOT. Have we solved the problem of Powerful Instruction?

No, but in the spirit of Pólya, we are making good progress.

Resources:

The TRU Math Suite and supporting documents are available on the

Teaching for Robust Understanding Framework

web site:

https://truframework.org/.

Also, be sure to visit

The **Mathematics Assessment Project** web site:

http://map.mathshell.org/

(Just Google the project names.)