Inequality and Welfare Analyses in the Harris-Todaro Model

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Abstract

We present inequality and welfare analyses of the Harris-Todaro migration model where rural sector wage depends on the level of rural labour force. As in the basic model we consider three policies: the modern sector enlargement; the traditional sector enrichment; and the modern sector wage restraint. A combination of the ¯rst two policies is also considered since it gives the same qualitative e®ects as the uniform wage subsidy given to both sectors. The wage elasticities of labour demand in urban and rural sectors play important roles in determining necessary and su±cient conditions for unambiguous changes in inequality and welfare. When the policy leads to ambiguity, the coe±cient of variation and variance have been used to improve ranking ability.

1 Introduction

In developing countries, economic growth has been biased towards the urban (modern) sector. Workers gradually migrate to the urban sector causing urban unemployment and other problems such as environmental degradation. The Harris-Todaro (H-T) (1970) model provides a powerful explanation of such phenomenon. There is a vast number of existing literature following H-T analysis that focus on labour market policies for the purpose of reducing the level of unemployment. Some also consider e®ects of labour market policies on income inequality and welfare.

The labour market policies can be classi¯ed as one of the following three types of growth policies or combination of them. The three policies are modern sector enlargement (MSENL)-a policy of modern sector job creation; traditional sector enrichment (TSENR)-a policy of rural development; and modern sector wage restraint (MSWR)-a policy of wage limitation in the urban sector.

Although income inequality and welfare analyses of these policies have been done in existing literature, the results are usually ambiguous. Fields (1979, 2001) and Temple (2002) ¯nd that, even in the simple H-T model, a labour market policy may lead to Lorenz crossing. Hence the e®ect on inequality is ambiguous. And it is likely that the policy would give ambiguous result in a more complicated model. Would it be possible to make statements on unambiguous changes in income inequality when Lorenz curves cross without relying on specific inequality indices?

Moreover Fields (1979) and Chakravarty and Dutta (1990) show that conditions for unambiguous changes in welfare depend on specific social welfare functions (SWFs). Fields (2001) measures welfare in a more general approach using an abbreviated SWF and first-order stochastic dominance (FSD). An abbreviated SWF is a function of labour market indicators namely unemployment, wage ratio, poverty, etc. FSD is just the cumulative income distribution. His results are also ambiguous. This is because for the abbreviated SWF, the results depend on the labour market indicators considered. And since income distributions tend to cross one another, FSD gives ambiguous results. Given the existing results, can we still make general statement on unambiguous changes in welfare when a labour market policy is implemented?

This paper addresses the above two questions. This is done by applying the concept of third-order stochastic dominance (TSD), i.e. the uses of coefficient of variation and variance for inequality and welfare analyses respectively.[1] When Lorenz curves cross just once, the rate of success in ranking income distributions is raised by an average of one third of those cannot be ranked according to Lorenz dominance, Shorrocks and Foster (1987).[2] We only consider application of TSD in cases of single Lorenz crossing since whenever ambiguity occurs in our analysis, Lorenz curves only cross once and from below.

Our analysis proceeds as follows. In Section 2, we present the H-T model. In Section 3, we discuss the effects of the three policies on the level of unemployment, unemployment rate, and mean income. A combined policy of MSENL and TSENR is also considered. This is because it can be shown that such policy leads to the same qualitative effects as uniform wage subsidy which is the first best policy in the H-T model, suggested by Bhagwati and Srinivasan (1974). In Section 4, we summarise theoretical results concerning inequality and welfare comparisons which will be applied to the H-T model in Section 5.

There is no ambiguity when TSENR is implemented. The policy unambiguously lowers inequality and increases welfare. We follow Fields (2001) and Temple (2002) and measure inequality in terms of Lorenz curves. Lorenz curves cross when the policy leads to an increase in the level of unemployment which depends on wage elasticities of labour demand in urban and rural sectors defined to be negative and denoted by δ_m and δ_r respectively.

For MSENL, the critical values of δ_r is $\delta_r^* = -\frac{n_m}{\upsilon n_r}$ where n_m and n_r denote the levels of employment in the urban and rural sectors respectively, and υ denotes the unemployment rate. Note that only δ_r^* is relevant since the urban wage (minimum wage) is unchanged. By definition of δ_r, it can be shown that the value of δ_r^* is achieved when the policy leads to the same percentage changes in urban labour force (n_u) and the unemployment rate in absolute terms. And this will occurs when the level of unemployment is unchanged. When the value of δ_r is lower than δ_r^*, the level of unemployment increases, and vice versa.

[1] See Shorrocks and Foster (1987) for the use of coefficient of variation in inequality analysis and Dardanoni and Lambert (1988) for the use of variance in welfare analysis.

[2] Atkinson (1973) finds that, using the Kuznets' data employed in his 1970 article, a further 71% of a possible 66 pairwise country involving single crossing Lorenz curves can be ranked using TSD. Davies and Hoy (1995) compare steady state distributions of after-tax life time income in the United States with and without changes introduced under the Tax Reform Act of 1986. They find that TSD does increase the ranking ability.

When MSWR is implemented, the policy leads to an increase in n_m which in turn may increase n_u. Both δ_m and δ_r will play a role in determining conditions for unambiguous changes. The critical value of δ_r is the same as in the case for MSENL. It can be shown that n_u increases if and only if δ_m is less than -1. The critical value of δ_m is δ_m^* as $-1 + \frac{n_m - (1-\pi)\delta_r n_r}{\delta_r \pi n_r + n_m}$ which is strictly less than -1 when δ_r is less than δ_r^*. Hence for the level of unemployment to increase, we need δ_m and δ_r to be sufficiently elastic. The condition for δ_m is also derived in Feldman (1989) and Temple (2002). However the role of δ_r is neglected. Although their analyses will not change, this could be informative for policy markers in choosing between different types of policies given the observed values of δ_m and δ_r.

When a combined policy is implemented, the critical value of δ_r is $\delta_r^{c*} = -\frac{(n_m + n_u)}{\pi n_r}$ where the level of unemployment is unchanged. Hence we need a more elastic value of δ_r for the level of unemployment to increase. This is not surprising since TSENR always reduces the level of unemployment whereas MSENL may increase it. Hence while MSENL increases the level of unemployment, this may not be the case for the combined policy. We then show that when Lorenz curves cross, the use of coefficient of variation enhances the ranking ability.

With respect to welfare analysis, we measure welfare in terms of generalised Lorenz curves. If the policy leads to generalised Lorenz dominance, all utilitarian SWFs, that favour progressive transfers, provide unanimous ranking. Ambiguity occurs when generalised Lorenz curves cross. This is the case where the policy leads to increases in the level of unemployment and mean income. There is a trade-off between equity (inequality) and efficiency (mean income). Moreover when MSWR leads to decreases in the level of unemployment and mean income, we have Lorenz dominance while the generalised Lorenz curves cross once and the equity-efficiency trade-off exists. We show that the variance can be used to enhance ranking ability subject to the constraint on inequality aversion of the utilitarian SWFs. Although ambiguity cannot be completely eliminated, the concept of TSD is worth considering under a simple framework such as the H-T model where ambiguity still occurs. We conclude our analysis in Section 6.

2 The Harris-Todaro (H-T) Migration Model

The H-T model studies labour market in a dualistic economy under the presence of sectoral wage differentials. Risk-neutral individuals decide between working in rural (agricultural) sector, where they receive a wage equal to their marginal productivity (w_r), or migrate to urban (manufacturing) sector, where they may be employed or unemployed. Those who are employed receive a minimum wage ($\bar{w}$) set at the level above the market clearing wage while the unemployed receive zero wage. The available urban jobs are assumed to be filled randomly. The probability of getting an urban job is the ratio of total urban employment (n_m) and total urban labour force (n_u). Hence the expected urban wage is uncertain and depends on a fixed income and endogenous probability determined by the model. On the other hand, the rural labour market is assumed to clear. The rural wage depends only on the total level of rural labour force (n_r).[3] Assume

[3] Other factors that lead to the wage uncertainty in the rural sector are, for example, random states of nature. However if we use only the expected rural wage without comparing

also that the price of agricultural goods relative to manufactures is exogenously fixed and normalised to 1.

Migration process continues until the expected urban wage equals the rural wage. In equilibrium:

$$w_r = \left(\frac{n_m}{n_u}\right)\bar{w} \tag{1}$$

where $\left(\frac{n_m}{n_u}\right)\bar{w}$ is the expected urban wage, n_m and n_u are defined as before, and w_r is the rural wage or rural marginal productivity which varies with the rural labour force (n_r)[4]:

$$w_r = g(n_r);\, g'(n_r) < 0; \tag{2}$$

where $g' = \frac{dg}{dn_r}$:

The constraint for labour endowment is

$$1 = n_u + n_r: \tag{3}$$

The total urban employment is given by[5]

$$n_m = e(\bar{w});\, e'(\bar{w}) < 0: \tag{4}$$

The mean income in the H-T model is

$$\mu = n_m\bar{w} + (1 - n_u)\,w_r$$

and from (1);

$$\mu = w_r: \tag{5}$$

In equilibrium, the level of urban unemployment (U) is

$$U = n_u - n_m \tag{6}$$

which is greater than zero as long as there exist sectoral wage differentials. From (1);

$$\begin{aligned} U &= n_u - n_m \\ &= \left(\frac{\bar{w}}{w_r} - 1\right) n_m: \end{aligned} \tag{7}$$

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the post and prior outcomes before the revelation of uncertainty, the original setting of the H-T model is sufficient and gives the same qualitative results.

[4] The rural output (Y_r) is determined by

$$Y_r = f_r(n_r) := F_r(n_r; \bar{L}; \bar{K}_r)$$

where $\bar{L}$ and $\bar{K}_r$; respectively, are fixed amounts of land and capital. Hence n_r is the only variable input and $f_r' > 0$, $f_r'' < 0$: The rural wage is the marginal productivity which is f_r':

[5] The urban production function is defined as

$$Y_m = f_m(n_m) := F_m\left[n_m; \bar{K}_m\right]$$

where $\bar{K}_m$ is fixed capital; n_m is the only variable input and $f_m' > 0$; $f_m'' < 0$: n_m can be written as a function of $\bar{w}$:

$$n_m = f_m'^{-1}(\bar{w}) = e(\bar{w});\, e'(\bar{w}) < 0:$$

The unemployment rate is defined as

$$\grave{A} = \frac{U}{n_u}: \tag{8}$$

In the existing literature of the H-T type model, an objective of labour market policies is to reduce the level of urban unemployment. We discuss different types of labour market policies in the next section.

3 Labour Market Policies in the H-T Model

First of all, we consider policy suggested by Harris and Todaro (1970) that is a combination of wage subsidy in the manufacturing sector and labour mobility restriction. Such policy could bring the economy back to its first best allocation of resources with full employment. In fact, there is no single policy that could lead to such allocation.

The manufacturing wage subsidy will increase the level of urban employment. From (7); the policy may result in an increase in the level of unemployment and may not lead to the first best allocation. Hence an increase in the level of urban employment alone may lead to a higher level of unemployment. Moreover any policy that imposes the restriction on labour mobility seems undesirable. Instead Bhagwati and Srinivasan (1974) argue that there exists an optimal level of wage subsidy given to both sectors that yields the first best solution.

The uniform wage subsidy has been criticised for two main reasons. First, such policy is costly. Krichel and Levine (1997) show that when the uniform wage subsidy is financed by income taxation, an increase in the income tax rate may be too high. This generates distortion which may offset the benefit of the policy. Second, it is very difficult to determine the optimal level of the wage subsidy. However Basu (1980, 1997) argues that any level of uniform wage subsidy will increase total production in both sectors, which is also accounted as an increase in social welfare, and reduce the level of urban unemployment. Although the policy will not eliminate the urban unemployment if the subsidy is not high enough, the policy is still desirable. Nevertheless Temple (2002) shows that an introduction of a small uniform wage subsidy may be harmful in terms of income inequality. This is because the policy has ambiguous effects on the level of unemployment and distribution of income.

When the manufacturing wage subsidy and the agriculture wage (or production) subsidy are implemented separately, each policy results in higher levels of employment in corresponding sectors. Moreover the first has the same effect as a particular type of growth in development typologies that is the modern sector enlargement growth (MSENL). On the other hand, the latter has the same effect as the traditional sector enrichment growth (TSENR).

Hence instead of considering the subsidy policies, we look at effects of three different types of labour market policies in terms of the levels of employment, inequality of income distribution, and total welfare. The first is MSENL where the level of urban employment increases without affecting the urban minimum wage. This may happen when there is excess demand for manufacturing good leading to biased urban development or when there is urban wage subsidy as discussed above.

The second policy is TSENR where the rural wage increases while the levels of urban employment and minimum wage stay unchanged. Examples of TSENR are rural wage subsidy, investment in land productivity and irrigation, etc. Moreover a combination of MSENL and TSENR can be used to achieve the same qualitative results as the uniform subsidy, see Section 3.1.4.

Finally, the third policy is the modern sector wage restraint (MSWR) where there is a reduction in the urban minimum wage affecting the levels of urban employment and total labour force. This policy is also suggested in Harris and Todaro (1970) as a plausible policy to raise employment in the urban sector. Each of the above policies affects the level of urban labour force and will also affect the rural wage, see equation (2):

Note that the modern sector wage enrichment (MSENR) would have opposite effects on the levels of urban employment and expected wage as MSWR.[6] MSENR increases the urban wage and reduces the level of urban employment. The effect on the level of unemployment will be ambiguous. In this paper, we will not consider MSENR explicitly. This is because such policy may worsen the problem of urban bias leading to an increase in unemployment rate.

There is an ample number of existing literature that consider the income inequality and welfare effects of different types of development policies, see Fields (1979, 2001) and Temple (2002) for inequality analysis, and Chakravarty and Dutta (1990) and Fields (1979, 2001) for welfare analysis.

Most of these works tie inequality and welfare measurements to some specific inequality indices and inequality-based welfare functions. Hence the results may be different depending on the measurement used in the analysis. However Fields (2001) and Temple (2002) consider inequality in terms of Lorenz curves allowing general statements to be made. Another weakness, except Temple (2002), is the assumption of constant rural wage. Although in most developing countries, the rural wage is inelastic, there is still evidence that it is not constant, see Abdulai and Delgado (1999) and references cited therein.

We base our analysis on Fields (2001) and Temple (2002). The conditions for unambiguous inequality and welfare changes depend on wage elasticities of labour demand in urban and rural sectors, denoted by δ_m and δ_r respectively. Although their values depend on endogenous variables, Temple argues that they should be seen as a description of long-run behaviour of inequality. This is because the H-T equilibrium wage condition is assumed to hold whenever the change in income distribution is analysed. Hence steady states are implicitly compared.

Contributions of this paper:

As in the original H-T model, we assume that individuals are risk neutral and the rural wage depends on the number of rural workers. This simply allows for the equity-efficiency trade-off with development process. We measure inequality in terms of Lorenz curves as in Fields (2001) and Temple (2002) and reexamine the three main policies in the H-T framework using Temple's results of unambiguous inequality changes. Both δ_m and δ_r play crucial role in determining condition for unambiguous changes. In the existing literature, the role of δ_r has been neglected either because the rural wage is constant or because the constraint on δ_r has not been specified. Moreover when ambiguity occurs,

[6] Chakravarty and Dutta (1990) consider the effect of MSENR on welfare using three specific social welfare functions (SWFs). For each SWF, they find condition for welfare improvement.

we attempt to find conditions for unambiguous inequality changes beyond the Lorenz criterion using the concept of third order stochastic dominance (TSD). This is done by the use of the coefficient of variation, see Sections 4 and 5.

Secondly, we consider welfare change for each of the policies in a more general approach using generalised Lorenz dominance. When there is a conflict between efficiency and equity, we look for unambiguous conditions for welfare changes using TSD approach. This will hold for all utilitarian social welfare functions (SWFs) satisfying the Principle of Transfer Sensitivity providing that individual utility function exhibits a sufficiently high degree of inequality aversion, see Section 4. The Atkinson and Kolm-Pollak welfare functions used in Charkravaty and Dutta (1990) are members of this class of SWFs.

The Gini welfare index is a member of a different class of SWFs, namely Yaari social welfare function (YSWF), see Zoli (1999) for the inequality and welfare analyses based on YSWFs. The direction of welfare changes for YSWFs in the H-T model may be found. However the calculation is more complicated and will not be presented in this paper.

Before ending this section, we show how different development policies affect the levels of unemployment, unemployment rate and mean income.

3.1 Effects of Labour Market Policies on Mean Income, Level of Unemployment, and Unemployment Rate

3.1.1 Modern Sector Enlargement

When a development policy is classified as a policy of MSENL, the urban wage is unchanged and the total number of urban jobs increases leading to a higher expected urban income. MSENL policies are, for examples, urban wage subsidy, and an import tariff when manufacturing good is considered as an import substitute. We have discussed the effect of the urban wage subsidy. As for the import tariff, the policy could lead to an increase in the demand of domestically produced import substitutes resulting in higher labour demand in the urban sector.

Fields (2001) assumes that w_r is constant. From (7), we see immediately that MSENL leads to an unambiguous increase in the level of urban unemployment. However in our setting, the rural wage increases when there is more migration. This helps to bring the economy back to its equilibrium at a faster rate than in the economy with constant rural wage. Since MSENL leads to further rural-to-urban migration, the mean income increases as the number of rural workers reduces. The effect on the level of unemployment is ambiguous depending on the wage elasticity of labour demand in the rural sector (δ_r). The value of δ_r tells us how much the demand for rural labour changes when the rural wage changes, i.e. $\delta_r = \left(\frac{\Delta n_r}{n_r}\right)\left(\frac{w_r}{\Delta w_r}\right)$ where Δ indicates total change in corresponding variable. From (1), (3), (5) and (8), $\delta_r = \left(\frac{\Delta n_u}{n_u}\right)\left(\frac{\upsilon}{\Delta \upsilon}\right)\frac{n_m}{\upsilon n_r}$. There is a critical value of δ_r (δ_r^*) at which the level of unemployment (U) is unchanged. By definition of δ_r and υ, δ_r^* is achieved when the percentages change in urban labour force (n_u) and unemployment rate (υ) are equal in absolute terms: given that $U = \upsilon n_u$ then $\Delta U = (\Delta \upsilon) n_u + (\Delta n_u)\upsilon = 0 \iff \frac{\Delta n_u}{n_u} = -\frac{\Delta \upsilon}{\upsilon}$.

The effects of the policy on μ, υ and U are summarised as in Lemma 1:

Lemma 1. *MSENL leads to*

(i) *an unambiguous increase in* μ;

(ii) *an unambiguous decrease in* υ;

(iii) *an ambiguous change in* U : *the critical value of* δ_r *is* $\delta_r^* = -\frac{n_m}{An_r}$ *where at* $\delta_r = \delta_r^*$; *the level of* U *is unchanged. Below this threshold, MSENL leads to an increase in the level of* U *and vice versa.*

Proof. See Appendix A. ■

Unlike Fields (2001), δ_r plays a role in determining the direction of changes in the level of unemployment. As expected, the expression of δ_r^* holds only when the percentages change in n_u and υ are equal in absolute terms. For high values of υ and/or n_r; the value of δ_r^* may be considerably low. In such case, MSENL can easily lead to higher level of unemployment. Note that the unemployment rate unambiguously decreases while it is unchanged if w_r is constant.

3.1.2 Traditional Sector Enrichment

This type of development policy leads to a higher rural wage without affecting either the level of urban employment or the minimum wage. There is no conflict between our results and those in Fields (2001). The policy unambiguously raises the rural employment lowering the level of unemployment and unemployment rate.

Lemma 2. *TSENR directly increases* μ *and always reduces the level of urban unemployment and unemployment rate.*

Proof. See Appendix A. ■

3.1.3 Modern Sector Wage Restraint

The policy reduces the minimum wage leading to an increase in the level of urban employment. The effect on the level of unemployment is ambiguous and will depend on the values of δ_m and δ_r: Therefore the change in the mean income is also ambiguous since its level depends on the number of urban labour force. δ_m determines how much the level of urban employment increases with respect to a change in the minimum wage, i.e. $\delta_m = \frac{\Delta n_m}{n_m} \frac{\bar{w}}{\Delta \bar{w}}$. For the level of unemployment to increase, it is necessary that δ_m is sufficiently elastic. This is because a small reduction in the minimum wage will be sufficient to raise significant number of urban jobs resulting in higher expected urban wage, further rural-to-urban migration, and possibly higher level of unemployment. However the condition for δ_m alone is insufficient. We need also the condition for δ_r as in the case of MSENL. Analytically it is possible that the policy leads to a lower number of urban labour force leading to a fall in the mean income. The effects of MSWR are summarised in Lemma 3.

Lemma 3. *MSWR leads to*

(i) *an unambiguous decrease in* υ;

(ii) *an unambiguous decrease in* U *while* n_u *and* μ *remain constant when* $\delta_m = -1$;

(iii) *unambiguous decreases in* U, n_u *and* μ *when* $\delta_m > -1$;

(iv) *unambiguous increases in* n_u *and* μ *and an ambiguous change in* U *when* $\delta_m < -1$:

$\frac{dU}{d\bar{w}} > 0 \iff \delta_m < -1$ *and* $\delta_r > \delta_r^*$

$\frac{dU}{d\bar{w}} > 0 \iff \delta_m^* \leq \delta_m < -1$ *and* $\delta_r < \delta_r^*$;

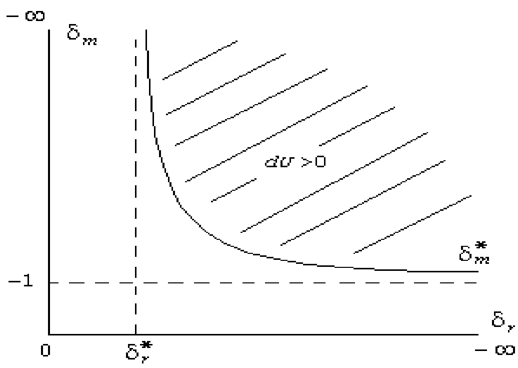

Figure 1: Representative area of elasticities where dU > 0 when MSWR is implemented. dU denotes the total change in the level of unemployment.

$$\frac{dU}{d\bar{w}} < 0 \iff \delta_m < \delta_m^* \text{ and } \delta_r < \delta_r^*;$$

where $\delta_r^* = -\frac{n_m}{\upsilon n_r}$ *and* $\delta_m^* = -\frac{\delta_r n_r}{\upsilon \delta_r n_r + n_m} = -1 + \frac{n_m - (1-\upsilon)\delta_r n_r}{\upsilon \delta_r n_r + n_m}$.

Proof. See Appendix A. ■

The values of δ_m and δ_r from above conditions for unambiguous changes in U and μ can be plotted as in Figure 1. The shaded area represents the values of δ_m and δ_r such that MSWR leads to an increase in the level of unemployment. That is when $\delta_m < \delta_m^*$ and $\delta_r < \delta_r^*$. Along the curve δ_m^*, the level of unemployment is unchanged. In other cases, it decreases unambiguously. This critical value of δ_m is also given in Temple (2002). Moreover the constraint on δ_r guarantees that the values of the elasticities are negative. The line $\delta_m = -1$ indicates that the mean income is unchanged. For $\delta_m > -1$, it decreases and vice versa. The mean decreases if δ_m is inelastic. Hence to increase n_m, we need to reduce the level of the minimum wage so much resulting in a reverse migration back to the rural sector. In such case, it is clear that U will decrease. Hence $\delta_m < \delta_m^*$ and $\delta_r < \delta_r^*$ are both necessary and sufficient conditions for stating unambiguous increase in the level of unemployment. However it is usually assumed that $\delta_m > -1$, as in Agénor (1996, fn. 21), then only Lemma 3(iii) will hold.

Feldman (1989) also provides similar plot of representative region of elasticities and states that two necessary conditions for increasing rural-urban migration are $\delta_r < 0$ and $\delta_m < -1$.

It is also shown in Appendix A that when w_r is fixed, $\delta_r \to -1$, $\frac{d\mu}{d\bar{w}} \to 0$ and $\frac{dU}{d\bar{w}} \to \frac{n_u}{\bar{w}}[1 + \delta_m \upsilon]$. Hence only δ_m determines the direction of the change in U for MSWR. This is consistent with Fields (2001).

3.1.4 Combining MSENL and TSENR

As discussed earlier, a combination of MSENL and TSENR could lead to the same qualitative results as the uniform wage subsidy in both sectors. MSENL increases n_m and may or may not increase U while TSENR unambiguously decreases U. Taking the increase in n_m as given, we can expect that the direction of change in the level of U depends on $\pm_r$. The critical value of $\pm_r$ under the combined policy ($\pm_r^{c¤}$) should be higher than that under MSENL ($\pm_r^{¤}$) in absolute terms.

Lemma 4. *A combined policy of MSENL and TSENR leads to*

(i) *an unambiguous decrease in* À;

(ii) *an unambiguous increase in* ¹*;*

(iii) *an ambiguous change in* U :

$$dU_c \; 6 \; 0 \;\; , \;\; \pm_r > \pm_r^{c¤};$$
$$dU_c > 0 \;\; , \;\; \pm_r < \pm_r^{c¤}$$

where $\pm_r^{c¤} = \pm_r^{¤} \; ¡ \; \frac{n_u}{An_r}$ *and* dU_c *represents the total change in the level of unemployment under the combined scheme.*

Proof. See Appendix A. ■

As suggested in Temple (2002) for the uniform wage subsidy, the combined policy also leads to an ambiguous change in the level of unemployment. We need a more elastic value of $\pm_r$ for unemployment to increase. Although this is more bene¯cial compared with the policy of MSENL, we have not discussed how to ¯nance such combined scheme.

Before turning to inequality and welfare comparisons, we summarise some useful results of inequality and welfare judgements applied to the H-T model in the next section.

4 Some useful results for inequality and welfare comparisons

This section summarises practical methods of comparing income distributions with respect to inequality and welfare. Since we make comparisons in general terms, we measure inequality and welfare in terms of Lorenz and generalised Lorenz curves, respectively.

Lorenz dominance implies unanimous agreement among all inequality indices obeying the Principle of Transfers (PT). PT requires that inequality does not increase when a given amount of income is transferred from a richer individual to a poorer individual, i.e. a progressive transfer. Atkinson (1970) points out that the procedures based on the comparison of Lorenz curves and second order stochastic dominance (SSD), drawing upon the theory of choice under uncertainty, are equivalent. In our analysis, the size of total population stays the same while the mean income can change as a result of policy implications. Thus we use mean-normalised distributions in inequality comparisons. However PT does not enable us to rank a pair of distributions when both a progressive and a regressive transfers are needed to convert one distribution into the other. Such transfers result in intersecting Lorenz curves. Therefore Lorenz dominance fails to provide a conclusive ranking.

Suppose we view that the lower in the distribution the progressive transfer occurs, the greater its impact on inequality reduction. Thus composite transfers which combine a progressive transfer with a regressive transfer at a higher income level is said to be favourable, i.e. inequality reducing. We call such transfer a *favourable composite transfer* (FACT). An inequality index is transfer sensitive if it decreases under the operation of any FACT. Shorrocks and Foster (1987) demonstrate that third order stochastic dominance (TSD) allows us to characterise unanimous agreement among the class of transfer sensitive inequality indices. This is done by imposing conditions on variance if mean incomes are the same and on coefficient of variation if mean incomes are different.

In terms of welfare comparisons, the class of social welfare functions (SWFs) that satisfies PT is the class of inequality-averse additive separable symmetric SWFs. Again the generalised Lorenz dominance is related to SSD. Nevertheless when the generalised Lorenz curves of two distributions cross an odd number of times, and mean incomes are unequal, efficiency-preference and the Rawlsian leximin (extreme inequality-aversion) criterion come into conflict. Under the equity-efficiency trade-off, the unanimous preference of all SWFs in this class is impossible. Dardanoni and Lambert (1988) show that an informed trade-off can be rationalised by imposing restrictions on the aforesaid class of SWFs. Thus equality will be preferred to efficiency by a subset of the aforesaid class of SWFs whose size, and inequality-posture, is a function of the means and variances of the distributions under examination. The procedure is related to TSD. We present the formal analytical framework, taken from Shorrocks and Foster (1987), Dardanoni and Lambert (1988), and Moyes (1999).

4.1 Analytical framework

Consider discrete, finite-population income distributions defined over positive income values, an income distribution for a homogeneous population consisting of N individuals ($N \geq 2$), ranked in increasing order, is

$$X := (x_1; x_2; \dots; x_N) \in D$$

where D represents the set of income distributions and $x_i \in [0; w^{max}]$ is the income of individual i, $x_i \leq x_{i+1}; i = 1; 2; \dots; N$:

The mean income and the variance of X are defined as

$$\mu_X = \frac{1}{N} \sum_{i=1}^{N} x_i \tag{9}$$

and

$$\sigma_X^2 = \frac{1}{N} \sum_{i=1}^{N} (x_i - \mu_X)^2 : \tag{10}$$

The coefficient of variation of X, C_X; is defined as

$$C_X = \frac{\sigma_X}{\mu_X} : \tag{11}$$

Let F_X be a discrete cumulative distribution for income X. Define

$$F_X^r(w) = \frac{1}{(r-1)!N(X)} \sum_{j=1}^{q(w;X)} (w - x_j)^{r-1} \text{ for all } w \in \Re$$

where $q(w; X) := \#\{j \in \{1; 2; \dots; N\} \mid x_j \leq w\}$ is the number of individuals having income equal or less than w in situation X: Given two income distributions $X; Y \in D$ with $\mu_X = \mu_Y$ then X stochastically dominates Y to the second order, written as $X \succcurlyeq_{SSD} Y$, if $F_X^2(w) \leq F_Y^2(w)$ for all $w \in \Re$: Similarly, X stochastically dominates Y to the third order, $X \succcurlyeq_{TSD} Y$, if $F_X^3(w) \leq F_Y^3(w)$ for all $w \in \Re$: Note that we can normalise any pair of income distributions such that their means are equal.

4.1.1 Inequality comparisons

Let $L_X(p)$ denote the Lorenz curve of distribution X where $p = F_X(w); 0 \leq p \leq 1$. It represents the aggregate income possessed by the p100% poorest individuals in situation X scaled down by the population size N. X Lorenz dominates Y, $X \succeq_L Y$, if $L_X(p) \geq L_Y(p)$ for all $p \in [0;1]$. It can be shown that Lorenz ordering is equivalent to SSD, see Moyes (1999).

Let $I : D \to \Re$ be an inequality index. Assume that I is a continuous function such that $I(X) \leq I(Y)$ implies that distribution X is no more unequal than distribution Y. For all $X;Y \in D$ with $N(X) = N(Y) \geq 2$; the following is a list of definitions of desirable properties of I:

Symmetry (S). $I(X) = I(Y)$ whenever X is obtained from Y by a permutation.

Principle of Transfer (PT). $I(X) < I(Y)$ whenever X is obtained from Y by a progressive transfer.

Population Principle (PP). $I(X) = I(Y)$ whenever X is obtained from Y by replication.

Scale Invariance (SI).[7] $I(X) = I(Y)$ whenever X is obtained from Y by a scale improvement.

Proposition 1 *[Shorrocks and Foster (1987)] Let $X;Y \in D$, the following statements are equivalent:*

(a) $\hat{X}$ can be obtained from $\hat{Y}$ by a non-empty finite sequence of rank-preserving progressive transfers.

(b) $I(X) \leq I(Y)$ *for all* I *satisfying* $S; PT; SI$ *and* PP.

(c) $X \succeq_{SSD} Y$.

(d) $X \succeq_L Y$.

However when Lorenz curves of distributions X and Y cross, we cannot rank X and Y using the Lorenz criterion nor SSD. Nevertheless, following Kolm (1976), we may be able to rank them if I also satisfies the Principle of Transfer Sensitivity.

Transfer Sensitivity (TS). For all $X;Y \in D$ with $N(X) = N(Y) \geq 3$; $I(X) \leq I(Y)$ whenever X is obtained from Y under the operation of FACT.

Proposition 2 *[Shorrocks and Foster (1987)] Let $X;Y \in D$, the following statements are equivalent:*

(a) $\hat{X}$ can be obtained from $\hat{Y}$ by a non-empty finite sequence of rank-preserving progressive transfers and/or FACTs.

(b) $I(X) \leq I(Y)$ *for all* I *satisfying* $S; PT; TS; SI$ *and* PP.

(c) $X \succeq_{TSD} Y$.

Shorrocks and Foster (1987) also show that when Lorenz curves cross just once the variance or the coefficient of variation plays a crucial role in ranking the distributions according to TSD.

Proposition 3 *[Shorrocks and Foster (1987)] Let $X;Y \in D$; if the Lorenz curve of X intersects that of Y once from above then $I(X) \leq I(Y)$ for all I satisfying $S; PT; TS; SI$ and PP if and only if*

$$C_Y > C_X.$$

[7] We only present the analysis considering the set of relative inequality indices. For the set of absolute inequality indices, SI will be replaced with translation invariance (TI); see Moyes (1999).

Note that if $\mu_X = \mu_Y$, the condition for the coefficient of variation in Proposition 3 will be replaced by $\sigma_Y^2 > \sigma_X^2$:

4.1.2 Welfare comparisons

In order to obtain robust welfare interpretation, the generalised Lorenz dominance becomes relevant. The generalised Lorenz curve of distribution X, GL_X; is defined as

$$GL_X(p) = \mu_X L_X(p): \tag{12}$$

X generalised Lorenz dominates Y if $GL_X(p) \geq GL_Y(p)$, all $p \in [0;1]$:

Let $V(x_i)$ be an individual i's utility of income function having positive marginal utility which declines with income: $V'(x_i) > 0$ and $V''(x_i) < 0; x_i \in [0; w^{max}]$: The utilitarian SWF of income distribution X is defined as

$$W(X) = \sum_{i=1}^{N} V(x_i): \tag{13}$$

W is an additively separable, symmetric and inequality-averse function of individual incomes. Let

$$\Omega = \{W : V'(x_i) > 0; V''(x_i) < 0 \text{ for all } x_i \in [0; w^{max}]\} \tag{14}$$

be the class of all such SWFs. Note that SWFs in this class satisfied PT:

Let's also define the Rawlsian leximin criterion. The Rawlsian leximin criterion ranks X higher than Y, $X \succ_R Y$; if under X the poorest income is greater than under Y, or if under X it is the same but occurs with a lower frequency.

Proposition 4 *[Rothschild and Stiglitz (1970)/ Hadar and Russell (1969)/ Shorrocks (1983)] Let* $X; Y \in D$, $W(X) \geq W(Y)$ *for all* $W \in \Omega \iff GL_X(p) \geq GL_Y(p)$; *all* $p \in [0;1]$:

It can be shown that the condition in Proposition 4 is equivalent to SSD, see Dardanoni and Lambert (1988). When generalised Lorenz curves cross an odd number of times, there is an equity-efficiency trade-off. To be able to rank the two distributions whose generalised Lorenz curves cross, we need to restrict the class of inequality-averse SWFs.

Let the sub-class Ω^* of Ω be defined by

$$\Omega^* = \{W \in \Omega : V'''(x_i) > 0 \text{ for all } x_i \in [0; w^{max}]\}: \tag{15}$$

Ω^* contains all SWFs in Ω with constant relative inequality aversion, and those with constant and decreasing absolute inequality aversion. It favours Kolm's principle of transfer sensitivity.

Dardanoni and Lambert (1988) show that when efficiency and equity are in conflict, in the particular case of a single generalised Lorenz crossing, a form of mean-variance analysis is decisive. The distribution with the lower mean can be recommended if its variance is sufficiently less than that of the efficiency-superior distribution. This is also related to TSD. In the H-T model, $w^{max} = \bar{w}$:

Proposition 5 *[Dardanoni and Lambert (1988)] Let* $X; Y \in D$, *suppose that* $\mu_Y > \mu_X; X \succ_R Y$ *and* $GL_X; GL_Y$ *cross once. If*

$$\sigma_X^2 \leq \sigma_Y^2 - (\mu_Y - \mu_X)(2\bar{w} - \mu_X - \mu_Y)$$

then $W(X) \geq W(Y)$ *for all* $W \in \Omega^*$ *such that*

$$-\frac{\bar{w}V''(\bar{w})}{V'(\bar{w})} \geq \frac{2\bar{w}(\mu_Y - \mu_X)}{(\sigma_Y^2 - \sigma_X^2) - (\mu_Y - \mu_X)(2\bar{w} - \mu_X - \mu_Y)} > 0.$$

The term $-\frac{V''(\bar{w})}{V'(\bar{w})}$ and $-\frac{\bar{w}V''(\bar{w})}{V'(\bar{w})}$ measures the degree of absolute and relative inequality aversion, respectively. The mean-variance expression is therefore the lower bound on inequality aversion which permits unanimous preference for X over Y for all utilities with constant and decreasing absolute inequality aversion and with constant relative inequality aversion.

5 Inequality and Welfare Analyses of Labour Market Policies

In the H-T model, the income distribution ranked in increasing order is

$$X = (0, 0, \ldots, 0, w_r, w_r, \ldots, w_r, \bar{w}, \bar{w}, \ldots, \bar{w}).$$

From (5), the mean income is

$$\mu = w_r.$$

From (10), and (11), the variance and the coefficient of variation are

$$\sigma^2 = \frac{n_m}{n_u}(n_u - n_m)\bar{w}^2 \qquad (16)$$

and

$$C = \left(\frac{n_u}{n_m}(n_u - n_m)\right)^{\frac{1}{2}}.$$

Without loss of generality the analysis will be made in terms of C^2. This is for convenience and simplicity purposes. Hence for the H-T type income distribution

$$C^2 = \frac{n_u}{n_m}(n_u - n_m). \qquad (17)$$

Notice that if n_u increases, other things being equal, C^2 increases unambiguously. This implies that rural investment will reduce inequality.

The Lorenz curve in the H-T model is drawn as in Figure 2. It consists of three linear segments and bends twice. The first segment, $0K_1$, is entirely flat because the unemployed receive the lowest income which is zero. The middle segment, K_1K_2, has slope of 1 since the rural wage is the mean income. The last segment, K_2K_3, is associated with the high income group and has slope of $\frac{\bar{w}}{w_r}$. Note that in the case of a perfect equal income distribution, the Lorenz curve lies along the forty-five degree line $0K_3$.

We restate Temple's necessary and sufficient conditions for an unambiguous decrease in wage inequality measured in terms of Lorenz curves as follows:

(D1) υ falls and U goes down;

(D2) υ is constant and U goes down. n_m falls and n_r rises;

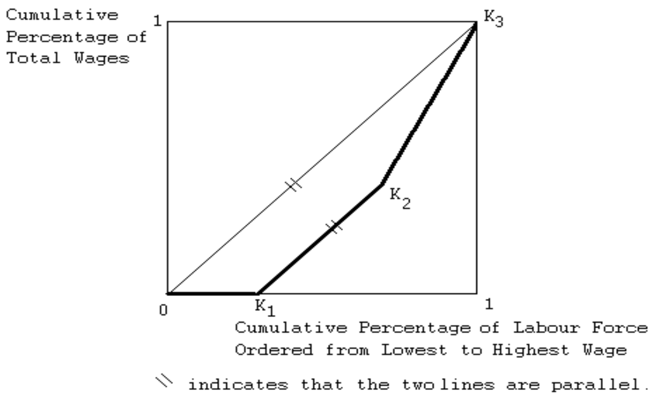

Figure 2: Lorenz Curve of the Original H-T Income Distribution, L_X (p)

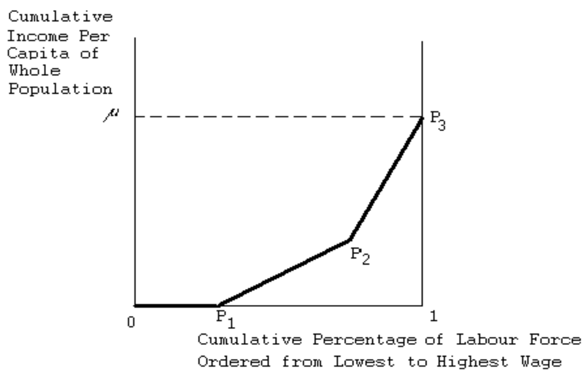

Figure 3: The Generalised Lorenz Curve of the H-T Income Distribution

(D3) À falls and U is constant. n_m rises and n_r falls.

Conditions (D1)-(D3) satisfy Proposition 1(d). And for all inequality indices satisfying S; PT; SI and PP, inequality unambiguously reduces. Note that an increase in À is equivalent to an increase in the slope of the third segment, K_2K_3: Hence to know what happens to inequality in the H-T model, we need to know the levels of À and U, Temple (2002).

The generalised Lorenz curve of a H-T type income distribution follows immediately from (12) and is drawn in Figure 3: It also consists of three segments. The ¯rst segment $0P_1$, associated with the unemployed, is also °at. The second segment P_1P_2 has slope of w_r while the last segment P_2P_3 has slope of $\bar{w}$:

The mean income becomes relevant when welfare is considered. Unambiguous statements of welfare improvement can be made when there is no con°ict between e±ciency and equity, see also Proposition 4. The conditions for unambiguous reduction in inequality, (D1)-(D3), do not always lead to unambiguous rise in welfare measured in terms of generalised Lorenz curves. In particular,

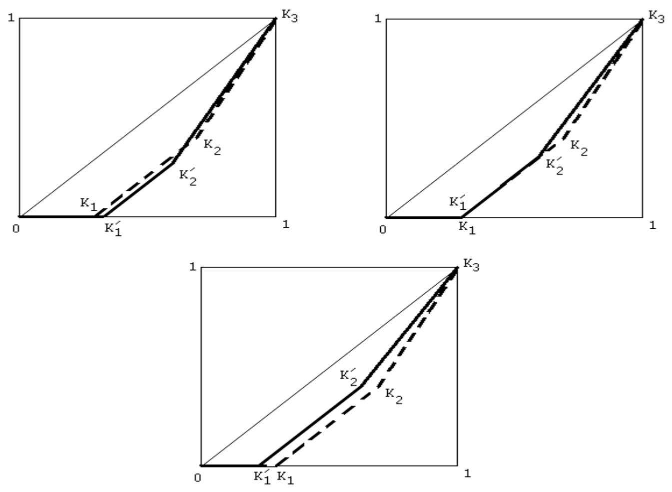

Figure 4: Lorenz Curves for the Modern Sector Enlargement: (a) dU > 0 ; (b) dU = 0 ; (c)dU < 0. Note that the dotted line in each case represents the Lorenz curve of the original income distribution.

(D2) implies that both U and ¹ fall resulting in equity-e±ciency trade-o®.

5.1 Modern Sector Enlargement

From Lemma 1, MSENL lowers À; increases ¹ and we have $\left(\frac{dU}{dn_m}\Big|_{4w=0}\right)$ R 0 , $\pm_r$ Q $\pm_r^{¤}$: In terms of the Lorenz curve, the ¯rst segment, $0K_1^0$; is still °at. The position of K_1^0 depends on the sign of $\frac{dU}{dn_m}\Big|_{4w=0}$: The second segment, $K_1^0K_2^0$; still has slope of 1. The slope of the third segment, $K_2^0K_3$; is lower: If the urban unemployment increases, K_1^0 lies to the right of K_1 and we have Lorenz crossing, MSENL case (a). In other cases, (b) and (c), where the level of unemployment is unchanged and when it decreases we have Lorenz improvement, see Figure 4. MSENL cases (b) and (c) are eqivalent to conditions (D3) and (D1), respectively.

Since MSENL leads to an increase in the mean income, the corresponding generalised Lorenz curves are drawn as in Figure 5. The third segment, $P_2^0P_3^0$, lies above P_2P_3 but still has the same slope. P_2^0 lies to the left of P_2 since there is an increase in n_m. The second segment, $P_1^0P_2^0$ is steeper than P_1P_2 since w_r increases. Figure 5 shows three possibilities of the new generalised Lorenz curves corresponding to the three possible Lorenz curves drawn in Figure 4.

We have generalised Lorenz dominance if the level of urban unemployment does not increase, Figures 5(b) and 5(c). Otherwise we have generalised Lorenz crossing, Figure 5(a).

Using Proposition 4 and let Y and X be the new and original income distributions, respectively, for all values of $\pm_r > \pm_r^{¤}$ we have W (Y) ¸ W (X) for all W 2 - since GL_Y ¸ GL_X:

The ambiguity occurs when U increases that is when $\pm_r < \pm_r^{¤}$. GL_Y crosses GL_X once from below and $¹_Y > ¹_X$; Figure 5(a). This implied that X $Â_R$ Y:

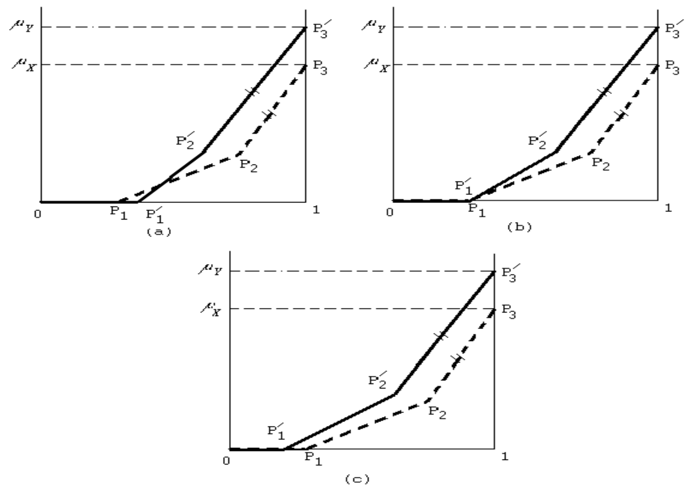

Figure 5: Generalised Lorenz Curves for the Modern Sector Enlargement: (a) $dU > 0$; (b) $dU = 0$; (c) $dU < 0$: Note that the dotted line in each case represents the generalised Lorenz curve of the original income distribution.

From Propositions 2, 3 and 5, it may be possible to make inequality and welfare comparisons making use of the coefficient of variation and the variance.

The directions of changes in income inequality and welfare for MSENL are given as in the following Proposition where $I; W; \Omega; \Omega^*$ are defined as in Section 4, $\delta_r^* = -\frac{n_m}{\lambda n_r}$ and $\hat{\delta}_r = -\frac{n_u}{\lambda n_r}$.

Proposition 6 *Let Y and X be the new and original H-T income distributions, respectively, MSENL leads to following results:*

(i) *When $\delta_r > \delta_r^*$; $Y \succ_L X$;*

When $\delta_r \leq \hat{\delta}_r$; $X \succ_{TSD} Y$ since $C_Y > C_X$;

When $\hat{\delta}_r < \delta_r < \delta_r^*$; the direction of the change in income inequality is ambiguous;

(ii) *When $\delta_r > \delta_r^*$; $GL_Y \geq GL_X$;*

When $\delta_r < \delta_r^*$; $X \succ_{TSD} Y$ for all $W \in \Omega^*$ if and only if

$$-\frac{\hat{w}V''(\hat{w})}{V'(\hat{w})} \geq \frac{-2(n_r\delta_r - n_u)}{\lambda(n_r\delta_r - n_u)^2 + (2\lambda + n_m)(n_r\delta_r - n_u) + 1}.$$

Proof. See Appendix B. ■

The use of TSD increases ranking ability. When MSENL leads to Lorenz crossing, inequality worsens if δ_r is significantly elastic. This is because MSENL raises significant number of rural-to-urban migrants while the rural wage does not increase much. The number of the rich and the unemployed increases leading to an increase in the coefficient of variation. Hence income inequality unambiguously increases according to TSD.

Moreover when MSENL leads to Lorenz crossing, generalised Lorenz curves also cross. The policy worsens welfare for some SWFs in subclass Ω^* if their

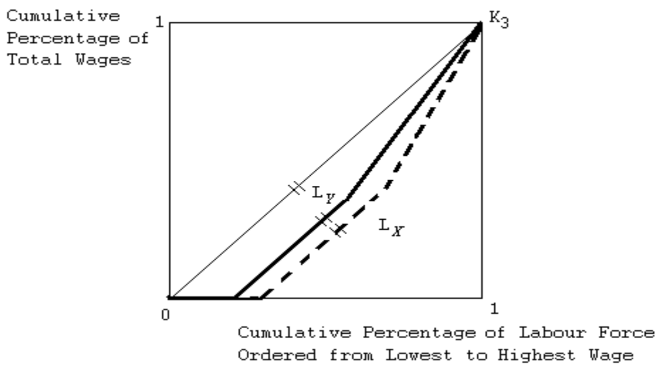

Figure 6: Lorenz improvement for the traditional sector enrichment: L_Y and L_X denote the Lorenz curves of the new and original income distributions.

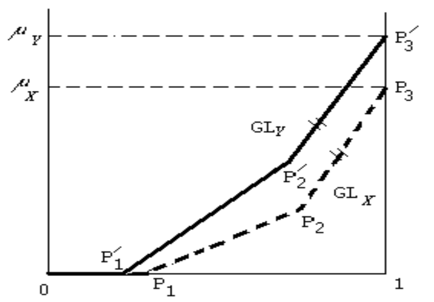

Figure 7: Generalised Lorenz Curve for Traditional Sector Enrichment: GL_Y and GL_X represent the new and original generalised Lorenz curves.

degree of inequality aversion is higher than its lower bound expressed as in Proposition 6(ii). However we need more information on endogenous variables in order to evaluate this lower bound.

5.2 Traditional Sector Enrichment

From Lemma 2, TSENR unambiguously lowers the level of unemployment leading to Lorenz and generalised Lorenz dominance as drawn in Figures 6 and 7, respectively.

The new generalised Lorenz curve is drawn as in Figure 7.

Using Propositions 1 and 4, we have unambiguous inequality and welfare improvement.

Proposition 7 *Let* Y *and* X *be the new and original H-T income distributions, respectively, TSENR leads to* $Y \succ_L X$; *and* $GL_Y \succ GL_X$.

5.3 Modern Sector Wage Restraint

MSWR leads to ambiguous changes in the level of unemployment and the rural wage, see Lemma 3. The Lorenz curves for MSWR can be drawn as those for MSENL. When the policy leads to an increase in the level of unemployment, the Lorenz curves cross. Otherwise we have Lorenz improvement.

Given the conditions in Lemma 3, there are five possibilities of total changes in the levels of mean income ($d\mu$) and unemployment (dU): The generalised Lorenz curves associated with the above five cases are respectively drawn in Figure 8(a)-(e), where GL_X and GL_Y denote the generalised Lorenz curves of the original H-T and the new income distributions, respectively.

a $dU = 0$ and $d\mu > 0 \iff \delta_m = \delta_m^*$ and $\delta_r < \delta_r^*$: In this case, the increase in the number of urban jobs is equal to the increase in the number of rural-urban migrants leading to an increase in mean income.

b $dU < 0$ and $d\mu = 0 \iff \delta_m = -1$ and $\delta_r < 0$: The increase in the number of urban jobs is fulfilled by those who are unemployed. Hence the level of rural population remains unchanged.

c $dU < 0$ and $d\mu > 0 \iff \delta_m^* < \delta_m < -1$ and $\delta_r < 0$: The number of urban jobs increases so much given a small reduction in $\bar{w}$: There will be further rural-to-urban migration leading to lower number of rural workers at the new equilibrium.

d $dU < 0$ and $d\mu < 0 \iff \delta_m > -1$: The elasticity of the minimum wage with respect to the urban labour demand is inelastic. This results in a reverse migration from urban to rural sector leading to lower mean income.

e $dU > 0$ and $d\mu > 0 \iff \delta_m < \delta_m^*$ and $\delta_r < \delta_r^*$: The policy leads to more migration to the urban sector at the level more than the number of new jobs. Thus there will be more unemployment. The rural wage thus increases.

Note that the direction of the change in inequality is ambiguous in case (e), whereas for welfare analysis ambiguity occurs in both cases (d) and (e). Using Propositions 1-5, the results of inequality and welfare changes for MSWR are summarised as in the following Proposition: where δ_m^* and δ_r^* are defined as before,

$$\hat{\delta}_m = -\frac{(1+\lambda)\,n_r\delta_r}{\lambda n_r\delta_r + n_u}$$

$$\hat{\delta}_r = -\frac{n_u}{\lambda n_r}$$

$$\delta_m^0 = -1 + \left(\frac{n_r\delta_r - n_u}{2n_m}\right)\left\{\bar{w}[n_m + \lambda(n_r\delta_r - n_u + 2)] - \left[\bar{w}^2(n_m + \lambda(n_r\delta_r - n_u + 2))^2 + 4\bar{w}n_m\right]^{\frac{1}{2}}\right\}$$

$$\delta_r^0 = \frac{(1+\lambda)\,n_u - 2\lambda n_r}{2(1+\lambda)\,n_r} - \frac{1}{2\bar{w}(1+\lambda)\,n_r}\left[\bar{w}^2(2\lambda + n_m)^2 + 4\bar{w}n_m(1+\lambda)\right]^{\frac{1}{2}}$$

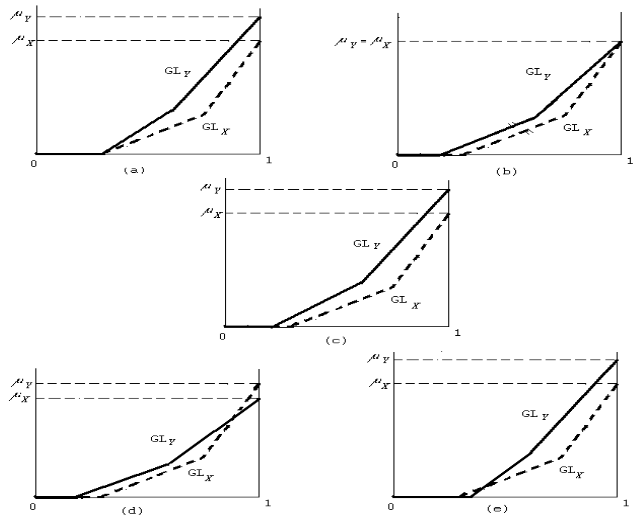

Figure 8: Generalised Lorenz curves for the modern sector wage restraint: (a) $dU = 0$ and $d\mu > 0$; (b) $dU < 0$ and $d\mu = 0$; (c) $dU < 0$ and $d\mu > 0$; (d) $dU < 0$ and $d\mu < 0$; (e) $dU > 0$ and $d\mu > 0$:

and

$$\psi = -\left[\frac{2\bar{w}}{\bar{w}(n_m + \upsilon(n_r\delta_r - n_u + 2)) - \frac{n_m(1+\delta_m)}{(n_r\delta_r - n_u)} + \frac{\bar{w}(n_r\delta_r - n_u)}{(1+\delta_m)}}\right]$$

Proposition 8 *Let* Y *and* X *be the new and original H-T income distributions, respectively, MSWR leads to following results:*

(i) *When* $\delta_m > \delta_m^*$ *and* $\delta_r < 0$; $Y \succeq_L X$;

When $\delta_m \leq \hat{\delta}_m$ *and* $\delta_r < \hat{\delta}_r$; $X \succeq_{TSD} Y$ *since* $C_Y > C_X$;
When $\hat{\delta}_m < \delta_m < \delta_m^*$ *and* $\delta_r < \delta_r^*$; *the direction of the change in income inequality is ambiguous;*

(ii) *When* $\delta_m^* \leq \delta_m \leq -1$ *and* $\delta_r < 0$; $GL_Y \geq GL_X$;

When $-1 < \delta_m < \delta_m^0$ *and* $\delta_r < 0$; $Y \succeq_{TSD} X$ *for all* $W \in \Omega^*(\;)$ $-\frac{\bar{w}V''(\bar{w})}{V'(\bar{w})} \geq \psi$;
When $\delta_m < \delta_m^*$ *and* $\delta_r < \delta_r^*$; $X \succeq_{TSD} Y$ *for all* $W \in \Omega^*(\;)$ $-\frac{\bar{w}V''(\bar{w})}{V'(\bar{w})} \geq \psi$;
When $\delta_m > \delta_m^0$ *and* $\delta_r^0 < \delta_r$; $Y \succ_R X$ *and the direction of the change in welfare is ambiguous.*

Proof. See Appendix B. ■

Proposition 8 can be summarised as in Figure 9. The shaded areas A and B represent ambiguous changes in inequality and welfare, respectively. Note that we do not know the exact positions of intersection points of δ_m^0 at both axes. For MSWR case (d), the area between the lines $\delta_m = -1$ and $\delta_m =$

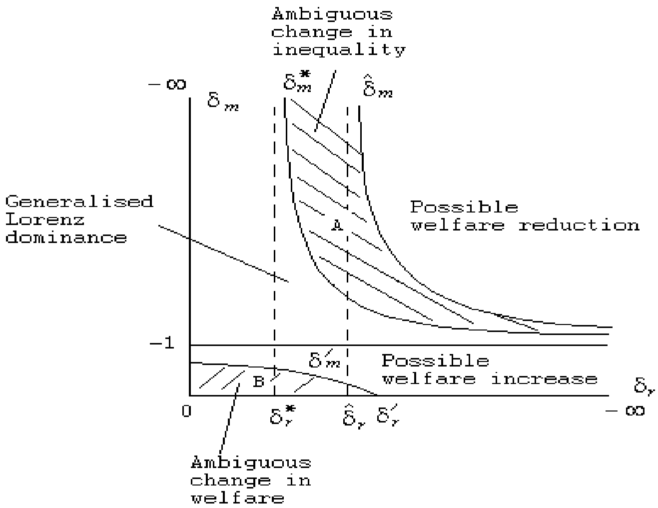

Figure 9: Representative areas of elasticities where changes in income inequality and welfare are ambiguous.

0; we have Lorenz dominance. However welfare may not increase because of the equity-efficiency trade-off. If the values of both elasticities are very low in absolute terms, we may have ambiguous change in welfare. Moreover although A indicates ambiguity in inequality analysis, we may be able to rank income distributions with respect to social welfare if the condition for the degree of inequality aversion holds.

5.4 Combined Policy

From Lemma 4, the combined policy leads to ambiguous change in the level of unemployment while the mean income unambiguously increases. The Lorenz curves and generalised Lorenz curves can be drawn as those for MSENL. The ambiguity occurs in case (a) where the level of unemployment increases leading to the conflict between equity and efficiency. Since the minimum wage is unchanged, δ_r plays a role in determining the conditions for unambiguous inequality and welfare changes. The results are given in the following Proposition where δ_r^{c*} is defined in Lemma 4, $\hat{\delta}_r^c = -\frac{(2+\pi)n_u}{\pi n_r}$; $\delta_r^c = \delta_r^{c*} + \frac{2}{n_r}\left[n_u + 2 - \frac{2}{\pi}(dw_r)\right]$; and $\rho = \frac{4}{[\pi(n_r\delta_r - n_u)+2n_m - 4\pi + \frac{4}{w}(dw_r)]}$.

Proposition 9 *Let* Y *and* X *be the new and original H-T income distributions, respectively, the combined policy of MSENL and TSENR leads to following results:*

(i) *When* $\delta_r > \delta_r^{c*}$; $Y \succ_L X$;

When $\delta_r \leq \hat{\delta}_r^c$; $X \succ_{TSD} Y$ *since* $C_Y > C_X$;

When $\hat{\delta}_r^c < \delta_r < \delta_r^{c*}$; *the direction of the change in income inequality is ambiguous;*

(ii) *When* $\delta_r > \delta_r^{c*}$; $GL_Y \geq GL_X$;

When $\delta_r \leq \delta_r^{c*}$ *and* $(dw_r) \leq \left(1 + \frac{n_u}{2}\right)\upsilon\bar{w}$; $X \succ_{TSD} Y$ *for all* $W \in \Omega^*(\,)$ $-\frac{\bar{w}V''(\bar{w})}{V'(\bar{w})} \geq \phi$;

When $\delta_r \leq \delta_r^{c}$ *and* $(dw_r) > \left(1 + \frac{n_u}{2}\right)\upsilon\bar{w}$; $X \succ_{TSD} Y$ *for all* $W_V \in \Omega^*(\,)$ $-\frac{\bar{w}V''(\bar{w})}{V'(\bar{w})} \geq \phi$;

When $\delta_r^{c} < \delta_r < \delta_r^{c*}$ *and* $(dw_r) > \left(1 + \frac{n_u}{2}\right)\upsilon\bar{w}$; *the direction of the change in welfare is ambiguous.*

Proof. See Appendix B. ■

The effects on inequality and welfare are similar to those caused by MSENL. Moreover if the magnitude of an increase in the rural wage is small, the combined policy may lead to welfare worsening and higher level of unemployment. However if such increase is significantly large, welfare change is ambiguous although the level of unemployment increases.

6 Conclusion

We have considered income inequality and welfare analyses in terms of Lorenz and generalised Lorenz curves respectively under different types of labour market policies. These are MSENL, TSENR, MSWR, and a combination of MSENL and TSENR. MSENL policies are, for example, urban wage subsidy, import tariff, and capital and technology accumulation. TSENR policies are also those involving the subsidy, capital and technology accumulation. Although the critical values of δ_m and δ_r are determined by endogenous variables of the model, they can be considered as long-run references.

We have derived formal conditions for unambiguous changes which depend on wage elasticities of labour demand in the urban and rural sectors, denoted by δ_m and δ_r respectively. When ambiguity occurs that is usually when the policy leads to an increase in the levels of unemployment and mean income, we use third-order stochastic dominance (TSD) to enhance the ranking ability.

Nonetheless the weakness of TSD involves restrictions on the domains of inequality indices and utilitarian SWFs. If the condition for the coefficient of variation is satisfied, only transfer sensitive inequality indices will provide unanimous ranking. With respect to welfare, the domain for utilitarian SWFs is restricted to include those that exhibit sufficient degree of inequality aversion.

The results also suggest that the policy makers can use the information of the values of δ_m and δ_r to decide which policy will be more suitable given the situation of the current economy. For example, suppose that the value of δ_r is slightly less than δ_r^* and δ_m is elastic but is greater than δ_m^*; i.e. $\delta_m^* < \delta_m < -1$. MSWR would be more desirable since it reduces inequality and increases welfare unambiguously. Whereas MSENL leads to an unambiguous increase in inequality and possibly reduces welfare. Although the combined policy may not increase inequality, the policy is more costly than MSWR. On the other hand, if δ_m and δ_r are sufficiently inelastic, it may be better to implement MSENL or the combined policy since it guarantees an increase in welfare and a fall in inequality while MSWR could lead to an ambiguous change in welfare.

Despite the above weakness, TSD seems to be a promising tool in inequality and welfare comparisons. In a simple H-T model, Lorenz and generalised Lorenz curves may cross. It is also likely that they will cross in a more complicated model. Thus Lorenz and generalised Lorenz criteria fail to provide unanimous ranking. However when ambiguity occurs, TSD gives some useful information for unambiguous inequality and welfare changes. Thus TSD enhances ranking ability.

References

[1] Abdulai, A. and Delgado, C. L. (1999) \An Empirical Investigation of Short and Long-run Agricultural Wage Formation in Ghana" Markets and Structural Studies Division Discussion Paper No. 37, International Food Policy Research Institute, Washington, D.C.

[2] Ag¶nor, P. R. (1996) \The labor market and economic adjustment" *IMF Sta® Papers*, 43, p261-335.

[3] Basu, K. (1980) \Optimal Policies in Dual Economies" *The Quarterly Journal of Economics*, vol. 95, issue 1, p, 187-196.

[4] Basu, K. (1997) Analytical development economies: the less developed economy revisited. Cambridge, Mass: MIT Press.

[5] Bhagwati, J.N., and Srinivasan, T.N. (1974) \On reanalyzing the Harris-Todaro model: Policy rankings in the case of sector-speci¯c sticky wages" *American Economic Review*, vol. 64, p502-508.

[6] Chakravarty, S.R. and Dutta, B. (1990) \Migration and Welfare" *European Journal of Political Economy*, vol. 6, p119-138.

[7] Dardanoni, V. and Lambert, P. (1988) \Welfare Rankings of Income Distributions: A Role for the Variance and Some Insights for Tax Reform" *Social Choice and Welfare*, vol. 5, p1-17.

[8] Davies, J. and Hoy, M. (1995) \Making Inequality Comparisons When Lorenz Curves Intersect" *The American Economic Review*, vol. 85, no. 4, p980-986.

[9] Feldman, D. H. (1989) \The trade-o® between GNP and unemployment in a dual economy" *Southern Economic Journal*, vol. 56, no. 1, p46-55.

[10] Fields, G. S. (1979) \A Welfare Economic Approach to Growth and Distribution in the Dual Economy" *The Quarterly Journal of Economics*, August 1979, p325-353.

[11] Fields, G. S. (2001) \A Welfare Economic Analysis of Labor Market Policies in the Harris-Todaro Model" Working paper, October 2001.

[12] Hadar, J. and Russell, W.R. (1969) \Rules for Ordering Uncertainty Prospects" *American Economic Review*, vol. 59, p25-34.

[13] Harris, J. and Todaro, M. (1970) \Migration, Unemployment and Development: A Two-Sector Analysis" *American Economic Review*, vol. 60, p126-142.

[14] Krichel, T., and Levine, P. (1997) "The Welfare Economics of Rural to Urban Migration: The Harris-Todaro Model Revisited", December 1997, Working Paper, University of Surrey.

[15] Moyes, P. (1999) "Stochastic Dominance and the Lorenz Curve" in Silber (1999) *Handbook of Income Inequality Measurement*, Kluwer Academic Publishers.

[16] Rothschild, M. and Stiglitz, J. (1970) "Increasing Risk: I.A Definition" *Journal of Economic Theory*, vol. 2, p225-243.

[17] Sen, A. (1997) On Economic Inequality. Oxford, Clarendon Press.

[18] Shorrocks, F. (1983) "Ranking Income Distributions" *Economica*, vol. 50, p3-17.

[19] Shorrocks, F., and Foster, J. E. (1987) "Transfer Sensitive Inequality Measures" *Review of Economic Studies*, vol. 54, p485-497.

[20] Temple, J. (1999) "Income distribution in the Harris-Todaro Model" Working paper, March 1999.

[21] Zoli, C. (1999) "Intersecting generalized Lorenz curves and the Gini index" *Social Choice and Welfare*, vol. 16, p183-196.

7 Appendix A

7.1 Proof of Lemma1

Lemma 1. *MSENL leads to*

(i) *an unambiguous increase in* μ;

(ii) *an unambiguous decrease in* υ;

(iii) *an ambiguous change in* U *: the critical value of* δ_r *is* $\delta_r^* = -\frac{n_m}{An_r}$ *where at* $\delta_r = \delta_r^*$; *the level of* U *is unchanged. Below this threshold, MSENL leads to an increase in the level of* U *and vice versa.*

Proof:

(iii) From (6);

$$U = n_u - n_m.$$

Differentiating both sides with respect to n_m to get

$$\left(\frac{dU}{dn_m}\right)_{\Delta \bar{w}=0} = \left(\frac{dn_u}{dn_m}\right)_{\Delta \bar{w}=0} - 1. \tag{18}$$

From the equilibrium condition, equation (1);

$$n_u w_r = n_m \bar{w}.$$

Differentiating both sides with respect to n_m fixing $\bar{w}$, we get

$$\left(w_r + n_u \frac{dw_r}{dn_r}\frac{dn_r}{dn_u}\right)\left(\frac{dn_u}{dn_m}\right)_{\Delta \bar{w}=0} = \bar{w}$$

$$\left(\frac{dn_u}{dn_m}\right)_{\Delta \bar{w}=0} = \frac{\bar{w}}{w_r\left(1 - \frac{n_u}{\delta_r n_r}\right)} = \frac{\delta_r n_r n_u}{n_m(\delta_r n_r - n_u)}. \qquad (19)$$

where $\frac{dn_r}{dn_u} = -1$ and

$$\delta_r = \frac{w_r}{n_r}\frac{dn_r}{dw_r} < 0.$$

Substituting (19) into (18), we get

$$\left(\frac{dU}{dn_m}\right)_{\Delta \bar{w}=0} = \frac{\delta_r n_r n_u}{n_m(\delta_r n_r - n_u)} - 1 = \frac{\delta_r n_r U + n_u n_m}{n_m(\delta_r n_r - n_u)}.$$

Thus

$$\left(\frac{dU}{dn_m}\right)_{\Delta \bar{w}=0} = 0 \iff \delta_r^* = -\frac{n_m}{\pi n_r}.$$

For $\delta_r < \delta_r^*$, $\left(\frac{dU}{dn_m}\right)_{\Delta \bar{w}=0} > 0$ and for $\delta_r > \delta_r^*$, $\left(\frac{dU}{dn_m}\right)_{\Delta \bar{w}=0} < 0$.

(ii) From (8),

$$\pi = \frac{U}{n_u}$$

differentiating both sides with respect to n_m to get

$$\left(\frac{d\pi}{dn_m}\right)_{\Delta \bar{w}=0} = \frac{1}{n_u}\left(\frac{dU}{dn_m}\right)_{\Delta \bar{w}=0} - \frac{U}{n_u^2}\left(\frac{dn_u}{dn_m}\right)_{\Delta \bar{w}=0}$$
$$= \frac{1}{\delta_r n_r - n_u} < 0.$$

Since $\delta_r < 0$, $\left(\frac{d\pi}{dn_m}\right)_{\Delta \bar{w}=0} < 0$ unambiguously.

(i) From (2) and (5),

$$\mu = w_r = g(n_r), g' < 0$$

differentiating both sides with respect to n_m to get

$$\left(\frac{d\mu}{dn_m}\right)_{\Delta \bar{w}=0} = -\frac{dw_r}{dn_r}\left(\frac{dn_u}{dn_m}\right)_{\Delta \bar{w}=0}$$

substituting for $\left(\frac{dn_u}{dn_m}\right)_{\Delta \bar{w}=0}$, from (19),

$$\left(\frac{d\mu}{dn_m}\right)_{\Delta \bar{w}=0} = -\frac{\bar{w}}{(\delta_r n_r - n_u)} > 0.$$

7.2 Proof of Lemma 2

Lemma 2. *TSENR directly increases μ and always reduces the level of urban unemployment and unemployment rate.*

Proof:
From (5)

$$\frac{d\mu}{dw_r} = 1 > 0.$$

From (6), by differentiating both sides with respect to w_r we get

$$\frac{dU}{dw_r} = \frac{dn_u}{dw_r}. \tag{20}$$

The expression for $\frac{dn_u}{dw_r}$ can be derived from the equilibrium condition. By differentiating both sides of equation (1) with respect to w_r we get

$$\frac{dn_u}{dw_r} w_r + n_u = 0$$

$$\frac{dn_u}{dw_r} = -\frac{n_u}{w_r} = -\frac{n_u^2}{n_m \bar{w}} < 0. \tag{21}$$

Substituting (21) into (20),

$$\frac{dU}{dw_r} = -\frac{n_u^2}{n_m \bar{w}} < 0.$$

From (8),

$$\frac{d\psi}{dw_r} = \frac{1}{n_u}\frac{dU}{dw_r} - \frac{U}{n_u^2}\frac{dn_u}{dw_r} = -\frac{(1-\psi)}{n_u}\frac{n_u^2}{n_m \bar{w}} < 0.$$

7.3 Proof of Lemma 3

Lemma 3. *MSWR leads to*

(i) *an unambiguous decrease in* ψ;

(ii) *an unambiguous decrease in* U *while* n_u *and* μ *remain constant when* $\delta_m = -1$;

(iii) *unambiguous decreases in* U, n_u *and* μ *when* $\delta_m > -1$;

(iv) *unambiguous increases in* n_u *and* μ *and an ambiguous change in* U *when* $\delta_m < -1$:

$\frac{dU}{d\bar{w}} > 0 \iff \delta_m < -1$ *and* $\delta_r > \delta_r^*$

$\frac{dU}{d\bar{w}} > 0 \iff \delta_m^* \leq \delta_m < -1$ *and* $\delta_r < \delta_r^*$;

$\frac{dU}{d\bar{w}} < 0 \iff \delta_m < \delta_m^*$ *and* $\delta_r < \delta_r^*$;

where $\delta_r^* = -\frac{n_m}{\psi n_r}$ *and* $\delta_m^* = -\frac{\delta_r}{\delta_r \psi + \frac{n_m}{n_r}} = -1 + \frac{n_m - (1-\psi)\delta_r n_r}{\psi \delta_r n_r + n_m}$.

Proof:
Differentiating both sides of (6) with respect to $\bar{w}$ to get

$$\frac{dU}{d\bar{w}} = \frac{dn_u}{d\bar{w}} - \frac{dn_m}{d\bar{w}}. \tag{22}$$

From (1),

$$n_u w_r(n_u) = n_m \bar{w}.$$

Differentiating both sides with respect to $\bar{w}$, we get

$$\frac{dn_u}{d\bar{w}} = \frac{(1+\delta_m)\,\delta_r n_u n_r}{(\delta_r n_r - n_u)\,\bar{w}} \tag{23}$$

where

$$\delta_m = \frac{dn_m}{d\bar{w}}\frac{\bar{w}}{n_m} < 0.$$

Substituting (23) into (22), we get

$$\frac{dU}{d\bar{w}} = \frac{n_u}{\bar{w}(\delta_r n_r - n_u)}\{\delta_m \delta_r \upsilon n_r + \delta_r n_r + \delta_m n_m\}. \tag{24}$$

The critical values of δ_r and δ_m are

$$\delta_r^* = -\frac{n_m}{\upsilon n_r}$$

$$\delta_m^* = -\frac{\delta_r n_r}{\delta_r \upsilon n_r + n_m} < -1.$$

If $\delta_r > \delta_r^*$, the level of unemployment decreases unambiguously:

$$\frac{dU}{d\bar{w}} > 0\ .$$

If $\delta_r < \delta_r^*$,

$$\frac{dU}{d\bar{w}} \gtreqless 0 \iff \delta_m \gtreqless \delta_m^*.$$

From (2) and (5),

$$\mu = w_r = g(n_r),\ g' < 0$$

differentiating both sides with respect to $\bar{w}$ to get

$$\frac{d\mu}{d\bar{w}} = -\frac{dw_r}{dn_r}\left(\frac{dn_u}{d\bar{w}}\right) = -\frac{(1+\delta_m)\,n_m}{(\delta_r n_r - n_u)}. \tag{25}$$

Hence

$$\frac{d\mu}{d\bar{w}} \gtreqless 0 \iff \delta_m \gtreqless -1.$$

From (8),

$$\frac{d\upsilon}{d\bar{w}} = \frac{1}{n_u}\frac{dU}{d\bar{w}} - \frac{U}{n_u^2}\frac{dn_u}{d\bar{w}} = \frac{\delta_r n_r n_m + \delta_m n_m n_u}{n_u \bar{w}(\delta_r n_r - n_u)} > 0.$$

Hence MSWR leads to a reduction in the level of unemployment rate.

Note that when μ is fixed, $\delta_r \to -1$; $\frac{d\mu}{d\bar{w}} \to 0$ and $\frac{dU}{d\bar{w}} \to \frac{n_u}{\bar{w}}[1+\delta_m \upsilon]$.

7.4 Proof of Lemma 4

Lemma 4. *A combined policy of MSENL and TSENR leads to*

(i) *an unambiguous decrease in* λ;

(ii) *an unambiguous increase in* μ;

(iii) *an ambiguous change in* U:

$dU_c \le 0 \Leftrightarrow \delta_r > \delta_r^{c*}$;

$dU_c > 0 \Leftrightarrow \delta_r < \delta_r^{c*}$

where $\delta_r^{c*} = \delta_r^{*} - \frac{n_u}{\lambda n_r}$ *and* dU_c *represents the total change in the level of unemployment under the combined scheme.*

Proof:

(i) The unemployment rate unambiguously reduces since both MSENL and TSENR lead to lower unemployment rate.

(ii) The mean income unambiguously increases since both MSENL and TSENR also lead to a rise in rural wage.

(iii) The total change in the level of unemployment is given as

$$
\begin{aligned}
dU_c &= (dU)_{MSENL} + (dU)_{TSENR} \\
&= \frac{n_u}{n_m \tilde{w}} (\delta_r \lambda n_r + n_m + n_u)\, dw_r
\end{aligned}
$$

where $(dn_m)_{\Delta \tilde{w}=0} = -\frac{(\delta_r n_r - n_u)}{\tilde{w}} dw_r$.

The critical value of δ_r is

$$\delta_r^{c*} = -\frac{n_m}{\lambda n_r} - \frac{n_u}{\lambda n_r} = \delta_r^{*} - \frac{n_u}{\lambda n_r}.$$

And

$$dU_c \gtreqless 0 \iff \delta_r \lesseqgtr \delta_r^{c*}.$$

8 Appendix B

8.1 Proof of Proposition 6

Proposition 6 *Let* Y *and* X *be the new and original H-T income distributions, respectively, MSENL leads to following results:*

(i) *When* $\delta_r > \delta_r^{*}$; $Y \succ_L X$;

When $\delta_r \le \hat{\delta}_r$; $X \succ_{TSD} Y$ *since* $C_Y > C_X$;

When $\hat{\delta}_r < \delta_r < \delta_r^{*}$; *the direction of the change in income inequality is ambiguous;*

(ii) *When* $\delta_r > \delta_r$; $GL_Y \ge GL_X$;

When $\delta_r < \delta_r^{*}$; $X \succ_{TSD} Y$ *for all* $W \in \Omega^{*}$ *if and only if*

$$-\frac{\tilde{w} V''(\tilde{w})}{V'(\tilde{w})} \ge \frac{-2(n_r\delta_r - n_u)}{\lambda (n_r\delta_r - n_u)^2 + (2\lambda + n_m)(n_r\delta_r - n_u) + 1}.$$

Proof:

(i) Given that $\delta_r > \delta_r^*$, $\left(\frac{dU}{dn_m}\right)_{\Delta\bar{w}=0} \leq 0$. Hence we have Lorenz dominance. Recall (17),

$$C^2 = \frac{n_u}{n_m}(n_u - n_m) = \frac{n_u}{n_m}U.$$

Differentiating both sides of (17) with respect to n_m keeping $\bar{w}$ fixed, we have

$$\left(\frac{d\left(C^2\right)}{dn_m}\right)_{\Delta\bar{w}=0} = \frac{n_u^2(\delta_r\lambda n_r + n_u)}{n_m^2(\delta_r n_r - n_u)}. \tag{26}$$

$$\left(\frac{d\left(C^2\right)}{dn_m}\right)_{\Delta\bar{w}=0} \gtreqless 0 \iff \delta_r \lesseqgtr \hat{\delta}_r = -\frac{n_u}{\lambda n_r}.$$

In case (a) where unemployment increases and ambiguity occurs, we have $\delta_r < \delta_r^*$. Since $\delta_r^* > \hat{\delta}_r$, using Propositions 2 and 3, MSENL leads to unambiguous inequality worsening if and only if $\delta_r \leq \hat{\delta}_r$. The ambiguity remains over the range of $\delta_r \in (\hat{\delta}_r, \delta_r^*)$.

(ii) From Proposition 4, we have generalised Lorenz dominance when $\delta_r > \delta_r^*$. In case (a), generalised Lorenz curves cross. We then use TSD approach to verify the direction of welfare change.

Setting $\bar{w} = 1$, the variance is defined as

$$\sigma^2 = \frac{n_m}{n_u}U\bar{w}^2$$

differentiating both sides with respect to n_m we have

$$\left(\frac{d\left(\sigma^2\right)}{dn_m}\right)_{\Delta\bar{w}=0} = \frac{\bar{w}^2[\lambda(\delta_r n_r - n_u) + n_m]}{(\delta_r n_r - n_u)n_u}. \tag{27}$$

It can be shown that when MSENL leads to an increase in U that is when $\delta_r < \delta_r^*$, $\left(\frac{d(\sigma^2)}{dn_m}\right)_{\Delta\bar{w}=0} > 0$.

From Proposition 5, if we can show that

$$0 \leq \left(\sigma_Y^2 - \sigma_X^2\right) - (\mu_Y - \mu_X)(2\bar{w} - \mu_Y - \mu_X)$$

$$0 \leq \left(\sigma_Y^2 - \sigma_X^2\right) - (\mu_Y - \mu_X)(2\bar{w} - (\mu_Y - \mu_X) - 2\mu_X) \tag{28}$$

then $W(X) \geq W(Y)$ for all $W \in \Omega^*$ such that

$$-\frac{\bar{w}V''(\bar{w})}{V'(\bar{w})} \geq \frac{2\bar{w}(\mu_Y - \mu_X)}{(\sigma_Y^2 - \sigma_X^2) - (\mu_Y - \mu_X)(2\bar{w} - \mu_Y - \mu_X)}. \tag{29}$$

Substituting the expressions for $\left(\sigma_Y^2 - \sigma_X^2\right) = \left(\frac{d(\sigma^2)}{dn_m}\right)_{\Delta\bar{w}=0}$; $(\mu_Y - \mu_X) = \left(\frac{d\mu}{dn_m}\right)_{\Delta\bar{w}=0}$; and $\mu_X = \frac{n_m}{n_u}\bar{w}$ in equation (28); we can show that for all values of $\delta_r < \delta_r^*$; (28) is satisfied and is expressed as

$$0 \leq \frac{\bar{w}^2}{(\delta_r n_r - n_u)^2}\left[\upsilon(\delta_r n_r - n_u)^2 + (2\upsilon + n_m)(\delta_r n_r - n_u) + 1\right]:$$

Hence if the condition for concavity of SWFs is satisfied, MSENL case (a) leads to unambiguous welfare reduction:

$$-\frac{\bar{w}V''(\bar{w})}{V'(\bar{w})} \geq \frac{-2(n_r\delta_r - n_u)}{\upsilon(n_r\delta_r - n_u)^2 + (2\upsilon + n_m)(n_r\delta_r - n_u) + 1} > 0:$$

8.2 Proof of Proposition 8

Proposition 8 *Let* Y *and* X *be the new and original H-T income distributions, respectively, MSWR leads to following results:*

(i) *When* $\delta_m > \delta_m^*$ *and* $\delta_r < 0$; $Y \succeq_L X$;

When $\delta_m \leq \hat{\delta}_m$ *and* $\delta_r < \hat{\delta}_r$; $X \succeq_{TSD} Y$ *since* $C_Y > C_X$;
When $\hat{\delta}_m < \delta_m < \delta_m^*$ *and* $\delta_r < \delta_r^*$; *the direction of the change in income inequality is ambiguous;*

(ii) *When* $\delta_m^* \leq \delta_m \leq -1$ *and* $\delta_r < 0$; $GL_Y \geq GL_X$;

When $-1 < \delta_m < \delta_m^0$ *and* $\delta_r < 0$; $Y \succeq_{TSD} X$ *for all* $W \in \Omega^*()\ -\frac{\bar{w}V''(\bar{w})}{V'(\bar{w})} \geq \psi$;
When $\delta_m < \delta_m^*$ *and* $\delta_r < \delta_r^*$; $X \succeq_{TSD} Y$ *for all* $W \in \Omega^*()\ -\frac{\bar{w}V''(\bar{w})}{V'(\bar{w})} \geq \psi$;
When $\delta_m > \delta_m^0$ *and* $\delta_r^0 < \delta_r < 0$; $Y \succ_R X$ *and the direction of the change in welfare is ambiguous.*

Proof:

(i) When $\delta_m > \delta_m^*$, we have Lorenz dominance. From Proposition 1, inequality unambiguously decreases. When $\delta_m < \delta_m^*$, Lorenz curves cross and we use TSD approach to rank income distributions.

Recall (17);

$$C^2 = \frac{n_u}{n_m}(n_u - n_m):$$

It can be shown that MSWR leads to unambiguous reduction in C^2: Differentiating both sides with respect to $\bar{w}$ to get

$$\frac{d\left(C^2\right)}{d\bar{w}} = \frac{n_u^2\left[(1+\upsilon)n_r\delta_r + \delta_m(\upsilon n_r\delta_r + n_u)\right]}{\bar{w}n_m(n_r\delta_r - n_u)}: \qquad (30)$$

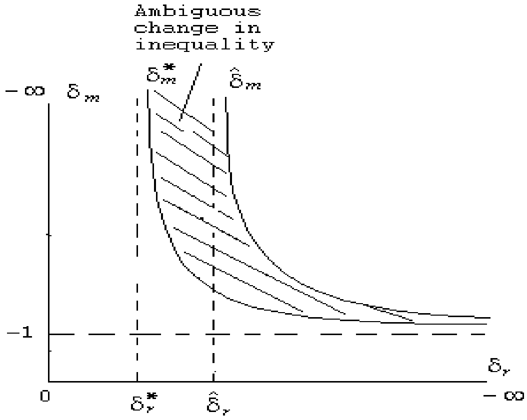

Figure 10: Plot of the values of δ_m and δ_r for unambiguous changes in C^2.

For $\delta_m < -1$:

$$\frac{d\left(C^2\right)}{d\bar{w}} > 0 \text{ when } \delta_r > \hat{\delta}_r$$

$$\frac{d\left(C^2\right)}{d\bar{w}} > 0 \iff \hat{\delta}_m \leq \delta_m < -1 \text{ when } \delta_r < \hat{\delta}_r$$

$$\frac{d\left(C^2\right)}{d\bar{w}} < 0 \iff \delta_m < \hat{\delta}_m \text{ when } \delta_r < \hat{\delta}_r$$

for $\delta_m > -1$:

$$\frac{d\left(C^2\right)}{d\bar{w}} > 0 \text{ for all values of } \delta_r$$

where $\hat{\delta}_r = -\frac{n_u}{\pi n_r}$ and $\hat{\delta}_m = -\frac{(1+\pi)n_r\delta_r}{\pi n_r\delta_r + n_u} < -1$.

The values of δ_r and δ_m for unambiguous changes in C^2 can be plotted as in Figure 10. Below the line $\hat{\delta}_m$, $\frac{d(C^2)}{d\bar{w}} > 0$; otherwise $\frac{d(C^2)}{d\bar{w}} \leq 0$.

In case (e) where both the levels of unemployment and rural wage increase, we have Lorenz crossing, Figure 4(e). From Propositions 2 and 3, inequality unambiguously increases for all inequality indices satisfying transfer sensitivity $\frac{d(C^2)}{d\bar{w}} < 0$ which hold for $\delta_r < \hat{\delta}_r$ and $\delta_m \leq \hat{\delta}_m$. However the shaded area in Figure 10 remains ambiguous. In this area, unemployment increases while the coefficient of variation decreases. When $\delta_m > \delta_m^*$ MSWR leads to unambiguous fall in inequality.

(ii) Figures 8(a)-(c) show cases where we have generalised Lorenz dominance. From Proposition 4, welfare unambiguously increases. On the other hand, Figures 8(d)-(e) show cases of generalised Lorenz crossing. Using Proposition 5, it may be possible to state unambiguous conditions for welfare changes.

From the expression of variance, $\sigma^2 = \frac{n_m}{n_u} U \bar{w}^2$, differentiating both sides with respect to $\bar{w}$ we have

$$\frac{d\left(\sigma^2\right)}{d\bar{w}} = \frac{\bar{w} n_m}{(\delta_r n_r - n_u)} \left\{\delta_m \left[n_m + \pi\left(\delta_r n_r - n_u\right)\right] + (1+\pi)\delta_r n_r - 2\pi n_u\right\}. \quad (31)$$

From Appendix A,

$$\frac{d\mu}{d\hat{w}} = -\frac{(1+\delta_m)\,n_m}{(\delta_r n_r - n_u)}.$$

For case (d), $\delta_m > -1$ and $W(Y) \geq W(X)$ for all $W \in \Omega^*$ if we can show that the following two conditions are satisfied:

(a)

$$0 \leq \left(\sigma_X^2 - \sigma_Y^2\right) - (\mu_X - \mu_Y)(2\hat{w} - \mu_Y - \mu_X) \tag{32}$$

that is

$$0 \leq \frac{n_m}{(\delta_r n_r - n_u)^2}\left\{\begin{array}{c} -n_m(1+\delta_m)^2 + \hat{w}(\delta_r n_r - n_u)^2 \\ +\hat{w}(\delta_r n_r - n_u)[n_m + 2\upsilon + \upsilon(\delta_r n_r - n_u)](1+\delta_m) \end{array}\right\}.$$

(b)

$$-\frac{\hat{w}V''(\hat{w})}{V'(\hat{w})} \geq \frac{2\hat{w}(\mu_X - \mu_Y)}{(\sigma_X^2 - \sigma_Y^2) - (\mu_X - \mu_Y)(2\hat{w} - \mu_Y - \mu_X)} \tag{33}$$

$$-\frac{\hat{w}V''(\hat{w})}{V'(\hat{w})} \geq \psi \tag{34}$$

where $\delta_m > -1$ and

$$\psi = -\frac{2\hat{w}}{\hat{w}[n_m + \upsilon(\delta_r n_r - n_u + 2)] - \frac{n_m(1+\delta_m)}{(\delta_r n_r - n_u)} + \frac{\hat{w}(\delta_r n_r - n_u)}{(1+\delta_m)}}.$$

Condition (a) will be satisfied when $-1 < \delta_m < \delta_m'$ where

$$\delta_m' = -1 + \left(\frac{n_r\delta_r - n_u}{2n_m}\right)\left\{\begin{array}{c} \hat{w}[n_m + \upsilon(n_r\delta_r - n_u + 2)] \\ -\left[\hat{w}^2(n_m + \upsilon(n_r\delta_r - n_u + 2))^2 + 4\hat{w}n_m\right]^{\frac{1}{2}} \end{array}\right\}.$$

The expression for δ_m' equals 0 when

$$\delta_r' = \frac{(1+\upsilon)\,n_u - 2\upsilon n_r}{2(1+\upsilon)\,n_r} - \frac{1}{2\hat{w}(1+\upsilon)\,n_r}\left[\hat{w}^2(2\upsilon + n_m)^2 + 4\hat{w}n_m(1+\upsilon)\right]^{\frac{1}{2}}.$$

For case (e), $\delta_m < \delta_m^*$ and $\delta_r < \delta_r^*$: This is similar to MSENL case (a), $W(X) \geq W(Y)$ for all $W \in \Omega^*$ if (28) and (29) are satisfied:

$$0 \leq \frac{n_m}{(\delta_r n_r - n_u)^2}\left\{\begin{array}{c} n_m(1+\delta_m)^2 - \hat{w}(\delta_r n_r - n_u)^2 \\ -\hat{w}(\delta_r n_r - n_u)[n_m + \upsilon(\delta_r n_r \varepsilon - n_u + 2)](1+\delta_m) \end{array}\right\} \tag{35}$$

and

$$-\frac{\hat{w}V''(\hat{w})}{V'(\hat{w})} \geq \psi$$

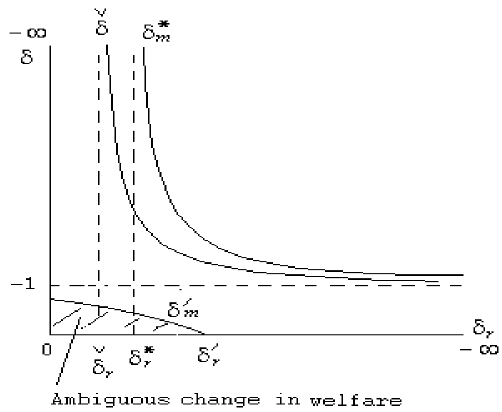

Figure 11: Representative area of elasticities for changences in variance and welfare.

where ψ is defined above.

Since the calculation is complicated, we show the plot of the values of δ_r and δ_m for changes in variance and welfare as drawn in Figure 11. The shaded areas represent the values of δ_r and δ_m for ambiguous welfare change. Between the lines $\delta_m = \delta_m^*$ and $\delta_m = -1$; MSWR leads to generalised Lorenz dominance and hence welfare improvement, Proposition 4, where $\delta_r^* = -\frac{n_m}{\psi n_r}$ and $\delta_m^* = -\frac{\delta_r n_r}{\psi \delta_r n_r + n_m}$.

8.3 Proof of Proposition 9

Proposition 9 *Let* Y *and* X *be the new and original H-T income distributions, respectively, the combined policy of MSENL and TSENR leads to following results:*

(i) *When* $\delta_r > \delta_r^{c*}$; $Y \succeq_L X$;

When $\delta_r \leqslant \hat{\delta}_r^c$; $X \succeq_{TSD} Y$ *since* $C_Y > C_X$;

When $\hat{\delta}_r^c < \delta_r < \delta_r^{c*}$; *the direction of the change in income inequality is ambiguous;*

(ii) *When* $\delta_r > \delta_r^{c*}$; $GL_Y \geq GL_X$;

When $\delta_r \leqslant \delta_r^{c*}$ *and* $(dw_r) \leqslant \left(1 + \frac{n_u}{2}\right)\upsilon\bar{w}$; $X \succeq_{TSD} Y$ *for all* $W \in \Omega^*\left(\right)$ $-\frac{\bar{w}V''(\bar{w})}{V'(\bar{w})} \geq \phi$;

When $\delta_r \leqslant \delta_r^c$ *and* $(dw_r) > \left(1 + \frac{n_u}{2}\right)\upsilon\bar{w}$; $X \succeq_{TSD} Y$ *for all* $W_V \in \Omega^*\left(\right)$ $-\frac{\bar{w}V''(\bar{w})}{V'(\bar{w})} \geq \phi$;

When $\delta_r^c < \delta_r < \delta_r^{c*}$ *and* $(dw_r) > \left(1 + \frac{n_u}{2}\right)\upsilon\bar{w}$; *the direction of the change in welfare is ambiguous.*

Proof:

The analysis of Lorenz curves is analogous to the case where MSENL is implemented. When $\delta_r > \delta_r^{c*}$; the level of unemployment decreases and we have Lorenz and generalised Lorenz dominance.

From (26)

$$\begin{aligned}\left(dC^2\right)_{MSENL} &= \frac{n_u^2(\delta_r \lambda n_r + n_u)}{n_m^2(\delta_r n_r - n_u)}(dn_m)_{\Delta \bar w=0}\\ &= -\frac{(\delta_r \lambda n_r + n_u)}{\bar w(1-\lambda)^2}(dw_r)\end{aligned}$$

where

$$\begin{aligned}(dn_m)_{\Delta \bar w=0} &= -\frac{1}{\bar w}(\delta_r n_r - n_u)(dw_r) > 0\\ \frac{n_m}{n_u} &= 1 - \lambda\end{aligned}$$

and it can be shown that

$$\left(dC^2\right)_{TSENR} = -\frac{(1+\lambda)n_u}{\bar w(1-\lambda)^2}(dw_r)$$

Hence the total change in the coefficient of variation is

$$\begin{aligned}\left(dC^2\right)_c &= \left(dC^2\right)_{MSENL} + \left(dC^2\right)_{TSENR}\\ &= -\frac{[\lambda n_r \delta_r + (2+\lambda)n_u]}{\bar w(1-\lambda)^2}(dw_r):\end{aligned}$$

$$\left(dC^2\right)_c \gtrless 0 \iff \delta_r \lessgtr \hat\delta_r^c$$

where $\hat\delta_r^c = -\frac{(2+\lambda)n_u}{\lambda n_r} < \delta_r^{c*}$.

Hence using Propositions 2 and 3, inequality increases for values of δ_r below $\hat\delta_r^c$ whereas for $\hat\delta_r^c < \delta_r < \delta_r^{c*}$, ambiguous inequality ranking remains.

From (27);

$$\left(d\sigma^2\right)_{MSENL} = -\bar w[\lambda(\delta_r n_r - n_u) + n_m](dw_r)$$

and it can be shown that

$$\left(d\sigma^2\right)_{TSENR} = -\bar w n_m(dw_r):$$

Hence the total effect on variance is

$$\begin{aligned}\left(d\sigma^2\right)_c &= \left(d\sigma^2\right)_{MSENL} + \left(d\sigma^2\right)_{TSENR}\\ &= -\bar w[\lambda(\delta_r n_r - n_u) + 2n_m](dw_r):\end{aligned}$$

$$\left(d\sigma^2\right)_c \gtrless 0 \iff \delta_r \lessgtr \delta_r^{c'}$$

where $\delta_r^{c'} = -\frac{2n_m}{\lambda n_r} + \frac{n_u}{n_r} = \delta_r^{c*} + \frac{2n_u}{n_r} > \delta_r^{c*}$: This implies that variance unambiguously increases when the policy leads to higher level of unemployment.

If (28) and (29) are satisfied, the combined policy may lead to welfare worsening when the level of unemployment increases. From (28),

$$0 \leq -(dw_r)\left[\upsilon(\delta_r n_r - n_u) + 2n_m - 4\upsilon + 4(dw_r)\right] \qquad (36)$$

which will be satisfied when

$$\delta_r \leq \delta_r^{c} = \delta_r^{c*} + \frac{2}{n_r}\left[2 + n_u - \frac{2}{\upsilon}(dw_r)\right]$$

and from (29),

$$-\frac{\bar{w}V''(\bar{w})}{V'(\bar{w})} \geq \left[\frac{4}{\upsilon(n_r\delta_r - n_u) + 2n_m - 4\upsilon + \frac{4}{\bar{w}}(dw_r)}\right]. \qquad (37)$$

If $(dw_r) \leqslant \left(1 + \frac{n_u}{2}\right)\upsilon\bar{w}$ then $\delta_r^{c} > \delta_r^{c*}$. The combined policy may lead to welfare worsening when $\delta_r \leq \delta_r^{c*}$ given that (37) is satisfied.

If $(dw_r) > \left(1 + \frac{n_u}{2}\right)\upsilon\bar{w}$ then $\delta_r^{c} < \delta_r^{c*}$ and ambiguity remains for the values of $\delta_r \in \left(\delta_r^{c}, \delta_r^{c*}\right)$.