Referendum-led Immigration Policy in the Welfare State

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Abstract

Preferences of heterogeneous citizens over the different levels of low-skilled immigration are examined in a two-period overlapping generation model in which an inflow of low-skilled immigrants affects the host economy via three channels — the labour market, the income support programme and the pay-as-you-go pension system. In most of the cases, the model can predict unique immigration policy determined by majority voting. However, a voting cycle can also arise in certain circumstances, subjecting a referendum outcome to manipulation.

Key words: international migration, majority voting, skill acquisition, welfare state

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1 Introduction

Immigration is a politically sensitive subject, for a certain level of a certain type of immigration is likely to have different impacts on different individuals in the host country. We restrict our attention to the economic impacts of low-skilled immigrant workers in the welfare state and their implications to the referendum-led policy formation with respect to the level of low-skilled labour immigration. We consider three channels via which the impacts of immigration become manifest in the host country. These are the labour market, the intragenerational transfer programme and the intergenerational transfer system.

This work is motivated by the fact that the existing literature on the political economy of immigration policymaking in the welfare state is roughly split into two. One studies the subject in the static context of intragenerational transfer, and the other in the dynamic context of intergenerational transfer.[1] However, intra- and intergenerational transfers often coexist in the welfare state. Therefore, we provide a framework which contains both types of transfers and enables us to examine the interactive effect of low-skilled immigration through these redistribution programmes.

We extend the frameworks provided by Razin & Sadka (2000) and Casarico & Devillanova (2003) in order to derive referendum outcomes. These studies examined the impact of an inflow of low-skilled immigrant workers in one period on the host country which is inhabited by heterogeneous agents of overlapping generations under a pay-as-you-go pension scheme. Razin & Sadka (2000) found that, if the production factor prices were fixed, the only change would be a gain by current pensioners — the result is driven by the specification of a fixed payroll tax rate. If the factor prices were flexible, however, the effect would be not only a gain by current pensioners but also a loss to all current workers and the subsequent generations. Immigration

[1] Kemnitz (2002), for instance, deals with an unemployment insurance scheme, although a rich model is developed for the static analysis. On the other hand, Scholten & Thum (1996) and Haupt & Peters (1998) focus on a pay-as-you-go pension scheme.

increases the supply of labour relative to capital, which reduces the wage rate. Since immigrant workers are assumed to enter in one period only, the wage rate, after a drop, starts rising to the pre-immigration stady state level over periods. Until then, the size of the per capita pension benefit is less than it would have been without immigration, as the tax base is smaller from the post-immigration period onwards than it would have been with no immigrants.[2]

Casarico & Devillanova (2003) provide a richer model in terms of the labour market as a transmission mechanism. First, they assume that high-skilled and low-skilled labour are imperfect substitutes of one another. Accordingly, two separate wage rates exist in the labour market.[3] This way of modelling the labour market seems to be sensible for our analysis because there is some evidence that low-wage earners are more likely to prefer reduced immigration than high-wage earners.[4] Second, they endogenise the

[2]A gain to current pensioners arises because, although the wage rate falls, immigration also increases the number of contributors in the period of entry.

[3]Razin & Sadka's (2000) model assumes high-skilled and low-skilled labour which are perfect substitutes of each other. Therefore, there is a single wage rate in the labour market. The difference between high-skilled and low-skilled workers is then the quantity of substitutable labour being endowed.

[4]See Scheve & Slaughter (2001, Table 2, p. 140) which examined the data from the National Election Studies surveys in the United States in the 1990s. They also found that those with low educational attainment are more likely to oppose immigration than more educated individuals. Since the type of immigrants was not specified in the survey questionnaire, their findings do not necessarily conform with the typical economic theoretical argument that an increase in the size of the labour force of one type would damage the labour market opportunities for workers who are already in the host country and belong to the same labour type. Nevertheless, if we assume that the survey respondents had a biased view that immigrants are typically poorly educated and are likely to work in low-wage sectors once arrived in the host country, the preferences of the respondents examined by Scheve & Slaughter (2001) may well reflect the fact that people anticipate the impacts of immigration just as the standard economic theoretical model would predict them. As people's perception matters in voting, whether influenced by media or learned through actual job market experiences, models which incorporate adverse labour market effects of immigration would be good predictors of majority voting outcomes on immigration policy.

skill acquisition decision of native workers. Since immigration affects the wage rates, the profitability of skill acquisition also changes according to the level of immigration.[5] As a result of an endogenised flow of native workers from the low-skilled to high-skilled labour pool, the impact of low-skilled immigration on the two wage rates is moderated.[6]

Their analysis reveals that currently low-skilled workers can be divided into three groups for a given level of low-skilled immigration: those who would remain low-skilled, those who would become high-skilled but would be better off without the immigration and those who would become high-skilled and would be better off with it. The second group is "pushed" out of the low-skilled labour force because, although becoming high-skilled with the immigration does not yield as high income as remaining low-skilled without it, remaining low-skilled with the immigation gives them even lower income than becoming high-skilled with it. They prefer no immigration to this level of immigration. On the other hand, the third group is "pulled" to the high-skilled labour force because becoming high-skilled with the immigration yeilds higher income than remaining low-skilled not only with it but also without it. They prefer this level of immigration to the status quo. Since the interest group division is important to the determination of majority voting outcomes, we follow Casarico & Devillanova (2003) and endogenise the skill acquisition decision.

These two studies offer a platform for our study of the formation of referendum-led immigration policy. We include an explicit intragenerational transfer when we describe our model in section 2.[7] After constructing

[5]See, for example, Chiswick (1989) for such a model.

[6]This is one potential reason why empirical evidence for the impact of immigrant workers on the host country's labour market is mixed. Endogenous skill acquisition decisions of native workers may lessen the labour market impact of immigration. LaLonde & Topel (1997), for instance, found that immigration would have a small impact on the labour market at destination.

[7]The two studies mentioned do include such a transfer but implicitly because their pay-as-you-go pension schemes are made of a single tax rate on wage earnings and a flat lump sum per capita benefit.

a model, we examine the preferences of individual voters so as to observe the impacts of low-skilled labour immigration on natives in section 3. We then derive majority voting outcomes determining quantitative immigration policy.[8] In addition to immigration occuring in one period only, we also examine the cases of immigration that takes place in every period. Section 4 concludes.

2 Model

Consider overlapping generations of agents who live for two periods in a country. In the first period, each agent supplies labour to earn wage income, saves a fraction of the disposable income for the second period and consumes the rest. In the second period, the agent does not work, receives a pension benefit and withdraws the savings which have earned interest over one period. She/he consumes all the income in the second period, i.e., no bequest. The pension scheme is balanced pay-as-you-go (PAYG). That is, the sum of pension benefits received by current pensioners equals the sum of contributions paid by current workers.

In addition to the unfunded pension scheme which is intergenerationally redistributive, an income support programme provides each low-wage earner with a flat lump sum benefit. The programme is financed by a linear tax on the gross wage of all workers, and hence intragenerational redistribution takes place from the rich to the poor at the same time as young agents support the elderly.

2.1 Population

Agents are categorised into two groups — natives and immigrants. We draw a line between these two groups by the voting right endowment: natives can

[8]Casarico & Devillanova (2003) revealed an interesting group division but did not examine politico-economic outcomes.

vote, but immigrants cannot. We assume that all natives exercise their voting rights in a referendum on quantitative immigration policy. On the other hand, all agents including immigrants are entitled to economic rights in the host country. Hence natives and immigrants are equally entitled to benefits of the PAYG pension scheme and the income support programme.

Assume for simplicity that there has been no immigration in the past. The total number of working natives in period t is denoted by N_t. The growth rate of the native population is assumed to be a positive constant, i.e., $\delta > 0$. Accordingly, $\frac{N_{t-1}}{N_t} = \frac{1}{1+\delta}$ where N_{t-1} is the total number of pensioners in period t. This ratio expresses the PAYG system dependency ratio in period t without immigration.

We assume that native agent i is born low-skilled with parameter e^i which indicates an idiosyncratic pecuniary cost to become high-skilled. The smaller the value of e^i is, the less costly it is for worker i to become high-skilled. We assume that $e^i \in [0, \overline{e}]$ where $\overline{e}$ is the highest cost of skill acquisition among the native workers. We further assume for ease of exposition that the cost parameter is uniformly distributed among young native workers.[9]

The existence of the idiosyncratic cost of skill acquisition implies that, while some native workers can afford to become high-skilled, skill acquisition is too costly for the others. Let $\widetilde{e}_t$ be the threshold level of the skill acquisition cost in period t. Young native i with $e^i > \widetilde{e}_t$ remains low-skilled in period t. Since we assume a uniform distribution for e^i, we can write the proportion of high-skilled workers in the native workforce by using its cumulative distribution function as

$$h_t := \frac{\widetilde{e}_t}{\overline{e}} \in [0, 1] \tag{1}$$

where $\widetilde{e}_t$ is later defined as a function of immigration.[10]

Consider now that immigrants enter the country. We assume that they are always low-skilled workers at the entry and are fully employed. Once

[9] It is possible to assume other, perhaps more realistic, distributions for e^i, though it is then likely that we need to resort to numerical simulation.

[10] See section 2.7 below.

entered, they remain in the host country for two periods of their lifetime. Low-skilled immigrants who enter in period t amount to M_t. We assume that immigrant workers do not bring in dependants when they arrive and have the same reproductive behaviour as natives during their working life. Furthermore, all children of immigrants who are born in the host country are given voting rights. That is, they are classified as natives in our model.

Definition 1. *By immigration, we mean low-skilled immigrant workers who are in the first period of their lifetime.*

If such immigration takes place in period t only, the PAYG system dependency ratio is $\frac{N_{t-1}}{N_t+M_t}$ in period t and is $\frac{N_t+M_t}{N_{t+1}}$ in period $t+1$ where $N_{t+1} = (1+\delta)(N_t + M_t)$. That is, immigrants reduce the pensioner-worker ratio in their working period, i.e., $\frac{N_{t-1}}{N_t+M_t} < \frac{1}{1+\delta}$, and the ratio returns to the pre-immigration level when they retire, i.e., $\frac{N_t+M_t}{N_{t+1}} = \frac{1}{1+\delta}$, if immigration is *temporary.*

Definition 2. *By temporary immigration, we mean that low-skilled immigrant workers enter the host country in one period only.*

We denote by m the growth rate of the low-skilled workforce due to immigration in period t, i.e.,

$$m_t := \frac{M_t}{(1-h_t)(1+\delta) N_{t-1}}. \tag{2}$$

The total supply of low-skilled labour in period t is $L_t := (1-h_t)(1+\delta) N_{t-1} + M_t$, assuming that each working agent provides one unit of labour. Since we assume that immigrant workers are always low-skilled, the total supply of high-skilled workers in period t is $H_t := h_t (1+\delta) N_{t-1}$.

2.2 Production

The production in the host country, Y, is characterised by the following Cobb-Douglas function which exhibits constant returns to scale:

$$Y_t(K_t, H_t, L_t) := K_t^{\gamma} H_t^{\varphi} L_t^{\varrho} \tag{3'}$$

where the output share parameters γ, φ and ϱ are all on the interval $(0,1)$ and $\gamma+\varphi+\varrho=1$. The marginal product of capital, K, is the interest rate, $r_t := \frac{\partial Y_t}{\partial K_t} = \gamma K_t^{\gamma-1} H_t^{\varphi} L_t^{\varrho}$. We assume international perfect mobility of capital, and the interest rate, r, is exogenously given. Accordingly, for a fixed interest rate, $K_t = \left(\frac{\gamma}{r}\right)^{\frac{1}{1-\gamma}} H_t^{\frac{\varphi}{1-\gamma}} L_t^{\frac{\varrho}{1-\gamma}}$. By substituting this expression back into the production function above, we get

$$Y_t\left(H_t, L_t\right) = A H_t^{\alpha} L_t^{1-\alpha}$$

where $A := \left(\frac{\gamma}{r}\right)^{\frac{1}{1-\gamma}}$ and $\alpha := \frac{\varphi}{\varphi+\varrho} \in (0,1)$. Thus, capital exists but does not explicitly enter the production function. The amount of capital perfectly adjusts to the interest rate which is exogenous. For ease of exposition, we normalise $A=1$. Therefore, our production function reduces to

$$Y_t\left(H_t, L_t\right) = H_t^{\alpha} L_t^{1-\alpha}. \tag{3}$$

Under perfect competition, firms make zero profit. Wages perfectly adjust for full employment. By differentiating the production function (3) with respect to H and L respectively, we obtain the marginal product of labour of each skill type, i.e.,

$$\frac{\partial Y_t}{\partial H_t} = \alpha \left(\frac{L_t}{H_t}\right)^{1-\alpha} \tag{4'}$$

and

$$\frac{\partial Y_t}{\partial L_t} = (1-\alpha) \left(\frac{L_t}{H_t}\right)^{-\alpha}. \tag{5'}$$

These are the gross wages, w^H and w^L, for one unit of high-skilled and low-skilled labour respectively. Using equation (2), we rewrite these marginal products of labour as follows:

$$w_t^H\left(m_t, h_t\left(m_t\right)\right) := \alpha \left[\frac{\left(1-h_t\left(m_t\right)\right)\left(1+m_t\right)}{h_t\left(m_t\right)}\right]^{1-\alpha} \tag{4}$$

and

$$w_t^L\left(m_t, h_t\left(m_t\right)\right) := (1-\alpha) \left[\frac{\left(1-h_t\left(m_t\right)\right)\left(1+m_t\right)}{h_t\left(m_t\right)}\right]^{-\alpha} \tag{5}$$

where it is implied that h_t is a function of m_t. As mentioned when we defined h_t in (1) above, we later define $\widetilde{e}_t$ as a function of m_t. We will then comment on the shapes of these wage functions.[11]

2.3 Income support

Our economy operates an income support programme for low-wage earners. We simply assume that all low-skilled workers receive such support which is flat lump sum, θ. The programme is financed through a programme-specific tax, μ. The budget constraint in period t is then

$$L_t\theta_t = \mu_t Y_t \tag{6'}$$

where $Y_t = w_t^H H_t + w_t^L L_t$. The tax is imposed on all workers, and the revenue is shared by only low-skilled workers. Hence pensioners are not affected by the programme, while high-skilled workers redistribute to low-skilled workers. We fix the per capita support exogenously. Therefore, the tax rate is residually determined by immigration policy, making the policy choice unidimensional. We then rewrite the constraint (6') by substituting (2), (4) and (5) into it as[12]

$$\mu_t\left(m_t, h_t\left(m_t\right)\right) = \theta\left[\frac{\left(1 - h_t\left(m_t\right)\right)\left(1 + m_t\right)}{h_t\left(m_t\right)}\right]^{\alpha}. \tag{6}$$

2.4 Pension

The pension scheme in the economy is balanced PAYG. We assume that the per capita pension benefit, b, is a flat lump sum payment for all pensioners. Accordingly, the following budget constraint must hold in any one period:

$$\left(N_{t-1} + M_{t-1}\right) b_t = \tau_t Y_t \tag{7'}$$

[11]See lemmata 3 and 4 below.

[12]See lemma 5 below for the behaviour of the programme-specific tax rate.

where τ is the payroll tax rate common to all workers. The left hand side of the equation represents the total amount of pension benefits to be paid to the pensioners in period t, and the right hand side the total amount of contributions to be collected from the workers in that period. Let us assume that the payroll tax rate is exogenously given. Accordingly, the amount of the per capita benefit adjusts residually to the level of immigration, again making the policy choice unidimensional. We continue to assume that no immigrant entered in period $t-1$, so we can rewrite the budget constraint (7') by substituting (2), (4) and (5) into it as[13]

$$b_t\left(m_t, h_t\left(m_t\right)\right) = \tau\left(1+\delta\right) h_t\left(m_t\right)\left[\frac{\left(1-h_t\left(m_t\right)\right)\left(1+m_t\right)}{h_t\left(m_t\right)}\right]^{1-\alpha}. \qquad (7)$$

Note that our flat lump sum pension scheme implies redistribution among pensioners of the same generation. That is, all agents receive the same amount of pension in the post-retirement period, whereas high-skilled ones contribute more to the pension system than the low-skilled during the working period. Hence our PAYG pension scheme is both inter- and intragenerationally redistributive.

If immigrants continue to enter the host country in the next period, the same expression (7) is applicable by changing the time subscript to $t+1$. The immigration rate is then $m_{t+1} = \frac{M_{t+1}}{(1-h_t(m_t))(1+\delta)[(1+\delta)N_{t-1}+M_t]}$. On the other hand, if immigration is *temporary*,[14] the per capita pension benefit in period $t+1$ becomes

$$b_{t+1}\left(0, h_{t+1}\left(0\right)\right) = \tau\left(1+\delta\right) h_{t+1}\left(0\right)\left(\frac{1-h_{t+1}\left(0\right)}{h_{t+1}\left(0\right)}\right)^{1-\alpha}.$$

Thus, when immigration is one-off, the per capita benefit returns to the pre-immigration level in the next period. That is, today's immigrants are not extra burdens in the next period under our PAYG pension system. This observation results from our assumption with regard to the fertility of immigrants.

[13]See lemmata 6 and 7 below for the behaviour of the per capita pension benefit.

[14]See definition 2 above.

2.5 Consumption

Worker i maximise their utility over the lifetime consumption, c. In the first period of life, the budget constraint is

$$c_t^i\left(m_t, h_t\left(m_t\right)\right) := \begin{cases} (1-s)\left[\left(1-\tau-\mu_t\right) w_t^H - e^i\right] & \text{if high-skilled;} \\ (1-s)\left[\left(1-\tau-\mu_t\right) w_t^L + \theta\right] & \text{otherwise,} \end{cases} \tag{8}$$

where s is the saving rate common to all workers. In the second period of life, the budget constraint is

$$\begin{aligned} & c_{t+1}^i\left(m_t, h_t\left(m_t\right), m_{t+1}, h_{t+1}\left(m_{t+1}\right)\right) \\ : \ = & \begin{cases} (1+r)\, s\left[\left(1-\tau-\mu_t\right) w_t^H - e^i\right] + b_{t+1} & \text{if high-skilled;} \\ (1+r)\, s\left[\left(1-\tau-\mu_t\right) w_t^L + \theta\right] + b_{t+1} & \text{otherwise.} \end{cases} \end{aligned} \tag{9}$$

By substituting into the saving term, we combine the constraints for two periods respectively, i.e.,

$$c_t^i + \frac{c_{t+1}^i}{1+r} = \begin{cases} \left(1-\tau-\mu_t\right) w_t^H - e^i + \frac{b_{t+1}}{1+r} & \text{if high-skilled;} \\ \left(1-\tau-\mu_t\right) w_t^L + \theta + \frac{b_{t+1}}{1+r} & \text{otherwise.} \end{cases} \tag{10'}$$

The left hand side of the equality sign expresses the lifetime consumption, while the right hand side expresses the lifetime income. Let us define the lifetime income as

$$z_t^i\left(w_t^H, w_t^L, \mu_t, b_{t+1}\right) := \begin{cases} \left(1-\tau-\mu_t\right) w_t^H - e^i + \frac{b_{t+1}}{1+r} & \text{if high-skilled;} \\ \left(1-\tau-\mu_t\right) w_t^L + \theta + \frac{b_{t+1}}{1+r} & \text{otherwise,} \end{cases} \tag{10}$$

where w_t^H, w_t^L, μ_t and b_{t+1} all depend on immigration.[15] In section 2.7 below, we model the skill acquisition decision of agent i.

2.6 Utility

We assume that all workers have an identical Cobb-Douglas utility function, i.e.,

$$u_t^i\left(c_t^i, c_{t+1}^i\right) := \left(c_t^i\right)^\beta \left(c_{t+1}^i\right)^{1-\beta} \tag{11}$$

[15] See lemmata 3 to 7 below.

where we assume $\beta \in (0,1)$. Form the Lagrangian using (11) with (10'). The first-order condition for utility maximisation suggests

$$\frac{c_t^i}{c_{t+1}^i} = \frac{\beta}{(1-\beta)(1+r)}$$

where we substitute into the Lagrangian multiplier for $\frac{\partial \mathsf{L}}{\partial c_t^i} = 0$ and $\frac{\partial \mathsf{L}}{\partial c_{t+1}^i} = 0$. Plug this into (10') to obtain the following demand functions:

$$c_t^{i*} = \beta z_t^i$$

and

$$c_{t+1}^{i*} = (1-\beta)(1+r) z_t^i.$$

By substituting these demands back into the utility function (11), we obtain the following indirect utility function:

$$v_t^i \left(z_t^i\right) := \beta^\beta (1-\beta)^{1-\beta} (1+r)^{1-\beta} z_t^i \tag{12}$$

where z_t^i is defined in (10) which indicates that immigration affects the preferences of agents via three channels — the labour market, the income support programme and the PAYG pension system. Since the relationship between v_t^i and z_t^i is positive linear in expression (12), we focus on z_t^i to examine the preferences of native workers in the subsequent analysis.

2.7 Skill acquisition

Young native i is assumed to become high-skilled if

$$(1-\tau-\mu_t) w_t^L + \theta + \frac{b_{t+1}}{1+r} \leq (1-\tau-\mu_t) w_t^H - e^i + \frac{b_{t+1}}{1+r}$$

or

$$e^i \leq (1-\tau-\mu_t)\left(w_t^H - w_t^L\right) - \theta \tag{13}$$

with given immigration. That is, to become high-skilled, she/he requires the lifetime income for high-skilled labour to be at least as high as that for low-skilled labour.[16] Note that we need to compare only the first-period incomes because both high-skilled and low-skilled workers receive the same amount of pension in the post-retirement period. The worker who pays the most to become high-skilled in period t is then born with the idiosyncratic cost of

$$\widetilde{e}_t\left(w_t^H, w_t^L, \mu_t\right) := \left(1-\tau-\mu_t\right)\left(w_t^H - w_t^L\right) - \theta. \tag{14'}$$

By substituting expressions (4), (5) and (6) into it, we obtain

$$\begin{aligned}\widetilde{e}_t\left(m_t, h_t\left(m_t\right)\right) = & \left\{1-\tau-\theta\left[\frac{\left(1-h_t\left(m_t\right)\right)\left(1+m_t\right)}{h_t\left(m_t\right)}\right]^{\alpha}\right\} \\ & \times\left\{\alpha\left[\frac{\left(1-h_t\left(m_t\right)\right)\left(1+m_t\right)}{h_t\left(m_t\right)}\right]^{1-\alpha} - (1-\alpha)\left[\frac{\left(1-h_t\left(m_t\right)\right)\left(1+m_t\right)}{h_t\left(m_t\right)}\right]^{-\alpha}\right\} \\ & -\theta.\end{aligned} \tag{14}$$

With this last expression, we are now able to determine how the skill acquisition decision is influenced by immigration and hence, via definition (1), how the skill composition of the native workforce changes over the range of policy alternatives $[0, \overline{m}]$ where $\overline{m}$ is the maximum feasible rate of immigration and is exogenously given.[17] We have the following two observations from (14).

Lemma 1. *Suppose no income support for low-skilled workers. The proportion of high-skilled workers in the native workforce is then monotonically increasing in immigration. Furthermore,* $\frac{dh_t}{dm_t} \in (0,1)$.

Proof. See appendix 1. ■

Lemma 2. *Suppose there is income support for low-skilled workers. The proportion of high-skilled workers in the native workforce is then initially*

[16] Refer to expression (10).

[17] This can be identified by assuming a given reservation wage for immigrants, $\underline{w}^L$, below which they would not accept to enter the host country, i.e.,

$$\overline{m} := \left(\frac{1-\alpha}{\underline{w}^L}\right)^{\frac{1}{\alpha}} \frac{h\left(\overline{m}\right)}{1-h\left(\overline{m}\right)} - 1$$

from expression (5).

increasing but subsequently decreasing in immigration, provided that (A) the size of per capita income support is "not excessively large", (B) the maximum feasible rate of immigration is "sufficiently high" and (C) the maximum cost of skill acquisition is "sufficiently high". Furthermore, $\frac{dh_t}{dm_t} \in (-1, 1)$.

Proof. See appendix 2. ■

Lemma 1 implies that the wage gap between high-skilled and low-skilled labour monotonically widens when immigration increases in the host country.[18] As a result, an increasing number of native workers pay e^i to become high-skilled, as the rate of immigration rises.

Lemma 2 suggests that the wage gap again widens as m increases, but it begins to shrink if m continues to increase beyond a certain level. That is, too many immigrants require excessive redistribution through the income support programme. Therefore, over a range of different rates of immigration, there is unique interior policy which maximises the proportion of high-skilled workers in the native workforce. As appendix 2 shows, this result is subject to three assumptions (A), (B) and (C). We assume that they hold throughout our subsequent analyses.

Note that, although h_t decreases with high m_t when $\theta > 0$, it does not fall below $h_t(0)$ because we assume that currently high-skilled workers cannot become low-skilled. That is, skill acquisition cannot be reversed. Hence we have a smaller range for h_t than defined in (1), i.e., $h_t(m_t) \in [h_t(0), 1] \, \forall m_t \in [0, \overline{m}]$.

Using these two lemmata about h_t, we now summarise the signs of $\frac{dw_t^H}{dm_t}$, $\frac{dw_t^L}{dm_t}$, $\frac{d\mu_t}{dm_t}$ and $\frac{db_t}{dm_t}$ by the following five lemmata.

Lemma 3. *Suppose no income support for low-skilled workers. The high-skilled wage rate is then monotonically increasing in immigration, and the low-skilled wage rate is monotonically decreasing in it.*

Proof. See appendix 3. ■

[18] When $\theta = 0$, expression (14') merely reflects the changing difference between the high-skilled and low-skilled wages, i.e., $\widetilde{e}_t := (1 - \tau) \left(w_t^H - w_t^L \right)$.

Lemma 4. *Suppose there is income support for low-skilled workers. Suppose assumptions (A), (B) and (C) of lemma 2 hold. The high-skilled wage rate is then monotonically increasing in immigration, and the low-skilled wage rate is monotonically decreasing in it.*

Proof. See appendix 4. ■

Lemma 5. *Suppose there is income support for low-skilled workers. Suppose assumptions (A), (B) and (C) of lemma 2 hold. The tax rate specific to the income support programme for low-skilled workers is then monotonically increasing in immigration.*

Proof. See appendix 5. ■

Lemma 6. *Suppose no income support for low-skilled workers. The per capita pay-as-you-go pension benefit is then monotonically increasing in immigration.*

Proof. See appendix 6. ■

Lemma 7. *Suppose there is income support for low-skilled workers. Suppose assumptions (A), (B) and (C) of lemma 2 hold. The per capita pay-as-you-go pension benefit is then monotonically increasing in immigration.*

Proof. See appendix 7. ■

These lemmata indicate that, as far as assumptions (A), (B) and (C) of lemma 2 hold, we always have $\frac{dw_t^H}{dm_t} > 0$, $\frac{dw_t^L}{dm_t} < 0$, $\frac{d\mu_t}{dm_t} > 0$ and $\frac{db_t}{dm_t} > 0$ regardless of whether the income support programme operates or not. We use lemmata 1 to 7 extensively when we prove the propositions in section 3.

2.8 Interest group division

Although the skill acquisition decision threshold (14') determines the skill composition of the native labour force if a given rate of immigration actually takes place, it does not indicate what proportion of the working native population prefer the given rate to the status quo. The reason is that the satisfaction of condition (13) does not necessarily mean

$$(1-\tau-\mu_t(0))\,w_t^L(0)+\theta+\frac{b_{t+1}(0)}{1+r}\leq(1-\tau-\mu_t(m_t))\,w_t^H(m_t)-e^i+\frac{b_{t+1}(m_{t+1})}{1+r}$$

or

$$\begin{aligned} e^i \;\leq\; & (1-\tau-\mu_t(m_t))\,w_t^H(m_t)-(1-\tau-\mu_t(0))\,w_t^L(0)-\theta \\ & +\frac{1}{1+r}\left(b_{t+1}(m_{t+1})-b_{t+1}(0)\right) \qquad (15) \end{aligned}$$

for given $m_t, m_{t+1} > 0$. In other words, worker i requires her/his lifetime income with given immigration to be at least as high as it would be without it, if the given immigration is to be preferred to the status quo. When condition (15) does not hold, she/he prefers the status quo to the given immigration, even if condition (13) holds. This point is made by Casarico & Devillanova (2003), and we formalise the idea within our model in order to examine majority voting outcomes. Let us define e_t^* as follows:[19]

$$\begin{aligned} e_t^*(m_t,h_t(m_t),m_{t+1},h_{t+1}(m_{t+1})) \;:\; & =[1-\tau-\mu_t(m_t,h_t(m_t))]\,w_t^H(m_t,h_t(m_t)) \qquad (16') \\ & -[1-\tau-\mu_t(0,h_t(0))]\,w_t^L(0,h_t(0))-\theta \\ & +\frac{1}{1+r}\left[b_{t+1}(m_{t+1},h_{t+1}(m_{t+1}))-b_{t+1}(0,h_{t+1}(0))\right]. \end{aligned}$$

By substituting (4), (5), (6) and (7) into it, we obtain

$$\begin{aligned} & e_t^*(m_t,h_t(m_t),m_{t+1},h_{t+1}(m_{t+1})) \qquad (16) \\ :\; & =\left\{1-\tau-\theta\left[\frac{(1-h_t(m_t))(1+m_t)}{h_t(m_t)}\right]^{\alpha}\right\}\alpha\left[\frac{(1-h_t(m_t))(1+m_t)}{h_t(m_t)}\right]^{1-\alpha} \\ & -\left[1-\tau-\theta\left(\frac{1-h_t(0)}{h_t(0)}\right)^{\alpha}\right](1-\alpha)\left(\frac{1-h_t(0)}{h_t(0)}\right)^{-\alpha}-\theta \\ & +\tau\frac{1+\delta}{1+r}\left\{h_{t+1}(m_{t+1})\left[\frac{(1-h_{t+1}(m_{t+1}))(1+m_{t+1})}{h_{t+1}(m_{t+1})}\right]^{1-\alpha}-h_{t+1}(0)\left(\frac{1-h_{t+1}(0)}{h_{t+1}(0)}\right)^{1-\alpha}\right\}. \end{aligned}$$

[19] Definition (16') assumes that 0 gives the highest z^i if skill acquisition does not take place and hence becomes irrelevant in proposition 8 towards the end of section 3 where a possibility of $z^i(0) < z^i(\overline{m})$ arises to influence the majority voting outcome.

The native agent who is born with e_t^* would experience no change in her/his lifetime income, whether given immigration takes place or not. Agents born with $e^i \in [0, e_t^*]$ would then become high-skilled and would be better off with the given immigration. Agents born with $e^i \in (e_t^*, \widetilde{e}_t]$ would also become high-skilled but prefer no immigration to the given immigration because she/he enjoys a higher level of lifetime income without it by remaining low-skilled. Let

$$h_t^* := \frac{e_t^*}{\overline{e}} \in [h_t(0), 1] \tag{17}$$

be the proportion of high-skilled workers who do not lose any in the lifetime income with given immigration.[20] Together with the skill acquisition rule (13), native workers can then be divided into three groups at each potential rate of immigration as follows:

$$\frac{\overline{e} - \widetilde{e}_t(m_t)}{\overline{e}} \equiv 1 - h_t(m_t);$$

$$\frac{\widetilde{e}_t(m_t) - e_t^*(m_t, m_{t+1})}{\overline{e}} \equiv h_t(m_t) - h_t^*(m_t, m_{t+1});$$

$$\frac{e_t^*(m_t, m_{t+1})}{\overline{e}} \equiv h_t^*(m_t, m_{t+1}).$$

Given immigration, the first group consists of those who remain low-skilled. The second group consists of those who become high-skilled with that immigration but then have lower lifetime income than when they remain low-skilled without it. This second group is, in other words, "pushed" to become high-skilled by the immigration. These two groups prefer no immigration to this given immigration policy.

For the same given immigration, the third group consists of those who become high-skilled and have at least as much lifetime income as without that immigration. This group includes those who are "pulled" to become high-skilled by the immigration. They prefer this immigration policy to the status quo.

[20]Note that, when $m_t = m_{t+1} = 0$, expression (16') is the same as expression (14') at $m = 0$. Hence $h_t^*(0) = h_t(0)$.

Definitions (1) and (17) mean that these three groups change in their relative size with each possible immigration policy because expressions (14) and (16) define both threshold costs of skill acquisition, $\widetilde{e}_t$ and e_t^*, as functions of immigration. We have already summarised the behaviour of h_t in lemmata 1 and 2 above. We now summarise the behaviour of h^* defined in (17).

Notice that, while expression (14) is determined by one rate of immigration, namely m_t, two rates of immigration are present in (16), i.e., m_t and m_{t+1}. For ease of exposition, we consider cases where $m_t = m_{t+1}$.

Definition 3. *By permanent immigration, we mean that the same rate of immigration occurs in every period.*

In the case of permanent immigration, we assume that, once a decision is taken, the policy becomes effective from period t onwards without fear of policy change in the future. The optimal choice of immigration is then the steady state solution, i.e., $m^* = m_{t+j} \forall j \geq 0$. Note that, if the chosen rate of immigration is positive in period t, the ratio of low-skilled to high-skilled labour changes permanently from $\left(\frac{L}{H}\right)_{t-1} = \frac{1-h_{t-1}(0)}{h_{t-1}(0)}$ to $\left(\frac{L}{H}\right)_{t+j} = \frac{(1-h_{t+j}(m^*))(1+m^*)}{h_{t+j}(m^*)} \forall j \geq 0$. Hereafter, we drop all the time subscripts because they are unnecessary in either temporary or permanent scenarios.[21]

Lemma 8. *Consider temporary immigration. Suppose no income support for low-skilled workers. The proportion of workers who prefer immigration to the status quo is then monotonically increasing in immigration.*

Proof. No income support for low-skilled workers means $\theta = 0$. Then, when immigration policy is temporary, (16') reduces to $e^*(m, h(m)) := (1-\tau)\left[w^H(m, h(m)) - w^L(0, h(0))\right]$. If $\theta = 0$, then $\frac{dw^H}{dm} > 0$ by lemma 3 and hence $\frac{de^*}{dm} > 0$. Since $\frac{dh^*}{de^*} > 0$ from (17), $\frac{dh^*}{dm} > 0$. ■

Lemma 9. *Consider permanent immigration. Suppose no income support for low-skilled workers. The proportion of workers who prefer immigration to the status quo is then monotonically increasing in immigration.*

[21] See definition 2 above for temporary immigration.

Proof. No income support for low-skilled workers means $\theta = 0$. Then, when immigration policy is permanent, (16') reduces to $e^*(m, h(m)) := (1-\tau)\left[w^H(m, h(m)) - w^L(0, h(0))\right] + \frac{1}{1+r}\left[b(m, h(m)) - b(0, h(0))\right]$. If $\theta = 0$, then $\frac{dw^H}{dm} > 0$ by lemma 3, $\frac{db}{dm} > 0$ by lemma 6 and hence $\frac{de^*}{dm} > 0$. Since $\frac{dh^*}{de^*} > 0$ from (17), $\frac{dh^*}{dm} > 0$. ■

Lemma 10. *Consider temporary immigration. Suppose there is income support for low-skilled workers. Suppose assumptions (A), (B) and (C) of lemma 2 hold. The proportion of workers who prefer immigration to the status quo is then initially increasing but subsequently decreasing in immigration, provided that condition (A.2.1') for assumption (A) is replaced by (A.8.1'). Furthermore, this proportion is at its maximum before the proportion of high-skilled workers reaches its peak.*

Proof. See appendix 8. ■

Lemma 11. *Consider permanent immigration. Suppose there is income support for low-skilled workers. Suppose assumptions (A), (B) and (C) of lemma 2 hold. The proportion of workers who prefer immigration to the status quo is then initially increasing but subsequently decreasing in immigration, provided that condition (A.2.1') for assumption (A) is replaced by (A.9.1'). Furthermore, this proportion is at its maximum before the proportion of high-skilled workers reaches its peak.*

Proof. See appendix 9. ■

These four lemmata indicate that, regardless of whether immigration is temporary or permanent, if the income support programme is absent, the proportion of workers who prefer immigration to the status quo monotonically increases in m. If the income support programme is in operation, it has a peak over $(0, m_h)$ where m_h maximises h, i.e., the solution for (A.2.1") in appendix 2. We use these four lemmata when we prove the propositions we state below.

3 Results

In the framework described above, we now examine the preferences of individuals over the rate of immigration and derive majority voting outcomes. We assume that a referendum takes place in the beginning of period t, and all natives rationally vote on the rate of immigration to be permitted into the country. Our focus is on the determination of the variable, m. An infinite number of potential policy alternatives over the interval $[0, \overline{m}]$ are compared pairwise where $\overline{m}$ is the maximum feasible rate of immigration.[22]

We examine impacts of both *temporary* and *permanent* immigration on individual preferences. Temporary immigration is defined in definition 2 above and is examined by Razin & Sadka (2000) and Casarico & Devillanova (2003). Permanent immigration is defined in definition 3 above. That is, we assume that, once a decision is taken, the policy becomes effective from period t onwards without fear of policy change in the future.

Native workers attempt to optimise their lifetime consumption/income at the beginning of their working period when the referendum takes place. The objective of working native i is

$$\max_{m} v^i \left(z^i \left(w^H (m, h(m)), w^L (m, h(m)), \mu (m, h(m)), b(m, h(m))\right)\right)$$
$$\text{s.t.} \quad m \in [0, \overline{m}] \quad \text{and} \quad h(m) \in [h(0), 1]. \tag{18}$$

Since expression (12) indicates that the utility is a positive monotonic transformation of the lifetime income, we examine the shape of z^i. The first-order total derivative of z^i with respect to permanent immigration is

$$\begin{aligned} \frac{dz^i (m, h(m))}{dm} &= [1 - \tau - \mu (m, h(m))] \frac{dw^k (m, h(m))}{dm} \\ &\quad - w^k (m, h(m)) \frac{d\mu (m, h(m))}{dm} + \frac{1}{1+r} \frac{db(m, h(m))}{dm} \end{aligned} \tag{19}$$

where $k = \{H, L\}$. For temporary immigration, the third term on the right hand side drops out. When the income support programme is absent, the second term disappears, and $\mu = 0$ in the first term.

[22] See footnote 17.

Finally, the objective of pensioners is

$$\max_{m} b(m, h(m)) \quad \text{s.t.} \quad m \in [0, \overline{m}] \quad \text{and} \quad h(m) \in [h(0), 1] \tag{20}$$

because immigration cannot affect their first-period income earned in the previous period in our model.[23] From objective (20), lemmata 6 and 7 indicate that pensioners always vote for $\overline{m}$, for immigration only increases their income and hence utility.[24] The preferences of workers are more complicated than pensioners', and we examine them in detail below.

3.1 Temporary immigration without income support

In this case, objective (18) and lemma 3 suggest that high-skilled workers vote for $\overline{m}$, while low-skilled workers vote for the status quo. Objective (20) and lemma 6 mean that pensioners vote for $\overline{m}$. Since currently high-skilled workers and pensioners exhibit the same ordinal preference over the interval $[0, \overline{m}]$, we consider the following two different scenarios regarding $h(0)$, i.e., the proportion of high-skilled workers in the native workforce at the voting

[23]If we relax our assumption of the fixed interest rate, immigration is likely to change the marginal product of capital and thus affect pensioners through savings as well as the PAYG pension benefit. However, since an increase in labour due to immigration would raise the interest rate, the preference of pensioners is unlikely to be modified even if we introduce flexibility for the interest rate.

[24]This is a consequence of using an unindexed flat lump sum pension and fixing the PAYG payroll tax rate. If the size of per capita pension is fixed instead, pensioners become indifferent in our model. Scholten & Thum (1996) and Haupt & Peters (1998) index the per capita pension benefit to the prevailing wage rate. In their model, pensioners are all against immigration that depresses the single wage rate because age is the only dimension of heterogeneity among agents. In our model, single-mindedness of retirees can be changed by, for instance, linking the per capita pension benefit to the contribution history, e.g., the benefit for high-skilled pensioners is indexed to the prevailing wage for high-skilled labour. In addition, incorporating their altruism towards the younger generation would make their preference non-monotonic.

stage:

$$\begin{array}{ll}\text{Scenario I} & (1+\delta)(1-h(0)) < (1+\delta)h(0)+1 \\ \text{Scenario II} & (1+\delta)(1-h(0)) > (1+\delta)h(0)+1\end{array}$$

In scenario I, currently high-skilled workers and pensioners form the majority when voting takes place. In scenario II, currently low-skilled workers form the majority. Scenario I is a straightforward case, as the following proposition states.

Proposition 1. *Consider temporary immigration policy. Suppose no income support for low-skilled workers. If currently high-skilled workers and pensioners form the majority at the voting stage, the referendum-led policy is to permit the maximum feasible rate of immigration.*

Proof. No income support for low-skilled workers means $\theta = 0$. Then, $\frac{dh}{dm} > 0$ by lemma 1. Under scenario I, $\frac{dh}{dm} > 0$ implies $\forall m \in [0, \overline{m}]$, $(1+\delta)(1-h(m)) < (1+\delta)h(m)+1$. That is, high-skilled workers and pensioners continue to form the majority over the interval $[0, \overline{m}]$. When immigration policy is temporary with $\theta = 0$, expression (10) for the high-skilled reduces to $z^i := (1-\tau)w^H - e^i$. Since $\theta = 0$, $\frac{dw^H}{dm} > 0$ by lemma 3 and hence $\frac{dz^i}{dm} > 0$ for the high-skilled. Also $\frac{db}{dm} > 0$ by lemma 6. Objectives (18) and (20) then imply that everyone in the majority monotonically increases her/his utility as m increases. The Condorcet winner is thus $\overline{m}$. ■

Under scenario II, there are two possibilities, as the following proposition states.

Proposition 2. *Consider temporary immigration policy. Suppose no income support for low-skilled workers. If currently low-skilled workers form the majority at the voting stage, the referendum-led policy is to permit (i) the maximum feasible rate of immigration if there exists $\ddot{m} \in [0, \overline{m}]$ satisfying*

$$e^*(\ddot{m}, h(\ddot{m})) \equiv h^*(\ddot{m}, h(\ddot{m}))\,\overline{e} = \frac{\delta}{2(1+\delta)}\overline{e}, \tag{21}$$

but (ii) no immigration if there is no such $\ddot{m} \in [0, \overline{m}]$.

Proof. See appendix 10. ■

According to this proposition, currently high-skilled workers and pensioners do not have to form the majority to achieve their most preferred policy when a referendum takes place, if there exists feasible policy that induces a sufficient number of currently low-skilled workers to have their lifetime income by becoming high-skilled at least as high as when remaining low-skilled. Condition (21) is a rearrangement of

$$(1+\delta)\left(1-h^{*}\left(\ddot{m},h\left(\ddot{m}\right)\right)\right)=\left(1+\delta\right)h^{*}\left(\ddot{m},h\left(\ddot{m}\right)\right)+1.$$

The left hand side of the inequality sign consists of those who remain low-skilled and those who become high-skilled but then have lower lifetime income at $\ddot{m}$ than when $m = 0$. The right hand side is the sum of all retired pensioners, currently high-skilled workers and those who are currently low-skilled but become high-skilled at $\ddot{m}$ and earn at least as high lifetime income as in the status quo.

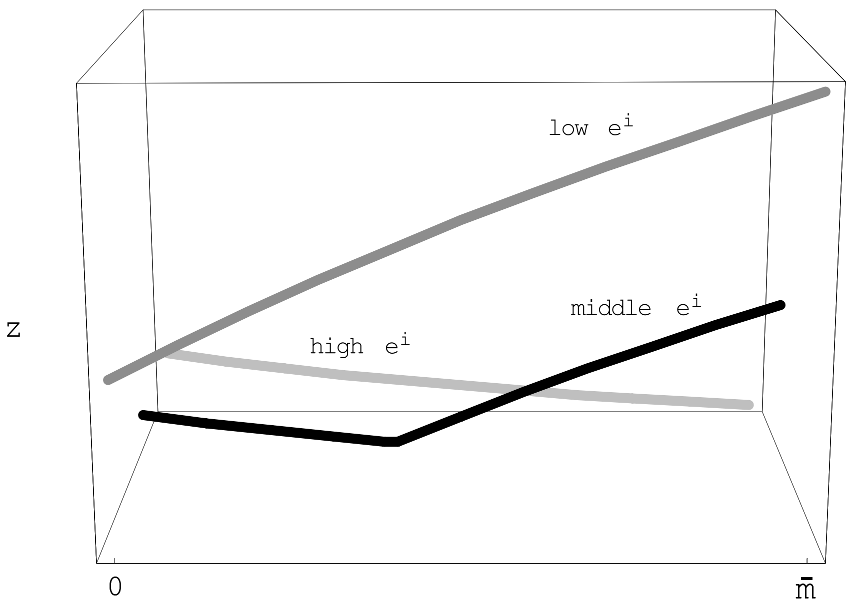

Figure 1. Lifetime Incomes of Workers with Different Costs of Skill Acquisition over Temporary Immigration Policy when the Income Support Programme is Absent

Satisfaction of condition (21) guarantees that there are those currently low-skilled workers who join high-skilled workers and pensioners to form the majority in favour of $\overline{m}$ against any other policy alternatives. In figure 1, the lifetime income of a worker with middle e^i implies that she/he most prefers $\overline{m}$ because $z^i(0) < z^i(\overline{m})$, although this worker is currently low-skilled.[25] If a sufficient number of currently low-skilled workers have this type of lifetime income projections with a kink and $z^i(0) < z^i(\overline{m})$, condition (21) is met, and outcome (i) occurs.

An implication of these two propositions is that there might be a tendency to decide on very liberal policy of temporary immigration if immigrants were not burdens in the welfare state, namely no income support in our model.[26]

3.2 Permanent immigration without income support

When immigration policy is permanent, the lifetime income for the low-skilled is no longer monotonically decreasing in immigration.

Lemma 12. *Suppose no income support for low-skilled workers. The lifetime income for the low-skilled is then initially decreasing but subsequently increasing in permanent immigration, provided that (D) the PAYG payroll tax rate is "not excessively high" and (B) the maximum feasible rate of immigration is "sufficiently high".*

Proof. See appendix 11. ■

On the other hand, the preference of currently high-skilled workers over policy alternatives remains the same as in the case of temporary immigration. Although lemmata 3 and 6 indicate that their lifetime income is higher than when immigration is temporary, their ordinal preference over policy

[25]The projections in figure 1 are calibrated with $\alpha = .6$, $\tau = .2$, $\overline{e} = 3$ and $\overline{m} = 1$. Low e^i is set to .6, and middle e^i to .73. Since z^L does not depend on the idiosyncratic cost of skill acquisition, any specific value is not assumed for high e^i.

[26]As we observed in section 2.4 above, immigrants are not burdens via the PAYG pension system in our model.

alternatives does not change and is monotonically increasing in permanent immigration. Since objective (20) and lemma 6 imply that the utility of pensioners is always increasing in immigration, they and currently high-skilled workers again share a same ordinal preference. Therefore, we continue to examine the majority voting outcomes under the same two scenarios as in the previous section. The outcome under scenario I is again straightforward.

Proposition 3. *Consider permanent immigration policy. Suppose no income support for low-skilled workers. If currently high-skilled workers and pensioners form the majority at the voting stage, the referendum-led policy is to permit the maximum feasible rate of immigration.*

Proof. No income support for low-skilled workers means $\theta = 0$. Then, $\frac{dh}{dm} > 0$ by lemma 1. Under scenario I, $\frac{dh}{dm} > 0$ implies $\forall m \in [0, \overline{m}]$, $(1+\delta)(1-h(m)) < (1+\delta)h(m) + 1$. That is, high-skilled workers and pensioners continue to form the majority over the interval $[0, \overline{m}]$. When immigration policy is permanent with $\theta = 0$, expression (10) for the high-skilled reduces to $z^i := (1-\tau)w^H - e^i + \frac{b}{1+r}$. Since $\theta = 0$, $\frac{dw^H}{dm} > 0$ by lemma 3, $\frac{db}{dm} > 0$ by lemma 6 and hence $\frac{dz^i}{dm} > 0$ for the high-skilled. Objectives (18) and (20) then imply that everyone in the majority monotonically increases her/his utility as m increases. The Condorcet winner is thus $\overline{m}$. ■

This result is the same as in proposition 1 for temporary immigration without income support under scenario I, since the preference of the majority over policy alternatives has not changed. Under scenario II, we again have the same outcome possibilities as for temporary immigration policy in proposition 2, but the conditions for these possibilities are now slightly different because we take into consideration the preferences of currently low-skilled workers that are not the same as in the case of temporary immigration.

Proposition 4. *Consider permanent immigration policy. Suppose no income support for low-skilled workers. If currently low-skilled workers form the majority at the voting stage, the referendum-led policy is to permit (i) the* *maximum feasible rate of immigration if either*

$$(1-\tau)\, w^{L}(0) + \frac{b(0)}{1+r} \leq (1-\tau)\, w^{L}(\overline{m}) + \frac{b(\overline{m})}{1+r} \tag{22}$$

hold or condition (22) does not hold and there exists $\ddot{m} \in [0, \overline{m}]$ *satisfying condition (21), but (ii) no immigration if condition (22) does not hold and there is no* $\ddot{m} \in [0, \overline{m}]$ *satisfying condition (21).*

Proof. See appendix 12. ■

What proposition 4 indicates is that the tendency to decide on very liberal policy of immigration is even stronger when policy is permanent than when it is temporary. First, all currently low-skilled workers may most prefer $\overline{m}$ when condition (22) is met, and everyone agrees on $\overline{m}$. Second, by comparing e^* in the proofs for lemmata 8 and 9, we observe that condition (21) is more easily met for permanent policy than for temporary policy even if condition (22) does not hold. This result is intuitive because immigration in the next period is beneficial for all current workers during their post-retirement period via the PAYG pension system.

3.3 Temporary immigration with income support

We now introduce the income support programme for low-skilled workers. As explained above, a common tax rate is applied to the earning of every worker as the source of the transfer fund, and every low-skilled worker, whether native or immigrant, receives a fixed lump sum benefit.

The preference of pensioners is not influenced via this welfare programme, objective (20) and lemma 7 imply that their utility is monotonically increasing in m.

Lemma 13. *Suppose there is income support for low-skilled workers. Suppose assumptions (A), (B) and (C) of lemma 2 holding. The lifetime income for the high-skilled is then initially increasing but subsequently decreasing in temporary immigration, provided that condition (A.2.1') for assumption (A)* *is replaced by (A.8.1'). The lifetime income for the low-skilled is monotonically decreasing in it.*

Proof. See appendix 13. ■

Remember that, as stated in lemma 10, h^* initially increases but subsequently decreases in m. Note that we observe in appendices 8 and 13 that z^i for the high-skilled and h^* experience their peaks with the same rate of immigration.

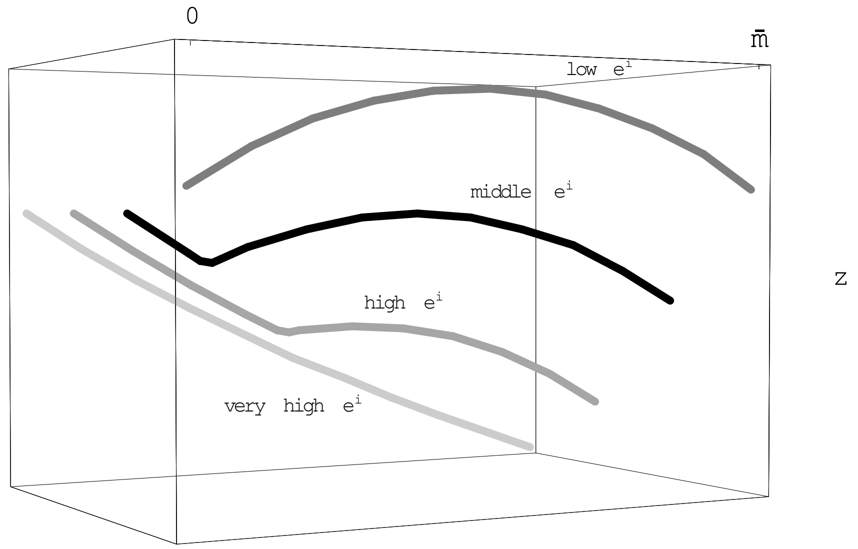

Figure 2. Lifetime Incomes of Workers with Different Costs of Skill Acquisition over Temporary Immigration Policy when the Income Support Programme Operates

The workers who remain low-skilled at any rate of immigration always have the lifetime income for the low-skilled defined in lemma 13. However, the lifetime income of worker i among the rest of the currently low-skilled workers is some combination of z^i for the low-skilled and high-skilled. When m is low, condition (13) does not hold. The lifetime income of currently low-skilled worker i is then identified by z^i for the low-skilled. However, as m

continues to increase, condition (13) is met. When it is satisfied by equality, her/his lifetime income switches from the low-skilled to the high-skilled without discontinuity. If condition (13) continues to hold until it reaches $\overline{m}$, the lifetime income exhibits only one kink at the switching point over $[0, \overline{m}]$. If condition (13) is violated with high m, the lifetime income exhibits two kinks because there are two switching points. Figure 2 illustrates two projections with one kink, i.e., the curves labelled as middle e^i and high e^i.[27]

Because of these non-single-peaked preferences, although z^i for the high-skilled and the low-skilled are respectively single-peaked, e.g., the curves labelled as low e^i and very high e^i respectively in figure 2, we cannot simply rely on Black's (1958) median voter theorem. Since we now observe that pensioners and currently high-skilled workers do not share a same ordinal preference over the policy interval $[0, \overline{m}]$, we consider the following two scenarios regarding $h(0) \in (0, 1)$ and $\delta > 0$ when a referendum takes place.

$$\begin{array}{ll} \text{Scenario III} & (1+\delta)(1-h(0)) + 1 < (1+\delta) h(0) \\ \text{Scenario IV} & (1+\delta) h(0) + 1 < (1+\delta)(1-h(0)) \end{array}$$

In scenario III, currently high-skilled workers form the majority. In scenario IV, currently low-skilled workers do so.[28]

Proposition 5. *Consider temporary immigration policy. Suppose there is income support for low-skilled workers. Suppose assumptions (A), (B) and (C) of lemma 2 hold. If currently high-skilled workers form the majority at the voting stage, the referendum-led policy is to permit*

$$\hat{m} := \left[\frac{(1-\alpha)(1-\tau)}{\theta}\right]^{\frac{1}{\alpha}} \frac{h(\hat{m})}{1-h(\hat{m})} - 1 \in (0, \overline{m}), \tag{23}$$

provided that condition (A.2.1') for assumption (A) is replaced by (A.8.1').

[27] The figure illustrates the case where $\alpha = .6$, $\tau = .2$, $\overline{e} = 3$, $\overline{m} = 1$ and $\theta = .095$. Low e^i is set to .46, middle e^i to .475 and high e^i to .49.

[28] The case where pensioners dominate the majority, i.e., $\delta < 1$, is trivial, for their utility monotonically increases in m, according to lemma 10. The chosen policy is then $\overline{m}$.

Proof. Since we assume that currently high-skilled workers cannot become low-skilled, they continue to form the majority over $[0, \overline{m}]$ under scenario III. By setting $\frac{dz^i}{dm} = 0$ for the high-skilled in (A.13.1), we solve for m which maximises z^i for the high-skilled because it is initially increasing but subsequently decreasing in m, as stated by lemma 13. The Condorcet winner is then this solution which we denote by $\hat{m}$. ■

Under scenario III, we find intermediate policy as a unique equiliblium when the income support programme redistributes from high-skilled to low-skilled workers including immigrants. Although immigration initially increases the utility of the majority, if it is too much their utility begins falling after a peak. This interior policy, $\hat{m}$, appears to be a strong candidate even if currently high-skilled workers do not dominate the majority, as the following proposition indicates.

Proposition 6. *Consider temporary immigration policy. Suppose there is income support for low-skilled workers. Suppose assumptions (A), (B) and (C) of lemma 2 hold, where condition (A.8.1') replaces (A.2.1') for assumption (A). If currently low-skilled workers form the majority at the voting stage, the referendum-led policy is to permit (i)* $\hat{m}$ *if there exists* $\ddot{m} \in (0, \hat{m})$ *satisfying condition (21), but (ii) no immigration if there is no such* $\ddot{m} \in (0, \hat{m})$*.*

Proof. See appendix 14. ■

Condition (21) assures that those who prefer the status quo to m do not form the majority, even though currently low-skilled workers form the majority. If this condition holds, a referendum leads to $\hat{m}$ defined in (23), as shown in appendix 14.

In sum, propositions 5 and 6 imply that the currently high-skilled workers' most preferred policy, $\hat{m}$, is a strong candidate in a referendum.

3.4 Permanent immigration with income support

Finally, we consider the interaction of the three channels which transmit the effects of permanent immigration. As before, the preference of pensioners continues to increase in immigration monotonically, as objective (20) and lemma 7 imply.

Lemma 14. *Suppose there is income support for low-skilled workers. Suppose assumptions (A), (B), (C) and (D) of lemmata 2 and 12 holding. The lifetime income for the high-skilled is then initially increasing but subsequently decreasing in permanent immigration, provided that condition (A.2.1') for assumption (A) is replaced by (A.9.1'). The lifetime income for the low-skilled is initially decreasing but subsequently increasing in it, provided that ϕ is replaced by ψ in condition (A.11.1') for assumption (D).*

Proof. See appendix 15. ■

As in the previous section, currently high-skilled workers and pensioners do not share a same ordinal preference over the policy interval $[0, \overline{m}]$. Hence we continue to consider the referendum outcomes under scenarios III and IV.

Proposition 7. *Consider permanent immigration policy. Suppose there is income support for low-skilled workers. Suppose assumptions (A), (B) and (C) of lemma 2 hold. If currently high-skilled workers form the majority at the voting stage, the referendum-led policy is to permit*

$$\acute{m} \; : \; = \left[\frac{(1-\alpha)(1-\tau)}{\theta} + \left(\frac{1-\alpha}{\alpha} \right) \frac{\tau}{\theta} \left(\frac{1+\delta}{1+r} \right) \frac{h(\acute{m})}{1-\psi(\acute{m})} \left(1 + \psi(\acute{m}) \frac{\alpha - h(\acute{m})}{1-\alpha} \right) \right]^{\frac{1}{\alpha}} \times \frac{h(\acute{m})}{1-h(\acute{m})} - 1 \in (0, \overline{m}), \tag{24}$$

provided that condition (A.2.1') for assumption (A) is replaced by (A.9.1').

Proof. Since we assume that currently high-skilled workers cannot become low-skilled, they continue to form the majority over $[0, \overline{m}]$ under scenario III. By setting $\frac{dz^i}{dm} = 0$ for the high-skilled in (A.15.1), we solve for m which maximises z^i for the high-skilled because it is initially increasing but subsequently decreasing in m, as stated by lemma 14. The Condorcet winner is then this solution which we denote by $\acute{m}$. ■

By comparing $\acute{m}$ in (24) to $\hat{m}$ in (23), we notice that the second term in the square brackets in the former is not present in the latter. This is because $\acute{m}$ takes into account the positive impact of immigration through the PAYG pension system in the next period, while $\hat{m}$ does not.

	$\theta = 0$		$\theta > 0$	
Scenario	Temporary	Permanent	Temporary	Permanent
I	$\overline{m}$	$\overline{m}$		
II	$\overline{m}, 0$	$\overline{m}, 0$		
III			$\hat{m}$	$\acute{m}$
IV			$\hat{m}, 0$	See table 2

Table 1. Referendum-led Immigration Policy

We have so far found a unique majority voting outcome in each circumstance, as summarised in table 1. Finally, we examine the referendum-led policy of permanent immigration with currently low-skilled workers forming the majority at the voting stage when the income support programme operates. The analysis reveals that the majority voting outcome might be manipulable under certain configurations of individual preferences because a voting cycle arises.

Proposition 8. *Consider permanent immigration policy. Suppose there is income support for low-skilled workers. Suppose assumptions (A), (B), (C) and (D) of lemmata 2 and 12 holding. Suppose that condition (A.2.1') for assumption (A) is replaced by (A.9.1'). Suppose that ϕ is replaced by ψ in condition (A.11.1') for assumption (D). If currently low-skilled workers form the majority at the voting stage, the referendum-led policy is either* 0, $\acute{m}$, $\overline{m}$ *or subject to manipulation due to the emergence of a voting cycle, depending on the configuration of individual preferences, as summarised in table 2.*

Proof. See appendix 16 and the text below. ■

Table 2 presents the referendum outcomes under four different configurations of high-skilled and low-skilled lifetime income curves. For instance, $z^i(\overline{m}) < z^i(\acute{m}) < z^i(0)$ for the low-skilled would lead the majority voting outcome to yield either $\check{m}$ or no immigration. Appendix 16 proves all the possibilities in the table except those under the situation where $z^i(\overline{m}) < z^i(0)$ for the high-skilled and $z^i(\acute{m}) < z^i(\overline{m}) < z^i(0)$ for the low-skilled coexist. We prove the outcomes under this situation here in order to illustrate the uncertainty.

Lifetime income for the low-skilled	Outcome possibilities
$z^i(0) < z^i(\acute{m}) < z^i(\overline{m})$	$\acute{m}, \overline{m}$
$z^i(\acute{m}) < z^i(0) < z^i(\overline{m})$	$\acute{m}, \overline{m}$
$z^i(\overline{m}) < z^i(\acute{m}) < z^i(0)$	$0, \acute{m}$
$z^i(\acute{m}) < z^i(\overline{m}) < z^i(0)$	$0, \acute{m}, \overline{m}, \textit{cycling}$

TABLE 2. PERMANENT POLICY WITH $\theta > 0$ UNDER SCENARIO IV

Proof for the case where $z^i(\overline{m}) < z^i(0)$ for the high-skilled and $z^i(\acute{m}) < z^i(\overline{m}) < z^i(0)$ for the low-skilled. Suppose $z^i(\overline{m}) < z^i(0)$ for the high-skilled and $z^i(\acute{m}) < z^i(\overline{m}) < z^i(0)$ for the low-skilled. Lemma 14 then implies $z^i(\overline{m}) < z^i(0) < z^i(\acute{m})$ for the high-skilled. Objective (20) and lemma 7 imply that the utility of pensioners monotonically increases in m. Let H denote the number of currently high-skilled workers, and R that of retired pensioners. Currently low-skilled workers are divided into four groups: L_0 denotes the number of those who would remain low-skilled over $[0, \overline{m}]$; L_1 the number of those who would undertake skill acquisition over some subset of $[0, \overline{m}]$ and have $z^i(\acute{m}) \leq z^i(\overline{m})$; L_2 the number of those who would undertake skill acquisition over some subset of $[0, \overline{m}]$ and have $z^i(\overline{m}) < z^i(\acute{m}) \leq z^i(0)$; L_3 the rest of the currently low-skilled who would undertake skill acquisition over some subset of $[0, \overline{m}]$ and have $z^i(0) < z^i(\acute{m})$.

Under scenario IV, $R+H < L_0+L_1+L_2+L_3$ at the voting stage. From the preference orderings configured above, we have the following information.

$$\begin{array}{lll} \text{a.} & L_0+L_1+R < L_2+L_3+H & \Rightarrow \acute{m} \succ m \in (\acute{m}, \overline{m}] \\ \text{b.} & L_0+L_1+R > L_2+L_3+H & \Rightarrow \overline{m} \succ m \in [\acute{m}, \overline{m}) \\ \text{c.} & L_0+L_1+L_2 < L_3+H+R & \Rightarrow \acute{m} \succ m \in [0, \acute{m}) \\ \text{d.} & L_0+L_1+L_2 > L_3+H+R & \Rightarrow 0 \succ m \in (0, \overline{m}] \\ \text{e.} & L_0+L_1+L_2+R < L_3+H & \Rightarrow \acute{m} \succ m \in [0, \overline{m}] \setminus \{\acute{m}\} \end{array}$$

Notice that, if at least one of conditions (b) and (d) holds, (e) does not hold. If condition (e) holds, both (a) and (c) hold. Accordingly, we have the following four cases.

Case 1 — conditions (a) and (c) hold: The Condorcet winner is $\acute{m}$, as it beats any other alternatives. Condition (e) does not need to hold for $\acute{m}$ to be the chosen policy.

Case 2 — conditions (a) and (d) hold: The status quo is the Condorcet winner, as condition (a) is redundant.

Case 3 — conditions (b) and (c) hold: The Condorcet winner is not $\acute{m}$ because $\acute{m} \succ m \in [0, \acute{m})$ but $\overline{m} \succ m \in [\acute{m}, \overline{m})$. For $\overline{m}$ to be the Condorcet winner, it has to beat all $m \in [0, \acute{m})$. However, $0 \succ \overline{m}$, since $z^i(\overline{m}) < z^i(0)$ for all workers.[29] Hence the Condorcet winner is not $\overline{m}$. As $\acute{m} \succ m \in [0, \acute{m})$, the Condorcet winner is not the status quo. The majority's preference is thus intransitive, and no policy can beat every other alternatives, leading to the emergence of a voting cycle.[30]

Case 4 — conditions (b) and (d) hold: The status quo is the Condorcet winner, as condition (b) is redundant.

[29] To be precise, there exists $[0, m'] \subset [0, \acute{m})$ which is preferred to $\overline{m}$ by the majority under scenario IV with conditions (b) and (c) holding.

[30] When $z^i(0) < z^i(\overline{m})$ for the high-skilled but everything else is the same, there are L_0+L_1 for whom $m \in [0, m'] \succ \overline{m}$ and L_2+L_3+H for whom $[m'', \acute{m}) \succ \overline{m}$. In this case, if $m'' < m'$, then $[m'', m'] \succ \overline{m}$ for all workers, leading to an emergence of a voting cycle.

Hence the referendum-led policy is either $\acute{m}$, the status quo or manipulable due to the intransitive preference of the majority.[31] ■

This section has shown the complexity of predicting a referendum outcome over immigration policy. When intragenerational and intergenerational transfers take place in the way we model in this paper, the majority's decision on the policy of permanent immigration might be subject to manipulation. For example, in case 3 above, the status quo can be maintained by setting the agenda: $\acute{m}$ against $\overline{m}$, then the winner against the status quo.

4 Concluding remarks

Propositions 1 to 4 are concerned with the cases in which there is no explicit intragenerational transfer. They imply that referendum-led immigration policy would be either the most liberal or the most restrictive policy. These extreme outcomes are observed regardless of whether policy is temporary or permanent. Since any individual obtains the highest utility either at 0 or $\overline{m}$, no interior policy arises as a winner. In spite of the unrealistic outcome possibilities that we obtained, propositions 2 and 4 indicate an interesting tendency. That is, currently high-skilled workers and pensioners may not form the majority, and still their most preferred policy may be chosen. This possibility is driven by the skill acquisition decision making which is specified as a function of immigration. If labour skills of individuals are exogenous, condition (21) becomes irrelevant, and propositions 2 and 4 would then state that the majority most prefer the status quo.

Propositions 5 to 8 are concerned with the cases where there is redistribution from high-skilled to low-skilled workers through the income support programme. As a result, in both cases of temporary and permanent immigration, currently high-skilled workers now exhibit a utility peak between

[31] Note that case 2 is unlikely because the satisfaction of conditions (a) and (d) implies $R < L_2$, which is likely to require a very high rate of population growth, i.e., $\delta > 1$. Since $(1+\delta) R = L_0 + L_1 + L_2 + L_3 + H$, we can express $R < L_2$ as $(L_0 + L_1 + L_3 + H)/L_2 < \delta$.

the most liberal and the most restrictive policy. Since immigrants are low-skilled workers by definition, they depend on the income support programme. Therefore, although immigrants increases the wage rate for the high-skilled, an excessive amount of them prevent currently high-skilled workers from achieving the utility maximisation.

Proposition 6 gives the same indication as propositions 2 and 4 did. That is, the domination of the majority by currently low-skilled workers does not guarantee the status quo. We observe a tendency towards currently high-skilled workers' most preferred policy. Proposition 8 also indicates the same tendency. However, if the utility curves of workers are configured in certain manners, the majority's preference becomes intransitive among policy alternatives. This raised possibilities to yield a voting cycle. In such a case, there is a room for manipulation, and agenda setters may have a control over the referendum outcome. Then, even a pure referendum may not give rise to the majority's most preferred policy.

References

Casarico, A., Devillanova, C., 2003, Social security and migration with endogenous skill upgrading, *Journal of Public Economics* **87**: 773-797.

Chiswick, C.U., 1989, The impact of immigration on the human capital of natives, *Journal of Labor Economics* **7**(4): 464-486.

Haupt, A., Peters, W., 1998, Public pensions and voting on immigration, *Public Choice* **95**: 403-413.

Kemnitz, A., 2002, On the political economy of low skilled immigration and the welfare state, *International Tax and Public Finance* **9**: 423-434.

LaLonde, R.J., Topel, R.H., 1997, Economic impact of international migration and the economic performance of migrants, in Rosenzweig, M.R., Stark, O. (eds.), *Handbook of Population and Family Economics*, Vol.1B, Chapter 14, Amsterdam: Elsevier Science.

Razin, A., Sadka, E., 2000, Unskilled migration: a burden or a boon for the welfare state?, *Scandinavian Journal of Economics* **102**(3): 463-479.

Scheve, K.F., Slaughter, M.J., 2001, Labor market competition and individual preferences over immigration policy, *Review of Economics and Statistics* **83**(1): 133-145.

Scholten, U., Thum, M., 1996, Public pensions and immigration policy in a democracy, *Public Choice* **87**: 347-361.

Appendix

1. Proof of lemma 1

No income support for low-skilled workers means $\theta = 0$. Expression (14') then reduces to

$$\widetilde{e}_t\left(m_t, h_t\left(m_t\right)\right) := \left(1-\tau\right)\left[w_t^H\left(m_t, h_t\left(m_t\right)\right) - w_t^L\left(m_t, h_t\left(m_t\right)\right)\right].$$

After the substitution of equations (4) and (5) into it and rearrangement, we obtain

$$\widetilde{e}_t\left(m_t, h_t\left(m_t\right)\right) = \left(1-\tau\right)\left\{\alpha\left[\frac{\left(1-h_t\left(m_t\right)\right)\left(1+m_t\right)}{h_t\left(m_t\right)}\right] - \left(1-\alpha\right)\right\}\left[\frac{h_t\left(m_t\right)}{\left(1-h_t\left(m_t\right)\right)\left(1+m_t\right)}\right]^{\alpha}.$$

By partially differentiating $\widetilde{e}_t$ with respect to immigration, we obtain

$$\frac{\partial \widetilde{e}_t}{\partial m_t} = \frac{\left(1-\tau\right)\alpha\left(1-\alpha\right)}{1+m_t}\left[1+\frac{\left(1-h_t\left(m_t\right)\right)\left(1+m_t\right)}{h_t\left(m_t\right)}\right]\left[\frac{h_t\left(m_t\right)}{\left(1-h_t\left(m_t\right)\right)\left(1+m_t\right)}\right]^{\alpha} > 0.$$

With regard to the proportion of high-skilled workers, we have

$$\frac{\partial \widetilde{e}_t}{\partial h_t\left(m_t\right)} = -\frac{\left(1-\tau\right)\alpha\left(1-\alpha\right)}{h_t\left(m_t\right)\left(1-h_t\left(m_t\right)\right)}\left[1+\frac{\left(1-h_t\left(m_t\right)\right)\left(1+m_t\right)}{h_t\left(m_t\right)}\right]\left[\frac{h_t\left(m_t\right)}{\left(1-h_t\left(m_t\right)\right)\left(1+m_t\right)}\right]^{\alpha} < 0.$$

The total differentiation of h_t with respect to m_t is

$$\frac{dh_t}{dm_t} = \frac{dh_t}{d\widetilde{e}_t}\left(\frac{\partial \widetilde{e}_t}{\partial m_t} + \frac{\partial \widetilde{e}_t}{\partial h_t}\frac{dh_t}{dm_t}\right)$$

or

$$\frac{dh_t}{dm_t} = \frac{dh_t}{d\widetilde{e}_t}\frac{\partial \widetilde{e}_t}{\partial m_t}\left(1-\frac{dh_t}{d\widetilde{e}_t}\frac{\partial \widetilde{e}_t}{\partial h_t}\right)^{-1} \tag{A.1.1}$$

where $\frac{dh_t}{d\widetilde{e}_t} = \frac{1}{\bar{e}} > 0$ from definition (1). Since we have $\frac{\partial \widetilde{e}_t}{\partial m_t} > 0$ and $\frac{\partial \widetilde{e}_t}{\partial h_t} < 0$ unambiguously, we conclude that $\frac{dh_t}{dm_t} > 0$. By substitution and rearrangement, we obtain

$$\frac{dh_t}{dm_t} = \phi_t \frac{h_t\left(1-h_t\right)}{1+m_t} \tag{A.1.2}$$

where

$$\phi_t := \frac{(1-\tau)\left[1+\frac{(1-h_t)(1+m_t)}{h_t}\right]\left[\frac{h_t}{(1-h_t)(1+m_t)}\right]^{\alpha}}{\overline{e}\frac{h_t(1-h_t)}{\alpha(1-\alpha)}+(1-\tau)\left[1+\frac{(1-h_t)(1+m_t)}{h_t}\right]\left[\frac{h_t}{(1-h_t)(1+m_t)}\right]^{\alpha}} \in (0,1). \quad \text{(A.1.3)}$$

Since $h_t \in (0,1)$, we have $\frac{dh_t}{dm_t} \in (0,1)$. ∎

2. Proof of lemma 2

When there is income support for low-skilled workers, $\theta > 0$. Then, full expression (14) applies. It can be rearranged as follows:

$$\begin{aligned}\widetilde{e}_t\left(m_t, h_t\left(m_t\right)\right) &= \left\{(1-\tau)\left[\frac{h_t\left(m_t\right)}{\left(1-h_t\left(m_t\right)\right)\left(1+m_t\right)}\right]^{\alpha}-\theta\right\} \\ &\times\left\{\alpha\left[\frac{\left(1-h_t\left(m_t\right)\right)\left(1+m_t\right)}{h_t\left(m_t\right)}\right]-(1-\alpha)\right\}-\theta.\end{aligned}$$

By partially differentiating $\widetilde{e}_t$ with respect to m_t, we obtain

$$\begin{aligned}\frac{\partial\widetilde{e}_t}{\partial m_t} &= \alpha\left(\frac{1-h_t\left(m_t\right)}{h_t\left(m_t\right)}\right) \\ &\times\left\{(1-\tau)(1-\alpha)\left[1+\frac{h_t\left(m_t\right)}{\left(1-h_t\left(m_t\right)\right)\left(1+m_t\right)}\right]\left[\frac{h_t\left(m_t\right)}{\left(1-h_t\left(m_t\right)\right)\left(1+m_t\right)}\right]^{\alpha}-\theta\right\}.\end{aligned}$$

Hence $\frac{\partial\widetilde{e}_t}{\partial m_t} > 0$ if

$$\theta < (1-\tau)(1-\alpha)\left[1+\frac{h_t\left(m_t\right)}{\left(1-h_t\left(m_t\right)\right)\left(1+m_t\right)}\right]\left[\frac{h_t\left(m_t\right)}{\left(1-h_t\left(m_t\right)\right)\left(1+m_t\right)}\right]^{\alpha}. \quad \text{(A.2.1)}$$

By partially differentiating $\widetilde{e}_t$ with respect to h_t, we obtain

$$\begin{aligned}\frac{\partial\widetilde{e}_t}{\partial h_t\left(m_t\right)} &= \alpha\left(\frac{1+m_t}{\left(h_t\left(m_t\right)\right)^2}\right) \\ &\times\left\{\theta-(1-\tau)(1-\alpha)\left[1+\frac{h_t\left(m_t\right)}{\left(1-h_t\left(m_t\right)\right)\left(1+m_t\right)}\right]\left[\frac{h_t\left(m_t\right)}{\left(1-h_t\left(m_t\right)\right)\left(1+m_t\right)}\right]^{\alpha}\right\}.\end{aligned}$$

Therefore, $\frac{\partial\widetilde{e}_t}{\partial h_t(m_t)} < 0$ if the condition (A.2.1) holds. The sign of expression (A.1.1) in this case depends on whether the condition (A.2.1) holds or not. Suppose that it holds with zero immigration, i.e.,

$$\theta < (1-\tau)(1-\alpha)\left(1+\frac{h_t(0)}{1-h_t(0)}\right)\left(\frac{h_t(0)}{1-h_t(0)}\right)^{\alpha}. \quad \text{(A.2.1')}$$

This is what we mean by saying that the size of per capita income support is not "excessively large", i.e., assumption (A) in the proposition. We then have $\frac{dh_t}{dm_t} > 0$ for $m_t = 0$.

However, as the level of immigration rises, the condition (A.2.1) would be violated if the maximum feasible rate of immigration is "sufficiently high", i.e., assumption (B). When

$$\theta = (1-\tau)(1-\alpha)\left[1 + \frac{h_t(m_t)}{(1-h_t(m_t))(1+m_t)}\right]\left[\frac{h_t(m_t)}{(1-h_t(m_t))(1+m_t)}\right]^{\alpha}, \tag{A.2.1"}$$

we have $\frac{dh_t}{dm_t} = 0$. When

$$\begin{aligned} & (1-\tau)(1-\alpha)\left[1 + \frac{h_t(m_t)}{(1-h_t(m_t))(1+m_t)}\right]\left[\frac{h_t(m_t)}{(1-h_t(m_t))(1+m_t)}\right]^{\alpha} \\ < \; & \theta \\ < \; & (1-\tau)(1-\alpha)\left[1 + \frac{h_t(m_t)}{(1-h_t(m_t))(1+m_t)}\right]\left[\frac{h_t(m_t)}{(1-h_t(m_t))(1+m_t)}\right]^{\alpha} \\ & + \frac{\overline{e}(h_t(m_t))^2}{\alpha(1+m_t)}, \end{aligned}$$

we have $\frac{dh_t}{dm_t} < 0$. By assuming "sufficiently high" $\overline{e}$, i.e., assumption (C), we guarantee that

$$\begin{aligned} \theta \; < \; & (1-\tau)(1-\alpha)\left[1 + \frac{h_t(m_t)}{(1-h_t(m_t))(1+m_t)}\right]\left[\frac{h_t(m_t)}{(1-h_t(m_t))(1+m_t)}\right]^{\alpha} \\ & + \frac{\overline{e}(h_t(m_t))^2}{\alpha(1+m_t)} \end{aligned}$$

always holds. We thus conclude that, if $\theta > 0$, and if assumptions (A), (B) and (C) hold, h_t is initially increasing but subsequently decreasing in m_t. By substitution and rearrangement, we obtain

$$\frac{dh_t}{dm_t} = \psi_t \frac{h_t(1-h_t)}{1+m_t} \tag{A.2.2}$$

where

$$\psi_t := \frac{(1-\tau)(1-\alpha)\left[1 + \frac{h_t}{(1-h_t)(1+m_t)}\right]\left[\frac{h_t}{(1-h_t)(1+m_t)}\right]^{\alpha} - \theta}{\overline{e}\frac{h_t^2}{\alpha(1+m_t)} + (1-\tau)(1-\alpha)\left[1 + \frac{h_t}{(1-h_t)(1+m_t)}\right]\left[\frac{h_t}{(1-h_t)(1+m_t)}\right]^{\alpha} - \theta} \in (-1, 1), \tag{A.2.3}$$

by assuming sufficiently high $\overline{e}$. Since $h_t \in (0,1)$, we have $\frac{dh_t}{dm_t} \in (-1,1)$. ■

3. Proof of lemma 3

From equation (4), the total differentiation of the high-skilled wage rate with respect to immigration gives

$$\frac{dw_t^H}{dm_t} = \frac{\partial w_t^H}{\partial m_t} + \frac{\partial w_t^H}{\partial h_t}\frac{dh_t}{dm_t}$$

where $\theta = 0$ implies $\frac{dh_t}{dm_t} > 0$ by lemma 1. After some manipulation, we obtain

$$\frac{dw_t^H}{dm_t} = (1-\phi_t)\frac{\alpha(1-\alpha)}{1+m_t}\left[\frac{(1-h_t)(1+m_t)}{h_t}\right]^{1-\alpha} > 0$$

where ϕ_t is defined in (A.1.3).

On the other hand, from equation (5), totally differetiating the low-skilled wage rate with respect to immigration gives

$$\frac{dw_t^L}{dm_t} = -(1-\phi_t)\frac{\alpha(1-\alpha)}{1+m_t}\left[\frac{h_t}{(1-h_t)(1+m_t)}\right]^{\alpha} < 0.$$

Hence w_t^H is monotonically increasing while w_t^L is monotonically decreasing in m_t if $\theta = 0$. ■

4. Proof of lemma 4

If $\theta > 0$ and assumption (A), (B) and (C) of lemma 2 holding, the sign of $\frac{dh_t}{dm_t} \in (-1,1)$ is initially positive but subsequently negative by lemma 2. For high-skilled labour, we have

$$\frac{dw_t^H}{dm_t} = (1-\psi_t)\frac{\alpha(1-\alpha)}{1+m_t}\left[\frac{(1-h_t)(1+m_t)}{h_t}\right]^{1-\alpha}$$

where ψ_t is defined in (A.2.3). Since $\psi_t \in (-1,1)$, we have $\frac{dw_t^H}{dm_t} > 0$. For low-skilled worker, we have

$$\frac{dw_t^L}{dm_t} = -(1-\psi_t)\frac{\alpha(1-\alpha)}{1+m_t}\left[\frac{h_t}{(1-h_t)(1+m_t)}\right]^{\alpha} < 0,$$

since $\psi_t \in (-1,1)$. ■

5. Proof of lemma 5

If $\theta > 0$ and assumption (A), (B) and (C) of lemma 2 holding, the sign of $\frac{dh_t}{dm_t} \in (-1, 1)$ is initially positive but subsequently negative by lemma 2. From equation (6), the total derivative of the tax rate for intragenerational transfer with respect to immigration is

$$\frac{d\mu_t}{dm_t} = (1 - \psi_t) \frac{\alpha\theta}{1 + m_t} \left[\frac{(1 - h_t)(1 + m_t)}{h_t} \right]^{\alpha}$$

where ψ_t is defined in (A.2.3). Since $\psi_t \in (-1, 1)$, we have $\frac{d\mu_t}{dm_t} > 0$. ■

6. Proof of lemma 6

From equation (7), the total derivative of the per capita PAYG pension benefit with respect to immigration is

$$\frac{db_t}{dm_t} = \tau (1 + \delta) \left[\frac{(1 - h_t)(1 + m_t)}{h_t} \right]^{1-\alpha} \left(\frac{(1 - \alpha) h_t}{1 + m_t} + \frac{\alpha - h_t}{1 - h_t} \frac{dh_t}{dm_t} \right)$$

where $\frac{dh_t}{dm_t} > 0$ by lemma 1 when $\theta = 0$. However, the sign of $\alpha - h_t$ is ambiguous. After some manipulation, we obtain

$$\frac{db_t}{dm_t} = \left(1 + \phi_t \frac{\alpha - h_t}{1 - \alpha} \right) \frac{(1 - \alpha) \tau (1 + \delta) h_t}{1 + m_t} \left[\frac{(1 - h_t)(1 + m_t)}{h_t} \right]^{1-\alpha} \quad \text{(A.6.1)}$$

where ϕ_t is defined in (A.1.3). Since we have

$$1 + \phi_t \frac{\alpha - h_t}{1 - \alpha} = \frac{\bar{e}\frac{h_t(1-h_t)}{\alpha(1-\alpha)} + (1 - \tau) \left[1 + \frac{(1-h_t)(1+m_t)}{h_t} \right] \left[\frac{h_t}{(1-h_t)(1+m_t)} \right]^{\alpha} \left(\frac{1-h_t}{1-\alpha} \right)}{\bar{e}\frac{h_t(1-h_t)}{\alpha(1-\alpha)} + (1 - \tau) \left[1 + \frac{(1-h_t)(1+m_t)}{h_t} \right] \left[\frac{h_t}{(1-h_t)(1+m_t)} \right]^{\alpha}} > 0,$$

we unconditionally have $\frac{db_t}{dm_t} > 0$. ■

7. Proof of lemma 7

When $\theta > 0$, ϕ_t in (A.6.1) has to be replaced by ψ_t defined by (A.2.3). Since

$$1+\psi_t \frac{\alpha - h_t}{1-\alpha} = \frac{\frac{\bar{e}h_t^2}{\alpha(1+m_t)} + \left\{ (1-\tau)(1-\alpha) \left[1 + \frac{h_t}{(1-h_t)(1+m_t)} \right] \left[\frac{h_t}{(1-h_t)(1+m_t)} \right]^\alpha - \theta \right\} \frac{1-h_t}{1-\alpha}}{\frac{\bar{e}h_t^2}{\alpha(1+m_t)} + (1-\tau)(1-\alpha) \left[1 + \frac{h_t}{(1-h_t)(1+m_t)} \right] \left[\frac{h_t}{(1-h_t)(1+m_t)} \right]^\alpha - \theta},$$

assumption (C) of lemma 2 can assure $\frac{db}{dm} > 0$. ■

8. Proof of lemma 10

When immigration is temporary, if $\theta > 0$, (16') reduces to

$$\begin{aligned} e^* (m, h(m)) \quad : &= [1 - \tau - \mu (m, h(m))] \, w^H (m, h(m)) \\ &\quad - [1 - \tau - \mu (0, h(0))] \, w^L (0, h(0)) - \theta. \end{aligned}$$

Substituting (4) and (6) into it, we obtain

$$\begin{aligned} e^* (m, h(m)) &= \alpha \frac{(1-h)(1+m)}{h} \left\{ (1-\tau) \left[\frac{h}{(1-h)(1+m)} \right]^\alpha - \theta \right\} \\ &\quad - [1 - \tau - \mu (0, h(0))] \, w^L (0, h(0)) - \theta. \end{aligned}$$

By partially differentiating e^* with respect to m, we obtain

$$\frac{\partial e^*}{\partial m} = \alpha \frac{1-h}{h} \left\{ (1-\tau)(1-\alpha) \left[\frac{h}{(1-h)(1+m)} \right]^\alpha - \theta \right\}.$$

With regard to h, we have

$$\frac{\partial e^*}{\partial h} = \alpha \frac{1+m}{h^2} \left\{ \theta - (1-\tau)(1-\alpha) \left[\frac{h}{(1-h)(1+m)} \right]^\alpha \right\}.$$

The total differentiation of e^* with respect to m is, by using $\frac{dh}{dm}$ from (A.2.2),

$$\frac{de^*}{dm} = (1-\psi) \, \alpha \frac{1-h}{h} \left\{ (1-\tau)(1-\alpha) \left[\frac{h}{(1-h)(1+m)} \right]^\alpha - \theta \right\}$$

where $\psi \in (-1, 1)$ is defined in (A.2.3). Since $\frac{dh^*}{de^*} = \frac{1}{\bar{e}} > 0$ by definition (17), we have $\frac{dh^*}{dm} > 0$ if

$$\theta < (1-\tau)(1-\alpha) \left[\frac{h}{(1-h)(1+m)} \right]^\alpha . \tag{A.8.1}$$

We assume

$$\theta < (1-\tau)(1-\alpha)\left(\frac{h(0)}{1-h(0)}\right)^{\alpha}, \tag{A.8.1'}$$

which requires a smaller value of θ than condition (A.2.1') does. Thus, if $\frac{dh^*}{dm}$ is to be initially positive, (A.8.1') should replace (A.2.1').

Suppose (A.8.1') holding. As m increases, we subsequently have

$$\begin{aligned}\theta &= (1-\tau)(1-\alpha)\left[\frac{h}{(1-h)(1+m)}\right]^{\alpha} \\ &< (1-\tau)(1-\alpha)\left[1+\frac{h}{(1-h)(1+m)}\right]\left[\frac{h}{(1-h)(1+m)}\right]^{\alpha}\end{aligned}$$

where (A.8.1) implies that h^* is at its peak, while (A.2.1) implies that h still continues to increase. As m further increases, we have

$$\theta = (1-\tau)(1-\alpha)\left[1+\frac{h}{(1-h)(1+m)}\right]\left[\frac{h}{(1-h)(1+m)}\right]^{\alpha}$$

where (A.8.1) implies that h^* is decreasing, while (A.2.1) implies that h is at its peak. ■

9. Proof of lemma 11

When immigration is permanent, if $\theta > 0$, full expression (16') applies. By partially differentiating e^* with respect to m in (16), we obtain

$$\begin{aligned}\frac{\partial e^*}{\partial m} &= \alpha\frac{1-h}{h}\left\{(1-\tau)(1-\alpha)\left[\frac{h}{(1-h)(1+m)}\right]^{\alpha}-\theta\right\} \\ &\quad +\tau\frac{1+\delta}{1+r}(1-\alpha)(1-h)\left[\frac{h}{(1-h)(1+m)}\right]^{\alpha}.\end{aligned}$$

With regard to h, we have

$$\begin{aligned}\frac{\partial e^*}{\partial h} &= \alpha\frac{1+m}{h^2}\left\{\theta-(1-\tau)(1-\alpha)\left[\frac{h}{(1-h)(1+m)}\right]^{\alpha}\right\} \\ &\quad +\tau\frac{1+\delta}{1+r}(\alpha-h)\frac{1+m}{h}\left[\frac{h}{(1-h)(1+m)}\right]^{\alpha}.\end{aligned}$$

The total differentiation of e^* with respect to m is, by using $\frac{dh}{dm}$ from (A.2.2),

$$\begin{aligned}\frac{de^*}{dm} &= (1-\psi)\,\alpha\frac{1-h}{h}\left\{(1-\tau)(1-\alpha)\left[\frac{h}{(1-h)(1+m)}\right]^{\alpha}-\theta\right\}\\ &\quad +\tau\frac{1+\delta}{1+r}\left(1+\frac{\alpha-h}{1-\alpha}\psi\right)(1-\alpha)(1-h)\left[\frac{h}{(1-h)(1+m)}\right]^{\alpha}\end{aligned}$$

where $1+\frac{\alpha-h}{1-\alpha}\psi>0$ by assumption (C), as shown in appendix 7 for the proof of lemma 7. Since $\frac{dh^*}{de^*}=\frac{1}{\bar{e}}>0$ by definition (17), we have $\frac{dh^*}{dm}>0$ if

$$\begin{aligned}\theta &< (1-\tau)(1-\alpha)\left[\frac{h}{(1-h)(1+m)}\right]^{\alpha}\\ &\quad\times\left\{1+\frac{\tau}{1-\tau}\frac{1+\delta}{1+r}\frac{1-\alpha}{\alpha}\left(1+\frac{\alpha-h}{1-\alpha}\psi\right)\frac{h}{1-\psi}\right\}.\end{aligned}\tag{A.9.1}$$

We assume

$$\begin{aligned}\theta &< (1-\tau)(1-\alpha)\left(\frac{h(0)}{1-h(0)}\right)^{\alpha}\\ &\quad\times\left\{1+\frac{\tau}{1-\tau}\frac{1+\delta}{1+r}\frac{1-\alpha}{\alpha}\left(1+\frac{\alpha-h(0)}{1-\alpha}\psi(0,h(0))\right)\frac{h(0)}{1-\psi(0,h(0))}\right\},\end{aligned}\tag{A.9.1'}$$

which requires a smaller value of θ than condition (A.2.1') does, but a larger value of it than condition (A.8.1') does. Thus, if $\frac{dh^*}{dm}$ is to be initially positive, (A.9.1') should replace (A.2.1').

Suppose (A.9.1') holding. As m increases, we then have

$$\begin{aligned}\theta &= (1-\tau)(1-\alpha)\left\{1+\frac{\tau}{1-\tau}\frac{1+\delta}{1+r}\frac{1-\alpha}{\alpha}\left(1+\frac{\alpha-h}{1-\alpha}\psi\right)\frac{h}{1-\psi}\right\}\left[\frac{h}{(1-h)(1+m)}\right]^{\alpha}\\ &< (1-\tau)(1-\alpha)\left[1+\frac{h}{(1-h)(1+m)}\right]\left[\frac{h}{(1-h)(1+m)}\right]^{\alpha}\end{aligned}$$

where (A.9.1) implies that h^* is at its peak, while (A.2.1) implies that h still continues to increase. As m further increases, we have

$$\theta=(1-\tau)(1-\alpha)\left[1+\frac{h}{(1-h)(1+m)}\right]\left[\frac{h}{(1-h)(1+m)}\right]^{\alpha}$$

where (A.9.1) implies that h^* is decreasing, while (A.2.1) implies that h is at its peak. ■

10. Proof of proposition 2

No income support for low-skilled workers means $\theta = 0$. Then, $\frac{dh}{dm} > 0$ by lemma 1. Also $\frac{dh^*}{dm} > 0$ by lemma 8 when immigration policy is temporary. Under scenario II, $(1+\delta) h^* (0, h(0)) + 1 < (1+\delta)(1 - h^* (0, h(0)))$.

Outcome (i): If $\exists \ddot{m} \in [0, \overline{m}]$, $(1+\delta)(1 - h^* (\ddot{m}, h(\ddot{m}))) = (1+\delta) h^* (\ddot{m}, h(\ddot{m})) + 1$, and if $\frac{dh^*}{dm} > 0$, then $\forall m \in (\ddot{m}, \overline{m}]$, $(1+\delta)(1 - h^* (m, h(m))) < (1+\delta) h^* (m, h(m)) + 1$. That is, there exists a range of policy over which pensioners, currently high-skilled workers and some currently low-skilled workers form the majority. The lifetime income of these currently low-skilled workers who join currently high-skilled workers and pensioners is identified by z^i for the high-skilled over $(\ddot{m}, \overline{m}]$. Then, condition (15) implies $m \in (\ddot{m}, \overline{m}]$ beats $m \in [0, \ddot{m}]$. When immigration policy is temporary with $\theta = 0$, expression (10) for the high-skilled reduces to $z^i := (1-\tau) w^H - e^i$. Since $\theta = 0$, $\frac{dw^H}{dm} > 0$ by lemma 3 and hence $\frac{dz^i}{dm} > 0$ for the high-skilled. Also $\frac{db}{dm} > 0$ by lemma 6. Objectives (18) and (20) then imply that everyone in the majority monotonically increases her/his utility as immigration increases. The Condorcet winner is thus $\overline{m}$.

Outcome (ii): If $\nexists \ddot{m} \in [0, \overline{m}]$, $(1+\delta)(1 - h^* (\ddot{m}, h(\ddot{m}))) = (1+\delta) h^* (\ddot{m}, h(\ddot{m})) + 1$, then $\forall m \in [0, \overline{m}]$, $(1+\delta) h^* (m, h(m)) + 1 < (1+\delta)(1 - h^* (m, h(m)))$ under scenario II. When immigration policy is temporary with $\theta = 0$, expression (10) for the low-skilled reduces to $z^i := (1-\tau) w^L$. Since $\theta = 0$, $\frac{dw^L}{dm} < 0$ by lemma 3 and hence $\frac{dz^i}{dm} < 0$ for the low-skilled. Objective (18) then implies that the Condorcet winner is $m = 0$. ■

11. Proof of lemma 12

When immigration policy is permanent, if $\theta = 0$, expression (10) for the low-skilled reduces to

$$z^i (m, h(m)) := (1-\tau) w^L (m, h(m)) + \frac{b(m, h(m))}{1+r}.$$

After substituting (5) and (7) into it, we partially differentiate z^i for the low-skilled to obtain ,

$$\frac{\partial z^i}{\partial m} = \frac{1-\alpha}{1+m}\left[\tau\frac{1+\delta}{1+r}(1-h)(1+m)-(1-\tau)\alpha\right]\left[\frac{h}{(1-h)(1+m)}\right]^{\alpha}$$

and

$$\frac{\partial z^i}{\partial h} = \left[\tau\frac{1+\delta}{1+r}\left(\frac{(\alpha-h)(1+m)}{h}\right)+(1-\tau)\frac{\alpha(1-\alpha)}{h(1-h)}\right]\left[\frac{h}{(1-h)(1+m)}\right]^{\alpha}.$$

The total differentiation of z^i for the low-skilled with respect to m is then, by using $\frac{dh}{dm}$ from (A.1.2),

$$\begin{aligned}\frac{dz^i}{dm} &= (1-\alpha)\left[\frac{h}{(1-h)(1+m)}\right]^{\alpha} \\ &\quad\times\left[\tau\frac{1+\delta}{1+r}\left(1+\phi\frac{\alpha-h}{1-\alpha}\right)(1-h)-(1-\tau)(1-\phi)\frac{\alpha}{1+m}\right].\end{aligned}$$

Its sign is ambiguous due to the expression in the second square brackets. It is negative if

$$\tau < \frac{\alpha(1-\phi)}{\alpha(1-\phi)+\frac{1+\delta}{1+r}\left(1+\phi\frac{\alpha-h}{1-\alpha}\right)(1-h)(1+m)}. \tag{A.11.1}$$

We assume

$$\tau < \frac{\alpha(1-\phi(0,h(0)))}{\alpha(1-\phi(0,h(0)))+\frac{1+\delta}{1+r}\left(1+\phi(0,h(0))\frac{\alpha-h(0)}{1-\alpha}\right)(1-h(0))}. \tag{A.11.1'}$$

In other words, we assume $\frac{dz^i}{dm}<0$ for the low-skilled at the pre-voting stage. This is what we mean by "not excessively high" τ, i.e., assumption (D). As m increases, however, the sign of $\frac{dz^i}{dm}$ for the low-skilled becomes positive if $\overline{m}$ is "sufficiently high", i.e., assumption (B). As we observed in appendix 1, $\frac{dh}{dm}\in(0,1)$ and $\phi\in(0,1)$. Hence the denominator on the right hand side of the inequality sign in (A.11.1) increases as m increases, eventually leading to $\frac{dz^i}{dm}=0$ and $\frac{dz^i}{dm}>0$ for the low-skilled. ■

12. Proof of proposition 4

No income support for low-skilled workers means $\theta = 0$. Then, $\frac{dh}{dm} > 0$ by lemma 1. Also $\frac{dh^*}{dm} > 0$ by lemma 9 when immigration policy is permanent. Expression (10) for the low-skilled reduces to $z^i := (1 - \tau) w^L + \frac{b}{1+r}$. Lemma 12 indicates that it is initially decreasing but subsequently increasing in m. For the high-skilled, $z^i := (1 - \tau) w^H - e^i + \frac{b}{1+r}$. Lemmata 3 and 6 then imply $\frac{dz^i}{dm} > 0$ for the high-skilled. Objectives (18) and (20) suggest that currently high-skilled workers and pensioners most prefer $\overline{m}$. Under scenario II, $(1 + \delta) h^* (0, h (0)) + 1 < (1 + \delta) (1 - h^* (0, h (0)))$.

Outcome (i): If condition (22) holds, even those who remain low-skilled over $[0, \overline{m}]$ most prefer $\overline{m}$. All natives then vote for $\overline{m}$, and the Condorcet winner is $\overline{m}$.

When condition (22) does not hold, if $\exists \ddot{m} \in [0, \overline{m}]$, $(1 + \delta) (1 - h^* (\ddot{m}, h (\ddot{m}))) = (1 + \delta) h^* (\ddot{m}, h (\ddot{m})) + 1$, and if $\frac{dh^*}{dm} > 0$, then $\forall m \in (\ddot{m}, \overline{m}]$, $(1 + \delta) (1 - h^* (m, h (m))) < (1 + \delta) h^* (m, h (m)) + 1$. That is, there exists a range of policy over which pensioners, currently high-skilled workers and some currently low-skilled workers form the majority. The lifetime income of those currently low-skilled workers who join currently high-skilled workers and pensioners is identified by z^i for the high-skilled over $(\ddot{m}, \overline{m}]$. Then, condition (15) implies $m \in (\ddot{m}, \overline{m}]$ beats $m \in [0, \ddot{m}]$. The utility of everyone in the majority monotonically increases in m over $(\ddot{m}, \overline{m}]$. Hence the Condorcet winner is $\overline{m}$.

Outcome (ii): If $\nexists \ddot{m} \in [0, \overline{m}]$, $(1 + \delta) (1 - h^* (\ddot{m}, h (\ddot{m}))) = (1 + \delta) h^* (\ddot{m}, h (\ddot{m})) + 1$, then $\forall m \in [0, \overline{m}]$, $(1 + \delta) h^* (m, h (m)) + 1 < (1 + \delta) (1 - h^* (m, h (m)))$ under scenario II. If condition (22) does not hold, objective (18) then implies that everyone in the majority most prefers $m = 0$. ■

13. Proof of lemma 13

When immigration policy is temporary, if $\theta > 0$, expression (10) for the high-skilled reduces to

$$z^i (m, h (m)) := [1 - \tau - \mu (m, h (m))] w^H (m, h (m)) - e^i.$$

By substituting (4) and (6) into it, we obtain

$$z^i(m, h(m)) = (1-\tau)\alpha\left[\frac{(1-h)(1+m)}{h}\right]^{1-\alpha} - \alpha\theta\frac{(1-h)(1+m)}{h} - e^i.$$

By partially differentiating it, we have

$$\frac{\partial z^i}{\partial m} = \alpha\frac{1-h}{h}\left\{(1-\tau)(1-\alpha)\left[\frac{h}{(1-h)(1+m)}\right]^{\alpha} - \theta\right\}$$

and

$$\frac{\partial z^i}{\partial h} = \alpha\frac{1+m}{h^2}\left\{\theta - (1-\tau)(1-\alpha)\left[\frac{h}{(1-h)(1+m)}\right]^{\alpha}\right\}.$$

The total differentiation of z^i for the high-skilled with respect to m is then, by using $\frac{dh}{dm}$ in (A.2.2),

$$\frac{dz^i}{dm} = (1-\psi)\alpha\frac{1-h}{h}\left\{(1-\tau)(1-\alpha)\left[\frac{h}{(1-h)(1+m)}\right]^{\alpha} - \theta\right\} \quad \text{(A.13.1)}$$

where $\psi \in (-1, 1)$ if $\overline{e}$ is "sufficiently high", i.e., assumption (C). Its sign is positive if condition (A.8.1) holds. We assume condition (A.8.1') holds. Then, since (A.8.1') requires a smaller θ than (A.2.1'), the satisfaction of (A.8.1') is what we mean by "not excessively large" θ, i.e., assumption (A). If $\overline{m}$ is "sufficiently high", i.e., assumption (B), we eventually have $\frac{dz^i}{dm} = 0$ and then $\frac{dz^i}{dm} < 0$ for the high-skilled.

For the low-skilled,

$$z^i(m, h(m)) := [1-\tau-\mu(m, h(m))]\, w^L(m, h(m)) + \theta.$$

By substituting (5) and (6) into it, we obtain

$$z^i(m, h(m)) = (1-\tau)(1-\alpha)\left[\frac{h}{(1-h)(1+m)}\right]^{\alpha} + \alpha\theta.$$

The total differentiation of this z^i with respect to m gives

$$\frac{dz^i}{dm} = -(1-\psi)\frac{(1-\tau)\alpha(1-\alpha)}{1+m}\left[\frac{h}{(1-h)(1+m)}\right]^{\alpha} < 0,$$

as long as assumptions (A), (B) and (C) of lemma 2 hold. Notice that $\frac{dz^i}{dm} = (1-\tau)\frac{dw^L}{dm}$ where $\frac{dw^L}{dm}$ is shown in appendix 4 for the proof of lemma 4. This indicates that immigration does not affect z^i for the low-skilled through the income support programme. ■

14. Proof of proposition 6

Lemmata 10 and 13 imply that it is not only z^i for the high-skilled but also h^* which experiences its peak at $\hat{m}$. Under scenario IV, $(1+\delta) h^* (0, h(0)) + 1 < (1+\delta)(1 - h^* (0, h(0)))$.

Outcome (i): First, lemma 13 implies $\frac{dz^i}{dm} < 0$ for all workers over $(\hat{m}, \overline{m}]$. Since $\delta > 0$, they form the majority, and $\hat{m} \succ m \in (\hat{m}, \overline{m}]$. Second, if $\exists \ddot{m} \in (0, \hat{m})$, $(1+\delta)(1 - h^* (\ddot{m}, h(\ddot{m}))) = (1+\delta) h^* (\ddot{m}, h(\ddot{m})) + 1$, lemma 10 implies $\forall m \in (\ddot{m}, \hat{m}]$, $(1+\delta)(1 - h^* (m, h(m))) < (1+\delta) h^* (m, h(m)) + 1$. Definition (16) then implies that workers in the majority over $(\ddot{m}, \hat{m}]$ experience higher lifetime income with $m \in (\ddot{m}, \hat{m}]$ than with $m \in [0, \ddot{m}]$. Lemmata 7 and 13 imply that everyone in the majority most prefers $\hat{m}$ over $(\ddot{m}, \hat{m}]$. Hence the Condorcet winner is $\hat{m}$.

Outcome (ii): If $\nexists \ddot{m} \in (0, \hat{m})$, $(1+\delta)(1 - h^* (\ddot{m}, h(\ddot{m}))) = (1+\delta) h^* (\ddot{m}, h(\ddot{m})) + 1$, then $\forall m \in [0, \overline{m}]$, $(1+\delta) h^* (m, h(m)) + 1 \leq (1+\delta)(1 - h^* (m, h(m)))$. Since $\frac{dz^i}{dm} < 0$ for the low-skilled over $[0, \overline{m}]$ by lemma 13, the Condorcet winner is $m = 0$. ■

15. Proof of lemma 14

When immigration policy is permanent, if $\theta > 0$, by substituting (4), (6) and (7), expression (10) for the high-skilled can be reexpressed as follows:

$$\begin{aligned} z^i (m, h(m)) &= \left[(1-\tau)\alpha + \tau \frac{1+\delta}{1+r} h \right] \left[\frac{(1-h)(1+m)}{h} \right]^{1-\alpha} \\ &\quad -\alpha\theta \frac{(1-h)(1+m)}{h} - e^i. \end{aligned}$$

By partially differentiating it, we have

$$\begin{aligned} \frac{\partial z^i}{\partial m} &= \frac{1-\alpha}{1+m} \left[(1-\tau)\alpha + \tau \frac{1+\delta}{1+r} h \right] \left[\frac{(1-h)(1+m)}{h} \right]^{1-\alpha} \\ &\quad -\alpha\theta \frac{1-h}{h} \end{aligned}$$

and

$$\frac{\partial z^i}{\partial h} = \left[\tau\left(\frac{1+\delta}{1+r}\right)\frac{\alpha-h}{1-h}-(1-\tau)\frac{\alpha(1-\alpha)}{h(1-h)}\right]\left[\frac{(1-h)(1+m)}{h}\right]^{1-\alpha} +\alpha\theta\frac{1+m}{h^2}.$$

The total differentiation of z^i for the high-skilled with respect to m is then, by using (A.2.2),

$$\frac{dz^i}{dm} = \frac{1-\alpha}{1+m}\left[\tau\frac{1+\delta}{1+r}h\left(1+\psi\frac{\alpha-h}{1-\alpha}\right)+(1-\psi)(1-\tau)\alpha\right]\left[\frac{(1-h)(1+m)}{h}\right]^{1-\alpha} -(1-\psi)\alpha\theta\frac{1-h}{h} \quad \text{(A.15.1)}$$

where $\psi \in (-1,1)$ if $\overline{e}$ is "sufficiently high", i.e., assumption (C). Its sign is positive if condition (A.9.1) holds. We assume that condition (A.9.1') holds. Then, since (A.9.1') requires a smaller θ than (A.2.1'), the satisfaction of (A.9.1') is what we mean by "not excessively large" θ, i.e., assumption (A). If $\overline{m}$ is "sufficiently high", i.e., assumption (B), we eventually have $\frac{dz^i}{dm} = 0$ and then $\frac{dz^i}{dm} < 0$ for the high-skilled.

For the low-skilled, by substituting expressions (5), (6) and (7) into (10), we obtain

$$z^i(m,h(m)) = (1-\tau)(1-\alpha)\left[\frac{h}{(1-h)(1+m)}\right]^{\alpha}-\alpha\theta +\tau\frac{1+\delta}{1+r}h\left[\frac{(1-h)(1+m)}{h}\right]^{1-\alpha}.$$

The total differentiation of it with respect to m is then, by using (A.2.2),

$$\frac{dz^i}{dm} = (1-\alpha)\left[\frac{h}{(1-h)(1+m)}\right]^{\alpha} \times\left[\tau\frac{1+\delta}{1+r}\left(1+\psi\frac{\alpha-h}{1-\alpha}\right)(1-h)-(1-\tau)(1-\psi)\frac{\alpha}{1+m}\right].$$

Hence its sign is negative if

$$\tau < \frac{\alpha(1-\psi)}{\alpha(1-\psi)+\frac{1+\delta}{1+r}\left(1+\psi\frac{\alpha-h}{1-\alpha}\right)(1-h)(1+m)}.$$

Therefore, assumption (D) requires ϕ to be replaced by ψ in condition (A.11.1'). As m increases, we have $\frac{dz^i}{dm} > 0$ for the low-skilled if $\overline{m}$ is "sufficiently high", i.e., assumption (B). ■

16. Proof of proposition 8

Objective (20) and lemma 7 imply that the utility of pensioners monotonically increases in m. Let H denote the number of currently high-skilled workers, and R that of retired pensioners. Let L_0 denote the number of those who remain low-skilled over $[0, \overline{m}]$.

When $z^i(0) < z^i(\acute{m}) < z^i(\overline{m})$ for the low-skilled, the coutcome is either $\acute{m}$ or $\overline{m}$. Divide the number of those who undertake skill aquisition at some $m \in [0, \overline{m}]$ into two groups: L_1 if $z^i(\acute{m}) < z^i(\overline{m})$ and L_2 if $z^i(\overline{m}) < z^i(\acute{m})$. Under scenario IV, $R + H < L_0 + L_1 + L_2$ at the voting stage. If $L_0 + L_1 + R < L_2 + H$, $\acute{m} \succ m \in [0, \overline{m}] \setminus \{\acute{m}\}$. If $L_0 + L_1 + R > L_2 + H$, $\overline{m} \succ m \in [0, \overline{m})$. To obtain this result, it does not matter whether $z^i(\overline{m}) < z^i(0)$ for the high-skilled or not. Thus, to prove the proposition, we do not use h^* because of the possibility of $z^i(0) < z^i(\overline{m})$ for the low-skilled.

When $z^i(\acute{m}) < z^i(0) < z^i(\overline{m})$ for the low-skilled, the outcome is either $\acute{m}$ or $\overline{m}$. Divide the number of those who undertake skill aquisition at some $m \in [0, \overline{m}]$ into three groups: L_1 if $z^i(\acute{m}) < z^i(0)$, L_2 if $z^i(0) < z^i(\acute{m}) < z^i(\overline{m})$ and L_3 if $z^i(\overline{m}) < z^i(\acute{m})$. We then have the follwoing information where if condition (a) holds, then (c) holds. If condition (d) holds, (b) also holds.

a. $L_0 + L_1 + L_2 + R < L_3 + H \;\Rightarrow\; \acute{m} \succ m \in [0, \overline{m}] \setminus \{\acute{m}\}$
b. $L_0 + L_1 + L_2 + R > L_3 + H \;\Rightarrow\; \overline{m} \succ m \in [0, \overline{m})$
c. $L_0 + L_1 < L_2 + L_3 + H + R \;\Rightarrow\; \acute{m} \succ m \in [0, \acute{m})$
d. $L_0 + L_1 > L_2 + L_3 + H + R \;\Rightarrow\; 0 \succ m \in (0, \acute{m}]$

Case 1 — condition (a) holds: The Condorcet winner is $\acute{m}$.

Case 2 — conditions (b) and (c) hold: The Condorcet winner is $\overline{m}$.

Case 3 — condition (d) holds: The Condorcet winner is $\overline{m}$.

To obtain this result, it does not matter whether $z^i(\overline{m}) < z^i(0)$ for the high-skilled or not.

When $z^i(\overline{m}) < z^i(\acute{m}) < z^i(0)$ for the low-skilled, the outcome is either 0 or $\acute{m}$. Divide the number of those who undertake skill aquisition at some $m \in [0, \overline{m}]$ into two groups: L_1 if $z^i(\acute{m}) < z^i(0)$ and L_2 if $z^i(0) < z^i(\acute{m})$. We then have the following information.

$$\begin{array}{lll} \text{a.} & R < L_0 + L_1 + L_2 + H & \Rightarrow \quad \acute{m} \succ m \in (\acute{m}, \overline{m}] \\ \text{b.} & L_0 + L_1 < L_2 + H + R & \Rightarrow \quad \acute{m} \succ m \in [0, \acute{m}) \\ \text{c.} & L_0 + L_1 > L_2 + H + R & \Rightarrow \quad 0 \succ m \in (0, \overline{m}] \\ \text{d.} & L_2 + H > L_0 + L_1 + R & \Rightarrow \quad \acute{m} \succ m \in [0, \overline{m}] \setminus \{\acute{m}\} \end{array}$$

Under scenario IV, $R + H < L_0 + L_1 + L_2$ at the voting stage. Hence condition (a) is true. Note that, if condition (d) holds, (b) also holds. If condition (c) holds, (d) does not hold.

Case 1 — condition (b) holds: Since condition (a) always holds, regardless of whether (d) holds or not, $\acute{m}$ is the Condorcet winner.

Case 2 — condition (c) holds: Since $\acute{m} \in (0, \overline{m}]$, the Condorcet winner is the status quo.

To obtain this result, it does not matter whether $z^i(\overline{m}) < z^i(0)$ for the high-skilled or not.

When $z^i(\acute{m}) < z^i(\overline{m}) < z^i(0)$ for the low-skilled, the outcome is either 0, $\acute{m}$, $\overline{m}$ or manipulable due to the emergence of a voting cycle. In the main text, we proved the possibility of resulting in either 0, $\acute{m}$ or a voting cycle when $z^i(\overline{m}) < z^i(0)$ for the high-skilled. As mentioned in footnote 30, the outcome possibilities when $z^i(0) < z^i(\overline{m})$ for the high-skilled include $\overline{m}$ because there may not exist $m \in [0, \acute{m})$ which can beat $\overline{m}$ by the majority. ■