Comparative Advantage and Heterogeneous Firms*

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Abstract

This paper examines how country, industry and firm characteristics interact in general equilibrium to determine nations' responses to trade liberalization. When firms possess heterogeneous productivity, countries differ in relative factor abundance and industries vary in factor intensity, falling trade costs induce reallocations of resources both within and across industries and countries. These reallocations generate substantial job turnover in all sectors, spur relatively more creative destruction in comparative advantage industries than comparative disadvantage industries, and magnify *ex ante* comparative advantage to create additional welfare gains from trade. The relative ascendance of high-productivity firms within industries boosts aggregate productivity and drives down consumer prices. In contrast with the neoclassical model, these price declines dampen and can even reverse the real wage losses of scarce factors as countries liberalize.

Keywords: Heckscher-Ohlin, inter-industry trade, intra-industry trade, trade costs, entry and exit, job creation and destruction, product variety, Ricardian productivity, creative destruction, exporting, missing trade

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1. Introduction

How do economies respond to trade liberalization? Neoclassical trade theory, with its emphasis on comparative advantage, stresses the reallocation of resources across industries and countries as well as changes in relative factor rewards, but provides no role for firm dynamics. More recent research on heterogeneous firms emphasizes the relative growth of high-productivity firms within industries but ignores comparative advantage by considering just a single factor and industry. Until now, very little has been known about how these two sources of reallocation combine in general equilibrium.

This paper derives new – and more realistic – predictions about trade liberalization by embedding heterogeneous firms in a model of comparative advantage and analyzing how firm, country and industry characteristics interact as trade costs fall. We report a number of new and often surprising results. In contrast with the neoclassical model, we find that simultaneous within- and across-industry reallocations of economic activity generate substantial job turnover in all sectors, even while there is net job creation in comparative advantage industries and net job destruction in comparative disadvantage industries. We show that steady-state creative destruction of firms also occurs in all sectors, but find that it is more highly concentrated in comparative advantage industries than comparative disadvantage industries. We demonstrate that the relative growth of high-productivity firms raises aggregate productivity in all industries, but that productivity growth is strongest in comparative advantage sectors. The price declines associated with these productivity increases inflate the real-wage gains of relatively abundant factors while dampening, or even potentially overturning, the real-wage losses of relatively scarce factors. Finally, we show that the behavior of heterogenous firms magnifies countries' comparative advantage and thereby creates a new source of welfare gains from trade.

Our analysis contributes to two literatures. We advance previous work on imperfect competition and comparative advantage, e.g. Helpman and Krugman (1985)[1], by relaxing the assumption that firms are identical. We extend more recent research on heterogeneous firms and monopolistic competition, e.g. Melitz (2003), by introducing an additional industry and factor and the complex interactions to which they give rise.[2] Our framework simultaneously explains why some countries export more in certain industries than in others (endowment-driven comparative advantage); why nonetheless two-way trade is observed within industries (firm-level horizontal product differentiation combined with increasing returns to scale); and why, within industries engaged in these two forms of trade, some firms export and others do not (self-selection driven by trade costs). These outcomes, as well as the assumptions underlying the model, are consistent with a host of stylized facts about firms and trade that have emerged across several empirical literatures.[3]

The framework we develop considers a world of two countries, two factors and two in-

[1] See also Krugman (1981), Helpman (1984) and Markusen and Venables (2000).

[2] Other international trade models incorporating heterogeneous firms include Bernard et al. (2003), Helpman et al. (2004), Melitz and Ottaviano (2003) and Yeaple (2005).

[3] Taken together, these facts document substantial variation in productivity across firms, frequent firm entry and exit, positive covariation in entry and exit rates across industries, higher productivity among exporting firms, the co-existence of exporting and non-exporting firms in all sectors, no feedback from exporting to firm productivity, and substantial sunk costs of entry into export markets. See, among others, Bartelsman and Doms (2000), Bernard and Jensen (1995, 1999, 2004), Clerides, Lach and Tybout (1998), Davis and Haltiwanger (1991), Dunne, Roberts and Samuelson (1989), and Roberts and Tybout (1997). For recent empirical evidence of Heckscher-Ohlin forces operating at the level of individual firms and products within industries, see Bernard, Jensen and Schott (2005).

dustries. Each industry is populated by a continuum of firms that each produce a single differentiated variety within their industry. Firms are heterogeneous in their level of productivity (which is constant during their lifetime), industries vary in relative factor intensity and countries differ in relative factor abundance. Firms from a competitive fringe may enter either industry by paying a sunk entry cost. After this sunk entry cost is paid, firm productivity is drawn from a fixed distribution and observed. The presence of fixed production costs means that firms drawing a productivity level below some lower threshold (the "zero-profit productivity cutoff") choose to exit the industry. Fixed and variable costs of exporting ensure that, of the active firms in an industry, only those who draw a productivity above a higher threshold (the "export productivity cutoff") find it profitable to export in equilibrium.

Consideration of the asymmetric export opportunities afforded by comparative advantage is key to understanding our results. When countries simultaneously transition from autarky to costly trade, firms' export opportunities increase and therefore the expected value of entering an industry rises. Entry from the competitive fringe increases with this expected value, and this entry drives down the *ex post* profitability of producing firms and increases the minimum level of productivity firms must possess in order to survive. This increase in the zero-profit productivity cutoff raises the average productivity of firms in an industry, thereby inducing aggregate (i.e., industry-level) productivity growth. In the presence of comparative advantage, the relative profitability of serving export and domestic markets varies with industry factor intensity and country factor abundance. As a result, comparative advantage industries experience greater productivity gains than comparative disadvantage industries as trade costs fall because firms in comparative advantage industries find it easier to export. The ensuing asymmetric aggregate productivity growth amplifies *ex ante* comparative advantage: pre-liberalization differences in the opportunity costs of production widen as trade costs fall because countries' comparative advantage industries become even more productive than their comparative disadvantage industries. This magnifies countries' original heterogeneity and thereby boosts the welfare gains from trade.

By increasing exporters' profits, falling trade costs also reduce the minimum level of productivity firms need to export successfully. Export productivity cutoffs decline relatively more in comparative advantage industries, where potential export profits are higher. As a result, the relative range of productivities over which firms export is higher in comparative advantage industries than in comparative disadvantage industries. Trade liberalization also raises average firm size by prompting exporters to sell more output abroad, and the increase in average firm output is largest in comparative advantage industries.

These findings contrast with the homogeneous-firm, imperfect-competition model of Helpman and Krugman (1985), where industry productivity remains constant and either all or no firms export following trade liberalization depending on the value of fixed and variable trade costs. They also differ from existing heterogeneous-firm models such as Melitz (2003) by demonstrating that the strength and importance of firm self-selection varies with the interaction of country and industry characteristics.

Our framework provides a rich setting for analyzing the distributional implications of international trade.[4] In the neoclassical model, falling trade costs lead to the well-known Stolper-Samuelson result, i.e., a rise in the real reward of the abundant factor and a decline in the real reward of the scarce factor. Here, there are two additional influences on real wages, and both are affected by the endogenous industry-level productivity gains noted above. The

[4] Distributional issues cannot be addressed in the single-factor models studied to date in the heterogeneous-firm literature.

first relates to consumers' taste for variety. Trade liberalization, as in Helpman and Krugman (1985), makes foreign varieties available to consumers. This increase in product variety reduces consumer price indices and raises real income. In our framework, however, there is an additional consideration: higher average firm productivity increases average firm size and reduces the mass of domestically produced varieties.

The second influence on real wages is unique to our approach and is a more direct consequence of aggregate productivity growth. Increases in industry productivity reduce the price of the average variety in each industry and thereby elevate the real income of both factors. Thus, even if the real wage of the scarce factor falls during liberalization in our model, its decline will be less than it would be in a neoclassical setting. Moreover, it is possible that the productivity gains associated with self-selecting heterogeneous firms are strong enough to raise the real wage of *both* factors, irrespective of the net change in varieties. The possibility of such an outcome, which also depends on model parameters, represents a sharp departure from the neoclassical model.

Our approach also generates novel predictions about the impact of trade liberalization on job turnover. In contrast to a neoclassical model, which predicts a simple flow of factors from comparative disadvantage to comparative advantage industries, we show that a reduction in trade barriers encourages simultaneous job creation and job destruction in all industries, but that gross and net job creation vary with country and industry characteristics. Comparative disadvantage industries exhibit net job destruction as the laying off of workers by exiting, lower-productivity firms exceeds the hiring by expanding, higher-productivity firms. Comparative advantage industries, on the other hand, enjoy net job creation as job loss due to exiting firms is dominated by the entrance and expansion of higher-productivity firms.

Surprisingly, steady-state firm failure, i.e., the creative destruction of firms, is highest in the comparative advantage industry. Each period a mass of incumbent firms dies exogenously while a separate mass of entrants fails to draw productivity levels above the zero-profit productivity cutoff and therefore exits after employing factors to pay their entry cost. Though the exogenous death rate is the same in both industries, overall steady-state creative destruction rises with the zero-profit productivity cutoff and is therefore higher in comparative advantage industries. This implication of the model may explain why workers in comparative advantage as well as comparative disadvantage industries report greater perceived job insecurity as countries liberalize.[5]

Finally, our framework offers a more useful benchmark than existing theory for predicting the pattern of trade. Recent research reveals that the poor empirical performance of neoclassical trade theory is driven by forces not captured in the standard Heckscher-Ohlin-Vanek model, including the existence of trade costs, factor price inequality and variation in technology and productivity across countries.[6] The model we develop here features these generalizations of the neoclassical model as endogenous outcomes of the interaction of firm, industry and country characteristics, and demonstrates how they give rise to both inter- and intra-industry trade.

The remainder of the paper is structured as follows. Section 2 develops the model and solves for general equilibrium under free trade. Section 3 explores the properties of the free trade equilibrium, highlighting the ways in which our analysis nests existing results for homogeneous-firm models of inter- and intra-industry trade. Section 4 introduces fixed and variable trade costs into the model and Section 5 examines the properties of the costly trade equilibrium. Section 6 provides a numerical solution to the model to illustrate the trajectory

[5] See Slaughter and Scheve (2004).

[6] See, for example, Bowen *et al.* (1987), Leamer (1984), Trefler (1993, 1995), Harrigan (1997), Davis and Weinstein (2001) and Schott (2003, 2004).

of endogenous variables for which closed-form analytical solutions do not exist. Section 7 concludes.

2. Free Trade

Throughout this section we maintain the assumption that international trade is costless. We consider a world of two countries, two industries, two factors and a continuum of heterogeneous firms. We make the standard Heckscher-Ohlin assumption that countries are identical in terms of preferences and technologies, but differ in terms of factor endowments. Factors of production can move between industries within countries but cannot move across countries. We use H to index the skill-abundant home country and F to index the skill-scarce foreign country, so that $\overline{S}^H/\overline{L}^H > \overline{S}^F/\overline{L}^F$ where the bars indicate country endowments.

2.1. Consumption

The representative consumer's utility depends on consumption of the output of two industries (i), each of which contains a large number of differentiated varieties (ω) produced by heterogeneous firms.[7] For simplicity, we assume that the upper tier of utility determining consumption of the two industries' output is Cobb-Douglas and that the lower tier of utility determining consumption of varieties takes the CES form[8],

$$U = C_1^{\alpha_1} C_2^{\alpha_2}, \qquad \alpha_1 + \alpha_2 = 1,\ \alpha_1 = \alpha \tag{1}$$

where, to simplify notation, we omit the country superscript except where important.

C_i is a consumption index defined over consumption of individual varieties, $q_i(\omega)$, with dual price index, P_i, defined over prices of varieties, $p_i(\omega)$,

$$C_i = \left[\int_{\omega\in\Omega_i} q_i(\omega)^\rho d\omega\right]^{\frac{1}{\rho}}, \qquad P_i = \left[\int_{\omega\in\Omega_i} p_i(\omega)^{1-\sigma} d\omega\right]^{\frac{1}{1-\sigma}}, \tag{2}$$

where $\sigma = 1/(1-\rho) > 1$ is the constant elasticity of substitution across varieties. For simplicity, we assume that the elasticity of substitution between varieties is the same in the two industries, but it is straightforward to allow this elasticity to vary.

2.2. Production

Production involves a fixed and variable cost each period. Both fixed and variable costs use multiple factors of production (skilled and unskilled labor) whose intensity of use varies across industries. All firms share the same fixed overhead cost but variable cost varies with

[7] Allowing one industry to produce a homogeneous good under conditions of perfect competition and constant returns to scale (e.g. Agriculture) is a special case of this framework where, in one industry, the elasticity of substitution between varieties is infinite and the fixed production and sunk entry costs are zero.

[8] We use the terms "good", "sector", and "industry" synonymously while variety is reserved for a horizontally differentiated version within an industry. All we require is a utility function with an upper tier where industries' outputs are substitutes and a lower tier where consumer preferences exhibit a love of variety. See Melitz and Ottaviano (2003) for a single industry model where love of variety takes the quasi-linear form. We concentrate on the CES case to focus on the effects of relative factor abundance with homothetic preferences and to make our results comparable with the existing inter- and intra-industry trade literature (Helpman and Krugman 1985).

firm productivity $\varphi \in (0, \infty)$. To avoid undue complexity, we assume that the cost function takes the Cobb-Douglas form,[9]

$$\Gamma_i = \left[f_i + \frac{q_i}{\varphi} \right] (w_S)^{\beta_i} (w_L)^{1-\beta_i}, \qquad 1 > \beta_1 > \beta_2 > 0 \tag{3}$$

where w_S is the skilled wage and w_L the unskilled wage, and industry 1 is assumed to be skill intensive relative to industry 2. We choose the skilled wage for the numeraire and so $w_S = 1$.

The presence of a fixed production cost implies that, in equilibrium, each firm will choose to produce a unique variety. The combination of monopolistic competition and variable costs that depend on firm productivity follows Melitz (2003). We augment that model by incorporating factor intensity differences across sectors and factor abundance differences across countries. As a result, Heckscher-Ohlin comparative advantage now plays an important role in shaping heterogeneous firms' adjustment to international trade.

Consumer love of variety and costless trade imply that all producing firms also export. Since firms face the same elasticity of demand in both the domestic market, d, and the export market, x, and trade is costless, profit maximization implies the same equilibrium price in the two markets, equal to a constant mark-up over marginal cost:

$$p_i(\varphi) = p_{id}(\varphi) = p_{ix}(\varphi) = \frac{(w_S)^{\beta_i}(w_L)^{1-\beta_i}}{\rho\varphi}. \tag{4}$$

With this pricing rule, firms' equilibrium domestic revenue, $r_{id}(\varphi)$, will be proportional to productivity:

$$r_{id}(\varphi) = \alpha_i R \left(\frac{\rho P_i \varphi}{(w_S)^{\beta_i}(w_L)^{1-\beta_i}} \right)^{\sigma-1}. \tag{5}$$

For given firm productivity φ, domestic revenue is increasing in the share of expenditure allocated to an industry, α_i, increasing in aggregate domestic expenditure (equals aggregate domestic revenue, R), increasing in the industry price index, P_i, which corresponds to an inverse measure of the degree of competition in a market, and increasing in ρ which is an inverse measure of the size of the mark-up of price over marginal cost. Firm revenue is decreasing in own price and hence in own production costs.

The equilibrium pricing rule implies that the relative revenue of two firms with different productivity levels within the same industry and market depends solely on their relative productivity, as is clear from equation (5): $r_{id}(\varphi'') = (\varphi''/\varphi')^{\sigma-1} r_{id}(\varphi')$.

Revenue in the export market is determined analogously to the domestic market and, with firms charging the same equilibrium price, relative revenue in the two markets for a firm of given productivity, φ, will depend on relative country size, R^F/R^H, and the relative price index, P_i^F/P_i^H. With the prices of individual varieties equalized and all firms exporting under costless trade, the price indices will be the same in the two countries, $P_i^F = P_i^H$, and relative revenue will depend solely on relative country size. Total firm revenue is the sum of revenue in the domestic and export markets. Under the equilibrium pricing rule, firm profits will equal

[9] The analysis generalizes in a relatively straightforward way to any homothetic cost function, for which the ratio of marginal cost to average cost will be a function of output alone. The assumption that fixed costs of production are independent of productivity captures the idea that many fixed costs, such as building and equipping a factory with machinery, are unlikely to vary substantially with firm productivity. All the analysis requires is that fixed costs are less sensitive to productivity than variable costs.

revenue from the two markets together scaled by the elasticity of substitution minus fixed costs of production:

$$\begin{aligned} r_i(\varphi) &= r_{id}(\varphi) + r_{ix}(\varphi) = \left[1 + \left(\frac{R^F}{R^H}\right)\right] r_{id}(\varphi) \\ \pi_i(\varphi) &= \frac{r_i(\varphi)}{\sigma} - f_i (w_S)^{\beta_i} (w_L)^{1-\beta_i}. \end{aligned} \tag{6}$$

To produce in an industry, firms must pay a fixed entry cost, which is thereafter sunk. The entry cost also uses skilled and unskilled labor, and we begin by assuming that the factor intensity of entry and production are the same, so that the industry sunk entry cost takes the form:

$$f_{ei}(w_S)^{\beta_i}(w_L)^{1-\beta_i}, \qquad f_{ei} > 0. \tag{7}$$

It is straightforward to relax the assumption of common factor intensities across the various stages of economic activity within industries. We discuss below how factor intensity differences between entry and production lead to additional interactions between country comparative advantage and the behavior of heterogeneous firms.

After entry, firms draw their productivity, φ, from a distribution, $g(\varphi)$, which is assumed to be common across industries and countries.[10] Firms then face an exogenous probability of death each period, δ, which we interpret as due to *force majeure* events beyond managers' control.[11]

A firm drawing productivity φ will produce in an industry if its revenue, $r_i(\varphi)$, at least covers the fixed costs of production, i.e. $\pi_i \geq 0$. This defines a **zero-profit productivity cutoff**, φ_i^*, in each industry such that:

$$r_i(\varphi_i^*) = \sigma f_i (w_S)^{\beta_i} (w_L)^{1-\beta_i}. \tag{8}$$

Firms drawing productivity below φ_i^* exit immediately, while those drawing productivity equal to or above φ_i^* engage in profitable production. The value of a firm, therefore, is equal to zero if it draws a productivity below the zero-profit productivity cutoff and exits, or equal to the stream of future profits discounted by the probability of death if it draws a productivity above the cutoff value and produces:

$$\begin{aligned} v_i(\varphi) &= \max\left\{0, \sum_{t=0}^{\infty} (1-\delta)^t \pi_i(\varphi)\right\} \\ &= \max\left\{0, \frac{\pi_i(\varphi)}{\delta}\right\}. \end{aligned} \tag{9}$$

[10] Combining the assumptions of identical cost functions within an industry across countries and a common productivity distribution yields the standard Heckscher-Ohlin assumption of common technologies across countries. It is straightforward to allow for differences in productivity distributions across countries and industries. As in previous trade models with heterogeneous firms, we treat each firm's productivity level as fixed after entry. This assumption matches the empirical findings of Bernard and Jensen (1999), Clerides, Lach, and Tybout (1998) and others that there is no feedback from exporting to productivity at the firm level.

[11] The assumption that the probability of death is independent of firm characteristics follows Melitz (2003) and is made for tractability to enable us to focus on the complex general equilibrium implications of international trade for firms, industries and countries. An existing literature examines industry dynamics in closed economies where productivity affects the probability of firm death (see, for example, Hopenhayn 1992 and Jovanovic 1982).

The *ex post* distribution of firm productivity, $\mu_i(\varphi)$, is conditional on successful entry and is truncated at the zero-profit productivity cutoff:

$$\mu_i(\varphi) = \begin{cases} \frac{g(\varphi)}{1-G(\varphi_i^*)} & \text{if} \quad \varphi \geq \varphi_i^* \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

where $G(\varphi)$ is the cumulative distribution function for $g(\varphi)$, and $1 - G(\varphi_i^*)$ is the *ex ante* probability of successful entry in an industry.

There is an unbounded competitive fringe of potential entrants and, in an equilibrium with positive production of both goods, we require the expected value of entry, V_i, to equal the sunk entry cost in each industry. The expected value of entry is the *ex ante* probability of successful entry multiplied by the expected profitability of producing the good until death, and the **free entry condition** is thus:

$$V_i = \frac{[1 - G(\varphi_i^*)]\,\overline{\pi}_i}{\delta} = f_{ei}(w_S)^{\beta_i}(w_L)^{1-\beta_i}, \tag{11}$$

where $\overline{\pi}_i$ is expected or average firm profitability from successful entry. Equilibrium revenue and profit in each market are constant elasticity functions of firm productivity (equation (5)) and, therefore, average revenue and profit are equal respectively to the revenue and profit of a firm with weighted average productivity, $\bar{r}_i = r_i(\tilde{\varphi}_i)$ and $\bar{\pi}_i = \pi_i(\tilde{\varphi}_i)$, where weighted average productivity is determined by the *ex post* productivity distribution and hence the zero-profit productivity cutoff below which firms exit the industry:

$$\widetilde{\varphi}_i(\varphi_i^*) = \left[\frac{1}{1 - G(\varphi_i^*)} \int_{\varphi_i^*}^{\infty} \varphi^{\sigma-1} g(\varphi) d\varphi\right]^{\frac{1}{\sigma-1}}. \tag{12}$$

It proves useful for the ensuing analysis to re-write the free entry condition in a more convenient form. The equation for equilibrium profits above gives us an expression for the profits of a firm with weighted average productivity, $\bar{\pi}_i = \pi_i(\tilde{\varphi}_i)$. Given the equilibrium pricing rule, the revenue of a firm with weighted average productivity is proportional to the revenue of a firm with the zero-profit productivity, $r_i(\tilde{\varphi}_i) = (\tilde{\varphi}_i/\varphi_i^*)^{\sigma-1} r_i(\varphi_i^*)$, where the latter is proportional to the fixed cost of production in equilibrium, $r_i(\varphi_i^*) = \sigma f_i(w_S)^{\beta_i}(w_L)^{1-\beta_i}$. Combining these results with the definition of weighted average productivity above, the free entry condition can be written so that it is a function solely of the zero-profit productivity cutoff and parameters of the model:

$$V_i = \frac{f_i}{\delta} \int_{\varphi_i^*}^{\infty} \left[\left(\frac{\varphi}{\varphi_i^*}\right)^{\sigma-1} - 1\right] g(\varphi) d\varphi = f_{ei}. \tag{13}$$

Terms in factor rewards have cancelled because average firm profitability and the sunk cost of entry are each proportional to factor costs, and entry and production have been assumed to have the same factor intensity. Since the expected value of entry in equation (13) is monotonically decreasing in φ_i^*, this relationship alone uniquely pins down the zero-profit productivity cutoff independent of factor rewards and other endogenous variables of the model. If entry and production have different factor intensities, this will no longer be the case. The free entry condition will then contain terms in factor rewards and, as discussed further below, movements in relative factor rewards will have important implications for heterogeneous firms' decisions about whether or not to exit the industry based on their observed productivity.

This way of writing the free entry condition also makes clear how the zero-profit productivity cutoff is increasing in fixed production costs, f_i, and decreasing in the probability of firm death, δ. Higher fixed production costs imply that firms must draw a higher productivity in order to earn sufficient revenue to cover the fixed costs of production. A higher probability of firm death reduces the mass of entrants into an industry, increasing *ex post* profitability, and therefore enabling firms of lower productivity to survive in the market.

2.3. Goods Markets

The steady-state equilibrium is characterized by a constant mass of firms entering an industry each period, M_{ei}, and a constant mass of firms producing within the industry, M_i. Thus, in steady-state equilibrium, the mass of firms that enter and draw a productivity sufficiently high to produce must equal the mass of firms that die:

$$[1 - G(\varphi_i^*)]M_{ei} = \delta M_i. \tag{14}$$

As noted above, under costless trade, firms charge the same price in the domestic and export markets and all firms export. Hence, the industry price indices are equalized across countries: $P_i^F = P_i^H$. A firm's equilibrium pricing rule implies that the price charged for an individual variety is inversely related to firm productivity, while the price indices are weighted averages of the prices charged by firms with different productivities, with the weights determined by the *ex post* productivity distribution. Exploiting this property of the price indices, we can write them as functions of the mass of firms producing in the home country multiplied by the price charged by a home firm with weighted average productivity, plus the mass of firms producing in the foreign country multiplied by the price charged by a foreign firm with weighted average productivity:

$$P_i = P_i^H = P_i^F = \left[M_i^H p_i^H (\tilde{\varphi}_i^H)^{1-\sigma} + M_i^F p_i^F (\tilde{\varphi}_i^F)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}. \tag{15}$$

The larger the mass of firms producing in the two countries, and the lower the price charged by a firm with weighted average productivity in the two countries, the lower the value of the common industry price index.

In equilibrium, we also require the goods market to clear at the world level, which requires the share of a good in the value of world production (in world revenue) to equal the share of a good in world expenditure:

$$\frac{R_1 + R_1^F}{R + R^F} = \alpha_1 = \alpha. \tag{16}$$

2.4. Labor Markets

Labor market clearing requires the demand for labor used in production and entry to equal labor supply as determined by countries' endowments:

$$\begin{aligned} S_1 + S_2 &= \overline{S}, \qquad & S_i &= S_i^p + S_i^e \\ L_1 + L_2 &= \overline{L}, \qquad & L_i &= L_i^p + L_i^e \end{aligned} \tag{17}$$

where S denotes skilled labor, L corresponds to unskilled labor, the superscript p refers to a factor used in production, and the superscript e refers to a factor used in entry.

2.5. Integrated Equilibrium and Factor Price Equalization

In this section, we describe the conditions for a free trade equilibrium characterized by factor price equalization (FPE). We begin by solving for the equilibrium of the integrated world economy, where both goods and factors are mobile, before showing that there exists a set of allocations of world factor endowments to the two countries individually such that the free trade equilibrium, with only goods mobile, replicates the resource allocation of the integrated world economy.

The integrated equilibrium is referenced by a vector of nine variables - the zero-profit cutoff productivities in each sector, the prices for individual varieties within each industry as a function of productivity, the industry price indices, aggregate revenue, and the two factor rewards: $\{\varphi_1^*, \varphi_2^*, P_1, P_2, R, p_1(\varphi), p_2(\varphi), w_S, w_L\}$. All other endogenous variables may be written as functions of these quantities. The equilibrium vector is determined by nine equilibrium conditions: firms' pricing rule (equation (4) for each sector), free entry (equation (13) for each sector), labor market clearing (equation (17) for the two factors), the values for the equilibrium price indices implied by consumer and producer optimization (equation (15) for each sector), and goods market clearing (equation (16)).

Proposition 1 *There exists a unique integrated equilibrium, referenced by the vector* $\{\hat{\varphi}_1^*, \hat{\varphi}_2^*, \hat{P}_1, \hat{P}_2, \hat{R}, \hat{p}_1(\varphi), \hat{p}_2(\varphi), \hat{w}_S, \hat{w}_L\}$. *Under free trade, there exists a set of allocations of world factor endowments to the two countries individually such that the unique free trade equilibrium is characterized by factor price equalization (FPE) and replicates the resource allocation of the integrated world economy.*

Proof. See Appendix ■

3. Properties of the Free Trade Equilibrium

Under autarky, the relative skill abundance in the home country leads to a lower relative price of skilled labor and of the skill-intensive good. The opening of trade leads to a convergence in relative goods prices and relative factor rewards, so that the relative skilled wage rises in the skill-abundant home country and falls in the labor-abundant foreign country. The rise in the relative price of the skill-intensive good in the home country results in a reallocation of resources towards the skill-intensive sector, as each country specializes according to its pattern of comparative advantage.

All four theorems of the Heckscher-Ohlin model (Rybczynski, Heckscher-Ohlin, Stolper-Samuelson, and Factor Price Equalization) continue to hold, with only minor modifications to take into account monopolistic competition, firm heterogeneity and increasing returns to scale. Factor price equalization requires that countries' endowments are sufficiently similar in the sense that their relative endowments of skilled and unskilled labor lie in between the integrated equilibrium factor intensities in the two sectors (Samuelson 1949; Dixit and Norman 1980).

Proposition 2 *A move from autarky to free trade leaves the zero-profit productivity cutoff and average industry productivity unchanged* (φ_i^* *and* $\widetilde{\varphi}_i$).

Proof. See Appendix ■

The intuition for this result is that, under free trade, all firms export and are affected symmetrically by the opening of trade. Firms of all productivities experience increased demand for their products in export markets and reduced demand in domestic markets as a result of entry by imported varieties. The mass of firms producing domestically in each industry, M_i, will change as countries specialize according to comparative advantage, and this will change the mass of firms producing at each level of productivity, $\mu_i(\varphi_i)M_i$, where $\mu_i(\varphi_i)$ is the *ex post* productivity distribution. Because firms of all productivities are affected symmetrically, there is no change in the zero-profit productivity cutoff below which firms exit the industry and no change in average industry productivity.

This result provides an important benchmark and clarifies the analysis of costly trade below, where we show that within-industry reallocations of resources between firms of different productivities are driven by the uneven effects of trade on exporters and non-exporters. However, we caution that the zero-profit productivity cutoff and average industry productivity will be affected even under free trade if entry and production have different factor intensities. Relative factor rewards change following the opening of trade, and if entry and production have different factor intensities, these changes in relative factor rewards will affect the value of entry (through firm profits) and entry costs differentially, resulting in a change in the equilibrium zero-profit productivity cutoff.[12]

As countries specialize according to comparative advantage, the mass of firms active in the comparative advantage industry will rise relative to the comparative disadvantage industry. Since in equilibrium the flow of successful entrants equals the flow of dying firms, the mass of entrants in an industry is proportional to the mass of firms, $M_{ei} = \left(\delta/\left[1 - G\left(\varphi_i^*\right)\right]\right) M_i$, and so the rise in the relative mass of firms in the comparative advantage industry will be reflected in a rise the relative amount of entry and exit.

4. Costly Trade

The assumption that trade is perfectly costless is at odds with recent empirical evidence of sizeable trade costs.[13] This empirical literature suggests that there are important fixed costs of entering foreign markets, such as the costs of acquiring information about foreign markets, developing appropriate marketing strategies and building distribution networks.[14]

In this section, we introduce fixed and variable costs of trade as in Melitz (2003). The basic setup remains the same as under free trade. However, in order to export a manufacturing variety to a particular market, a firm must incur a fixed export cost, which uses both skilled and unskilled labor with the same factor intensities as production. In addition, the firm may also face variable trade costs, which take the standard iceberg form whereby a fraction $\tau_i > 1$ units of a good must be shipped in industry i in order for one unit to arrive. These fixed and variable trade costs mean that, depending on their productivity, some firms may choose not to export in equilibrium.

We show how these trade costs interact with comparative advantage to determine responses to trade liberalization that vary across firms, industries and countries. Factor intensity and fac-

[12] For example, if entry is more skill-intensive than production, the fall in the relative skilled wage in the labor abundant country following the opening of trade will reduce the sunk costs of entry relative to the expected value of entry, inducing increased entry, an increase in the zero-profit productivity cutoff, and an improvement in aggregate industry productivity. See Flam and Helpman (1987) for an exploration of factor intensity differences between fixed and variable production costs in a homogeneous firm model of trade.

[13] See, in particular, Anderson and van Wincoop (2004) and Hummels (2001).

[14] See, for example, Roberts and Tybout (1997) and Bernard and Jensen (2004).

tor abundance, which have traditionally been viewed as determining reallocations of resources between industries, also play an important role in shaping within-industry reallocations of resources from less to more productive firms.

4.1. Consumption and Production

Profit maximization implies that equilibrium prices are again a constant mark-up over marginal cost, with export prices a constant multiple of domestic prices due to the variable costs of trade:[15]

$$p_{ix}^H(\varphi) = \tau_i p_{id}^H(\varphi) = \frac{\tau_i (w_S^H)^{\beta_i} (w_L^H)^{1-\beta_i}}{\rho\varphi}. \tag{18}$$

Given firms' pricing rules, equilibrium revenue in the export market is proportional to that in the domestic market. However, the price differences between the two markets mean that relative revenue in the export market now depends on variable trade costs. Furthermore, price indices now vary across the two countries due to variation in the mass of firms producing in an industry, different prices charged by firms in domestic and export markets (variable trade costs), and the existence of both exporters and non-exporters (fixed and variable trade costs). As a result, relative price indices enter as a determinant of relative revenue in the export market:

$$r_{ix}^H(\varphi) = \tau_i^{1-\sigma} \left(\frac{P_i^F}{P_i^H}\right)^{\sigma-1} \left(\frac{R^F}{R^H}\right) r_{id}^H(\varphi). \tag{19}$$

The wedge between revenue in the export and domestic markets in equation (19) will typically vary across countries and industries, and will prove important in determining how trade liberalization increases the expected value of entry into an industry. Total revenue received by a home firm is:

$$r_i^H(\varphi) = \begin{cases} r_{id}^H(\varphi) & \text{if it does not export} \\ r_{id}^H(\varphi)\left[1 + \tau_i^{1-\sigma}\left(\frac{P_i^F}{P_i^H}\right)^{\sigma-1}\left(\frac{R^F}{R^H}\right)\right] & \text{if it exports.} \end{cases} \tag{20}$$

Consumer love of variety and fixed production costs imply that no firm will ever export without also producing for the domestic market. Therefore, we may separate each firm's profit into components earned from domestic sales, $\pi_{id}^H(\varphi)$, and foreign sales, $\pi_{ix}^H(\varphi)$, where we apportion the entire fixed production cost to domestic profit and the fixed exporting cost to foreign profit:[16]

$$\begin{aligned} \pi_{id}^H(\varphi) &= \frac{r_{id}^H(\varphi)}{\sigma} - f_i (w_S^H)^{\beta_i} (w_L^H)^{1-\beta_i} \\ \pi_{ix}^H(\varphi) &= \frac{r_{ix}^H(\varphi)}{\sigma} - f_{ix} (w_S^H)^{\beta_i} (w_L^H)^{1-\beta_i} \end{aligned} \tag{21}$$

[15] In the analysis below, we write out expressions for home explicitly; those for foreign are analogous.

[16] This is a convenient accounting device which simplifies the exposition. Rather than comparing revenue from exporting to the fixed cost of exporting, we could equivalently compare the sum of domestic and export revenue to the sum of fixed production and exporting costs.

where the fixed cost of exporting requires both skilled and unskilled labor, $f_{ix}(w_S^H)^{\beta_i}(w_L^H)^{1-\beta_i}$.[17] A firm which produces for its domestic market also exports if $\pi_{ix}^H(\varphi) > 0$, and total firm profit is given by:

$$\pi_i^H(\varphi) = \pi_{id}^H(\varphi) + \max\left\{0, \pi_{ix}^H(\varphi)\right\}. \tag{22}$$

4.2. Decision to Produce and Export

After firms have paid the sunk cost of entering an industry, they draw their productivity, φ, from the distribution $g(\varphi)$. There are now two cutoff productivities, the **costly-trade zero-profit productivity cutoff**, φ_i^{*H}, above which firms produce for the domestic market and the **costly-trade exporting productivity cutoff**, φ_{ix}^{*H}, above which firms produce for both the domestic and export markets:

$$\begin{aligned} r_{id}^H(\varphi_i^{*H}) &= \sigma f_i (w_S^H)^{\beta_i}(w_L^H)^{1-\beta_i} \\ r_{ix}^H(\varphi_{ix}^{*H}) &= \sigma f_{ix} (w_S^H)^{\beta_i}(w_L^H)^{1-\beta_i}. \end{aligned} \tag{23}$$

Combining these two expressions, we obtain one equation linking the revenues of a firm at the zero-profit productivity cutoff to those of a firm at the exporting productivity cutoff. A second equation is obtained from the relationship between the revenues of two firms with different productivities within the same market, $r_{id}(\varphi'') = (\varphi''/\varphi')^{\sigma-1} r_{id}(\varphi')$, and from the relationship between revenues in the export and domestic markets, equation (19). The two equations together yield an equilibrium relationship between the two productivity cutoffs:

$$\varphi_{ix}^{*H} = \Lambda_i^H \ \varphi_i^{*H} \qquad \text{where} \qquad \Lambda_i^H \equiv \tau_i \left(\frac{P_i^H}{P_i^F}\right)\left(\frac{R^H}{R^F}\frac{f_{ix}}{f_i}\right)^{\frac{1}{\sigma-1}}. \tag{24}$$

The exporting productivity cutoff will be high relative to the zero-profit productivity cutoff, i.e. only a small fraction of firms will export, when the fixed cost of exporting, f_{ix}, is large relative to the fixed cost of production, f_i. In this case, the revenue required to cover the fixed export cost is large relative to the revenue required to cover the fixed production cost, implying that only firms of high productivity will find it profitable to serve both markets. The exporting productivity cutoff will also be high relative to the zero-profit productivity cutoff when the home price index, P_i^H, is high relative to the foreign price index, P_i^F, and the home market, R^H, is large relative to the foreign market, R^F. Again, only high-productivity firms receive enough revenue in the relatively small and competitive foreign market to cover the fixed cost of exporting. Finally, higher variable trade costs increase the exporting productivity cutoff relative to the zero-profit productivity cutoff by increasing prices and reducing revenue in the export market.

For values of $\Lambda_i^k > 1$, there is selection into markets, i.e. only the most productive firms export. Since empirical evidence strongly supports selection into export markets and the interior equilibrium is the most interesting one, we focus throughout the rest of the paper on parameter values where $\Lambda_i^k > 1$ across countries k and industries i.[18]

[17] We assume that fixed export costs use domestic factors of production, consistent with the idea that resources must be set aside to acquire information about and to enter foreign markets. One could also introduce a component of fixed export costs that employ factors of production in the foreign market. However, this would introduce Foreign Direct Investment (FDI) into the model and distract from our focus on the relationship between international trade, heterogeneous firms and comparative advantage. See Helpman, Melitz and Yeaple (2004) for an analysis of FDI in a single-factor model of heterogeneous firms.

[18] For empirical evidence on selection into export markets, see Bernard and Jensen (1995, 1999, 2004), Clerides, Lach and Tybout (1998), and Roberts and Tybout (1997).

Firms' decisions concerning production for the domestic and foreign markets are summarized graphically in Figure 1. Of the mass of firms M_{ei}^H who enter the industry each period, a fraction, $G(\varphi_i^{*H})$, draw a productivity level sufficiently low that they are unable to cover fixed production costs and exit the industry immediately; a fraction, $G(\varphi_{ix}^{*H}) - G(\varphi_i^{*H})$, draw an intermediate productivity level such that they are able to cover fixed production costs and serve the domestic market, but are not profitable enough to export; and a fraction, $G(\varphi_{ix}^{*H})$, draw a productivity level sufficiently high that it is profitable to serve both the home and foreign markets in equilibrium.

The *ex ante* probability of successful entry is $[1 - G(\varphi_i^{*H})]$ and the *ex ante* probability of exporting conditional on successful entry is:

$$\chi_i^H = \frac{[1 - G(\varphi_{ix}^{*H})]}{[1 - G(\varphi_i^{*H})]}. \tag{25}$$

4.3. Free Entry

In an equilibrium with positive production of both goods, we again require the expected value of entry, V_i^H, to equal the sunk entry cost in each industry. The expected value of entry is now the sum of two terms: the *ex ante* probability of successful entry times the expected profitability of producing the good for the domestic market until death and the *ex ante* probability of successful entry times the probability of exporting times the expected profitability of producing the good for the export market until death:

$$V_i = \frac{[1 - G(\varphi_i^*)]}{\delta} \left[\bar{\pi}_{id}^H + \chi_i^H \bar{\pi}_{ix}^H\right] = f_{ei}(w_S)^{\beta_i}(w_L)^{1-\beta_i} \tag{26}$$

where average profitability in each market is equal to the profit of a firm with weighted average productivity, $\bar{\pi}_{id}^H = \pi_{id}^H(\widetilde{\varphi}_i^H)$ and $\bar{\pi}_{ix}^H = \pi_{ix}^H(\widetilde{\varphi}_{ix}^H)$. Some lower-productivity firms do not export which leads to higher weighted average productivity in the export market than in the domestic market. Weighted average productivity is defined as in equation (12), where the relevant cutoff for the domestic market is the zero-profit productivity, φ_i^*, and the relevant cutoff for the export market is the exporting productivity, φ_{ix}^*.

Following the same line of reasoning as under free trade, we can write the free entry condition as a function of the two productivity cutoffs and model parameters:

$$V_i^H = \frac{f_i}{\delta} \int_{\varphi_i^{*H}}^{\infty} \left[\left(\frac{\varphi}{\varphi_i^{*H}}\right)^{\sigma-1} - 1\right] g(\varphi)d\varphi + \frac{f_{ix}}{\delta} \int_{\varphi_{ix}^{*H}}^{\infty} \left[\left(\frac{\varphi}{\varphi_{ix}^{*H}}\right)^{\sigma-1} - 1\right] g(\varphi)d\varphi = f_{ei}. \tag{27}$$

The expected value of entry under costly trade equals the expected value of entry under autarky plus a second term reflecting the expected profits to be derived from serving the export market. The closer the exporting productivity, φ_{ix}^*, lies to the zero-profit productivity cutoff, φ_i^*, the larger this second term and the larger the increase in the expected value of entry from opening the economy to costly trade. In equilibrium, φ_{ix}^* and φ_i^* are related according to equation (24). The distance in productivity between the least productive firm able to survive in the domestic market and the least productive firm able to survive in the export market depends on industry price indices and country size, and will hence vary systematically across countries and industries as considered further below.

4.4. Goods and Labor Markets

Again, in steady-state, the mass of firms that enter an industry and draw a productivity high enough to produce equals the mass of firms that die.

Using the equilibrium pricing rule, the industry price indices may be written as:

$$P_i^H = \left[M_i^H \left(p_{id}^H(\widetilde{\varphi}_i^H) \right)^{1-\sigma} + \chi_i^F M_i^F \left(\tau_i \; p_{id}^F(\widetilde{\varphi}_{ix}^F) \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}. \tag{28}$$

In general, the price indices for an industry will now vary across countries because of differences in the mass of domestic and foreign firms, differences in domestic and export prices (variable trade costs captured by τ_i), and differences in the proportion of exporting firms (fixed and variable trade costs reflected in χ_i^F and $\widetilde{\varphi}_{ix}^F$).

In equilibrium, we also require that the sum of domestic and foreign expenditure on domestic varieties equals the value of domestic production (total industry revenue, R_i) for each industry and country:

$$R_i^H = \alpha_i R^H M_i^H \left(\frac{p_{id}^H(\tilde{\varphi}_i^H)}{P_i^H} \right)^{1-\sigma} + \alpha_i R^F \chi_i^H M_i^H \left(\frac{\tau_i \; p_{id}^H(\tilde{\varphi}_{ix}^H)}{P_i^F} \right)^{1-\sigma} \tag{29}$$

where, with free entry into each industry, total industry revenue equals total labor payments, $R_i^H = w_S^H S_i^H + w_L^H L_i^H$. Requiring that equation (29) holds for all countries and industries implies that the goods markets clear at the world level.

The first term on the right-hand side of equation (29) captures home expenditure on home varieties, which equals the mass of varieties sold domestically, M_i^H, times expenditure on a variety with weighted average productivity.[19] The second term on the right-hand side of equation (29) captures foreign expenditure on home varieties. The key differences between the two terms are that only some of the varieties produced in home are exported to foreign (captured by the probability of exporting χ_i^H), the price charged by home producers in the export market is higher than in the domestic market (variable trade costs τ_i), and the weighted average productivity in the export market is greater than in the domestic market because of entry into export markets only by higher productivity firms.

4.5. Costly Trade Equilibrium

The costly trade equilibrium is referenced by a vector of thirteen variables in home and foreign: $\{\varphi_1^{*k}, \varphi_2^{*k}, \varphi_{1x}^{*k}, \varphi_{2x}^{*k}, P_1^k, P_2^k, p_1^k(\varphi), p_2^k(\varphi), p_{1x}^k(\varphi), p_{2x}^k(\varphi), w_S^k, w_L^k, R^k\}$ for $k \in \{H, F\}$. All other endogenous variables may be written as functions of these quantities. The equilibrium vector is determined by the following equilibrium conditions for each country: firms' pricing rule (equation (18) for each industry and for the domestic and export market separately), free entry (equation (27) for each sector), the relationship between the two productivity cutoffs (equation (24) for each sector), labor market clearing (equation (17) for the two factors), the values for the equilibrium price indices implied by consumer and producer optimization (equation (28) for each sector), and world expenditure on a country's varieties equals the value of their production (equation (29) for each sector).

[19] Expenditure on a variety with weighted average productivity depends negatively on the domestic price of such a variety, $p_{id}^H(\tilde{\varphi}_i^H)$, positively on the price of competing varieties (including those produced in foreign and exported to home) as summarized in the domestic price index, P_i^H, positively on the share of consumer expenditure devoted to a good in equilibrium, α_i, and positively on aggregate home expenditure (equals aggregate home revenue, R^H).

Proposition 3 *There exists a unique costly trade equilibrium referenced by the pair of equilibrium vectors,* $\{\hat{\varphi}_1^{*k}, \hat{\varphi}_2^{*k}, \hat{\varphi}_{1x}^{*k}, \hat{\varphi}_{2x}^{*k}, \hat{P}_1^k, \hat{P}_2^k, \hat{p}_1^k(\varphi), \hat{p}_2^k(\varphi), \hat{p}_{1x}^k(\varphi), \hat{p}_{2x}^k(\varphi), \hat{w}_S^k, \hat{w}_L^k, \hat{R}^k\}$ *for* $k \in \{H, F\}$.

Proof. See Appendix. ∎

5. Properties of the Costly Trade Equilibrium

The combination of multiple factors, multiple countries, country asymmetry, firm heterogeneity, and trade costs means that there are no longer closed form solutions for several key endogenous variables of the model. Nonetheless, we are able to derive a number of analytical results concerning the effects of opening a closed economy to costly trade. We begin by developing these analytical results. In section 6, we numerically solve the model, illustrate the analytical results for a particular parameterization of the model, and trace the evolution of the endogenous variables for which no closed form solution exists.

5.1. Productivity and Exporting

Proposition 4 *The opening of costly trade increases the zero-profit productivity cutoff and average industry productivity in both industries.*
(a) *Other things equal, the increase in the zero-profit productivity cutoff and average industry productivity will be greater in a country's comparative advantage industry.*
(b) *Other things equal, the exporting productivity cutoff will be closer to the zero-profit productivity cutoff in a country's comparative advantage industry.*

Proof. See Appendix ∎

This proposition is driven by the fact that, with trade costs, not all firms find it profitable to export. As a result, trade has a differential effect on the profits of exporting and non-exporting firms. Moving from autarky to costly trade, the *ex post* profits of more productive exporting firms rise. This increases the expected value of entry in each industry because there is a positive *ex ante* probability of drawing a productivity sufficiently high to export. This induces more entry and so raises the mass of active firms in the industry. As a result, the industry becomes more competitive and the *ex post* profits of low-productivity firms that only serve the domestic market are reduced. Therefore, some low-productivity domestic firms no longer receive enough revenue to cover fixed production costs and exit the industry. The zero-profit productivity cutoff, φ_i^*, and average industry productivity, $\widetilde{\varphi}_i$, both rise.[20]

Profits in the export market are larger relative to profits in the domestic market in comparative advantage industries. Therefore, following the opening of trade, the *ex post* profits

[20] Starting from autarky and reducing trade costs, the zero profit productivity cutoff will rise as long as there is selection into export markets. As trade costs continue to fall, there will eventually come a point where all firms export. From this point onwards, further reductions in trade costs increase *ex post* profitability for all firms, *reducing* the value of the zero profit productivity above which firms can profitably produce, until free trade is attained at which point the cutoff takes the same value as under autarky. The same values for the zero profit productivity cutoff under autarky and free trade follow from the cutoff being independent of market size and relative factor prices (under free trade, the world is a single integrated market). We focus on parameter values where there is selection into export markets since this is the empirically relevant case.

of more productive exporting firms rise by more in comparative advantage industries. As a result, the expected value of entering the industry rises further in comparative advantage industries, which induces more entry, and so leads to a larger increase in the zero-profit productivity cutoff and average industry productivity in comparative advantage industries. Finally, since exporting is relatively more attractive in comparative advantage industries, the exporting productivity will lie closer to the zero-profit productivity cutoff, as shown graphically in Figure 2.

Another way to gain intuition for the greater exit of low-productivity firms and the greater increase in average productivity in comparative advantage industries comes from the general equilibrium implications for the labor market. Opening to costly trade leads to an increase in labor demand at exporters. This increase in labor demand bids up factor prices, reducing the *ex post* profits of non-exporters, and increasing the zero-profit productivity below which firms exit the industry.

The increase in labor demand at exporters is larger in the comparative advantage industry than in the comparative disadvantage industry, resulting in a rise in the relative price of the abundant factor. This rise in the relative price of the abundant factor leads to a greater reduction in the *ex post* profits of firms only serving the domestic market in the comparative advantage industry which uses the abundant factor intensively. As a result, the zero-profit productivity cutoff and average industry productivity rise by more in the comparative advantage industry. This does not occur under free trade because firms of all productivities benefit from the increase in demand generated by access to export markets.

The relationship between the zero-profit productivity cutoff and the exporting productivity cutoff can be analyzed more formally using equation (24). Dividing this relationship in one industry by the same relationship in the other industry, we obtain:

$$\frac{\Lambda_1^H}{\Lambda_2^H} \equiv \frac{\varphi_{1x}^{*H}/\varphi_1^{*H}}{\varphi_{2x}^{*H}/\varphi_2^{*H}} = \frac{\tau_1}{\tau_2}\left(\frac{f_{1x}/f_1}{f_{2x}/f_2}\right)^{\frac{1}{\sigma-1}} \frac{P_1^H/P_2^H}{P_1^F/P_2^F}. \tag{30}$$

Under costly trade, the relative price index for the two goods varies across countries. In a country's comparative advantage industry, the mass of firms producing at home will be larger relative to the mass of firms producing abroad. Since firms charge higher prices for varieties in export markets and not all firms export, this causes the relative price index for a country's comparative advantage good to be lower at home than abroad. This difference in relative price indices is reinforced by the lower relative reward for a country's abundant factor under costly trade, which reduces the relative price of the good using the abundant factor intensively. Again this causes the relative price for a country's comparative advantage good to be lower at home than abroad.

Equating the value of variable trade costs and the ratio of fixed exporting to fixed production costs in the two industries, the relationship between the exporting and zero-profit productivity cutoffs depends solely on relative price indices. A lower relative price for the skill-intensive good in the skill-abundant home country implies a ratio of relative prices on the far right of the equation less than one, which requires the exporting productivity cutoff to be closer to the zero-profit productivity cutoff in the comparative advantage industry ($\Lambda_1^H < \Lambda_2^H$ and conversely $\Lambda_2^F < \Lambda_1^F$). Since the exporting cutoff is closer to the zero-profit cutoff in the comparative advantage industry, opening the economy to international trade leads to a larger rise in the expected value of entry in the comparative advantage industry (equation (27)), and so leads to a larger increase in the zero-profit productivity cutoff in this industry.

In interpreting this result, it is important to distinguish between the overall amount of exit in an industry and the relative productivity of exiting and surviving firms. Following the opening of costly trade, there will be substantial exit in comparative *disadvantage* industries due to a fall in the mass of firms, M_i, as economies specialize according to comparative advantage. This fall in the overall mass of firms will be reflected in a decline in the mass of firms observed at each value of productivity, $\mu_i(\varphi_i) M_i$, in comparative disadvantage industries. However, selection on productivity during the entry and exit process is more intense in comparative *advantage* industries because of the greater draw of export opportunities, i.e. low-productivity firms are less likely to survive trade liberalization in the comparative advantage industry.

5.2. Heterogeneous and Homogeneous Firm Models

The endogenous increase in average industry productivity, the fact that only some firms export, and the variation in these responses to the opening of costly trade across comparative advantage and disadvantage industries are the central differences between the model of heterogeneous firms developed here and the homogeneous-firm model of inter and intra-industry trade in Helpman and Krugman (1985), henceforth HK. They are important not only in themselves but also because of their general equilibrium implications as analyzed below.

The homogeneous-firm model can be viewed as a special case of the framework developed here where all firms have the same constant value of productivity, equal to weighted average productivity, and the sunk cost of entry is absorbed into the period by period fixed production cost. Setting productivity in the homogeneous firm model equal to weighted average productivity in the heterogeneous firm model under autarky, the two models yield identical autarky equilibrium values of price indices, production, factor rewards and factor allocations.[21]

The real difference between the two models emerges when the closed economy is opened to costly trade. In the homogeneous-firm model, productivity is a common parameter across firms and remains unchanged following the opening of trade. Either all firms export or no firms export depending on the value of fixed and variable costs of trade. In contrast, in our model the opening of trade leads to a rise in the zero-profit productivity cutoff and average industry productivity. In the interior equilibrium of our model, there is selection into export markets whereby higher productivity firms export and lower productivity firms only serve the domestic market. Furthermore, these differences between the two models vary with comparative advantage. The increase in average industry productivity and the degree of participation in export markets are stronger in comparative advantage industries.

5.3. Firm Size and the Mass of Firms

Proposition 5 *The opening of costly trade increases average firm output in both industries, and other things equal the largest increase occurs in the comparative advantage industry.*

Proof. See Appendix ■

As in the single-sector model of Melitz (2003), the opening of costly trade has two sets of effects on equilibrium firm output. More intense entry following the opening of costly trade enhances domestic product market competition, and so reduces equilibrium firm output for the domestic market. At the same time, the potential to trade generates additional output for the export market at firms with a sufficiently high productivity to export. Unlike Melitz (2003)

[21] See the numerical solutions appendix for further discussion. There remain some differences between the two frameworks under autarky, since in our heterogeneous firm model there is dispersion of productivity across firms and ongoing entry and exit in steady-state.

the presence of factor intensity differences across industries and factor endowment differences across countries will cause these effects to vary systematically across sectors and countries with comparative advantage.

The change in average firm output depends on the expected value of lower output for the domestic market and higher output for the export market. Since the opening of costly trade increases the zero-profit productivity cutoff, it reduces the probability of drawing a productivity sufficiently high to produce. In equilibrium, the expected value of entry must equal the unchanged sunk entry cost, and therefore average profits conditional on producing must rise. This rise in average profits implies an increase in average firm output:

$$\bar{q}_i = \left(\frac{\tilde{\varphi}_i}{\varphi_i^*}\right)^{\sigma} \varphi_i^* f_i (\sigma - 1) + \chi_i \left(\frac{\tilde{\varphi}_{ix}}{\varphi_{ix}^*}\right)^{\sigma} \varphi_{ix}^* f_{ix} (\sigma - 1). \tag{31}$$

where the first term captures expected output for the domestic market and the second term captures expected output for the export market.

Since the zero-profit productivity cutoff rises by more in the comparative advantage industry, the increase in average firm output will be greater in the comparative advantage industry. This again contrasts with the homogeneous-firm model, where average firm output under costly trade is determined solely by the productivity parameter, fixed production costs, fixed exporting costs and the elasticity of substitution.

The equilibrium mass of domestically produced varieties equals aggregate industry revenue divided by average firm revenue:

$$M_i = \frac{R_i}{\bar{r}_i} \tag{32}$$

where average firm revenue depends on average variety prices and average output, while aggregate industry revenue depends on factor prices and the equilibrium allocation of labor to the two sectors. Other things equal, the rise in average productivity and hence average firm output in our model following the opening of costly trade reduces the equilibrium mass of domestically produced varieties compared to the homogeneous-firm model.

5.4. Welfare and Income Distribution

Proposition 6 *The opening of costly trade magnifies ex ante cross-country differences in comparative advantage by inducing endogenous Ricardian productivity differences at the industry level that are positively correlated with Heckscher-Ohlin-based comparative advantage.*

Proof. See Appendix. ■

Although parameters are identical across countries, the more intensive selection of high-productivity firms in comparative advantage industries following the opening of costly trade gives rise to endogenous Ricardian technology differences at the industry level that are non-neutral across sectors. The opening of costly trade increases average productivity in comparative advantage industries relative to comparative disadvantage industries, and therefore magnifies Heckscher-Ohlin based comparative advantage.

We capture this magnification effect with the following measure of relative productivity in the two industries and countries, which we refer to as the magnification ratio:

$$\frac{\widetilde{\varphi}_1^H / \widetilde{\varphi}_2^H}{\widetilde{\varphi}_1^F / \widetilde{\varphi}_2^F} \geq 1.$$

The larger rise in average productivity in a country's comparative advantage industry magnifies cross-country differences in relative opportunity costs of production and therefore provides a new source of welfare gains from trade.

Proposition 7 *The opening of costly trade has four sets of effects on the real income of skilled and unskilled workers:*
(a) *The relative nominal reward of the abundant factor will rise and the relative nominal reward of the scarce factor will fall.*
(b) *The rise in average industry productivity reduces average variety prices in both industries and so reduces consumer price indices.*
(c) *The rise in average firm size reduces the equilibrium mass of domestically produced varieties and so increases consumer price indices.*
(d) *The opportunity to import foreign varieties reduces consumer price indices.*

Proof. See Appendix ■

The opening of costly trade leads to an increase in the relative demand for a country's comparative advantage good. As production of the comparative advantage good expands, relative demand for the country's abundant factor increases, since the comparative advantage good uses the abundant factor intensively. The result is a rise in the relative reward of the abundant factor, as in the familiar Stolper-Samuelson Theorem. Compared to homogeneous-firm models with comparative advantage, the magnitude of the change in relative factor rewards and hence the impact on income distribution will differ in our model due to the endogenous emergence of average industry productivity differences, which will affect the size of the reallocation of factors across industries.

The real wage of each factor is the nominal factor reward divided by the consumer price index, which depends on the price indices for the two sectors:

$$W_S^H = \frac{w_S^H}{\left(P_1^H\right)^{\alpha}\left(P_2^H\right)^{1-\alpha}}, \qquad W_L^H = \frac{w_L^H}{\left(P_1^H\right)^{\alpha}\left(P_2^H\right)^{1-\alpha}}. \tag{33}$$

In addition to the change in factor rewards associated with the Stolper-Samuelson Theorem, the opening of costly trade has three other effects on welfare and income distribution. The first of these is absent from both the Heckscher-Ohlin and HK models and results from the increases in average industry productivity following the opening of costly trade. Average productivity rises in both sectors, which reduces the average price of varieties, and so reduces the price index for each good.

The two other welfare effects operate through the mass of varieties available for consumption. As in HK, the opening of costly trade gives domestic consumers access to foreign varieties, although with selection into export markets only a fraction of foreign varieties are exported. This increase in product variety reduces consumer price indices and raises real income. In our framework, however, there is an additional consideration: higher average firm productivity increases average firm size and reduces the mass of domestically-produced varieties. The net effect on the total mass of varieties (domestic plus foreign) that are available for consumption is ambiguous.

If the net welfare gains from the variety and productivity effects taken together are sufficiently large relative to changes in nominal factor rewards, it becomes possible for the opening of costly trade to lead to an increase in the real reward of the scarce factor. This stands in marked contrast to the Heckscher-Ohlin model, where the real reward of the scarce factor necessarily falls. When the two countries have relatively similar factor endowments, the change in nominal factor rewards following the opening of trade will be relatively small, enhancing the potential for the real reward of the scarce factor to rise. More generally, even if the real reward of the scarce factor falls in our model, it will fall by less than in the Heckscher-Ohlin model.[22]

5.5. Job Creation and Job Destruction

Proposition 8 ***(a)*** *The opening of costly trade results in net job creation in the comparative advantage industry and net job destruction in the comparative disadvantage industry.*
(b) *The opening of costly trade results in simultaneous gross job creation and gross job destruction in both industries, so that gross job changes exceed net job changes, and both industries experience excess job reallocation.*

Proof. See Appendix ■

Our heterogeneous-firm model with comparative advantage has the same general pattern of net job creation and net job destruction as in the standard Heckscher-Ohlin model. The opening of costly trade results in net job creation in the comparative advantage industry and net job destruction in the comparative disadvantage industry. The magnitude of the net job creation and destruction will differ as a result of the endogenous changes in average industry productivity which shape the extent of the reallocation of factors across industries.

In our model, there is an important distinction between gross and net job creation and destruction that is completely absent from the Heckscher-Ohlin model. In both industries, there is gross job creation at high-productivity firms that expand to serve the export market combined with simultaneous gross job destruction at surviving firms that produce just for the domestic market. Therefore, even within the same sector, some firms gain while other firms lose from reductions in trade costs.[23]

Within each industry, the change in employment following the opening of costly trade can be decomposed into: (a) the change in employment used in the sunk costs of entry; (b) the change in employment for domestic production due to firm exit or entry; (c) the change in employment for domestic production at continuing firms; (d) the change in employment for the export market due to entry of firms into exporting; and (e) the change in employment for the export market at continuing exporters. Gross job creation and destruction are large relative to their net values because these individual terms have different signs within the same industry.

5.6. Steady-State Creative Destruction of Firms

Proposition 9 *The opening of costly trade leads to a larger increase in steady-state creative destruction of firms in comparative advantage industries than in comparative disadvantage industries.*

[22] The combination of these forces affecting the real wages of the two factors may help explain the absence of a clear empirical relationship between trade liberalization, real wages and poverty. See, for example, the survey in Goldberg and Pavcnik (2004).

[23] This excess job reallocation even within comparative advantage sectors may help explain why even skilled workers in skill-intensive industries in skill-abundant countries report anxiety about trade liberalization. See, for example, Scheve and Slaughter (2004).

Proof. See Appendix ■

The costly trade equilibrium displays steady-state creative destruction which in our model varies systematically across countries and industries with comparative advantage. Each period a mass of existing firms δM_i dies and there is a cohort of new entrants M_{ei}, of whom $[1 - G(\varphi_i^*)] M_{ei}$ draw a productivity sufficiently high to produce, and of whom $G(\varphi_i^*) M_{ei}$ exit. In steady-state equilibrium, the flow of successful entrants equals the flow of dying firms so that the mass of firms within the industry remains constant. The steady-state rate of creative destruction corresponds to the steady-state probability of firm failure, which equals the flow of exiting and dying firms divided by the flow of new entrants and existing firms:

$$\Psi_i = \frac{G(\varphi_i^*) M_{ei} + \delta M_i}{M_{ei} + M_i}. \tag{34}$$

The higher the zero-profit productivity cutoff, φ_i^*, the greater the probability of a firm drawing a productivity below the cutoff and exiting, and so the greater the steady-state probability of firm failure. Since the opening of costly trade leads to a larger increase in the zero-profit productivity cutoff in the comparative advantage industry, the steady-state rate of creative destruction rises by more in comparative advantage industries.[24]

5.7. International Trade

Endogenous increases in average industry productivity and selection into export markets imply that the volume of trade in our model will systematically depart from the volume of trade in the homogeneous-firm model of HK with fixed and variable trade costs.

Proposition 10 *Under costly trade at the same values of aggregate revenue, price indices and factor rewards, there are three sources of differences in the volume of trade relative to a homogeneous-firm benchmark:*
(a) *the volume of trade is lower because only a fraction of domestically produced varieties are traded*
(b) *the volume of trade is lower due to the rise in average firm size and fall in the mass of domestically produced varieties induced by endogenous increases in average firm productivity*
(c) *the volume of trade is higher due to selection on productivity in export markets.*

Proof. See Appendix ■

We review these three sources of differences in turn. First, in our framework only a fraction of varieties produced are exported, while in the homogeneous-firm model all varieties are traded if any trade occurs. Since consumers value variety, the reduction in the volume of trade due to fewer domestically-produced varieties exceeds the increase in trade due to higher productivity among the remaining varieties produced.

Second, with heterogeneous firms, the opening of costly trade increases average firm productivity and reduces average variety prices. With an unchanged mass of domestically produced varieties, this would increase the volume of trade. However, the rise in average firm productivity also leads to an increase in average firm output and so reduces the mass of varieties

[24] Firm failure is closely related to concerns about job insecurity as firm deaths are responsible for a large share of job destruction. We discuss the flow of jobs associated with this creative destruction of firms in the numerical solutions section.

produced by each country. Since consumers value variety, this again leads to a decline in the overall volume of trade.

Third, in our model only the most productive firms export. Therefore, average productivity among exported varieties is higher than among varieties sold in the domestic market. This selection effect reduces the average free on board price of exported varieties relative to the average price of varieties sold in the domestic market and so increases the volume of trade. Aggregate revenue, price indices and factor rewards will also differ between the equilibria of the two models and we analyze the effect of this variation on the volume of trade when we numerically solve the model below.

Proposition 11 *The net factor content of trade systematically departs from the predictions of the Heckscher-Ohlin-Vanek model due to:*
(a) Trade costs
(b) Selection into export markets
(c) The magnification of comparative advantage
(d) Non-factor price equalization.

Proof. See Appendix ■

The empirical literature on the net factor content of trade has established a substantial difference between the measured net factor content of trade and that predicted by the Heckscher-Ohlin-Vanek model under the standard assumptions (see, for example, Bowen *et al.* 1987). The current consensus in this literature emphasizes the importance of trade costs, non-factor price equalization and non-neutral technology differences across industries in explaining why the measured net factor content of trade is so much smaller than that predicted by the Heckscher-Ohlin-Vanek model (Leamer 1984; Bowen *et al.* 1987; Trefler 1993, 1995; Harrigan 1997; Davis and Weinstein 2001). The heterogeneous-firm model developed here integrates all of these features. Trade costs not only reduce the volume of trade but also reduce the set of varieties traded whenever there is selection into export markets. The non-neutral technology differences are endogenous and are driven by the differential selection of high-productivity firms across countries and industries with comparative advantage. Trade costs, selection into export markets and endogenous non-neutral technology differences together generate the violation of factor price equality.

Finally, since there is non-factor price equalization, varieties in the same industry are produced with different factor intensities across countries, and so there is factor content to intra-industry trade as emphasized in recent research, e.g. Davis and Weinstein (2004) and Schott (2003, 2004). This is consistent with the idea that focusing on the factor content of *net* trade may understate the true degree to which factor services are traded across countries.

6. Numerical Solutions

In this section, we parameterize the costly trade model and solve it numerically. These solutions serve three purposes. First, they provide a visual representation of the equilibria described in the previous sections as well as reinforce the intuition behind them. Second, they enable us to contrast the outcomes of our model to a homogeneous-firm benchmark. Third, they allow us to trace out the evolution of variables, such as job turnover, that cannot be characterized explicitly as trade barriers fall across costly trade equilibria.

We assume a Pareto distribution for *ex ante* firm productivity,

$$g(\varphi) = ak^a \varphi^{-(a+1)}, \tag{35}$$

where $k > 0$ is the minimum value for productivity ($\varphi \geq k$), and $a > 0$ is a shape parameter that determines the skewness of the Pareto distribution. In addition to being tractable, this distribution provides a reasonable approximation of observed variation in firm productivity.[25] We assume $a > \sigma - 1$ so that log firm sales have a finite variance.

To focus on comparative advantage, we assume that all industry parameters except factor intensity (β_i) are the same across industries. We consider symmetric differences in country factor endowments and symmetric differences in industry factor intensities. The share of each good in consumer expenditure is assumed to equal one half. A more detailed discussion of other parameter values is included in the Appendix.

We compare our results to a modified HK model with the same preference and technology structure used in this paper. To render this benchmark meaningful, we augment it to include both fixed and variable costs of exporting.[26] In the results that follow, numerical solutions for the HK model are labelled "HK Benchmark"; all other results, sometimes labelled "our model" for clarity, are for the framework we present in this paper. To conserve space, we do not report results for the HK Benchmark model where the evolution of variables is clear from the discussion above (e.g. firm productivity is a constant and the probability of exporting is unity with sufficiently low trade costs). Parameters common to the two models, such as the elasticity of substitution, are assumed to take the same value. For firm productivity, we set the HK Benchmark productivity parameter equal to weighted average productivity in our model so that the two models yield identical outcomes under autarky.[27]

To study the impact of trade liberalization, we consider symmetric reductions in variable trade costs from autarky to a range of 60% to 20% (i.e. from $\tau = 1.6$ to $\tau = 1.2$).[28] This range is chosen to ensure that there is an interior equilibrium in the HK Benchmark model where the representative firm finds it profitable to incur the fixed and variable trade costs necessary to engage in international trade. Under the assumptions outlined in this section, outcomes for foreign country F are the mirror image of outcomes for home country H, and so we may characterize outcomes in both countries with a single figure, distinguishing between comparative advantage and disadvantage industries and between abundant and scarce factors as necessary.

6.1. Productivity and Exporting

In our model, trade liberalization leads to a rise in the zero-profit productivity cutoff and a decline in the exporting productivity cutoff as shown in the top panel of Figure 3. The relatively large increase in the comparative-advantage industry's zero-profit productivity cutoff generates relatively large gains in average industry productivity, as shown in the second panel of the figure. The relatively large decline in the comparative advantage industry's export

[25] See also Helpman *et al.* (2004) and Ghironi and Melitz (2004).

[26] The introduction of fixed exporting costs into the Helpman and Krugman (1985) model causes average firm output to rise when countries open to costly trade, as firms need to sell more output to cover the sum of fixed production and exporting costs and still earn zero profits.

[27] Our model includes a sunk entry cost which is not present in Helpman and Krugman (1985). However, as discussed in the appendix, absorbing the sunk entry cost into the fixed per period production cost, the HK Benchmark model yields autarky equilibrium outcomes identical to our framework.

[28] Qualitatively similar results are obtained if we reduce fixed rather than variable costs of trade, as long as fixed trade costs remain sufficiently high as to induce selection into export markets.

productivity cutoff, on the other hand, leads to a relatively large rise in the probability of exporting in that industry, as seen in the third panel of the figure.

6.2. Firm Size and the Mass of Firms

Increases in average firm productivity result in rising average firm output and declining masses of firms, as shown in the top two panels of Figure 4. Because productivity gains are larger in the comparative advantage industry, firms in this sector grow relatively bigger as trade costs fall.

During trade liberalization, countries specialize according to comparative advantage and devote an increasing share of resources to the comparative advantage industry. As a result, the mass of domestic firms (i.e., domestic varieties) active in the comparative advantage industry rises relative to the comparative disadvantage industry. This outcome is illustrated in the middle panel of Figure 4. Since in equilibrium the flow of successful entrants equals the flow of dying firms, the mass of entrants in an industry is proportional to the mass of firms. As resources are reallocated toward the comparative advantage industry, the mass of entrants therefore rises in the comparative advantage industry and falls in the comparative disadvantage industry, as illustrated in the bottom panel of Figure 4.

6.3. Welfare

The left panel of Figure 5 illustrates the magnification of *ex ante* comparative advantage induced by heterogeneous firms. As trade costs fall, initial differences in industries' relative opportunity costs of production widen endogenously as aggregate productivity rises faster in the comparative advantage industry. For the parameter values we have chosen, the magnification ratio,

$$\frac{\widetilde{\varphi}_1^H/\widetilde{\varphi}_2^H}{\widetilde{\varphi}_1^F/\widetilde{\varphi}_2^F},$$

rises from unity under autarky to 1.09 when variable trade costs equal 20%. There is no magnification of comparative advantage in the HK Benchmark model because industry productivity is constant for all values of costly trade.

Magnification of comparative advantage contributes to higher welfare in our model relative to that of the HK Benchmark, as shown in the right panel of Figure 5. While the models are calibrated to yield identical levels of welfare under autarky, our model generates roughly 2 percent higher welfare for the chosen parameter values under costly trade.

6.4. Income Distribution

Both factors of production see larger increases in real income in our model than in the HK Benchmark. These differences are due to aggregate productivity gains that drive down average variety prices in our model. As indicated in the left panel of Figure 6, the real wages of the abundant and scarce factors are roughly 3 and 2 percent higher in our model than in the HK Benchmark under the parameter values we have chosen. Note that the autarky real wages of both factors are equal in the two models.

Figure 6 is noteworthy for providing an example where the real wage of both factors rises as countries liberalize. This result occurs in both our model and in the HK Benchmark, and it contrasts sharply with the Stolper-Samuelson Theorem of the neoclassical model. In our model it is driven by aggregate productivity gains, which reduce consumer price indices below the level that would otherwise be achieved under trade liberalization. In the HK Benchmark, real wages rise because of a net increase in the number of domestically available varieties. A key difference between our model and Helpman and Krugman (1985) is that both factors' real wages can rise in our framework due to productivity-driven price declines even if the net change in varieties is negative.

As trade costs fall. relative factor prices increase less in our model than in the HK Benchmark model (Figure 6 right panel). By enhancing access to foreign markets, trade liberalization leads to an increase in the relative demand for a country's comparative advantage good. As production of the comparative advantage good expands, relative demand for the country's abundant factor increases. However, in our model the increase in the relative productivity of the comparative advantage industry raises the relative supply of the comparative advantage good, so that some of the increase in demand for this good can be satisfied by increased productivity rather than by factor reallocation. As a result, there is less reallocation of factors between sectors in our model than in the HK Benchmark model, and therefore a smaller change in relative factor demand.

6.5. Job Creation and Job Destruction

A central difference between our model and homogeneous-firm models is that, in our framework, trade liberalization results in gross job creation and gross job destruction in all industries, with the magnitude of these gross job flows varying across countries and industries with comparative advantage.

Table 1 reports job turnover as variable trade costs decline from autarky to 20 percent. Each time a worker moves between firms, one job is lost and another is gained. The table is arranged into four panels, each of which reports total, between- and within-industry job turnover as a percent of the total labor force (skilled plus unskilled) for a different factor-industry combination.

Between-industry turnover represents transfers of jobs across industries, while within-industry turnover corresponds to switches of jobs across firms within the same industry. Total job turnover is the sum of the absolute value of between- and within-industry turnover. Due to specialization, between-industry turnover is positive for comparative advantage industries and negative for comparative disadvantage industries. Labor market clearing implies that these between-industry reallocations are equal in magnitude across sectors for a particular factor. As indicated in the table, 14.2 percent of the labor force switches industries as the economy opens to trade, with greater between-industry reallocation for the abundant factor.

Within-industry turnover is driven by a reallocation of economic activity across firms inside industries as zero-profit productivity cutoffs rise, export productivity cutoffs fall, production for the domestic market declines, and production for the export market expands. Across all factors and industries, within-industry reallocation is substantially larger than between-industry shifts: an additional 21.1 percent of the labor force change jobs within sectors.[29] The degree of intra-industry reallocation of the two factors taken together is highest (11.0 percent of all workers) for the comparative advantage industry during liberalization. Within-industry job turnover is highest for the abundant factor in the comparative advantage industry (6.7 percent of all workers) and the scarce factor in the comparative disadvantage industry (6.6

[29]Because each separated worker reflects both one job destroyed and one job created, the share of the labor force changing jobs within industries is half the sum of the within-industry turnovers, (13.3 + 7.0 + 8.7 + 13.2) / 2 = 21.1). Similarly, the share of the labor force changing jobs between or within industries is half the sum of the total turnovers noted in the table.

percent of the labor force). This result is driven by the interaction of country and industry characteristics: within-industry turnover is highest for the industries and factors that have an affinity for one another in terms of the factor being used relatively intensely by the industry.[30]

6.6. Steady-State Creative Destruction of Jobs

Another distinctive feature of our model is that steady-state creative destruction varies systematically across countries and industries with comparative advantage. Figure 7 summarizes steady-state factor churning by industry in autarky and with trade costs ranging from 60 percent to 20 percent. This churning is defined as the share of the total labor force in an industry engaged in entry or being used by firms that fail. Steady-state churning rises in the comparative advantage industry for both factors, reflecting the rise in the mass of entrants in that industry, changes in the productivity cutoffs, and changes in relative wages which affect factor demand.

6.7. International Trade

As trade costs fall, countries specialize according to comparative advantage, leading to increased inter-industry trade and enhanced net trade in factor services. At the same time, trade liberalization raises demand for foreign varieties and induces increased participation in export markets, resulting in higher volumes of intra-industry trade. Larger increases in average industry productivity in comparative advantage industries give rise to non-neutral industry technology differences across countries that influence patterns of trade in goods and factor services. Trade costs and non-neutral technology differences result in factor price inequality that again influences trade in both goods and factors.

As shown in the top-left panel of Figure 8, the overall volume of trade is lower than the HK benchmark. In the HK model, all firms are identical and therefore all firms export whenever trade occurs. In our model, only a fraction of firms export, reducing the volume of trade in our model relative to the HK Benchmark. In our model increases in average industry productivity would increase trade volume if the mass of varieties produced were held constant. However, increases in average productivity raise average firm output at the expense of the number of varieties (see Figure 4). Since consumers value variety, the increased productivity of remaining varieties does not fully compensate for the smaller range of varieties produced, reducing the volume of trade. The higher productivity of exported varieties relative to those sold domestically is not sufficient to offset these other effects, and the overall impact is to reduce the volume of trade.

Though our model generates lower values of both inter and intra-industry trade than the HK Benchmark, the relative importance of inter-industry trade is enhanced in our model by the magnification of comparative advantage.[31] This is seen in the top-right panel of Figure 8 which displays intra-industry trade, as measured by the minimum value of exports and imports within each industry, summed across industries. The disparity in the extent of intra-industry

[30] While intra-industry job reallocation is greatest for the abundant factor in the comparative advantage industry, the rise in the relative reward of the abundant factor augments the amount of within-industry reallocation for the scarce factor in both industries.

[31] Lower inter-industry trade is due to both sectors having love of variety preferences, so that trade in both sectors is suppressed by only a fraction of varieties being exported and by the reduction in the mass of varieties produced. If one industry produced a homogeneous product under conditions of perfect competition with no uncertainty regarding productivity, the magnification of comparative advantage in the other sector might raise inter-industry trade relative to the HK benchmark.

trade between our model and the HK benchmark is greater than the disparity in the overall volume of trade, as revealed by a comparison of the top two panels. In our model, varieties produced in the skill-intensive industry in the skill-abundant country have higher productivity and lower prices than varieties produced in the skill-intensive industry in the labor-abundant country (and vice versa), promoting inter-industry trade.

Surprisingly, the lower overall volume of trade in our model is combined with greater welfare gains from trade. This is explained by the fact that, with heterogeneous firms, the *ex ante* potential to export plays an important independent role. Because there is a positive probability of drawing a productivity high enough to export, this drives increased entry, the exit of low-productivity firms and increases in average industry productivity. These increases in average industry productivity reduce the average prices of all varieties, including those only sold domestically, thereby raising real income and welfare.

The lower volume of inter-industry trade in our model than in the HK benchmark is reflected in a smaller net trade in the services of abundant and scarce factors, as shown in the lower two panels of Figure 8. On the one hand, trade costs, factor price inequality and endogenous non-neutral technology differences move us toward explaining the "mystery of the missing trade" as identified by Trefler (1995). On the other hand, the use of different factor intensities to make varieties within the same industry and the existence of intra-industry trade suggests that focusing on the factor content of *net* (rather than gross) trade flows may understate the true extent of trade in factor services.

7. Conclusions

The reallocation of resources as countries liberalize has been a primary concern of economists since at least the time of Ricardo. Until recently, however, trade economists have neglected to provide a meaningful role for firms in these reallocations. We develop a model of comparative advantage that incorporates heterogeneous firms in order to study how firm, country and industry characteristics all interact in general equilibrium as trade costs fall. Our approach generates a number of novel implications that are worthy of further inquiry.

We find that within- and across-industry reallocations of economic activity during trade liberalization raise average industry productivity and average firm output in all sectors but relatively more so in comparative advantage industries than in comparative disadvantage industries. The endogenous emergence of these non-neutral productivity gains magnifies *ex ante* comparative advantage and provides a new source of welfare gains from trade. In contrast to the Heckscher-Ohlin and Helpman-Krugman (1985) models, trade results in gross job creation and gross job destruction in both comparative advantage and disadvantage industries. Unlike existing heterogeneous-firm frameworks such as Melitz (2003), the magnitude of these gross job flows and the extent of steady-state creative destruction varies systematically across countries and industries with comparative advantage.

In the model we develop, trade not only generates aggregate welfare gains but also has distinct implications for the distribution of income across factors. Increases in average industry productivity arising from trade liberalization drive down goods prices and therefore benefit both factors. If productivity declines are strong enough, the real wage of the scarce factor may even rise during trade liberalization, a contradiction of the well-known Stolper-Samuelson Theorem. More generally, the productivity gains induced by the behavior of heterogeneous firms dampen the decline of the scarce factor's real wage relative to its decline in more neoclassical settings. Our analysis also provides intuition for recent findings on the empirical shortcomings of the Heckscher-Ohlin-Vanek model by including features, such as trade costs, factor price inequality and non-neutral technology differences, that subvert neoclassical trade-flow predictions. Since factor prices are not equalized through trade in our model, varieties within industries are produced with different factor intensities across countries. As a result, there is both intra- and inter-industry trade in factor services.

Interesting areas for further research include the empirical testing of these theoretical predictions on the role of comparative advantage in industry productivity dynamics and extensions of the theory to introduce additional sources of firm heterogeneity, dynamic firm productivity, and multiple products within industries. While the focus in this paper has been on symmetric reductions of trade costs across industries and countries, the model easily allows the analysis of asymmetric trade cost reductions, as well as variation in the skill intensity of entry, production, and fixed export costs.

More generally, our analysis provides an example of the rich insights to be gained by combining microeconomic modelling of firms with general equilibrium analyses of trade. It points to the fruitfulness of placing individual firm behavior at the center of economies' adjustment to trade.

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A Appendix

A1. Proof of Proposition 1

Proof. (a) Begin with the existence of a unique integrated equilibrium.
Choose the skilled wage as numeraire, so $w_S = 1$.
From the free entry condition (13), $V_i \to \infty$ as $\varphi_i^* \to 0$; $V_i \to 0$ as $\varphi_i^* \to \infty$; and V_i is monotonically decreasing in φ_i^*. Thus, equation (13) defines a unique equilibrium value of the zero-profit productivity cutoff, $\hat{\varphi}_i^*$, as a function of parameters.
From equation (12), φ_i^* uniquely determines weighted average productivity, $\widetilde{\varphi}_i(\varphi_i^*)$. Combining $\bar{r}_i = r_i(\tilde{\varphi}_i) = (\tilde{\varphi}_i/\varphi_i^*)^{\sigma-1} r_i(\varphi_i^*)$ with the zero-profit cutoff condition (8), average revenue and profitability may be expressed as functions of φ_i^* and factor rewards alone:

$$\begin{aligned} \bar{r}_i &= r_i(\widetilde{\varphi}_i) = \left(\frac{\widetilde{\varphi}_i(\varphi_i^*)}{\varphi_i^*}\right)^{\sigma-1} \sigma f_i (w_S)^{\beta_i}(w_L)^{1-\beta_i} \qquad (36) \\ \overline{\pi}_i &= \pi_i(\widetilde{\varphi}_i) = \left[\left(\frac{\widetilde{\varphi}_i(\varphi_i^*)}{\varphi_i^*}\right)^{\sigma-1} - 1\right] f_i (w_S)^{\beta_i}(w_L)^{1-\beta_i}. \end{aligned}$$

In each sector, the total value of payments to labor used in production equals total revenue minus total profits:

$$w_S S_i^p + w_L L_i^p = R_i - \Pi_i \qquad (37)$$

while, combining free entry (11) and steady-state stability (14), the total value of payments to labor used in entry equals total profits:

$$\Pi_i = M_i \overline{\pi}_i = M_{ei} f_{ei} (w_S)^{\beta_1}(w_L)^{1-\beta_1} = w_S S_i^e + w_L L_i^e. \qquad (38)$$

Thus, total payments to labor in each sector equal total revenue:

$$w_S S_i + w_L L_i = R_i. \qquad (39)$$

where $S_i = S_i^p + S_i^e$ and $L_i = L_i^p + L_i^e$. Since this is true for both sectors, aggregate revenue equals aggregate income:

$$w_S \overline{S} + w_L \overline{L} = R. \qquad (40)$$

Since production technologies in each sector are Cobb-Douglas with the same factor intensities for production and entry, payments to skilled and unskilled labor are a constant share of total industry revenue, yielding the following equilibrium labor demands:

$$S_i = \frac{\beta_i R_i}{w_S}, \qquad L_i = \frac{(1-\beta_i) R_i}{w_L}. \tag{41}$$

Combining these expressions with labor market clearing, and using the fact that the representative consumer allocates constant shares of expenditure $\{\alpha, (1-\alpha)\}$ to the two sectors, we obtain the integrated equilibrium labor allocation:

$$S_1 = \left(\frac{\beta_1 \alpha}{\beta_1 \alpha + \beta_2 (1-\alpha)}\right) \overline{S}, \quad S_2 = \left(\frac{\beta_2 (1-\alpha)}{\beta_1 \alpha + \beta_2 (1-\alpha)}\right) \overline{S} \tag{42}$$

$$L_1 = \left(\frac{(1-\beta_1)\alpha}{(1-\beta_1)\alpha + (1-\beta_2)(1-\alpha)}\right) \overline{L}, \; L_2 = \left(\frac{(1-\beta_2)(1-\alpha)}{(1-\beta_1)\alpha + (1-\beta_2)(1-\alpha)}\right) \overline{L}. \tag{43}$$

Substituting equilibrium employment into the expression for unskilled labor demand, and simplifying using equilibrium consumer expenditure shares and our choice of numeraire, yields the integrated equilibrium unskilled wage:

$$w_L = \left(\frac{\overline{S}}{\overline{L}}\right) \left(\frac{1 - \beta_2 - \alpha\beta_1 + \alpha\beta_2}{\beta_2 + \alpha\beta_1 - \alpha\beta_2}\right). \tag{44}$$

The integrated equilibrium vector is $\{\varphi_1^*, \varphi_2^*, P_1, P_2, R, p_1(\varphi), p_2(\varphi), w_S, w_L\}$. We have solved for $\{\varphi_1^*, \varphi_2^*\}$ and $\{w_S, w_L\}$. $\{p_1(\varphi), p_2(\varphi)\}$ follow immediately from the pricing rule (4) and equilibrium wages. $\{R\}$ follows from equation (40) and equilibrium wages. $\{P_1, P_2\}$ may be determined from the analogue of equation (15) for the integrated world economy: $P_i = M_i^{1/(1-\sigma)} p_i(\tilde{\varphi}_i)$, where $\tilde{\varphi}_i$ is uniquely determined by φ_i^* and $M_i = R_i/\bar{r}_i$. From equation (36), $\bar{r}_i$ is determined by φ_i^* and wages for which we have solved. From equation (39), R_i is determined by wages and labor allocations for which we have also solved. We have thus fully characterized the integrated equilibrium vector.

(b) Now establish the existence of a FPE equilibrium which replicates the integrated equilibrium resource allocation.
FPE and our choice of the skilled wage in one country as numeraire implies:

$$w_S^H = w_S^F = w_S = 1, \qquad w_L^H = w_L^F = w_L. \tag{45}$$

Cost minimization implies the same equilibrium factor intensities in the two countries:

$$\frac{S_i^H}{L_i^H} = \frac{S_i^F}{L_i^F} = \frac{\beta_i}{1-\beta_i} \frac{w_L}{w_S} = \frac{\beta_i}{1-\beta_i} w_L. \tag{46}$$

The factor market clearing conditions in each country $k \in \{H, F\}$ may be expressed as follows:

$$\begin{aligned} \lambda_{L1}^k \left(\frac{S_1^k}{L_1^k}\right) + \left(1 - \lambda_{L1}^k\right) \left(\frac{S_2^k}{L_2^k}\right) &= \frac{\overline{S}^k}{\overline{L}^k}, \qquad \lambda_{Li}^k \equiv \frac{L_i^k}{\overline{L}^k} \\ \lambda_{S1}^k \left(\frac{L_1^k}{S_1^k}\right) + \left(1 - \lambda_{S1}^k\right) \left(\frac{L_2^k}{S_2^k}\right) &= \frac{\overline{L}^k}{\overline{S}^k}, \qquad \lambda_{\text{Si}}^k \equiv \frac{S_i^k}{\overline{S}^k}. \end{aligned} \tag{47}$$

Substituting for equilibrium factor intensities in the above and rearranging yields the free trade equilibrium labor allocations in each country as a function of endowments and the common unskilled relative wage for which we solve below:

$$L_1^k = \frac{\frac{1}{w_L^k}\overline{S}^k - \left(\frac{\beta_2}{1-\beta_2}\right)\overline{L}^k}{\left(\frac{\beta_1}{1-\beta_1}\right) - \left(\frac{\beta_2}{1-\beta_2}\right)}, \quad L_2^k = \frac{\left(\frac{\beta_1}{1-\beta_1}\right)\overline{L}^k - \frac{1}{w_L}\overline{S}^k}{\left(\frac{\beta_1}{1-\beta_1}\right) - \left(\frac{\beta_2}{1-\beta_2}\right)} \tag{48}$$

$$S_1^k = \frac{\left(\frac{\beta_1}{1-\beta_1}\right)\overline{S}^k - \left(\frac{\beta_1}{1-\beta_1}\right)\left(\frac{\beta_2}{1-\beta_2}\right)w_L\overline{L}^k}{\left(\frac{\beta_1}{1-\beta_1}\right) - \left(\frac{\beta_2}{1-\beta_2}\right)}, \; S_2^k = \frac{\left(\frac{\beta_1}{1-\beta_1}\right)\left(\frac{\beta_2}{1-\beta_2}\right)w_L\overline{L}^k - \left(\frac{\beta_2}{1-\beta_2}\right)\overline{S}^k}{\left(\frac{\beta_1}{1-\beta_1}\right) - \left(\frac{\beta_2}{1-\beta_2}\right)}. \tag{49}$$

Applying the same arguments as in the integrated equilibrium, aggregate income in each country equals aggregate revenue:

$$R^k = \overline{S}^k + w_L\overline{L}^k. \tag{50}$$

In both countries, total industry payments to unskilled labor are a constant share $(1-\beta_i)$ of total industry revenue, while world expenditure on a good equals a constant share of world revenue:

$$w_L\left(L_1^H + L_1^F\right) = (1-\beta_1)\alpha\left[(\overline{S}^H + \overline{S}^F) + w_L(\overline{L}^H + \overline{L}^F)\right]. \tag{51}$$

Substituting for free trade equilibrium employment levels $\{L_1^H,\ L_1^F\}$ and rearranging yields the equilibrium unskilled wage, which equals the value for the integrated world economy in equation (44).

The FPE set is characterized by the requirement that both countries' relative endowments of skilled and unskilled workers lie in between the integrated equilibrium factor intensities of the two sectors:

$$\left(\frac{\beta_1}{1-\beta_1}\right)\widehat{w}_L^{IE} > \frac{\overline{S}^H}{\overline{L}^H} > \frac{\overline{S}^F}{\overline{L}^F} > \left(\frac{\beta_2}{1-\beta_2}\right)\widehat{w}_L^{IE} \tag{52}$$

where the superscript IE indicates an integrated equilibrium value.

The free trade equilibrium is referenced by the vector $\{\varphi_1^{*k}, \varphi_2^{*k}, P_1^k, P_2^k, R^k, p_1^k(\varphi), p_2^k(\varphi), w_S^k, w_L^k\}$ for each country $k \in \{H, F\}$.

We have already solved for $\{w_S^H = w_S^F = 1,\ w_L^H = w_L^F\}$. $\{p_1^H(\varphi) = p_1^F(\varphi),\ p_2^H(\varphi) = p_2^F(\varphi)\}$ follow immediately from the pricing rule (4) and equilibrium wages. $\{R^k\}$ follows from equation (50) and equilibrium wages. $\{\varphi_1^{*H} = \varphi_1^{*F},\ \varphi_2^{*H} = \varphi_2^{*H}\}$ are determined by the free entry condition alone (equation (13)). $\{P_1^H = P_1^F,\ P_2^H = P_2^F\}$ are determined from equation (15), where $\tilde{\varphi}_i$ is uniquely determined by φ_i^* and $M_i^k = R_i^k/\bar{r}_i^k$. From equation (36), $\bar{r}_i^H = \bar{r}_i^F$ is determined by equilibrium φ_i^{*k} and factor rewards for which we have solved. From equation (39), R_i^k is determined by wages and labor allocations in each country for which we have also solved. We have thus completed our characterization of the FPE equilibrium vector.

We have already established that, for country factor endowments within the FPE set, free trade equilibrium wages equal their value in the integrated equilibrium. Since the free entry condition is the same in the two countries and a function of parameters alone, the free trade zero-profit cutoff productivities also equal their integrated equilibrium values. Thus, $\{\varphi_1^{*k}$, φ_2^{*k}, P_1^k, P_2^k, $p_1^k(\varphi)$, $p_2^k(\varphi)$, w_S^k, $w_L^k\}$ are the same as in integrated equilibrium. Aggregate revenue, industry revenue, the mass of firms, and labor allocations will vary across countries in the free trade equilibrium. However, their sum across countries equals the values in the integrated equilibrium. ■

A2. Proof of Proposition 2

Proof. The zero-profit productivity cutoff remains unchanged in the move from autarky to free trade because the free entry condition (13) uniquely pins down φ_i^{*k} as a function of model parameters alone. With the zero-profit cutoff productivity unchanged, weighted average productivity, $\widetilde{\varphi}_i^k$, in equation (12) will also remain the same. ■

A3. Proof of Proposition 3

Proof. We choose the skilled wage in one country as numeraire, $w_S^H = 1$.
Suppose that the equilibrium wage vector $\{1,\, w_L^H,\, w_S^F,\, w_L^F\}$ is known.
The free trade equilibrium allocations of skilled and unskilled labor in equations (48) and (**??**) were determined using labor market clearing (equation (17)) and equilibrium industry factor intensities (equation (46)). The expressions for the costly trade equilibrium allocations of skilled and unskilled labor are the same, except that the relative unskilled wage will now generally vary across countries, so that terms in what was previously the common unskilled wage, w_L, need to be replaced with country-specific values for the relative unskilled wage, w_L^k / w_S^k for $k \in \{H, F\}$.
Using the costly trade analogues of equations (48) and (**??**), the wage vector uniquely pins down equilibrium labor allocations in home and foreign: $\{L_1^H,\, L_2^H,\, L_1^F,\, L_2^F,\, S_1^H,\, S_2^H,\, S_1^F,\, S_2^F\}$. These equilibrium allocations now include labor used in entry, production and exporting: $L_i^k = L_i^{kp} + L_i^{ke} + L_i^{kx}$ and $S_i^k = S_i^{kp} + S_i^{ke} + S_i^{kx}$.
Following the same line of reasoning as in the proof of Proposition 1, it may be shown that total industry payments to labor used in production, entry and exporting equal total industry revenue:

$$R_1^k = w_S^k S_1^k + w_L^k L_1^k, \qquad R_2^k = w_S^k S_2^k + w_L^k L_2^k. \tag{53}$$

Thus, the wage vector and equilibrium labor allocations uniquely pin down total industry revenue $\{R_1^H,\, R_2^H,\, R_1^F,\, R_2^F\}$ and hence each country's aggregate revenue $\{R^H,\, R^F\}$.
The pricing rule (18) determines equilibrium variety prices in the domestic and export markets for each country $\{p_{1d}^H(\varphi),\, p_{1x}^H(\varphi),\, p_{2d}^H(\varphi),\, p_{2x}^H(\varphi),\, p_{1d}^F(\varphi),\, p_{1x}^F(\varphi),\, p_{2d}^F(\varphi),\, p_{2x}^F(\varphi)\}$ as a function of the wage vector.
With wages, variety prices, total industry revenue, and aggregate revenue known, the equilibrium zero-profit cutoff productivities $\{\varphi_1^{*k},\, \varphi_2^{*k}\}$, the exporting-cutoff productivities $\{\varphi_{1x}^{*k},\, \varphi_{2x}^{*k}\}$, and price indices $\{P_1^k,\, P_2^k\}$ are the solution to the system of six simultaneous equations in each country k defined by (27), (24) and (28) for the two industries. In solving this system of six simultaneous equations in each country, we substitute out for the equilibrium mass of firms, $M_i^k = R_i^k / \bar{r}_i^k$, probability of exporting, $\chi_i^k = \frac{[1-G(\varphi_{ix}^{*k})]}{[1-G(\varphi_i^{*k})]}$, and average firm revenue, $\bar{r}_i^k = \left(\frac{\widetilde{\varphi}_i^k(\varphi_i^{*k})}{\varphi_i^{*k}}\right)^{\sigma-1} \sigma f_i (w_S^k)^{\beta_i} (w_L^k)^{1-\beta_i}$, using the fact that these are functions of elements of the six unknowns $\{\varphi_1^{*k},\, \varphi_2^{*k},\, \varphi_{1x}^{*k},\, \varphi_{2x}^{*k},\, P_1^k,\, P_2^k\}$ as well as the known wage vector and equilibrium industry revenue for which we have already solved.
Thus, given the wage vector $\{1,\, w_L^H,\, w_S^F,\, w_L^F\}$, we have solved for all other elements of the equilibrium vector $\{\varphi_1^{*k},\, \varphi_2^{*k},\, \varphi_{1x}^{*k},\, \varphi_{2x}^{*k},\, P_1^k,\, P_2^k,\, p_1^k(\varphi),\, p_2^k(\varphi),\, p_{1x}^k(\varphi),\, p_{2x}^k(\varphi),\, R^k\}$ for $k \in \{H, F\}$.
The equilibrium wage vector itself is pinned down by the requirement that the value of total industry revenue, $R_i^k = w_S^k S_i^k + w_L^k L_i^k$, equals the sum of domestic and foreign expenditure on domestic varieties (equation (29) for each country and industry). ■

A4. Proof of Proposition 4

Proof. Under autarky, the free entry condition is given by the expression in equation (13) for the integrated world economy. Under costly trade, the free entry condition becomes (27), where the relationship between the productivity cutoffs is governed by equation (24), so that $\varphi_{ix}^{*k} = \Lambda_i^k \varphi_i^*$. The expected value of entry, V_i^k, in equation (27) equals its value in the closed economy (equation (13)), plus a positive term reflecting the probability of drawing a productivity high enough to export.
Since, using equation (24), V_i^k is monotonically decreasing in φ_i^{*k}, it follows that equilibrium φ_i^{*k} must be higher in both industries under costly trade than under autarky. This is required in order for V_i^k to equal the unchanged sunk entry cost f_{ei}.
Since weighted average productivity $\tilde{\varphi}_i$ is monotonically increasing in φ_i^*, it follows that the opening of costly trade must lead to an increase in $\tilde{\varphi}_i$ in both industries.
(a) At the free trade equilibrium, the relative price indices of the two sectors are the same in the two countries and are determined according to equation (15). Under autarky, the relative price indices generally differ across countries k and are given by:

$$\frac{P_1^k}{P_2^k} = \left(\frac{M_1^k}{M_2^k}\right)^{\frac{1}{1-\sigma}} \frac{p_1^k(\widetilde{\varphi}_1^k)}{p_2^k(\widetilde{\varphi}_2^k)}. \tag{54}$$

The mass of firms M_i^k is determined according to $M_i^k = R_i^k/\bar{r}_i^k$, equilibrium average revenue $\bar{r}_i$ is given by equation (36), while, under autarky, $R_i^k = \alpha_i R^k$. Substituting for the relative mass of firms in the above, and simplifying using the pricing rule (18), the autarky relative price index becomes:

$$\frac{P_1^k}{P_2^k} = \left(\frac{\alpha}{1-\alpha}\right) \frac{\varphi_2^{*k}}{\varphi_1^{*k}} \left(\frac{f_2}{f_1}\right)^{\frac{1}{1-\sigma}} \left(\frac{w_L^k}{w_S^k}\right)^{\frac{\sigma(\beta_1-\beta_2)}{1-\sigma}}. \tag{55}$$

To make comparisons across the two countries under autarky, we require consistent units of measurement and we choose skilled labor as the numeraire in each country, so that $w_S^H = 1$ and $w_S^F = 1$. The closed economy relative unskilled wage is given by equation (44), substituting a country's relative endowments for world relative endowments (exploiting the fact that the integrated world economy is closed). From equation (44), the closed economy with a larger relative supply of skilled labor is characterized by a higher relative wage of unskilled workers, w_L^k.
In equation (55), $\beta_1 > \beta_2$ and $\sigma > 1$, while identical technologies implies $\varphi_i^{*H} = \varphi_i^{*F}$. Hence, the higher relative wage of unskilled workers in the skill-abundant closed economy is reflected in a lower relative price index for the skill-intensive good: $P_1^H/P_2^H < P_1^F/P_2^F$.
Under costly trade, from equation (28), the relative price indices may be expressed as:

$$\frac{P_1^k}{P_2^k} = \left[\frac{M_1^k \left(p_{1d}^k(\widetilde{\varphi}_1^k)\right)^{1-\sigma} + \chi_1^j M_1^j \left(\tau_1\, p_{1d}^j(\widetilde{\varphi}_{1x}^j)\right)^{1-\sigma}}{M_2^k \left(p_{2d}^k(\widetilde{\varphi}_2^k)\right)^{1-\sigma} + \chi_2^j M_2^j \left(\tau_2\, p_{2d}^j(\widetilde{\varphi}_{2x}^j)\right)^{1-\sigma}}\right]^{1/(1-\sigma)}, \tag{56}$$

for $k, j \in \{H, F\}$, $j \neq k$. As $\tau_i \to \infty$ and $f_{ix} \to \infty$ for $i \in \{1, 2\}$, the costly trade relative price index converges to its autarkic value. In equation (56), $\chi_i^k \to 0$, while M_i^k and $p_{1d}^k(\widetilde{\varphi}_1^k)$ converge to their autarky values.
As $\tau_i \to 1$ and $f_{ix} \to 0$ for $i \in \{1, 2\}$, the costly trade relative price index converges to its common free trade value. In equation (56), $\chi_i^k \to 1$, while M_i^k, $p_{id}^k(\widetilde{\varphi}_i^k)$ and $p_{id}^k(\widetilde{\varphi}_{ix}^k)$ converge

to their free trade values, where $p_{id}^{k}(\widetilde{\varphi}_{i}^{k}) = p_{id}^{k}(\widetilde{\varphi}_{ix}^{k}) = p_{id}^{j}(\widetilde{\varphi}_{i}^{j}) = p_{id}^{j}(\widetilde{\varphi}_{ix}^{j})$.
For intermediate fixed and variable trade costs where selection into export markets occurs, the relative price indices will lie in between the two countries' autarky values and the common free trade value: $P_1^H/P_2^H < P_1^F/P_2^F$.
In the absence of cross-industry differences in τ_i or f_{ix}/f_i, this pattern of variation in relative price indices implies, from equation (30), that Λ_i^k will be smaller in a country's comparative advantage industry than in the country's comparative disadvantage industry ($\Lambda_1^H < \Lambda_2^H$ and $\Lambda_2^F < \Lambda_1^F$). Therefore, the additional positive term in the costly trade free entry condition, not present under autarky, will be larger in a country's comparative advantage industry. Because V_i^k is monotonically decreasing in φ_i^{*k}, the rise in φ_i^{*k} following the opening of costly trade must be greater in a country's comparative advantage industry in order for V_i^k to equal the unchanged sunk entry cost f_{ei}.
Since weighted average productivity $\tilde{\varphi}_i$ is monotonically increasing in φ_i^*, the larger increase in φ_i^* results in a larger increase in $\tilde{\varphi}_i$ in a country's comparative advantage industry.
(b) Since $\varphi_{ix}^{*k} = \Lambda_i^k \varphi_i^{*k}$ and we have established that Λ_i^k will be smaller in a country's comparative advantage industry, it follows that the exporting productivity cutoff will be closer to the zero-profit productivity cutoff in a country's comparative advantage industry. ■

A5. Proof of Proposition 5

Proof. Under autarky, the free entry condition is given by the expression in equation (13) for the integrated world economy. Using $\bar{\pi}_i = \bar{r}_i/\sigma - f_i w_S^{\beta_i} w_L^{1-\beta_i}$, $\bar{r}_i = \bar{p}_i \bar{q}_i$, the equilibrium pricing rule in equation (4), and cancelling terms in factor prices, the autarky free entry condition may be rewritten as:

$$\left[1 - G\left(\varphi_i^{*Aut}\right)\right]\left[\frac{\bar{q}_i^{Aut}}{(\sigma-1)\,\tilde{\varphi}_i^{Aut}} - f_i\right] = \delta f_{ei}$$

where the superscript ^{Aut} denotes autarky. The costly trade free entry condition is given by equation (26). Using $\bar{\pi}_{id} = \bar{r}_{id}/\sigma - f_i w_S^{\beta_i} w_L^{1-\beta_i}$, $\bar{\pi}_{ix} = \bar{r}_{ix}/\sigma - f_{ix} w_S^{\beta_i} w_L^{1-\beta_i}$, $\bar{r}_{id} = \bar{p}_{id}\bar{q}_{id}$, $\bar{r}_{ix} = \bar{p}_{ix}\bar{q}_{ix}$, the equilibrium pricing rule in equation (18), and cancelling terms in factor prices, the costly trade free entry condition can be rewritten as:

$$\left[1 - G\left(\varphi_i^{*CT}\right)\right]\left[\frac{\bar{q}_i^{CT}}{(\sigma-1)\,\tilde{\varphi}_i^{CT}} - f_i + \chi_i\left[\frac{\tau_i \bar{q}_{ix}^{CT}}{(\sigma-1)\,\tilde{\varphi}_{ix}^{CT}} - f_{ix}\right]\right] = \delta f_{ei}$$

where the superscript ^{CT} indicates costly trade. Comparing the two expressions, we know: $\varphi_i^{*CT} > \varphi_i^{*Aut}$, $\tilde{\varphi}_i^{CT} > \tilde{\varphi}_i^{Aut}$, $\tilde{\varphi}_{ix}^{CT} > \tilde{\varphi}_i^{CT}$ and $f_{ix} + f_i > f_i$. In order for the expected value of entry on the left-hand side of each expression to equal the sunk entry cost times the probability of firm death on the right-hand side, we require $\bar{q}_i^{CT} + \chi_i \tau_i \bar{q}_{ix}^{CT} > \bar{q}_i^{Aut}$. That is, average output sold in the domestic market plus average output produced for the export market (including the output lost as a result of variable trade costs) under costly trade exceeds average output produced for the domestic market under autarky. This is true for both industries, but since from Proposition 4 the increase in the zero-profit productivity cutoff following the opening of costly trade is greatest in the comparative advantage industry, the increase in average firm size is greatest in the comparative advantage industry. ■

A6. Proof of Proposition 6

Proof. The larger increase in the zero-profit productivity cutoff, φ_i^{*k}, in the country's comparative advantage industry results in a larger increase in weighted average productivity, $\widetilde{\varphi}_i^k$, in the comparative advantage industry. Since this is true for both countries, the opening of costly trade results in the emergence of endogenous Ricardian productivity differences at the industry level, which are positively correlated with Heckscher-Ohlin-based comparative advantage ($\widetilde{\varphi}_1^H/\widetilde{\varphi}_2^H > \widetilde{\varphi}_1^F/\widetilde{\varphi}_2^F$). ■

A7. Proof of Proposition 7

Proof. (a) The relative unskilled wage under autarky is given by equation (44). The relative unskilled wage under free trade with factor price equalization is given by equation (44) for the integrated world economy, where world relative factor endowments are substituted for the country's own relative factor endowments. The relative unskilled wage under costly trade lies in between these two values, converging to the autarkic value as trade costs become infinite and the free trade value as trade costs approach zero. Since the home country is skill abundant and the foreign country is labor abundant ($S^H/L^H > S^{World}/L^{World} > S^F/L^F$), the opening of costly trade leads to a rise in the relative skilled wage and a reduction in the relative unskilled wage in the skill-abundant home country.
(b) Industry price indices under costly trade are given by equation (28). The corresponding expression under autarky is:

$$P_i^H = \left[M_i^H \left(p_i^H \left(\tilde{\varphi}_i^H\right)\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}. \tag{57}$$

Since variety prices are monotonically decreasing in productivity, the rise in weighted average productivity in both industries and countries following the opening of costly trade reduces consumer price indices.
(c) The mass of domestically produced varieties is equal to $M_i = R_i/\bar{r}_i$ where $\bar{r}_i = \bar{p}_i\bar{q}_i$. Other things equal, the rise in average firm output, $\bar{q}_i$, reduces the mass of domestically produced varieties. Since consumer price indices are monotonically decreasing in the mass of varieties, this raises consumer price indices.
(d) Industry price indices under costly trade and autarky are given by equation (28) and equation (57) respectively. Other things equal, the potential to import foreign varieties expands the range of varieties available to domestic consumers which reduces consumer price indices. ■

A8. Proof of Proposition 8

Proof. (a) Combining cost minimization and factor market clearing, the equilibrium allocation of skilled and unskilled labor to the two sectors is determined according to:

$$\begin{aligned}
\lambda_{L1}^k\left(\frac{\beta_1}{1-\beta_1}\frac{w_L^k}{w_S^k}\right)+\left(1-\lambda_{L1}^k\right)\left(\frac{\beta_2}{1-\beta_2}\frac{w_L^k}{w_S^k}\right) &= \frac{\overline{S}^k}{\overline{L}^k}, \qquad \lambda_{Li}^k \equiv \frac{L_i^k}{\overline{L}^k}\\
\lambda_{S1}^k\left(\frac{1-\beta_1}{\beta_1}\frac{w_S^k}{w_L^k}\right)+\left(1-\lambda_{S1}^k\right)\left(\frac{1-\beta_2}{\beta_2}\frac{w_S^k}{w_L^k}\right) &= \frac{\overline{L}^k}{\overline{S}^k}, \qquad \lambda_{\mathrm{Si}}^k \equiv \frac{S_i^k}{\overline{S}^k}.
\end{aligned}$$

where superscript ^{k} denotes countries. Since $\beta_1 > \beta_2$, the fall in the relative unskilled wage in the skill-abundant country following the opening of costly trade leads to a rise in the share of both skilled and unskilled labor employed in the skill-intensive industry. With unchanged factor endowments, this implies net job creation in the comparative advantage industry and net job destruction in the comparative disadvantage industry.

(b) The zero-profit productivity cutoff condition and the exporting productivity cutoff condition in equation (23) imply the following values for the output of the least productive firm active in the domestic and export markets respectively: $q_{id}(\varphi_i^*) = \varphi_i^*(\sigma-1) f_i$ and $q_{ix}(\varphi_{ix}^*) = \varphi_{ix}^*(\sigma-1) f_{ix}/\tau_i$. The equilibrium pricing rule (18) implies that the relative output of two firms with different productivities within the same market depends solely on their relative productivities: $q(\varphi'') = (\varphi''/\varphi')^\sigma q(\varphi')$. Therefore, equilibrium firm output under autarky may be expressed as follows:

$$q_{id}^{Aut}(\varphi) = (\varphi)^\sigma \left(\varphi_i^{*Aut}\right)^{1-\sigma}(\sigma-1) f_i$$

where superscript ^{Aut} indicates autarky. Equilibrium output under costly trade depends on whether a firm exports and may be expressed as:

$$q_i^{CT}(\varphi) = \begin{cases} (\varphi)^\sigma \left(\varphi_i^{*CT}\right)^{1-\sigma}(\sigma-1) f_i & \text{no exports} \\ (\varphi)^\sigma \left(\varphi_i^{*CT}\right)^{1-\sigma}(\sigma-1) f_i + (\varphi)^\sigma \left(\varphi_{ix}^{*CT}\right)^{1-\sigma}(\sigma-1) f_{ix}/\tau_i & \text{exports} \end{cases}$$

where superscript ^{CT} denotes costly trade.

Since $\varphi_i^{*CT} > \varphi_i^{*Aut}$ the opening of costly trade results in the exit of low-productivity firms and gross job destruction in both industries. Furthermore, comparing the expressions above and noting that $\varphi_i^{*CT} > \varphi_i^{*Aut}$ and $\sigma > 1$, the opening of costly trade reduces equilibrium output at surviving firms that only serve the domestic market in both industries. This provides another source of gross job destruction. Finally, we established in the proof of Proposition 5 that the opening of costly trade increases average firm output in both industries. Therefore, the output of some surviving exporters must rise in both industries, providing a source of gross job creation. ■

A9. Proof of Proposition 9

Proof. The steady-state rate of creative destruction corresponds to the steady-state probability of firm failure:

$$\Psi_i = \frac{G(\varphi_i^*) M_{ei} + \delta M_i}{M_{ei} + M_i}.$$

From the steady-state stability condition:

$$M_{ei}\left[1 - G(\varphi_i^*)\right] = \delta M_i.$$

Substituting for M_{ei} in the expression for the probability of firm failure and rearranging:

$$\Psi_i = \frac{\delta}{\delta + \left[1 - G(\varphi_i^*)\right]}$$

which is monotonically increasing in the zero-profit productivity cutoff φ_i^*. From the proof of Proposition 4, the zero-profit productivity cutoff is higher in the comparative advantage industry, which implies a greater steady-state rate of creative destruction in the comparative advantage industry. ■

B Proof of Proposition 10

Proof. Exports of country k in industry i in the heterogeneous-firm model are:

$$\begin{aligned} \left(X_i^k\right)^{Het} &= \alpha_i R^F \left(P_i^F\right)^{\sigma-1} \chi_i^H M_i^H \tau_i^{1-\sigma} p_{id}(\tilde{\varphi}_{ix})^{1-\sigma} \qquad (58) \\ &= \alpha_i R^F \left(P_i^F\right)^{\sigma-1} R_i^H \tau_i^{1-\sigma} \frac{(\sigma-1)^{\sigma-1}}{\sigma^\sigma} \left[w_S^{\beta_i} w_L^{1-\beta_i}\right]^{-\sigma} \Phi_i^H \end{aligned}$$

$$\text{where} \qquad \Phi_i^H = \frac{(\varphi_i^*)^{\sigma-1}}{\left(1/\chi_i^H\right)(\tilde{\varphi}_i/\tilde{\varphi}_{ix})^{\sigma-1} f_i + f_{ix}} \tag{59}$$

where the second equation in (58) uses $M_i = R_i/\bar{r}_i$, the pricing rule (18) and the expression for $\bar{r}_i$ under costly trade. Exports of country k in industry i in the homogeneous-firm model are:

$$\begin{aligned}\left(X_i^k\right)^{Homog} &= \alpha_i R^F \left(P_i^F\right)^{\sigma-1} M_i^H \tau_i^{1-\sigma} p_{id} \left(\tilde{\varphi}_i^{Aut}\right)^{1-\sigma} \\ &= \alpha_i R^F \left(P_i^F\right)^{\sigma-1} R_i^H \tau_i^{1-\sigma} \frac{(\sigma-1)^{\sigma-1}}{\sigma^\sigma} \left[w_S^{\beta_i} w_L^{1-\beta_i}\right]^{-\sigma} \Omega_i^H\end{aligned} \tag{60}$$

$$\text{where} \qquad \Omega_i^H = \frac{\left(\tilde{\varphi}_i^{Aut}\right)^{\sigma-1}}{f_i + f_{ix}} \tag{61}$$

where the second equation in (60) uses analogous relationships to those above in the homogeneous-firm model, and where the productivity of the representative firm in the homogeneous-firm model is set equal to weighted average productivity in the heterogeneous-firm model under autarky.
(a) Since $0 < \chi_i^H < 1$ in equation (59), this reduces Φ_i^H relative to Ω_i^H. The volume of trade in the heterogeneous-firm model is reduced due to only a subset of varieties being traded.
(b) We require $\varphi_i^* < \varphi_{ix}^* < \tilde{\varphi}_i^{Aut}$ in order for the representative firm in the homogeneous-firm model to export. From equations (59) and (61), this reduces Φ_i^H relative to Ω_i^H. The volume of trade in the heterogeneous-firm model is reduced because the increase in average productivity increases average firm output and reduces the mass of varieties produced.
(c) Since $\tilde{\varphi}_{ix} > \tilde{\varphi}_i$ in equation (59), this increases Φ_i^H relative to Ω_i^H. The volume of trade in the heterogeneous-firm model is increased, reflecting the higher average productivity of the varieties exported in the heterogeneous-firm model. ■

C Proof of Proposition 11

Proof. The measured net factor content of trade for country k equals:

$$\mathbf{A}^k \mathbf{m}^k = \mathbf{A}^k \mathbf{c}^k - \mathbf{A}^k \mathbf{y}^k$$

where bold face denotes a vector or matrix; $\mathbf{A}$ is the matrix of unit factor input requirements; $\mathbf{m}$ is the net import vector; $\mathbf{c}$ is the consumption vector; $\mathbf{y}$ is the output vector; s will be used below to denote a country's share of world consumption; and $\mathbf{V}$ will be used below to indicate the factor endowment vector.
Under the assumptions of the Heckscher-Ohlin-Vanek model, the predicted net factor content of trade may be derived as follows:

$$\begin{aligned}\mathbf{A}^k \mathbf{m}^k &= \mathbf{A}^k \mathbf{c}^k - \mathbf{V}^k &(62)\\ &= \mathbf{A}^k s^k \mathbf{c}^{World} - \mathbf{V}^k &(63)\\ &= s^k \mathbf{A}^k \mathbf{y}^{World} - \mathbf{V}^k &(64)\\ &= s^k \mathbf{V}^{World} - \mathbf{V}^k. &(65)\end{aligned}$$

Equation (63) exploits identical and homothetic preferences and frictionless trade. This equation no longer holds in the heterogeneous-firm model due to trade costs and the trade of only a fraction of varieties which acts as an additional trade friction. Equation (65) exploits identical technologies and factor price equalization. This no longer holds in the heterogeneous-firm model for two reasons. First, the magnification of comparative advantage results in variation in average productivity and hence unit factor input requirements that is non-neutral across industries. Second, trade costs and the magnification of comparative advantage result in non-factor price equalization and induce cross-country variation in unit factor input requirements. ■

C1. Numerical Solutions

We calibrate key parameters of the model to match features of the plant-level U.S. manufacturing data reported in Bernard *et al.* (BEJK) (2003). We set the elasticity of substitution $\sigma = 3.8$. The standard deviation of log US plant sales in BEJK (2003) is 1.67, and since the standard deviation of log sales in the model is $1/(a-\sigma+1)$, we set the Pareto shape parameter $a = 3.4$. This satisfies the requirement for the standard deviation of log firm sales to be finite with a Pareto distribution: $a > \sigma - 1$.

To focus on comparative advantage, we assume that all industry parameters except factor intensity (β_i) are the same across industries and countries. We consider symmetric differences in country factor endowments $\{\bar{S}^H = 1200,\ \bar{L}^H = 1000,\ \bar{S}^F = 1000,\ \bar{L}^F = 1200\}$ and symmetric differences in industry factor intensities $\{\beta_1 = 0.6,\ \beta_2 = 0.4\}$. The share of each good in consumer expenditure is assumed to equal one half ($\alpha_1 = \alpha = 0.5$).

Changing the fixed cost of entry, f_{ei}, rescales the mass of firms in an industry and, without loss of generality, we set $f_{ei} = f_e = 2$. We set the minimum value for productivity $k = 0.2$. Fixed production costs are set equal to 5% of fixed entry costs, $f = f_i = 0.1$. As a convenient normalization, we set fixed exporting costs equal to fixed production costs, $f_x = f_{ix} = f$, since this normalization ensures that all firms export when there are no variable trade costs ($\tau = 1$).

Exit in the model includes both the endogenous decision of firms with low productivity draws to leave the industry and exogenous death due to *force majeure* events. Changes in the probability of exogenous firm death, δ, rescale the mass of entrants relative to the mass of firms and, without loss of generality, we set $\delta = 0.025$.

Parameters that are common to the homogeneous and heterogeneous-firm models of imperfect competition and comparative advantage, such as the elasticity of substitution and factor intensities, are assumed to take the same value. In addition, we make the following two normalizations which ensure that the heterogeneous and homogeneous-firm models yield identical autarky equilibrium values of price indices, production, factor rewards and factor allocations. First, the common productivity parameter in the homogeneous-firm model is set equal to weighted average productivity under autarky in the heterogeneous-firm model. Second, there is no entry process in the homogeneous-firm model and no uncertainty regarding productivity. Therefore the fixed production cost in the homogeneous-firm model is set equal to the fixed production cost in the heterogeneous-firm model, plus the per period value of the sunk entry cost over the heterogeneous firm's expected lifetime scaled by the probability under autarky of a heterogeneous firm drawing a productivity above the exit threshold. The scaling takes account of the fact that the entry cost is paid by all heterogeneous firms not only those who successfully produce, so that we obtain: $f_i^{\text{Hom}} = f_i^{\text{Het}} + \delta f_{ei} / \left[1 - G\left(\varphi_i^{*\text{AutarkyHet}}\right)\right]$.

	Comparative Advantage Industry		Comparative Disadvantage Industry	
	Job Turnover	Decline From Autarky to 20%	Job Turnover	Decline From Autarky to 20%
Abundant Factor	Total	20.7	Total	14.3
	Between-Industry	7.3	Between-Industry	-7.3
	Within-Industry	13.3	Within-Industry	7.0
Scarce Factor	Total	15.6	Total	20.1
	Between-Industry	6.9	Between-Industry	-6.9
	Within-Industry	8.7	Within-Industry	13.2

Notes: Table displays jobs added and lost as a percent of countries' total labor force in response to noted decline in variable trade costs (i.e. one worker changing jobs results in one job loss plus one job gain). Between-industry (i.e. net) job turnover refers to the net number of jobs added to (+) or lost from (-) the industry. Within-industry turnover refers to jobs lost and gained in the same industry. Total (i.e. gross) job turnover is the sum of the absolute value of the between- and within-industry components.

Table 1: Job Turnover as Trade Costs Fall

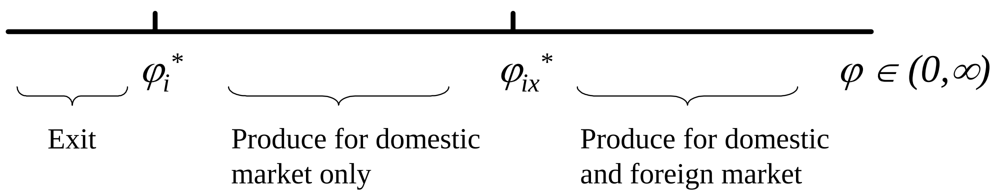


Figure 1: Zero-profit and exporting productivity cutoffs with costly trade

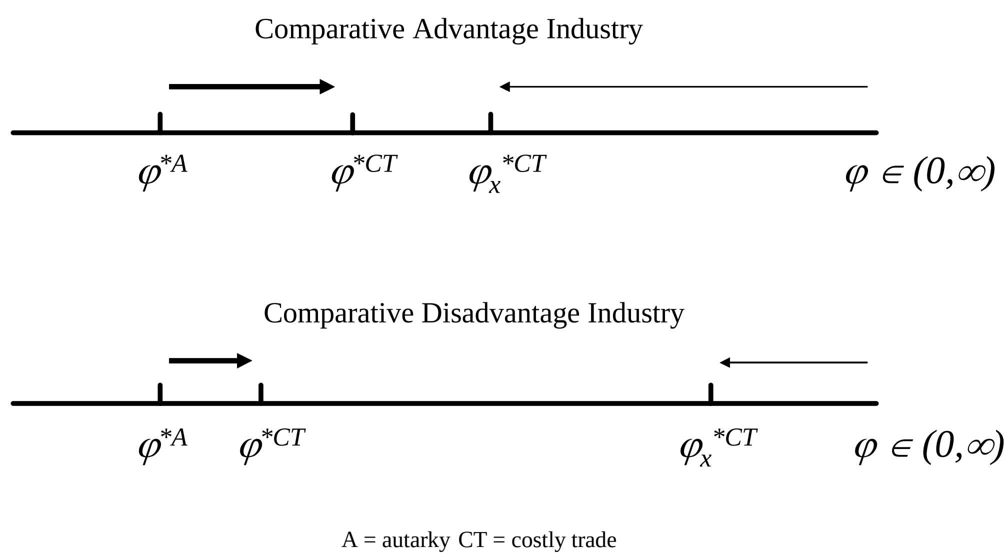


Figure 2: From autarky to costly trade: differential movements of the productivity cutoffs across industries

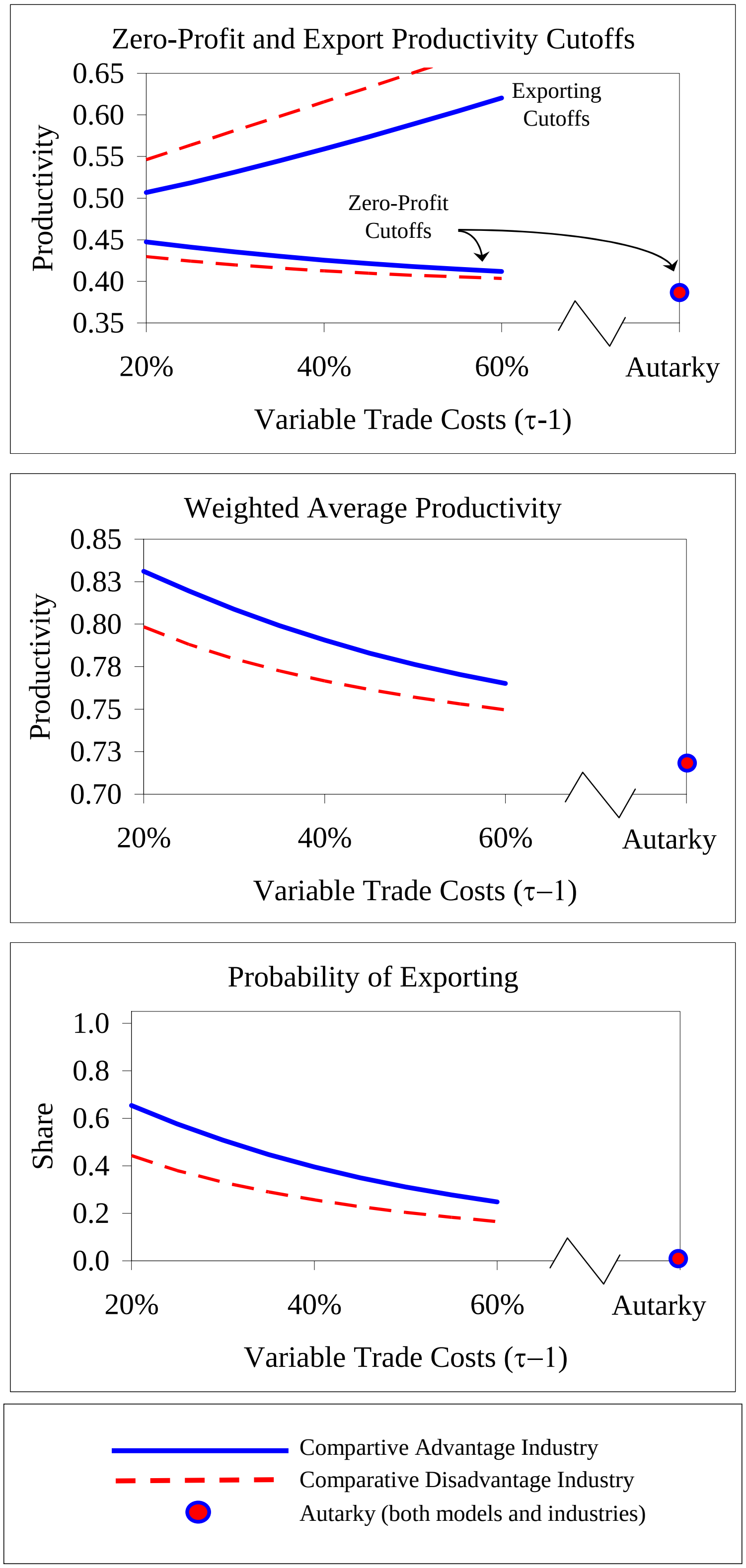


Figure 3: Productivity Cutoffs, Average Productivity and Probability of Exporting as Trade Costs Fall

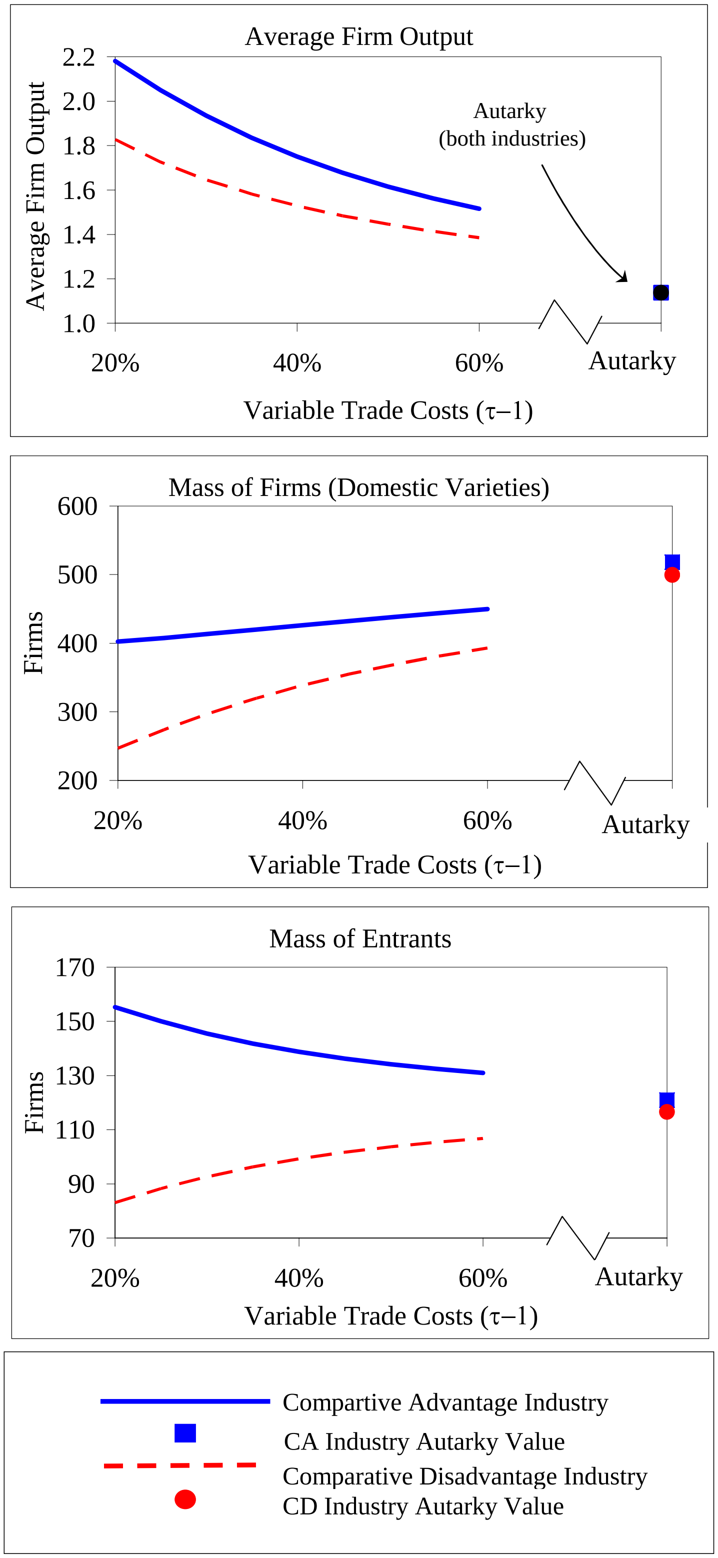
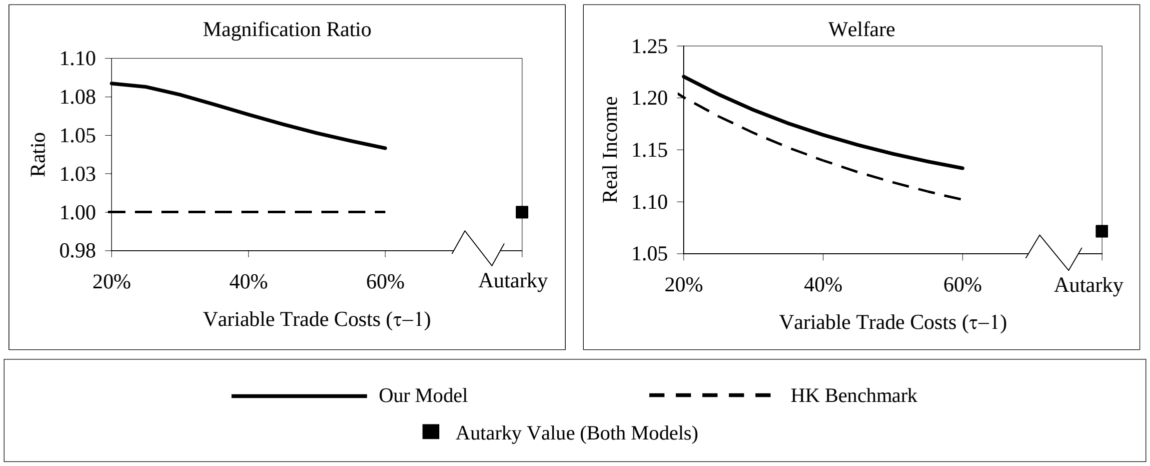
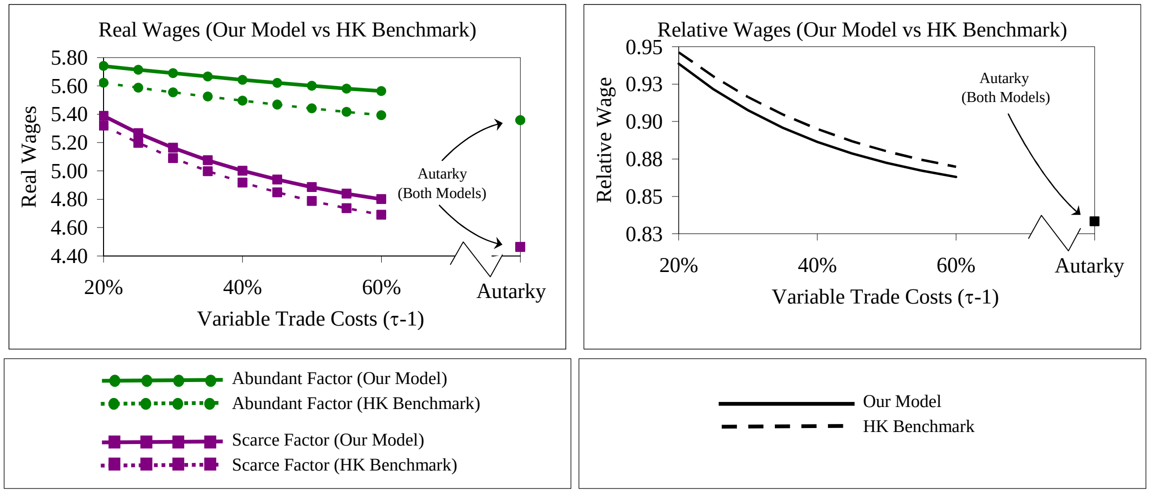


Figure 4: Firm Size and Number of Firms as Trade Costs Fall



Note: The HK Benchmark trends are based on a Helpman-Krugman (1985) model with fixed and variable costs of exporting. Magnification Ratio is the ratio of Home versus Foreign average industry productivity in the comparative advantage industry to the same relative quantity for the comparative disadvantage industry. See text for formal definition.

Figure 5: Welfare and the Magnification of Comparative Advantage as Trade Costs Fall



Notes: The HK Benchmark trends are based on a Helpman-Krugman (1985) model with fixed and variable costs of exporting.

Figure 6: Real Wages as Trade Costs Fall

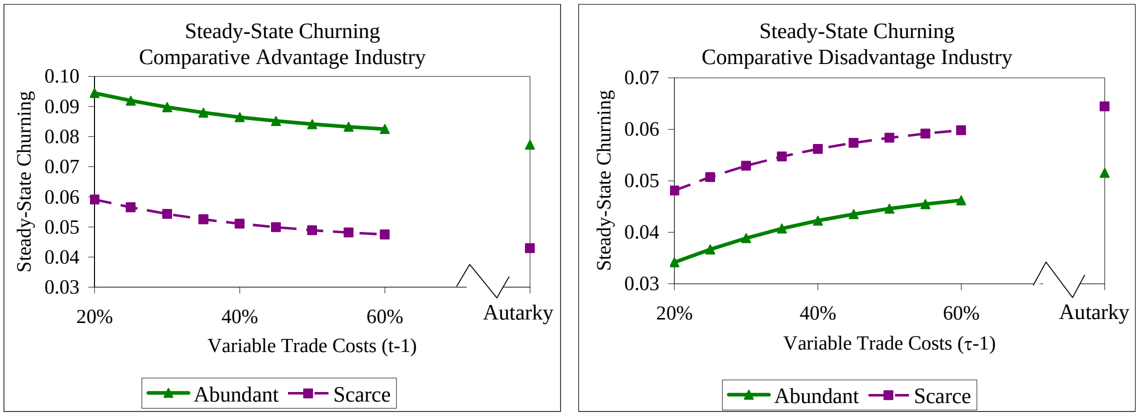
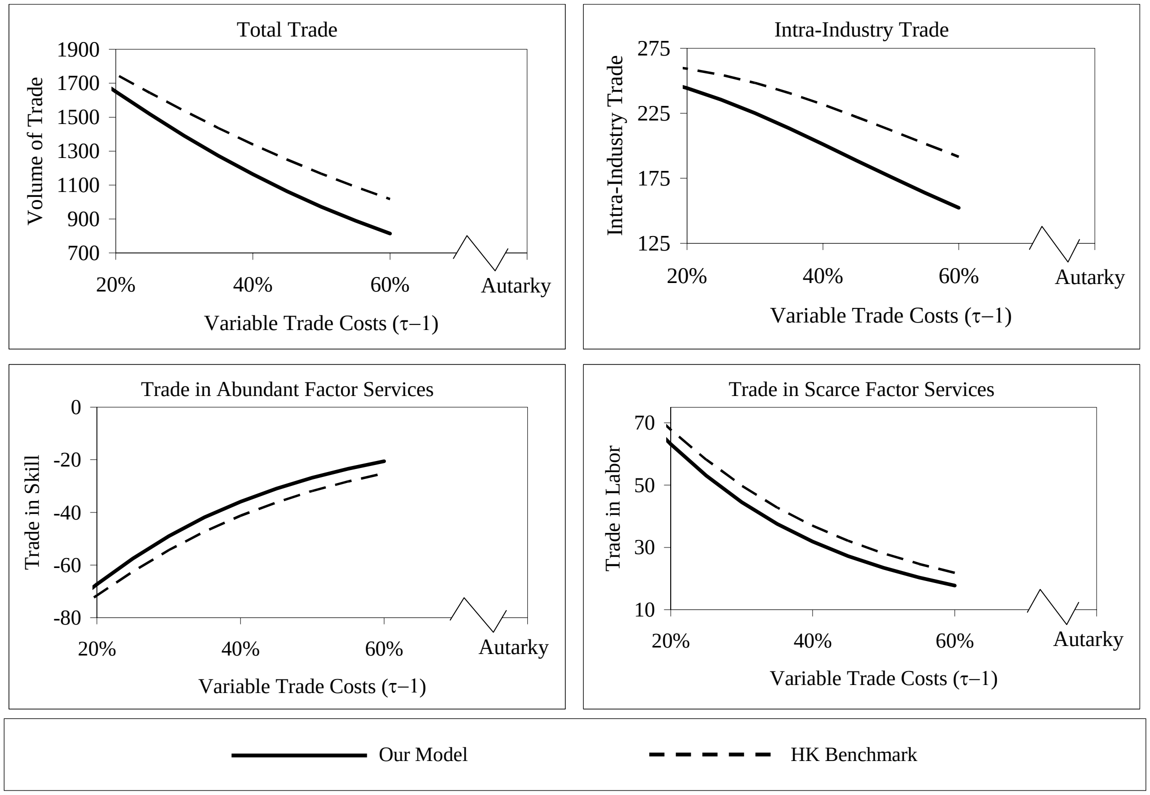


Figure 7: Steady State Employment Churning as Trade Costs Fall



Notes: The HK Benchmark trends are based on a Helpman-Krugman (1985) model with fixed and variable costs of exporting. Autarky trade flows are zero in each graph.

Figure 8: Inter- and Intra-Industry Trade as Trade Costs Fall