International Capital Flows meet Corporate Liquidity Demand[*]

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Abstract

There is a huge literature on corporate liquidity demand. The implications of this theory for international macro, however, are poorly studied. We extend the Holmstroem-Tirole (1998) model on corporate liquidity demand to a two country world. Introducing dual agency problems as well as differing agency costs allows us to link the share of foreign capital holdings with the extent of liquidity shock resistance of the domestic economy. When capital is scarce, a higher share of foreign capital implies a more vulnerable domestic economy. Thus countries that want to open up their capital account face a trade-off: a higher level of investment versus an increased vulnerability with respect to liquidity shocks. We find that less developed countries are more severely affected by this trade-off and are thus more likely to resort to policy instruments such as capital controls. We further show that domestic capital scarcity will place restrictions on foreign capital inflows. The smaller the domestic capital base, the smaller the amount of foreign capital that can be attracted. This explains why most international capital flows occur between rich countries and offers a fresh view on the missing catch-up predicted in neoclassical growth models.

JEL Classification: F34, F41, G30

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1 Introduction

There is a quite extensive literature on corporate liquidity demand. This literature explains the failure of the financial markets to provide sufficient liquidity in times of crisis using assumptions and modelling devices like asymmetric information, principal agents structure, moral hazard and adverse selection.[1] In this sense these model have a strong industrial organizations and contract theory background and it is not surprising that most of these models are micro models. Questions that have been posed (and answered) are, for example, "What is the role of the government in supplying liquidity?" Holmstroem and Tirole (1998), "What are the implications of the distribution of wealth for investment?" Holmstroem and Tirole (1997) and "What are asset prices like under liquidity constraints?" Holmstroem and Tirole (2001). On the macro level, however, applications of the main results of this literature are hard to find. The most prominent model here is perhaps the Kyotaki-Moore model (Kiyotaki and Moore (1997)), that does not deal with liquidity demand directly but rather assumes it, by introducing a Leontief technology. Firms thus produce using two types of capital, using the illiquid one as collateral to borrow against. Other examples that use elements of corporate liquidity demand in a macro context are Bernake, Gertler and Gilchrist (1998) and Bernake and Gilchrist (1996). Here liquidity demand is brought into the picture by focusing on the financing decision of a firm that uses two types of capital: fixed installed and a variable form. This demand in turn is used to link micro conditions, i.e. a weak financial sector or a low net-worth, with macroeconomic performance. Overall, however, we find that the implications of the findings of this strand of literature for international macro, although interesting and of high relevance, are poorly studied and rarely used in macro models. In particular, the specific form of liquidity that is modelled by Holmstroem and Tirole has to date, to the best of our knowledge, not been brought into a macro framework.

This paper brings the sound micro structure of the corporate liquidity demand models into a framework that is suited to address macroeconomic questions. To this end we extend the Holmstroem-Tirole (see Holmstroem and Tirole (1998)) model to allow for international capital movements. First we study the implications for the second-best contract if the home country is in autarky and investment resources are scarce. This will lead to a redistribution of the net-social surplus and under some additional conditions to a less vulnerable economy with respect to liquidity shocks. We then in-

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[1]Bengt Holmstroem and Jean Tirole have been very important for this literature. Main contributions include Holmstroem and Tirole (1998),Holmstroem and Tirole (1997),Holmstroem and Tirole (1996)and Holmstroem and Tirole (2002).

troduce international capital flows into our model. We use the concept of a dual-agency problem (for an introduction with some empirical evidence see chapter 5 in Tirole (2002)) to distinguish between domestic and foreign investors. Investors in general not only have a contract with the entrepreneurs but also an implicit one with the home government that protects their rights. Domestic interests will usually be better protected than foreign ones. Thus foreign investors face higher agency costs when investing abroad. It turns out that under this setup the vulnerability of the domestic economy with respect to liquidity shocks increases relative to the autarky case: the more foreign money firms have to raise the narrower will be the bandwidth of feasible liquidity shocks. If a large enough liquidity shock materializes, foreigners will withdraw all their money from domestic projects, even though home investors would have remained.

Our second result is that domestic capital scarcity can be the reason for limited foreign capital supply. In the context of our analysis this amounts to saying that if the domestic economy does not have a sufficiently large capital base, the level of aggregate investment will be constrained, even if there is foreign capital abundance. This result, again, hinges on the dual-agency structure assumption, i.e. that investments made by foreigners are less well protected than those made by domestic investors. Countries for which this result bears most relevance are, of course, less developed countries that have a relatively small capital base. Using our second result we are able to address a broad range of questions in international economics and growth theory, for example the phenomenon that most capital flows occur between rich countries(Lucas (1990)) or the missing catch-up that is predicted in neoclassical growth models.

The results of our paper match quite well with recent findings in the empirical literature on the effects of capital account liberalization. If we were to use the classification of the World Bank for low, middle and high income countries, our results would predict that most of the benefits of capital account liberalization accrue to middle income countries. The reason for this is that low income countries would only have restricted access to foreign capital and high income countries would have a sufficiently large capital base to begin with. This is in accordance to recent findings that point to an inverted U-shape distribution of the gains from capital account liberalization, i.e. low and high income countries gaining insignificantly in the process. This view is voiced for example in Klein (2003) and Edison, Klein, Ricci and Sloek (2002).

We see our work as contribution to the huge literature that discusses the

effects of capital account liberalization[2]. We show, that from a corporate liquidity demand perspective the composition of debt matters. In a world where capital is scarce a high ratio of foreign capital to overall domestic investment renders the economy more vulnerable to liquidity shocks. Thus the good that international capital flows bring about, i.e. an increased level of investment also has some severe side effects. This side effects are especially strong in countries that either receive a lot of foreign capital (relative to domestic capital) or suffer from particularly high agency costs for foreign investors (or both). Natural candidates for this type of countries are, again, less developed countries, which renders the results of our work relevant for the globalization issue and may provide a rationale why capital controls are a widespread policy instrument in this group of countries.

The paper is organized as follows. The first section will shed some light on the workings of the Holmstroem-Tirole (HT) model that serves as our starting and reference point throughout. The second chapter then extends the HT-model. The first extension deals with the implications of the HT-model under autarky. The following section derives the main results of our paper under free capital flows. In the next section we turn to discuss the severity of the assumptions with respect to the investment levels and show that foreign capital scarcity may arise without limits placed by foreigners. Section four then turns to discuss the empirical implications of our model and section five concludes.

2   The workings of the Holmstroem-Tirole model

In what follows we will devote some space to replicate and illustrate some of the results of the original HT-model. Our analysis will draw heavily on this results, therefore we discuss them in some length. We make use of the notation of the original paper. This will facilitate understanding for readers already familiar with this class of models.

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[2]There are a great many contributions to this literature from many different perspectives. The majority appears to take a growth perspective see, inter alia, Klein (2003), Bekaert, Harvey and Lundblad (2004) and Singh (2003). Recent empirical evidence is captured in Eichengreen (2001) and Edison et al. (2002) provide a thourough survey. Other aspects are discussed in Bacchetta (1992) (for the interaction of capital account liberalization and domestic financial liberalization), Kim (2003) (on how the budget deficit is influenced by opening up the capital account), Gruben and McLeod (2002) (on inflation dampening effects) and Bartolini and Drazen (1997) (the information content of policies regarding the capital account).

2.1 Description of the model

The economy is populated by two types of agents: entrepreneurs (or firms) and investors (or consumers). There is a continuum of entrepreneurs with unit mass. Holmstroem-Tirole assume that all entrepreneurs possess a constant returns technology so the assumption of identical endowments for each firm can be made. The discussion will then be, of course, in terms of the representative entrepreneur. The model has three periods. In period t_0 the entrepreneurs can begin a project that pays off a return R in period t_2 if it succeeds and nothing in case of failure. To start the project, the entrepreneurs require funds. There are two sources of these funds in the model: inside finance, i.e. the endowment of the entrepreneurs A and outside finance, i.e. what the investors have to add to A in order to reach the overall amount of investment I. In t_2 the project, in the case of success, thus pays out RI. In period t_1 there is a liquidity shock ρ that requires the investors to pay an additional amount ρI for the project to be continued - there is no further endowment the entrepreneurs have, i.e. no second period endowment. The liquidity shock is stochastic and follows a density function $f(\rho)$. If the liquidity shock cannot be paid, the project is terminated and pays off nothing. If, in period t_1, continuation of the project is decided the entrepreneurs have the choice about the effort they put into the project. This effort in turn results in two distinct success probabilities: p_H if effort is exerted and p_L if they shirk, where $p_H > p_L$. There are private benefits from shirking that accrue to the entrepreneurs of the amount $BI > 0$. A necessary condition for an interior solution to this problem is that the net present value of an investment stream $(t_0 = I; t_1 = \rho I)$ is positive when effort is exerted in period t_2 (otherwise there would not be much point in investing in the first place). Formally this amounts to

$$I \int_0^\infty \max\left(p_H R - \rho, 0\right) f(\rho)\, d\rho > I. \tag{1}$$

This condition implicitly defines the so-called first best cutoff: $\rho_1 = p_H R$. This cutoff defines a range of liquidity shocks $[0, \rho_1]$ within which continuation of the project generates a surplus and hence is socially desirable. If in turn the entrepreneur shirks, the net present value of the project is negative:

$$I \int_0^\infty \max\left(p_L R + B - \rho, 0\right) f(\rho)\, d\rho < I.$$

This two conditions taken together ensure that only contracts that implement the action p_H are feasible.

2.2 Solving the model

It is clear from this setup that the first-best cutoff ρ_1 cannot be reached, since outside liquidity is needed in t_1 and the firms have to be promised a certain share of the expected profits to implement the action p_H. This in turn means that in t_1 not all expected unit profits can credibly be promised to outside investors, rendering the (theoretically possible) dilution of outside claims up to the amount ρ_1 not feasible anymore. We are therefore looking for a second-best solution. The optimal (second-best) contract specifies:

1. what the entrepreneurs (and hence investors) will get in case of success: $p_H R_f(\rho)$

2. the amount of investment undertaken in period t_0: I and

3. a state contingent rule what to do in period t_1 when the shock materializes: $\lambda(\rho)$.

Formally the second-best contract solves

$$\max_{R_f(\rho),I,\lambda(\rho)} I \int_0^\infty p_H R_f(\rho) \lambda(\rho) f(\rho) d\rho - A \tag{2}$$

$$s.t.$$

$$I \int_0^\infty (p_H[R - R_f(\rho)] - \rho) \lambda(\rho) f(\rho) d\rho \geq I - A$$

$$R_f(\rho) p_H \geq B + R_f(\rho) p_L \quad \forall \rho.$$

The first condition simply states that the entrepreneur's goal is to maximize his profits. The second condition is the participation constraint for the investors (simply a zero profit condition for the investors). This condition will bind at the optimal solution.[3] The last inequality is the incentive compatibility constraint. This condition states that in order for the entrepreneur not to shirk, he has to be paid a certain amount. It later turns out that this condition will be binding as well. The optimal contract hence boils down to specify a cutoff value $\overline{\rho}$[4] that can be interpreted in the following way: continue if and only if $\rho \leq \overline{\rho}$.[5] The resulting objective function of the entrepreneur can be written as follows:[6]

$$\max_{\overline{\rho}} U_f(\overline{\rho}) = m(\overline{\rho}) I \tag{3}$$

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[3]Note that the assumed underlying utility function $U = c_1 + c_2 + c_3$ is linear and implies no discount for future consumption.

[4]See 6.2 for proof of the optimality of a cut-off probability.

[5]The proof for this is in the appendix 6.2.

[6]The exact derivation is relegated to appendix 6.1.

where $m(\rho) = \int_0^{\overline{p}} (\rho_1 - \rho) f(\rho) dp - 1$. Equation (3) shows the expected share of each unit of investment that the firms can appropriate as a function of the threshold shock, i.e. the expected return per unit of gross investment for the firm. Investors in turn will, on average, only get their money back, which directly follows from the participation constraint holding with equality. It can be seen from (3) that the objective function reaches its maximum at the first-best cutoff ρ_1. Further condition (1) implies that $m(\rho_1) > 0$, hence expected profits will be positive. Using again the second equation of (2), we can derive the optimal amount of investment:

$$I = k(\overline{\rho}) A \tag{4}$$

where

$$k(\overline{\rho}) = \frac{1}{1 + \int_0^{\overline{p}} \rho f(\rho) d\rho - F(\overline{\rho}) \rho_0}.$$

Holmstroem-Tirole name the function $k(\overline{p})$ the equity multiplier. It holds in general (there is another condition necessary for that, namely $\int_0^{\rho_o} (\rho_o - \rho) f(p) d\rho < 1$) that $k(\rho_o) > 1$. This in turn means that the initial investment I is bigger than the endowment of the entrepreneurs at the cutoff ρ_0, formally that $I(\rho_0) - A > 0$. This cutoff is defined as

$$\rho_o = \left(p_H - \frac{B}{p_H - p_L} \right) R = \left(p_H - \frac{B}{\Delta p} \right) R$$

and marks the *pledegeable date 1 unit return*, i.e. the amount of money that can be in t_1 *at most* promised to outside investors, given that there are private benefits to the entrepreneurs B that cannot be appropriated by the outside investors. This moral hazard problem creates the shortfall of liquidity provision from the social optimal value ρ_1. The equity multiplier can also be less than one, if the cutoff value is set sufficiently high. This would in turn imply that in period t_0 we have $I - A < 0$ meaning that in period t_0 the entrepreneurs would not borrow. Total investment, i.e. $I - A + \rho I$ would, of course, still be positive for otherwise the discussion would be pointless. In their analysis Holmstroem-Tirole rule out this possibility, i.e. they assume that $I > A$ with the argument that "it seems natural to have the firm a net borrower in period 0"[7].

Within this very simple framework it remains to determine the second-best cutoff, i.e. the liquidity shock up to which projects are continued. Intuitively this second best solution should be somewhere between the first-best cutoff ρ_1 and the pledgeable unit return at date 1 ρ_0. This becomes clear from

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[7]See Holmstroem and Tirole (1998, p. 10.)

the following reasoning: as long as the cutoff value of the liquidity shock is chosen $\rho < \rho_o$ both investors and entrepreneurs choose to continue after the shock has materialized. If it is chosen to be above ρ_1, the net-present value of continuing the project is negative (by condition 1), and it would be better to leave the project. Within the interval things are less obvious. By lowering the cutoff the firms can increase the amount of money they can raise, by increasing it, they can withstand higher liquidity shocks and thus raise the marginal expected return on the initial investment, but at the same time they decrease initial outside investment, i.e.

$$\frac{dk\left(\rho\right)}{d\rho} < 0, \quad \frac{dm\left(\rho\right)}{d\rho} > 0.$$

The firms choose $\overline{\rho}$ to maximize their marginal gain per unit invested, formally

$$\max_{\overline{\rho}} U\left(\overline{\rho}\right) = m\left(\overline{\rho}\right) k\left(\overline{\rho}\right) A. \tag{5}$$

This cutoff value $\overline{\rho}$ then is determined by the first order condition[8]:

$$\int_{o}^{\overline{\rho}} F\left(\rho\right) d\rho = 1. \tag{6}$$

and hence gives an utility level of

$$U\left(\overline{\rho}\right) = \frac{\rho_1 - \overline{\rho}}{\overline{\rho} - \rho_o} A. \tag{7}$$

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[8]The necessary steps to derive (6) are in the appendix .

The following picture captures the basic properties of the second-best contract of the HT-world:

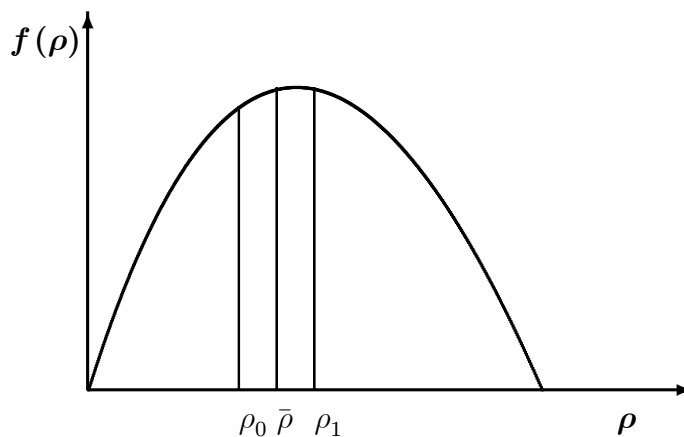$$Liquidity\ Shock\ and\ Cutoff\ Values$$

Figure 1: The Holmstroem-Tirole Model graphically

Figure 1 shows as an example a particular distribution of a liquidity shock. Three cutoff values are depicted, ρ_0, $\overline{\rho}$ and ρ_1. If no contract would be set up beforehand, entrepreneurs would "overinvest" and could not, once in period t_1 raise more than $\rho_0 I$, i.e. the expected net-worth of the project minus private benefit payments. If negotiations were to take place (and implementation issues were solved) in period t_0, investment *per project* would be less, however, due to the increased equity quota, the range of feasible liquidity shocks would increase and hence $\overline{\rho} I$ could be raised in period t_1. However, the first-best outcome ρ_1 cannot be implemented.

2.3   Properties of the second-best contract

From equation (7) it becomes clear that the higher the first-best threshold, the higher is the return of investment for the entrepreneurs. Also, higher inside benefits B that accrue only to the entrepreneurs, will reduce their return. This results are more or less standard and not overly surprising. Holmstroem-Tirole point to another, more surprising result: If the distribution of the liquidity shock becomes riskier (in the sense of a mean preserving spread), the cutoff will go down. This can be seen from (6). Profits, however, will go up. This is a consequence of the fact that the option to terminate the project becomes more valuable when the distribution is more risky.

3 Introducing International Capital Flows

We are now in the position to extend the HT-analysis to a macro framework. We consider a two open country setup, where both, home and foreign are also Holmstroem-Tirole worlds. This essentially means that the capital allocation is ruled by the same principles as outlined above. Our crucial assumption, however, will be that there are not enough funds in the home country to meet the period 0 investment demand

$$I_D\left(\overline{\rho}\right) = k\left(\overline{\rho}\right) A$$

where $\overline{\rho}$ is the second-best cutoff in the unconstrained HT-world. This short-fall of investment supply can thought of being either a result from domestic portfolio optimization or capital scarcity (or both). With free capital flows this shortfall of domestic investment supply will (at least partially) be matched by capital inflows. In contrast, under autarky this is not possible, the optimal solution of our Holmstroem-Tirole world will be different: the cutoff ρ goes up. We derive the behavior of the HT-world under autarky and capital scarcity in detail below. We then turn to analyze our two country world with free capital flows. .

3.1 Autarky

A natural starting point for our analysis is to consider the implications of the HT-model under autarky. Autarky itself is only interesting when we have

$$\overline{I}_H \leq I_D\left(\overline{\rho}\right) - A = \left(k\left(\overline{\rho}\right) - 1\right) A \tag{8}$$

where $\overline{I}_H$[9] is the exogenously given amount of investment home investors are willing (or able) to undertake and I_D denotes the amount of investment that would prevail in an unconstrained world. In principle we could think of the investment-shortfall $\overline{I}_H - I_D + A$ as also being dependent on the parameter vector of the domestic economy, however, to keep matters simple we simply assume this excess demand for investments. The original HT-world result will now be altered. As long as there were, by assumption, enough funds to meet any arising investment demand, the investors got zero expected returns. This result was brought about by the assumptions of risk neutrality, the absence of time preference of the investors and the zero returns of the outside asset cash. Investors would then compete away any positive profit - hence the net-social surplus accrues to the entrepreneurs. With limited outside funds,

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[9]Note that in order to distinguish fixed quantities from quantities that are variables we use the bar, hence $\overline{I}$ denotes a fixed amount as opposed to I.

however, this result changes. Now the entrepreneurs compete for the limited funds and investors will be able to appropriate a part or all of the net-social surplus. Formally the optimal contract now solves

$$\max_{R_f(\rho),\lambda(\rho),I} I \int_0^\infty \left(p_H[R - R_f(\rho)] - \rho\right) \lambda(\rho) f(\rho) d\rho - I + A$$

$$s.t$$

$$R_f(\rho) p_H \geq B + R_f(\rho) p_L \quad \forall \rho$$

$$I \int_0^\infty p_H R_f(\rho) \lambda(\rho) f(\rho) d\rho \geq A$$

$$I \leq \overline{I}_H + A$$

Note the similarity of this program to the HT-model optimization problem. First of all, what was the participation constraint of the entrepreneurs now is the objective function. Secondly, the incentive compatibility constraint remains unaltered. There is now a new participation constraint relevant for the investors. The only new part is the capacity constraint, i.e. that investments undertaken cannot be greater than an exogenously given amount $\overline{I}_H + A$. Two cases that arise out of this optimization problem can now be distinguished: $I p_H \frac{B}{\Delta p} F(\rho^A) > A$ and $I p_H \frac{B}{\Delta p} F(\rho^A) \leq A$. We will analyze each case in turn.

3.1.1 $I p_H \frac{B}{\Delta p} F(\rho) > A$

If this condition were to hold, entrepreneurs would be able to get a part of the net social surplus, as they will recover more than their initial investment, implying a rate of interest above zero. The optimization problem then reduces to

$$\max_{\rho^A} I \int_0^{\rho^A} \left(p_H[R - R_f(\rho)] - \rho\right) f(\rho) d\rho - I + A.$$

where ρ^A refers to the cutoff value under autarky. It is easy to see that letting $\rho = \rho_o = p_H[R - R_f(\rho)]$ maximizes this expression. However, this case is in the original paper assumed away by the condition

$$\int_0^{\rho_0} (\rho_0 - \rho) f(\rho) d\rho < 1,$$

i.e. that investors would realize a negative expected return on their investment. Holmstroem and Tirole in the original paper stress that without this condition investment would be self-financing and the problem under consideration would be trivial. Without this condition, the allowable liquidity

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shock in t_1 would always be less than the pledgeable income and thus financing it can always be achieved by diluting capital. Hence, this case must be excluded from the set of possible solutions.

3.1.2 $Ip_H \frac{B}{\Delta p} F(\rho) \leq A$

Under this condition entrepreneurs now simply get a zero expected return on their investment A. Since the moral hazard constraint will always be binding, ρ adjusts to ρ^A so as to bring about $\overline{I} p_H \frac{B}{\Delta p} F(\rho^A) = A$. To see this, simply note that entrepreneurs would not start projects with negative expected return but instead keep their initial endowment for themselves. Hence $\overline{I} p_H \int_0^{\rho^A} \frac{B}{\Delta p} f(\rho) \, d\rho = A$ must always hold. The second-best contract under autarky and capital scarcity will thus specify an optimal cutoff value ρ^A following the program:

$$\max_{\rho^A, R_f(\rho), I} I \left(\int_0^{\rho^A} (p_H R - \rho) f(\rho) \, d\rho - 1 \right) \tag{9}$$

$$s.t.$$

$$I \leq \overline{I}_H + A$$

$$\int_0^{\rho^A} R_f(\rho) f(\rho) \, d\rho = \frac{A}{\overline{I}_H + A} \frac{1}{p_H}.$$

Note that the solution to this program always implies that the entrepreneurs get an expected return of zero on their investment. Further, if $I \int_0^{\overline{\rho}} p_H \frac{B}{\Delta p} f(\rho) \, d\rho < A$, there would be in principle two ways to implement the participation constraint of the entrepreneurs: increase the payment to the entrepreneurs, i.e. chose $R_f(\rho) > \frac{B}{\Delta p}$ or increase ρ for $F(\rho)$ is increasing in ρ. Clearly, there is no incentive for the investors to give more away than absolutely necessary, hence ρ will increase and the payments for the entrepreneurs will remain $\frac{B}{\Delta p}$. The investors will be able to appropriate the full net-social surplus. In this sense this solution is exactly the opposite of the original HT-world solution.

From the perspective of the original HT-world solution, two cases have to be distinguished: $Ip_H \frac{B}{\Delta p} F(\overline{\rho}) = A$ and $Ip_H \frac{B}{\Delta p} F(\overline{\rho}) < A$. In the first case the solution to (9)is $\rho^A = \overline{\rho}$, just as in the HT-world. However, due to the restricted amount of investment, the overall utility will be strictly less than in the unrestricted world. We have

$$U_A < U_{HT}{}^{10}$$

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[10] Appendix 6.3 derives this solution in more detail.

with

$$U_A = \frac{\rho_1 - \overline{\rho}}{\rho_1 - \rho_0} A < \frac{\rho_1 - \overline{\rho}}{\overline{\rho} - \rho_o} A = U_{HT}. \qquad (10)$$

The solution for the second case is then simply a cutoff value ρ^A that solves:

$$F\left(\rho^A\right) = \frac{A}{\overline{I}_H + A} \frac{\Delta p}{B p_H} \; \forall \rho \leq \rho_1.$$

A different way of stating the results under autarky is the following graphic:

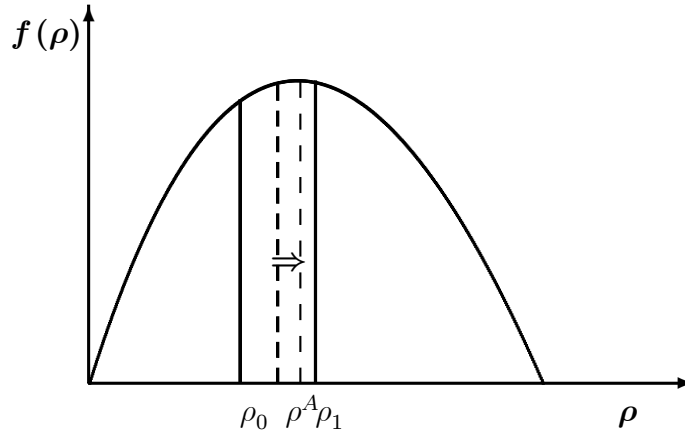$$Liquidity \; Shock \; and \; Cutoff \; Values : \; Autarky$$

Figure 2: The Autarky Case

Figure 2 above makes clear that forcing the economy to overall invest less, expands the range of admissible liquidity shocks from $\overline{\rho}$ to ρ^A. Thus, even though the economy as a whole invests less, a greater proportion of projects will survive any given liquidity shock. Hence the economy becomes less vulnerable with respect to liquidity shocks.

We conclude the discussion of the autarky solution of the HT-model with the following proposition:

Proposition 1 *In a HT-model under autarky the following holds true: The representative firm with endowment A will invest $I\left(\rho^A\right) = k\left(\rho^A\right) A < I\left(\overline{\rho}\right)$. The second-best cutoff under autarky ρ^A will lie in the interval $[\rho_1, \overline{\rho}]$. For sufficiently small values of $\overline{I_H} = I\left(\rho^A\right) - A$ we will have $\rho^A > \overline{\rho}$. In that situation the domestic economy will be less vulnerable with respect to liquidity shocks. This comes at the cost of a smaller scale for each and every project. The net-social surplus accrues to the investors. The utility level that can be reached upon implementation of the second-best contract is strictly less than in the original HT-world.*

3.2 Free Capital Flows

In this section we will introduce the possibility that foreigners can (and will) add some amount I_F to make up for (some of) the shortfall of investment supply to investment demand. hence

$$I_F \leq I_D\left(\overline{\rho}\right) - \overline{I}_H - A.$$

Foreign investors will be investing their money under exactly the same conditions as home investors with one exception: In the spirit of a dual agency problem (seeTirole (2002)) we introduce higher agency costs, that is a different B for foreign investors.[11] The main implication of this assumption is that foreigners will face a different pledgeable income in period t_1 than home investors. This can be justified on grounds of several reasons. The first that comes to mind are monitoring costs. Even though not explicitly modelled here, one could think of the amount of private benefits that accrue only to the firm also as being the outcome of a monitoring process - the more efficient the latter the lesser the former. It is reasonable to assume that foreigners possess a less efficient monitoring technology. Secondly there is the dual agency perspective. Foreign investors do not only have a contract with the firms they invest in but also, implicitly, with the home government that sets the rules (bankruptcy laws etc.) within its domain. There may be a tendency in case of failure of the firm to protect home interests more efficient than foreign ones. From this we conjecture that $B^F > B$, i.e. that the firm will need more from foreign investors to be kept diligent. Recently there have been contributions that endogenize the choice of B by the government (see for example Tirole (2003)) and in general this would be possible in our model as well. To keep matters simple here, however, we stick with the assumed exogenous nature of the agency costs. The analysis of the economy under free capital movements bears close resemblance with the setups discussed above. To use our previous results we need to clarify and discuss in turn the different cases that can arise when foreign capital is allowed to augment the domestic capital stock. In what follows we will abstract from exchange rate issues or terms of trade uncertainty. Home and foreign thus share the same commodity that is not subject to any change in its price. Further, as outlined above, home and foreign agents have the same utility

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[11]There is a literature of its own that concludes that the amount B is indeed country specific. For example Porta, Lopez-De-Silanes, Shleifer and Vishny (2002) tries to assess the influence this differences have on corporate valuation and Porta, Lopez-De-Silanes, Shleifer and Vishny (1998) identifies four different legal systems, each with a different degree of investor protection. Finally Porta, Lopez-De-Silanes, Shleifer and Vishny (1997) finds that countries with lower investor protection tend to have narrower capital markets.

function, namely $U(c_0, c_1, c_2) = c_0 + c_1 + c_2$. If agents decide to simply hold their endowments without investing them returns will be zero, there is a loss-free storage technology. The minimum expected return for any investor (and entrepreneur) in this model is therefore zero and cannot be less. In order to gain clarity over the implications of the model we distinguish in what follows two different cases. Both cases, of course, involve domestic capital scarcity, but in the first the additional capital that is provided from abroad is sufficient to bridge the gap between demand and supply. In the second case we will discuss the implications if capital scarcity as measured by the gap $\overline{I}_F + \overline{I}_H < I(\overline{\rho}) - A$ remains even with foreign capital flowing in.

3.2.1 $I_F + \overline{I}_H > I(\overline{\rho}) - A$

When the supply of foreign capital is large enough, capital scarcity no longer exists and, from a contractual point of view we are back in the original HT-world. The net-social surplus then accrues to the entrepreneurs. The second-best contract solves

$$\max_{R_H(\rho), I_H, I_F, \lambda(\rho), R_F(\rho)} \alpha I \int_0^\infty p_H R_H(\rho) \lambda(\rho) f(\rho) d\rho \tag{11}$$

$$+ (1 - \alpha) I \int_0^\infty p_H R_F(\rho) \lambda(\rho) f(\rho) d\rho - A$$

$$s.t.$$

$$\alpha I \int_0^\infty \left(p_H \left(R - \alpha R_H(\rho) - (1 - \alpha) R_F(\rho) \right) - \rho \right) \lambda(\rho) f(\rho) d\rho \geq I_H \tag{12}$$

$$(1 - \alpha) I \int_0^\infty \left(p_H \left(R - \alpha R_H(\rho) - (1 - \alpha) R_F(\rho) \right) - \rho \right) \lambda(\rho) f(\rho) d\rho \geq I_F. \tag{13}$$

$$R_H(\rho) p_H \geq B + R_H(\rho) p_L \quad \forall \rho \tag{14}$$

$$R_F(\rho) p_H \geq B^F + R_F(\rho) p_L \quad \forall \rho \tag{15}$$

$$I_H \leq \overline{I}_H \tag{16}$$

where $\alpha = \frac{I_H}{I - A}$ the denotes the share of domestic outside capital relative to total outside capital and the overall level of investment is given by $I = I_F + I_H + A$. Note that in order to make the description of the optimization problem complete we need to introduce two types of returns here the entrepreneur will get $R_H(\rho)$ for the investment that is financed with domestic outside money and $R_F(\rho)$ for the foreign financed part. If the project is successful investors will share the proceeds $R - \alpha R_H - (1 - \alpha) R_F$ according to their respective shares in the investment. This is what constitutes the

left hand side of equation (12) and (13). The implicit assumption in writing this type of contract is that every project is financed in part by foreigners and in part by domestic investors (joint-venture). An alternative to that assumption clearly is that a certain share of all projects will receive finance from domestic investors and the remainder will receive foreign outside funding (pure-origin finance). Mathematically both problems are equivalent.[12] In this section we therefore restrict the discussion to the aforementioned case. The implications for the vulnerability of the domestic economy with respect to liquidity shocks, however, a quite different under each setup and will be discussed in more detail below.

The solution to program (11) is the same as in the original HT-world and specifies a cutoff value $\overline{\rho}$[13].This is not surprising as the only parameter that has changed is the composition of the net-social surplus. Note also that the difference in agency costs does not directly matter here. This, at the first glance, a bit strange result follows from the independence of the optimal cutoff $\overline{\rho}$ from $\rho_0^F < \rho_o < \overline{\rho} < \rho_1$ as long as it does not hit one of this boundaries. However, this independence is not complete. The higher agency costs that foreigner face will place a constraint on the equity multiplier (equation 4)for this type of capital. Depending on the parameters of the economy, especially the amount of domestically provided outside capital and the size of the endowment of the entrepreneurs, there may well be *endogenous foreign capital scarcity*. We will in a later section analyze this case in more detail, i.e. under which conditions the increased B^F is going to induce foreigners to limit their investments.

The level of utility that can be reached is higher than in the original HT-model. This is follows directly from the assumption of the higher agency costs foreigner investors face, that is domestic entrepreneurs can extract higher private benefits from foreign investors than from domestic ones. Formally this is stated below

$$U_{FA} = \frac{\rho_1 - \overline{\rho}}{\overline{\rho} - (\alpha \rho_0 + (1 - \alpha) \rho_0^F)} > U_{HT}$$

where $\rho_0^F = p_H \left(R - \frac{B^F}{\Delta p} \right) < \rho_0$. Note that this result implies that entrepreneurs always prefer foreign investors financing their projects. Program (11) has 5 choice variable but only 4 restrictions, since the second and third equation are equivalent. Hence without a further condition, α remains undetermined within the bounds defined by $\overline{I}_H$. However, as argued above the size of the domestic capital base restricts the maximum amount of foreign

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[12]This is shown in the Appendix (6.5).

[13]A detailed derivation can be found in the appendix 6.4.1.

capital that will flow into the domestic economy, especially implying that α will never fall to zero. This together with the result that entrepreneurs will seek a share of outside finance as high as possible provides the last restriction on our optimization problem. This is, of course, an artefact of the linear objective function. For the purposes of this paper it is sufficient to note that the composition of outside debt is not vital for our main results as long as we assume parameter values for which there will always be both, foreign and domestic outside finance.

The results of this section are summarized by the following proposition.

Proposition 2 *In a HT-model with domestic capital scarcity defined as $I_D\left(\overline{\rho}\right)- A > \overline{I}_H$, international capital flows and foreign capital abundance defined as $I_F > I\left(\rho\right) - \overline{I}_H - A$ the following holds true: the second-best contract has the same properties as in the original HT-model, namely firms invest $I_D\left(\overline{\rho}\right) = k\left(\overline{\rho}\right)A$, the net-social surplus accrues to the entrepreneurs and investors break even. The utility level that entrepreneurs can reach will be higher than in the original HT-model. This is due to the higher private benefit that domestic firms can extract from foreign investors.*

3.2.2 $\quad \overline{I}_F + \overline{I}_H \leq I\left(\overline{\rho}\right) - A$

We now turn to determine the second-best contract under both domestic and foreign capital scarcity. Similar to the analysis under autarky, this situation again calls for a different approach. The first implication is, of course, a redistribution of the domestic net-social surplus in favour of the investors. At the first glance it seems as if there are now two second-best programs to be solved, one for the domestic investors and the second for the foreign ones.:

$$\max_{R_H(\rho),\lambda^H(\rho),I_H} \alpha I \int_0^\infty \left(p_H\left(R - \alpha R_H\left(\rho\right) - \left(1-\alpha\right)R_F\left(\rho\right)\right) - \rho\right)\lambda\left(\rho\right)f\left(\rho\right)d\rho - I_H$$
(17)

$$s.t.$$
$$\alpha I \int_0^\infty p_H R_H\left(\rho\right)\lambda\left(\rho\right)f\left(\rho\right)d\rho \geq \left(\frac{I_H}{I-A}\right)A$$
$$R_H\left(\rho\right)p_H \geq B + R_H\left(\rho\right)p_L \quad \forall\rho$$
$$I_H \leq \overline{I}_H$$
(18)

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The program for the foreign capital then reads

$$\max_{R_F(\rho),\lambda^F(\rho),I_F} (1-\alpha)\,I\int_0^\infty \left(p_H\left(R-\alpha R_H\left(\rho\right)-\left(1-\alpha\right)R_F\left(\rho\right)\right)-\rho\right)\lambda\left(\rho\right)f\left(\rho\right)d\rho - I_F$$
(19)

$$s.t.$$

$$(1-\alpha)\,I\int_0^\infty p_H R_F\left(\rho\right)\lambda\left(\rho\right)f\left(\rho\right)d\rho \geq \left(\frac{I_F}{I-A}\right)A$$

$$R_F\left(\rho\right)p_H \geq B^F + R_F\left(\rho\right)p_L \quad \forall\rho$$

$$I_F \leq \overline{I}_F.$$

However, merging the two maximization problems into one is mathematically equivalent and leads to the same results. The solution technique employed is the same as in section (3.1.2). First we observe that any meaningful solution must involve binding incentive compatibility constraints, i.e. $R_F = \frac{B^F}{\Delta p}$ and $R_H = \frac{B^H}{\Delta p}$. Secondly the cutoff values have to adjust to fulfill the participation constraint of the entrepreneurs. This implies in general two different cutoff values for home and foreign investment contracts respectively. To see this write out the two participation constraints as

$$F\left(\rho^H\right) = \frac{1}{I}\frac{A}{B}\frac{\Delta p}{p_H}$$
(20)

and

$$F\left(\rho^F\right) = \frac{1}{I}\frac{A}{B^F}\frac{\Delta p}{p_H}.^{14}$$
(21)

The first implication is that contracts with foreigners will always have a lower cutoff value than contracts with domestic investors. In this setup this result is completely driven by the assumption of the higher agency costs that foreigners face and the fulfillment requirement of the participation constraint. The second implication is that under free (but limited) capital inflows the domestic contract cutoff value will be less than under autarky as long as foreigners contribute a small fraction of the domestic capital stock. Condition (20) makes this immediately clear.

The resulting contracts under all our setups considered are depicted for

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[14]The exact derivation is again relegated to appendix (6.4.2).

reference in the table below:

Free Capital Flows	Contract Home Investors
$I_D(\overline{\rho}) - A \leq \overline{I}_H$	$\{\overline{\rho}, I(\overline{\rho}), R_f = \frac{B}{\Delta p}\}$
$I_D(\overline{\rho}) - A > \overline{I}_H$, $I_D(\overline{\rho}) - A < \overline{I_H} + I_F(\overline{\rho})$	$\{\overline{\rho}, \overline{I}, R_H = \frac{B}{\Delta p}\}$
$I_D(\overline{\rho}) - A > \overline{I}$, $I_D(\overline{\rho}) - A \geq \overline{I_H} + \overline{I}_F$	$\{\rho^H, \overline{I}, R_H = \frac{B}{\Delta p}\}$
	Contract Foreign Investors
$I_D(\overline{\rho}) - A \leq \overline{I}_H$	-
$I_D(\overline{\rho}) - A > \overline{I}_H$, $I_D(\overline{\rho}) - A < \overline{I_H} + I_F(\overline{\rho})$	$\{\overline{\rho}, I_F(\overline{\rho}, \overline{I}), R_F = \frac{B^F}{\Delta p}\}$
$I_D(\overline{\rho}) - A > \overline{I}$, $I_D(\overline{\rho}) - A \geq \overline{I_H} + \overline{I}_F$	$\{\rho^F, \overline{I_F}, R_F = \frac{B^F}{\Delta p}\}$

The implications of our results are straightforward. Foreign capital inflows will render the domestic economy more vulnerable to liquidity shocks. This effect has two sources. The first is the share of foreign capital invested in domestic firms, the second is the higher agency costs that foreign investors face. We can decompose these two sources by comparing the different cutoff values that prevail in an economy with foreign and domestic capital with another with the same level of investment but only domestic capital:

$$\frac{I_F}{I - A}\rho^F + \frac{I_H}{I - A}\rho^H \quad < \quad \rho^A_{|\overline{I} - A = I_H + I_F} = \rho^H \tag{22}$$

$$|\left(\rho^F - \rho^H\right)\frac{I_F}{I - A}| \quad = \quad FCCP > 0.$$

Relationship (22) defines a "foreign capital cutoff premium" (FCCP) and makes clear that the higher the share of foreign outside finance in domestic firms, the higher the premium the economy pays in terms of increased vulnerability. What is depicted in (22) is thus the effect stemming from the *share* of foreign capital, for any given ratio $\frac{B^F}{B}$ multiplied by $\left(\rho^F - \rho^H\right)$ that stands for the direct contribution of the differing agency costs to the higher vulnerability of the domestic economy with respect to liquidity shocks. Note that the impact of a given liquidity shocks also depend on the whether firms seek outside finance as a joint-venture or as pure-origin. Under joint venture the impact is the greatest, if we assume that projects cannot be divided, implying that all projects that experience a liquidity shock higher than ρ^F are abandoned. Under pure-origin finance only a fraction $(1 - \alpha)$ of all projects is subject to this lower threshold, the remainder being continued up to ρ^H.
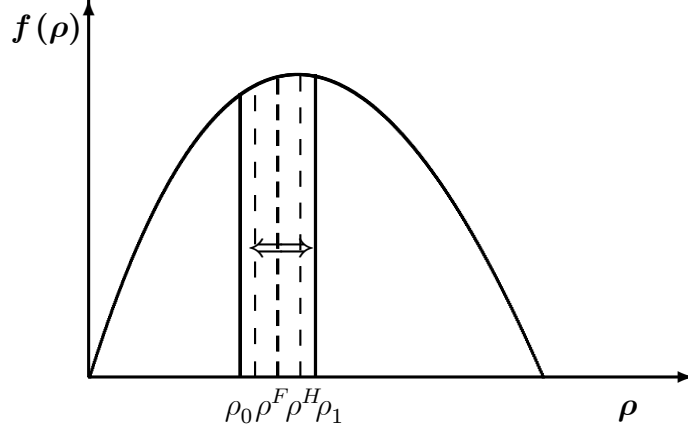
Figure 3: Free Capital Flows

Figure 3 states (one of) the results under free capital flows graphically. The thick dotted line represents the reference point $\overline{\rho}$ of our benchmark HT-world. As the above analysis made clear, international capital flows will, under certain conditions, lead to two distinct cutoff values: ρ^H for domestic outside capital and ρ^F for the foreign counterpart. The part of the project (or the part of the projects, depending on pure origin vs. joint-venture finance) that is financed from abroad will only be continued up to liquidity shocks of ρ^F. This will, in general, be less than $\overline{\rho}$ and thus worse than in an otherwise comparable HT-world. The domestically financed part will be better protected than the part that is financed from abroad. Whether or not the domestically financed part is even better protected than it would be in the benchmark HT-world (as shown in Figure 3) depends on the parameter vector of the model.

The following proposition sums up the results of this section:

Proposition 3 *In a HT-model with domestic capital scarcity defined as $I_D(\overline{\rho}) - A > \overline{I}_H$, international capital flows and foreign capital scarcity as defined by $I_D(\overline{\rho}) - \overline{I}_H - A > \overline{I}_F$ the following hold true: the representative firm invests $I = \overline{I}_H + \overline{I}_F + A < I_D(\overline{\rho})$, the net-social surplus now accrues to domestic and foreign investors, the second-best contract specifies two distinct cutoff values, one for the foreign investors ρ^F and one for the domestic investors ρ^H where $\rho^F < \rho^H < \rho^A$. The domestic economy is thus more vulnerable to liquidity shocks than under autarky, i.e. a larger part of the firms will have to abandon the project in period t_1 for any given liquidity shock $\rho > \rho_0$.*

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3.3 Domestic capital scarcity versus international capital flows

To derive our results, we so far had to make some strong assumptions on the relative size of capital supply to demand without giving a justification for the respective sizes. It turns out, however, that even if we stick with the exogenous nature of the size of the outside capital, we can say a bit more about the relevant ranges. Consider equation (4) that serves as our benchmark throughout the analysis. This equation determines the second-best capital stock in an otherwise unconstrained world. Hence it yields the amount of investment in the HT-model that maximizes the expected (second-best) net-social surplus. The term $\frac{1}{1+\int_0^{\overline{\rho}} \rho f(\rho) d\rho - F(\overline{\rho})\rho_0}$ is called (by Holmstroem and Tirole) equity multiplier and is in the original HT-model assumed to be greater than 1. Under free capital flows there are two scenarios that are of interest. First we consider domestic capital scarcity and foreign capital abundance. This, as stated in section (3.2.1), implies a contract $\{\overline{\rho}, \overline{I}_H, I_F(\overline{\rho}), R_H = \frac{B}{\Delta p}, R_F = \frac{B^F}{\Delta p}\}$. The properties of this contract are the same as in the HT-model, with the exception that foreigners face a smaller period t_1 pledgeable ρ_0^F income as do home investors. In order for this contract to be feasible, we need the following condition to hold

$$
\begin{aligned}
I_F(\overline{\rho}) &\leq \frac{1}{1 + \int_0^{\overline{\rho}} \rho f(\rho) d\rho - F(\overline{\rho})\rho_0^F} A - A \qquad (23)\\
I_F(\overline{\rho}) &\leq I^F(\overline{\rho}) - A,
\end{aligned}
$$

where $I^F(\overline{\rho})$ denotes the level of investment that would prevail in a world in which all investors were foreigners. Clearly, as $\rho_o^F < \rho_o$ by assumption, we have $I_D(\overline{\rho}) > I^F(\rho)$. This places a restriction on the term "foreign capital abundance" in the sense that foreign capital cannot indefinitely augment the domestic economy, even if there is no shortage in supply. This in turn renders our results from section (3.2.2), the second case we considered, more relevant: *foreign capital scarcity, as we defined it, will not only follow from a lack of foreign supply but also from an especially low domestic capital base.* To see this note that the overall level of initial outside investment demand under free capital flows was given by $I_D(\overline{\rho}) - A = \overline{I}_H + I_F(\overline{\rho})$. For small enough values of $\overline{I}_H$, however, constraint (23) will be binding and thus we face domestic *and* foreign capital scarcity even though there is no restriction placed on the level of foreign capital from outside. The reason for this "self-restriction" of the model is the different (and higher) agency costs foreigners face. The higher these costs are, relative to the costs domestic investors face, the earlier this "self-restriction" is going to bind. Further, the higher this

relative agency costs are, the greater the "distance", as measured by $\rho^H - \rho^F$, between domestic and foreign contracts. Formally the constraint on the level of domestic outside investment reads

$$\overline{I}_H < \left(k\left(\overline{\rho}\right) - k^F\left(\overline{\rho}\right) \right) A \qquad (24)$$

where $k\left(\overline{\rho}\right)$ is the equity multiplier of the original HT-model and $k^F\left(\overline{\rho}\right)$ is the equity multiplier that would prevail in the case in which all outside finance would be stemming from abroad. Condition (24) states that if domestic outside capital supply fails in making up for the difference between unconstrained investment demand and maximum foreign outside capital, foreign outside capital will be restricted endogenously. Note that the endowment has a different role. In general, as (24) makes clear, the smaller the domestic endowments, the smaller the domestic outside capital supply must be in order to induce foreign capital scarcity (and vice versa). This is, of course, a pure demand effect. The higher the endowment, the higher is the demand for outside capital. The results of this section are summarized in the following proposition.

Proposition 4 *In a HT-model with international capital flows domestic outside capital scarcity may induce foreign capital scarcity. This will be the case when the amount of domestic outside finance investors are willing (or able) to invest falls short of the difference between the investment level that would prevail in an unconstrained world and the amount of outside finance foreigners will be willing to provide at given agency costs. The resulting second-best contract in this situation is the one described in section* (3.2.2).

4   Empirical Implications

Our extensions to the original HT model have some interesting empirical implications. According to our results, depending on the domestic capital base, countries may have limited scope for borrowing abroad. Naturally, the poorer the country under consideration, the lower the domestic capital base and hence the more limited its borrowing potential will be. This in turn would imply, that the effects of international capital flows on poor countries are ambiguous in our model. As the analysis in section (3.2.2) makes clear the positive effect of augmenting the domestic capital base has the (potentially) negative side effect of rendering the domestic economy more vulnerable to liquidity shocks. The final evaluation thus depends on the distribution of the liquidity shocks. On the other hand, rich countries that are capital abundant to begin with, will not gain anything from opening their capital

account, the original HT-contract remains in place unaltered. However, the main effects of international capital flows according to our model will occur in the middle-income group of countries, i.e. those countries that have an insufficient capital base to begin with but that can significantly augment their capital base with the help of foreign capital inflows. If this augmentation is complete (in the sense that the complete excess demand for outside capital can be contracted abroad), only gains will accrue. In the case of incomplete augmentation it remains unclear whether there will be any gains or not.

It is clear that our model does not account for all possible motives of international capital flows. Hence it will, in general, not be able to explain all these flows between countries nor the empirical findings often found in studies on the subject. What is remarkable, however, is the models ability to rationalize two main findings of the empirical literature in the field. First, in recent studies on the effect of international capital flows on growth find, although far from conclusive, evidence that the growth effects of capital account liberalization are distributed in a U-shape manner (see Klein (2003) and Edison et al. (2002)). The main gains accrue to middle-income countries with poor and rich countries benefitting only marginally. This is in line with the predictions of our model. Secondly, it has become a stylized fact that most net-capital flows occur between developed countries, much the opposite to what neoclassical growth theory would predict.[15] Our model offers a very simple explanation for this: poor countries do not have enough collateral to borrow up to their needs. They are in effect credit constrained.

With respect to income volatility our analysis bears another empirical implication. As a result of foreign capital inflows, the poorest countries *may* witness an (relative) increase in output variability due to the increased vulnerability with respect to liquidity shocks. However, this is only a possibility, depending on the liquidity shocks that hit the economy, that may or may not materialize. Surprisingly, although many contributions to the discussion on the effects of capital account liberalization name increasing volatility as a possible negative consequence, empirical studies on this topic are scarce. We are only aware of two such papers, namely Bekaert et al. (2004) and Mukerji (2003). The first (and most recent) study finds that *overall* the impact of financial liberalization in real consumption growth volatility is negligible. However, if we are viewing the effects Bekaert et al. (2004) find at the country level there are significant negative effects for some countries. The second study finds, in accordance with our model, negative effects for less developed

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[15]Lucas (1990) in widely cited paper draws on this point and sparked a literature of its own on that topic. Examples of follow up papers are Kray (2000) and Reinhardt and Rogoff (2004).

countries but negligible output effects for more developed countries.

5   Conclusion

In this paper we extended a model of corporate liquidity demand in order
to be able to address questions of macroeconomic importance. This was
achieved by introducing international capital flows into the model of Holm-
stroem and Tirole that can augment the domestic capital base. Foreign in-
vestors, as opposed to domestic ones, face higher agency costs when investing
in domestic firms. We justify this assumption in the spirit of a dual-agency
problem: principals will not only have a contract with their agents but also
an implicit one with the authorities that set the frame for any economic activ-
ity. However, we could think of many more possible reasons why agency costs
should be higher for foreigners, monitoring costs being one of these reasons.
The model then allows to discuss and answer several interesting questions
that relate to the field of international macroeconomics, like the phenomenon
that net capital flows are substantially higher between developed countries
than between developed countries and less developed countries.

When outside capital is scarce domestically the economy will invest less
but will also be able to withstand higher liquidity shocks. Foreign capital
that flows in then brings about two effects: more investments but at the cost
of higher vulnerability with respect to liquidity shocks. This effect would
be especially pronounced in less developed countries. This may provide a
rationale why several countries from this group chose to introduce capital
controls as a policy option.

When outside capital is in limited supply both, from domestic investors
and foreign ones, the economy will be even more vulnerable to liquidity shocks
as the foreign share of the outside finance cannot sustain liquidity shocks as
high as the domestic share. The consequence is that a larger fraction of
projects will be abandoned in period t_1. Foreign outside capital in this sense
is less robust than domestic outside capital. This result has some interesting
implications. First, countries that have to draw heavily on foreign capital,
should try hard to lower agency costs for foreigners. This would mitigate
the negative effects foreign capital inflows have. To the extent that foreign
outside capital is rendered scarce by the low domestic capital base our model
offers an explanation why most net-capital flows occur between developed
countries, i.e. those with a relatively high domestic capital base. The second
implication then of course would be that LDC's may well fare well introducing
capital controls or restrictions like a Tobin Tax, for the amount of capital that
can flow into an especially poor economy is restricted by what it can provide

domestically. Lastly, by drawing on the same result, our model provides a rationale why the catch-up prediction of neoclassical growth models does not work empirically: Foreign capital simply cannot augment the domestic capital base indefinitely. There are two reason for this: the higher agency costs that foreign investors end up paying and the credit rationing itself that is already part of the original HT-model.

Our analysis has several shortcomings that could be the subject of further research. First, the crucial point of capital scarcity is exogenous in our model. An extension to endogenize domestic capital supply thus seems natural. Secondly agency costs may also be made endogenous by introducing a government. Even though our results would not change qualitatively, this step would allow to analyze the effect of different policy regimes on domestic welfare. Lastly an extension of the present framework to an overlapping generation model and thus ultimately the possibility to answer the question whether or not trading off vulnerability against scale is worth the effort may prove an exiting avenue for further research.

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6   Mathematical Appendix

6.1   Derivation of the optimal HT-world contract

In this first part of the mathematical appendix we will devote some space to go in some detail through the steps necessary to solve the optimization problem of the original HT-paper. We want to determine $\lambda(\rho)$, I, $R_f(\rho)$ that solve the second-best contract:

$$\max_{R_f(\rho), I, \lambda(\rho)} I \int_0^\infty p_H R_f(\rho) \lambda(\rho) f(\rho) \, d\rho - A$$

$$s.t.$$

$$I \int_0^\infty (p_H[R - R_f(\rho)] - \rho) \lambda(\rho) f(\rho) \, d\rho \geq I - A$$

$$R_f(\rho) p_H \geq B + R_f(\rho) p_L \quad \forall \rho.$$

Noting first that the second equation must hold with equality for otherwise the solution would be unconstrained and substituting into the objective function we get

$$\max I \int_0^\infty p_H R_f(\rho) \lambda(\rho) f(\rho) \, d\rho - I$$

$$+ I \int_0^\infty (p_H[R - R_f(\rho)] - \rho) \lambda(\rho) f(\rho) \, d\rho$$

$$\implies \quad \max I \left(\int_0^\infty (p_H R - \rho) \lambda(\rho) f(\rho) \, d\rho - 1 \right)$$

Note the that $R_f(\rho)$ disappears. Now making use of the cutoff value $\overline{\rho}$, i.e. rewriting $\lambda(\rho)$ as $\lambda(\rho) = \left\{ \begin{array}{c} 1 \forall \rho \leq \overline{\rho} \\ 0 \; otherwise \end{array} \right\}$ we arrive at

$$\max_{\overline{\rho}} I \left(\int_0^{\overline{\rho}} (p_H R - \rho) f(\rho) \, d\rho - 1 \right)$$

Further using that $p_H R I = \rho_1 I$ is the social optimal cutoff value we can find the marginal net social return on investment as a more concise objective function of the entrepreneur:

$$\max_{\overline{\rho}} U_f(\overline{\rho}) = m(\overline{\rho}) I.$$

Now using again the second equation of (2), we can derive the amount of investment:

$$I \int_0^\infty p_H [R - R_f(\rho) - \rho] \lambda(\rho) f(\rho) \, d\rho = I - A$$

$$\frac{1}{1 + \int_o^{\overline{\rho}} \rho f(\rho) \, d\rho - \rho_o F(\overline{\rho})} A = I$$

$$k(\overline{\rho}) A = I.$$

To determine the second-best cutoff $\overline{\rho}$ we maximize (5) with respect to $\overline{\rho}$. Writing out (5) gives

$$\max_{\overline{\rho}} U(\overline{\rho}) = \frac{F(\overline{\rho}) \rho_1 - \int_o^{\overline{\rho}} \rho f(\rho) \, d\rho - 1}{1 + \int_o^{\overline{\rho}} \rho f(\rho) \, d\rho - F(\overline{\rho}) \rho_o} A$$

$$\max_{\overline{\rho}} U(\overline{\rho}) = \frac{\rho_1 - \left(1 + \int_o^{\overline{\rho}} \rho f(\rho) \, d\rho\right) \frac{1}{F(\rho)}}{\left(1 + \int_o^{\overline{\rho}} \rho f(\rho) \, d\rho\right) \frac{1}{F(\rho)} - \rho_o} A$$

$$= \min_{\overline{\rho}} \left(1 + \int_o^{\overline{\rho}} \rho f(\rho) \, d\rho\right) \frac{1}{F(\rho)}.$$

The first-order condition for this problem is simply $\int_o^{\rho^*} F(\rho) \, d\rho = 1$, since

$$\frac{d \left(1 + \int_o^{\overline{\rho}} \rho f(\rho) \, d\rho\right) \frac{1}{F(\rho)}}{d\rho} = \rho f(\rho) \frac{1}{F(\rho)} - \left(1 + \int_o^{\overline{\rho}} \rho f(\rho) \, d\rho\right) \frac{1}{(F(\rho))^2} f(\rho) = 0$$

this, together with the result that $F(\rho) \rho - \int_0^{\overline{\rho}} \rho f(\rho) \, d\rho = \int_0^{\overline{\rho}} F(\rho) \, d\rho$ gives (6). Using this last equality once more in the numerator and the denominator of (5) and assuming an interior solution, we finally arrive at (7).

6.2 Proof of the optimality of a cutoff rule

Here we prove that $\lambda(\rho)$, the state-contingent continuation policy, takes the form of a cutoff rule, i.e. continue iff $\rho \leq \overline{\rho}$. Suppose we would propose a continuation policy that says continue for $\forall \rho$. In this case we would continue projects that have a negative net present value, hence also those that are socially undesirably. Now suppose we would propose a rule that no project should be continued for any positive ρ. Then, of course, there are projects with positive net present value that are abandoned by this rule. We can improve both situations by imposing a cut-off value ρ upon reaching which projects are terminated. The value of $\overline{\rho}$ is left to be determined endogenously. We can see this result also from the concavity of the objective function of the second-best program with respect to a cut-off $\overline{\rho}$:[16]

$$\max_{\lambda(\rho)} \int_0^\infty (p_H R - \rho) \lambda(\rho) f(\rho) \, d\rho.$$

As long as ρ is increasing but less than $p_H R$ the value of the objective function increases. As soon as $p_H R$ is surpassed, additional contributions will be negative and hence undesirable. As long as the support of the liquidity shock ρ comprises also of $p_H R$ it will be optimal to define a cutoff rule.

6.3 Derivation of the second best contracts under autarky

We start solving the program (9) subject to the condition of the first case: $I \int_0^{\rho^A} p_H \frac{B}{\Delta p} f(\rho) = A$. This condition states, that the amount of funds provided by the investors is such that moral hazard constraint just binds. We can then note that this amount of investment is simply given by

$$I = \frac{A}{p_H \frac{B}{\Delta p} F(\rho)}.$$

Inserting this into the first equation yields

$$\max_{\rho^A} \frac{A\left(\int_0^{\rho^A} (p_H R - \rho) f(\rho) \, d\rho - 1\right)}{p_H \frac{B}{\Delta p} F(\rho)}. \tag{25}$$

This is equivalent to the maximization problem in the original setup, i.e. maximizing (25) is equivalent to $\min_{\overline{\rho}} \left(1 + \int_o^{\overline{\rho}} \rho f(\rho) \, d\rho\right) \frac{1}{F(\rho)}$. To see this

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[16]This follows from the optimization.

rewrite (25) as

$$\max_{\rho^A} \frac{\rho_1 F\left(\rho\right) - \int_0^{\rho^A} \rho f\left(\rho\right) d\rho - 1}{p_H \frac{B}{\Delta p} F\left(\rho\right)} A$$

$$\max_{\rho^A} \frac{\rho_1 - \left(1 + \int_0^{\rho^A} \rho f\left(\rho\right) d\rho\right) \frac{1}{F\left(\rho\right)}}{p_H \frac{B}{\Delta p}} A.$$

The solution is thus the same as in the original HT-world, namely $\int_0^{\rho^A} F\left(\rho\right) = 1$. Using this and the definition of $\rho_0 = p_H \left(R - \frac{B}{\Delta p}\right)$ we can rewrite utility of the investors as (10).

6.4 Derivation of the second-best contracts under free capital flows

6.4.1 Domestic capital scarcity, foreign capital abundance

We start with the program (11). First we observe that the program is linear in I and hence participation constraints of the investors have to bind in order to obtain an interior solution. Thus we can consolidate the second and the third equation to

$$\alpha I \int_0^\infty \left(p_H \left(R - \alpha R_H\left(\rho\right) - \left(1 - \alpha\right) R_F\left(\rho\right)\right) - \rho\right) \lambda\left(\rho\right) f\left(\rho\right) d\rho$$

$$+ \left(1 - \alpha\right) I \int_0^\infty \left(p_H \left(R - \alpha R_H\left(\rho\right) - \left(1 - \alpha\right) R_F\left(\rho\right)\right) - \rho\right) \lambda\left(\rho\right) f\left(\rho\right) d\rho = I - A$$

Solving for A and inserting into the objective function yields the familiar expression

$$\max_{\rho} I \left(\rho_1 F\left(\rho\right) - \int_0^{\rho^F} \rho f\left(\rho\right) d\rho - 1\right)$$

and thus $\rho^F = \overline{\rho}$.

6.4.2 Domestic and foreign capital scarcity

Under this section we derive the form of the participation constraints (20) and (21). We start out with stating the second equation of program (17) as an example:

$$\left(I_H + \frac{I_H}{I_F + I_H} A\right) \int_0^{\rho^H} p_H \frac{B}{\Delta p} f\left(\rho\right) d\rho = \left(\frac{I_H}{I_F + I_H}\right) A.$$

This is equivalent to

$$\left(\frac{I_H (I - A)}{I - A} + \frac{I_H}{I - A} A \right) p_H \frac{B}{\Delta p} F \left(\rho^H \right) d\rho = \left(\frac{I_H}{I_F + I_H} \right) A.$$

Simplifying and solving for $F \left(\rho^H \right)$ then yields $F \left(\rho^H \right) = \frac{1}{I} \frac{A}{B} \frac{\Delta p}{p_H}$.

6.5 The equivalence of joint-venture and pure-origin finance

In section (3.2.1) we state the problem for the entrepreneur under the assumption that each project is in part financed by domestic investors and receives also a share from foreign investors. Alternatively the economy could also consist of a share of projects α that is financed entirely by home investors, leaving a share of projects $(1 - \alpha)$ that is financed by foreign investors. This then gives rise to two programs to be solved:

$$\max_{R_H(\rho), \lambda(\rho), I_H} \alpha I \int_0^\infty p_H R_H (\rho) \lambda (\rho) f (\rho) d\rho - \alpha A$$

$$s.t.$$

$$\alpha I \int_0^\infty (p_H (R - R_H (\rho)) - \rho) \lambda (\rho) f (\rho) d\rho \geq I_H$$

$$R_H (\rho) p_H \geq B + R_H (\rho) p_L \quad \forall \rho$$

$$I_H \leq \overline{I}_H$$

and

$$\max_{R_F(\rho), \lambda(\rho), I_F} (1 - \alpha) I \int_0^\infty p_H R_F (\rho) \lambda (\rho) f (\rho) d\rho - (1 - \alpha) A$$

$$s.t.$$

$$(1 - \alpha) I \int_0^\infty (p_H (R - R_H (\rho)) - \rho) \lambda (\rho) f (\rho) d\rho \geq I_F$$

$$R_F (\rho) p_H \geq B^F + R_F (\rho) p_L \quad \forall \rho.$$

Clearly, the two programs can be merged into one by simply adding the objective functions. This yields (11).