PREDICTING THE PATTERN OF INTERNATIONAL SPECIALIZATION IN THE NEOCLASSICAL TRADE MODEL: A SYNTHESIS

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Abstract

I propose a framework for thinking about the predictability of a theory. A set of conceivable outcomes is taken as the primitive and a theory is said to make a prediction by identifying a subset on the set of conceivable outcomes. I use this notion of predictability as an organizing principle for characterizing pattern of international specialization in the neoclassical trade model. I identify "local/global efficiency" as the unifying subset selection criterion for the different formulations of the neoclassical trade model, ranging from Ricardo's (1817) labour content formulation to the empirically relevant multi-cone Heckscher-Ohlin specification with multiple countries, goods and factors.

JEL classification: F11.

1. Introduction

The purpose of this paper is to examine pattern of trade predictions in neoclassical trade theory. I propose a simple framework for thinking about predictability and use this as an organizing device for characterizing pattern of trade predictions in various formulations of the neoclassical trade model. In this framework a set of conceivable outcomes is taken as the primitive and a theory is said to make a prediction by identifying a subset on the set of conceivable outcomes.

The paper makes the following contributions. First, building on Ruffin's (2002) reinterpretation of Ricardo's (1817) "four magic numbers" as labour embodied in trade rather than labour unit coefficients, I show that Ricardo implicitly used this intuitive notion of predictability in what is arguably the first formal model in the history of economic thought. I discuss Ricardo's labour content formulation of

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comparative advantage in a graphical framework which illustrates the intellectual continuity between Ricardo's first prediction and the higher dimensional formulations which were developed over one and a half centuries later.

Second, I provide a model taxonomy which is organized around the different specifications of the set of conceivable outcomes. I distinguish between small open economy predictions (class (i) models) and integrated equilibrium predictions (class (ii) models). In class (i) models, the terms of trade defines the set of conceivable trading patterns and autarky prices impose a single restriction on the pattern of an economy's multilateral trade. Gains from trade, or local efficiency, are shown to be the subset selection criterion for predicting the pattern of commodity and factor content of trade. From this perspective the structure of the small open economy predictions is invariant to dimensionality in goods and in factor content space. This questions the popular perception that 2-dimensional formulations provide strong predictions whereas the n-dimensional extensions provide only weak restrictions.

In integrated equilibrium predictions, the set of conceivable outcomes is the set of goods or industries in which countries could specialize in equilibrium. In this framework lack of international factor price equalization is a prerequisite for the ability to predict in which industries countries will specialize. Free trade factor prices are shown to impose restrictions on predictive specialization based on global efficiency in productive allocation, independent of preferences. The global efficiency criterion implies that in an *n*-country world, the exports or factor content of exports of a single country face *n*-1 restrictions incorporating factor price comparisons with *all* of its trading partners. An important implication of this is that Helpman's (1984) well-known bilateral specification in which a country's factor content of bilateral trade is

only restricted by a single free trade factor price comparison is a misspecified comparative advantage prediction in a world with more than 2 countries.

The paper is organized as follows. Section 2 provides a definition of predictability and relates it to the concept of comparative statics. Section 3 revisits Ricardo's (1817) first pattern of trade prediction. Section 4 provides a taxonomy of pattern of trade predictions distinguishing between small open economy (section 4.1) and integrated equilibrium (section 4.2) formulations. Section 5 concludes.

2. Predictabilty: A definition

Let us motivate the definition of predictability with a situation outside of economics. A month prior to the 2006 Football World Cup tournament in Germany, a school teacher poses the following question to his students: Who do you predict will win the world cup? Assume the teacher gets the following three answers. Answer A: Brazil will win. Answer B: A European team will win. Answer C: Wales will win. Which of these answers are valid predictions? Clearly, Answer A is a valid prediction. However, Answer B is a valid prediction, too. Although Answer B does not identify a single country as a winner, it provides a prediction in the sense of reducing the set of conceivable winners to a European one.[2] On the other hand, Answer C is not a valid prediction. Since Wales did not qualify for the tournament, this country was not a conceivable winner.

The example illustrates that there are two components to a prediction. The specification of a set of conceivable outcomes and the prediction which identifies a subset on this set of conceivables. Formally:

[2] In fact, historically Answer B turned out to be the best prediction since, with the exception of Brazil in 1958, a European team has always won when the tournament was played in Europe.

Definition:

Given a set Ω *of outcomes that are either directly observed or estimated, a theory T is said to make a prediction on the set of conceivable outcomes through the specification of a subset* Ω_P *of* Ω. Ω_P *is called the prediction set and* $\Omega_A = \Omega/\Omega_P$ *is called the alternative.*

The advantage of this notion of predictability is that it leaves room for the specification of "an alternative" which is often ignored in empirical tests that aim to link theoretical formulations to data. For example, if Ω_A is identified by an alternative theory T_A, then the theories T and T_A can be distinguished by whether the observed/estimated outcomes fall either in Ω_P or Ω_A. If there is no alternative theory that restricts Ω, which is more common, one can postulate "chance" as the alternative hypothesis.

Let us apply this framework to the well-known question of how the imposition of an excise tax affects the volume of sales in a well-defined market. Prior to any economic theorizing, there are four conceivable outcomes: the tax will increase sales, it will decrease sales, it will keep sales unchanged or the relationship is ambiguous. Denoting sales by *x* and the excise tax by *t*, the set of conceivable outcomes is given by $\Omega=\{\partial x/\partial t>0, \partial x/\partial t<0, \partial x/\partial t=0, \text{ambiguous}\}$. Given the standard ceteris paribus assumptions, partial equilibrium theory predicts that the sales volume will decline, i.e $\Omega_P = \{\partial x^*/\partial t<0\}$.

This example illustrates that the standard comparative statics logic can be viewed as a special case of this predictability framework. Assume we are interested in how changes in a variable α affect a variable x, where the focus is on the direction of the effect, rather than the magnitude. We construct then a theory *T* which is

characterized by f(x,α)=0 or a fixed point equation x=g(x,α), where *x* is the equilibrium variable and α is a parameter of the model. In comparative statics we consider the functional relationship $x^*(\alpha)$ where x^* is the solution to the fixed point equation. Given that *T* predicts that x^* is increasing in α, the theory's comparative statics prediction can be written as follows: $\Omega=\{\partial x/\partial\alpha>0, \partial x/\partial\alpha<0, \partial x/\partial\alpha=0,$ ambiguous$\}$ and $\Omega_P = \{\partial x^*/\partial\alpha>0\}$.[3]

Although the comparative statics framework is extremely powerful when the variables of interest are univariate, its applications are limited in higher dimensional settings, which are particularly important in international trade theory.[4]

3. Revisiting Ricardo

The first pattern of trade prediction can be found in Ricardo's (1817) famous passage in his *Principles of Political Economy and Taxation*:

"The quantity of wine which she [Portugal] shall give in exchange for the cloth of England, is not determined by the respective quantities of labour devoted to the production of each, as it would be, if both commodities were manufactured in England, or both in Portugal.

England may be so circumstanced, that to produce the cloth may require the labour of **100** men if she attempted to make the wine, it might require the labour of **120** men for the same time. England would therefore find it her interest to import wine, and to purchase it by the exportation of cloth.

To produce the wine in Portugal, might require only the labour of **80** men for one year, and to produce the cloth in the same country, might require the labour of **90** men for the

[3] If an alternative theory T_A were to predict that $\partial x/\partial\alpha<0$, then the two theories could be distinguished from each other.
[4] See Milgrom and Roberts (1994) for developing an ordinal approach to comparing equilibria to remedy some of the shortcomings of the comparative statics framework.

same time, It would therefore be advantageous for her to export wine in exchange for cloth." (Ricardo, 1817, p.82)

Traditionally, Ricardo's four magic numbers have been interpreted as the labour units necessary to produce one unit of cloth and wine in England and Portugal, i.e. $a_c^{Eng}=100$, $a_w^{Eng}=120$, $a_c^{Por}=90$, $a_w^{P}=80$. [5] Given this interpretation, England is predicted to export cloth and import wine because the relative cost of cloth is lower in England than in Portugal: i.e. $a_c^{Eng}/a_w^{Eng}<a_c^{Por}/a_w^{Por}$. However, a major shortcoming of this interpretation is that the underlying cost comparison logic is incompatible with the historical text. In particular, Ricardo provides a prediction about England's pattern of trade based entirely on the first two numbers, but the cost comparison logic requires the knowledge of all four numbers.

In a recent paper, Ruffin (2002) has argued that Ricardo's numbers are the labour units embodied in actual trade rather than the country's per unit labour coefficients. Building on Ruffin (2002), Maneschi (2004) and Bernhofen (2007a), I argue that Ricardo made implicit use of the framework discussed in section 2.[6]

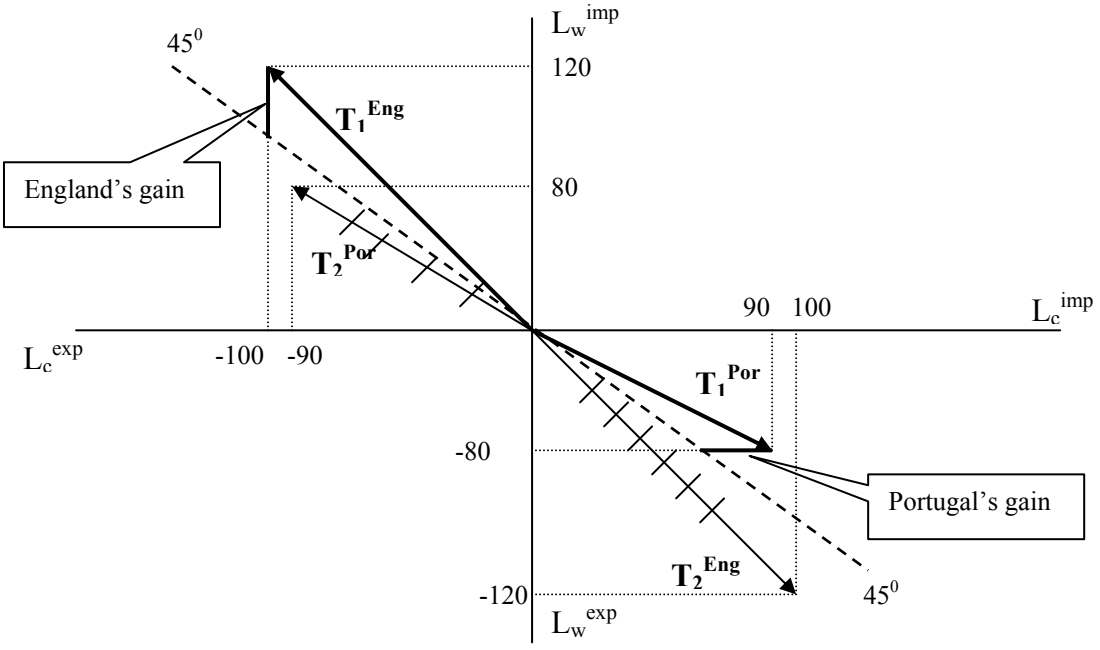
The first key assumption behind Ricardo's prediction is that he postulated a given terms of trade, or international commodity exchange ratio between cloth and wine, T_c/T_w. However, since Ricardo's formulation of comparative advantage was based on his labour theory of value, he expressed the terms of trade in domestic labour units.[7] If England is able to exchange T_c^{Eng} units of cloth for T_w^{Eng} units of wine, this is equivalent to trading a_c^{Eng} T_c^{Eng} English workers embodied in cloth for

[5] The term "magic" has been coined by Paul Samuelson.
[6] Following Ruffin's interpretation, Maneschi (2004) has investigated the link between Ricardo's measure of the gains from trade in terms of labour units and the equivalent and compensating variations measure of welfare. However, neither Ruffin nor Maneschi discuss the structure and generality of Ricardo's pattern of trade prediction.
[7] Ruffin (2002), among others, has argued that Ricardo's labour theory of value was essential to his discovery of comparative advantage.

$a_w^{Eng}T_w^{Eng}$ English workers embodied in wine. Ricardo's first two numbers pertain then to the English labour content of trade, i.e. 100= $a_c^{Eng}\, T_c^{Eng}$ and 120= $a_w^{Eng}T_w^{Eng}$. Similarly, Ricardo's second two numbers pertain to the Portugese labour content of trade: $90=a_c^{Por}T_c^{Por}$ and $80=a_w^{Por}T_w^{Por}$.[8]

In the case of two goods, there are two trading possibilities for each country: (i) export cloth ($T_c^i<0$) and import wine ($T_w^i>0$) or (ii) import cloth ($T_c^i>0$) and export wine ($T_w^i<0$), (where i=Eng, Por). Each pair of numbers defines then a country-specific set of two conceivable trading possibilities in labour content space. For England $\Omega^{Eng}=\{\, \mathbf{T_1^{Eng}}, \mathbf{T_2^{Eng}} \}$ where $\mathbf{T_1^{Eng}} =(a_c^{Eng} (-T_c^{Eng}), a_w^{Eng}T_w^{Eng}) = (-100, 120)$ and $\mathbf{T_2^{Eng}} = (a_c^{Eng}\, T_c^{Eng}, a_w^{Eng} (-T_w^{Eng})) = (100,-120)$ and for Portugal $\Omega^{Por}=\{$ $\mathbf{T_1^{Por}}, \mathbf{T_2^{Por}} \}$ where $\mathbf{T_1^{Por}} =(a_c^{Por}\, T_c^{Por}, a_w^{Por} (-T_w^{Por})) = (90, -80)$ and $\mathbf{T_2^{Por}} =(a_c^{Por} (-T_c^{Por}), a_w^{Por}\, T_w^{Por}) = (-90, 80)$. The four labour content vectors $\mathbf{T_1^{Eng}}$, $\mathbf{T_2^{Eng}}$, $\mathbf{T_1^{Por}}$, $\mathbf{T_2^{Por}}$ are illustrated in Figure 1.

Figure 1: Ricardo's four numbers and the labour content of trade



[8] The fact that England and Portugal face the same international commodity exchange ratio implies that $T_c^{Eng}/T_w^{Eng}=T_c^{Por}/T_w^{Por}$.

The second assumption inherent in Ricardo's prediction is that he postulated labour gains as a criterion to select among the set of conceivable trading possibilities. If England imports cloth and exports wine, i.e. chooses $\mathbf{T_1}^{\mathbf{Eng}}$, it will incur a labour loss of 20; if it exports cloth and imports wine, i.e. chooses $\mathbf{T_2}^{\mathbf{Eng}}$, it will incur a labour gain of 20. Hence, *"England would therefore find it her interest to import wine and to purchase it by the exportation of cloth*". Similarly, Portugal incurs a labour gain of 10 if it chooses $\mathbf{T_1}^{\mathbf{Por}}$ and a labour loss of 10 if it chooses $\mathbf{T_2}^{\mathbf{Por}}$. "*It would therefore be advantageous for (Portugal) to export wine in exchange for cloth.*"

Several things are worth noticing. Although the domain of Ricardo's prediction pertains to the pattern of commodity trade, the logic is inherently tied to the labour content of trade. Hence, the idea of trade in factor services, or the factor content of trade, is not a 20th century invention, but has its genesis in Ricardo. Second, by taking the terms of trade as given Ricardo linked its pattern of trade prediction to the gains from trade without requiring information about its trading partner. The next section will show that the underlying logic accommodates more general formulations of comparative advantage.

4. A taxonomy of pattern of trade predictions

4.1 Small open economy formulations

Consider the case of a small open economy that faces an exogenous set of world prices. Building on Deardorff (1980, 1982) and Neary and Schweinberger (1986), we apply our predictability framework to commodity and factor content predictions and show that the underlying structure of the prediction is invariant to dimensionality in goods and factor content space. In addition, the analysis reveals that Ricardo's formulation is a special case of either formulation.

(i) Commodity trade predictions

We start out with the 2-good formulation of comparative advantage for a small open economy that considers trading with the rest of the world. In this formulation, the world prices p_1^w, p_2^w are exogenously given and determine the terms at which any trade with the rest of the world will take place. The familiar relative price comparison (or opportunity cost) formulation is then:

$$\text{if } \frac{p_1^a}{p_2^a} < (>) \frac{p_1^w}{p_2^w} \text{ then } T_1<(>)0 \text{ and } T_2>(<)0, \qquad (1)$$

where p_1^a and p_2^a denote the economy's autarky prices and T_1 and T_2 the corresponding net import quantities. A shortcoming of the price comparison formulation is that it is not extendable to higher dimensions (see Ethier, 1984). However, the price comparison formulation (1) can be rewritten in terms of a restriction on the set of conceivable outcomes. The set of conceivable outcomes is then given by the balanced trade condition, i.e. $\Omega=\{T \in R^2 | \ p^w_1T_1+p^w_2T_2=0\}$, and the prediction set is given by:

$$\Omega_P=\{T \in R^2 | \ p^w_1T_1+p^w_2T_2=0 \text{ and } p_1^aT_1+p_2^aT_2 >0\}. \qquad (2)$$

It is easily verified that (1) are (2) are equivalent. However an advantage of (2) is that its underlying structure is invariant to dimensionality:

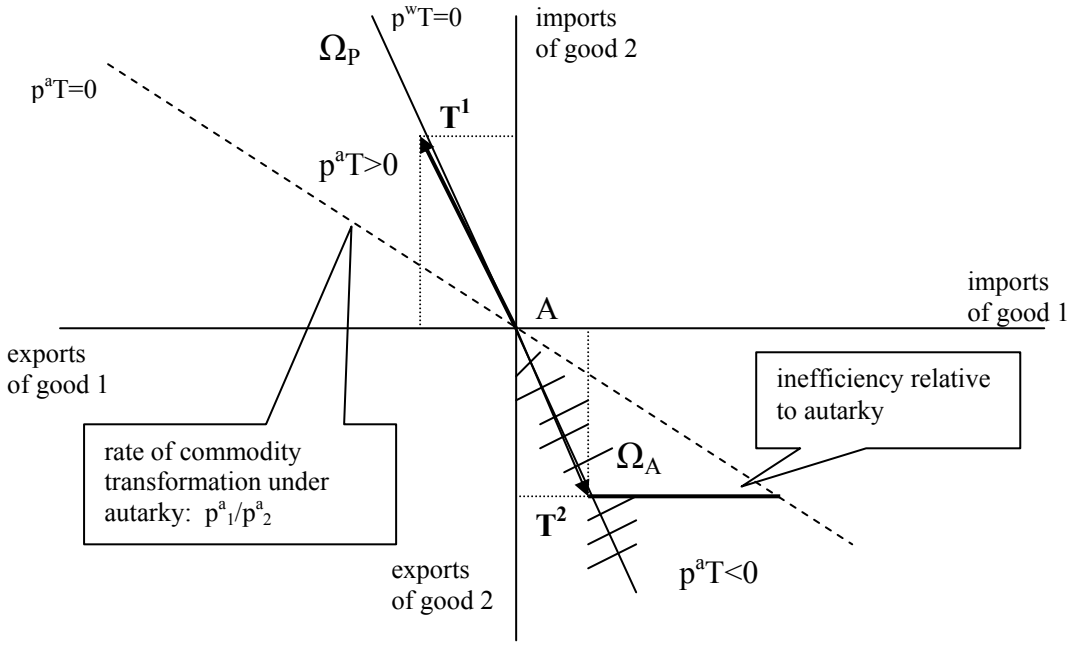
$$\Omega_P=\{T \in R^n | \ p^w_1T_1+\ldots+p^w_nT_n=0 \text{ and } p^aT_1+\ldots+p_2^aT_n >0\}, \qquad (2')$$

where (2') is the n-dimensional comparative advantage formulation developed by Deardorff (1980). The underlying nature of the prediction is illustrated in Figure 2.

The balance trade condition defines a hyperplane in R^n, which is cut into half by the restriction $p^aT>0$. In the two-good case, the hyperplane is a line with only two conceivable directions for trade, which is illustrated by the vectors T^1 and T^2. In this special case, the restriction predicts a unique trading configuration, e.g. T^1. In higher dimensions, the set of conceivable permissible trading outcomes are also cut into half, however, this does not identify which goods are exported or imported.

In Ricardo's one factor formulation, a country's relative autarky prices are given by the labour input coefficients: $p_c^a=a_c$, $p_w^a=a_w$.[9] Ricardo's numbers pertain then to the formulation in (2) rather than (1). The restriction on conceivable trading possibilities in the 2-commodity (cloth-wine) world is then $T_ca_c+T_wa_w>0$, postulating that there must be labour savings from international trade.

Figure. 2: Commodity pattern of trade prediction



The selection criteriona is intimately related to the gains from trade resulting from a more efficient allocation of resources. In particular, the trading vector T^2 in

[9] Because the labour coefficients determine only relative prices, we would have to include a factor of proportionality k. However, without loss of generality, we assume that k=1.

Figure 1 is excluded since it is associated with an international transformation of good 2 (i.e. the exportable) into good 1 (i.e. the importable) that is inefficient relative to the domestic transformation (along the dotted line) without international trade.

Finally, from a "testing perspective", the *n* and 2-good formulations are completely equivalent with regard to the specification of the alternative hypothesis. As there exist no alternative theory that imposes restrictions on the set of conceivable outcomes, we can postulate "chance" as the alternative. Under the assumption that "under chance" each element of the set of conceivable outcomes is equally likely, we can define the null and the alternative hypothesis:

$$H_0: \ \Pr(T \in \Omega_P)=1; \quad H_1: \Pr(T \in \Omega_P)=0.5, \tag{3}$$

where Pr(.) denotes the probability measure. The key point here is that the probability statement in the alternative hypothesis is independent of dimensionality.[10]

(ii) Factor content prediction

Alternatively, we can investigate predictions pertaining to the factor content of trade. Technologies are such that *n* goods are produced from *l* factors under standard CRS production functions. A key point in factor content analysis is how to define the factor content of trade in a world with unequal technologies.[11] In the context of our framework, we calculate the economy's factor content using the domestic technology matrix A. We can then define then the set of conceivable outcomes as:

[10] Using autarky price data from 19th century Japan, Bernhofen and Brown (2004) were able to reject the alternative hypothesis at a 99% significance level.
[11] Deardorff (1982) considers three different variations of the factor content of trade, but assumes identical technologies. Neary and Schweinberger (1986) define the factor content of trade based on domestic techniques of production.

$$\Omega=\{F\in R^l|F=AT \text{ and } p^wT=0\}. \quad (4)$$

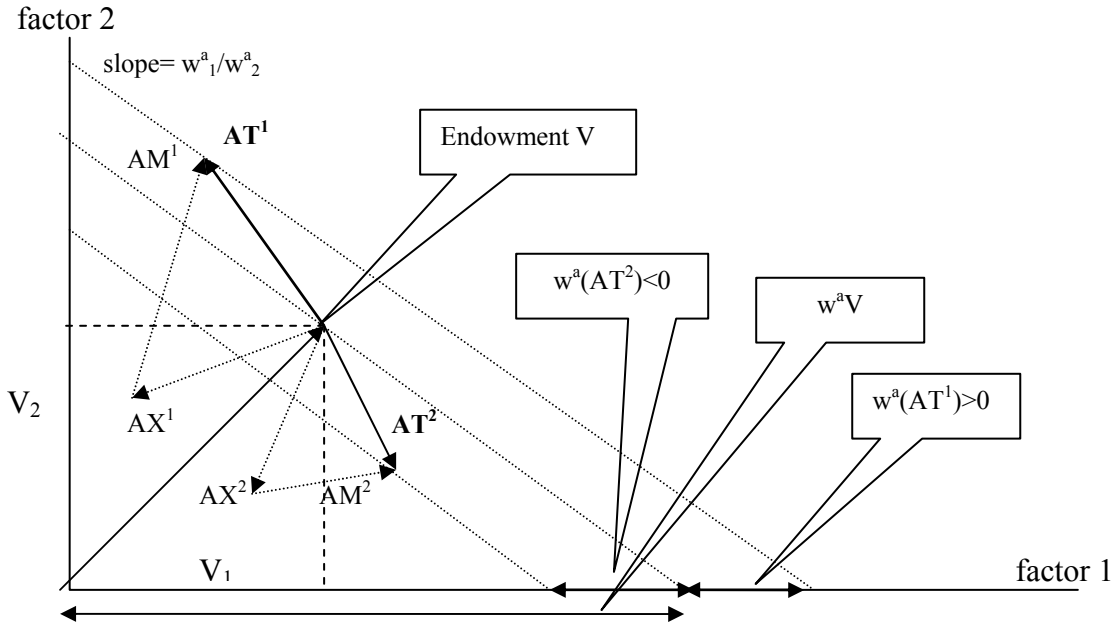
The prediction or selection criterion identifies again the trading configurations that are efficient for the economy. Now it will be convenient to split the net import vector into its individual components: T=M-X, where M is the *n*-good import vector and X is the *n*-good export vector.[12] Given a particular trading vector T, the economy is giving up actual factor services AX embodied in its exports in exchange for the factor services embodied in its imports. AM are the domestic resource gains embodied in imports.. Interpreting the autarky factor price vector w^a as the shadow prices at which the economy evaluates factor services embodied in trade, the economy would be willing to engage in the trading opportunity T only if the 'gain from factor imports' exceed the losses from factor exports, i.e. $w^a(AM)>w^a(AX)$. The corresponding prediction can be stated as follows:

$$\Omega_P=\{AT\in R^l|\ w^a(AT)>0\}. \quad (5)$$

Figure 3 illustrates factor content of trade triangles in the two-factor case. Given two conceivable factor content of trade vectors AT^1 and AT^2, trade in factor services can be thought of an augmentation of the country's endowment vector $V=(V_1,V_2)$. The factor content of trade is decomposed in the factor content of exports AX^i and the factor content of imports AM^i (i=1,2). The factor content vector AT^1 leads to a welfare gain since $w^a(V+AT^1)>w^aV$. By comparison, the factor content vector AT^2 leads to a welfare loss since $w^a(V+AT^2)<w^aV$.

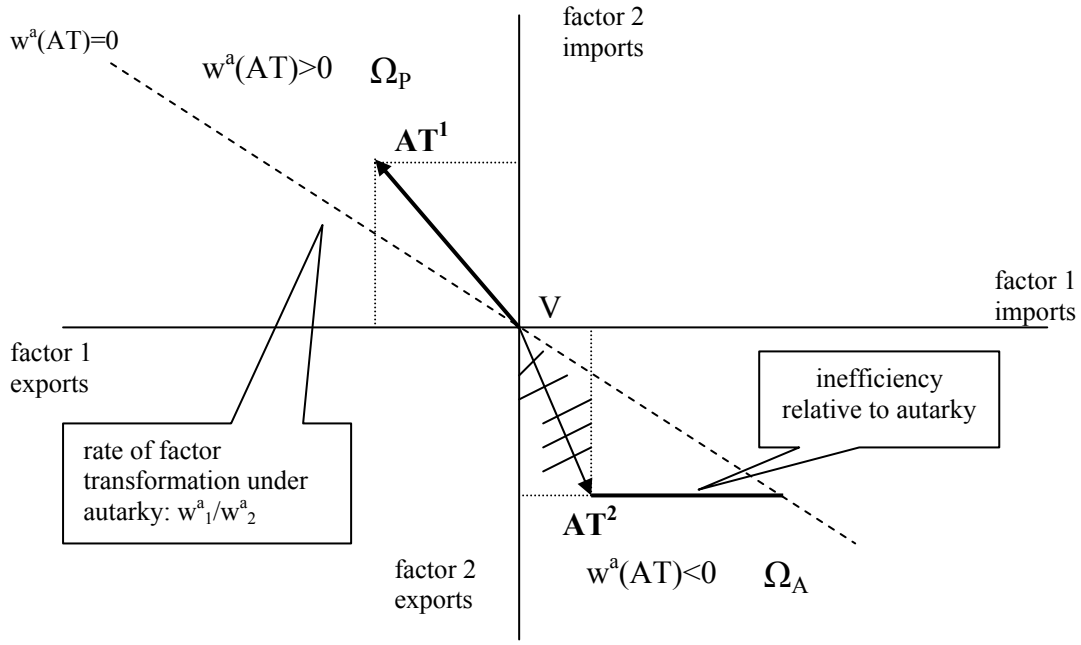
[12] The entries in X and M are now all non-negative. Since a particular good is either imported or exported, X will have entries of "0"for goods that are imported and M will have entries of "0" for goods that are exported.

Figure 3: Factor content triangles and welfare



Alternatively, the factor content prediction is illustrated in Figure 4, which can be viewed as the factor content dual to Figure 2. The factor content of trade vector AT^2 is excluded from the set of conceivable outcomes as it leads to an inefficient international factor transformation relative to the situation of no trade (dashed line). Ricardo's prediction can be viewed as a special formulation of (5): $w^a(T_c a_c + T_w a_w)>0$. In the case of a single factor, the magnitude of the autarky price w^a does not matter for the sign of the left-hand side, so w^a can be normalized to be 1.

Figure 4: Factor content prediction



4.2 Integrated equilibrium formulations

In this section we characterize restrictions on the pattern of international specialization in an integrated equilibrium. The analysis is motivated by an emerging empirical literature claiming evidence in favor of the neoclassical trade model by testing restrictions on bilateral trade flows.[13] The theoretical foundation of these studies is based on Helpman (1984), who has shown that in an integrated equilibrium without international factor price equalization, the factor content $F^{ij}$of any bilateral trade flow from country *i* to country *j*, is restricted (or predicted) by the corresponding factor price difference (w^j-w^i) between these two countries: $(w^j-w^i)F^{ij} \geq 0$. In their empirical implementation Choi and Krishna (2004, p. 889) recognize the empirical attractiveness of Helpman's bilateral specification: "*A further and equally important contrast with the existing literature derives from the fact that while most empirical*

[13] See for instance the recent papers by Choi and Krishna (2004), Lai and Zhu (2007) and the earlier work by Brecher and Choudhri (1993).

tests of the theory have focused on the net factor content of a country's multilateral trade, our tests concern bilateral trade flows, thereby enabling the examination of trade flows between only a subset of countries for which quality data (relatively speaking) are available. In what follows, I will show two things. First, I show that in an integrated equilibrium the predictive domain of the theory is a country's exports or factor content of exports, independent of destination. Second, a country's equilibrium exports (or factor content of exports) is restricted by the factor prices of all trading partners.[14]

Our analytical framework is based on the continuum of goods formulations pioneered by Dornbusch, Fischer and Samuleson, DFS (1977, 1980). In this set-up, the set of conceivable outcomes Ω is the set of industries in the world economy, characterized by the unit interval $\Omega = [0,1]$. Free trade factor prices impose restrictions on $\Omega = [0,1]$ which predict in which industries an economy will specialize. Since the emphasis is on the production side of the economy, one does not need to make any specific assumptions about the demand side of the economy, except that preferences are such that an equilibrium exists. To build the intuition, we first characterize predictions in the Ricardian specification (DFS, 1977) and then move on to the Heckscher-Ohlin specifications (DFS, 1980).

(i) Ricardian continuum of good formulation

In the the Ricardian formulation $\Omega = [0,1]$ is a continuum of goods which are exogenously ranked according to their relative labour productivity $A(z)=a_2(z)/a_1(z)$, where $A(z)$ is decreasing in z so that country 1 (home) has productivity advantage in

[14] In a companion paper (Bernhofen (2007b)), I have derived these additional restrictions using Helpman's analytical apparatus and applied them to the data domain of Choi and Krishna. However, the focus of this paper is just to characterize these restrictions and identify the links to other specifications.

low-indexed industries and country 2 (foreign) has a productivity advantage in high-indexed industries. In this specification, home and foreign free trade factor prices w_1 and w_2 determine the dividing, or marginal, good m, defined as $A(m)=w_1/w_2$. The prediction set for the home economy is $\Omega_1= [0,m(w_1/w_2)]$, which can be characterized by

$$\Omega_1=\{z\in[0,1]|\ a_2(z)w_2-a_1(z)w_1\geq 0\}. \qquad (6)$$

The prediction in (6) can be interpreted as saying that free trade factor prices impose a restriction on $\Omega = [0,1]$ that guarantee that the country 1 specializes in those goods in which it is most efficient relative to country 2, i.e. the left-side of the interval.[15] The exact location of the border good m will depend on w_1/w_2 which embodies information about preferences, endowments etc. In sum, the pattern of specialization is characterized by a single restriction.[16]

(ii) Multi-cone Heckscher-Ohlin formulation: 2 countries

Consider now a Heckscher-Ohlin specification with 2 factors (capital and labour), 2 countries (country 1 and 2) and identical CRS technologies.[17] The set of conceivable outcomes is again a continuum of industries in the unit interval $\Omega = [0,1]$, where each industry z in Ω is characterized by its capital-labour ratio $a_K(z)/a_L(z)$. Industries are ranked in order of decreasing capital intensity, i.e. $a_K(z)/a_L(z)$ is

[15] Alternatively, $\Omega_2= [m(w_1/w_2),1]$. If one incorporates uniform iceberg transportation costs, the efficiency criteria is modified such that there are two border goods, m_1 and m_2, with country 1 specializing in $[0,m_1]$, country 2 specializing in $[m_2,1]$ and both countries producing the non-traded goods $[m_1, m_2]$.

[16] Here we focus only on the two-country specification since it has been a challenge to extend DFS (1977) to multiple countries in a tractable way. See Matsuyama (2007) for an excelllent survey on the Ricardian trade literature.

[17] It is important to note that the identical technology assumption is not central to argument. We could, for instance, incorporate Hicks-neutral technological differences. But this is beyond the scope of this paper.

decreasing in z. An example of a specific cost function that parameterizes the underlying technology is $c(w,r)=w^z r^{1-z}$. For this technology, the capital-labour ratio is $a_K(w,r,z)/a_L(w,r,z)=(w/r)(1-z)/z$, which is decreasing in z.

Assume that country 1 is relatively capital abundant, i.e. $K_1/L_1>K_2/L_2$. If the free trade equilibrium is characterized by factor price equalization, i.e. $w_1=w_2$ and $r_1=r_2$, then the model does not provide any prediction on sectoral specialization. The reason for this is because of identical factor prices, it is equally efficient to produce the goods in either country. Therefore, there is no global efficiency criterion that imposes a restriction on the location of production.[18] Lack of international factor price equalization is central to predictability.

Assume now that factor endowments are sufficiently dissimilar so that factor prices are different in equilibrium. Because country 1 is assumed to be relatively capital abundant, country 1 will have a higher wage-rental ratio in equilibrium: $w_1/r_1 \geq w_2/r_2$. Then there exists again a border good m_1, such that home specializes in $\Omega_1=[0,m_1]$ and foreign specializes in $\Omega_2=[m_1,1]$. Ω_1 is characterized by the following restriction

$$\Omega_1=\{z \in [0,1] |\ a_K(z,w_2/r_2)r_2+a_L(z,w_2/r_2)w_2-a_K(z,w_1/r_1)r_2-a_L(z,w_1/r_1)w_1 \geq 0\}. \qquad (7)$$

The underlying structure in (7) is quite similar to (6). Factor prices in (7) impose a restriction which ensures that the more capital abundant country 1 will specialize in the more capital-intensive goods. The border good m_1 can be viewed as an implicit function of both factor prices: $m_1=m_1(w_1/r_1, w_2/r_2)$.

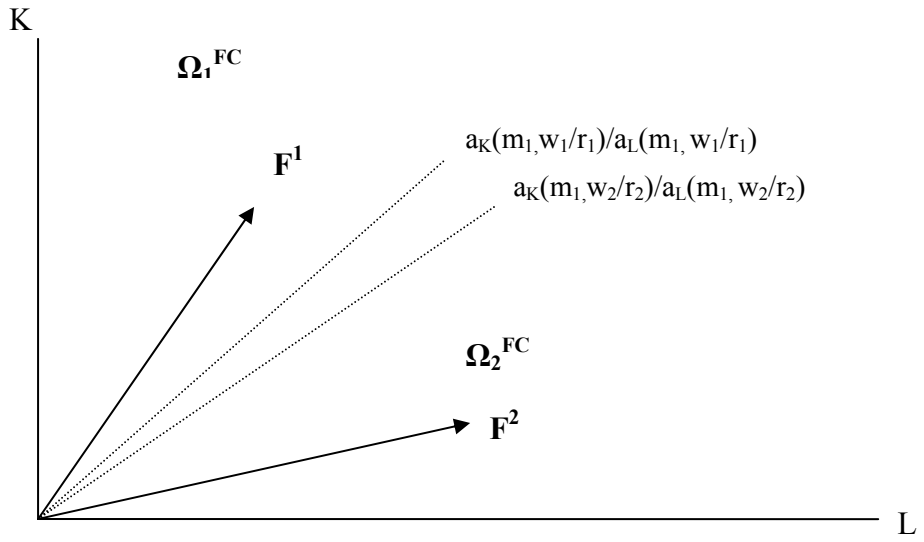
[18] Under the assumption of identical homothetic preferences, we obtain the Heckscher-Ohlin-Vanek prediction, which has been the workhorse equation for testing the neoclassical trade model.

Alternatively, we can characterize the factor content formulation of (7). The set of conceivable outcomes in factor content space is $\Omega^{FC}=\{(K,L)|\ 0\leq K/L\leq 1\}$ and the factor content equivalent of (7) is

$$\Omega_1^{FC}= \{(K/L)|\ a_K(m_1,w_1/r_1)/a_L(m_1,w_1/r_1) \leq K/L \leq 1\ \}. \qquad (8)$$

By the same logic, the factor content prediction for country 2 is given by $\Omega_2^{FC}= \{(K,L)|\ 0\leq K/L \leq a_K(m_1,w_2/r_2)/a_L(m_1,w_2/r_2)\}$. The factor content prediction is illustrated in Figure 5, which shows that Ω_1^{FC} and Ω_2^{FC} define country-specific cones. This specification predicts that any factor content of export vector F^i of country i must lie in Ω_i^{FC} and is implictly restricted by home and foreign factor prices. Since country 1 specializes in the most capital abundant goods the restrictions imply that any of its exports have a higher capital labour ratio than any of its imports. Since there are only two countries, exports are by definition bilateral.

Figure 5: Factor content predictions with a single restriction



(iii) Multi-cone Heckscher-Ohlin formulation: n countries.

Consider now what happens to the equilibrium if, like manna from heaven, n-2 additional countries are dropped on the world economy. Assume that these countries are capable of producing the goods in [0,1] with the same CRS technologies and differ only in their relative factor endowments. Without loss of generality, assume the following factor endowment ranking: $K_1/L_1 > K_2/L_2 > K_3/L_3 > \ldots > K_n/L_n$. Assuming again that factor endowments are sufficiently dissimilar, the equilibrium factor price ratios will reflect the endowment ranking: $w_1/r_1 > w_2/r_2 > w_3/r_3 > \ldots > w_n/r_n$. The integrated equilibrium will then be characterized by n-1 border goods $m_1, m_2, \ldots, m_{n-1}$ which define the ranges of specialization for the individual countries: $\Omega_1=[0,m_1]$, $\Omega_2=[m_1, m_2], \ldots \Omega_i=[m_{i-1},m_i] \ldots \Omega_n=[m_{n-1},1]$. Ω_i is characterized by the following global efficiency criterion:

$$\Omega_i = \{z \in [0,1] | \, a_K(z,w_j/r_j)r_j + a_L(z,w_j/r_j)w_j - a_K(z,w_i/r_i)r_i - a_L(z,w_i/r_i)w_i \geq 0 \text{ for all } j \neq i\}. \quad (9)$$

The key characteristic of the specification in (9) is that the prediction set Ω_i of country i is determined by *n-1* restrictions involving the free trade factor prices of all trading partners. Consequently, any border good m_i can be viewed an implicit function of all factor prices: $m_i=m_i(w_1/r_1, w_2/r_2, \ldots, w_n/r_n)$. The intuition for this is that since the factor price ratios embody information on countries' relative factor scarcities, efficient multilateral specialization requires information on the factor scarcities of all trading partners.

We can again characterize the factor content dual to (9). The factor content space is characterized by *n* country-specific cones $\Omega_1^{FC}, \Omega_2^{FC}, \ldots, \Omega_{n-1}^{FC}$, where Ω_i^{FC} is given by:

$$\Omega_i^{FC} = \{(K,L) | \ a_K(m_i,w_i/r_i)/a_L(m_i,w_i/r_i) \leq K/L \leq a_K(m_{i-1},w_i/r_i)/a_L(m_{i-1},w_i/r_i) \ ] \quad (10)$$

Figure 6: Factor content predictions with *n*-1 restrictions (*n*>2):

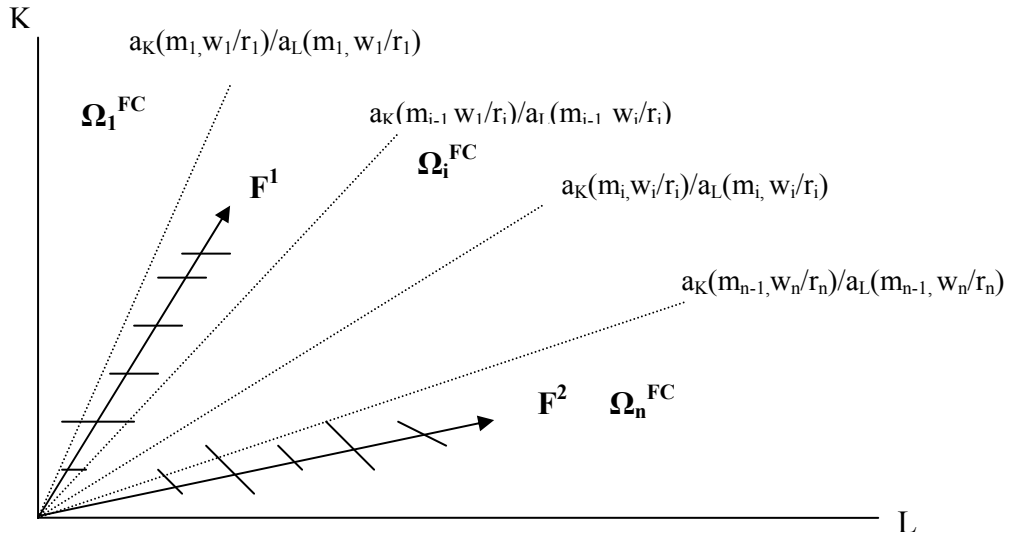


Figure 6 illustrates the multi-cone factor content prediction in the *n*-country case. A few comments are in order. First, since a factor content set Ω_i^{FC} characterizes the production side of the economy, the prediction pertains to the factor content vector F^i of a country's exports, independent of where the exports are shipped. Second, the number of trading partners matters.[19] In particular, the theory predicts that the size of the cones becomes smaller, the more trading partners there are. Ω_1^{FC} is smaller in Figure 6 than in Figure 5 since the additional trading partners enables country 1, which is most capital abundant, to specialize in a smaller set of the most capital-intensive goods. A comparison of Figures 5 and Figure 6 also illustrates the misspecification that occurs when testing only a single restriction in a multi-country world. Assuming we observe the factor content of export vectors F^1 and F^2 for

[19] Assuming, of course, that countries' factor endowments are sufficiently dissimilar.

countries 1 and 2. The 2-country specification in Figure 5 would suggest that these data vectors are in accord with the theoretical prediction, whereas the multi-lateral specification in Figure 6 suggests otherwise.

5. Concluding remarks

Using a single analytical framework, I tried to characterize a whole class of pattern of trade predictions, from Ricardo (1817) to the multi-cone specification. One of the key messages of this paper is that the pattern of trade predictions can be linked to efficiency. For small open economy predictions, efficiency is directly related to the gains from trade as an economy will be only willing to engage in trading activities that are more efficient than what it can do under autarky. As a result, autarky goods and factor prices impose restrictions on observable trading patterns. For integrated equilibrium predictions, the pattern of international specialization is governed by global efficiency. Lack of factor price equalization is central and free trade factor prices of all trading partners restrict patterns of specialization in goods and factor content space.

The message that pattern of trade predictions are directly related to efficiency gains in models without factor price equalization provides an important justification for testing these models. For instance, if empirical tests confirm these predictions, they provide implicit evidence for efficiency gains resulting from international specialization.

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