Federal Reserve Bank of Minneapolis
Preliminary

June 2007

Trade Liberalization, Growth, and Productivity*

Claustre Bajona
Ryerson University

Mark J. Gibson
Washington State University

Timothy J. Kehoe
University of Minnesota,
Federal Reserve Bank of Minneapolis,
and National Bureau of Economic Research

Kim J. Ruhl
University of Texas at Austin

ABSTRACT

There is a lively debate about the impact of trade liberalization on economic growth measured as growth in real gross domestic product (GDP). Most of this literature focuses on the empirical relation between trade and growth. This paper investigates the theoretical relation between trade and growth. We show that standard models — including Ricardian models, Heckscher-Ohlin models, monopolistic competition models with homogeneous firms, and monopolistic competition models with heterogeneous firms — predict that opening to trade increases welfare, not necessarily real GDP. In a dynamic model where trade changes the incentives to accumulate factors of production, trade liberalization may lower growth rates even as it increases welfare. To the extent that trade liberalization leads to higher rates of growth in real GDP, it must do so primarily through mechanisms outside of those analyzed in standard models.

*This paper was prepared for the conference "New Directions in International Trade Theory" at the University of Nottingham. The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

1. Introduction

How does trade liberalization affect a country's growth and productivity? How does it affect a country's social welfare? As Rodriguez and Rodrik (2001) point out, "growth and welfare are not the same thing. Trade policies can have positive effects on welfare without affecting the rate of economic growth."

There is a lively debate about the impact of trade liberalization on economic growth measured as growth in real gross domestic product (GDP). Most of this literature focuses on the empirical relation between trade and growth. The findings are mixed. Many studies find a connection between trade, or some other measure of openness, and growth. But Rodriguez and Rodrik (2001), among others, are skeptical that these studies find a connection between *trade policy* and growth. (We provide an overview of these literatures below.) A further criticism of the empirical literature, posed by Slaughter (2001), is that it largely does not address the specific mechanisms through which trade may affect growth.

This paper investigates the theoretical relation between trade policy and growth. We do so using simple versions of some of the most common international trade models, including Heckscher-Ohlin models, Ricardian models, monopolistic competition models with homogeneous firms, monopolistic competition models with heterogeneous firms, and dynamic Heckscher-Ohlin models. These models allow us to investigate a number of specific mechanisms by which trade liberalization is commonly thought to enhance growth or productivity: improvements in the terms of trade, increases in product variety, reallocation toward more productive firms, and increased incentive to accumulate capital.

For each model we provide an analytical solution for the autarky equilibrium and for the free trade equilibrium. We then look at the extreme case of trade liberalization by comparing autarky and free trade. To be consistent with empirical work, we measure *real GDP* in each of these models as real GDP is typically measured in the data, as GDP at constant prices. In each model the supply of labor is fixed, so changes in real GDP are also changes in measured *labor productivity*. We then contrast real GDP with a theoretical measure of *real income*, or social welfare.

In each model, trade liberalization increases social welfare. This is to be expected, but our results on real GDP may come as a surprise to many economists. In the static models, there is no general connection between trade liberalization and growth of real GDP per capita — the relationship may even be negative. Moreover, in a dynamic model with capital accumulation, some countries will have slower rates of growth under free trade than under autarky. Opening to trade improves welfare, but does not necessarily increase real GDP per capita or speed up growth. If openness does in fact lead to large increases in real GDP, these increases do not come from the standard mechanisms of international trade.

There is a vast empirical literature on the relationship between trade and growth. We can classify these papers into two groups depending on their interpretation of trade. The first line of research interprets trade as export volume and uses the growth rate of exports or of exports relative to GDP as its measure of changes in trade. The second line of research interprets trade as "trade policy" and studies the correlation of a variety of "openness" indicators and economic growth.

Early papers in the first line of research include Michaely (1977) and Balassa (1978). Lewer and Van den Berg (2003) present an extensive survey of this literature. They argue that most studies in this literature find a positive relationship between trade volume and growth and that they are fairly consistent on the size of this relationship.

In the second line of research, findings are mixed. Studies that find a positive relationship between trade openness and growth (using different techniques and openness measures) include, among others, World Bank (1987), Dollar (1992), Sachs and Warner (1995), Frankel and Romer (1999), Hall and Jones (1999), and Dollar and Kraay (2004).

Rodriguez and Rodrik (2001) question the findings of these studies. In particular, they argue that the indicators of openness used in these studies are either bad measures of trade barriers or are highly correlated with variables that also affect the growth rate of income. In the latter case, the studies may be attributing to trade the negative effects on growth of those other variables. Following this argument, Rodrik, Subramanian, and Trebbi (2002) find that openness has no significant effect on growth once institution-related variables are added in the regression analysis. Several studies using tariff rates as their specific measures of openness have found the relationship between trade policy and growth to depend on a country's level of development. In particular, Yanikkaya (2003) and DeJong and Ripoll (2006) find a *negative* relationship between trade openness and growth for developing countries.

A further criticism of the empirical literature, posed by Slaughter (2001), is that the literature does not, in general, address the specific mechanisms through which trade may affect growth. Exceptions are Wacziarg (2001) and Hall and Jones (1999). They find that trade affects growth mainly through capital investment and productivity.

A smaller set of papers study the relationship between openness to trade and productivity. Examples are Alcalá and Ciccone (2004) and Hall and Jones (1999), which find a significant positive relationship between trade and productivity.

Theoretical studies on the relationship between trade and growth do not offer a clear view on whether there should be a relationship between trade openness (measured as lower trade barriers) and growth in income. Models following the endogenous growth literature with increasing returns, learning-by-doing, or knowledge spillovers predict that opening to trade increases growth in the world as a whole, but may decrease growth in developing countries if they specialize in the production of goods with less potential for learning. Young (1991), Grossman and Helpman (1991), and Lucas (1988) are examples of examples of papers in this area. By contrast, Rivera-Batiz and Romer (1991) find that trade leads to higher growth for all countries by promoting investment in research and development.

Models of trade using the Dixit-Stiglitz theory of industrial organization have typically focused on welfare. Krugman shows, for instance, that trade liberalization leads to welfare increases because of increases in product variety. Melitz (2003) incorporates heterogeneous firms into a Krugman model and finds that trade liberalization increases a theoretical measure of productivity. Chaney (2006) also considers a simple model of heterogeneous firms, close to the one we study here. When productivity is measured in the model as in the data, Gibson (2006) shows that trade liberalization does not, in general, increase productivity in these sorts of models. The increase is, rather, in welfare. Gibson (2006) finds that adding mechanisms to allow for technology adoption generate increases in measured productivity from trade liberalization.

Models following the exogenous growth literature do not have a clear prediction for the relationship between trade and growth. In particular, in dynamic Heckscher-Ohlin models — models that integrate a neoclassical growth model with a Heckscher-Ohlin model of trade — opening to trade may increase or decrease a country's growth rate of income depending on parameter values. Trade may slow down growth in the capital-scarce country even while it raises welfare. Papers in this literature are Ventura (1997), Cuñat and Maffezzoli (2004), and Bajona and Kehoe (2006).

The transition from theoretical to empirical results is not straightforward. Besides the lack of consensus on whether there should be a relationship, the variables studied in the empirical and theoretical analysis do not necessarily coincide. The main problem is the measure of real GDP. Empirical studies use measures of real GDP reported in the national income and product accounts, which use either base-year prices or a chain-weighting method. Kehoe and Ruhl (2007) show that these differences in measurement methods may lead to model predictions not being reflected in the measured data. In particular, they show that in standard models income effects due to changes in the terms of trade are not reflected in data-based measures of real GDP. Similar issues are addressed by Diewert and Morrison (1986) and Kohli (1983, 2004).

2. Do improvements in the terms of trade increase real GDP?

A country's *terms of trade* is the price of its imports relative to the price of its exports. By decreasing the relative price of imports, trade liberalization acts as a positive terms-of-trade shock. We consider how improvements in the terms of trade affect real GDP in both a Heckscher-Ohlin model and a Ricardian model with a continuum of goods. (Kehoe and Ruhl (2006) consider the same issue in a small open economy model and arrive at similar conclusions.) In the Heckscher-Ohlin model, countries differ in their relative factor endowments but not in their technologies. In the Ricardian model, countries differ in their technologies but use a common factor of production.

2.1. A static Heckscher-Ohlin model

In each country i, $i = 1, 2, ..., n$, there is measure L_i of consumers. Each consumer is endowed with one unit of labor and $\bar{k}_i$ units of capital. There are two tradable goods, $j = 1, 2$. All quantities are in per capita terms.

A consumer in country i chooses c_{ij}, $j=1,2$, to maximize

$$a_1 \log c_{i1} + a_2 \log c_{i2}, \tag{1}$$

where $a_1 + a_2 = 1$, subject to the budget constraint

$$p_{i1}c_{i1} + p_{i2}c_{i2} = w_i + r_i \bar{k}_i. \tag{2}$$

Here p_{ij} is the price of good j, r_i is the rental rate of capital, and w_i is the wage rate.

Good j is produced by combining capital and labor according to the Cobb-Douglas production function

$$y_{ij} = \theta_j k_{ij}^{\alpha_j} \ell_{ij}^{1-\alpha_j}, \tag{3}$$

where $\alpha_1 > \alpha_2$ (that is, good 1 is relatively capital-intensive). The zero-profit conditions are

$$p_{ij}\alpha_j k_{ij}^{\alpha_j - 1} \ell_{ij}^{1-\alpha_j} - r_i \leq 0, \quad = 0 \text{ if } k_{ij} > 0 \tag{4}$$

$$p_{ij}\left(1-\alpha_j\right) k_{ij}^{\alpha_j} \ell_{ij}^{-\alpha_j} - w_i \leq 0, \quad = 0 \text{ if } \ell_{ij} > 0. \tag{5}$$

Clearing in the factors markets requires that

$$k_{i1} + k_{i2} = \bar{k}_i \tag{6}$$

$$\ell_{i1} + \ell_{i2} = 1. \tag{7}$$

Under autarky,

$$c_{ij} = y_{ij}. \tag{8}$$

Under free trade,

$$\sum_{i=1}^{n} L_i c_{ij} = \sum_{i=1}^{n} L_i y_{ij}. \tag{9}$$

There are analytical solutions for the autarky and free trade equilibria of this model. To simplify the notation, let

$$A_1 = a_1\alpha_1 + a_2\alpha_2 \tag{10}$$

$$A_2 = 1 - A_1 = a_1\left(1-\alpha_1\right) + a_2\left(1-\alpha_2\right) \tag{11}$$

$$D_j = \frac{\theta_j a_j \alpha_j^{\alpha_j}\left(1-\alpha_j\right)^{1-\alpha_j}}{A_1^{\alpha_j} A_2^{1-\alpha_j}} \tag{12}$$

$$D = D_1^{a_1} D_2^{a_2} \tag{13}$$

$$\mu_{i1} = \frac{A_1\gamma_i\left(1-\alpha_2\right) - A_2\alpha_2}{a_1\left(\alpha_1 - \alpha_2\right)} \tag{14}$$

$$\mu_{i2} = \frac{A_2\alpha_1 - A_1\gamma_i\left(1-\alpha_1\right)}{a_2\left(\alpha_1 - \alpha_2\right)}. \tag{15}$$

Throughout the paper we denote autarky equilibrium objects by a superscript A and denote free trade equilibrium objects by a superscript T.

Autarky

Prices are

$$p_{ij}^A = \frac{a_j D}{D_j}\bar{k}_i^{A_1-\alpha_j} \tag{16}$$

$$r_i^A = A_1 D\bar{k}_i^{A_1-1} \tag{17}$$

$$w_i^A = A_2 D\bar{k}_i^{A_1}. \tag{18}$$

Allocations are

$$c_{ij}^A = y_{ij}^A = D_j\bar{k}_i^{\alpha_j} \tag{19}$$

$$k_{ij}^A = \frac{a_j\alpha_j}{A_1}\bar{k}_i \tag{20}$$

$$\ell_{ij}^A = \frac{a_j\left(1-\alpha_j\right)}{A_2}. \tag{21}$$

In order to compare the model with the data, we measure GDP in the model as it is measured in the data. The standard way of calculating real GDP in the data is as GDP at constant prices (as opposed to GDP at current prices). In the static models in this paper, our measure of real GDP is simply GDP at autarky prices. Here GDP at autarky prices is

$$\begin{aligned} GDP_i^A &= p_{i1}^A y_{i1}^A + p_{i2}^A y_{i2}^A \\ &= D\bar{k}_i^{A_1} \end{aligned}. \tag{22}$$

For each model, we contrast our data-based measure of real GDP with a theoretical index of real income, or social welfare. Throughout the paper we calculate *real income* as a homogeneous-of-degree-one transformation of period utility. Here real income is

$$\begin{aligned} v_i^A &= \left(c_{i1}^A\right)^{a_1}\left(c_{i2}^A\right)^{a_2} \\ &= D\bar{k}_i^{A_1} \end{aligned}. \tag{23}$$

Free trade

To obtain an analytical solution, we focus on the case where all countries are in the cone of diversification. That is, letting

$$\bar{k} = \frac{\sum_{i=1}^{n} L_i \bar{k}_i}{\sum_{i=1}^{n} L_i} \tag{24}$$

$$\gamma_i = \frac{\bar{k}_i}{\bar{k}} \tag{25}$$

$$\kappa_j = \left(\frac{\alpha_j}{1-\alpha_j}\right)\frac{A_2}{A_1}, \tag{26}$$

we examine the case where $\kappa_2 \le \gamma_i \le \kappa_1$, $i = 1,...,n$.

World prices are

$$p_j^T = \frac{a_j D}{D_j}\bar{k}^{A_1-\alpha_j} \tag{27}$$

$$r^T = A_1 D\bar{k}^{A_1-1} \tag{28}$$

$$w^T = A_2 D\bar{k}^{A_1}. \tag{29}$$

Allocations are

$$c_{ij}^T = \left(A_1\gamma_i + A_2\right)D_j\bar{k}^{\alpha_j} \tag{30}$$

$$y_{ij}^T = \mu_{ij} D_j \bar{k}^{\alpha_j} \tag{31}$$

$$k_{ij}^T = \mu_{ij}\frac{a_j\alpha_j}{A_1}\bar{k} \tag{32}$$

$$\ell_{ij}^T = \mu_{ij}\frac{a_j\left(1-\alpha_j\right)}{A_2}. \tag{33}$$

Notice that setting $\gamma_i = 1$ gives the same values as for autarky.

GDP at current prices is

$$\begin{aligned} gdp_i^T &= p_1^T y_{i1}^T + p_2^T y_{i2}^T \\ &= \left(A_1\gamma_i + A_2\right) D\bar{k}^{A_1} \end{aligned}. \tag{34}$$

Notice that we use the lowercase gdp to distinguish current prices under free trade so as not to confuse it with our measure of real GDP, GDP at autarky prices. Here GDP at autarky prices is

$$\begin{aligned} GDP_i^T &= p_{i1}^A y_{i1}^T + p_{i2}^A y_{i2}^T \\ &= \frac{\gamma_i^{-\alpha_1}\left(A_1\gamma_i\left(1-\alpha_2\right) - A_2\alpha_2\right) + \gamma_i^{-\alpha_2}\left(A_2\alpha_1 - A_1\gamma_i\left(1-\alpha_1\right)\right)}{\alpha_1 - \alpha_2} D\bar{k}_i^{A_1} \end{aligned}. \tag{35}$$

Real income is

$$\begin{aligned} v_i^T &= \left(c_{i1}^T\right)^{a_1}\left(c_{i2}^T\right)^{a_2} \\ &= \left(A_1\gamma_i + A_2\right) D\bar{k}^{A_1} \end{aligned}. \tag{36}$$

Effect of trade liberalization

It is straightforward to show that each country's terms of trade improve following trade liberalization (simply compare relative prices under autarky and under free trade). This leads to an increase in real income. But real GDP actually decreases following trade liberalization.

Proposition 1. If $\gamma_i \neq 1$, real income strictly increases following trade liberalization.

Proof. We want to show that

$$\left(A_1\gamma_i + A_2\right) D\bar{k}^{A_1} > D\bar{k}_i^{A_1}, \tag{37}$$

or equivalently that

$$A_1\gamma_i + 1 - A_1 > \gamma_i^{A_1}. \tag{38}$$

Define the functions

$$f\left(z\right) = \varsigma z + 1 - \varsigma \tag{39}$$

$$g\left(z\right) = z^{\varsigma}. \tag{40}$$

Then

$$f'(z) = \varsigma \tag{41}$$

$$g'(z) = \varsigma z^{\varsigma - 1}. \tag{42}$$

The linear function f is tangent to the function g at $z = 1$ (both functions have the same value and slope at $z = 1$). Since g is strictly concave, $f(z) > g(z)$ if $z \neq 1$. ■

Proposition 2. If $\gamma_i \neq 1$, GDP at autarky prices strictly decreases following trade liberalization.

Proof. We want to show that

$$p_{i1}^A y_{i1}^A + p_{i2}^A y_{i2}^A > p_{i1}^A y_{i1}^T + p_{i2}^A y_{i2}^T. \tag{43}$$

Define the function

$$\begin{gathered} \pi(p_1, p_2, k_i) = \max\ p_1 \theta_1 k_{i1}^{\alpha_1} \ell_{i1}^{1-\alpha_1} + p_2 \theta_2 k_{i2}^{\alpha_2} \ell_{i2}^{1-\alpha_2} \\ \text{s.t. } k_{i1} + k_{i2} \leq k_i \\ \ell_{i1} + \ell_{i2} \leq 1 \\ k_{ij} \geq 0,\ \ell_{ij} \geq 0 \end{gathered} . \tag{44}$$

Since $\alpha_1 > \alpha_2$, this function is strictly concave. Notice that

$$\pi(p_{i1}^A, p_{i2}^A, \bar{k}_i) = p_{i1}^A y_{i1}^A + p_{i2}^A y_{i2}^A. \tag{45}$$

The free trade allocation also satisfies the feasibility constraints in (44), so

$$p_{i1}^A y_{i1}^A + p_{i2}^A y_{i2}^A > p_{i1}^A y_{i1}^T + p_{i2}^A y_{i2}^T, \tag{46}$$

where the strict inequality follows from the strict concavity of π. ■

Figure 1 illustrates the proof.

2.2. A Ricardian model with a continuum of goods

There are two symmetric countries. In each country i, $i = 1, 2$, the representative consumer is endowed with $\overline{\ell}$ units of labor. There is a continuum of tradable goods, $z \in [0,1]$.

The representative consumer chooses $c_i(z)$, $z \in [0,1]$, to maximize

$$\int_0^1 \log c_i(z)\, dz \tag{47}$$

subject to the budget constraint

$$\int_0^1 p_i(z) c_i(z)\, dz = w_i \overline{\ell}\,. \tag{48}$$

Here $p_i(z)$ is the price of good z and w_i is the wage rate.

The technology for producing good z in country i is

$$y_i(z) = \ell_i(z)/a_i(z), \tag{49}$$

where

$$a_1(z) = e^{\alpha z} \tag{50}$$

$$a_2(z) = e^{\alpha(1-z)}, \tag{51}$$

where $\alpha > 0$. Here $a_i(z)$ is the quantity of labor required to produce one unit of good z in country i.

The zero-profit conditions are

$$p_i(z) - a_i(z) w_i \le 0, \ \ = 0 \text{ if } y_i(z) > 0\,. \tag{52}$$

Clearing in the labor market requires that

$$\int_0^1 \ell_i(z)\, dz = \overline{\ell}_i\,. \tag{53}$$

Under autarky,

$$c_i(z) = y_i(z)\,. \tag{54}$$

Under free trade,

$$c_1(z) + c_2(z) = y_1(z) + y_2(z)\,. \tag{55}$$

Autarky

We normalize $w_i = 1$. The prices of the goods are

$$p_1^A(z) = e^{\alpha z} \tag{56}$$

$$p_2^A(z) = e^{\alpha(1-z)}. \tag{57}$$

The consumption and production levels are

$$c_i^A(z) = y_i^A(z) = \frac{\overline{\ell}}{p_i^A(z)}. \tag{58}$$

The allocation of labor is

$$\ell_i^A(z) = \overline{\ell}. \tag{59}$$

GDP at current prices is

$$\begin{aligned} GDP_i^A &= \int_0^1 p_i^A(z)\, y_i^A(z)\, dz \\ &= \overline{\ell} \end{aligned}. \tag{60}$$

Real income is

$$\begin{aligned} v_i^A &= \exp \int_0^1 \log c_i^A(z)\, dz \\ &= \overline{\ell} e^{-\alpha^2/2} \end{aligned}. \tag{61}$$

Free trade

Since the countries are symmetric, we normalize $w_1 = w_2 = 1$. Country 1 produces and exports goods $z \in [0, 0.5]$ and country 2 produces and exports goods $z \in (0.5, 1]$. The prices of the goods are

$$p^T(z) = \begin{cases} e^{\alpha z} & z \in [0, 0.5] \\ e^{\alpha(1-z)} & z \in (0.5, 1] \end{cases}. \tag{62}$$

The consumption levels are

$$c_1^T(z) = c_2^T(z) = \frac{\overline{\ell}}{p^T(z)}. \tag{63}$$

For goods $z \in [0, 0.5]$, the production plans are

$$y_1^T(z) = \frac{2\overline{\ell}}{p^T(z)},\ \ell_1^T(z) = 2\overline{\ell} \tag{64}$$

$$y_2^T(z) = \ell_2^T(z) = 0\,. \tag{65}$$

For goods $z \in (0.5,1]$, the production plans are

$$y_1^T(z) = \ell_1^T(z) = 0 \tag{66}$$

$$y_2^T(z) = \frac{2\overline{\ell}}{p^T(z)},\ \ell_2^T(z) = 2\overline{\ell} \tag{67}$$

GDP at current prices is

$$\begin{aligned} gdp_i^T &= \int_0^1 p^T(z)\, y_i^T(z)\, dz \\ &= \overline{\ell} \end{aligned}\,. \tag{68}$$

GDP at autarky prices is

$$\begin{aligned} GDP_i^T &= \int_0^1 p_i^A(z)\, y_i^T(z)\, dz \\ &= \overline{\ell} \end{aligned}\,. \tag{69}$$

Real income is

$$\begin{aligned} v_i^T &= \exp\int_0^1 \log c_i^T(z)\, dz \\ &= \overline{\ell} \end{aligned}\,. \tag{70}$$

Effect of trade liberalization

After trade liberalization, the prices of each country's imports decrease, resulting in an improvement in the terms of trade. Real income increases from $\overline{\ell}e^{-\alpha^2/2}$ to $\overline{\ell}$. GDP at autarky prices remains constant at $\overline{\ell}$.

3. Do increases in product variety from trade liberalization increase real GDP?

It is well known that, in standard monopolistic competition models with homogeneous firms, trade liberalization leads to an increase in the number of product varieties available to the consumer. This increase in product variety leads to an increase in real income, but does it lead to an increase in real GDP? We find that this depends on the nature of competition in the product market. If there is a continuum of product varieties, then real GDP does not change. If there is a finite number of product varieties, then real GDP increases. The reason is that, with Cournot (or Bertrand) competition among firms, markups over marginal cost decrease when the number of firms supplying goods to a market increases. We make this point using a monopolistic competition model with a finite number of product varieties.

A monopolistic competition model with homogeneous firms

In each country i, $i = 1, 2, ..., n$, the representative consumer is endowed with $\bar{\ell}_i$ units of labor. Let J_i be the number of goods available to the consumer in country i. Consumer i chooses c_{ij}, $j = 1, 2, ..., J_i$, to maximize

$$(1/\rho)\log \sum_{j=1}^{J_i} c_{ij}^{\rho} \tag{71}$$

subject to the budget constraint

$$\sum_{j=1}^{J_i} p_{ij} c_{ij} = w_i \bar{\ell}_i . \tag{72}$$

Here p_{ij} is the price of good j and w_i is the wage rate.

A firm producing good j in country i has the increasing-returns-to-scale technology

$$y_{ij} = (1/b)\max\left[\ell_{ij} - f,\ 0\right], \tag{73}$$

where f is the fixed cost, in units of labor, of operating.

There is Cournot competition among firms. Taking as given the consumer's demand function and the decisions of all other firms, a firm's problem is to choose the quantity of output that maximizes its profits. There is free entry of firms, so there are no aggregate profits.

Clearing in the labor market requires that

$$\sum_{j=1}^{J_i} \ell_{ij} = \bar{\ell}_i . \tag{74}$$

Under autarky,

$$c_{ij} = y_{ij} . \tag{75}$$

Under free trade, if good j is produced in country i,

$$y_{ij} = \sum_{i=1}^{n} c_{ij} \ . \tag{76}$$

Autarky

Normalize $w_i = 1$. Each firm takes the consumer's indirect demand function as given. Consumer i's indirect demand function for good j is

$$p_{ij} = \frac{c_{ij}^{\rho-1}}{\sum_{m=1}^{J_i} c_{im}^{\rho}} \overline{\ell}_i \ . \tag{77}$$

The firm in country i producing good j chooses y_{ij} to maximize profits,

$$p_{ij} y_{ij} - b y_{ij} - f \ . \tag{78}$$

Plugging (77) into (78), the expression for profits becomes

$$\frac{y_{ij}^{\rho-1}}{\sum_{m=1}^{J_i} y_{im}^{\rho}} \overline{\ell}_i y_{ij} - b y_{ij} - f \ . \tag{79}$$

Profit maximization implies that marginal revenue is equal to marginal cost, so

$$\left(\frac{y_{ij}^{\rho-1}}{\sum_{m=1}^{J_i} y_{im}^{\rho}} \right) \left(\frac{\left(\sum_{m=1}^{J_i} y_{im}^{\rho} \right) \rho y_{ij}^{\rho-1} - y_{ij}^{\rho} \rho y_{ij}^{\rho-1}}{\left(\sum_{m=1}^{J_i} y_{im}^{\rho} \right)^2} \right) \overline{\ell}_i = b \, . \tag{80}$$

Imposing symmetry across firms (the j subscripts are omitted), we obtain

$$c_i^A = y_i^A = \frac{\rho \left(J_i^A - 1 \right) \overline{\ell}_i}{\left(J_i^A \right)^2 b} \tag{81}$$

$$p_i^A = \frac{b J_i^A}{\rho \left(J_i^A - 1 \right)} \, . \tag{82}$$

The profits of a firm are

$$p_i^A y_i^A - b y_i^A - f = \frac{\overline{\ell}_i}{J_i^A} - \frac{\rho \left(J_i^A - 1 \right) \overline{\ell}_i}{\left(J_i^A \right)^2} - f \ . \tag{83}$$

Since there is free entry, firm profits must be zero in equilibrium:

$$f \left(J_i^A \right)^2 - \left(1 - \rho \right) \overline{\ell}_i J_i^A - \rho \overline{\ell}_i = 0 \tag{84}$$

Let N_i be the number of firms in country i. Using the quadratic formula, we solve for the number of varieties and firms:

$$J_i^A = N_i^A = \frac{(1-\rho)\overline{\ell}_i + \sqrt{(1-\rho)^2 \overline{\ell}_i^2 + 4f\rho\overline{\ell}_i}}{f^2}. \tag{85}$$

Notice that the number of goods is not necessarily an integer. Alternatively, we could allow for aggregate profits and calculate N_i as the integer such that there are nonnegative profits but that, if one more firm entered, profits would be negative.

GDP at current prices is

$$\begin{aligned} GDP_i^A &= N_i^A p_i^A y_i^A \\ &= \overline{\ell}_i \end{aligned}. \tag{86}$$

Real income is

$$\begin{aligned} v_i^A &= \left(J_i^A \left(c_i^A \right)^\rho \right)^{1/\rho} \\ &= \left(J_i^A \right)^{\frac{1-\rho}{\rho}} \frac{\rho\left(J_i^A - 1 \right)}{J_i^A b} \overline{\ell}_i \end{aligned}. \tag{87}$$

Free trade

We can use the above approach to solve for the integrated equilibrium of the world economy, in which the supply of labor is $\overline{\ell} = \sum_{i=1}^n \overline{\ell}_i$. We again normalize $w = 1$ and obtain

$$J^T = \frac{(1-\rho)\overline{\ell} + \sqrt{(1-\rho)^2 \overline{\ell}^2 + 4f\rho\overline{\ell}}}{f^2} \tag{88}$$

$$y^T = \frac{\rho\left(J^T - 1\right)\overline{\ell}}{\left(J^T\right)^2 b} \tag{89}$$

$$p^T = \frac{bJ^T}{\rho\left(J^T - 1\right)}. \tag{90}$$

Disaggregating proportionally,

$$c_i^T = \frac{\overline{\ell}_i}{\overline{\ell}} \overline{y}^T. \tag{91}$$

$$N_i^T = \frac{\overline{\ell}_i}{\overline{\ell}} J^T. \tag{92}$$

Notice that the equilibrium values for free trade are the same as those for autarky if $\overline{\ell}_i = \overline{\ell}$.

GDP at current prices is

$$\begin{aligned} gdp_i^T &= N_i^T p^T y^T \\ &= \overline{\ell}_i \end{aligned}. \tag{93}$$

GDP at autarky prices is

$$\begin{aligned} GDP_i^T &= N_i^T p_i^A y^T \\ &= \frac{J_i^A}{\left(J_i^A - 1\right)} \frac{\left(J^T - 1\right)}{J^T} \overline{\ell}_i \end{aligned}. \tag{94}$$

Real income is

$$\begin{aligned} v_i^T &= \left(J^T \left(c_i^T\right)^\rho\right)^{1/\rho} \\ &= \left(J^T\right)^{\frac{1-\rho}{\rho}} \frac{\rho\left(J^T - 1\right)}{J^T b} \overline{\ell}_i \end{aligned}. \tag{95}$$

Effect of trade liberalization

Proposition 3. If $\overline{\ell}_i < \overline{\ell}$, then real income in country i strictly increases following trade liberalization.

Proof. We want to show that

$$\left(J^T\right)^{\frac{1-\rho}{\rho}} \frac{\rho\left(J^T - 1\right)}{J^T b} \overline{\ell}_i > \left(J_i^A\right)^{\frac{1-\rho}{\rho}} \frac{\rho\left(J_i^A - 1\right)}{J_i^A b} \overline{\ell}_i. \tag{96}$$

It suffices to show that $J^T > J_i^A$, which is evident from comparing (85) and (88). ■

Proposition 4. If $\overline{\ell}_i < \overline{\ell}$, then GDP at autarky prices in country i strictly increases following trade liberalization.

Proof. We want to show that

$$\frac{J_i^A}{\left(J_i^A - 1\right)} \frac{\left(J^T - 1\right)}{J^T} \overline{\ell}_i > \overline{\ell}_i . \tag{97}$$

Again, this follows from the fact that $J^T > J_i^A$. ■

Real GDP increases because markups decrease. Since $J^T > J_i^A$, there are more firms competing in each market. With Cournot competition, this lowers the markup over marginal cost:

$$\frac{J^T}{\rho\left(J^T - 1\right)} < \frac{J_i^A}{\rho\left(J_i^A - 1\right)} . \tag{98}$$

If there is a continuum, rather than a finite number, of product varieties, then the markup over marginal cost is constant at $1/\rho$, regardless of trade policy. In this case, GDP at autarky prices remains constant following trade liberalization.

4. Does reallocation across heterogeneous firms following trade liberalization increase measured productivity?

With heterogeneous firms and fixed costs of exporting, trade liberalization can lead to a reallocation of resources across firms. In a simple model, trade liberalization causes the least productive firms to exit and the most productive firms to become exporters. Intuitively, this reallocation of resources toward more productive firms should increase aggregate productivity. But we find that it does not. The finding here is explored further in Gibson (2007), where a positive mechanism is also provided.

A monopolistic competition model with heterogeneous firms

There are two symmetric countries. In each country i, $i = 1, 2$, the representative consumer is endowed with $\bar{\ell}$ units of labor and measure μ of potential firms (potential firms may choose not to operate). Each firm produces a differentiated good.

Let Z_i be the set of goods available to consumer i. The consumer chooses $c_i(z)$, $z \in Z_i$, to maximize

$$(1/\rho)\log\int_{Z_i} c_i(z)^{\rho}\, dz \tag{99}$$

subject to the budget constraint

$$\int_{Z_i} p_i(z)c_i(z)\,dz = w_i\bar{\ell} + \pi_i\,. \tag{100}$$

Here $p_i(z)$ is the price of good z, w_i is the wage rate, and π_i is the profits of firms.

Firms differ in their productivity levels. Let $x(z)$ be the productivity level of the firm that produces good z. The firm producing good z in country i has the increasing-returns-to-scale technology

$$y_i(z) = \max\left[x(z)\left(\ell_i(z) - f_d\right),\ 0\right], \tag{101}$$

where f_d is the fixed cost, in units of labor, of operating. If the economies are open to trade, then a firm can choose to export by paying an additional fixed cost of f_e units of labor.

Potential firms draw their productivities from a Pareto distribution

$$F(x) = 1 - x^{-\gamma}\,, \tag{102}$$

$x \geq 1$. The choice of one as the lower bound on the Pareto distribution can be thought of as a normalization. For reasons that will be clear later, we impose the restriction that $\gamma > \max\left[2,\ \rho/(1-\rho)\right]$.

Taking the consumer's demand functions as given, the firm's problem is to choose the profit-maximizing price. Each firm decides whether to operate. If there is free trade, each firm decides whether to export.

Clearing in the labor market requires that

$$\int_{Z_i} \ell_i(z)\,dz = \overline{\ell}\,. \tag{103}$$

Autarky

There are two possibilities: Either all potential firms choose to produce or not. We examine the latter case. In this case, there is a cutoff $\overline{x}_d$, $\overline{x}_d > 1$, such that a firm with productivity x produces if $x \geq \overline{x}_d$.

Since the countries are symmetric, country subscripts are omitted. Set $w = 1$. The profit-maximizing prices are

$$p^A(x) = \frac{1}{\rho x}\,. \tag{104}$$

The aggregate price index is

$$\begin{aligned} P^A &= \left(\mu \int_{\overline{x}_d^A}^{\infty} p^A(x)^{\frac{-\rho}{1-\rho}} dF(x)\right)^{\frac{-(1-\rho)}{\rho}} \\ &= \left(\frac{\gamma(1-\rho)-\rho}{\rho^{\frac{\rho}{1-\rho}}(1-\rho)\gamma\mu\left(\overline{x}_d^A\right)^{\frac{\rho-\gamma(1-\rho)}{1-\rho}}}\right)^{\frac{1-\rho}{\rho}}. \end{aligned} \tag{105}$$

The demand for a good produced by a firm with productivity $x \geq \overline{x}_d^A$ is

$$\begin{aligned} c^A(x) = y^A(x) &= p^A(x)^{\frac{-1}{1-\rho}}\left(P^A\right)^{\frac{\rho}{1-\rho}}\left(\overline{\ell}+\pi^A\right) \\ &= \frac{\rho\left(\gamma(1-\rho)-\rho\right)\left(\overline{\ell}+\pi^A\right)x^{\frac{1}{1-\rho}}}{(1-\rho)\gamma\mu\left(\overline{x}_d^A\right)^{\frac{\rho-\gamma(1-\rho)}{1-\rho}}}\,. \end{aligned} \tag{106}$$

A firm with productivity $\overline{x}_d^A$ must make zero profits in equilibrium, so

$$p^A\left(\overline{x}_d^A\right)c^A\left(\overline{x}_d^A\right) - \frac{c^A\left(\overline{x}_d^A\right)}{\overline{x}_d^A} - f_d = 0\,. \tag{107}$$

Plugging (104) and (106) into (107), we obtain

$$\overline{x}_d^A = \left(\frac{\mu\gamma f_d}{\left(\gamma(1-\rho)-\rho\right)\left(\overline{\ell}+\pi^A\right)}\right)^{1/\gamma}, \tag{108}$$

where

$$\begin{aligned}\pi^A &= \mu\int_{\bar{x}_d^A}^{\infty}\left(p^A(x)c^A(x)-\frac{c^A(x)}{x}-f_d\right)dF(x)\\&=\frac{\rho\bar{\ell}}{\gamma-\rho}\end{aligned}. \tag{109}$$

Plugging (109) into (108), the cutoff for operating is

$$\bar{x}_d^A=\left(\frac{\mu(\gamma-\rho)f_d}{(\gamma(1-\rho)-\rho)\bar{\ell}}\right)^{1/\gamma}. \tag{110}$$

GDP at current prices is

$$\begin{aligned}GDP^A &= \mu\int_{\bar{x}_d^A}^{\infty}p^A(x)y^A(x)dF(x)\\&=\frac{\gamma}{\gamma-\rho}\bar{\ell}\end{aligned}. \tag{111}$$

Real income is

$$\begin{aligned}v^A &= \left(\mu\int_{\bar{x}_d^A}^{\infty}c^A(x)^{\rho}\,dF(x)\right)^{1/\rho}\\&=\frac{\gamma}{(\gamma-\rho)P^A}\bar{\ell}\end{aligned}. \tag{112}$$

Free trade

We again examine the case in which not all firms choose to produce. That is, firm z produces if $x(z)\geq\bar{x}_d$, $\bar{x}_d>1$. With free trade, each firm faces an additional decision: whether to pay the fixed cost f_e to export. There is a cutoff $\bar{x}_e$, $\bar{x}_e>\bar{x}_d$, such that firm z exports if $x(z)\geq\bar{x}_e$.

Since the countries are symmetric, we set $w_1=w_2=1$. The profit-maximizing prices are

$$p^T(x)=\frac{1}{\rho x}. \tag{113}$$

The aggregate price index is

$$P^T = \left(\mu \int_{\bar{x}_d^T}^{\infty} p^T(x)^{\frac{-\rho}{1-\rho}} dF(x) + \mu \int_{\bar{x}_e^T}^{\infty} p^T(x)^{\frac{-\rho}{1-\rho}} dF(x) \right)^{\frac{-(1-\rho)}{\rho}}$$

$$= \left(\frac{\gamma(1-\rho)-\rho}{\rho^{\frac{\rho}{1-\rho}}(1-\rho)\gamma\mu\left(\left(\bar{x}_d^T\right)^{\frac{\rho-\gamma(1-\rho)}{1-\rho}} + \left(\bar{x}_e^T\right)^{\frac{\rho-\gamma(1-\rho)}{1-\rho}} \right)} \right)^{\frac{1-\rho}{\rho}} . \quad (114)$$

The demand in a country for a good produced by a firm with productivity $x \geq \bar{x}_d^T$ is

$$c^T(x) = p^T(x)^{\frac{-1}{1-\rho}} \left(P^T\right)^{\frac{\rho}{1-\rho}} \left(\bar{\ell} + \pi^T\right)$$

$$= \frac{\rho\left(\gamma(1-\rho)-\rho\right)\left(\bar{\ell}+\pi^T\right) x^{\frac{1}{1-\rho}}}{(1-\rho)\gamma\mu\left(\left(\bar{x}_d^T\right)^{\frac{\rho-\gamma(1-\rho)}{1-\rho}} + \left(\bar{x}_e^T\right)^{\frac{\rho-\gamma(1-\rho)}{1-\rho}} \right)} . \quad (115)$$

Then

$$y^T(x) = \begin{cases} c^T(x) & \bar{x}_d^T \leq x < \bar{x}_e^T \\ 2c^T(x) & x \geq \bar{x}_e^T \end{cases} . \quad (116)$$

The cutoff for operating, $\bar{x}_d^T$, must satisfy

$$p^T\left(\bar{x}_d^T\right) c^T\left(\bar{x}_d^T\right) - \frac{c^T\left(\bar{x}_d^T\right)}{\bar{x}_d^T} - f_d = 0 , \quad (117)$$

so

$$\frac{\left(\gamma(1-\rho)-\rho\right)\left(\bar{\ell}+\pi^T\right)\left(\bar{x}_d^T\right)^{\frac{\rho}{1-\rho}}}{\gamma\mu\left(\left(\bar{x}_d^T\right)^{\frac{\rho-\gamma(1-\rho)}{1-\rho}} + \left(\bar{x}_e^T\right)^{\frac{\rho-\gamma(1-\rho)}{1-\rho}} \right)} - f_d = 0 . \quad (118)$$

Similarly, the cutoff for exporting, $\bar{x}_e^T$, must satisfy

$$p^T\left(\bar{x}_e^T\right) c^T\left(\bar{x}_e^T\right) - \frac{c^T\left(\bar{x}_e^T\right)}{\bar{x}_e^T} - f_e = 0 , \quad (119)$$

so

$$\frac{\left(\gamma(1-\rho)-\rho\right)\left(\overline{\ell}+\pi^T\right)\left(\overline{x}_e^T\right)^{\frac{\rho}{1-\rho}}}{\gamma\mu\left(\left(\overline{x}_d^T\right)^{\frac{\rho-\gamma(1-\rho)}{1-\rho}}+\left(\overline{x}_e^T\right)^{\frac{\rho-\gamma(1-\rho)}{1-\rho}}\right)}-f_e=0. \tag{120}$$

Here

$$\begin{aligned}\pi^T &= \mu\int_{\overline{x}_d^T}^{\infty}\left(p^T(x)c^T(x)-\frac{c^T(x)}{x}-f_d\right)dF(x)\\ &+\mu\int_{\overline{x}_e^T}^{\infty}\left(p^T(x)c^T(x)-\frac{c^T(x)}{x}-f_e\right)dF(x)\\ &=(1-\rho)\left(\overline{\ell}+\pi^T\right)-\mu\left(\left(\overline{x}_d^T\right)^{-\gamma}f_d+\left(\overline{x}_e^T\right)^{-\gamma}f_e\right)\end{aligned}. \tag{121}$$

Notice that (118), (120), and (121) give us a system of 3 equations in 3 unknowns to be solved for $\overline{x}_d^T$, $\overline{x}_e^T$, and π^T. The solution is

$$\overline{x}_d^T=\left(\frac{\mu(\gamma-\rho)f_d\left(1+\left(f_e/f_d\right)^{\frac{\rho-\gamma(1-\rho)}{\rho}}\right)}{\left(\gamma(1-\rho)-\rho\right)\overline{\ell}}\right)^{1/\gamma} \tag{122}$$

$$\overline{x}_e^T=\left(\frac{f_e}{f_d}\right)^{\frac{1-\rho}{\rho}}\left(\frac{\mu(\gamma-\rho)f_d\left(1+\left(f_e/f_d\right)^{\frac{\rho-\gamma(1-\rho)}{\rho}}\right)}{\left(\gamma(1-\rho)-\rho\right)\overline{\ell}}\right)^{1/\gamma} \tag{123}$$

$$\pi^T=\frac{\rho\overline{\ell}}{\gamma-\rho}. \tag{124}$$

GDP at current prices is

$$\begin{aligned}gdp^T &= \mu\int_{\overline{x}_d^T}^{\infty}p^T(x)y^T(x)dF(x)\\ &=\frac{\gamma}{\gamma-\rho}\overline{\ell}\end{aligned}. \tag{125}$$

GDP at autarky prices is

$$\begin{aligned}GDP^T &= \mu\int_{\overline{x}_d^T}^{\infty}p^A(x)y^T(x)dF(x)\\ &=\frac{\gamma}{\gamma-\rho}\overline{\ell}\end{aligned}. \tag{126}$$

Real income is

$$
\begin{aligned}
v^T &= \left(\mu \int_{\bar{x}_d^T}^{\infty} c^T(x)^\rho \, dF(x) + \mu \int_{\bar{x}_e^T}^{\infty} c^T(x)^\rho \, dF(x) \right)^{1/\rho} \\
&= \frac{\gamma}{(\gamma - \rho) P^T} \bar{\ell}
\end{aligned}
. \qquad (127)
$$

Effect of trade liberalization

Proposition 5. The cutoff for operating strictly increases following trade liberalization.

Proof. Compare (110) and (122). ■

Proposition 6. GDP at autarky prices does not change following trade liberalization.

Proof. Compare (111) and (126). ■

Proposition 7. Real income increases following trade liberalization.

Proof. Comparing (112) and (127), it suffices to show that $P^A > P^T$. Comparing (105) and (114), we see that this follows from Proposition 5. ■

The effect of reallocation across firms — the exit of the least productive firms and the movement of resources toward the most productive firms which start exporting — increases welfare, not real GDP.

5. How does trade liberalization affect growth rates?

Trade liberalization can change the incentives to accumulate capital, which in turn affects growth rates. Does trade liberalization have any effect on growth rates? To analyze this, we consider a dynamic Heckscher-Ohlin model with endogenous capital accumulation as in Bajona and Kehoe (2006).

A dynamic Heckscher-Ohlin model

In each country i, $i = 1, 2, ..., n$, there is measure L_i of consumers. Each consumer is endowed with one unit of labor and $\bar{k}_{i0}$ units of capital. There are two tradable goods, $j = 1, 2$.

A consumer in country i chooses $\{c_{ijt}, x_{ijt}, k_{it}\}$, $j = 1, 2$, $t = 0, 1, ...$, to maximize

$$\sum_{t=0}^{\infty} \beta^t \left(a_1 \log c_{i1t} + a_2 \log c_{i2t}\right), \tag{128}$$

where $a_1 + a_2 = 1$, subject to the budget constraint

$$p_{i1t}\left(c_{i1t} + x_{i1t}\right) + p_{i2t}\left(c_{i2t} + x_{i2t}\right) = w_{it} + r_{it}k_{it} \tag{129}$$

and the law of motion of capital

$$k_{i,t+1} = \left(1-\delta\right)k_{it} + x_{i1t}^{a_1} x_{i2t}^{a_2}, \tag{130}$$

given $k_{i0} = \bar{k}_{i0}$. Here p_{ijt} is the price of good j, w_{it} is the wage rate, and r_{it} is the rental rate of capital.

Each country has the Cobb-Douglas technologies

$$y_{ijt} = \theta_j k_{ijt}^{\alpha_j} \ell_{ijt}^{1-\alpha_j}, \tag{131}$$

where $\alpha_1 > \alpha_2$. The zero-profit conditions are

$$p_{ijt}\alpha_j k_{ijt}^{\alpha_j - 1} \ell_{ijt}^{1-\alpha_j} - r_{it} \le 0, \ = 0 \text{ if } k_{ijt} > 0 \tag{132}$$

$$p_{ijt}\left(1-\alpha_j\right) k_{ijt}^{\alpha_j} \ell_{ijt}^{-\alpha_j} - w_{it} \le 0, \ = 0 \text{ if } \ell_{ijt} > 0. \tag{133}$$

Clearing in the factors markets requires that

$$k_{i1t} + k_{i2t} = k_{it} \tag{134}$$

$$\ell_{i1t} + \ell_{i2t} = 1. \tag{135}$$

Under autarky,

$$c_{ijt} + x_{ijt} = y_{ijt}. \tag{136}$$

Under free trade,

$$\sum_{i=1}^{n} L_i\left(c_{ijt} + x_{ijt}\right) = \sum_{i=1}^{n} L_i y_{ijt}. \tag{137}$$

To obtain an analytical solution, we assume that there is complete depreciation, $\delta = 1$. We use the same notational conventions that we used for the static Heckscher-

Ohlin model. Given k_{it}, the equilibrium values in the dynamic model for period t are the same as in the static model, except that output is split between consumption and investment.

Autarky

The analytical solution is

$$r_{it}^A = A_1 D\left(k_{it}^A\right)^{A_1 - 1} \tag{138}$$

$$w_{it}^A = A_2 D\left(k_{it}^A\right)^{A_1} \tag{139}$$

$$p_{ijt}^A = \frac{a_j D}{D_j}\left(k_{it}^A\right)^{A_1 - \alpha_j} \tag{140}$$

$$y_{ijt}^A = D_j\left(k_{it}^A\right)^{\alpha_j} \tag{141}$$

$$k_{ijt}^A = \frac{a_j \alpha_j}{A_1} k_{it}^A \tag{142}$$

$$\ell_{ijt}^A = \frac{a_j\left(1-\alpha_j\right)}{A_2} \tag{143}$$

$$c_{ijt}^A = \left(1 - \beta A_1\right) D_j\left(k_{it}^A\right)^{\alpha_j} \tag{144}$$

$$x_{ijt}^A = \beta A_1 D_j\left(k_{it}^A\right)^{\alpha_j} \tag{145}$$

where

$$k_{it}^A = \beta A_1 D\left(k_{i,t-1}^A\right)^{A_1} = \left(\beta A_1 D\right)^{\frac{1-A_1^t}{1-A_1}} \bar{k}_{i0}^{A_1^t}. \tag{146}$$

GDP at current prices is

$$\begin{aligned} gdp_{it}^A &= p_{i1t}^A y_{i1t}^A + p_{i2t}^A y_{i2t}^A \\ &= D\left(k_{it}^A\right)^{A_1} \\ &= \left(\beta A_1 D\right)^{\frac{A_1 - A_1^{t+1}}{1-A_1}} D\bar{k}_{i0}^{A_1^{t+1}} \end{aligned}. \tag{147}$$

Notice that GDP at current prices is equal to $\left(y_{i1t}^A\right)^{a_1}\left(y_{i2t}^A\right)^{a_2}$.

GDP at period-0 prices is

$$
\begin{aligned}
GDP_{it}^{A} &= p_{i10}^{A} y_{i1t}^{A} + p_{i20}^{A} y_{i2t}^{A} \\
&= \left(a_1 \left(\frac{k_{it}^{A}}{\bar{k}_{i0}} \right)^{\alpha_1} + a_2 \left(\frac{k_{it}^{A}}{\bar{k}_{i0}} \right)^{\alpha_2} \right) D \bar{k}_{i0}^{A_1} \\
&= \left(a_1 \left(\left(\beta A_1 D \right)^{\frac{1-A_1^t}{1-A_1}} \bar{k}_{i0}^{A_1^t - 1} \right)^{\alpha_1} + a_2 \left(\left(\beta A_1 D \right)^{\frac{1-A_1^t}{1-A_1}} \bar{k}_{i0}^{A_1^t - 1} \right)^{\alpha_2} \right) D \bar{k}_{i0}^{A_1}
\end{aligned} \quad . \qquad (148)
$$

Real income is

$$
\begin{aligned}
v_{it}^{A} &= \left(c_{i1t}^{A} \right)^{a_1} \left(c_{i2t}^{A} \right)^{a_2} \\
&= \left(1 - \beta A_1 \right) D \left(k_{it}^{A} \right)^{A_1} \\
&= \left(1 - \beta A_1 \right) \left(\beta A_1 D \right)^{\frac{A_1 - A_1^{t+1}}{1-A_1}} D \bar{k}_{i0}^{A_1^{t+1}}
\end{aligned} \quad . \qquad (149)
$$

Free trade

To obtain an analytical solution, we assume that the initial factor endowments are such that factor prices are equalized in the first period. Bajona and Kehoe (2006) show that, in this case, factor price equalization occurs along the entire equilibrium path for the Cobb-Douglas model. This implies that the model can be solved by calculating the equilibrium of the integrated economy — the economy with initial endowments equal to the world endowments — and then splitting production, consumption, and investment across countries in each period. If all countries are in the cone of diversification, then

$$k_{it} = \gamma_i k_t \qquad (150)$$

where

$$\gamma_i = \frac{\bar{k}_{i0}}{\bar{k}_0} \qquad (151)$$

$$\bar{k}_0 = \frac{\sum_{i=1}^{n} L_i \bar{k}_{i0}}{\sum_{i=1}^{n} L_i} . \qquad (152)$$

The analytical solution for the case where $\kappa_2 \le \gamma_i \le \kappa_1$, $i = 1, \ldots, n$, is

$$r_t^T = A_1 D \left(k_t^T \right)^{A_1 - 1} \qquad (153)$$

$$w_t^T = A_2 D\left(k_t^T\right)^{A_1} \tag{154}$$

$$p_{jt}^T = \frac{a_j D}{D_j}\left(k_t^T\right)^{A_1-\alpha_j} \tag{155}$$

$$c_{ijt}^T = \left(\gamma_i A_1 + A_2 - \gamma_i \beta A_1\right) D_j \left(k_t^T\right)^{\alpha_j} \tag{156}$$

$$x_{ijt}^T = \gamma_i \beta A_1 D_j \left(k_t^T\right)^{\alpha_j} \tag{157}$$

$$y_{ijt}^T = \mu_{ij} D_j \left(k_t^T\right)^{\alpha_j} \tag{158}$$

$$k_{ijt}^T = \mu_{ij} \frac{a_j \alpha_j}{A_1} k_t^T \tag{159}$$

$$\ell_{ijt}^T = \mu_{ij} \frac{a_j\left(1-\alpha_j\right)}{A_2}, \tag{160}$$

where

$$k_t^T = \beta A_1 D\left(k_{t-1}^T\right)^{A_1} = \left(\beta A_1 D\right)^{\frac{1-A_1^t}{1-A_1}} \bar{k}_0^{A_1^t}. \tag{161}$$

GDP at current prices is

$$\begin{aligned} gdp_{it}^T &= p_{1t}^T y_{i1t}^T + p_{2t}^T y_{i2t}^T \\ &= \left(\gamma_i A_1 + A_2\right) D\left(k_t^T\right)^{A_1} \\ &= \left(\gamma_i A_1 + A_2\right)\left(\beta A_1 D\right)^{\frac{A_1 - A_1^{t+1}}{1-A_1}} D\bar{k}_0^{A_1^{t+1}} \end{aligned} \quad . \tag{162}$$

GDP at period-0 prices is

$$\begin{aligned} GDP_{it}^T &= p_{10}^T y_{i1t}^T + p_{20}^T y_{i2t}^T \\ &= \left(a_1 \mu_{i1}\left(\frac{k_t^T}{\bar{k}_0}\right)^{\alpha_1} + a_2 \mu_{i2}\left(\frac{k_t^T}{\bar{k}_0}\right)^{\alpha_2}\right) D\bar{k}_0^{A_1} \\ &= \left(a_1 \mu_{i1}\left(\left(\beta A_1 D\right)^{\frac{1-A_1^t}{1-A_1}} \bar{k}_0^{A_1^t - 1}\right)^{\alpha_1} + a_2 \mu_{i2}\left(\left(\beta A_1 D\right)^{\frac{1-A_1^t}{1-A_1}} \bar{k}_0^{A_1^t - 1}\right)^{\alpha_2}\right) D\bar{k}_0^{A_1} \end{aligned} \quad . \tag{163}$$

If the countries are initially in autarky, we can measure real GDP as GDP at period-0 autarky prices:

$$
\begin{aligned}
GDP_{it}^{T} &= p_{10}^{A} y_{i1t}^{T} + p_{20}^{A} y_{i2t}^{T} \\
&= \left(a_1 \mu_{i1} \left(\frac{k_t^T}{\bar{k}_{i0}} \right)^{\alpha_1} + a_2 \mu_{i2} \left(\frac{k_t^T}{\bar{k}_{i0}} \right)^{\alpha_2} \right) D \bar{k}_{i0}^{A_1} \\
&= \left(a_1 \mu_{i1} \left(\frac{\left(\beta A_1 D\right)^{\frac{1-A_1^t}{1-A_1}} \bar{k}_0^{A_1^t}}{\bar{k}_{i0}} \right)^{\alpha_1} + a_2 \mu_{i2} \left(\frac{\left(\beta A_1 D\right)^{\frac{1-A_1^t}{1-A_1}} \bar{k}_0^{A_1^t}}{\bar{k}_{i0}} \right)^{\alpha_2} \right) D \bar{k}_{i0}^{A_1}
\end{aligned}
\quad . \qquad (164)
$$

Real income is

$$
\begin{aligned}
v_i^T &= \left(c_{i1t}^T\right)^{a_1} \left(c_{i2t}^T\right)^{a_2} \\
&= \left(\gamma_i A_1 + A_2 - \gamma_i \beta A_1\right) D \left(k_t^T\right)^{A_1} \\
&= \left(\gamma_i A_1 + A_2 - \gamma_i \beta A_1\right) \left(\beta A_1 D\right)^{\frac{A_1 - A_1^{t+1}}{1-A_1}} D \bar{k}_0^{A_1^{t+1}}
\end{aligned}
\quad . \qquad (165)
$$

Effect of trade liberalization

We begin with the analysis of real income and discuss real GDP later. First we analyze rates of growth of real income under both autarky and free trade.

Proposition 8. Under autarky, if $\bar{k}_{i0} < \bar{k}_{j0}$, then the growth rate of real income is higher in country i than in country j in every period.

Proof. Under autarky, the growth rate of real income in country i is

$$
\begin{aligned}
\frac{v_{i,t+1}^A}{v_{it}^A} - 1 &= \left(\frac{k_{i,t+1}^A}{k_{it}^A} \right)^{A_1} - 1 \\
&= \left(\beta A_1 D\right)^{A_1} \left(k_{it}^A\right)^{A_1 (A_1 - 1)} - 1 . \\
&= \left(\beta A_1 D\right)^{A_1^{t+1}} \bar{k}_{i0}^{A_1^{t+1} (A_1 - 1)} - 1
\end{aligned}
\qquad (166)
$$

This is decreasing in $\bar{k}_{i0}$. ■

Proposition 9. Under free trade, real income grows at the same rate in every country.

Proof. With free trade, the growth rate of real income is

$$\frac{v_{i,t+1}^T}{v_{it}^T} - 1 = \left(\beta A_1 D\right)^{A_1^{t+1}} \bar{k}_0^{A_1^{t+1}(A_1-1)} - 1 . \tag{167}$$

This is independent of i. ■

Notice that, under free trade, income in country i relative to income in the world is constant over time.

Proposition 10. If $\bar{k}_{i0} > \bar{k}_0$, then real income in country i grows at a faster rate under free trade than under autarky in every period. If $\bar{k}_{i0} < \bar{k}_0$, then real income in country i grows at a slower rate under free trade than under autarky in every period.

Proof. This follows directly from the previous two propositions. ■

Despite the fact that trade liberalization leads to slower growth of real income in some countries, trade liberalization increases welfare in every country.

Proposition 11. If $\gamma_i \neq 1$, welfare is strictly higher under free trade than under autarky.

Proof. Welfare in country i under autarky is

$$\begin{aligned} W_i^A &= \sum\nolimits_{t=0}^{\infty} \beta^t \log v_{it}^A \\ &= \sum\nolimits_{t=0}^{\infty} \beta^t \log\left[\left(1-\beta A_1\right)\left(\beta A_1 D\right)^{\frac{A_1 - A_1^{t+1}}{1-A_1}} D\bar{k}_{i0}^{A_1^{t+1}} \right] \qquad . \\ &= \frac{\log\left[\left(1-\beta A_1\right)D\right]}{1-\beta} + \frac{\beta A_1 \log\left(\beta A_1 D\right)}{\left(1-\beta\right)\left(1-\beta A_1\right)} + \frac{A_1 \log \bar{k}_{i0}}{1-\beta A_1} \end{aligned} \tag{168}$$

Welfare in country i under free trade is

$$W_i^T = \sum_{t=0}^{\infty} \beta^t \log v_{it}^T$$

$$= \sum_{t=0}^{\infty} \beta^t \log\left[\left(A_1\gamma_i + A_2 - \beta A_1\right)\left(\beta A_1 D\right)^{\frac{A_1 - A_1^{t+1}}{1-A_1}} D\bar{k}_0^{A_1^{t+1}} \right] \quad . \tag{169}$$

$$= \frac{\log\left[\left(A_1\gamma_i + A_2 - \beta A_1\right)D\right]}{1-\beta} + \frac{\beta A_1 \log\left(\beta A_1 D\right)}{\left(1-\beta\right)\left(1-\beta A_1\right)} + \frac{A_1 \log \bar{k}_0}{1-\beta A_1}$$

We want to show that $W_i^T > W_i^A$, or equivalently that

$$\frac{A_1\left(1-\beta\right)}{1-\beta A_1}\gamma_i + 1 - \frac{A_1\left(1-\beta\right)}{1-\beta A_1} > \gamma_i^{\frac{A_1(1-\beta)}{1-\beta A_1}} \quad . \tag{170}$$

From here, the proof is the same as that for Proposition 1. ■

What happens to real GDP following trade liberalization? We can infer from the static model that, if a country is initially in autarky, then trade liberalization initially causes a decrease, or at least a decrease in the growth rate of, real GDP in that country.

Proposition 12. If $\gamma_i \neq 1$, GDP at period-0 autarky prices is strictly lower under free trade than under autarky in period 0.

Proof. This follows from Proposition 2. ■

At this point we would like to analyze the growth rates of GDP at period-0 prices under both autarky and free trade. The expressions for these growth rates are not analytically comparable, however. We instead provide an illustrative numerical example. There are two countries, and country 1 is relatively capital-rich. We set $L_1 = L_2 = 1$, $\beta = 0.96$, $a_1 = a_2 = 0.5$, $\theta_1 = \theta_2 = 1$, $\alpha_1 = 0.6$, $\alpha_2 = 0.4$, $\bar{k}_{10} = 0.05$, and $\bar{k}_{20} = 0.03$. The results on growth rates of real GDP are similar to those on growth rates of real income. As Figure 2 shows, under autarky the capital-poor country grows much faster than the capital-rich country, just as we would expect from a standard growth model. This completely changes under free trade. Figure 3 shows that the capital-rich country grows faster than the capital-poor country. Figures 4 and 5 reiterate this finding from the perspective of each individual country.

6. Conclusion

To the extent that trade liberalization leads to higher productivity or higher rates of growth in real GDP, it does so through mechanisms that are, for the most part, outside of those analyzed in standard models. Determining the relation between trade liberalization and growth is not just a challenge for empirical research but also for theoretical research.

References

Alcalá, F., and A. Ciccone (2004), "Trade and Productivity," *Quarterly Journal of Economics,* 119, 613-46.

Atkeson, A., and P. J. Kehoe (2000), "Paths of Development for Early and Late Bloomers in a Dynamic Heckscher-Ohlin Model," Federal Reserve Bank of Minneapolis Staff Report 256.

Bajona, C., and T. J. Kehoe (2006), "Demographics in Dynamic Heckscher-Ohlin Models: Overlapping Generations versus Infinitely Lived Consumers," Federal Reserve Bank of Minneapolis Staff Report.

Bajona, C., and T. J. Kehoe (2006), "Trade, Growth, and Convergence in a Dynamic Heckscher-Ohlin Model," Federal Reserve Bank of Minneapolis Staff Report.

Balassa, B. (1978), "Exports and Economic Growth: Further Evidence," *Journal of Development Economics,* 5, 181-189.

Baldwin, L. E. (2003), "Openess and growth: what is the empirical relationship?" In L. E. Baldwin and L. A. Winters (eds) *Challenges to Globalization*. University of Chicago Press. Chicago. (NBER 9578)

Chaney, T. (2006), "Distorted Gravity: Heterogeneous Firms, Market Structure, and the Geography of International Trade," University of Chicago.

Cuñat A., and M. Maffezzoli (2004), "Neoclassical Growth and Commodity Trade," *Review of Economic Dynamics* 7, 3: 707-736.

DeJong, D. N. and M. Ripoll (2006) "Tariffs and growth: comparing relationships among the rich and poor." *Review of Economics and Statistics* 88: 625-40.

Diewert, W. E. and C. J. Morrison (1986) "Adjusting output and productivity indexes fro changes in the terms of trade." *Economic Journal* 96: 659-79.

Dollar, D. (1992) "Outward-oriented developing economies really do grow more rapidly: evidence from 95 LDCs, 1976-1985." *Economic Development and Cultural Change* 40: 523-544.

Dollar, D. and A. Kraay (2004) "Trade, growth, and poverty." *The Economic Journal* 114: 22-49.

Feenstra, R. C. (1994), "New Product Varieties and the Measurement of International Prices," *American Economic Review*, 84, 157-177.

Ferreira and Trejos (2006) "On the output effects of barriers to trade." *International Economic Review* 47(4): 1319-1340.

Frankel, J. A. and D. Romer (1999) "Does trade cause growth?" *American Economic Review* 89, 379-399.

Gibson, M. J. (2007), "Trade Liberalization, Reallocation, and Productivity," University of Minnesota.

Grossman, G. and E. Helpman (1991) *Innovation and growth in the global economy*. MIT Press, Cambridge, Massachusetts.

Hall, R. and C. Jones (1999) "Why do some countries produce so much more output per worker than others?" *Quarterly Journal of Economics* 114: 83-116.

Holmes, T. and J. Schmitz Jr. (1995) "Resistance to new technology and trade between areas." *FRB of Minneapolis Quarterly Review* Winter: 2-18.

Kehoe, T. J., and K. J. Ruhl (2006), "Are Shocks to the Terms of Trade Shocks to Productivity?" Federal Reserve Bank of Minneapolis Staff Report.

Kohli, U. (1983) "technology and the demand for imports," *Southern Economic Journal* 50: 137-150.

Kohli, U. (2004) "Real GDP, real domestic income, and terms of trade changes." *Journal of International Economics* 62: 83-106.

Levine, R. and D. Renelt (1992) "A sensitivity analysis of cross-country growth." *American Economic Review* 82: 942-963.

Lewer, Joshua J. and Hendrik Van den Berg (2003). "How large is international trade's effect on economic growth?" *Journal of Economic Surveys 17(3): 363-396*

Lewer, Joshua J. and Hendrik Van den Berg (2007). *International trade and economic growth*. M.E. Sharpe. Armonk. New York.

Lucas, R. (1988) "On the mechanics of economic development."

McGrattan, E. R., and E. C. Prescott (2007), "Openness, Technology Capital, and Development," Federal Reserve Bank of Minneapolis.

Melitz, M. J. (2003), "The Impact of Trade on Intra-industry Reallocations and Aggregate Industry Productivity," *Econometrica*, 71, 1695-1725.

Michaely M. (1977) “Exports and growth: an empirical investigation.” *Journal of Development Economics* : 49-53.

Rivera-Batiz, L. A. and P. Romer (1991) “Economic integration and endogenous growth.” *Quarterly Journal of Economics* 106: 531-56.

Rodriguez, F. and D. Rodrik (2001) “Trade policy and economic growth: a skeptics guide to the cross-national evidence.” In Bernanke, B., and K. S. Rogoffs (eds) *NBER Macroeconomics Annual 2000,* MIT Press, Cambridge, Massachusetts.

Rodrik D., A. Subramanian and F. Trebbi (2002) “Institutions rule: the primacy of institutions over geography and integration in economic development.” *NBER Working Paper* 9305.

Sachs, J., and A. Warner (1995), “Economic Reform and the Process of Global Integration,” *Brookings Papers on Economic Activity*, 1, 1-118.

Sala-i-Martin, X. (1997) “I just run two million regressions.” *American Economic Review* 87: 178-183.

Slaughter (2001)

Ventura (1997) “Growth and interdependence.” *Quarterly Journal of Economics* 112: 57-84.

Wacziarg, R. (2001), “Measuring the dynamic gains from trade.” *World Bank Economic Review* 15: 393-429.

World Bank (1987), *The World Bank World Development Report*. The World Bank, New York.

Yanikkaya, H. (2003), “Trade openness and economic growth: a cross-country empirical investigation.” *Journal of Development Economics* 72: 57-89.

Young, A. (1991), “Leaning by doing and the dynamic effects of international trade.” *Quarterly Journal of Economics* 106: 369-405.

Figure 1

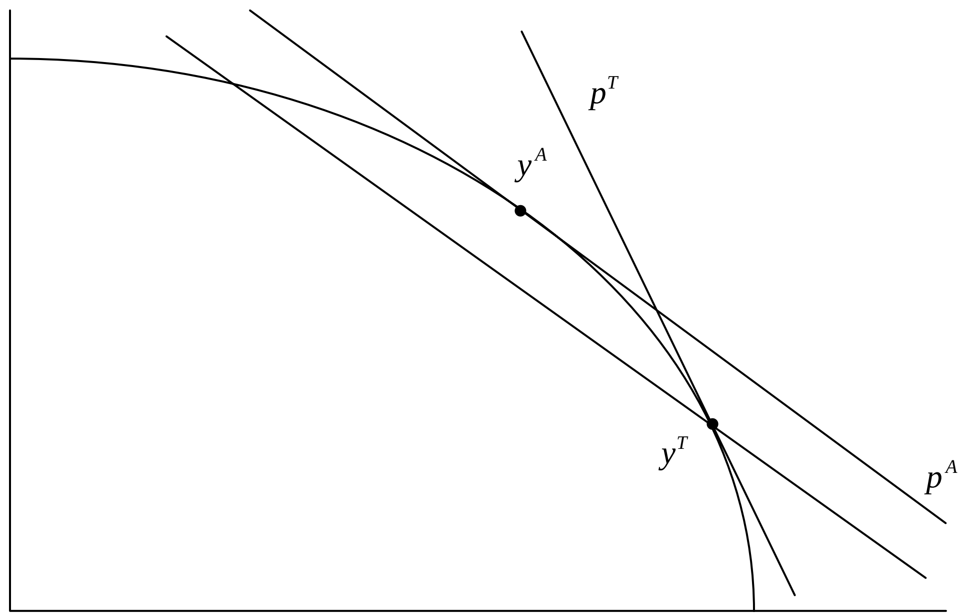
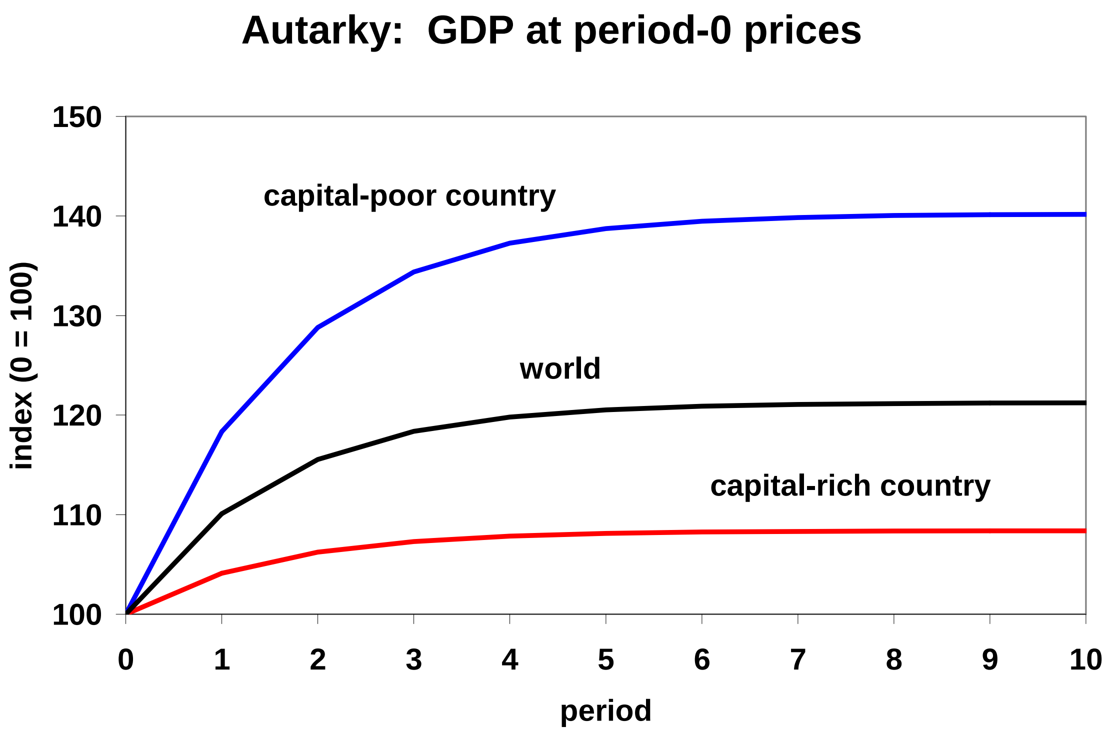


Figure 2



Autarky: GDP at period-0 prices
capital-poor country
world
capital-rich country
index (0 = 100)
150
140
130
120
110
100
0
1
2
3
4
5
6
7
8
9
10
period

Figure 3

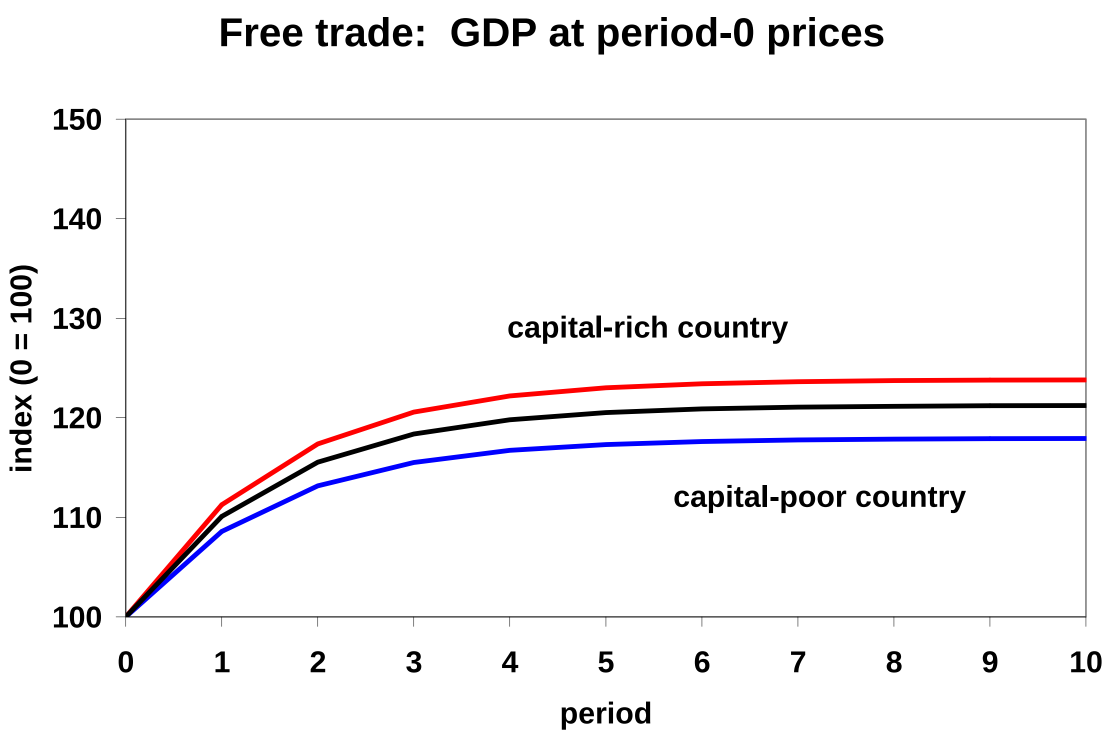
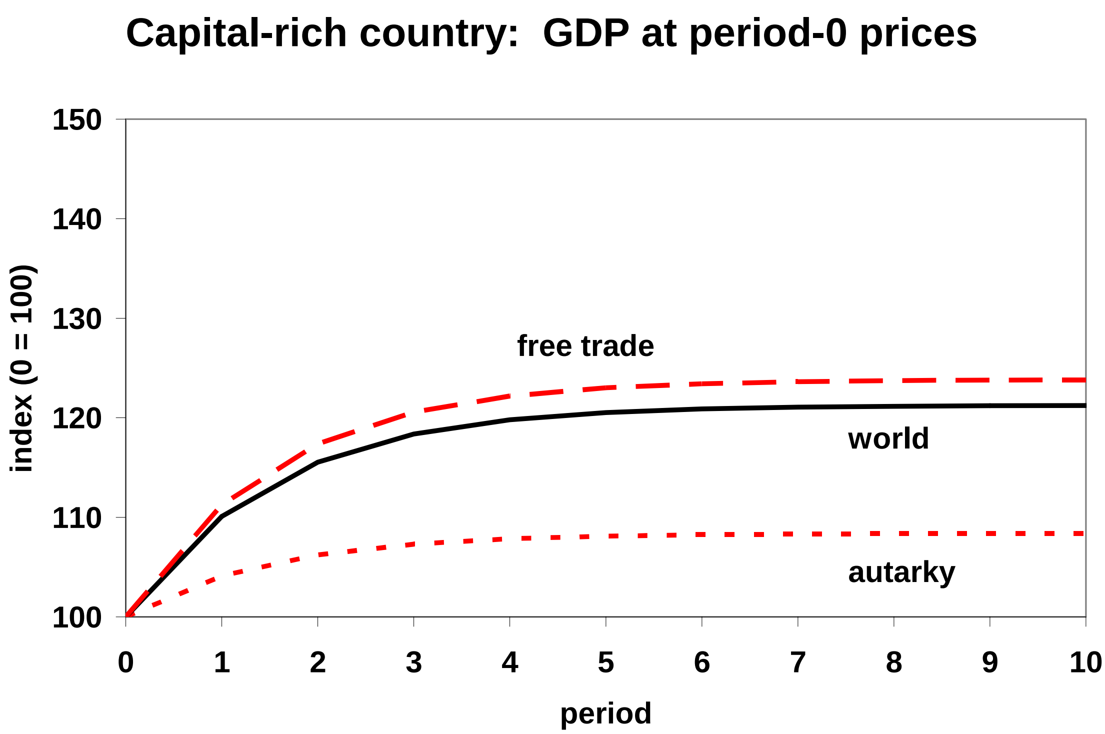
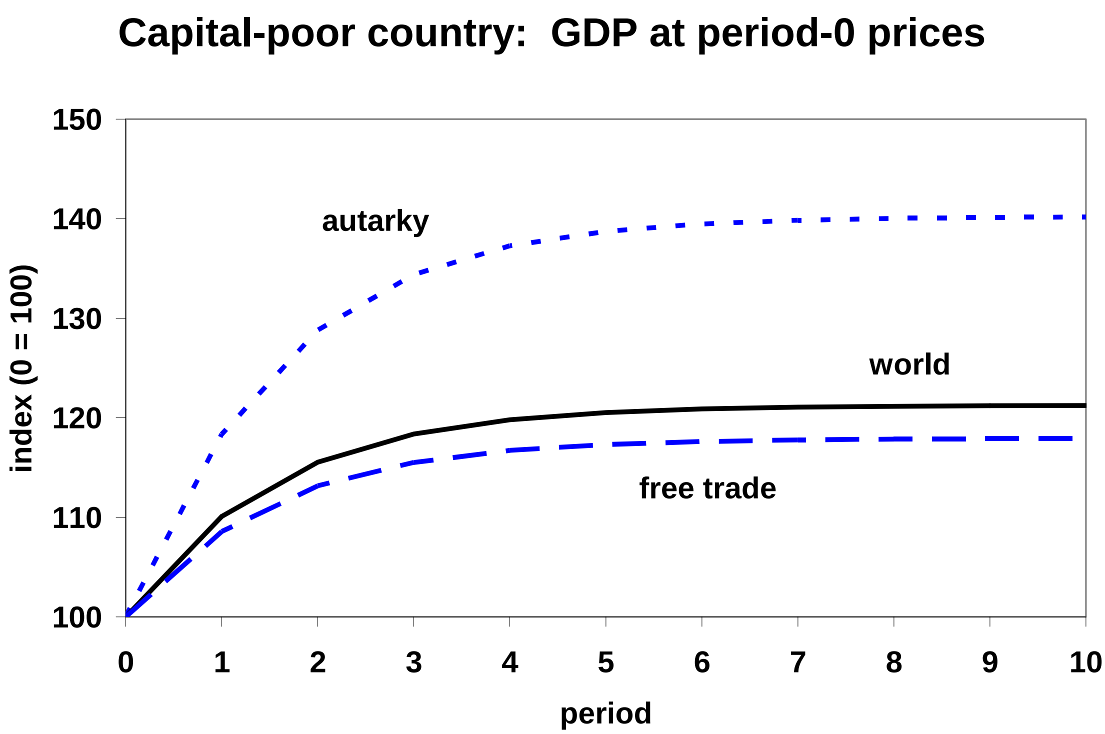

Free trade: GDP at period-0 prices
150
140
130
120
110
100
index (0 = 100)
capital-rich country
capital-poor country
0
1
2
3
4
5
6
7
8
9
10
period

Figure 4



Capital-rich country: GDP at period-0 prices
150
140
130
120
110
100
index (0 = 100)
0
1
2
3
4
5
6
7
8
9
10
period
free trade
world
autarky

Figure 5


Capital-poor country: GDP at period-0 prices
150
140
130
120
110
100
index (0 = 100)
0
1
2
3
4
5
6
7
8
9
10
period
autarky
world
free trade