A Multi-Country Dynamic Heckscher-Ohlin Model with Physical and Human Capital Accumulation*

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Abstract

We study a multi-country endogenous growth model in which the long-run growth is propelled by the accumulation of physical and human capital. In the integrated world economy, we illustrate that a unique balanced growth equilibrium exists which is locally saddlepoint stable. We also found that incorporating Penrose effect in the human capital accumulation process leads to a lower long-run growth rate. In the 2-country world economy with international trade, we conclude that asymmetric balanced growth path can be pinned down in the present 2-country framework. Lastly, we build the dynamic Heckscher-Ohlin theorem of international trade pattern.

1 Introduction

Historical data illustrate that the world economic growth has been accompanied by more than proportional growth in world trade (Ventura, 2005). This is most obviously so in the East Asian economies, where an oriented policy has been adopted. Table 1 reports the two rates of growth in income and trade in China. Though the causality of openness and economic growth is still under discussion, the internaional trade and economic growth issues

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should not be considered independently.[1] This paper examines the simultaneous determination of the long-run economic growth rate and international trade pattern in an integrated framework. We are interested in how the continuing growth in income and trade would affect the conventional understanding of international trade issues, such as the comparative advantage theory of factor endowments.

Table 1. China's Growth in Income and Trade

Average growth rate	1979-96 (%)	1997-2002 (%)	2003-05 (%)
Real GDP	9.9	7.9	9.5
Total trade volume	27.8	13.9	32.5
Exports	28.6	14	32

Source: NBS(2005b), State Informational Center (2005)

There have been many contributions in the literature on economic growth and international trade which focus mostly on innovation.[2] While research activities are responsible for the economic growth of advanced economies, however, this is not case for developing countries, in particular at the early stage of their economic development, see Young (1995) and Ventura (1997). As Young has plausibly argued, the accumulation of produced means of production has been the main determinant of the Asian Miracle. To understand the economic growth and international trade facts in the developing countries, we assume in this paper that the long-run economic growth engine is the accumulation of both physical and human capital.[3] Given our growth engine assumption, an appropriate international trade framework for the discussion should be the conventional Heckscher-Ohlin model. The aim of this paper is to discuss the dynamic and static comparative advantage results of factor endowments.

To our knowledge, the literature along this direction is relatively small. The analytical difficulty stems from the existence of two stocks, of physical

[1] See Frenkel and Romer (1999), for example, for the discussion on the causality of openess and economic growth.

[2] Among others, see Grossman and Helpman (1991), for example.

[3] Empirical evidence relating human capital with productivity growth can be found, for example, in Barro (1991), Benhabib and Spiegel (1994), Dinopoulos and Thompson (2000). As in Lucas (1993), human capital in this model can be understood as general knowledge which can be attained by accepting school education, training at workplaces, or learning by doing. Since the acquisition of all those knowledge needs resource inputs, we put them into one general "education" sector, and assume that agents raise their human capital level by buying the "education" services.

and human capital, structure. Given that both state variables are changeable over time, there will be no difference in investing on the two capitals in the long run. Hence the stock of physical and human capital are generally indeterminate, which in turn creats difficulties for any attempt to dynamise traditional trade theory with its emphasis on factor endowments. Bond, Trask and Wang (2003) is an important exception, which discusses the free trade equilibrium mimic the corresponding integrated word economy, and present the conditions that H-O theorem holds or not. We reexamine this issue by introducing *Penrose effect* in human capital accumulation. We illustrate that this extension brings out new insights. For example, the difference in human capital formation efficiency between countries can make us to pin down one unique steady growth path.

The idea of the *Penrose effect* is that there are increasing costs of growth in production. According to Penrose (1980), production is prevented from growing as fast as firms may like because certain inputs may be difficult to vary. This idea is in line with what Barro and Sala-i-Martin (1995, p119) emphasized from a different point of view: adjustment costs

> "If we distinguish between physical and human capital, then we anticipate that adjustment costs would be especially important for increases in human capital through process of education. The learning experience fundamentally takes time, and attempts to accelerate the educational process are likely to encounter rapid diminish rates of return."

Departing from the existing literature, we assume that one unit of education service does not necessarily correspond to one unit increase in human capital stock level. Instead, the attained education services raise the agents' human capital in a nonlinear concave way. On the other hand, we predict in advance that, if there is difference between countries on human capital formation efficiency, then this difference affect the long-run growth rate and the trade pattern between countries, which is confirmed ex post.

Our main results are as follows. In the autarky economy, there is a unique BGP which is locally saddlepoint stable. The introduction of Penrose effect affects the long-run growth in a negative way. That is, comparing with the case of linear human capital formation function, a relative smaller long-run growth rate is realized. Furthermore, in a 2-country world economy, difference in human capital formation efficiency between the two countries leads to asymmetric Balance Growth Path possible. A modified Heckscher-Ohlin result is obtained conditioned on the human capital formation process.

2 The autarky economy

We firstly consider the case of autarky economy, which can also be understood as an integrated world economy. There are two kinds of agents, identical firms and identical households. Different from Kemp and Shimomura (1995), we assume here that representative agents are unaware that they are identical, therefore they will be price takers.

2.1 Production

There are two factors of production, physical capital K and human capital H, which can be used to produce three different goods, a pure consumption good, a pure investment good, and educational service. In the following, we denote the variables in these three sectors with the ones with subscripts c, i and e respectively.

The standard constant-returns-to-scale neoclassical production technology prevails in all three sectors. Specifically, production function in sector-j is $Y_j = F^j(K_j, H_j) \ \ (j = c, i, e)$ where K_j, H_j are physical and human capital inputs to sector j. When production factors can move freely between sectors, all sectors face the same factor prices. Denote the rates of returns to physical and human capitals as r and w; then the cost of producing one unit of commodity j can be expressed as a function of r and w, $\Lambda^j(r, w)$, for example. Because of the neoclassical technology assumption, $\Lambda^j(r, w)$ owns the usual properties that it is increasing in each augment, and linearly homogeneous and quasiconcave in both augments. In the case of incomplete specialization in production, markets are perfectly competitive, with each commodity price equal to the unit-cost in that sector. That is

$$p = \Lambda^c(r, w), \ 1 = \Lambda^i(r, w), \ q = \Lambda^e(r, w). \tag{1}$$

Here we have assumed that the investment good is the numeraire, so that the prices of the consumption good p and the educational services q are prices in units of the investments goods.

Solving (1) for r, w and q given p, we obtain the factor prices and education services price with respect to p alone, that is, $r(p)$, $w(p)$ and $q(p) \equiv \Lambda^e(r(p), w(p))$. Using these functions, we can write the GDP function as $r(p)K + w(p)H$. We then have the following lemma.

Lemma 1 *The supply of the consumption goods is*

$$Y_c = r'(p)K + w'(p)H - q'(p)Y_e, \tag{2}$$

where $q'(p) = \Lambda^e_r(r(p), w(p))r'(p) + \Lambda^e_w((r(p), w(p))w'(p) = \frac{\Lambda^e_w(k_{it}-k_{et})}{\Lambda^c_w(k_{it}-k_{ct})}$ *with* k_{jt} *being the factor ratio in sector* j $(j = c, i, e)$.

Proof. See Appendix. ■

Notice that when, in the standard 2×2 model, K and H are allocated to the two sectors, the supply of the consumption good $Y_c = r'(p)K + w'(p)H$ obtained from the properties of the GDP function. This is no longer the case in the present setting since educational services are produced by using physical and human capital inputs. Therefore we have to delete the parts used in education sector, this is the third term in (2).

2.2 Households

Households, as factors owners, supply physical and human capitals to factor markets, and use the income to buy consumption good and to invest in human capital by buying educational services. Consumption contributes to current utility, while educational services add to human capital holdings and therefore to future consumption and utility. In fact, expenditure on educational services normally changes not only human capital but also human preferences. For analytical tractability, however, we ignore the effect of educational services on preference.

The optimization problem of the representative household is, for $K(0), H(0)$ given,[4]

$$\max \int_0^\infty (\ln C) \exp(-\rho t) dt, \quad \rho > 0$$

[4] We use a logrithmic utility function for simplicity. A general CAAC utility function like $\left(C^{1-\sigma} - 1\right) / (1 - \sigma)$ will bring no new results on this papers main concerns on steady growth path and trade pattern.

subject to

$$\dot{K} = r(p)K + w(p)H - pC - q(p)E \tag{3}$$

$$\dot{H} = Hf(E/H) \tag{4}$$

Here E is the demand for educational services, while $f(\cdot)$ is the human capital formation function which represents the efficiency of transforming educational services into human capital. Following Penrose (1980), we assume that $f(\cdot)$ has the following properties:

$$\begin{aligned} f(0) &= 0,\ f'(z) > 0,\ f''(z) < 0 \ \text{ for any } z > 0 \\ \lim_{z\to\infty} f(z) &= \infty,\ \lim_{z\to 0} f'(z) = 1,\ \lim_{z\to\infty} f'(z) = 0 \end{aligned}$$

where $z \equiv E/H$ is the ratio of the flow of educational services to the human-capital stock.

Notice that, unlike existing studies[5], which rely on a one-one relation between educational services and the increment of human capital, here we introduce a *Penrose Effect* in human capital accumulation.[6] This effect is reflected in $f(z)$. An example of $f(z)$ which satisfies these properties is $f(z) = \ln(1 + z)$.

The Hamiltonian of the representative household is

$$\mathcal{H} = \ln C + \lambda[r(p)K + w(p)H - pC - q(p)zH] + \mu H f(z),$$

where λ and μ are the shadow prices of the physical and human capital in utility units. The first-order conditions for interior solutions are

$$C \ :\ 1/C - p\lambda = 0 \tag{5a}$$

$$z \ :\ H(\mu f'(z) - \lambda q(p)) = 0 \tag{5b}$$

$$\dot{\lambda} = \lambda\,[\rho - r(p)] \tag{5c}$$

$$\dot{\mu} = \rho\mu - w(p)\lambda + \lambda q(p)z - \mu f(z) \tag{5d}$$

Here (5a) and (5b) are the efficiency conditions for consumption demand C

[5] Among others, see Lucas (1988), Caballe and Santos (1993), Bond, Wang and Yip (1996) and Mino (1996), for example.

[6] Uzawa (1969) introduces the Penrose effect into the investment of physical capital.

and educational services demand z. Recall the assumptions on the human capital formation function, we know $f'(z) < 1 \ \forall z \in (0, +\infty)$. That is, one unit education service can only lead to less than one unit increase in the human capital level. This property in human capital formation can also be understood as the existence of adjustment costs in human capital, which are much more important than those in physical capital as stressed by Barro and Sala-i-Martin (1995).

Lastly, the transversality condition is

$$\lim_{t\to\infty} [\lambda(t)K(t) + \mu(t)H(t)] \exp[-\rho t] = 0.$$

2.3 The market-clearing conditions

At each point of time, the aggregate demand for the pure-consumption good is C, while the total supply of this good is Y_c; when the market reaches an equilibrium, it must be the case $C = Y_c$. Similarly, in the educational sector, the demand for educational services is E while the aggregate supply of this service is Y_e, at equilibrium it must be that $E = Y_e$ holds true. The equilibrium of the remaining market (for the capital good) is taken care of by the Walras' Law.

2.4 Dynamic system

Combining the resource constraint conditions, the optimal conditions of the representative firm and the representative household and the market-clearing conditions, we conclude that the world economy as a whole can be described as the following system:

$$\begin{aligned}
\dot{K} &= r(p)K + w(p)H - pC - q(p)zH \\
\dot{H} &= Hf(z) \\
\dot{\lambda} &= \lambda(\rho - r(p)) \\
\dot{\mu} &= \rho\mu - w(p)\lambda + \lambda q(p)z - \mu f(z)
\end{aligned}$$

where C, z, p satisfy

$$
\begin{aligned}
0 &= 1 - p\lambda C \\
0 &= \mu f'(z) - \lambda q(p) \\
0 &= C + q'(p)zH - r'(p)K - w'(p)H
\end{aligned}
$$

respectively.

Let $k \equiv K/H$, $\theta \equiv \lambda/\mu$, and $c \equiv pC/H = 1/H\lambda$, then the above system can be rearranged as

$$
\begin{aligned}
\dot{k} &= (r(p) - f(z))k + w(p) - c - q(p)z && \text{(6a)} \\
\dot{\theta} &= \theta[f(z) - r(p) + \theta\,(w(p) - q(p)z)] && \text{(6b)} \\
\dot{c} &= c[r(p) - \rho - f(z)] && \text{(6c)}
\end{aligned}
$$

with $z = z(\theta, p)$ and p satisfying

$$
\begin{aligned}
0 &= \theta - f'(z)/q(p), && \text{(6d)} \\
0 &= c + pq'(p)z - pr'(p)k - pw'(p). && \text{(6e)}
\end{aligned}
$$

from which we can express z as $z = z(\theta, p)$. Therefore this is a complete dynamic system on (k, θ, c). If denote y_c as the per-unit-of-human-capital output of consumption goods: $y_c = r'(p)k + w'(p) - q'(p)z$, then the last relation can be rewritten as $c = py_c$.

2.5 Balanced growth equilibrium

When all variables grow at constant rates, possibly in zero rate, the economy is said to be at the balanced growth path (BGP henceforth). Given the constant-returns-to-scale technology, at the BGP, $\dot{k} = \dot{\theta} = \dot{c} = 0$; that is, the steady state k, θ, c, z, p must satisfy

$$
\begin{aligned}
0 &= [r(p) - f(z)]\,k + w(p) - c - q(p)z \\
0 &= f(z) - r(p) + \theta\,(w(p) - q(p)z) \\
\rho &= r(p) - f(z) \\
\theta &= f'(z)/q(p) \\
c &= py_c
\end{aligned}
$$

where $y_c = r'(p)k + w'(p) - q'(p)z$.

Given the steady state price $\bar{p}$, $\dot{c} = 0 : \rho = r(p) - f(z)$, reveals the steady-state ratio of educational service flow to the human capital stock $\bar{z} = z(\bar{p})$, which satisfies that $dz/dp = f'(z)/r'(p)$. Substituting this $\bar{z}$ into the first order condition on z, we obtain $\bar{\theta} = \theta(\bar{p})$. Since

$$d\theta/dp = \left[f''(z)z'(p)q(p) - f'(z)q'(p)\right]/q^2(p)$$

then as long as $q'(p) > 0$, this is the case when $k_{it} > k_{ct}$ and $k_{it} > k_{et}$, $d\theta/dp < 0$. Furthermore, $\dot{\theta} = 0$ implies that

$$\theta\left(w(p) - q(p)z\right) = \rho$$

which yields a unique steady state price $\bar{p}$ because $d\left[\theta\left(w(p) - q(p)z\right)\right]/dp > 0$ and

$$\begin{aligned} \lim_{p \to 0^+} \left[\theta\left(w(p) - q(p)z\right)\right] &= f'(z)\left[w(p)/q(p) - z\right] < 0, \\ \lim_{p \to +\infty} \left[\theta\left(w(p) - q(p)z\right)\right] &= +\infty. \end{aligned}$$

Finally, the steady-state values of c and k can be straightforwardly obtained by substituting $\bar{p}, \bar{z}$ and $\bar{\theta}$ into $\dot{k} = 0$ and the market-clearing condition (6e).

Proposition 2 *There is a unique steady state* $(\bar{k}, \bar{c}, \bar{\theta}, \bar{z}, \bar{p})$ *which is locally saddle point stable.*

Proof. See Appendix. ■

The effects of the Penrose effect of human capital accumulation on the long-run economy can be found by conducting comparative statics. When Penrose effect is introduced to human capital formation, a higher steady state price $\bar{p}$ should be the case; this higher steady state price in turn corresponds to a lower rate of return to physical capital if the capital sector is more physical capital intensive. Hence a lower steady state growth rate should be the case. This result is consistent with the intuitive understanding that efficiency in human capital formation will affect the long-run tate of growth.

Example 3 *If we specify the production technology as* $\Lambda^j(r, w) = r^{\alpha_j} w^{1-\alpha_j}$

and $f(z) = \ln(1+z)$, then the steady state price shall satisfy

$$r(p) - \rho = \ln\left[1 + w(p)/q(p)\right] - \ln(1+\rho).$$

If ρ is close to zero then $\ln(1+\rho) \approx \rho$. On the other hand, Cobb-Douglas technology implies that $r(p) = p^{\varepsilon_r}$, $w(p) = p^{\varepsilon_w}$ and $q(p) = p^{\varepsilon_q}$, where ε_j is the elasticity of factor price to commodity price p. Since $\ln\left[1 + w(p)/q(p)\right] < w(p)/q(p)$, we have the steady state price with Penrose effect satisfies

$$p^{\varepsilon_w - \varepsilon_q - \varepsilon_r} > 1.$$

When the capital sector is the most physical capital intensive one, $\varepsilon_w > 1$, $\varepsilon_r < 0$. From $\varepsilon_q = \alpha_e \varepsilon_r + (1-\alpha_e)\varepsilon_w$, we have $\varepsilon_w - \varepsilon_q - \varepsilon_r > 0$. Then the steady state price with Penrose effect is larger than one. It is straightforward to obtain that, when there is no Penrose effect, that is, $f(z) = z$, the steady state value of p is one.

3 The World Economy with International Trade

There are two countries in the world, home and foreign, which have the same production technology and preference function but differ in their human capital formation efficiency and the initial factor endowments. The produced capital good and consumption good are internationally tradable without cost, while educational services cannot cross borders. Following the convention of neoclassical international trade theory, we exclude the factor movement between countries. Furthermore, in order to have incomplete specialization of production in both countries, we assume that the human capital formation functions in the two countries are close to each other efficiently. Given these assumptions, factor price equalization can be realized through international trade in goods.

3.1 Production, households, and market equillibrium

The variablies of the foreign country are distinguished by asterisks. As before, the resource constraints for the home country are

$$\begin{aligned} \dot{K} &= r(p)K + w(p)H - pC - q(p)zH, \\ \dot{H} &= Hf(z), \end{aligned}$$

while these for the foreign country are

$$\begin{aligned} \dot{K}^* &= r(p)K^* + w(p)H^* - pC^* - q(p)z^*H^*, \\ \dot{H}^* &= H^*f^*(z^*), \end{aligned}$$

where zH, z^*H^* are the demand for the educational service in the home, foreign countries. Because of Penrose effects, one unit of bought educational service will lead to a less than one unit increase in the human capital level in both countries. In addition, the home and foreign countries may differ in the efficiency of their human capital formation. We will show in the following that this difference in human capital formation will affect the long-run trade pattern between the two countries.

Parallel to the analysis in the previous section, agents in each country maximize their life time utility. The respective static optimization conditions for consumption and education service C, C^*, z and z^* are

$$\begin{aligned} C &: 1/C - p\lambda = 0 \\ C^* &: 1/C^* - p\lambda^* = 0 \\ z &: H(\mu f'(z) - \lambda q(p)) = 0 \\ z^* &: H^*(\mu^* f^{*\prime}(z^*) - \lambda^* q(p)) = 0 \end{aligned}$$

the intertemporal conditions for physical capital and human capital are

$$\begin{aligned} \dot{\lambda} &= \lambda(\rho - r(p)) \\ \dot{\lambda}^* &= \lambda^*(\rho - r(p)) \\ \dot{\mu} &= \rho\mu - w(p)\lambda + \lambda q(p)z - \mu f(z) \\ \dot{\mu}^* &= \rho\mu^* - w(p)\lambda^* + \lambda^* q(p)z^* - \mu^* f^*(z^*) \end{aligned}$$

and the equilibrium conditions are $zH = Y_e$, $z^*H^* = Y_e^*$ and

$$C + C^* = r'(p)(K + K^*) + w'(p)(H + H^*) - q'(p)(Y_e + Y_e^*). \tag{7}$$

Finally, the transversality condiitons are

$$\begin{aligned} \lim_{t\to\infty} [\lambda(t)K(t) + \mu(t)H(t)] \exp[-\rho t] &= 0, \\ \lim_{t\to\infty} [\lambda^*(t)K^*(t) + \mu^*(t)H^*(t)] \exp[-\rho t] &= 0. \end{aligned}$$

Notice that, $\lambda/\lambda^* =$ constant, hence so is $(pC)/(pC^*)$ =constant$\equiv \xi$.

3.2 Balanced growth equilibrium in the traded world

Denoting the world aggregate human capital level as $H^w = H + H^*$, and rearranging all the variables in the form of per unit of human capital ones. Hence $k = K/H^w$ and $k^* = K^*/H^w$ are physical capital per unit of human capital in the home and foreign country respectively; $h = H/H^w$ is the human capital ratio of the home country to the world aggregate economy, and that of the foreign country is $h^* = H^*/H^w = 1 - h$; $c = (pC)/H^w$ is the value of consumption good per unit of aggregate human capital in the home country, and the one for the foreign country is $c^* = (pC^*)/H^w = \xi c$; $m = \mu^{-1}/H^w$, $m^* = (\mu^*)^{-1}/H^w$.

Using $\dot{H}^w/H^w = hf(z)+h^*f^*(z^*)$, then the dynamic system of the world trading economy can be expressed as:

$$\begin{aligned} \dot{k} &= \{r(p) - [hf(z) + h^*f^*(z^*)]\}\, k + h\, [w(p) - q(p)z] - c \\ \dot{k}^* &= \{r(p) - [hf(z) + h^*f^*(z^*)]\}\, k^* + h^*\, [w(p) - q(p)z^*] - c^* \\ \dot{h} &= h(1-h)[f(z) - f^*(z^*)] \\ \dot{c} &= c\,\{r(p) - \rho - [hf(z) + h^*f^*(z^*)]\} \\ \dot{m} &= m\,\{(1-h)[f(z) - f^*(z^*)] - \rho + [w(p) - q(p)z]\, m/c\} \\ \dot{m}^* &= m^*\,\{(-h)[f(z) - f^*(z^*)] - \rho + [w(p) - q(p)z^*]\, m^*/c^*\} \end{aligned}$$

together with the market-clearing condition for the consumption goods:

$$c + c^* = pr'(p)(k + k^*) + pw'(p) - pq'(p)\,[zh + z^*h^*] \tag{8}$$

where the FOCs of the representative house hold reveal that z and z^* satisfy

$$f'(z) = mq(p)/c,\ f^{*\prime}(z^*) = m^* q(p)/c^*.$$

A BGP equilibrium is that along which all variables grow at constant (maybe zero) rates. Because of the neoclassical assumptions about production in both countries, at the BGP, $\dot{k} = \dot{k}^* = \dot{h} = \dot{c} = \dot{m} = \dot{m}^* = 0$.

Since $\dot{h} = \dot{c} = 0$ the steady state price and educational services satisfy

$$\begin{aligned} 0 &= r(p) - \rho - f(z) \\ 0 &= r(p) - \rho - f^*(z^*) \end{aligned}$$

That is, when the home country are more efficient in transforming educational services into human capital, relatively smaller educational services consumption is needed. From these two equations, the steady-state values of z and z^* can be expressed as $z = z(p)$ and $z^* = z^*(p)$. On the other hand, $\dot{m} = \dot{m}^* = 0$ yields

$$\begin{aligned} \rho &= [w(p) - q(p)z]\, m/c, \\ \rho &= [w(p) - q(p)z^*]\, m^*/(\xi c) \end{aligned}$$

from which we solve out the steady-state values $m = m(c,p)$ and $m^* = m^*(c,p)$. Substituting $z = z(p)$, $z^* = z^*(p)$, $m = m(c,p)$ and $m^* = m^*(c,p)$ into the FOCs of z and z^* yields

$$f'(z(p))/q(p) = m(c,p)/c,\ \ f^{*\prime}(z^*(p))/q(p) = m^*(c,p)/(\xi c)$$

from which a unique solution $(\bar{c}, \bar{p})$ can be solved out.

Lemma 4 *Given a value of ξ, the FOCs of z and z^* and $\dot{h} = \dot{c} = \dot{m} = \dot{m}^* = 0$ determine a unique vector of steady-state values $(\bar{p}, \bar{c}, \bar{m}, \bar{m}^*, \bar{z}, \bar{z}^*)$.*

Proof. See Appendix. ■

The steady state values of h, k and k^* satisfy the market clearing condition (8) and $\dot{k} = \dot{k}^* = 0$; that is

$$\begin{aligned} 0 &= \rho k + h\,[w(p) - q(p)z] - c, \\ 0 &= \rho k^* + h^*\,[w(p) - q(p)z^*] - c^* \end{aligned}$$

from which steady-state values of h, k and k^* are uniquely determined, for a given value of ξ.

Proposition 5 *In the case of incomplete specialization in production, for each given value of ξ, there is a unique balanced growth path in the world economy. The steady growth rate along the balance growth path is $\bar{\nu} = r(\bar{p}) - \rho$; with $\bar{p}$ the steady-state price of the consumpiton good and $r(\bar{p})$ the steady-state rate of return to physical capital. $\bar{p}$ satisfies*

$$f'(z)/q(p) = \rho/\left[w(p) - q(p)z\right]$$

where z is a function of p given by $f'(z) = r(p) - \rho$.

Similar to the local stability analysis in the autarkic economy, we can obtain the local stability of the BGP in the world economy with internaitonal trade. Therefore, given the initial state variables $K(0)$, $H(0)$, $K^*(0)$, $H^*(0)$, then the world economy will grow along a transitional path before arriving the Balanced Growth Path.

The introduction of Penrose effect helps us to determine a BGP differing from that of the integrated world. It is easily seen from the above reasoning that, without differences in human capital formation efficiency, the only possible BGPs are these replicating the autarkic economy.

3.3 Trade pattern

In order to derive the H-O result of international trae pattern, we rearrange the BGP equilbrium path as the follows:

$$pC/H - pC^*/H^* = \rho(K/H - K^*/H^*) - q(p)(z - z^*).$$

Substituting this relation into (7) and rearranging terms, yields

$$\chi\left[\left(\rho - pr'(p)\right)(K/H - K^*/H^*) + (pq'(p) - q(p))(z - z^*)\right] = pY_c^*/H^* - pC^*/H^*$$

where $\chi = H/(H + H^*)$.Hence, depending on the educational pattern and the human capital accumulation process, Heckscher-Ohlin result may hold or not.

Proposition 6 *If the home country is relatively capital intensive and if the home country invests more (less) in human capital when the educational* *service has an elastisity larger (smaller) than one, then the home country will export the capital good and import the consumption good.*

It is worth noting that, when term $(pq'(p) - q(p))(z - z^*)$ has a different sign from $(\rho - pr'(p))(K/H - K^*/H^*)$, it is possible that the Heckscher-Ohlin result may fail to hold.

4 Concluding Remarks

Combining Penrose effect with the human capital formation process, we examine the long-run growth rate and international trade pattern in a unified framework. One main result is that this addition lowers the long-run growth rate in both autarky and the world economies. Furthermore, we have shown that international difference in the efficiency with which educaitonal service is transformed into human capital affects the pattern of trade. Conditions are provided for a modified Heckscher-Ohlin result on trade patterns.

Appendix

1. *Proof of Lemma 1*

Considering the properties of the unit-cost functions, we have th full employment conditions

$$\begin{aligned} K - \Lambda_r^e Y_e &= \Lambda_r^c Y_c + \Lambda_r^i Y_i \\ H - \Lambda_w^e Y_e &= \Lambda_w^c Y_c + \Lambda_w^i Y_i \end{aligned}$$

from which we obtain

$$Y_c = \frac{\Lambda_w^i (K - \Lambda_r^e Y_e) - \Lambda_r^i (H - \Lambda_w^e Y_e)}{\Delta}$$

where $\Delta \equiv \Lambda_r^c \Lambda_w^i - \Lambda_r^i \Lambda_w^c$. Next, differentiate the identities $p = \Lambda^c(r(p), w(p))$ and $1 = \Lambda^i(r(p), w(p))$ with respect to p, yield

$$\begin{aligned} 1 &= \Lambda_r^c r'(p) + \Lambda_w^c w'(p) \\ 0 &= \Lambda_r^i r'(p) + \Lambda_w^i w'(p) \end{aligned}$$

Solving the system for $r'(p)$ and $w'(p)$, we obtain $r'(p) = \Lambda_w^i/\Delta$ and $w'(p) = -\Lambda_r^i/\Delta$. Substituting $r'(p), w'(p)$ into Y_c yields

$$\begin{aligned} Y_c &= r'(p)K + w'(p)H - \{\Lambda_r^e(r(p), w(p))r'(p) + \Lambda_w^e((r(p), w(p))w'(p)\}Y_e \\ &= r'(p)K + w'(p)H - q'(p)Y_e. \end{aligned}$$

2. *Proof of Proposition 1*

First, from the first-order condition on z, we have $f'(z) = \theta q(p)$, hence

$$\begin{aligned} z_\theta(\theta, p) &\equiv \partial z(\theta, p)/\partial\theta = q(p)/f''(p) < 0, \\ z_p(\theta, p) &\equiv \partial z(\theta, p)/\partial p = \theta q'(p)/f''(p) \end{aligned}$$

from the properties of human capital formation function $f(\cdot)$. Second, linearizing the dynamic system on (k, c, θ, p) around the steady state, yields

$$\begin{pmatrix} \dot{k} \\ \dot{c} \\ \dot{\theta} \\ 0 \end{pmatrix} = X \begin{pmatrix} k - \bar{k} \\ c - \bar{c} \\ \theta - \bar{\theta} \\ p - \bar{p} \end{pmatrix}$$

Here the coefficient matrix X is evaluated at the steady state

$$X = \begin{vmatrix} \rho & -1 & d_1 z_\theta & y_c + d_1 z_p \\ 0 & 0 & -cf'z_\theta & c(r' - f'z_p) \\ 0 & 0 & \rho & d_2 \\ -pr' & 1 & pq'z_\theta & -y_c - p(y_c)'_p \end{vmatrix}$$

with $f' = f'(z)$, $r' = r'(p)$, $y_c = r'(p)k + w'(p) - q'(p)z$ and

$$\begin{aligned} d_1 &= (\partial\dot{k}(k, c, \theta, p, z)/\partial z) = -f'(z), \\ d_2 &= (\partial\dot{\theta}(k, c, \theta, p, z(p))/\partial p), \\ (y_c)'_p &= r''(p)k + w''(p) - q''(p)z - q'(p)z_p(\theta, p). \end{aligned}$$

The eigenvalues xs of X satisfy the following eigen equation

$$a_3x^3 + a_2x^2 + a_1x + a_0 = 0$$

where

$$
\begin{aligned}
a_3 &= \left[y_c + p(y_c)'_p\right] \\
a_2 &= (-2\rho)\left[y_c + p(y_c)'_p\right] + pr'\left(y_c + d_1 z_p\right) - c\left(r' - f'z_p\right) - pq'z_\theta d_2 \\
a_1 &= \rho^2\left[y_c + p(y_c)'_p\right] - \rho\left[pr'\left(y_c + d_1 z_p\right) - c(r' - f'z_p) - pq'z_\theta d_2\right] \\
&\quad -(pr' - \rho)\left[c(r' - f'z_p) + pq'z_\theta d_2\right] + pr'\left(d_1 + pq'\right)z_\theta d_2 + cf'z_\theta d_2 \\
a_0 &= (pr' - \rho)\left[\rho c\left(r' - f'z_p\right) + d_2 c f' z_\theta\right]
\end{aligned}
$$

Since $a_3 > 0$ and $a_0 = -\text{Det} > 0$, then there are one or three negative eigenvalues. On the other hand, $a_2 = -\text{trace}(X) < 0$ reveal that there are at least one positive root,. Combining the two, one stable eigenvalue should be the case. Since k is the only state variable in the dynamic system, then this is the case that the dynamic system (9) is locally saddlepoint stable.

3. Proof of Lemma 2.

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