The effect of trade openness on optimal government size under endogenous firm entry*

Sandra Hanslin†

February 2008

Abstract

This paper analyzes the effect of trade liberalization on government spending in a general equilibrium model with a continuum of industries supplying tradable and nontradable goods under monopolistic competition. Trade liberalization is modeled as the opening up of product markets between two countries, which may differ in total factor productivity, factor endowment and fix cost technology. In this setup, I show that the optimal provision of a public consumption good depends positively on the degree of openness. Moreover, the richer and more productive country chooses a lower optimal government share.

Keywords: International trade, monopolistic competition, trade openness, public expenditure

JEL Classification: F12, H40

---

*I am very grateful to Josef Falkinger and Harmut Egger for many insightful discussions and suggestions. I also thank Gino Gancia, Peter Neary and Fabrizio Zilibotti for many helpful comments.

†University of Zurich, Socioeconomic Institute, Zürichbergstrasse 14, CH-8032 Zurich. E-mail: sandra.hanslin@wwi.uzh.ch

1 Introduction

Through the increasing international integration of goods markets new challenges for the public sector arise. There is a large literature supporting the view that trade liberalization between countries puts pressure on governments. On the one hand, the literature on tax competition points out that the integration of markets erodes the tax base and therefore tends to increase the costs of public goods provision. While this mechanism has been mainly discussed in the context of capital mobility, it is clearly also at work when goods market integrate (see Haufler, 2001). On the other hand, in a seminal paper Rodrik (1998) argues that openness may expose a country to greater risk, due to terms of trade volatility. In this case, there may be a need to extend government spending after trade liberalization in order to provide a social insurance against the external risk.[1] Devereux (1991), Anderson et al. (1996) and Epifani and Gancia (2007) point to a further channel of influence. They argue that in an open economy, the costs of taxation can be exported if changes in public spending influence the terms of trade. Similar to Rodrik (1998), these studies point to a positive impact of trade liberalization on public expenditure.[2]

While the idea that a country can reduce its costs of public good provision is intuitively appealing, I show, that it is clearly not only the changes in terms of trade that can explain this result. Rather, if consumers have a love of variety, an increase in public expenditures - which lowers domestic employment in the private sector and the number of domestic firms producing a variety - affects the number of consumed varieties in open economies by less than in closed ones due to the imports of foreign varieties. It is clear, that this love of variety effect is larger the more a country is integrated into the world market.

The variety debate is clearly a central issue in the newer literature. Therefore, it seems obvious to discuss the effects of trade liberalization on government expenditure in a model which takes into account the existence of varieties within a country. There is some empirical evidence provided by Epifani and Gancia (2007) that the positive effect of openness on government consumption depends on the degree of

[1]The idea that governments provide an insurance against external risk has first come up by Cameron (1978).

[2]There is convincing empirical evidence for a positive relationship between the openness for trade and public expenditure. See for example Cameron (1978), Rodrik (1998), Garrett (2001) and Epifani and Gancia (2007).

the share of differentiated products on exports. However, in their theoretical model Epifani and Gancia (2007) assume that each country produces one variety in every industry (Armington, 1969) in order to isolate the terms of trade effect. I assume endogenous product differentiation within countries, i.e., firms produce under monopolistic competition and there is free entry for firms until profits are equal to zero. A Dixit and Stiglitz (1977) framework - applied to intraindustry trade in horizontally differentiated goods by Krugman (1980) - is used to model monopolistic competition. For example, in context of the automobile industry, I assume that consumers decide between BMW, Audi, Ford and Fiat rather than between German, American or Italian cars, i.e., firms are global players and not countries.[3]

The effect of openness on government spending is analyzed within a general equilibrium framework with two possibly asymmetric countries. The countries may differ in total factor productivity, capital and labor endowment and fix cost technology. The measure for openness to trade is obtained, following Epifani and Gancia (2007), by assuming a continuum of industries of measure 1 whereof an exogenous fraction splits the industries into tradable and nontradable ones. The advantage of this measure is to discuss the effects of a marginal increase of openness on public expenditure - rather than to make only a comparison between autarky and open economy. I assume, as Epifani and Gancia (2007), that government expenditures are used for the production of a public consumption good.[4]

Both production factors, capital and labor, are employed for production of the private and public consumption goods. A higher government share implies that more capital and labor is employed for public good production while the labor force and capital stock for private production is reduced. Hence, fewer firms enter the market and fewer varieties are produced. This lowers utility of the representative household if the household has a love of variety.

It follows that, in absence of the effects discussed in the literature so far, optimal public good provision depends positively on the degree of openness. Because of the

[3]Empirical investigation of the Armington assumption (product differentiation by country) versus the Helpman-Krugman assumption (IRS and endogenous product differentiation within country) found more support for the Helpman and Krugman (1985) assumption. See for example, Feenstra et al. (2001), Evenett and Keller (2002) and Schott (2004).

[4]There is some literature examining the role of public infrastructure - in contrast to public consumption good - in a globalizing world. Under the assumption of monopolistic competition and increasing returns to scale, Anwar (2001) investigates the effect of public infrastructure on the pattern of international trade and Egger and Falkinger (2006) discuss the role of public infrastructure and subsidies for international outsourcing and firm location.

love of variety, household's utility is larger the more industries are open to trade. An increase of government spending in home lowers indeed the number of varieties in home, however, since the number of foreign varieties consumed remains constant (for given foreign public expenditure), utility loss in open industries is lower than in closed ones. This positive love of variety effect induces governments to set a higher public spending if the country is more open. Furthermore, since a decline in domestic varieties reduces utility in the foreign country as well, foreign governments lowers optimal public good provision due to an increase in domestic public good provision. In addition, I find that in closed economies, the optimal government share is invariant with respect to the size of the respective economies. Moreover, if the two countries engage in trade, the richer and more productive country has a lower optimal government share.

While the costs of public good provision in terms of the number of firms are equal for closed and open economies, the costs in terms of welfare are lower in open economies than in closed ones. In contrast to autarky, both governments increase public spending in open economies on expense of the other country. Since the countries do not internalize the costs on the welfare of the other country, both governments act too expansionary in equilibrium. Thus, aggregate welfare would be higher with cooperation than it is without.

The rest of the paper is organized as follows. In the next Section the theoretical model is introduced. Section 3 derives the equilibrium values of the variables for a given government share. In Section 4 benevolent governments in both countries choose their optimal government share. The Nash equilibrium of the public sector is analyzed with respect to the asymmetry between the two countries and the parameters of the economy. Section 5 provides some extensions. On the one hand the importance of the love of variety versus the market power is illustrated and on the other hand additional effects with a public sector producing more or less capital intensive than the private sector are discussed. The last section concludes.

2 The model

There are two countries, home (H) and foreign (F). In both countries there is a private and a public sector. The private sector is characterized by a continuum of industries of measure 1 indexed by $j \in [0,1]$. In each industry and country various firms produce differentiated goods with capital and labor under monopolistic competition. Each firm is monopolist for one variety, after having incurred some fixed cost. There is free entry, that is, the equilibrium number of firms in an industry is endogenously determined. I assume free trade between the two countries in an exogenous fraction $\tau \in [0,1]$ of the industries and no trade for the remaining fraction $1-\tau$. In addition to intraindustry trade, there can be interindustry trade. Without loss of generality I refer to trading industries with index $j \leq \tau$ and to the nontrading industries with index $j > \tau$. Thus, τ is the measure for openness. In each country, there is one country-specific public good which is produced with capital and labor and which is not tradable. The countries have an endowment $\bar{K}_i$ of capital and $\bar{L}_i$ of labor where I assume that both production factors are perfectly mobile within each country. The subscript i refers to the two countries H and F.

2.1 Endowments, preferences and demand

The representative household owns total endowment of capital ($\bar{K}_i$) and labor ($\bar{L}_i$). Hence, the household's income is given by $w_i\bar{L}_i + r_i\bar{K}_i$, where w_i denotes the wage rate and r_i the rental rate in country i.[5] Net income is given by $I_i := w_i\bar{L}_i + r_i\bar{K}_i - T_i$, where T_i denotes the income tax imposed by the government.

The representative household derives utility from consumption of the different varieties in each industry and a country specific public good denoted by G_i. Household's preference for private goods versus the public good is captured in the parameter $\eta \in (0,1)$.

$$U_i = \eta \int_0^1 \log Y_{ij} dj + (1-\eta) \log G_i \quad for \quad i = H, F \tag{1}$$

where subutility Y_{ij} is a CES aggregator of the varieties consumed in industry j

$$Y_{ij} = \left(\sum_{k \in \mathcal{N}_{ij}} \left(y_{kj}^i\right)^\nu \right)^{\frac{1}{\nu}}, \quad i = H, F \tag{2}$$

with $\nu \in (0,1)$. y_{kj}^i denotes consumption of variety k in industry j by the repre-

[5] Note that I assume perfect mobility of capital and labor within each country.

sentative household in country i.[6] The elasticity of substitution between any two varieties from industry j is given by $\sigma = \frac{1}{1-\nu}$. The assumption $\nu \in (0,1)$ implies $\sigma > 1$. A higher value of ν implies a lower love of variety, a higher elasticity of substitution and less market power for any individual firm.[7] For $\nu \to 1$ goods become perfect substitutes, so that the firms' monopoly power vanishes, the goods are perfect substitutes and the households have no love of variety. Within any industry $j > \tau$, the household consumes only the varieties produced in the own country, within an industry $j \leq \tau$, the household consumes all varieties produced in country H and F. $\mathscr{N}_{ij}$ is the index set of all varieties from industry j which are available for consumption in country i. $N_{ij} = |\mathscr{N}_{ij}|$ denotes the number of varieties from industry j consumed in country i. If an industry produces tradables, households in both countries consume the same varieties. Therefore, for $j \leq \tau$, $\mathscr{N}_{Hj} = \mathscr{N}_{Fj}$ and $N_{Hj} = N_{Fj}$.

Since the elasticity of substitution between the subutilities Y_{ij} is equal to 1, the household allocates its expenditures equally among the industries. Moreover, since the measure of all industries is equal to 1, this amount equals net income I_i. Thus, the budget constraint for purchasing varieties from an industry is given by the equation:

$$I_i = \sum_{k \in \mathscr{N}_{ij}} p_{kj} y^i_{kj} \tag{3}$$

where p_{kj} is the price of variety k in industry j. In a traded industry the household spends the budget I_i on all industry specific varieties produced in country H and F, whereas in nontraded industries the household spends I_i only on domestic varieties.

Household's subutility (2) is maximized with respect to y^i_{kj}, subject to the budget constraint per industry (3). The resulting demand curve for each variety is given by

$$y^i_{kj} = \left(\frac{p_{kj}}{P_{ij}}\right)^{\frac{-1}{1-\nu}} Y_{ij} \ , \qquad \forall \ k \in \mathscr{N}_{ij} \ \wedge \ \forall \ j \in [0,1] \tag{4}$$

[6]The location of production does not matter for the household's optimal consumption choice.

[7]Benassy (1998) introduced a generalization of the Dixit-Stiglitz assumption (eq. 2) which allows to disentangle the degree of love of variety on the one hand and the elasticity of substitution and market power respectively on the other hand. For the tractability of the analysis the Dixit-Stiglitz assumption is used having in mind that the parameter ν comprises various measures. For illustration of the importance of the love of variety Section 5.1 provides an analysis using the Benassy assumption.

where $P_{ij} := \left(\sum_{k\in\mathcal{N}_{ij}} (p_{ki})^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$ is the consumer price index for industry j.[8] It may be interpreted as the unit cost function of subutility Y_{ij}. Because of free trade, home and foreign have the same consumer price index in tradable industries: $P_{Hj} = P_{Fj}$ for $j \leq \tau$.

2.2 Production and supply

2.2.1 Public good

The country-specific public good is produced also with capital and labor. Like firms the public sector takes factor prices as given, which is a common assumption in the literature. For simplicity, it is assumed that the public good is produced with a Leontief production function:

$$G_i = \min\left\{\beta_i g_{Ki}\bar{K}_i, g_{Li}\bar{L}_i\right\} \tag{5}$$

where $\beta_i > 0$ describes the productivity of capital relative to labor in public good production and $g_{Ki} \in (0,1)$, $g_{Li} \in (0,1)$ the fraction of the economy's capital and labor endowments respectively, which are employed by the public sector. Cost minimizing production of the public good implies $\beta_i g_{Ki}\bar{K}_i = g_{Li}\bar{L}_i$. Throughout this paper I assume that $\beta_i = \frac{\bar{L}_i}{\bar{K}_i}$. This implies that capital intensity in public good production is equal to the proportion of capital endowment to labor endowment in the country. Hence, $g_{Li} = g_{Ki} = g_i$ so that public production is given by $G_i = g_i\bar{K}_i = g_i\bar{L}_i$ and government expenditures amount to $g_i(w_i\bar{L}_i + r_i\bar{K}_i)$.

At first sight, this simplification may seem arbitrary. However, it has the important virtue to isolate the interaction between international integration and size of the public sector from interactions between private and public production, which arise when the factor proportion supplied to the private sector is distorted by public production.[9]

The public good is financed by a lump-sum tax T_i. A balanced budget requires

$$T_i = g_i(w_i\bar{L}_i + r_i\bar{K}_i).$$

[8] The derivation of the demand curve can be found in Appendix A.1.

[9] Deviations from this assumption imply additional effects which are discussed in Section 5.1.

2.2.2 Private goods

Each firm in an industry produces one variety with capital and labor according to a Cobb-Douglas production function with increasing returns to scale

$$x_{kj} = \begin{cases} A_i(K_{kj} - K_i^*)^{1-\gamma}(L_{kj})^{\gamma} & \text{if } K_{kj} \geq K_i^* \text{ and } L_{kj} \geq 0 \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

where x_{kj} denotes output of firm k in industry j located in country H or F, K_{kj} (L_{kj}) is the input of capital (labor) of an individual firm, A_i denotes total factor productivity in country i and K_i^* the overhead capital needed to run the plant. The corresponding cost function is a linear function in x_{kj} and the fix costs are given by the rental rate times overhead capital:[10]

$$c(w_i, r_i, x_{kj}) = \underbrace{r_i K_i^*}_{a_{0i}(r_i)} + \underbrace{\left(\frac{r_i}{1-\gamma}\right)^{1-\gamma}\left(\frac{w_i}{\gamma}\right)^{\gamma}\frac{1}{A_i}}_{\equiv a_{1i}(r_i, w_i)} x_{kj} \tag{7}$$

The firms in a tradable industry face demand of both countries while firms in a nontradable sector face only domestic demand:

$$x_{kj} = \begin{cases} y_{kj}^i & = \quad Y_{ij}\left(\frac{p_{kj}}{P_{ij}}\right)^{\frac{-1}{1-\nu}} & j > \tau \\ y_{kj}^i + y_{kj}^{i'} & = \quad \left(Y_{ij} + Y_{i'j}\right)\left(\frac{p_{kj}}{P_j}\right)^{\frac{-1}{1-\nu}} & j \leq \tau \end{cases} \tag{8}$$

(Recall that in the trading industries $P_{Hj} = P_{Fj}$. Therefore, the country index can be skipped.) Assuming that the potential number of firms is large, an individual firm takes as given aggregate industry supply and the price index. Firms maximize profit for given Y_{ij} and P_{ij}, $i = H, F$

$$\max_{x_{kj}} \; p_{kj}x_{kj} - a_{1i}x_{kj} - a_{0i}. \tag{9}$$

subject to market demand (8). Solving the profit maximization problem yields for the optimal price:

$$p_{kj} = p_i = \frac{a_{1i}}{\nu}\ , \tag{10}$$

which is independent of firm and industry. Free firm entry implies zero profits

[10] The derivation of the cost function can be found in Appendix A.2.

in equilibrium. This determines the quantity produced by each firm x_{kj} which is identical for all firms within a country, independent of industry.

$$x_{kj} = x_i = \frac{a_{0i}\nu}{a_{1i}(1-\nu)} \tag{11}$$

Using equation (11) and the definition for marginal cost a_{1i}, conditional factor demand simplifies to

$$L_i\left(\frac{r_i}{w_i}\right) = \frac{\gamma\nu}{1-\nu}\frac{r_i}{w_i}K_i^* \tag{12}$$

$$K_i = K_i^*\frac{1-\gamma\nu}{1-\nu} \ . \tag{13}$$

While the demand for labor depends on the relative factor prices, demand for capital is independent of the factor prices.

2.3 Macroeconomic equilibrium conditions

2.3.1 Capital and labor market

Since government employs $g_i\bar{K}_i$ and $g_i\bar{L}_i$ for public good production, $(1-g_i)\bar{K}_i$ and $(1-g_i)\bar{L}_i$ remains available for production of private goods. The full employment condition for capital and labor in $i = H, F$ is given by

$$(1-g_i)\bar{K}_i = \int_0^1 n_{ij}K_i dj \tag{14}$$

$$(1-g_i)\bar{L}_i = \int_0^1 n_{ij}L_i dj \tag{15}$$

where n_{ij} is the equilibrium number of firms in industry j and country i which has not yet been determined. Combining equation (13) and (14) we obtain the absolute number of firms in each country which is given by the following expression

$$\int_0^1 n_{ij}dj = \frac{(1-g_i)\bar{K}_i}{K_i^*}\frac{1-\nu}{1-\gamma\nu}. \tag{16}$$

Using (16) and (12) in equation (15), we have for the factor price ratio

$$\frac{r_i}{w_i} = \frac{1-\nu\gamma}{\nu\gamma}\frac{\bar{L}_i}{\bar{K}_i} \ , \tag{17}$$

which gives for the labor demand of a firm

$$L_i = \frac{1-\gamma\nu}{1-\nu} K_i^* \frac{\bar{L}_i}{\bar{K}_i} \ . \tag{18}$$

Equation (17) implies that $\gamma\nu = \frac{w_i \bar{L}_i}{w_i \bar{L}_i + r_i \bar{K}_i}$, the share of labor income in total income is identical in the two economies. The higher the elasticity of output with respect to labor input (γ) and the lower the market power of firms (i.e. the higher ν) the higher is the wage share. Moreover, substituting (17) for w/r in the expression defining $a_{1i}(r, w)$ and using the result in (11), we obtain for the output of an individual firm

$$x_i = \frac{\phi}{1-\nu} A_i K_i^* \left(\frac{\bar{L}_i}{\bar{K}_i}\right)^{\gamma} \quad i = H, F \tag{19}$$

with $\phi := (\nu(1-\gamma))^{1-\gamma}(1-\nu\gamma)^{\gamma}$.

2.3.2 Demand

In a closed industry, consumption of a variety is equal to its production, given by (19). It remains to show how consumption of a variety in a open industry is split between the two representative households. From now on I denote consumption of a variety in closed industries with x_H and x_F, respectively, and demand for varieties in open industries by y_k^i, $k = H, F$ and $i = H, F$, where k refers to the location of production and i to the location of consumption of the variety. Dividing household's H budget constraint for a tradable industry through the one of household F

$$\frac{I_H}{I_F} = \frac{n_{Hj} p_H y_H^H + n_{Fj} p_F y_F^H}{n_{Hj} p_H y_H^F + n_{Fj} p_F y_F^F}$$

and using equation (A1) to replace y_F^H by $\left(\frac{p_H}{p_F}\right)^{\frac{1}{1-\nu}} y_H^H$ and y_F^F by $\left(\frac{p_H}{p_F}\right)^{\frac{1}{1-\nu}} y_H^F$, we see that consumption of in H relative to consumption in F of a variety produced in H is equal to the relative income:

$$\frac{y_H^H}{y_H^F} = \frac{I_H}{I_F} \ .$$

Using the equation for market clearing in a tradable industry, $x_H = y_H^H + y_H^F$, and the relation $\frac{y_H^H}{y_F^H} = \left(\frac{p_F}{p_H}\right)^{\frac{1}{1-\nu}}$ we get the demand for tradable varieties:

$$\begin{aligned}(y_H^H, y_F^H) &= \left(\frac{I_H x_H}{I_F + I_H}, \left(\frac{p_H}{p_F}\right)^{\frac{1}{1-\nu}} \frac{I_H x_H}{I_F + I_H}\right) \\ (y_H^F, y_F^F) &= \left(\frac{I_F x_H}{I_F + I_H}, \left(\frac{p_H}{p_F}\right)^{\frac{1}{1-\nu}} \frac{I_F x_H}{I_F + I_H}\right)\end{aligned} \tag{20}$$

2.3.3 Trade account

Trade occurs because households love varieties. In open industries, a household spreads its consumption over all produced varieties in both countries. In equilibrium, the value of exports must equal the value of imports. In other words, the number of varieties times price times consumption of the varieties produced in country i and consumed in country i', integrated over all traded industries, must equal the number of varieties times price times consumption of the varieties produced in country i' and consumed in country i, integrated over all traded industries.

$$\int_0^\tau p_i y_i^{i'} n_{ij} dj = \int_0^\tau p_{i'} y_{i'}^i n_{i'j} dj \tag{21}$$

We may rewrite the trade account condition by using the budget constraint for purchases from a tradable industry, $I_i = n_{ij} p_i y_i^i + n_{i'j} p_{i'} y_{i'}^i$, taking the integral from 0 to τ on both sides, and combining the result with (21). This yields $\tau I_i = \int_0^\tau n_{ij} p_i \left(y_i^i + y_i^{i'}\right) dj$. Since $y_i^i + y_i^{i'} = x_i$, this reduces to:

$$\int_0^\tau n_{ij} dj = \frac{\tau I_i}{p_i x_i}. \tag{22}$$

Using (22) in (21) the trade account becomes $y_H^F \frac{I_H}{x_H} = y_F^H \frac{I_F}{x_F}$. Combining this with (20), we finally get

$$\frac{x_F}{x_H} = \left(\frac{p_H}{p_F}\right)^{\frac{1}{1-\nu}} . \tag{23}$$

3 Positive analysis

In each nontraded industry an equal number of firms enters the market in equilibrium. The households spend an equal amount on each industry and prices and quantities produced are constant for all industries, $I_i = n_{ij} p_i x_i$. In the tradable industries only the average number of firms per industry is determined. According to (22), $\frac{1}{\tau}\int_0^\tau n_{ij}dj = \frac{I_i}{p_i x_i}$. It follows that the number of firms in each nontradable industry (denoted as $n_{i,j>\tau}$) is equal to the total number of firms per country (see equation (16)) denoted by n_i. We have

$$\frac{1}{\tau}\int_0^\tau n_{ij}dj = n_{i,j>\tau} = n_i = \frac{(1-g_i)\bar{K}_i}{K_i^*}\frac{1-\nu}{1-\gamma\nu}\ . \tag{24}$$

The location of the firms in single tradable industries is undetermined. Interindustry trade allows that firms in one tradable industry are mainly located in H while in others are located more in F, as long as the location pattern is consistent with (22).

The higher capital endowment or the lower overhead capital required to run the plant the higher is the number of firms in equilibrium. The endowment left to the private sector is decisive for the number of firms in the in the market. If the size of the public sector expands, less firms are active in the private sector.

Each firm producing a positive amount in equilibrium supplies

$$x_i = \frac{\phi}{1-\nu}A_i K_i^*\left(\frac{\bar{L}_i}{\bar{K}_i}\right)^\gamma \quad i = H, F$$

where I defined $\phi = (\nu(1-\gamma))^{1-\gamma}(1-\nu\gamma)^\gamma$.[11] The supply of each variety depends positively on the amount of overhead capital and productivity, and declines if capital supply is scarce relative to labor endowment. Note that only the factor proportion matters - not the absolute level of endowments. The assumption that the public sector does not distort the factor proportion implies that the size of the public sector does not affect equilibrium output per firm - in contrast to the number of firms.

For the numéraire I choose the price of home varieties. Setting $p_H = 1$, the marginal cost is equal to $a_{1H} = \lambda_H = \nu$. Hence the equilibrium rental and wage rate are[12]

$$r_H = \phi A_H\left(\frac{\bar{L}_H}{\bar{K}_H}\right)^\gamma$$
$$w_H = \frac{\phi\nu\gamma}{1-\nu\gamma}A_H\left(\frac{\bar{K}_H}{\bar{L}_H}\right)^{1-\gamma}\ ,$$

[11]Note that $x_i = \frac{\nu}{1-\nu}(1-\gamma)^{1-\gamma}\gamma^\gamma A_i K_i^*\left(\frac{r_i}{w_i}\right)^\gamma$.

[12]Note that $r_i = \lambda_i(1-\gamma)^{1-\gamma}\gamma^\gamma A_i\left(\frac{r_i}{w_i}\right)^\gamma$,and $w_i = \lambda_i(1-\gamma)^{1-\gamma}\gamma^\gamma A_i\left(\frac{w_i}{r_i}\right)^{1-\gamma}$.

respectively. The trade account condition $\frac{x_F}{x_H} = \left(\frac{p_H}{p_F}\right)^{\frac{1}{1-\nu}}$ determines the price of foreign varieties in terms of the fundamentals of the economy:

$$p_F = \left(\frac{A_H}{A_F}\left(\frac{\bar{L}_H\bar{K}_F}{\bar{K}_H\bar{L}_F}\right)^{\gamma}\frac{K_H^*}{K_F^*}\right)^{1-\nu}.$$

Note, since the price of domestic varieties is normalized, p_F is equal to the relative price of foreign to domestic varieties. The more productive and the more capital-rich is F relative to H, the lower is the relative price of foreign goods. Moreover, prices in foreign are low, ceteris paribus, if overhead capital is high in F. High overhead capital reduces the number of firms in F and increases output x_F per firm, which can only be sold if the price of the foreign varieties decreases as well. A low price requires that marginal cost is low. Through this channel high fixed capital requirements depress factor returns.

Equation (10) together with (17) determines the rental rate and wage rate in country F: $r_F = \phi A_F\left(\frac{\bar{L}_F}{\bar{K}_F}\right)^{\gamma}p_F$ and $w_F = \frac{\phi\nu\gamma}{1-\nu\gamma}A_F\left(\frac{\bar{K}_F}{\bar{L}_F}\right)^{1-\gamma}p_F$. Substituting p_F, we have

$$\begin{aligned} r_F &= \phi A_F^{\nu}A_H^{1-\nu}\left(\frac{K_H^*}{K_F^*}\right)^{1-\nu}\left(\frac{\bar{L}_H}{\bar{K}_H}\right)^{\gamma(1-\nu)}\left(\frac{\bar{L}_F}{\bar{K}_F}\right)^{\gamma\nu} \\ w_F &= \frac{\phi\nu\gamma}{1-\nu\gamma}A_F^{\nu}A_H^{1-\nu}\left(\frac{K_H^*}{K_F^*}\right)^{1-\nu}\left(\frac{\bar{L}_H}{\bar{K}_H}\right)^{\gamma(1-\nu)}\left(\frac{\bar{K}_F}{\bar{L}_F}\right)^{1-\nu\gamma} \end{aligned}$$

Comparing this with H, we see that both factor returns are low if overhead capital in F is high relative to overhead capital in H. The net incomes are given by

$$I_i(g_i) = (1-g_i)\frac{\phi}{1-\nu\gamma}A_i(\bar{K}_i)^{1-\gamma}(\bar{L}_i)^{\gamma}p_i$$

To summarize, an increase in public spending (higher g_i) has only a variety effect - the equilibrium number of firms decreases. Supply of commodities and its prices, as well as wage and rental rates do not depend on the public good provision. This feature depends on the very special assumption of $\beta_i = \frac{\bar{L}_i}{\bar{K}_i}$ which implies that the capital intensity for public good production is equal to the capital intensity country endowments. Since the factor prices depend on the relative factor availability for private production and as long as government does not affect this relative availability, government won't have an effect on market prices.

4 Optimal public good provision

4.1 Governments optimization problem

I assume a benevolent government whose aim is to provide a quantity of the public good that maximizes the utility of the representative household. Government's choice parameter is g_i, the fraction of capital and labor used for public good production. Having determined the equilibrium values in the last section we can describe the indirect subutilities for traded and nontraded industries as follows:[13]

$$\begin{aligned} Y_{i,j>\tau}(g_i) &= (n_i(g_i))^{\frac{1}{\nu}} x_i \\ Y_{i,j\leq\tau}(g_H, g_F) &= \left(\tfrac{I_H(g_H)+I_F(g_F)}{I_i(g_i)}\right)^{\frac{1}{\nu}-1} (n_i(g_i))^{\frac{1}{\nu}} x_i \end{aligned} \tag{25}$$

where $i = H, F$. Note that $\left(\frac{I_H(g_H)+I_F(g_F)}{I_i(g_i)}\right)^{\frac{1}{\nu}-1} > 1$. Since the household loves varieties, subutility in open industries is higher than in closed ones. Ceteris paribus, this difference is higher the poorer the country. It follows that richer countries do gain less from trade than poorer countries.

Government in country i chooses g_i such as to maximize the utility of the representative household in country i taken as given government spending in the other country $g_{i'}$, $i, i' \in H, F$ and $i \neq i'$.

$$\max_{g_i} \; \eta\tau \log Y_{i,j\leq\tau}(g_i) + \eta(1-\tau)\log Y_{i,j>\tau}(g_H, g_F) + (1-\eta)\log G_i(g_i) \tag{26}$$

Governments maximization of the household's utility yields the following first order condition:

$$\vartheta_i := \underbrace{\eta\tau\frac{1-\nu}{\nu}\left(\frac{1}{1-g_i} - \frac{1}{1-g_i+(1-g_{i'})\Omega_{i'}}\right)}_{>0} \underbrace{-\frac{\eta}{\nu}\frac{1}{1-g_i}}_{<0} + \underbrace{(1-\eta)\frac{1}{g_i}}_{>0} = 0 \tag{27}$$

where I defined

$$\Omega_{i'} := \left(\frac{A_{i'}}{A_i}\right)^{\nu} \left(\frac{K_i^*}{K_{i'}^*}\right)^{1-\nu} \left(\frac{\bar{K}_{i'}}{\bar{K}_i}\right)^{1-\gamma\nu} \left(\frac{\bar{L}_{i'}}{\bar{L}_i}\right)^{\gamma\nu}$$

for $i, i' \in H, F$ and $i \neq i'$. The asymmetry of the two countries is captured in Ω_i. Note that only relative differences in the fundamentals affects government expendi-

[13]See Section A.3 in Appendix for the derivation of the subutility in open industries.

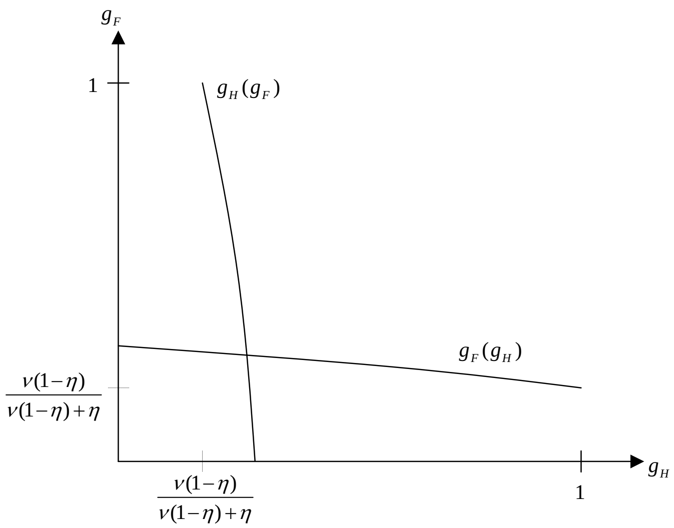

Figure 1: Governmental reaction functions

tures. The third term in equation (27) represents the positive marginal utility of a higher supply of the public good. The second term represents the marginal utility loss due to the crowding out of private firms in tradable and nontradable industries. As we discussed before, the average number of firms in equilibrium is lower with higher public production. The first bracket is positive and dampens the negative effect of the second term. This positive effect comes from the fact that domestic public production affects only the number of domestic firms - not the number of available foreign varieties. Since households have a love for varieties, utility in open industries is higher than in closed ones and due to the crowding out of domestic firms, the number of domestic relative to foreign varieties decreases which increases the relative utility gain in open industries. This dampening effect is larger the higher the integration of the goods market which is measured by τ. By inspecting equation (27), it can be seen, that the effect of the asymmetry between countries on optimal government share vanishes if $\nu \to 1$, i.e. in a situation under perfect competition and no love of variety. Then, optimal government share is given by $g = 1 - \eta$, identical for both countries.

For any given $g_{i'}$ equation (27) has one solution at which the second order condition for a maximum holds. The two reaction functions $g_H(g_F, \tau, \nu, \eta, \Omega_F)$ and $g_F(g_H, \tau, \nu, \eta, \Omega_H)$ which correspond to the maximum are illustrated in figure 1.[14]

[14]The proof of the existence and uniqueness of the equilibrium is found in the Appendix A.5.

Proposition 1. *Domestic optimal government share depends negatively on foreign optimal government share if $\tau > 0$.*

Proof. This can be easily shown by applying the implicit function theorem on the first order condition (equation (27)).

$$\frac{dg_i}{dg_{i'}} = -\frac{-\eta\tau\frac{1-\nu}{\nu}\frac{\Omega_{i'}}{((1-g_i)+(1-g_{i'})\Omega_{i'})^2}}{\frac{\partial\vartheta_i}{\partial g_i}}$$

The expression $\frac{\partial\vartheta_i}{\partial g_i}$ is the second order condition which is negative at the optimum level of g_i. Therefore, $\frac{dg_i}{dg_{i'}} < 0$. □

The higher government expenditure in the foreign country the less firms enter the market to produce a variety in country F. Hence, households utility in country H and F is lower since they consume less varieties. Taking this into account, government in H sets a lower g_H.

For a first investigation of the optimal government share in a single country it is interesting to look at the symmetric case. If the two countries are identical, we are able to calculate explicitly the equilibrium value of the Nash game of the public sector.

4.2 Symmetric countries

The asymmetry is captured in the parameter $\Omega_{i'}$ which is equal to 1 for fully symmetric countries. In this case $g_H = g_F = g$ is a mutually best response of both governments. This optimal g is given by

$$g = \frac{1-\eta}{1+\eta\frac{1-\nu}{\nu}(1-\frac{\tau}{2})} \tag{28}$$

Since households love varieties, subutility in open industries is higher than in a closed industry. Thus, the more open a country the higher is aggregate utility. This lowers the trade-off for governments, which results in a higher g_i.

$$\frac{\partial g}{\partial\tau} = \frac{(1-\eta)\eta(1-\nu)\nu}{2(\nu+\eta(1-\nu)(1-\tau/2))^2} > 0$$

Optimal government expenditure in a closed economy ($\tau = 0$) is equal to $g = \frac{1-\eta}{1-\eta+\frac{\eta}{\nu}}$. The effect of openness on government expenditure plays together with the elasticity of substitution or love of variety respectively. The higher ν the higher is government spending. A higher ν means lower love of variety and a higher elasticity of substitution

$$\frac{\partial g}{\partial \nu} = \frac{(1-\eta)\eta(1-\tau/2)}{(\nu+\eta(1-\nu)(1-\tau/2))^2} > 0$$

To conclude (proof in the text)

Proposition 2. *In a symmetric equilibrium* $\frac{dg}{d\tau} > 0$ *and* $\frac{dg}{d\nu} > 0$.

While in my analysis optimal public good provision depends positively on the elasticity of substitution between varieties, in Epifani and Gancia (2007) public good provision depends negatively on the elasticity of substitution between export and import goods. Note, that Epifani and Gancia assume product differentiation by country and hence, the elasticity of substitution is a measure for the monopoly power of countries vis à vis other countries. This power is important for the terms of trade effect. They argue that a lower elasticity of substitution between exports and imports allows to a country higher wages and prices in respond to higher labor demand by governments. There is no variety effect in Epifani and Gancia, since the number of varieties (equal to the number of countries) is exogenous in their model. In contrast, in my analysis the number of firms in equilibrium is endogenous and depend on public spending. As a consequence, optimal expenditure in my model is driven through the variety effect.[15] There is no terms of trade effect of the public sector in my model (since market prices do not react on government activity). In my model, the terms of trade are determined by the relative productivity, the capital overhead and the factor proportion of the two countries.

4.3 Asymmetric countries

Having obtained an insight for the effect of the different parameters on government spending in a symmetric equilibrium, I now turn to the asymmetric case. Note, that only the relative difference between the two countries enters the first order condition. I first consider some limiting cases where the countries are infinitely different. If $\Omega_{i'} \to 0$, i.e. country i is infinitely richer than country i', the solution

[15] Remind that the parameter ν captures different measures: on the one hand the love of variety and on the other the elasticity of substitution and market power respectively. A higher love of variety goes hand in hand with a lower elasticity of substitution which is a lower ν. Section 5.1 disentangles the two effects and illustrates the importance of the love of variety for a positive correlation between openness and optimal government share.

for g_i (given by (27)) reduces to $\frac{1-\eta}{1+\eta\frac{1-\nu}{\nu}}$. Since $(\Omega_i)^{-1} = \Omega_{i'}$, $\Omega_{i'} \rightarrow 0$ implies $\Omega_i \rightarrow \infty$. Therefore, the solution for $g_{i'}$ is given by $\frac{1-\eta}{1+\eta\frac{1-\nu}{\nu}(1-\tau)}$. To summarize, while for the infinitely rich country the degree of openness does not affect government spending, the infinitely poor country does benefit from more openness. Further, in this limiting cases, domestic optimal government share is independent of foreign government share. This result is intuitively clear.

Proposition 3 summarizes the effects of the asymmetry between the two countries on the Nash equilibrium of the public sector.

Proposition 3. *The richer and more productive is country i relative to the country i' (lower $\Omega_{i'}$), the lower is optimal government share in country i and the higher in country i'.*

Proof. Again, the implicit function theorem is applied on the first order condition for given public foreign expenditure to get

$$\left.\frac{dg_i}{d\Omega_{i'}}\right|_{g_{i'} constant} = -\frac{-\eta\tau\frac{1-\nu}{\nu}\frac{-(1-g_{i'})}{((1-g_i)+(1-g_{i'})\Omega_{i'})^2}}{\frac{\partial\vartheta_i}{\partial g_i}} > 0$$

The expression $\frac{\partial\vartheta_i}{\partial g_i}$ is the second order condition which is negative at the optimum level of g_i. Neglecting the effect of $\Omega_{i'}$ on $g_{i'}$ the result above means that for given $g_{i'}$ optimal g_i is higher the higher $\Omega_{i'}$. Since $\Omega_{i'} = (\Omega_i)^{-1}$ and because of the symmetry, a higher $\Omega_{i'}$ leads to a lower $g_{i'}$ for any given g_i. The lower $g_{i'}$ due to the higher $\Omega_{i'}$ has an additional positive effect on g_i as I have shown in proposition 1. Hence, any change in the fundamentals, which makes country i' bigger (higher $\bar{K}_{i'}/\bar{K}_i$ and $\bar{L}_{i'}/\bar{L}_i$) and more productive (higher $A_{i'}/A_i$ and lower $K^*_{i'}/K^*_i$) lowers the equilibrium value of g_i and increases $g_{i'}$. Graphically the reaction function $g_i(g_{i'})$ rotates outwards and $g_{i'}(g_i)$ rotates inwards. □

A lower overhead capital in H, for instance, raises the number of varieties in H. The number of firms entering the market increases, output per variety falls and prices increases due to the increase in marginal costs. Since income in country H increases the demand for varieties in country F, while production in F stays constant, prices in country F, wage and rental rate increase.

It is obvious from equation (27) that for given foreign government share a higher τ results in a higher optimal domestic government share. That is, the reaction functions rotate outwards if τ increases. However, since the curves are negatively sloped it is not a priori clear whether openness does increase government spending in both countries. The symmetric case already showed that openness has a positive effect on optimal government spending in equilibrium. The limiting asymmetry cases show that the infinitely rich country does not benefit from openness and therefore, government spending does not depend on openness. While the optimal government share of the infinitely poor country depends positively on τ. It follows that

$$\left.\frac{dg_i}{d\tau}\right|_{\Omega_{i'}\to\infty} > \left.\frac{dg_i}{d\tau}\right|_{\Omega_{i'}=1} > \left.\frac{dg_i}{d\tau}\right|_{\Omega_{i'}\to 0} = 0 \ .$$

which let conjecture that the effect of openness on optimal government share is decreasing in the relative size of of the country. I provide some simulation outcomes (see figure 2) which support the conjecture that $\frac{d^2 g_i}{d\tau d\Omega_{i'}} > 0$ and $\frac{dg_i}{d\tau} > 0 \; \forall \; \Omega_{i'} \neq 0$.

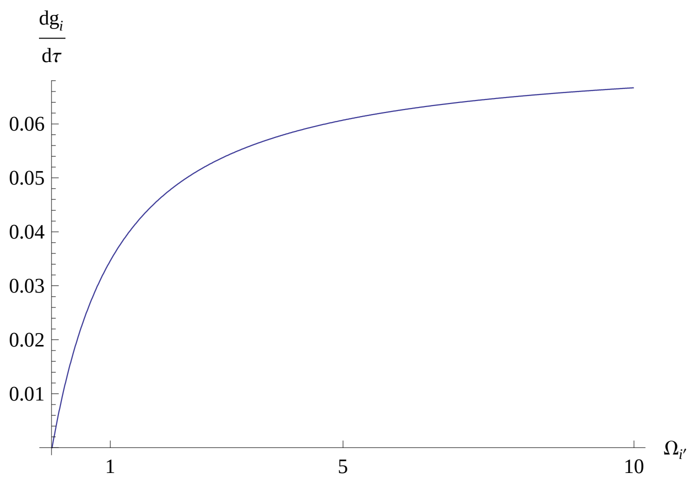

Figure 2: Effect of τ on optimal g_i subject to $\Omega_{i'}$ ($\nu = 0.7, \tau = 0.3, \eta = 0.6$). The plot looks qualitatively the same for various values of $\nu, \tau, \eta \in (0, 1)$.

4.4 Welfare

In the previous section we discussed optimal public good provision from a country's point of view. Optimal public good provision increases with openness since the more a country is integrated into the world market (higher τ), the more costs of public good provision are exported. Since governments do not internalize the costs which are on expense of the other country, there is overprovision of the public good in equilibrium. If the governments in the two countries cooperate they would set optimally $g_H = g_F = \frac{\nu(1-\eta)}{\nu(1-\eta)+\eta}$ which is equal to the level in autarky. Comparing welfare under non-cooperative and cooperative governments, aggregate welfare is higher with cooperation than without.

5 Extensions

5.1 Love of variety vs. market power

The aim of this Section is to examine the role of market power and love of variety. In the standard Dixit-Stiglitz case we are not able to distinguish between the two effects. Therefore, I follow Benassy (1998) who introduced a generalization of the Dixit-Stiglitz assumption. The nice and special feature of this extension is to disentangle the parameter for monopolistic power from the one for "love of variety". Under the Benassy extension the subutility is given by

$$Y_{ij} = (N_{ij})^{\rho+1-\frac{1}{\nu}} \left(\sum_{k \in \mathcal{N}_{ij}} \left(y^i_{kj}\right)^{\nu} \right)^{\frac{1}{\nu}}, \quad i = H, F. \tag{29}$$

The "love of variety" is captured in the parameter ρ. If ρ is equal to zero there is no love of variety and if $\rho = \frac{1-\nu}{\nu}$ we are back to the standard Dixit-Stiglitz case.[16]

The demand curve for a variety changes as follows

$$y^i_{kj} = (N_{ij})^{\rho\frac{\nu}{1-\nu}-1} \left(\frac{p_{kj}}{P_{ij}}\right)^{\frac{-1}{1-\nu}} Y_{ij} \quad \forall \ k \in \mathcal{N}_{ij} \ \wedge \ \forall \ j \in [0,1] \tag{30}$$

where $P_{ij} := (N_{ij})^{-\rho+\frac{1}{\sigma-1}} \left(\sum_{k\in\mathcal{N}_{ij}} (p_{ki})^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$ is the price index per industry and may be interpreted as the unit cost function of the subutility Y_{ij} (29).

Compared to the results presented so far, the subutilities for traded and non tradable industries change as follows:

$$\begin{aligned} Y_{i,j>\tau}(g_i) &= (n_i)^{\rho+1} x_i \\ Y_{i,j\leq\tau}(g_H, g_F) &= \left(\frac{n_H+n_F}{n_i}\right)^{\rho} \left(\frac{(I_H+I_F)n_i}{I_i(n_H+n_F)}\right)^{\frac{1}{\nu}-1} Y_{i,j>\tau}(g_i) \end{aligned} \tag{31}$$

for $i = H, F$ and note that n_i and I_i are both functions of g_i. Subutility in closed industries depends positively on the love of variety while the parameter for market

[16] See Montagna (2001) for a discussion of the welfare effects from trade under the two cases, Dixit-Stiglitz versus no love of variety. Blanchard and Giavazzi (2003) discuss the case where $\rho = 0$.

power has no direct effect. Note however that ν affects n_i. When there is no love of variety, $\rho = 0$, it is not clear anymore whether subutility in open industries is higher than in closed ones. Whether $\frac{(I_H+I_F)n_i}{I_i(n_H+n_F)}$ is greater or smaller than 1, depends on the relative prices of foreign and domestic varieties. Substituting $I_i = n_i p_i x_i$, $i = H, F$, and using (23)

$$\frac{(I_i + I_{i'})n_i}{I_i(n_i + n_{i'})} = \frac{n_i + n_{i'}\left(\frac{p_i}{p_{i'}}\right)^{\frac{\nu}{1-\nu}}}{n_i + n_{i'}} \tag{32}$$

which is greater than 1 if $p_i > p_{i'}$ and smaller than 1 if $p_i < p_{i'}$. In words, if the average price in open industries is lower than in closed ones, subutility in open industries is higher than in closed industries. Recalling that the relative price of foreign to domestic varieties is equal to $p_F = \left(\frac{A_H}{A_F}\left(\frac{\bar{L}_H\bar{K}_F}{\bar{K}_H\bar{L}_F}\right)^{\gamma}\frac{K_H^*}{K_F^*}\right)^{1-\nu}$, the term (32) for a household in H is greater than 1 if, anything equal, H is more capital rich, less productive in A_H or overhead capital requirement is lower.

Governments maximization of the household's utility (equation (26) subject to (31)) yields following first order condition:

$$\begin{aligned}
\phi_i \; &:= \eta\tau\rho\left(\frac{1}{1-g_i} - \frac{1}{(1-g_i)+(1-g_{i'})\kappa_{i'}}\right) \\
&\quad + \eta\tau\frac{1-\nu}{\nu}\left(\frac{1}{(1-g_i)+(1-g_{i'})\kappa_{i'}} - \frac{1}{(1-g_i)+(1-g_{i'})\Omega_{i'}}\right) \\
&\quad - \eta(\rho+1)\frac{1}{1-g_i} + (1-\eta)\frac{1}{g_i} \; = \; 0
\end{aligned} \tag{33}$$

where we defined

$$\begin{aligned}
\Omega_{i'} \; &:= \; \left(\frac{A_{i'}}{A_i}\right)^{\nu}\left(\frac{K_i^*}{K_{i'}^*}\right)^{1-\nu}\left(\frac{\bar{K}_{i'}}{\bar{K}_i}\right)^{1-\gamma\nu}\left(\frac{\bar{L}_{i'}}{\bar{L}_i}\right)^{\gamma\nu} \\
\kappa_{i'} \; &:= \; \frac{\bar{K}_{i'}K_i^*}{\bar{K}_i K_{i'}^*}
\end{aligned}$$

for $i, i' \in \{H, F\}$ and $i \neq i'$. As before, the last term in equation (33) captures the positive marginal utility of a higher supply of the public good. The third term captures the marginal utility loss due to the crowding out of private firms in each tradable and nontradable industry. It illustrates, that the crowding out effect is driven by the love of variety and not by the elasticity of substitution. The first term in (27) (derived under the assumption of $\rho = \frac{1-\nu}{\nu}$) combined the first two terms in (33). The first term in (33) is positive and captures the love of variety effect in open industries. The second term, which I call the income effect, is positive if the inequality $\kappa_{i'} < \Omega_{i'}$ holds which is equal to the inequality $p_{i'} < p_i$.[17] If the fundamentals of the economy are such that the price of foreign varieties is smaller than the price of domestic varieties, the income effect is positive since a relative low price of foreign varieties implies a decrease of the average price in opening industries. If foreign varieties are more expensive than domestic ones the income effect will be negative. The price differential of the two countries has an effect as long as the firms have market power, i.e. $\nu < 1$. It is obvious that if the income effect is positive for country H, it is negative for country F and vice versa.

Consider for a moment the case without love of variety, hence $\rho = 0$. The first term in (33) disappears. Openness affects government expenditure only through the income effect. Openness to trade does not necessarily affect government spending positively. If foreign prices are lower than domestic ones, there are gains from trade for the domestic country and the domestic optimal government share does increase in openness. In contrast optimal government share of the foreign country decreases in openness.

Equation (33) illustrates that if the love of variety is sufficiently large (which is definitely the case under the Dixit-Stiglitz assumption (see Section 4)) there are gains from trade for both countries and government spending is larger in open economies than in closed ones.

If the countries are symmetric, then $\kappa_i = 1$ and $\Omega_i = 1$ and the second term in the first order condition (eq. (33)) disappears. Thus, in a world of identical countries the degree of market power or elasticity of substitution has no effect on optimal public good provision. The optimal g is given by

$$g = \frac{1-\eta}{1+\eta\rho(1-\tau/2)} \tag{34}$$

The effect of openness on government expenditure plays together with ρ, the parameter for "love of variety". If individuals have no love of variety, openness won't have an effect on government spending. The higher ρ, i.e. the higher the "love of variety", the lower is optimal government spending. The effect of trade on government spending is stronger the higher the love of variety since marginal utility of open industries is higher if ρ is high: $\frac{\partial^2 g}{\partial\rho\partial\tau} > 0$. This result makes clear that the

[17]This can be seen after some rearranging:
$\kappa_{i'} < \Omega_{i'} \Leftrightarrow \frac{\bar{K}_{i'} K_i^*}{\bar{K}_i K_{i'}^*} < \left(\frac{A_{i'}}{A_i}\right)^{\nu} \left(\frac{K_i^*}{K_{i'}^*}\right)^{\nu} \left(\frac{\bar{K}_{i'}}{\bar{K}_i}\right)^{1-\gamma\nu} \left(\frac{\bar{L}_{i'}}{\bar{L}_i}\right)^{\gamma\nu} \Leftrightarrow 1 < \left(\frac{p_i}{p_{i'}}\right)^{\frac{\nu}{1-\nu}}$.

positive relation between ν and public spending derived before is driven by the love of variety and not by the elasticity of substitution.

To conclude (proof in the text)

Proposition 4. *In a symmetric equilibrium where both countries are totally identical, optimal government spending depends negatively on the parameter for love of variety. Further, optimal government spending depends not on the elasticity of substitution. Openness has a positive effect on optimal government size if consumers have a love of variety.*

5.2 $\beta_i \neq \frac{\bar{L}}{\bar{K}}$

The equilibrium derived in this paper was under the assumption that government activity does not distort the relative factor endowment available for private production. This was an important assumption in order to focus on interactions between openness on the government sector without having a distortion of the government sector on prices and supply. The only effect of an increase in government activity was due to the crowding out of private firms. The aim of this Section is to analyze the additional effects of government activity if public good production is more labor intensive or more capital intensive than the private sector. Consider equation (5). A large β_i implies that productivity of capital is large which in turn implies that employment of capital in public good production is low.

Remind that cost minimizing production implies that

$$g_{Li} = g_{Ki}\frac{\beta_i \bar{K}_i}{\bar{L}_i}$$

The share of labor endowment is increasing in the share of capital endowment. In contrast to before, β_i is allowed to differ from $\frac{\bar{L}_i}{\bar{K}_i}$ and $g_{Ki} = g_{Li}$ does no longer hold. The inequality $\beta_i < \frac{\bar{L}_i}{\bar{K}_i}$ implies that $g_{Li} < g_{Ki}$ which means that the public sector is more capital intensive than the private sector. If $\beta_i > \frac{\bar{L}_i}{\bar{K}_i}$ the reverse holds.

Production of a public good is thus given by

$$G_i = g_{Ki}\frac{\beta \bar{K}_i}{\bar{L}_i} = g_{Li}\bar{L}_i.$$

The factor proportions supplied to the private sector is $\frac{(1-g_{Ki})\bar{K}_i}{(1-g_{Li})\bar{L}_i}$ with $g_{Li} =$

$g_{Ki}\frac{\beta_i \bar{K}_i}{\bar{L}_i}$. As before, there is one public choice parameter since there is a one-to-one correspondance between g_{Ki} and g_{Li}. Having determined the share of capital endowment for public good production the share of labor endowment is determined. Therefore, in the following g_{Li} is replaced by $g_{Ki}\frac{\beta_i \bar{K}_i}{\bar{L}_i}$. This implies for the relative factor prices

$$\frac{r_i}{w_i} = \frac{1-\nu\gamma}{\nu\gamma}\frac{\bar{L}_i}{\bar{K}_i}B_i \ ,$$

where I defined $B_i := \frac{1-g_{Ki}\beta\frac{\bar{K}_i}{\bar{L}_i}}{1-g_{Ki}}$. Since the supply of a variety and labor demand of a firm depends on the relative factor prices, it is now affected by government activity.

$$x_i = \frac{\phi}{1-\nu}A_i K_i^* \left(\frac{\bar{L}_i}{\bar{K}_i}\right)^{\gamma} B_i^{\gamma}$$

and

$$L_i = \frac{1-\gamma\nu}{1-\nu}K_i^*\frac{\bar{L}_i}{\bar{K}_i}B_i \ .$$

The fact that supply and labor demand are affected by the relative factor prices and therefore by government activity depends on the assumption of the non-homothetic cost function. A firm's demand for capital is still given by (13) and the number of firms is given by equation (24) with g_i replaced by g_{Ki}. The rental and wage rates are given by

$$r_i = \phi A_i \left(\frac{\bar{L}_i}{\bar{K}_i}\right)^{\gamma} B_i^{\gamma} p_i \qquad w_i = \frac{\phi\nu\gamma}{1-\nu\gamma}A_i\left(\frac{\bar{K}_i}{\bar{L}_i}\right)^{1-\gamma} B_i^{\gamma-1}p_i$$

The price of foreign varieties depends on the public share of home and foreign:

$$p_F = \left(\frac{A_H}{A_F}\left(\frac{B_H\bar{L}_H\bar{K}_F}{B_F\bar{K}_H\bar{L}_F}\right)^{\gamma}\frac{K_H^*}{K_F^*}\right)^{1-\nu} \ .$$

Under the assumption of $\beta_i = \frac{\bar{L}_i}{\bar{K}_i}$, government activity does only influence the number of firms producing a variety in equilibrium. However, if we allow for $\beta_i \neq \frac{\bar{L}_i}{\bar{K}_i}$, there is no equilibrium variable which stays unaffected by the government share. For analyzing the effect of government activity we have to distinguish two cases: $\beta_i > \frac{\bar{L}_i}{\bar{K}_i}$ and $\beta_i < \frac{\bar{L}_i}{\bar{K}_i}$.

Case 1: $\beta_i > \frac{\bar{L}_i}{\bar{K}_i}$

Recall that if $\beta_i > \frac{\bar{L}_i}{\bar{K}_i}$, the public sector produces more labor intensive than the private sector, that is $g_{Ki} < g_{Li}$. There is proportionally less labor available for the private sector which implies a lower relative wage rate compared to the case where $\beta_i = \frac{\bar{L}_i}{\bar{K}_i}$. Consider an increase in government activity. An expansion of the public sector increases g_{Li} by more than g_{Ki} and hence the the factor proportions supplied to the private sector (given by $B_i^{-1}\frac{\bar{K}_i}{\bar{L}_i}$) increases. This implies that the rental rate decreases and the wage rate increases. Of course, the number of firms decreases with an increase in public good production.[18] The supply of each variety is decreasing in the factor proportion $B_i^{-1}\frac{\bar{K}_i}{\bar{L}_i}$ and hence decreases with an expansion of the public sector if $\beta_i > \frac{\bar{L}_i}{\bar{K}_i}$. In addition, for given public spending in country F, an expansion of the government sector in H lowers the price of foreign varieties. Hence, government spending has a terms of trade improvement if the public sector produces more labor intensive. The same holds from the perspective of the foreign country. An increase in the public sector in F for given government size in H increases p_F.

Case 2: $\beta_i < \frac{\bar{L}_i}{\bar{K}_i}$

If $\beta_i < \frac{\bar{L}_i}{\bar{K}_i}$, the public sector produces more capital intensive than the private sector: $g_{Li} < g_{Ki}$. An expansion of the public sector increases g_{Li} by less than g_{Ki} and hence the factor proportions supplied to the private sector (given by $B_i^{-1}\frac{\bar{K}_i}{\bar{L}_i}$) decreases if the government share increases. This implies that the rental rate increases and the wage rate decreases. Again, the number of firms decreases with an increase in public good production. The supply of each variety is decreasing in the factor proportion $B_i^{-1}\frac{\bar{K}_i}{\bar{L}_i}$ and hence increases with an expansion of the public sector if $\beta_i < \frac{\bar{L}_i}{\bar{K}_i}$. In contrast to the case 1 where the government sector produces more capital intensive than the private sector, an increases in the government size (given foreign government size) results in a deterioration of the terms of trade.

6 Conclusion

I set up a trade model with two heterogeneous countries, tradable and nontradable industries, endogenous product differentiation within each country and consumers with a love of variety. In this framework, the effect of openness to trade on optimal

[18] As long as the public sector employs capital for production, there is a crowding out effect of private firms.

public good provision is analyzed. The measure for openness is obtained by assuming a continuum of industries of measure one whereof an exogenous fraction is tradable. This allows to explicitly express optimal government spending - from a country's perspective - as a function of the trade openness. In addition, due to the assumption of heterogeneous countries, we are able discuss the effects of heterogeneity between countries on optimal public good provision.

Different channels how openness may affect government spending are discussed in the literature. The idea that in open economies government expansion may be achieved on expense of the welfare of other countries due to terms of trade effect is familiar. However, I show that under the assumption of endogenous firm entry and Dixit-Stiglitz preferences, the cost of public good provision may be exported due to the love of variety effect. Since a governmental expansion crowds out only domestic varieties the number of available varieties for consumption decreases by less in open economies. The costs of public good provision are lower the more industries are open. Hence, government spending depends positively on openness as long as consumers have a love of variety. Further, since the gains from trade are larger for a small and poor country, the smaller country's optimal government share is higher. The possibility of governments to export the costs of public good provision results in an equilibrium where both governments act too expansionary. An agreement between the two governments reducing government activity in both countries would lead to a higher aggregate welfare.

A Appendix

A.1 Derivation of the demand curve (eq. (4))

Maximization of household's subutility (2) with respect to y^i_{kj}, subject to the budget constraint per industry (3), yields the following first order conditions

$$\left(\frac{Y_{ij}}{y^i_{kj}}\right)^{1-\nu} = \mu_{ij} p_{kj} \quad \forall \;\; k \in \mathscr{N}_{ij} \;\wedge\; \forall \;\; j \in [0,1]$$

where μ_{ij} denotes the Lagrangian multiplier associated with the budget constraint per industry (3). It can be interpreted as marginal utility of income spent on industry j. Taking the first order conditions for any two varieties k and k' we get

$$\frac{y^i_{kj}}{y^i_{k'j}} = \left(\frac{p_{k'j}}{p_{kj}}\right)^{\frac{1}{1-\nu}} \tag{A1}$$

Consumption of variety k equals the one of variety k' if the prices of the two varieties are identical. Using equation (A1) and (2) we get for each variety the demand curve.

A.2 Derivation of the cost function (eq. (7))

A necessary condition for profit maximization of firms is to minimize the costs, $r_i K_{kj} + w_i L_{kj}$, subject to technology (6). For an interior solution ($K_{kj} > K^*_i$ and $L_{kj} > 0$), the cost minimizing problem yields the following first order conditions

$$r_i = \lambda_i (1-\gamma) A_i (K_{kj} - K^*_i)^{-\gamma} (L_{kj})^{\gamma}$$
$$w_i = \lambda_i \gamma A_i (K_{kj} - K^*_i)^{1-\gamma} (L_{kj})^{\gamma-1}$$

where λ_i is the Lagrange multiplier associated with equation (6), that is, marginal cost of output. For the MRTS follows

$$\frac{r_i}{w_i} = \frac{1-\gamma}{\gamma} \frac{(L_{kj})}{(K_{kj} - K^*_i)}.$$

Hence conditional factor demand is given by

$$L_{kj}(w_i, r_i, x_{kj}) = \frac{1}{A_i} \left(\frac{\gamma r_i}{(1-\gamma) w_i}\right)^{1-\gamma} x_{kj}$$
$$K_{kj}(w_i, r_i, x_{kj}) = K^*_i + \frac{1}{A_i} \left(\frac{(1-\gamma) w_i}{\gamma r_i}\right)^{\gamma} x_{kj}.$$

The cost function is thus given by

$$c(w_i, r_i, x_{kj}) = r_i K_i^* + \left(\frac{r_i}{1-\gamma}\right)^{1-\gamma} \left(\frac{w_i}{\gamma}\right)^{\gamma} \frac{1}{A_i} x_{kj}$$

Note that marginal cost, $\left(\frac{r_i}{1-\gamma}\right)^{1-\gamma} \left(\frac{w_i}{\gamma}\right)^{\gamma} \frac{1}{A_i}$, is equal to the Lagrange multiplier λ_i.

A.3 Derivation of the subutility in open industries

$$\begin{aligned} Y_{H,j\leq\tau} &= \left(n_{Hj}\left(y_H^H\right)^{\nu} + n_{Fj}\left(y_F^H\right)^{\nu}\right)^{\frac{1}{\nu}} \\ &= y_H^H \left(n_{Hj} + n_{Fj}\left(\frac{p_H}{p_F}\right)^{\frac{\nu}{1-\nu}}\right)^{\frac{1}{\nu}} \end{aligned}$$

Using $I_H = p_H n_H x_H = n_{Hj} p_H y_H^H + n_{Fj} p_F y_F^H$ and $y_F^H = \left(\frac{p_H}{p_F}\right)^{\frac{1}{1-\nu}} y_H^H$ implies

$$\frac{n_H x_H}{y_H^H} = n_H + n_F \left(\frac{p_H}{p_F}\right)^{\frac{\nu}{1-\nu}}.$$

The same argument holds for country F. Hence $\frac{n_F x_F}{y_H^F} = n_{Hj} + n_{Fj}\left(\frac{p_H}{p_F}\right)^{\frac{\nu}{1-\nu}}$. After substituting y_H^i, $i = H, F$, we get the subutility in open industries in expression (25).

A.4 Derivation of the first order condition (eq. (27))

We first rewrite the objective function of the government in the form:

$$\max_{g_i} \;\; \eta\tau\frac{1-\nu}{\nu}\left(\log(I_H(g_H) + I_F(g_F)) - \log I_i(g_i)\right) + \eta \log\left(n_i(g_i)\right)^{\frac{1}{\nu}} x_i + (1-\eta)\log G_i(g_i)$$

The first order condition of this problem is given by:

$$0 \;=\; \eta\tau\frac{1-\nu}{\nu}\left(\frac{\partial \log(I_H + I_F)}{\partial g_i} - \frac{\partial \log I_i}{\partial g_i}\right) + \frac{\eta}{\nu}\frac{\partial n_i}{\partial g_i} + (1-\eta)\frac{\partial \log G_i}{\partial g_i}$$

This yields equation (27). After rearranging terms the following quadratic equation results:

$$
\begin{aligned}
0 \; &= g_i^2(\nu(1-\eta)+\eta) \\
&\quad + g_i((\eta\tau(1-\nu)-\eta)(1-g_{i'})\Omega_{i'} - \eta - (1-\eta)\nu - \nu(1-\eta)(1+(1-g_{i'})\Omega_{i'})) \\
&\quad + \nu(1-\eta)(1+(1-g_{i'})\Omega_{i'}) \\
&=: g_i^2 a + g_i b_{i'} + c_{i'}
\end{aligned}
\tag{A2}
$$

A.5 Proof of existence and uniqueness of equilibrium

Equation (A2) has two solutions:

$$
g_{1i}(g_{i'}) = \frac{-b_{i'} + \sqrt{b_{i'}^2 - 4ac_{i'}}}{2a} \quad \text{and} \quad g_{2i}(g_{i'}) = \frac{-b_{i'} - \sqrt{b_{i'}^2 - 4ac_{i'}}}{2a}
$$

where $g_{1i}(g_{i'}) > g_{2i}(g_{i'})$. The two solutions at $g_{i'} = 1$ are $g_{1i}(g_{i'} = 1) = 1$ and $g_{2i}(g_{i'} = 1) = \frac{\nu(1-\eta)}{\nu(1-\eta)+\eta} < 1$. While the second order condition at $g_i = 1$ and $g_{i'} = 1$ is positive, it is negative at $g_i = \frac{\nu(1-\eta)}{\nu(1-\eta)+\eta}$ and $g_{i'} = 1$. Hence, the solution g_{2i} corresponds to the maximum. In proposition 1 it is shown that $\frac{dg_i}{dg_{i'}} > 0$ which implies that the solution $g_{1i} > 1$ for $g_{i'} \in [0, 1)$. Proposition 1 implies also that $g_{2i} > \frac{\nu(1-\eta)}{\nu(1-\eta)+\eta}$ for $g_{i'} \in [0, 1)$. Therefore, it remains to show that $g_{2i} < 1$ at $g_{i'} = 0$. The proof follows by contradiction.

Suppose that $g_{2i} \geq 1$ at $g_{i'} = 0$, then $g_{2i}(g_{i'}) = 1$ for $g_{i'} \in [0, 1)$ because of $g_{2i}(g_{i'} = 1) = \frac{\nu(1-\eta)}{\nu(1-\eta)+\eta} < 1$ and the result of proposition 1. However, solving $g_{2i}(g_{i'}) = 1$ for $g_{i'}$ yields $g_{i'} = 1$, which is a contradiction.

References

Anderson, T. M., Rasmussen, B. S., and Sorensen, J. R. (1996). Optimal fiscal policy in open economies with labor market distortions. *Journal of Political Economics*, 63:103–117.

Anwar, S. (2001). Government spending on public infrastructure, prices, production and international trade. *The Quarterly Review of Economics and Finance*, 41:19–31.

Armington, P. S. (1969). A Theory of Demand for Products Distinguished by Place of Production. *IMF Staff Papers*, 16:159–176.

Benassy, J.-P. (1998). Is there always too little research in endogenous growth with expanding product variety? *European Economic Review*, 42(1):61–69.

Blanchard, O. and Giavazzi, F. (2003). Macroeconomic effects of regulation and deregulation in goods and labor markets. *The Quarterly Journal of Economics*, 118(3):879–907.

Cameron, D. R. (1978). The Expansion of the Public Economy: A Comparative Analysis. *The American Political Science Review*, 72(4):1243–1261.

Devereux, M. B. (1991). The terms of trade and the international coordination of fiscal policy. *Economic Inquiry*, 29(4):720–736.

Dixit, A. K. and Stiglitz, J. E. (1977). Monopolistic Competition and Optimum Product Diversity. *The American Economic Review*, 67(3):297–308.

Egger, H. and Falkinger, J. (2006). The role of public infrastructure and subsidies for firm location and international outsourcing. *European Economic Review*, 50(8):1993–2015.

Epifani, P. and Gancia, G. (2007). On Globalization and the Growth of Governments. Discussion Paper No. 6065, Centre of Economic Policy Research.

Evenett, S. J. and Keller, W. (2002). On theories explaining the success of the gravity equation. *Journal of Political Economy*, 110:281–316.

Feenstra, R. C., Markusen, J. R., and Rose, A. K. (2001). Using the gravity equation to differentiate among alternative theories of trade. *Canadian Journal of Economics*, 32(2):430–447.

Garrett, G. (2001). Globalization and Government Spending around the World. *Studies in Comparative International Development*, 35(4):3–29.

Haufler, A. (2001). *Taxation in a Global Economy.* Cambridge University Press, Cambridge UK.

Helpman, E. and Krugman, P. R. (1985). *Market structure and foreign trade: increasing returns, imperfect competition, and the international economy.* MIT Press, Cambridge, Massachusetts.

Krugman, P. (1980). Scale Economies, Product Differentiation, and the Pattern of Trade. *The American Economic Review*, 70(5):950–959.

Montagna, C. (2001). Efficiency gaps, love of variety and international trade. *Economica*, 68:27–44.

Rodrik, D. (1998). Why Do More Open Economies Have Bigger Governments. *Journal of Political Economy*, 106(5):997–1032.

Schott (2004). Across-product versus within-product specialization in international trade. *Quarterly Journal of Economics*, 119(2):646–677.