Globalization and Multiproduct Firms*

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Abstract

We present an international trade model of multiproduct firms that focuses on a new source of firm heterogeneity: organizational capability. All firms have a limited span of control, but the resulting trade off between firm scope and marginal cost is less severe for firms with greater organizational capability. The model generates a negative relationship between firm size and market-to-book ratio. Multilateral trade liberalization (globalization) induces a merger wave and results in a reduction of average industry production costs and in a flattening of the size distribution of firms.

Keywords: multiproduct firms, firm size distribution, trade liberalization, size discount, firm heterogeneity, merger wave, organizational capability

JEL Classification: F12, F15

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1 Introduction

With the increasing availability of huge firm-level datasets, economists have documented enormous variation across firms along many dimensions, such as productivity, scope (number of products), technology, and managerial practices. Much of this interest, particularly within the field of international trade, has been driven by the model of Melitz (2003), which posits that firms vary in their inherent productivity. The key insight of the Melitz-model is that when firms vary in their productivity, they sort into serving foreign markets with the implication that trade liberalization induces within-industry selection effects: trade liberalization leads to a reallocation of resources from less productive to more productive firms. More recently, the Melitz-model has been adapted so as to incorporate multiproduct firms (e.g., Bernard, Redding and Schott, 2010b; Melitz, Mayer and Ottaviano, 2010). In these models, trade liberalization can cause within-firm selection effects that are akin to the within-industry selection effects that appear in the original Melitz model.

While these selection-driven models have yielded many important insights into the allocation of resources across firms and the productivity effects of trade liberalization, they are remarkably narrow in the sources of firm heterogeneity that they allow, i.e., random differences in marginal costs. Given the abundance of research involving models that focus on this one particular source of firm heterogeneity, it seems important to expand the scope of the analysis to other dimensions along which firms differ.

In this paper, we present a model of firm heterogeneity in which firms differ in their ability to manage many different products. We explore the implications of firm heterogeneity in the "span of control," where the relevant span is not defined on firm size per se but rather on the number of products that the firm produces. Accordingly, the more products a firm chooses to manage, the greater is the burden on management, and so the higher are the marginal costs for all products. To focus on firm heterogeneity in organizational capability relating to the span of control, we posit that all firms are equally efficient when constrained to produce a single product, but the rate at which the marginal cost of production increases with firm scope is lower for firms with greater organizational capability.

Our focus on the span of control thus defined is natural given the huge variation in the number of products offered by different firms (Bernard, Redding and Schott, 2010b) and the constant churn in the product lines across firms (Bernard, Redding and Schott, 2010a). We argue that the concept of span of control, which has a long tradition in economics (Lucas, 1978), is most naturally interpreted as involving the increased burden to firm-level organizational capabilities posed by a large range of products. Indeed, according to the "resource-based view of the firm" in the management literature, greater diversification reflects an abundance of organizational capabilities that is optimally directed towards increasing firm scope (Matsusaka, 2001; Collis and Montgomery, 2005).

We embed a simple parametric version of our mechanism into a model of monopolistic competition à la Asplund and Nocke (2006) and Melitz and Ottaviano (2008), and obtain a variety of insights into the organization of industries, and the effects of trade shocks onto this organization. We discuss three implications of the model that provide an interesting contrast with pure-selection driven models in the spirit of Melitz (2003). First, in our model, firms with greater organizational capability choose to manage a larger number of products, so much larger in fact that they end up having higher marginal costs even though they are intrinsically more efficient. The model therefore predicts a negative relationship between market-to-book value (Tobin's Q) and firm size. Second, in our model, trade liberalization induces a merger wave as it has an asymmetric effect on firms' incentives to adjust their scope. Following a reduction in trade costs, some firms have an incentive to reduce their product range while other firms have an incentive to expand their scope, thereby affecting the firm size distribution in a systematic way. Perhaps surprisingly, a multilateral trade liberalization induces a flattening of the distribution of domestic sales. Third, our results suggest that some of the effects of the observed productivity effects of trade liberalization may be due to the reallocation of products across firms. While our paper gives rise to some unique empirical predictions, the main contribution of the paper does not consist in hitting any particular empirical fact about the structure of production or the response to trade liberalization, but rather in exploring the effects of a different dimension of firm heterogeneity.

Our paper is closely related to two strands in the theoretical trade literature. First, it contributes to the recent and growing literature that is concerned with the within-industry reallocation effects of trade liberalization (Melitz, 2003; Melitz and Ottaviano, 2008). Second, and more specifically, our paper contributes to the nascent literature that is concerned with multiproduct firms in international trade (Bernard, Redding and Schott, 2010b; Eckel and Neary, 2010; Mayer, Melitz and Ottaviano, 2010). In all of these papers, firms essentially draw a distribution of marginal costs for various products of different degrees of substitutability so that the marginal cost of any given product is exogenous. Instead, we explore a rather different mechanism, namely one where a firm's marginal cost of production for any given product depends on the firm's choice of scope, and firms differ in the extent of this relationship. In our model, and in contrast to Eckel and Neary (2010), there are no cannibalization effects on the demand side. Instead, extending a firm's range of products induces higher marginal costs for the firm's existing products by diverting managerial attention due to the limited span of control. Another important difference to Eckel and Neary (2010) is that firms are heterogeneous. As emphasized above, the model differs from all of the other trade models with heterogeneous firms in that it focuses on a novel dimension of heterogeneity – organizational capability.

The plan of the paper is as follows. In the next section, we present our model in a closed-economy setting. There, we analyze how firms' choice of scope is affected by firms' organizational capability, and derive predictions on the relationship between firm size on the one hand and marginal costs and the market-to-book ratio on the other. In Section 3, we embed the model in a two-country international trade setting. There, we explore the effects of trade shocks on the industrial organization of production and the induced size distribution of firms. We conclude in Section 4.

2 The Closed Economy

In this section, we consider a closed economy where monopolistically competitive firms differ exclusively in their organizational capabilities and choose how many products to manage. To sharpen our focus on firm heterogeneity in organizational capabilities, we treat products as perfectly symmetric. We then analyze how, in equilibrium, firms with different organizational capabilities solve the fundamental trade off between firm scope and marginal costs.

2.1 The Model

We consider a closed economy with a mass L of identical consumers with the following linear-quadratic utility function:

$$U = \int x(k)dk - \int [x(k)]^2 \, dk - 2\sigma \left[\int x(k)dk \right]^2 + H, \tag{1}$$

where $x(k)$ is consumption of product k in the differentiated goods industry, H is consumption of the outside good, and $\sigma > 0$ is a parameter that measures the degree of product differentiation. Assuming that consumer income is sufficiently large, each consumer's inverse demand for product k is then given by

$$p(k) = 1 - 2x(k) - 4\sigma \int x(l)dl. \tag{2}$$

The outside goods industry is perfectly competitive and uses a constant returns to scale technology. In the differentiated goods industry, there is a mass M of atomless firms that differ in their organizational capabilities.[1] A firm's organizational capability is denoted by $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$, where $\overline{\theta} > \underline{\theta} > 0$, and the distribution of organizational capabilities in the population of firms is given by the distribution function G. Each firm can manage any number $n \geq 1$ of products.

[1] In this section, we remain agnostic about the determinants of M. In the next section, we will consider a long-run equilibrium in which the mass of firms is endogenous.

Each product is of measure zero and so is each firm. We assume that firms have constant marginal costs at the product level but decreasing returns to the span of control at the firm level: the more products a firm manages, the higher are its marginal costs for each product.

The firm faces two types of costs. First, there is a fixed cost r per product. This cost can be thought of as the market price of the property rights over an existing product. Second, there is a constant marginal cost $c(n;\theta)$ associated with the production of each unit of output, which is the same for all n products. (For simplicity, we will ignore integer constraints on n.) This marginal cost function has the following properties. First, an increase in the number of products increases a firm's marginal cost. This induced increase in the firm's marginal cost may be due to a reduction in productivity or due to a need to pay higher wages as a result of diminished managerial attention. The induced higher wage may be due to a variety of reasons, e.g., because of the need to hire better (and therefore more costly) workers or because the additional burden on management makes monitoring workers less effective.[2] Second, we assume that the trade off between firm scope and marginal cost is independent of scope: there is a constant elasticity of marginal cost with respect to the number of product lines. Third, we abstract from exogenous cost differences amongst single-product firms and focus instead on the idea that organizational capability is about how good firms are at coordinating the production of multiple products. We therefore assume that the marginal cost of producing a single product is independent of organizational capability and that the elasticity of marginal costs with respect to the number of products is smaller for firms with greater organizational capability. In sum, we impose the following simple functional form:

$$c(n;\theta) = c_0 n^{1/\theta}. \tag{3}$$

Each firm's optimization problem consists in choosing the number n of products and the quantity (output) q_k of product $k \in [1, n]$ so as to maximize its profit. (Since firms are monopolistically competitive, each firm could equivalently choose price p_k rather than quantity.)

2.2 Equilibrium Analysis

As we show in the Appendix, each firm in the differentiated goods industry faces – in equilibrium – a linear residual demand curve for each one of its products:

$$D(p) = \frac{L}{2}(a - p),$$

[2] Schoar (2002) empirically finds that those firms who add new products experience a reduction in total factor productivity of existing products. She also finds some evidence that firms with a greater product range pay higher wages.

where p is price and a the endogenous demand intercept, and so inverse demand is

$$P(q) = a - \frac{2q}{L},$$

where q denotes output.

Consider a firm with organizational capability θ. We first analyze the firm's quantity-setting problem, holding fixed the number n of products. Recall that each firm is of measure zero (which means that each firm takes the demand intercept a as given), and so there are no cannibalization effects. Symmetry of demand and costs then implies that the firm will optimally sell the same quantity of each product. The profit-maximizing output per product of a firm with marginal cost $c(n;\theta)$ is then given by

$$\begin{aligned} q(c(n;\theta)) &\equiv \arg\max_q \left[P(q) - c(n;\theta)\right] q \\ &= \frac{L}{4}(a - c(n;\theta)), \end{aligned}$$

and gross profit *per product* is

$$\begin{aligned} \pi(c(n;\theta)) &\equiv \left[P(q(c(n;\theta))) - c(n;\theta)\right] q(c(n;\theta)) \\ &= \frac{L}{8}(a - c(n;\theta))^2. \end{aligned} \tag{4}$$

In the Appendix, we derive the equilibrium value of the demand intercept:

$$a = \frac{1 + \sigma M \int \left[n(\theta) c(n(\theta);\theta)\right] dG(\theta)}{1 + \sigma N},$$

where $n(\theta)$ is the number of products offered by a firm with organizational capability θ, and N is the aggregate number of products offered by firms in the economy,

$$N = M \int_{\underline{\theta}}^{\overline{\theta}} n(\theta) dG(\theta).$$

We consider now the firm's optimal choice of the number of products. Given the optimal output policy, a firm of organizational capability θ chooses n so as to maximize its net profit over all products, and so the solution is given by:

$$n(\theta) \equiv \arg\max_n n \left[\pi(c(n;\theta)) - r\right].$$

From the envelope theorem, $\pi'(c(n(\theta);\theta)) = -q(c(n(\theta);\theta))$, and so the first-order condition for $n(\theta)$ can be written as

$$\left[\pi(c(n(\theta);\theta)) - r\right] - n(\theta) q(c(n(\theta);\theta)) \frac{\partial c(n(\theta);\theta)}{\partial n} = 0. \tag{5}$$

The impact of an additional product on the firm's profit can be decomposed into two effects. The first term on the l.h.s. of equation (5) is the net profit of the marginal product. The second term summarizes the negative effect that the marginal product imposes on the $n(\theta)$ inframarginal products: the production cost of each product increases by $q(c(n(\theta);\theta))\partial c(n(\theta);\theta)/\partial n$ because of the additional burden on management. We will henceforth refer to this second term – which is the consequence of the firm's loss of focus – as the "inframarginal cost effect".

From the cost function (3), $n(\theta)\partial c(n(\theta);\theta)/\partial n = (1/\theta)c(n(\theta);\theta)$. Hence, the optimal choice of the number of products, $n(\theta)$, enters the first-order condition (5) only through the induced marginal cost $c(n(\theta);\theta)$. This means that the firm's problem can equivalently be viewed as one of choosing c rather than n. Indeed, using the gross profit function (4), the first-order condition can be rewritten as

$$\Psi(c(\theta);\theta) \equiv [P(q(c(\theta))) - c(\theta)]\, q(c(\theta)) - r - \frac{c(\theta)}{\theta} q(c(\theta)) = 0, \tag{6}$$

where $c(\theta) \equiv c(n(\theta);\theta)$.

Henceforth, we will assume that the fixed cost r is not too large so that the firm can make a strictly positive profit by managing a single product, i.e.,

$$\pi(c_0) = [P(q(c_0)) - c_0]\, q(c_0) > r. \tag{7}$$

We now state our central result on the relationship between a firm's organizational capability and its marginal cost.

Proposition 1 *The optimal choice of products is such that the induced marginal cost $c(\theta)$ is increasing in the firm's organizational capability θ. Specifically, there exists a cutoff $\widetilde{\theta}$ such that $c(\theta) = c_0$ for all $\theta \leq \widetilde{\theta}$, and $c(\theta)$ is strictly increasing in θ for all $\theta \geq \widetilde{\theta}$.*

Proof. See Appendix. ∎

For a given number of products n, the inframarginal cost effect that the marginal product exerts is the smaller, the greater is the firm's organizational capability. Not surprisingly then, firms with greater organizational capability will optimally choose a larger number of products than firms with inferior organizational capability: $n(\theta) = 1$ for $\theta \leq \widetilde{\theta}$, and $n(\theta)$ is strictly increasing in θ for $\theta \geq \widetilde{\theta}$. Perhaps surprisingly, however, for $\theta \geq \widetilde{\theta}$, $n(\theta)$ is increasing so fast with θ that firms with greater organizational capability will, in fact, exhibit higher unit costs. To see this, consider two firms, firm 1 and firm 2, with organizational capability $\theta_1 \geq \widetilde{\theta}$ and $\theta_2 > \theta_1$, respectively. From the first-order condition (6), firm 1 will optimally choose $n(\theta_1)$ such that its marginal cost $c(\theta_1)$ satisfies $\Psi(c(\theta_1);\theta_1) = 0$. Suppose now firm 2 were to choose the number of products such that its induced marginal cost is also equal to $c(\theta_1)$. If so, the two firms

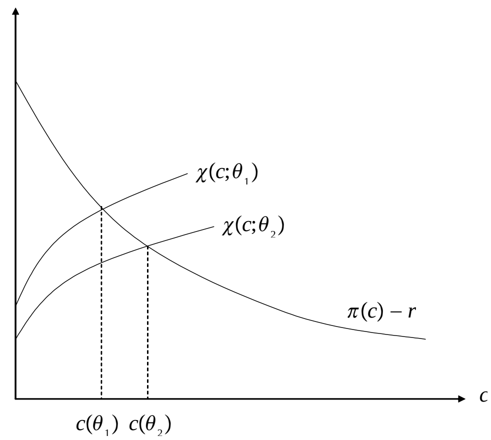

Figure 1: The induced choice of marginal cost balances the net profit per product line, $\pi(c)-r$, and the inframarginal cost effect, $\chi(c;\theta)$. A firm with greater organizational capability, $\theta_2 > \theta_1$, chooses to have higher marginal costs, $c(\theta_2) > c(\theta_1)$.

would sell the same quantity $q(c(\theta_1))$ per product, and thus fetch the same price $P(q(c(\theta_1)))$. Hence, the net profit of the marginal product, $[P(q(c(\theta))) - c(\theta)]\, q(c(\theta)) - r$, would be the same for the two firms. However, as can be seen from equation (6), the absolute value of the inframarginal cost effect imposed by the marginal product, $\chi(c(\theta);\theta) \equiv (1/\theta)c(\theta)q(c(\theta))$, is smaller for the firm with the greater organizational capability, and so $\Psi(c(\theta_1);\theta_2) > 0$. Hence, firm 2 can increase its profit by further adding products, even though this implies higher unit costs, $c(\theta_2) > c(\theta_1)$. This is illustrated graphically in Figure 1.

Remark 1 *For convenience, we have chosen a particular functional form for marginal cost $c(n;\theta)$ that permits a simple interpretation of organizational capability θ as the inverse of the (constant) elasticity of marginal cost with respect to the number of products. But Proposition 1 holds more generally. Let*

$$\varepsilon(n;\theta) \equiv \frac{\partial c(n;\theta)}{\partial n}\frac{n}{c(n;\theta)}$$

denote the elasticity of marginal cost with respect to n. It can be shown that Proposition 1 holds if (i) $c(n;\theta)$ is strictly increasing in n, and decreasing in θ; and (ii) $\varepsilon(n;\theta)$ is strictly *decreasing in* θ *and not increasing at too fast a rate with* n*:*

$$\frac{\partial \varepsilon(n;\theta)}{\partial n}\left(\frac{-\frac{\partial c(n;\theta)}{\partial \theta}}{\frac{\partial c(n;\theta)}{\partial n}}\right)+\frac{\partial \varepsilon(n;\theta)}{\partial \theta}<0 \text{ for all } n \geq 1 \text{ and } \theta \in [\underline{\theta},\overline{\theta}].$$

Proposition 1 shows that a firm's unit cost is *inversely* related to its intrinsic efficiency (its organizational capability θ). In practice, it is often hard to measure costs correctly.[3] Since we do not take a stance on what causes marginal costs to rise with firm scope, we do not make any claim about the correlation between marginal costs and productivity, as conventionally measured. However, our model does make clear-cut predictions on an alternative summary measure of firm efficiency, which is frequently invoked in the corporate finance literature: a firm's market-to-book ratio, or Tobin's Q. This ratio is given by

$$T(\theta) \equiv \frac{m(\theta)}{b(\theta)},$$

where $m(\theta)$ is the market value of the firm (including its assets) and $b(\theta)$ the book value of the assets used by the firm (independently of whether the assets are rented or owned). The firm's assets are the products that it manages as well as any capital it uses for production. As we now show, our model predicts that more intrinsically efficient firms will have lower market-to-book ratios.

Suppose each firm has a Cobb-Douglas production function and α is the capital share in production costs. Then, the firm's book value is given by

$$b(\theta) = n(\theta)r + n(\theta)\alpha c(\theta)q(c(\theta)),$$

where the first term is the book value of the property rights over the $n(\theta)$ products and the second term the book value of the capital used for production. The market value of the firm (and its assets) is given by

$$m(\theta) = n(\theta)P(q(c(\theta)))q(c(\theta)) - n(\theta)(1-\alpha)c(\theta)q(c(\theta)),$$

where the first term is revenue and the second term labor costs. The next lemma shows that the market-to-book ratio is negatively related to a firm's intrinsic efficiency.

[3] In the presence of diseconomies of scope measuring a firm's intrinsic efficiency is not straightforward because a reduction in scope will lower marginal cost. In our model, even if unit costs were observable the mapping from marginal cost to intrinsic efficiency requires the following correction for the number of products:

$$\theta = \frac{\ln(n)}{\ln\left(\frac{c}{c_0}\right)}.$$

Lemma 1 *A firm's market-to-book ratio (Tobin's* Q*), $T(\theta)$, is decreasing in the firm's organizational capability θ.*

Proof. See Appendix. ■

Tobin's Q in a sense accurately reflects variation across firms in their marginal costs. Because more intrinsically efficient firms have higher marginal costs in our model, they have lower market-to-book ratios. More intrinsically efficient firms are in fact larger, however. Let

$$S(\theta) \equiv n(\theta)q(c(\theta))P(q(c(\theta)))$$

denote the sales of a firm with organizational capability θ.

Lemma 2 *A firm's sales $S(\theta)$, book value $b(\theta)$, and market value $m(\theta)$ are increasing in the firm's organizational capability θ.*

Proof. See Appendix. ■

Lemma 1 establishes a negative relationship between the market-to-book ratio (Tobin's Q) and organizational capability, while Lemma 2 establishes a positive relationship between firm size and organizational capability. Our model thus predicts that, in a cross section of firms, we should find a negative correlation between market-to-book ratio and firm size:

Proposition 2 *A firm's market-to-book ratio (Tobin's* Q*), $T(\theta)$, is inversely related to various measures of firm size: sales $S(\theta)$, book value $b(\theta)$, and market value $m(\theta)$.*

Proof. This follows immediately from Lemmas 1 and 2. ■

Propositions 1 and 2 summarize the implications of heterogeneity in the ability of firms to manage multiple product lines for the allocation of resources across firms. There are important similarities and differences in these implications relative to standard models where firms differ in their (exogenously given) constant marginal cost of production, e.g., Asplund and Nocke (2006) and Melitz and Ottaviano (2008). For instance, in both types of models intrinsically more efficient firms are larger and generate higher absolute profits than intrinsically less efficient firms. The source of the variation in firm size and profit is very different in the two types of models, however. In standard models of firm heterogeneity, such as Melitz and Ottaviano (2008), intrinsically more efficient firms have lower marginal costs and so can sell profitably more units of the same product. In multi-product versions of the standard model, such as Bernard, Redding, and Schott (2010), larger firms have on average lower marginal costs across the spectrum of products and so sell a larger range of products and sell on average more of each product. In our model, intrinsically efficient firms expand so far along the extensive margin (number of product lines) that they end up being larger despite actually being smaller on the intensive margin, i.e., they have fewer sales at the product level ($P(q(c(\theta)))q(c(\theta)) = S(\theta)/n(\theta)$ is decreasing in θ) as they end up with higher marginal costs.[4]

Differences across firms in their marginal costs in both types of models should show up as variation in Tobin's Q: lower marginal cost firms have higher values of Tobin's Q than higher marginal cost firms. Hence, standard models of marginal cost heterogeneity predict a positive relationship between Tobin's Q and firm size, while our model makes the opposite prediction. Interestingly, the raw correlation between firm size and Tobin's Q has been shown to be negative (Lang and Stulz 1994; Eeckhout and Jovanovic, 2002). In the corporate finance literature, this stylized fact is known as the "size discount puzzle" or "diversification discount puzzle" as it is commonly interpreted as saying that larger or more diversified firms trade at a discount compared to smaller, less diversified firms.[5]

Our focus on a new dimension of firm heterogeneity – one that is orthogonal to the dimension considered in the existing literature – provides interesting insights that complement those of the literature. In the presence of heterogeneity in the strength of diseconomies of scope there is a force that pushes intrinsically more efficient firms to expand the number of product lines even if this expansion raises the marginal cost and lowers the profitability of individual products. An exciting avenue for future work therefore consists in building richer models of firm heterogeneity – combining various dimensions of firm heterogeneity – that can help explain a wider range of the empirical findings.

Remark 2 *While we have assumed that consumers have linear-quadratic preferences, resulting in linear demand, our results hold more generally. In our earlier working paper, Nocke and Yeaple (2006), we show that Propositions 1 and 2 continue to hold as long as the residual inverse demand for each product, $P(q)$, is not too convex: $P'(q) + qP''(q) < 0$ for all q such that $P(q) > 0$.*

[4] Bernard, Redding, and Schott (2010b) provide evidence that larger firms expand along both extensive and intensive margins. Empirically testing the prediction on the relationship between firm size and sales per product is fraught with difficulties that stem from finding data at the appropriate level of aggregation, however. For instance, divisional sales data (which is more readily available than product sales data) aggregates over many products and our model is silent about the relationship between organizational capability and sales at some intermediate level of aggregation.

[5] While our model might help explain the size-discount puzzle, it is not unique in that regard: several explanations of the size-discount puzzle have been proposed in the corporate finance literature. For instance, Rajan, Servaes, and Zingales (2000) provide an explanation based on agency costs that result in the misallocation of resources across divisions. Maksimovic and Phillips (2002) argue that the size-discount puzzle can better be explained by comparative advantage across sectors. There are also some who argue that the discount may in fact be a statistical artifact of selection (see, for instance, Villalonga, 2004).

3 The Open Economy

In this section, we turn to the effects of trade liberalization and market integration on firm scope and the size distribution of firms. To this end, we extend the model of the previous section to a two-country setting with trade costs. We are concerned with the effects of globalization both in the short run, where the number of firms and the aggregate number of products are fixed, and the long run, where the number of firms and the aggregate number of products are endogenous.

3.1 The Model and Preliminary Analysis

There are two countries, country 1 and country 2. In each country, there is a mass L of identical consumers with linear-quadratic utility, as given by (1). As before, we assume that the outside goods industry is perfectly competitive and faces constant returns to scale. Further, the outside good is freely traded and is produced in both countries. Consequently, the wage rate is the same in both countries and is independent of the equilibrium in the differentiated goods sector.

Let us now turn to the differentiated goods industry. In country $i = 1, 2$, there is a mass M_i of firms. In both countries, the distribution of organizational capabilities in the population of firms is given by the distribution function G with support $\left[\underline{\theta}, \overline{\theta}\right]$, $\overline{\theta} > \underline{\theta} > 0$. Firms can sell in both countries but can produce only in their home country; there are no multinational firms. The cost function of a firm with organizational capability θ is again given by (3). In addition to production costs, firms need to incur a specific tariff (or transport cost). This transport cost or tariff is indexed by a country pair (i, j): t_{ij} is the transport cost or tariff per unit of output from country i to country j. Transport costs and tariffs have to be incurred only for exports from one country to the other, and so $t_{11} = t_{22} = 0$, $t_{12} \geq 0$, and $t_{21} \geq 0$.

We assume that each firm can segment the two markets. Since each product is of measure zero, a firm's choice of output for one product does not affect its choice of output for another product. Residual inverse demand for each product in country j is

$$P_j(q) = a_j - \frac{2q}{L},$$

where a_j is the endogenous demand intercept in country j. The profit-maximizing output in country j of a firm from country i with organizational capability θ and n products is given by

$$\begin{aligned} q_j(c(n;\theta) + t_{ij}) &\equiv \arg\max_q \left[\left(a_j - \frac{2q}{L} \right) - (c(n;\theta) + t_{ij}) \right] q \\ &= \frac{L}{4}(a_j - t_{ij} - c(n;\theta)), \; i, j = 1, 2, \end{aligned}$$

where $t_{ij} = 0$ if $i = j$. The resulting gross profit per product from sales in country j is

$$\pi_j(c(n;\theta) + t_{ij}) = \frac{L}{8}(a_j - t_{ij} - c(n;\theta))^2,\ i, j = 1, 2.$$

We will focus on the case where the tariffs imposed by the two countries are initially the same, $t_{12} = t_{21} = t$, so that the demand intercepts are also the same, $a_1 = a_2 = a$. If t is sufficiently small, as we will henceforth assume, then each firm will find it optimal to sell in both countries.[6]

A firm from country i with organizational capability θ chooses the number $n_i(\theta)$ of products so as to maximize its total net profit:

$$n_i(\theta) \equiv \arg\max_n n\left[\pi_i(c(n;\theta)) + \pi_j(c(n;\theta) + t_{ij}) - r_i\right],\ j \neq i,$$

where r_i is the market price of the property rights over a product from the firm's home country i. (In principle, r_1 and r_2 may differ because we do not allow for multinational companies.) The associated first-order condition can be written as

$$\begin{aligned} \Omega^i(c_i(\theta);\theta;t_{12},t_{21}) &\equiv \{\pi_i(c_i(\theta)) + \pi_j(c_i(\theta) + t_{ij}) - r_i\} \\ &\quad - \frac{c_i(\theta)}{\theta}\{q_i(c_i(\theta)) + q_j(c_i(\theta) + t_{ij})\} \\ &= 0, \end{aligned} \tag{8}$$

where $c_i(\theta) = c_0\left[n_i(\theta)\right]^{1/\theta}$ is the implicit choice of marginal cost by a firm with organizational capability θ based in country i. For convenience, we assume that the domain of organizational capabilities, $\left[\underline{\theta}, \overline{\theta}\right]$, is such that this first-order condition determines the optimal choice of $c_i(\theta)$ for all $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$. Applying the implicit function theorem to (8), we obtain

$$\frac{dc_i(\theta)}{d\theta} = -\frac{\partial\Omega^i(c_i(\theta);\theta;t_{12},t_{21})/\partial\theta}{\partial\Omega^i(c_i(\theta);\theta;t_{12},t_{21})/\partial c_i}. \tag{9}$$

Since $\partial\Omega^i(c_i(\theta);\theta;t_{12},t_{21})/\partial c_i < 0$ (as $c_i(\theta)$ maximizes the firm's profit) and

$$\frac{\partial\Omega^i(c_i(\theta);\theta;t_{12},t_{21})}{\partial\theta} = \frac{c_i(\theta)}{\theta^2}\left[q_i(c_i(\theta)) + q_j(c_i(\theta) + t_{ij})\right] > 0,$$

we obtain that $c_i'(\theta) > 0$. That is, Proposition 1 carries over to the two-country setting: firms with greater organizational capability choose to have higher marginal costs.

Let N_i denote the mass of products managed by firms from country i. The endogenous demand intercept in country i can then be written as

$$a_i = \frac{1 + \sigma\int\{M_i n_i(\theta)c_i(\theta) + M_j n_j(\theta)\left[c_j(\theta) + t_{ji}\right]\}dG(\theta)}{1 + \sigma(N_1 + N_2)},\ i \neq j,\ i = 1, 2. \tag{10}$$

[6] A sufficient condition is that $t < 2a/(2 + \overline{\theta})$.

In equation (10), firms from the home country i enter differently than firms from the foreign country j because the latter have to incur transport costs or tariffs. Aggregating the endogenous numbers of products over all M_i firms from country i yields the mass N_i of products managed by these firms:

$$N_i = M_i \int_{\underline{\theta}}^{\overline{\theta}} n_i(\theta) dG(\theta),\ i = 1, 2. \tag{11}$$

A change in trade costs will lead to different responses across firms in their choice of the number of products, and these different responses will alter the distribution of induced marginal costs and, hence, the endogenous demand intercept a. The following lemma shows how a and average marginal costs change when high-θ firms divest products while low-θ firms increase the number of their products.

Lemma 3 *Suppose there exist marginal types $\widehat{\theta}_1$ and $\widehat{\theta}_2$ such that all firms in country $i \in \{1, 2\}$ with organizational capability $\theta > \widehat{\theta}_i$ divest products, $\Delta n_i(\theta) < 0$ for $\theta > \widehat{\theta}_i$, while all other firms in country i add products, $\Delta n_i(\theta) > 0$ for $\theta < \widehat{\theta}_i$, holding the total mass of products in each country i fixed, $\int \Delta n_i(\theta) dG(\theta) = 0$. Then, the (weighted by the number of products) average marginal costs of firms producing in country i strictly decreases:*

$$\int \frac{d}{dn} \left[n c_i(n; \theta) \right] \bigg|_{n = n_i(\theta)} \Delta n_i(\theta) dG(\theta) < 0.$$

Hence, the endogenous demand intercept a_i strictly decreases, $\Delta a_i < 0$.

Proof. See Appendix. ■

We now turn to the short-run and long-run effects of trade liberalization and market integration. These correspond to two different ways of closing the model: in the short-run equilibrium, the mass of firms and products (M_i and N_i) is fixed, but the market price r_i is endogenous, while in the long-run equilibrium, both M_i and N_i are endogenous, but the market price r_i is effectively given by the fixed cost of inventing a new product.

3.2 Comparative Statics: Short-Run Effects of Globalization

In short-run equilibrium, the mass of firms producing in country i, M_i, is fixed, as is the mass of products managed by these firms, N_i. However, following a trade shock, property rights over the products can be traded in a perfectly competitive market for corporate assets. Trade in products (and their corresponding divisions) corresponds to partial firm acquisitions and divestitures, which are more than half of all M&A activity in the U.S. (Maksimovic and Phillips, 2001). In our model, the location of production of a product is assumed to be fixed in the short run (and there is no foreign direct investment), so that the endogenous (short-run) market price of a product, r_i, may differ across countries. We define a *short-run equilibrium* as a collection $\{c_i(\cdot), n_i(\cdot), a_i, r_i\}_{i=1}^2$ satisfying the cost equation (3), the first-order condition for the optimal choice of the number of products, (8), the equation for the endogenous demand intercept, (10), and the merger market condition (11).

We now analyze the short-run effects of multilateral and unilateral tariff changes on firm scope. For this purpose, we assume that, prior to the change in tariffs, the two countries are identical: $N_1 = N_2 = N$, $M_1 = M_2 = M$, and $t_{12} = t_{21} = t$. We first consider a small symmetric reduction in the common tariff t.

Proposition 3 *Suppose that the countries impose identical tariffs, $t_{12} = t_{21} = t$, and consider the short-run effects of a small symmetric trade liberalization, $dt < 0$. There exists a marginal type $\widehat{\theta} \in (\underline{\theta}, \overline{\theta})$ such that all firms with organizational capability $\theta > \widehat{\theta}$ respond by divesting products, while all firms with organizational capability $\theta < \widehat{\theta}$ respond by purchasing additional products.*

Proof. See Appendix. ■

The proposition implies that a (multilateral) change in trade costs induces a "merger wave" in the short run. In response to a multilateral trade liberalization, large firms downsize by selling property rights over products (and the associated divisions) to small firms. A reduction in trade costs has two countervailing effects on the "effective market size" faced by each firm. Holding the endogenous demand intercept a fixed, the "direct effect" of the reduction in trade frictions is to increase the effective size of the foreign market. The "indirect effect" of a multilateral trade liberalization is to increase the intensity of competition (by decreasing the endogenous demand intercept a), which reduces the effective market size of both countries. A crucial step in the proof consists in showing that the "direct effect" outweighs the "indirect effect" so that, if the market price of the property rights over a product, r, were unchanged, all firms would actually want to add products. But the aggregate number of products is fixed, and so r increases in response to a multilateral trade liberalization. Given this endogenous price increase, only the firms with the lowest marginal costs (i.e., the firms with inferior organizational capability) find it optimal to add products. The following corollary is an immediate implication of Proposition 3 and Lemma 1.

Corollary 1 *Consider a multilateral reduction in trade costs, i.e., a decrease in t. Then, firms with large market-to-book ratios $T(\theta)$ purchase property rights over products from firms with small market-to-book ratios.*

To the extent that much of the merger and acquisition activity is due to "globalization" (or, alternatively, positive productivity shocks), our model predicts that firms with high values of

Tobin's Q buy corporate assets from firms with low Tobin's Q.[7] Proposition 3 in conjunction with Lemma 3 implies that a multilateral trade liberalization reduces the weighted (by number of products) average production costs in the industry. Our model gives thus gives rise to a particular link between trade liberalization and aggregate production costs that is not based on selection effects but rather on firm-level adjustments of scope.

Next, we consider a small unilateral reduction in the tariff imposed by country 1 on imports from country 2, t_{21}.

Proposition 4 *Suppose that the countries initially impose identical tariffs, $t_{12} = t_{21} = t$, and consider the short-run effects of a small unilateral trade liberalization by country 1, $dt_{21} < 0$. In the liberalizing country 1, there exists a marginal type $\widehat{\theta}_1 \in (\underline{\theta}, \overline{\theta})$ such that all firms with organizational capability $\theta > \widehat{\theta}_1$ respond by purchasing additional products, while all firms with organizational capability $\theta < \widehat{\theta}_1$ respond by divesting products. In contrast, in country 2, there exists a marginal type $\widehat{\theta}_2 \in (\underline{\theta}, \overline{\theta})$ such that all firms with organizational capability $\theta > \widehat{\theta}_2$ respond by divesting products, while all firms with organizational capability $\theta < \widehat{\theta}_2$ respond by purchasing additional products.*

Proof. See Appendix. ■

The short-run effects of a unilateral trade liberalization are very different from those of a multilateral trade liberalization. In the liberalizing country 1, increased competition with foreign firms in the home country 1 (the "indirect effect") induces the largest firms to add products while the smallest firms become even smaller as they divest products. (There is no "direct effect" for firms from country 1 since their access to country 2 does not improve.) The improved access of country-2 firms to country 1's market (the "direct effect") has the opposite impact on firms in that country: large firms reduce and small firms increase the number of products that they manage. That is, for the non-liberalizing country 2, the qualitative effects are the same as for a multilateral trade liberalization.

3.3 Comparative Statics: Long-Run Effects of Globalization

In our analysis of the effects of trade liberalization on firm scope, we have assumed so far that the mass of firms and the aggregate mass of products produced in each country is fixed. Here, we consider a different set of assumptions: we assume that both the mass of firms and the aggregate mass of products will adjust in response to changes in tariffs. We are thus concerned with the long-run effects of trade liberalization.

[7] This is consistent with the empirical evidence summarized by Andrade, Mitchell, and Stafford (2001).

Specifically, there is a sufficiently large mass of ex ante identical potential entrants. If a firm decides to enter, it has to pay a fixed entry cost ϕ; if it decides not to enter, it obtains a payoff normalized to zero. After paying the entry cost, a firm receives a random draw of its organizational capability θ from the c.d.f. $G(\cdot)$. A firm then decides on the number of its products. In both countries, the fixed development cost per product is r. We assume that the life span of each product is limited, which implies that, in long-run equilibrium, the market price of each product is equal to the exogenous development cost r, and the merger market does not play any allocative role. Since potential entrants are ex ante identical, the expected net profit of each entrant in country i must be equal to zero in long-run equilibrium:

$$\int_{\underline{\theta}}^{\overline{\theta}} n_i(\theta) \left\{ \pi_i(c_i(\theta)) + \pi_j(c_i(\theta) + t_{ij}) - r \right\} dG(\theta) - \phi = 0, \; i = 1, 2. \tag{12}$$

We define a *long-run equilibrium* as a collection $\{c_i(\cdot), n_i(\cdot), a_i, N_i, M_i\}_{i=1}^{2}$ satisfying the cost equation (3), the first-order condition (8), the equation for the endogenous demand intercept, (10), the adding-up condition (11), and the free-entry condition (12).

We now analyze the long-run effects of multilateral and unilateral tariff changes on firm scope. For this purpose, we assume that the industry is in a long-run equilibrium, both before and after the change in tariffs. As before, we assume that, prior to the change in tariffs, the two countries are identical, and so $N_1 = N_2 = N$, $M_1 = M_2 = M$, and $t_{12} = t_{21} = t$. We first consider a small symmetric reduction in the common tariff t.

Proposition 5 *Suppose that the countries impose identical tariffs, $t_{12} = t_{21} = t$, and consider the long-run effects of a small symmetric trade liberalization, $dt < 0$. There exists a marginal type $\widehat{\theta} \in \left[\underline{\theta}, \overline{\theta}\right]$ such that all firms with organizational capability $\theta > \widehat{\theta}$ have a reduced number of products, $dn(\theta) < 0$, while all firms with organizational capability $\theta < \widehat{\theta}$ have an increased number of products, $dn(\theta) > 0$.*

Proof. See Appendix. ■

Qualitatively, the long-run effects of a trade liberalization are similar to the short-run effects: there is a tendency for small firms with inferior organizational capability (but low marginal cost) to increase the number of products, while the reverse holds for large firms with superior organizational capability (but high marginal cost). In contrast to the short run, however, it is conceivable that $n(\theta)$ moves in the same direction for all firms, namely when $\widehat{\theta} = \underline{\theta}$ or $\widehat{\theta} = \overline{\theta}$.

Next, we consider the long-run effects of a small unilateral reduction in the tariff imposed by country 1 on imports from country 2, t_{21}.

Proposition 6 *Suppose that the countries initially impose identical tariffs, $t_{12} = t_{21} = t$, and consider the long-run effects of a small unilateral trade liberalization by country 1, $dt_{21} < 0$.*

In the liberalizing country 1, there exists a marginal type $\widehat{\theta}_1 \in [\underline{\theta}, \overline{\theta}]$ such that all firms with organizational capability $\theta > \widehat{\theta}_1$ have an increased number of products, $dn_1(\theta) > 0$, while all firms with organizational capability $\theta < \widehat{\theta}_1$ have a reduced number of products, $dn_2(\theta) < 0$. In contrast, in country 2, there exists a marginal type $\widehat{\theta}_2 \in [\underline{\theta}, \overline{\theta}]$ such that all firms with organizational capability $\theta > \widehat{\theta}_2$ have a reduced number of products, $dn_2(\theta) < 0$, while all firms with organizational capability $\theta < \widehat{\theta}_2$ have an increased number of products, $dn_2(\theta) > 0$.

Proof. See Appendix. ■

The long-term implications of a unilateral trade liberalization for the size distribution of firms are similar to those of the short-run. In the liberalizing country, production becomes more concentrated in the largest firms while production becomes less concentrated in the other country. As was the case for multilateral liberalization, it is conceivable that all firms within a country increase or decrease the number of their products.

3.4 Globalization and the Size Distribution of Firms

We now turn to the effects of globalization on the size distribution of firms. To allow for a clean comparison between our model and other recent models of trade with heterogeneous firms, we will focus on the effects of a multilateral reduction in trade costs on the distribution of firm sales in firms' home country.[8] Specifically, we will consider the *fractional* change in domestic firm sales by (initial) firm size.

Recall that the domestic sales of a firm with organizational capability θ are given by

$$S(\theta) = n(\theta)\frac{L}{8}\left[a^2 - c(n(\theta);\theta)^2\right]. \tag{13}$$

Hence, there are two channels in which a multilateral reduction in trade costs will affect the size distribution of firms. First, holding fixed firm scope $n(\theta)$ (and, therefore, the firm's marginal cost $c(n(\theta);\theta)$), a reduction in t will change the intensity of competition by affecting the endogenous demand intercept a. We call this the "competition effect" of globalization. Second, a multilateral reduction in trade costs will lead to a reallocation of ownership of products across firms, and thus change firm scope $n(\theta)$. We call this the "scope effect" of globalization.

Let us first consider the competition effect of globalization. From (13), the fractional change of domestic sales through the induced change in the endogenous demand intercept is

$$\frac{\left[n(\theta)\frac{L}{4}a\right]\frac{da}{dt}}{n(\theta)\frac{L}{8}\left[a^2 - c(n(\theta);\theta)^2\right]} = \frac{2a\frac{da}{dt}}{a^2 - c(n(\theta);\theta)^2}.$$

[8] While in our model all firms are exporters, this is not true in the models of Melitz (2003) and Melitz and Ottaviano (2008).

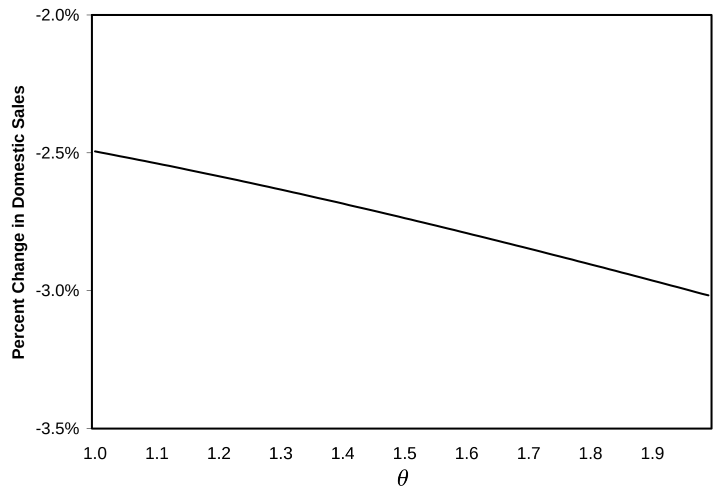

Figure 2: The Short-Run Effects of a Symmetric Trade Liberalization on Domestic Sales by Firm Type. (The parameters are: $M = 100$, $N = 1000$, $L = 800$, $\sigma = 0.0008$, $\theta \sim U[1, 2]$, and $c_0 = 0.05$; the change in tariff is from $t = 0.02$ to $t = 0$.)

Since $da/dt > 0$ (a multilateral reduction in trade costs leads to more intense competition), the fractional change in sales is increasing with marginal cost c. In our model, larger firms have higher marginal costs. Hence, a reduction in trade costs causes a *flattening* of the size distribution of firms in that larger firms decrease their domestic sales by a larger fraction than smaller firms.

Let us now turn to the scope effect of globalization. Propositions 3 and 5 show that a multilateral trade liberalization induces large firms to shed products and small firms to add products. That is, following a multilateral trade liberalization, the induced change in firm scope reinforces the competition effect in flattening the size distribution of firms.

The total effect of globalization on the domestic size distribution is illustrated in Figures 2 and 3. Figure 2 considers the short-run effect while Figure 3 illustrates the long-run effect. As both figures show, a multilateral trade liberalization induces a larger percentage decline in domestic sales for a (large) high-θ firm than for a (small) low-θ firm. That is, a multilateral trade liberalization results in a flattened distribution of domestic sales.

The predictions of our model on the effects of globalization on the size distribution of firms provide an interesting contrast to those that would obtain in the model of Melitz and Ottaviano (2008).[9] The main difference between our model and that of Melitz and Ottaviano is the manner in which firm heterogeneity is modeled. In Melitz and Ottaviano, each firm

[9] Melitz and Ottaviano (2008) do not consider the effects of globalization on the size distribution of firms.

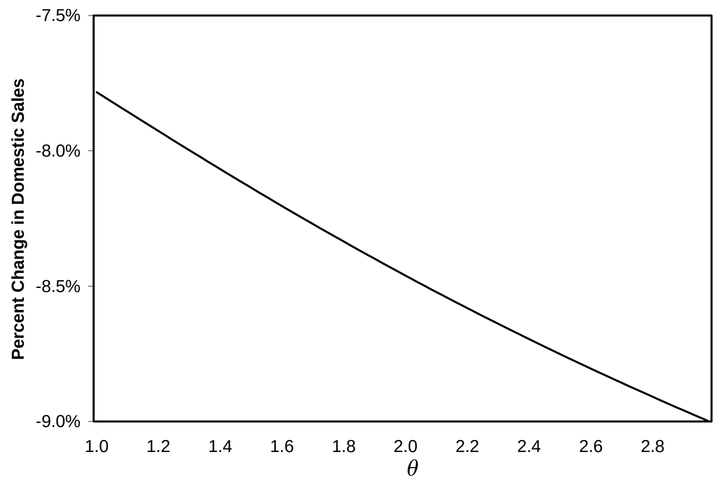

Figure 3: The Long-Run Effects of a Symmetric Trade Liberalization on Domestic Sales by Firm Type. (The parameters are: $L = 800$, $\sigma = 0.01$, $r = 0.5$, $\theta \sim U[1, 3]$, and $c_0 = 0.03$; the change in tariff is from $t = 0.01$ to $t = 0$.)

can produce only one product but firms differ in their (constant) marginal cost of production, c, which is drawn from a random distribution upon entry. Since all firms manage a single product, the scope effect of globalization is absent in Melitz and Ottaviano. Furthermore, since larger firms have lower marginal costs in their model, the sign of the competition effect of globalization is reversed. Hence, in Melitz and Ottaviano (2008), a multilateral reduction in trade costs would lead to a *steepening* of the (domestic) size distribution of firms.[10]

4 Conclusion

In this paper, we have developed a theory of multiproduct firms and endogenous firm scope that focuses on a new dimension of firm heterogeneity – organizational capability. In our model, firms have constant marginal costs of production for each of their products. But there are decreasing returns to the span of control at the firm level: the more products a firm manages, the higher are its marginal costs. Firms differ in their organizational capability: the greater is a firm's organizational capability, the less responsive are its marginal costs to an increase in the number of its products. A key feature of our model is that marginal costs are

[10] In Melitz (2003), a multilateral trade liberalization would not affect the gradient of the (domestic) size distribution. Since all firms produce a single product, there is no scope effect of globalization. Moreover, due to the CES preference structure, changes in trade costs do not affect markups, and so there is no competition effect either.

endogenously determined and depend on the firm's inherent organizational capability and its profit-maximizing choice of scope.

The model has empirical implications that provide an interesting contrast to those of standard, selection-driven models of firm heterogeneity. As in these standard models, intrinsically more efficient firms end up being larger in terms of their sales and their market value relative to less efficient firms, but the mechanism through which this occurs is very different. In our model, firms with greater organizational capability expand their product scope to such an extent that they end up having higher marginal costs than firms with lower organization capability. Consistent with a well-known empirical regularity, the higher marginal costs of larger firms manifest themselves as a negative relationship between firm size and the market-to-book ratio.

Because each product's marginal cost depends on the firm's choice of scope (and its inherent organizational capability), globalization affects production costs not just at the industry level but also at the product level as it leads to a reallocation of products across firms. Unlike standard, selection-driven models of firm heterogeneity, different firms respond differently to market integration, inducing a merger wave as large firms divest products and small firms expand their product range. As globalization progresses and product ownership is transferred from high marginal cost firms to low marginal cost firms, average industry marginal costs fall.

The model has been designed to explore the implications of a new dimension of firm heterogeneity and has focused on this dimension alone. We believe that an exciting avenue for future work consists in building richer trade models that combine several dimensions of firm heterogeneity and can help explain more features of the data.

5 Appendix

Derivation of the Linear Demand System. As discussed in the main text, each consumer's inverse demand for product k is given by

$$p(k) = 1 - 2x(k) - 4N\sigma\widehat{x}, \tag{14}$$

where

$$\widehat{x} \equiv \frac{1}{N}\int_0^n x(k)dk$$

is the average consumption over all varieties. Consider now the maximization problem of the firm producing variety k. Since consumers are identical and firms have constant marginal costs for each one of their products, we can think of the firm choosing the average output per consumer, $x(k)$. (Note that in models of monopolistic competition with a continuum of firms, price and quantity competition yield the same equilibrium allocation.) There are L consumers, and so the firm's total output will be $q(k) = Lx(k)$. The firm's problem is given by:

$$\max_{x(k)} \left[1 - 2x(k) - 4N\sigma\widehat{x} - c(k)\right] Lx(k).$$

Dropping arguments for notational simplicity, the first-order condition yields

$$x = \frac{1 - 4N\sigma\widehat{x} - c}{4}.$$

Taking averages, we obtain

$$\widehat{x} = \frac{1 - \widehat{c}}{4(1 + N\sigma)},$$

where

$$\widehat{c} = \frac{M}{N} \int n(\theta) c(n(\theta);\theta) dG(\theta)$$

is the average marginal cost of a product in the differentiated goods industry. Hence, the output of a firm with marginal cost c is

$$q(c) = Lx = L\left(\frac{1 - N\sigma\frac{1-\widehat{c}}{(1+N\sigma)} - c}{4}\right) = \frac{L}{4}\left(a - c\right), \tag{15}$$

where

$$a \equiv \frac{1 + \sigma M \int n(\theta) c(n(\theta);\theta) dG(\theta)}{1 + N\sigma} \tag{16}$$

is the endogenous demand intercept (or choke-off price). Finally, we derive the inverse market demand for each product in equilibrium. From (14),

$$\begin{aligned} P(q) &= 1 - 4\sigma\frac{M}{L}\int q(c(n(\theta);\theta))n(\theta)dG(\theta) - \frac{2q}{L} \\ &= 1 - 4\sigma\frac{M}{L}\int \frac{L}{4}\left[a - c(n(\theta);\theta)\right]n(\theta)dG(\theta) - \frac{2q}{L} \\ &= a - \frac{2q}{L}, \end{aligned}$$

where the second equality follows from inserting (15) and the third from using (16).

Proof of Proposition 1. Recall that

$$\begin{aligned} \Psi(c;\theta) &\equiv \left[P(q(c)) - c\right]q(c) - r - \frac{c}{\theta}q(c) \\ &= \frac{L}{8}(a-c)^2 - r - \frac{c}{\theta}\frac{L}{4}(a-c). \end{aligned}$$

The first-order condition (6) then states that $\Psi(c(\theta);\theta) = 0$. We will show that $c(\theta) = c_0$ for $\theta \leq \widetilde{\theta}$ and $c'(\theta) > 0$ for $\theta \geq \widetilde{\theta}$.

To see this, note first that $\Psi(0;\theta) = (L/8)a^2 - r$, which is strictly positive by (7), and $\Psi(a;\theta) = -r < 0$. Since $\Psi(c;\theta)$ is continuous, this implies that there exists a $\widetilde{c}(\theta) \in (0, a)$ such that $\Psi(\widetilde{c}(\theta);\theta) = 0$. We claim that $\widetilde{c}(\theta)$ is unique. To see this, note that $\Psi(\widetilde{c}(\theta);\theta) = 0$ and $r > 0$ imply that $\frac{L}{8}(a - \widetilde{c}(\theta))^2 > \frac{\widetilde{c}(\theta)}{\theta}\frac{L}{4}(a - \widetilde{c}(\theta))$, and so

$$a - \widetilde{c}(\theta) > \frac{2\widetilde{c}(\theta)}{\theta}. \tag{17}$$

Taking the partial derivative of $\Psi(\widetilde{c}(\theta);\theta)$ with respect to c, yields

$$\begin{aligned}
\Psi_c(\widetilde{c}(\theta);\theta) &= -\frac{L}{4}\left\{\left(1 + \frac{1}{\theta}\right)(a - \widetilde{c}(\theta)) - \frac{\widetilde{c}(\theta)}{\theta}\right\} \\
&< -\frac{L}{4}\left\{\left(1 + \frac{1}{\theta}\right)\frac{2\widetilde{c}(\theta)}{\theta} - \frac{\widetilde{c}(\theta)}{\theta}\right\} \\
&= -\frac{L}{4}\left\{1 + \frac{2}{\theta}\right\}\frac{\widetilde{c}(\theta)}{\theta} \\
&< 0,
\end{aligned}$$

where the first inequality follows from (17). The uniqueness of $\widetilde{c}(\theta)$ follows from $\Psi_c(\widetilde{c}(\theta);\theta) < 0$. Since $c_0 = \min_n c(n;\theta)$, the equilibrium value of marginal cost of a firm with organizational capability θ is given by

$$c(\theta) = \max\{c_0, \widetilde{c}(\theta)\}.$$

(The corner solution $c(\theta) = c_0$ obtains if and only if the firm finds it optimal to manage a single product only.)

Finally, we show that $d\widetilde{c}(\theta)/d\theta > 0$. From the implicit function theorem,

$$\frac{d\widetilde{c}(\theta)}{d\theta} = -\frac{\Psi_\theta(\widetilde{c}(\theta);\theta)}{\Psi_c(\widetilde{c}(\theta);\theta)},$$

where

$$\Psi_\theta(\widetilde{c}(\theta);\theta) = \frac{\widetilde{c}(\theta)}{\theta^2}\frac{L}{4}(a - \widetilde{c}(\theta)).$$

Since $\Psi_\theta(\widetilde{c}(\theta);\theta) > 0$ and $\Psi_c(\widetilde{c}(\theta);\theta) < 0$, it follows that $d\widetilde{c}(\theta)/d\theta > 0$. Hence, $c(\theta) = c_0$ for $\theta \leq \widetilde{\theta}$ and $c'(\theta) = d\widetilde{c}(\theta)/d\theta > 0$ for $\theta \geq \widetilde{\theta}$. ∎

Proof of Lemma 1. Tobin's Q is given by

$$\begin{aligned}
T(\theta) &\equiv \frac{[P(q(c(\theta))) - (1-\alpha)c(\theta)]\, q(c(\theta))}{r + \alpha c(\theta) q(c(\theta))} \\
&= \frac{\pi(c(\theta)) + \alpha c(\theta) q(c(\theta))}{r + \alpha c(\theta) q(c(\theta))},
\end{aligned} \tag{18}$$

which is independent of θ for $\theta \leq \widetilde{\theta}$ since then $c(\theta) = c_0$. Assume now that $\theta \geq \widetilde{\theta}$ so that $c'(\theta) > 0$. Substituting for $\pi(c(\theta))$ and $q(c(\theta))$ in (18) yields

$$T(\theta) = \frac{L}{8} \frac{(a - c(\theta))^2 + 2\alpha(a - c(\theta))c(\theta)}{r + \alpha\frac{L}{4}(a - c(\theta))c(\theta)}.$$

Differentiating this expression with respect to θ and combining terms yields

$$\frac{dT(\theta)}{d\theta} = -\frac{Lc'(\theta)}{4} \frac{r\left[(1-\alpha)(a - c(\theta)) + \alpha c(\theta)\right] + (\alpha L/8)(a - c(\theta))^2 a}{\left[r + \alpha(L/4)(a - c(\theta))c(\theta)\right]^2} < 0.$$

From $c'(\theta) > 0$, it follows that the firm's market-to-book ratio is strictly decreasing in θ for $\theta > \widetilde{\theta}$. ■

Proof of Lemma 2. It is straightforward to see that all single-product firms have the same sales, book value, and market value; that is, $S(\theta)$, $b(\theta)$, and $m(\theta)$ are all constant for $\theta \leq \widetilde{\theta}$. In the following, we assume $\theta \geq \widetilde{\theta}$.

Step 1. We first show that a firm's sales are increasing in θ. These sales can be written,

$$\begin{aligned} S(\theta) &= n(\theta)q(c(\theta))P(q(c(\theta))) \\ &= \left(\frac{c(\theta)}{c_0}\right)^{\theta} q(c(\theta))P(q(c(\theta))) \\ &= \left(\frac{c(\theta)}{c_0}\right)^{\theta} \frac{L}{8}\left(a^2 - c(\theta)^2\right), \end{aligned}$$

where we have substituted for $n(\theta)$, $q(c(\theta))$, and $P(q(c(\theta)))$. Differentiating this expression with respect to θ yields

$$S'(\theta) = \left(\frac{c(\theta)}{c_0}\right)^{\theta} \frac{L}{8}\left\{\ln\left(\frac{c(\theta)}{c_0}\right)\left(a^2 - c(\theta)^2\right) + c'(\theta)\left(\frac{\theta}{c(\theta)}\left(a^2 - c(\theta)^2\right) - 2c(\theta)\right)\right\} \tag{19}$$

The first term in brackets in equation (19) is nonnegative because $c(\theta) \geq c_0$ and $a > c(\theta)$. To show that the second term in brackets is positive, consider the first order condition for the optimal choice of the number of product lines:

$$\frac{L}{8}(a - c(\theta))^2 - \frac{L}{4}\frac{c(\theta)}{\theta}(a - c(\theta)) = r.$$

Reorganizing this expression, we have

$$a - c(\theta) = \frac{2c(\theta)}{\theta} + \frac{8r}{L(a - c(\theta))} > \frac{2c(\theta)}{\theta}.$$

Multiplying both sides of this inequality with $a + c(\theta)$ and reorganizing the resulting expression establishes

$$\begin{aligned} \frac{\theta}{c(\theta)}(a^2 - c(\theta)^2) &> 2[a + c(\theta)] \\ &> 2c(\theta). \end{aligned}$$

Hence, the second term in brackets in equation (19) is positive, so that $S'(\theta) > 0$.

Step 2. We now show that a firm's book value,

$$b(\theta) = n(\theta)r + n(\theta)\alpha c(\theta)q(c(\theta)),$$

is increasing in θ. Since $n'(\theta) \geq 0$ (with a strict inequality if and only if $\theta \geq \widetilde{\theta}$), it suffices to show that

$$\frac{d}{d\theta}\left\{n(\theta)c(\theta)q(c(\theta))\right\} > 0$$

for $\theta \geq \widetilde{\theta}$. This inequality can be rewritten as

$$\frac{d}{d\theta}\left\{S(\theta)\left(\frac{c(\theta)}{P(q(c(\theta)))}\right)\right\} = \frac{d}{d\theta}\left\{S(\theta)\left(\frac{2c(\theta)}{a + c(\theta)}\right)\right\} > 0.$$

But $S(\theta)$ is increasing in θ, as we have shown in step 1 and $c(\theta)/(a + c(\theta))$ is clearly increasing in θ. Hence, the inequality does indeed hold.

Step 3. Finally, we show that a firm's market value,

$$m(\theta) = n(\theta)P(q(c(\theta)))q(c(\theta)) - n(\theta)(1-\alpha)c(\theta)q(c(\theta)),$$

is increasing in θ. It is immediate to see that $m(\theta)$ is constant for $\theta \leq \widetilde{\theta}$. We need to show that $m(\theta)$ is strictly increasing in θ for $\theta \geq \widetilde{\theta}$. We can rewrite the market value as the sum of the firm's net profit and its book value:

$$m(\theta) = n(\theta)\left\{\left[P(q(c(\theta))) - c(\theta)\right]q(c(\theta)) - r\right\} + b(\theta).$$

Clearly, a high-θ firm can always replicate the choice of products by a low-θ firm, but at lower unit costs, and so a firm's net profit is increasing in θ. Moreover, $b'(\theta) > 0$ for $\theta \geq \widetilde{\theta}$, as we have shown in step 2. Hence, the firm's market value is strictly increasing in θ for $\theta \geq \widetilde{\theta}$. ■

Proof of Lemma 3. The first step consists in showing that $\frac{d}{dn}\left[nc_i(n;\theta)\right]\Big|_{n=n_i(\theta)}$ is positive. To see this, note that

$$\begin{aligned}
\frac{d}{dn}\, nc_i(n;\theta)\big|_{n=n(\theta)} &= \frac{d}{dn}\, c_0\,[n]^{(1+\theta)/\theta}\Big|_{n=n_i(\theta)} \\
&= \left(\frac{1+\theta}{\theta}\right) c_0\,[n_i(\theta)]^{1/\theta} \\
&= \left(\frac{1+\theta}{\theta}\right) c_i(\theta) \\
&> 0.
\end{aligned}$$

The second step consists in showing that $(1+\theta)c_i(\theta)/\theta$ is strictly increasing in θ. We have

$$\frac{d}{d\theta}\left(\frac{1+\theta}{\theta}\right)c_i(\theta) = \left(\frac{1+\theta}{\theta}\right)c_i'(\theta) - \frac{c_i(\theta)}{\theta^2}.$$

Using equation (9), it can easily be seen that $c_i'(\theta) > \theta^{-1}(1+\theta)^{-1}c_i(\theta)$. The claim then follows. We have thus shown that $\frac{d}{dn}\left[nc_i(n;\theta)\right]\big|_{n=n_i(\theta)}$ is positive and strictly increasing in θ.

The next step consists in showing that $\int \frac{d}{dn}\left[nc_i(n;\theta)\right]\big|_{n=n_i(\theta)} \Delta n_i(\theta)dG(\theta) < 0$. But this follows immediately from the following observations: (i) $\frac{d}{dn}\left[nc_i(n;\theta)\right]\big|_{n=n_i(\theta)}$ is positive and strictly increasing in θ, (ii) $\Delta n_i(\theta) > 0$ for $\theta < \widehat{\theta}$ and $\Delta n_i(\theta) < 0$ for $\theta > \widehat{\theta}$, and (iii) $\int \Delta n_i(\theta)dG(\theta) = 0$.

The final step consists in showing that $\Delta a_i < 0$ for each country i. But this follows immediately from the previous results and the equilibrium condition for a_i, equation (10). ■

Proof of Proposition 3. We need to show that $dc(\theta)/dt$ is positive for high-θ (i.e., high-c) firms and negative for low-θ (i.e., low-c) firms. Under symmetric tariffs, the first-order condition (8) can be rewritten as

$$\begin{aligned}\Omega(c(\theta);\theta;t) &\equiv \frac{L}{8}\left[(a-c(\theta))^2 + (a-t-c(\theta))^2\right] - r - \frac{c(\theta)}{\theta}\frac{L}{4}\left[(a-c(\theta)) + (a-t-c(\theta))\right] \\ &= 0, \qquad (20)\end{aligned}$$

Applying the implicit function theorem to this equation, yields

$$\frac{dc(\theta)}{dt} = -\frac{\Omega_t(c(\theta);\theta;t)}{\Omega_c(c(\theta);\theta;t)},$$

where the subscript $s \in \{t, c\}$ indicates the partial derivative with respect to variable s. Note that $\Omega_c(c(\theta);\theta;t) < 0$ since $\Omega(c(\theta);\theta;t) = 0$ is a profit maximum. Consequently, the sign of $dc(\theta)/dt$ is equal to the sign of $\Omega_t(c(\theta);\theta;t)$. Market clearing for products requires that some firms sell products while others purchase products, and so the sign of $\Omega_t(c(\theta);\theta;t)$ will vary with θ. In the following, we will show that $d\Omega_t(c(\theta);\theta;t)/d\theta > 0$.

Taking the partial derivative of $\Omega(c(\theta);\theta;t)$, as defined by equation (20), with respect to the cost parameter t, yields

$$\Omega_t(c(\theta);\theta;t) = \frac{L}{8}\left\{(a-c(\theta)) + (a-t-c(\theta)) - \frac{2c(\theta)}{\theta}\right\}\left[2\frac{da}{dt} - 1\right] + \frac{L}{8}t - \frac{dr}{dt}. \qquad (21)$$

From the first-order condition (20),

$$\frac{2c(\theta)}{\theta} = \frac{(a-c(\theta))^2 + (a-t-c(\theta))^2 - (8/L)r}{(a-c(\theta)) + (a-t-c(\theta))}.$$

Inserting this expression into equation (21), we obtain

$$\Omega_t(c(\theta);\theta;t) = \frac{L}{8}\left\{\frac{2(a-c(\theta))(a-t-c(\theta)) + (8/L)r}{(a-c(\theta)) + (a-t-c(\theta))}\right\}\left[2\frac{da}{dt} - 1\right] + \frac{L}{8}t - \frac{dr}{dt}.$$

Observe that θ enters this equation only through the endogenous marginal cost $c(\theta)$. Hence,

$$\begin{aligned}\frac{d\Omega_t(c(\theta);\theta;t)}{d\theta} &= \frac{d}{dc}\frac{L}{8}\left\{\frac{2(a-c(\theta))(a-t-c(\theta))+(8/L)r}{(a-c(\theta))+(a-t-c(\theta))}\right\}\left[2\frac{da}{dt}-1\right]\frac{dc(\theta)}{d\theta} \\ &= -\frac{L}{4}\left\{\frac{\left[(a-c(\theta))^2+(a-t-c(\theta))^2\right]-(8/L)r}{\left[(a-c(\theta))+(a-t-c(\theta))\right]^2}\right\}\left[2\frac{da}{dt}-1\right]\frac{dc(\theta)}{d\theta}.\end{aligned}$$

From the first-order condition (20), the expression in curly brackets is strictly positive. Since $dc(\theta)/d\theta > 0$, the sign of $d\Omega_t(c(\theta);\theta;t)/d\theta$ is thus equal to the sign of $[1-2da/dt]$.

We claim that $da/dt < 1/2$. To see this, suppose first that $da/dt = 1/2$. Then, $d\Omega_t(c(\theta);\theta;t)/d\theta = 0$, and so three cases may arise: (i) $dc(\theta)/dt > 0$ for all θ, (ii) $dc(\theta)/dt < 0$ for all θ, or else (iii) $dc(\theta)/dt = 0$ for all θ. But cases (i) and (ii) cannot occur since there is a fixed number of products. Hence, case (iii) must apply: $dc(\theta)/dt = 0$ for all θ; that is, there is no trade in products. But then, from equation (10), $da/dt = \sigma N/[1+2\sigma N] < 1/2$. A contradiction. Next, suppose that $da/dt > 1/2$. Then, $d\Omega_t(c(\theta);\theta;t)/d\theta < 0$. Hence, there exists a threshold type $\widehat{\theta} \in (\underline{\theta},\overline{\theta})$ such that – following a small *increase* in t – all firms with $\theta < \widehat{\theta}$ acquire products (and so $dc(\theta)/d\theta > 0$) while all firms with $\theta > \widehat{\theta}$ divest products (and so $dc(\theta)/d\theta < 0$). From Lemma 3, it follows that this "reshuffling" of products reduces the endogenous demand intercept a. From (10), the direct effect of an increase in t on a, holding $n(\theta)$ fixed, satisfies $\partial a/\partial t < 1/2$. Hence, the total effect of a small increase in t on a satisfies $da/dt < 1/2$. A contradiction. We have thus shown that $da/dt < 1/2$, and so there exists a threshold type $\widehat{\theta}$, such that – in response to a small increase in t – all firms with $\theta < \widehat{\theta}$ sell products while all firms with $\theta > \widehat{\theta}$ acquire products. The reverse conclusion holds if $dt < 0$. ■

Proof of Proposition 4. We need to show that $dc_1(\theta)/dt_{21}$ is negative for high-θ (i.e., high-c) firms and positive for low-θ (i.e., low-c) firms, while the opposite holds for $dc_2(\theta)/dt_{21}$. From the first-order condition (8), $\Omega^i(c_i(\theta);\theta;t_{12},t_{21}) = 0$, and so

$$\frac{2c_i(\theta)}{\theta} = \frac{(a_i-c_i(\theta))^2+(a_j-t_{ij}-c_i(\theta))^2-(8/L)r_i}{(a_i-c_i(\theta))+(a_j-t_{ij}-c_i(\theta))}. \tag{22}$$

Applying the implicit function theorem to the first-order condition, we obtain

$$\frac{dc_i(\theta)}{dt_{21}} = -\frac{\Omega^i_{t_{21}}(c_i(\theta);\theta;t_{12},t_{21})}{\Omega^i_c(c_i(\theta);\theta;t_{12},t_{21})},$$

where the subscript $s \in \{t,c\}$ indicates the partial derivative with respect to variable s. Note that $\Omega^i_c(c_i(\theta);\theta;t_{12},t_{21}) < 0$ since $\Omega^i(c_i(\theta);\theta;t_{12},t_{21}) = 0$ is a profit maximum. Consequently, the sign of $dc_i(\theta)/dt_{21}$ is equal to the sign of $\Omega^i_{t_{21}}(c_i(\theta);\theta;t_{12},t_{21})$. Market clearing for products requires that some firms sell products while others purchase products, and so the sign of $\Omega^i_{t_{21}}(c_i(\theta);\theta;t_{12},t_{21})$ will vary with θ. In the following, we will show that $d\Omega^1_{t_{21}}(c_i(\theta);\theta;t_{12},t_{21})/d\theta<0$ and $d\Omega^2_{t_{21}}(c_i(\theta);\theta;t_{12},t_{21})/d\theta>0$.

Consider first country 1. Using the first-order condition (8) and initial symmetry between countries, we obtain

$$\begin{aligned}\Omega^1_{t_{21}}(c(\theta);\theta;t_{12},t_{21}) &= \frac{L}{8}\left[2(a-c(\theta))-\frac{2c(\theta)}{\theta}\right]\left[\frac{da_1}{dt_{21}}+\frac{da_2}{dt_{21}}\right]-\frac{L}{4}t\frac{da_2}{dt_{21}}-\frac{dr_1}{dt_{21}}\\ &= \frac{L}{8}\left[2(a-c(\theta))-\frac{(a-c(\theta))^2+(a-t-c(\theta))^2-(8/L)r}{(a-c(\theta))+(a-t-c(\theta))}\right]\left[\frac{da_1}{dt_{21}}+\frac{da_2}{dt_{21}}\right]\\ &\quad -\frac{L}{4}t\frac{da_2}{dt_{21}}-\frac{dr_1}{dt_{21}},\end{aligned}$$

where the second equality follows from equation (22). Taking the derivative of this expression with respect to θ, yields

$$\frac{d\Omega^1_{t_{21}}(c(\theta);\theta;t_{12},t_{21})}{d\theta}=-\frac{L}{4}\left\{\frac{(a-c(\theta))^2+(a-t-c(\theta))^2-(8/L)r}{[(a-c(\theta))+(a-t-c(\theta))]^2}\right\}\left[\frac{da_1}{dt_{21}}+\frac{da_2}{dt_{21}}\right]\frac{dc(\theta)}{d\theta}.$$

From the first-order condition, the expression in curly brackets is strictly positive. Since $dc(\theta)/d\theta>0$, the sign of $d\Omega^1_{t_{21}}(c(\theta);\theta;t_{12},t_{21})/d\theta$ is thus equal to the sign of $-[da_1/dt_{21}+da_2/dt_{21}]$.

Consider now country 2. We have

$$\begin{aligned}\Omega^2_{t_{21}}(c(\theta);\theta;t_{12},t_{21}) &= \frac{L}{8}\left[2(a-c(\theta))-\frac{2c(\theta)}{\theta}\right]\left[\frac{da_1}{dt_{21}}+\frac{da_2}{dt_{21}}-1\right]+\frac{L}{4}t\left[1-\frac{da_1}{dt_{21}}\right]-\frac{dr_2}{dt_{21}}\\ &= \frac{L}{8}\left[2(a-c(\theta))-\frac{(a-c(\theta))^2+(a-t-c(\theta))^2-(8/L)r}{(a-c(\theta))+(a-t-c(\theta))}\right]\left[\frac{da_1}{dt_{21}}+\frac{da_2}{dt_{21}}-1\right]\\ &\quad +\frac{L}{4}t\left[1-\frac{da_1}{dt_{21}}\right]-\frac{dr_2}{dt_{21}},\end{aligned}$$

where the second equality follows again from equation (22). Taking the derivative of this expression with respect to θ, yields

$$\frac{d\Omega^2_{t_{21}}(c(\theta);\theta;t_{12},t_{21})}{d\theta}=-\frac{L}{4}\left\{\frac{(a-c(\theta))^2+(a-t-c(\theta))^2-(8/L)r}{[(a-c(\theta))+(a-t-c(\theta))]^2}\right\}\left[\frac{da_1}{dt_{21}}+\frac{da_2}{dt_{21}}-1\right]\frac{dc(\theta)}{d\theta}.$$

From the first-order condition, the expression in curly brackets is strictly positive. Since $dc(\theta)/d\theta>0$, the sign of $d\Omega^2_{t_{21}}(c(\theta);\theta;t_{12},t_{21})/d\theta$ is thus equal to the sign of $[1-da_1/dt_{21}-da_2/dt_{21}]$.

We claim that $0<da_1/dt_{21}+da_2/dt_{21}<1$, so that $d\Omega^1_{t_{21}}(c(\theta);\theta;t_{12},t_{21})/d\theta<0$ and $d\Omega^2_{t_{21}}(c(\theta);\theta;t_{12},t_{21})/d\theta>0$ To see this, suppose first that $da_1/dt_{21}+da_2/dt_{21}\geq 1$. Then, $d\Omega^1_{t_{21}}(c(\theta);\theta;t_{12},t_{21})/d\theta<0$ and $d\Omega^2_{t_{21}}(c(\theta);\theta;t_{12},t_{21})/d\theta\leq 0$. Hence, there exists a threshold type $\widehat{\theta}_1 \in (\underline{\theta}, \overline{\theta})$ in country 1 such that firms of type $\theta > \widehat{\theta}_1$ in country 1 will sell products to firms of type $\theta < \widehat{\theta}_1$. In country 2, either $n_2(\theta)$ remains unchanged, namely if $da_1/dt_{21}+da_2/dt_{21} = 1$, or else there also exists a threshold type $\widehat{\theta}_2 \in (\underline{\theta}, \overline{\theta})$ such that firms of type $\theta > \widehat{\theta}_2$ in country 2 will sell products to firms of type $\theta < \widehat{\theta}_2$. From Lemma 3, it follows that this "reshuffling" of products reduces the endogenous demand intercepts a_1 and a_2. Moreover, from (10), the "direct" effect of an increase in t_{21} on the demand intercepts satisfies $\partial a_1/\partial t_{21} < 1/2$ and $\partial a_2/\partial t_{21} = 0$. It follows that the total effect of a small increase in t_{21} on the demand intercepts satisfies $da_1/dt_{21} + da_2/dt_{21} < 1$. A contradiction. A similar argument can be used to show that $da_1/dt_{21} + da_2/dt_{21} \leq 0$ leads to a contradiction. ■

Proof of Proposition 5. We need to show that there exists a $\widehat{\theta} \in [\underline{\theta}, \overline{\theta}]$ such that $dc(\theta)/dt$ is positive for $\theta > \widehat{\theta}$ and negative for $\theta < \widehat{\theta}$. As shown in the proof of Proposition 3, the sign of $dc(\theta)/dt$ is equal to the sign of $\Omega_t(c(\theta); \theta; t)$, where

$$\Omega_t(c(\theta);\theta;t) = \frac{L}{8}\left\{(a-c(\theta)) + (a-t-c(\theta)) - \frac{2c(\theta)}{\theta}\right\}\left[2\frac{da}{dt} - 1\right] + \frac{L}{8}t$$

since $dr/dt = 0$ in the long run. Using the same steps as in the proof of Proposition 3,

$$\Omega_t(c(\theta);\theta;t) = \frac{L}{8}\left\{\frac{2(a-c(\theta))(a-t-c(\theta)) + (8/L)r}{(a-c(\theta)) + (a-t-c(\theta))}\right\}\left[2\frac{da}{dt} - 1\right] + \frac{L}{8}t,$$

and

$$\frac{d\Omega_t(c(\theta);\theta;t)}{d\theta} = -\frac{L}{4}\left\{\frac{\left[(a-c(\theta))^2 + (a-t-c(\theta))^2\right] - (8/L)r}{\left[(a-c(\theta)) + (a-t-c(\theta))\right]^2}\right\}\left[2\frac{da}{dt} - 1\right]\frac{dc(\theta)}{d\theta}. \tag{23}$$

We now claim that $da/dt < 1/2$ in the long run. To see this, suppose otherwise that $da/dt \geq 1/2$. Then, the profit of each firm of type θ would strictly increase following a small increase in t, even holding fixed the choice of the number of products, $n(\theta)$:

$$\begin{aligned}
\frac{d}{dt}\left\{(a-c(\theta))^2 + (a-t-c(\theta))^2\right\}_{c(\theta)=const.} &= 2\left[(a-c(\theta)) + (a-t-c(\theta))\right]\frac{da}{dt} - 2(a-t-c(\theta)) \\
&\geq \left[(a-c(\theta)) + (a-t-c(\theta))\right] - 2(a-t-c(\theta)) \\
&> 0
\end{aligned}$$

for all θ. But this is inconsistent with free entry.

Since $da/dt < 1/2$, equation (23) implies that $d\Omega_t(c(\theta);\theta;t)/d\theta > 0$. Hence, the assertion of the proposition follows. ■

Proof of Proposition 6. We need to show that there exist thresholds $\widetilde{\theta}_1 \in [\underline{\theta}, \overline{\theta}]$ and $\widetilde{\theta}_2 \in [\underline{\theta}, \overline{\theta}]$ such that $dc_1(\theta)/dt_{21}$ is negative for $\theta > \widetilde{\theta}_1$ and positive for $\theta < \widetilde{\theta}_1$, while the opposite holds for $dc_2(\theta)/dt_{21}$. As shown in the proof of Proposition 4, the sign of $dc_i(\theta)/dt_{21}$ is equal to the sign of $\Omega^i_{t_{21}}(c_i(\theta);\theta;t_{12},t_{21})$, where

$$\begin{aligned}\Omega^1_{t_{21}}(c(\theta);\theta;t_{12},t_{21}) &= \frac{L}{8}\left[2(a-c(\theta))-\frac{(a-c(\theta))^2+(a-t-c(\theta))^2-(8/L)r}{(a-c(\theta))+(a-t-c(\theta))}\right]\left[\frac{da_1}{dt_{21}}+\frac{da_2}{dt_{21}}\right]\\ &\quad -\frac{L}{4}t\frac{da_2}{dt_{21}},\end{aligned}$$

and

$$\begin{aligned}\Omega^2_{t_{21}}(c(\theta);\theta;t_{12},t_{21}) &= \frac{L}{8}\left[2(a-c(\theta))-\frac{(a-c(\theta))^2+(a-t-c(\theta))^2-(8/L)r}{(a-c(\theta))+(a-t-c(\theta))}\right]\left[\frac{da_1}{dt_{21}}+\frac{da_2}{dt_{21}}-1\right]\\ &\quad +\frac{L}{4}t\left[1-\frac{da_1}{dt_{21}}\right],\end{aligned}$$

since r is fixed in the long run. As we have shown in the proof of Proposition 4,

$$\frac{d\Omega^1_{t_{21}}(c(\theta);\theta;t_{12},t_{21})}{d\theta}=-\frac{L}{4}\left\{\frac{\left[(a-c(\theta))^2+(a-t-c(\theta))^2\right]-(8/L)r}{\left[(a-c(\theta))+(a-t-c(\theta))\right]^2}\right\}\left[\frac{da_1}{dt_{21}}+\frac{da_2}{dt_{21}}\right]\frac{dc(\theta)}{d\theta}.$$

and

$$\frac{d\Omega^2_{t_{21}}(c(\theta);\theta;t_{12},t_{21})}{d\theta}=-\frac{L}{4}\left\{\frac{\left[(a-c(\theta))^2+(a-t-c(\theta))^2\right]-(8/L)r}{\left[(a-c(\theta))+(a-t-c(\theta))\right]^2}\right\}\left[\frac{da_1}{dt_{21}}+\frac{da_2}{dt_{21}}-1\right]\frac{dc(\theta)}{d\theta}.$$

We now claim that $da_1/dt_{21}+da_2/dt_{21}<1$ in the long run. To see this, suppose otherwise that $da_1/dt_{21}+da_2/dt_{21}\geq 1$. Consider the change in the profit per product of a country-1 firm with marginal cost $c(\theta)$:

$$\left.\frac{d\left[\pi_1(c(\theta))+\pi_2(c(\theta)+t_{12})\right]}{dt_{21}}\right|_{\substack{t_{12}=t_{21}=t\\ c(\theta)=const.}}=\frac{L}{4}\left\{(a-c(\theta))\left[\frac{da_1}{dt_{21}}+\frac{da_2}{dt_{21}}\right]-t\frac{da_2}{dt_{21}}\right\}.$$

Free entry implies that this expression cannot be strictly positive for all values of $c(\theta)$. Hence, $da_2/dt_{21}>0$. Consider now change in the profit per product of a country-2 firm with marginal cost $c(\theta)$:

$$\left.\frac{d\left[\pi_2(c(\theta))+\pi_1(c(\theta)+t_{21})\right]}{dt_{21}}\right|_{\substack{t_{12}=t_{21}=t\\ c(\theta)=const.}}=\frac{L}{4}\left\{(a-t-c(\theta))\left[\frac{da_1}{dt_{21}}+\frac{da_2}{dt_{21}}-1\right]+t\frac{da_2}{dt_{21}}\right\}.$$

Free entry implies that this expression cannot be strictly positive for all values of $c(\theta)\leq a-t$ (which holds by assumption). Hence, $da_2/dt_{21}\leq 0$. A contradiction.

We now claim that $da_1/dt_{21}+da_2/dt_{21}>0$ in the long run. To see this, suppose otherwise that $da_1/dt_{21}+da_2/dt_{21}\leq 0$. Free entry implies that $d\left[\pi_1(c(\theta))+\pi_2(c(\theta)+t_{12})\right]/dt_{21}$

cannot be strictly negative for all values of $c(\theta)$. Hence, $da_2/dt_{21} \leq 0$. Free entry also implies that $d\left[\pi_2(c(\theta)) + \pi_1(c(\theta) + t_{21})\right]/dt_{21}$ cannot be strictly negative for all values of $c(\theta)$. Hence, $da_2/dt_{21} > 0$. A contradiction.

Since $0 < da_1/dt_{21} + da_2/dt_{21} < 1$, it then follows that $d\Omega^1_{t_{21}}(c(\theta); \theta; t_{12}, t_{21})/d\theta < 0 < d\Omega^2_{t_{21}}(c(\theta); \theta; t_{12}, t_{21})/d\theta$. ■

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