Uncertain Productivity Growth and the Choice between FDI and Export

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Abstract

With aggregate sales by foreign affiliates exceeding world exports, determinants of FDI patterns have received great attention, while the timing of their surge has been understudied. Recent evidence indicates that transportation costs of goods have fallen too little to explain these figures based on the *proximity-concentration* trade-off argument alone. Contextually, other changes have occurred: in particular, the uncertainty that firms bear has increased. Enriching the classical choice problem of a multinational firm with insights from the investment literature, we show that increased uncertainty along with the sizable fixed costs characterizing engagements on international markets can explain a surge in FDI.

JEL: F17, F21, F23

Key Words: Proximity-Concentration Hypothesis, Stochastic Processes, Real Option

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1 Introduction

A prominent feature of the data in the last decades is the double digit growth of foreign direct investment (FDI) (Navaretti & Venables, 2004; UNCTAD, 2009), with aggregate sales by foreign affiliates having exceeded world exports (Raff & Ryan, 2008). To explain these patterns, the *proximity-concentration* trade-off, first presented in Brainard's (1993, 1997) seminal contributions, has become the workhorse model within Dunning's "*ownership, location, internalization - OLI*" framework. Recent advances in the trade theory have extended this basic framework to include a more involved treatment of the firm: not simply a multinational firm with two-plant operations, but also with multiple platforms (Ekholm et al., 2007), or with different efficiency levels (Helpman et al., 2004), or with more complex acquisition strategies of foreign affiliates (Nocke and Yeaple, 2007).

While these contributions have successfully explained the observed patterns of exporting and FDI strategies at any point in time, even in very narrowly defined industries, they cannot fully account for the dynamics and timing of alternative internationalization strategies. According to the proximity-concentration trade-off argument, only a drastic fall in transportation costs could explain this wave of FDI. But the available evidence generally points only to a slow, although steady, decline of air and ocean fares (Hummels, 2007). If anything, such patterns of transportation costs should imply the consolidation of exporting.

The aim of our paper is to reconcile this puzzle with the theory based on the proximity-concentration trade-off. Clearly, changes other than transportation costs can affect this trade-off, too. Neary (2009) proposes to look at regionalism and foreign acquisition as driving forces behind this change. Our paper proposes to investigate productivity evolvements and uncertainty as determinants of this change.

Indeed, common in this literature is the assumption of an unchanged productivity throughout the firm's life span. This assumption is an unsatisfactory treatment of the firm and it is at odds with the empirical evidence on technological adoption, emphasizing steep learning curves and productivity disruptions (Holmes et al., 2008). Moreover, recent evidence discloses that uncertainty which firms bear has risen: the volatility of earnings, sales, employment, and capital expenditure at the firm level have all increased in the last decades in relation to R&D investments and industry deregulations (Comin and Philippon, 2005; Comin and Mulani, 2006). What these figures imply for FDI investments remains unexplored in the literature. Yet, a lesson that clearly stems from the investment literature of the last decades is that fixed and sunk costs have important consequences for the value of action and inaction of firms in an uncertain and dynamic context (Bertola, 2010; Stokey, 2009; Dixit and Pindyck, 1994). Given that fixed costs associated with either exporting or FDI are considered sizable and partly sunk, and their role is prominent in all the recent trade literature (Das et al., 2007), it is perhaps not surprising that these costs ought to have important consequences for firms' strategies in an uncertain context.

Taking an economic environment with shocks that are stochastic and idiosyncratic to the firm and possibly of adverse nature as our starting point, we revise the classical choice problem of a multinational firm between exporting and FDI. Following the real option methodology, we model these shocks as an (exogenous) stochastic evolvement of the firm's productivity. Such a modeling choice is wise in terms of uncovering the effects of uncertainty on the predictions of the *proximity-concentratation* trade-off framework, while preserving considerable analytical tractability.

The novel element in our model is the stochastic productivity that cannot be fully anticipated by the firm when deciding both the entry mode and the timing of entry in a new foreign market. We expressly restrict ourselves to the case of exporting and FDI being perfect substitute modes: in this sense, our work complements Rob and Vettas (2003) and reflects our distinct aims. While we revise the proximity concentration framework in a dynamic and uncertain setting, they analyze conditions under which complementarity between exporting and FDI modes may emerge in a context of a stochastic non-decreasing demand. Similarly to them, and to Neary (2009), we opt for a partial equilibrium approach, in an otherwise standard monopolistic competitive setting. Although this approach disregard "crowding-out" effects, which are investigated in Head and Ries (2004), it makes possible to endogenously determine the timing of FDI, opening the perspective of explaining FDI waves within the *proximity-concentration* trade off framework.

The main result of our analysis is that uncertainty biases the choice of the firm toward the FDI strategy. In an uncertain environment, waiting for a firm is valuable to partially resolve uncertainty. But because fixed costs in the real option framework are irreversible and only incurred upon entry, waiting, in a dynamic setting, also reduces the prospective fixed costs. These cost adjustments over time reduce the relative fixed cost disadvantage of the FDI mode enhancing it as an optimal foreign market serving mode.

The remainder of this paper is organized as follows. The next section delineates the model and motivates our assumptions. We derive the optimal investment mode in three different scenarios each in a subsection. The final section concludes.

2 The Model

We consider the classical choice problem of a firm between exporting (mode E hereafter) and greenfield FDI (mode F hereafter): a risk neutral single firm, already operating in the domestic market, has to decide upon serving a new foreign market either through export or through a new foreign affiliate (horizontal FDI). The demand side is rather standard: the firm faces the iso-elastic demand function

$$p_t = Z_t X_t^{\nu-1}, \quad 0 \leq \nu \leq 1, \tag{1}$$

where p_t represents the price of the firm's output X_t offered in the new destination country at time t.[1] ν is the degree of competition in the market: for $\nu = 1$ the demand curve is perfectly elastic (i.e. the market is perfectly competitive), whereas as $\nu \to 0$ the firm's monopoly power increases. Z_t represents a shift factor, including for instance the destination country's income.

The only production factor is labor L, inelastically supplied. The output in mode i is

$$X_{it}(L_{it}) = \phi_t L_{it}^{\theta}, \qquad 0 < \theta \leq 1, \qquad \phi_t > 0, \quad i \in \{E, F\}, \tag{2}$$

[1] Such a demand for a variety ω in the variety set Ω can be derived from a CES utility function $U = \left(\int_{\omega\in\Omega} X_t(\omega)^\rho d\omega\right)^{\frac{1}{\rho}}$, where $\eta = \frac{1}{1-\rho} > 1$ is the elasticity of substitution. The inverse mark-up is then $\nu = \frac{\eta-1}{\eta}$.

where ϕ_t represents the firm-specific productivity.[2] Therefore, we are implicitly assuming a perfect technology transfer: the firm is equally productive regardless of the production location.

Transportation costs are also standard and of the iceberg type, so that $\tau_t > 1$ units of output have to be shipped for one unit of output to be delivered at destination. Only FDI can economize on this cost, so that $\tau_{Ft} = 1$, and $\tau_{Et} = \tau_t$.

Normalizing domestic market sales, without loss of generality, to zero, the firm's profit maximizing behavior implies

$$\pi_{it} = \max_{L_{it}} p_t \; X_{it} - w_{it} L_{it} \quad \text{s.t.} \quad X_{it} = \frac{\phi_t L_{it}^{\theta}}{\tau_{it}} \quad \text{s.t.} \quad p_t = Z_t X_{it}{}^{(\nu-1)}, \quad i \in \{E, F\}, \tag{3}$$

where w_{Et} and w_{Ft} are the wage rates prevailing respectively on the domestic and foreign labor markets, and τ_{it} is the iceberg transport costs incurred in mode i. Thus, the per-period profit in either mode is

$$\pi_{it}(\phi_t) = M_{it}\phi_t^{\kappa}, \quad i \in \{E, F\}, \tag{4}$$

where $M_{it} = Z_t^{\frac{1}{1-\nu\theta}} \left(\frac{\nu\theta}{w_{it}\tau_{it}^{\frac{1}{\theta}}} \right)^{\kappa\theta} (1 - \nu\theta)$ and $\kappa = \frac{\nu}{1-\nu\theta}$. π_{it} is increasing in the level of productivity ϕ_t, and convex for $\kappa > 1$. It is also stochastic as we depart from the literature and assume that ϕ_t is uncertain and evolves over time according to the following Geometric Brownian motion:[3]

$$d\phi_t = \alpha\phi_t dt + \sigma\phi_t dz_t, \tag{5}$$

where the drift parameter $\alpha \geq 0$ represents the firm's persistent productivity growth. The diffusion parameter $\sigma \geq 0$ measures the extent of continuous productivity shocks, which can arise e.g. from technology shocks (similar to Irarrazabal and Opromolla, 2008). α and σ are

[2] ϕ could also be re-interpreted as a quality index of the good produced.

[3] The term "uncertainty" will be used in an interchangeable manner with the term risk. In a concise way, risk refers to a known probability distribution, whereas uncertainty refers to events in which the numerical probabilities cannot be specified. As in Dixit and Pindyck (1994) we don't follow this distinction.

time invariant.[4] dz_t is the increment of a standard Wiener process and uncorrelated across time, with $dz_t = \epsilon_t \sqrt{dt} \quad \text{and} \quad \epsilon_t \sim N(0,1)$.

To enter into the foreign market the firm incurs a sizeable constant upfront fixed cost.[5] Entry is irreversible, but postponable, a typical feature of the real option approach, which sets it apart from the monopolistic competitive model.[6] Therefore, entry does not necessarily take place in the initial period, but the firm can execute the investment at any future time and only then incur the entire fixed cost of entry. The timing of entry is endogenously determined in the initial period, but once the investment is executed in any subsequent period, it is - by assumption - irreversible.[7] The lapse of time between the moment the investment decision is taken and the moment the investment is executed is the waiting time to entry.

The proximity-concentration trade-off argument dictates that, relative to exports, FDI economizes transport costs, but expands production facilities, resulting in higher fixed costs. To substantiate this argument we assume

$$\frac{I_E}{I_F} < 1 \quad \text{and} \quad \frac{w_F}{w_E \tau_E} < 1 \tag{6}$$

or, consequently, that $M_E < M_F$. Before proceeding with the formal analysis, some of our assumptions deserve a brief discussion.

The firm is a price taker on the factor market, the price of labor is exogenously given, and we abstain from considering market entry. This is practically a single firm model and, following Bertola (1998), all firm interactions and the competitive pressure are simply parametrically indexed by ν. These features are simplification over the standard monopolistic competitive model, rendering our approach a partial equilibrium model, but allowing us to complicate the production side to include firm's idiosyncratic productivity shocks in a tractable manner. In particular, given the multiplicative structure of the profit, an advantage of this formulation is

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[4] This is a needed simplification over the empirical evidence presented in Comin and Mulani (2006) to keep the framework tractable. Nevertheless, we shall perform a comparative static analysis on σ below in our analysis.

[5] See also Bernard et al. (2003), Helpman et al. (2004), and Yeaple (2009).

[6] See also McDonald and Siegel (1986), and Pindyck (1991).

[7] This assumption can be relaxed and additionally switching behavior can be considered, which would necessitate numerical simulations. As we focus only on first time market entry decisions, we omit this extension.

that our model can be extended to include other sources of uncertainty (e.g. foreign demand, factor prices, transportation cost), even simultaneously.[8]

It is worth noting that the period in which the timing of entry is determined does not coincide generally with the period in which effective entry occurs. We will show that in a dynamic setting, with the timing of the investment and the timing of the decision that are asynchronous, the rate at which the future time is discounted is of primary relevance. In a risk-free environment, the interest rate represents an adequate discount rate, as it expresses the opportunity cost of any real investment in terms of forgone returns on a safe bond. But in a risky environment such an interest rate falls short of the risk that even a risk-neutral investor has to bear and for which she should be compensated. Therefore, an appropriate discount rate should also comprise a compensation for the risk, i.e. a risk-premium, and exceed the safe bond return. The risk adjusted expected return rate is in general terms

$$\mu = r + R(\sigma), \tag{7}$$

where $R(\sigma)$ is the positive risk premium and $R(0) = 0$, so that $\mu = r$ for a risk free environment. We shall be more specific about the exact derivation of $R(\sigma)$ below when we shall introduce uncertainty. For the time being, it suffices to note that its derivation necessitates the assumption of a complete capital market.

For expositional clarity, we find it convenient to present first all the challenges that the dynamic formulation of the proximity-concentration trade-off poses in the case of deterministic productivity growth, and only subsequently analyze the same trade-off with stochastic productivity growth. The reason for such a convenience is two-fold. First, the main results can be established without the complication of stochastic dynamic optimization. Second, we can disentangle the dynamic effects from the uncertainty effects and show that uncertainty will amplify the effects derived in the deterministic case. Therefore, we proceed presenting

[8] If the profit of a firm is the product of several stochastic variables, but each variable's motion is individually described by a Geometric Brownian motion, then the profit itself behaves as a Geometric Brownian motion. The resulting drift and diffusion parameters are linear combinations of the corresponding single process parameters (Bertola, 1998).

three cases: the static case ($\alpha = 0$, $\sigma = 0$), as a benchmark to compare to the dynamic deterministic case ($\alpha > 0$, $\sigma = 0$) before the stochastic dynamic case ($\alpha > 0$, $\sigma > 0$) is elaborated.

For the remainder of the paper we deliberately limit our analysis to $\theta = 1$, i.e., the monopolistic competitive setting in which the *proximity-concentration* trade-off is typically analyzed. Furthermore, we consider profits to be strictly convex in productivity ($\kappa > 1$), which is motivated by empirical estimates of the elasticity of substitution that are usually larger than two.[9] From a theoretical point of view, in a dynamic setting convexity gives rise to additional interesting effects which we elaborate on. Time is clearly continuous and the time horizon is infinite. Throughout the paper for notational simplicity we set the initial period to be $t = 0$ and $\phi_0 = \phi$. We also drop the time index for all time-invariant variables. Finally, as a standard real option assumption, for bounded solutions $\mu > \alpha$.

2.1 Export versus FDI without Productivity Growth

In a static scenario ($\alpha = \sigma = 0$) the discount rate is just equal to r, the interest rate on a riskless bond, and all variables are constant. Therefore, waiting has no value to the firm: either entry is profitable from the initial period or it will never become so. Entry is profitable if the gross value of the firm $V_{i_c}(\phi)$, the present discounted value of the firm's stream of profits, exceeds the fixed entry cost, or

$$V_{i_c}(\phi) = \int_0^\infty M_i \phi^\kappa e^{-rs} ds = \frac{M_i \phi^\kappa}{r} \geq I_i, \quad i \in \{E, F\}. \tag{8}$$

(8) is the standard Marshallian entry condition. Because the value function of the firm is increasing in productivity while the fixed costs are constant, there is a productivity level at which the firm is indifferent between entry in mode i or staying out of the market, namely

[9] In monopolistic competition models the elasticity of substitution needs to be larger than 1. However, for values smaller than two the resulting mark-ups are far too high compared to empirical findings (see e.g. Broda and Weinstein, 2006). With an elasticity of substitution larger than two we impose $\nu > 0.5 \rightarrow \kappa > 1$. Qualitatively, our equilibrium results are the same for linear profits in ϕ.

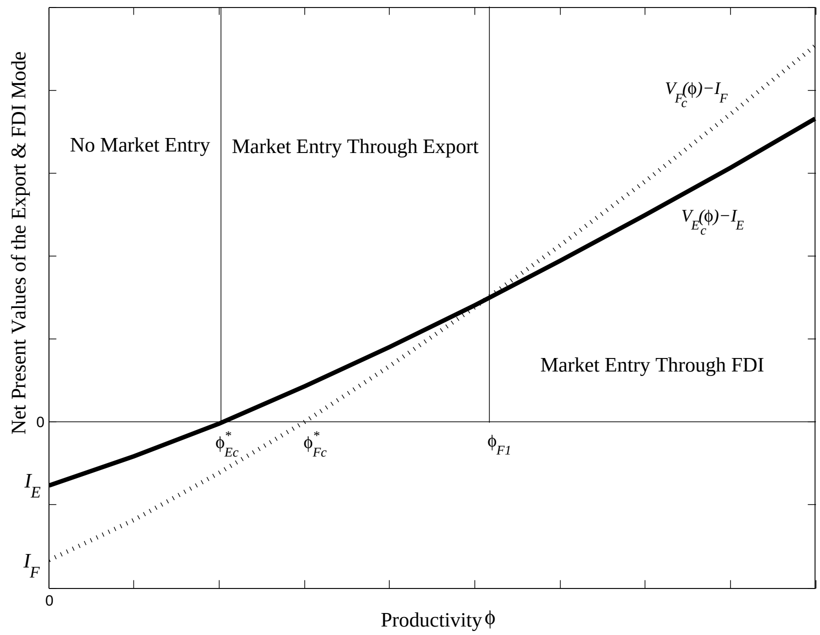

Figure 1: Value Functions within the Proximity-Concentration Trade-Off

when (8) holds with equality. These cutoffs for exporting and FDI are, respectively,

$$\phi^*_{Ec} = \sqrt[\kappa]{\frac{rI_E}{M_E}} \quad \text{and} \quad \phi^*_{Fc} = \sqrt[\kappa]{\frac{rI_F}{M_F}} \tag{9}$$

and are shown in figure 1, where each net present value function $V_{i_c}(\phi) - I_i$ cuts the productivity axis. Because $M_E < M_F$, if $\phi^*_{Fc} > \phi^*_{Ec}$, the case depicted in figure 1, the sorting patterns into FDI or exporting depends on the level of productivity, as in Helpman et al. (2004). Firms with productivity lower than ϕ^*_{Ec} stay out of the market and with productivity higher than ϕ_{FI} serve the foreign market through FDI. Firms with intermediate productivity between these two values export to the foreign market. On the contrary, if $\phi^*_{Fc} \leq \phi^*_{Ec}$, the FDI strategy dominates the exporting strategy, as the upper envelope function in figure 1 would be the FDI net present value. Therefore, depending on the productivity level, the firm would either perform FDI or not enter at all. Interestingly, since these sorting patterns depend on the ordinal rank between the two cutoffs, they ultimately hinge also on the proximity

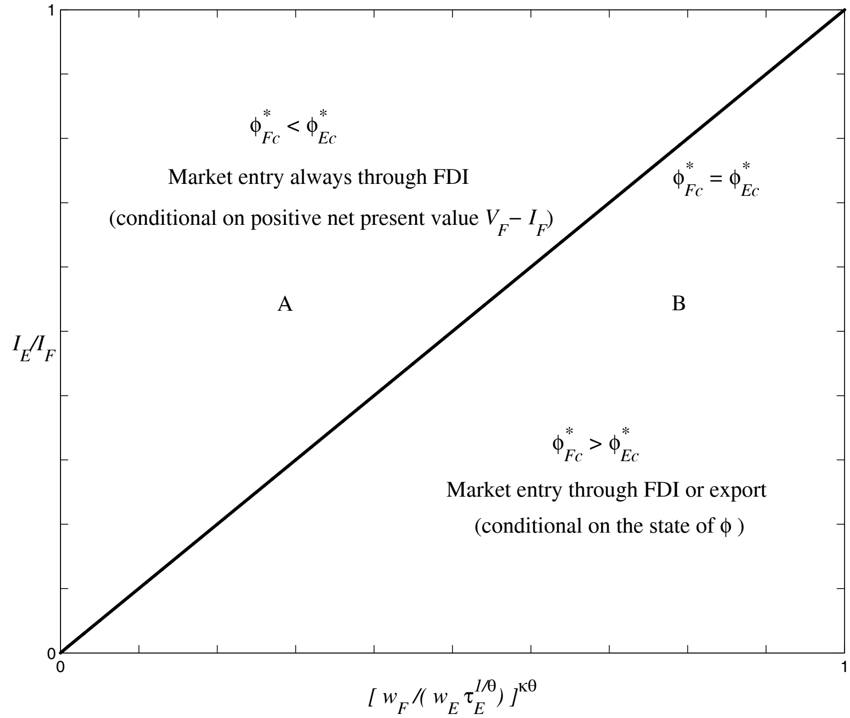

Figure 2: Relative Proximity-Concentration Cost Space

Since within the proximity-concentration trade-off framework relative fixed costs and relative variable costs never exceed unity, it is possible to depict all relevant cost patterns in a unit relative cost box.

concentration trade-off, as

$$\phi^*_{Ec} < \phi^*_{Fc} \quad \text{if} \quad \frac{I_E}{I_F} < \left(\frac{w_F}{w_E \tau_E^{\frac{1}{\theta}}} \right)^{\kappa\theta} . \tag{10}$$

In figure 2 we can depict a box with the vertical axis measuring the relative fixed cost advantage of the export mode and the horizontal axis measuring the relative variable cost advantage of the FDI mode. The diagonal of this box expresses the equal cut-off condition and borders the FDI-dominance region given by the upper triangle of the box. In this area denoted by A, a firm either serves the foreign market through FDI or remains out of the market. Area B, the lower triangle in our box, represents instead the area with more complex sorting patterns where a firm either performs FDI or exports or stays out of the market according to its productivity level.[10]

We summarize these findings in the following proposition.

Proposition 1:

Assuming that $I_E < I_F$, $w_F < w_E \tau_E^{\frac{1}{\theta}}$, and $\theta = 1, \nu \neq 1$,

- *if $\frac{I_E}{I_F} \geq \left(\frac{w_F}{w_E \tau_E^{\frac{1}{\theta}}}\right)^{\kappa\theta}$, FDI is the firm's dominant strategy, if entry is profitable;*
- *if $\frac{I_E}{I_F} < \left(\frac{w_F}{w_E \tau_E^{\frac{1}{\theta}}}\right)^{\kappa\theta}$, the mode choice depends on the firm's specific productivity level ϕ, conditional on entry being profitable.*

2.2 Export versus FDI with Deterministic Productivity Growth

In a scenario with deterministic productivity growth ($\alpha > 0, \sigma = 0$), the firm's market entry decision becomes an inter-temporal optimization problem and the discount rate is still the interest rate r. Recall that in our framework, entry is irreversible, but postponable. The timing of events is as follows. In the initial time ($t = 0$) the firm has to decide the timing of entry as well as the mode of entry. Accordingly, entry in either mode will then happen at $t = T \geq 0$, after a waiting time of T periods.[11] In a dynamic setting, the option to wait has to be weighed against the value of action: for a firm with productivity ϕ that enters the foreign market at period T with mode i, the value of action is the value of the investment evaluated in the initial period defined as

$$V_i(\phi, T) = \int_T^{\infty} M_i \phi^{\kappa} e^{-(r-\alpha_g)s} ds = \frac{M_i \phi^{\kappa}}{r - \alpha_g} e^{-(r-\alpha_g)T}, \quad i \in \{E, F\}, \tag{11}$$

[10] For cost patterns in area B export is the optimal serving mode if the firm's initial productivity level ϕ fulfills the following condition: $\phi_{Fc1} > \phi > \sqrt[\kappa]{\left(\frac{I_E - I_F}{M_E - M_F}\right) r} \geq \phi_{Ec}^*$. Note also that as in Helpman et al. (2004) the firm never enters a new market with a combined export-FDI strategy due to constant returns to scale. This results changes e.g. if $\theta < 1$.

[11] This is one advantage of normalizing the initial period to be $t = 0$. The period of entry and the waiting time coincide and can be used interchangeably.

where $\alpha_g = \alpha\kappa$ is the growth rate of the per-period profit and $r > \alpha_g$ for a well defined problem.[12] Inaction stemming from exercising the option to wait also generates a value for the firm, as it delays the costs of entry. Therefore, entry can be optimal only provided that the difference between the benefit of entry (the value of the investment) and the fixed cost associated to it, evaluated in the initial period, is at its maximum. Formally, the optimal timing solves the following maximization problem:

$$F_i(\phi) = \max_T \left(\frac{M_i \phi^\kappa}{r - \alpha_g} \, e^{-(r-\alpha_g)T} - I_i e^{-rT} \right), \quad i \in \{E, F\}. \tag{12}$$

The FOC to this problem offers the best intuition for this optimal behavior. Equating $\partial F_i(\phi)/\partial T$ to zero, we obtain:[13]

$$M_i \phi^\kappa e^{\alpha_g T} = I_i r \quad , \quad i \in \{E, F\}. \tag{13}$$

which implicitly states that entry is optimal for the firm only if the per-period profit equates the user cost of capital, (Jorgenson, 1963). Otherwise, either anticipating or postponing entry is preferable.[14]

At the base of the firm's optimal decision, there is a no-arbitrage condition: the firm correctly anticipates that the per-period profits are growing at the rate α_g, and therefore finds it optimal to postpone its market entry till the periodical profits, i.e., the yield of the real investment, are not as big as the yield earnable from investing an amount equivalent to

[12] From (4), the growth rate of the per-period profit is $\frac{d\phi^\kappa}{\phi^\kappa} = \alpha\kappa = \alpha_g$. Note also that the value of the investment corresponds to the one derived in (8) if $T = 0$ and $\alpha = 0$.

[13] The second order condition for this maximization problem is fulfilled for $\alpha > 0$.

[14] It is interesting to compare the implications of this optimal condition for immediate market entry reexpressed in terms of the value of the investment. Setting $T = 0$ and rearranging equation (13) leads to $V_i(\phi, 0) = \frac{r}{(r-\alpha_g)} I_i$, which states that immediate market entry should occur only if the gross value of an investment is much larger than the fixed costs. This is the crucial difference between the Jorgensonian and Marshallian rules.

the market entry cost on the capital market.[15]

Crucial to this result is a positive growth rate of productivity: for $\alpha = 0$ the Jorgensonian rule coincides with the Marshallian rule, and the firm will never postpone a profitable market entry. And, conditional on a positive α, the higher the growth rate, the shorter will be the waiting time as the value of inaction depreciates relative to the value of action.

To develop our analysis let $V_i(\phi, 0) \equiv V_i(\phi)$, $V_i(\phi) - I_i$ be the net value function representing the net present value of strategy i executed at $T = 0$, and let ϕ_i^* be the productivity threshold which triggers immediate entry. Setting $T = 0$ in (13), this productivity cut-off level is

$$\phi_i^* = \sqrt[\kappa]{\frac{rI_i}{M_i}}. \tag{14}$$

The value function $F_i(\phi)$ has an interesting interpretation and its discussion here, in an easier setting, will make the comprehension of its fundamental role in the uncertain case easier. To explicitly determine it, solve (13) for the optimal entry time

$$T_i = \max\left(\frac{1}{\alpha_g}\ln\left[\frac{rI_i}{M_i\phi^\kappa}\right], 0\right), \qquad i \in \{E, F\} \tag{15}$$

and substitute (15) into (12) to obtain

$$F_i(\phi) = \begin{cases} A_{i_g}\phi^{\beta_g} & \phi < \phi_i^* \\ V_i(\phi) - I_i & \phi \geq \phi_i^* \end{cases}, \qquad i \in \{E, F\}, \tag{16}$$

where $A_{i_g} = \frac{\alpha_g}{r-\alpha_g}\, I_i \left(\frac{M_i}{rI_i}\right)^{\frac{r}{\alpha_g}}$, $\beta_g = \frac{r}{\alpha}$, and F_i is increasing and convex in productivity. Economically, F_i represents the option value of implementing strategy i delayed by $T_i \geq 0$ as opposed to the alternative strategy of immediate entry represented by the net value function. For $\alpha = 0$, no profitable investments are postponed and therefore the option value is null. By

[15] To develop further the intuition, think of the problem of the firm as one of a farmer deciding upon the right time to pick a cherry from the tree. Even in the absence of uncertainty so that the cherry will not be eaten by others, the farmer is confronted with two picking strategies: either pick it today at a cost I, or pick it when it has grown bigger, incurring only then the cost I of gathering it. Compare, in discrete time, the benefits of harvesting immediately to the discounted benefits of delaying picking: the farmer will choose action (reaping) only when the latter is smaller than the former. See Bertola (2010).

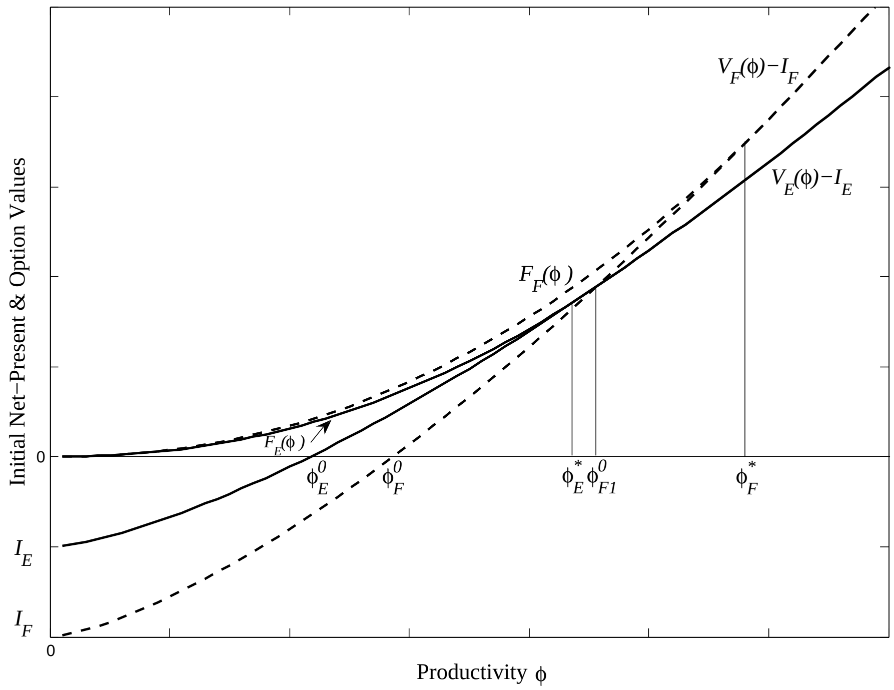

Figure 3: Value & Option Functions of Exporting and FDI

definition, whenever $F_i(\phi) \geq V_i(\phi) - I_i$, waiting is valuable to the firm, and only when the two functions become tangent, the execution of the investment is optimal. Investment i is optimal right from the initial period only for firms with productivity level above this cut-off level, while firms with productivity lower than this threshold wait T_i periods to enter because a higher investment value can be realized postponing the investment into the future relative to executing it right from the initial time.

Note that, as it is relevant for the uncertain case, the following two conditions, known respectively as the *value matching* and *the smooth pasting* condition, hold at ϕ_i^*:

$$F_i(\phi_i^*) = V_i(\phi_i^*) - I_i \tag{17}$$

$$\frac{\partial F_i(\phi_i^*)}{\partial \phi} = \frac{\partial V_i(\phi_i^*)}{\partial \phi} \qquad \text{with} \quad i \in \{E, F\} \tag{18}$$

and (13) is necessarily satisfied.

The option curves are introduced in figure 3: the continuous line for the export mode and the dashed line for the FDI mode, each situated above its respective net present value curve. At ϕ_i^0 the net value function of mode i crosses the horizontal axis, representing the productivity threshold beyond which investment i has a positive net value.[16]

To interpret this graph, it is instructive to consider first each entry mode separately. In the range of productivity between ϕ_E^0 and ϕ_{F1}^0, the net present value of exporting is positive, implying that exporting would be a profitable strategy. Applying the standard Marshallian rule, a firm within this productivity range would therefore enter the market already in the initial period. But it is the Jorgensonian rule, not the Marshallian rule, that implies optimal behavior: for all productivity levels below ϕ_E^* the export mode's option value $F_E(\phi)$ lies always above its net value function $V_E(\phi) - I_E$. Hence, firms with these productivity levels are better off postponing exporting into the future at T_E. Analogously, for any initial productivity level between $\phi_F^0 < \phi < \phi_F^*$ there exists for the firm a positive *value of waiting*, and its profitable FDI strategy is postponed until T_F.

The productivity cut-off levels ϕ_E^* and ϕ_F^*, which trigger the firm's optimal investment already in the initial period are at the tangency point between the net present value with the option value.[17] At these points, there is no value of waiting as the value of the option executed in $T_i = 0$ is equivalent to the investment value carried on from the initial period.

The ordinal relation between the two threshold levels is described by the same condition as in (10). However, such a relation is no longer sufficient to determine the optimal market entry mode. As figure 3 exemplifies, the exporting strategy is profitable with productivity level ϕ_E^*, but given the higher option value $F_F(\phi_E^*)$, the FDI strategy executed at T_F^* generates a higher investment value. In using equation (16) the relative position of the two option values can again be related to the proximity concentration trade-off as

$$F_E(\phi) < F_F(\phi) \quad \text{if} \quad \left(\frac{w_F}{w_E \tau_E^{\frac{1}{\theta}}} \right)^{\kappa\theta} < \frac{I_E}{I_F} \left(\frac{I_E}{I_F} \right)^{-\frac{\kappa}{\beta_g}}, \tag{19}$$

so that condition (10) and (19) can be represented in the same box introduced in the previous section. The locus of points on the dashed line in figure 4 is the level curve for equal option

[16] Becuase of the lower discount rate $r - \alpha_g$, $\phi_i^0 < \phi_{i_c}^*$.

[17] This is indeed the geometric interpretation of equations (17) and (18).

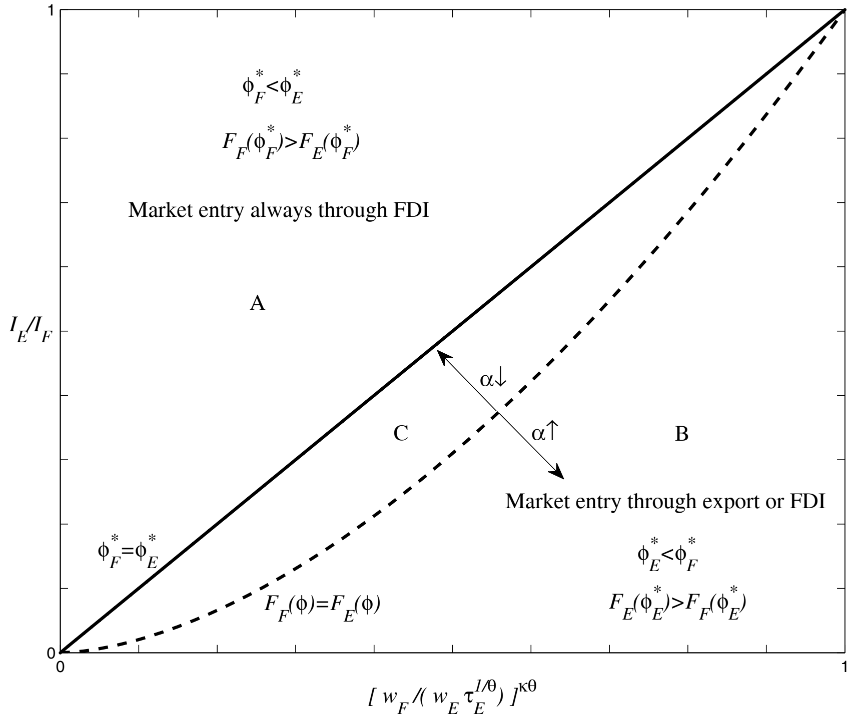

Figure 4: Relative Proximity-Concentration Cost Space with Growth

values and partitions the box into three areas. Area C between the diagonal line and the dashed curve depicts the case analyzed in figure 3 with $\phi_E^* < \phi_F^*$ and $F_F(\phi_E^*) > F_E(\phi_E^*)$. For these cost constellations a firm chooses to serve the market with FDI. Area A - $\phi_E^* > \phi_F^*$ and $F_F(\phi_F^*) > F_E(\phi_F^*)$ - is the FDI dominance region because, under these conditions, the FDI's option and net present values are the only envelope of all the curves. Indeed, we prove in appendix A that for cost constellations in area A and C

$$\left.\frac{F_F(\phi)}{V_E(\phi)}\right|_{\phi > \phi_E^*} > 1. \tag{20}$$

Finally, area B - $\phi_E^* < \phi_F^*$ and $F_F(\phi_E^*) < F_E(\phi_E^*)$ - is the ambiguous region where either FDI or exporting are possible outcomes. In all cases, whether the firm executes the investment immediately or waits some periods depends on its initial productivity level ϕ.

Relative to the static case, the FDI region expands, gaining area C at the expense of area B. The relative contraction of the latter depends on the growth rate of productivity, as for α that approaches zero, the dashed level curve becomes a straight diagonal line and conditions (10) and (19) coincide. A positive α is after all the sole source of the firm's profit growth, as opposed to constant fixed costs. Therefore, fixed costs also become, in discounted terms, relatively smaller, reducing the FDI's relative cost disadvantage. In the proximity-concentration trade-off, the choice of the firm then becomes biased toward the FDI mode. This intuition can be developed scrutinizing (19): an increase in α reduces the comparative fixed cost advantage of the export mode and increases the comparative variable cost advantage of the FDI strategy. Hence, in an inter-temporal setting productivity growth compensates for the relatively higher fixed costs of the FDI mode. This compensation increases the likeliness of market entry through FDI. In fact, for area C, $T_E < T_F$ follows from equation (15).

We summarize these results in the following proposition.

Proposition 2:
Assume $I_E < I_F$, $w_F < w_E \tau_E^{\frac{1}{\theta}}$, $\theta = 1$, $\nu \neq 1$, and $r > \alpha$. For $\alpha > 0$,

- *if $\left(\frac{w_F}{w_E \tau_E^{\frac{1}{\theta}}}\right)^{\kappa\theta} \leq \frac{I_E}{I_F}\left(\frac{I_E}{I_F}\right)^{-\frac{\kappa}{\beta_g}}, \left.\frac{F_F(\phi)}{V_E(\phi)}\right|_{\phi > \phi_E^*} \geq 1$, FDI is the firm's dominant strategy. $T_F = 0 \Leftrightarrow \phi \geq \phi_F^*$, $T_F > 0 \Leftrightarrow \phi < \phi_F^*$;*

- *if $\left(\frac{w_F}{w_E \tau_E^{\frac{1}{\theta}}}\right)^{\kappa\theta} > \frac{I_E}{I_F}\left(\frac{I_E}{I_F}\right)^{-\frac{\kappa}{\beta_g}}$, both mode-choices are possible. $T_i = 0 \Leftrightarrow \phi \geq \phi_i^*$, $T_i > 0 \Leftrightarrow \phi < \phi_i^*$;*

- *if $\phi_E^* < \phi_F^*$, $T_E < T_F$.*

Proof: *See appendix A and above.*

2.3 Export versus FDI with Stochastic Productivity Growth

In the stochastic case ($\alpha > 0, \sigma > 0$), the firm's market entry decision becomes a stochastic inter-temporal optimization problem and the relevant discount rate should account for risk. The perfect capital market assumption assures that, when the firm neglects immediate entry, it can invest into a financial asset which is perfectly correlated with the Geometric Brownian motion (5). This replication asset is assumed to pay no dividends and, therefore, its complete return can be attributed to its capital gain.[18] With reference to the capital asset pricing model (CAPM) the risk adjusted expected return rate μ comprises the risk-premium

$$R(\sigma) = \frac{(r_M - r)}{\sigma_M} \upsilon_{cM} \sigma, \tag{21}$$

where r_M and σ_M are the expected return rate and the volatility of the market portfolio, and υ_{cM} specifies the correlation between the replication asset and the market portfolio (Sharpe, 1964).[19]

Because of convex expected per-period profits in productivity, further adjustments are necessary. Similarly to the deterministic case, for ϕ^κ the expected growth rate is

$$\mathbb{E}\left(\frac{d\phi^\kappa}{\phi^\kappa}\right) = \alpha_u = \alpha\kappa + \frac{1}{2}\kappa(\kappa - 1)\sigma^2, \tag{22}$$

and we prove in appendix B that the equivalent risk-adjusted return rate on the capital market becomes[20]

$$\mu_u = r + \kappa(\mu - r). \tag{23}$$

Practically, this just means that the new discount rate μ_u is larger than r, so that the future is discounted faster than in the deterministic case.

[18] Within the option theory such a procedure is referred to as asset spanning or asset replication (Schwartz and Trigeorgis, 2004).

[19] The market portfolio consists of all assets in the capital market, where each asset is weighted by its market capitalization.

[20] Because of such convexity ($\kappa > 1$), the spanned asset likewise has to incorporate convex profit streams.

To determine the immediate entry productivity cut-off, in analogy with the deterministic case, the net value function of investment i, $V_{i_u}(\phi) - I_i$ has to be equated, in accordance with the Jorgensonian rule, with the option value $F_{i_u}(\phi)$. Again for bounded solutions we assume $\mu_u > \alpha_u$. The value of investment i evaluated at $T = 0$ is[21]

$$V_{i_u}(\phi) - I_i = \frac{M_i \phi^\kappa}{\mu_u - \alpha_u} - I_i \geq 0, \qquad i \in \{E, F\} \tag{24}$$

and we show in appendix C that the analytical derivation of the option value of investment i with the guess and verify method leads, similarly to (16), to

$$F_{i_u}(\phi) = A_{i_u} \phi^{\beta_u}, \qquad i \in \{E, F\}, \tag{25}$$

also increasing and convex in productivity. A_{i_u}, and β_u are, differently from the deterministic case, unknown parameters to be calculated jointly with the cut-off productivity level $\phi^*_{i_u}$ (see appendix C). At this aim, the three equilibrium conditions we impose are the *value-matching* condition (17), the *smooth-pasting* condition (18), and the following *limiting* boundary condition:

$$F_{i_u}(0) = 0, \quad i \in \{E, F\}. \tag{26}$$

The cut-off productivity levels for immediate entry are

$$\phi^*_{Eu} = \sqrt[\kappa]{\frac{\beta_u}{\beta_u - \kappa} \frac{I_E(\mu_u - \alpha_u)}{M_E}} \quad \text{and} \quad \phi^*_{Fu} = \sqrt[\kappa]{\frac{\beta_u}{\beta_u - \kappa} \frac{I_F(\mu_u - \alpha_u)}{M_F}}. \tag{27}$$

It is interestingly to note that their ordinal rank is again preserved - as described by (10).[22] This is because uncertainty influences both productivity cut-offs proportionally. As in the deterministic case, these conditions have to be evaluated jointly with the relative position of

[21] Set $V_{i_u}(\phi, T) = \int_T^\infty M_i \phi^\kappa e^{-(\mu_u - \alpha_u)s} ds$ to $V_{i_u}(\phi, 0)$.

[22] To compare them to the case of deterministic growth, rewrite (14) as $\phi^*_i = \sqrt[\kappa]{\frac{\beta_g}{\beta_g - \kappa} \frac{I_i(r - \alpha_g)}{M_i}}$.

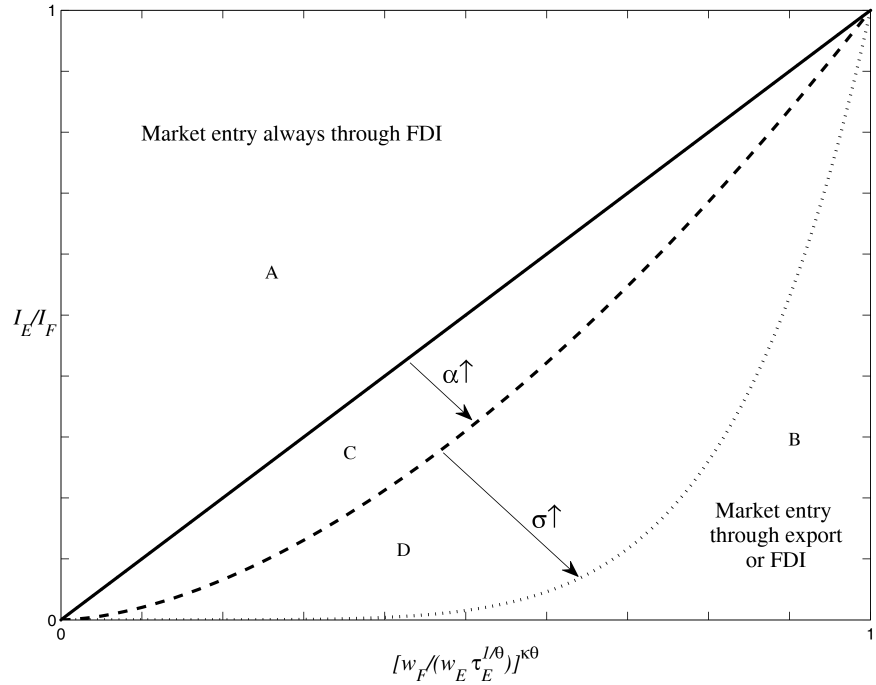

Figure 5: Relative Proximity-Concentration Cost Space with Stochastic Growth

the two option values to establish the firm's entry mode. The equivalent condition to (19) is

$$F_{E_u}(\phi) < F_{F_u}(\phi) \quad \text{if} \quad \left(\frac{w_F}{w_E \tau_E^{\frac{1}{\theta}}} \right)^{\kappa\theta} < \frac{I_E}{I_F} \left(\frac{I_E}{I_F} \right)^{-\frac{\kappa}{\beta_u}}. \tag{28}$$

β_u is the only difference between these two conditions and therefore defines the additional impact that uncertainty has on the optimal market entry mode. Using the fundamental quadratic equation, we show in appendix D that β_u is strictly decreasing in σ, $\beta_u > \kappa$, and $\beta_u = \beta_g$ only if $\sigma = 0$.

These effects are represented graphically in figure 5: the dotted level curve depicting the locus of points of equal option values extends, relative to the deterministic case, further to the bottom right corner of our box. It would coincide with the level curve derived in the deterministic case only for $\sigma = 0$. Therefore, the FDI-dominance region further extends from areas A and C to include additionally also area D, at the expense of area B, which is squeezed further. The indication we derive from this result is that uncertainty simply compounds the effects of productivity growth.

The economic intuition for this increase in FDI dominance is as follows. As a common real option result, uncertainty increases the option values of both market entry strategies. The higher value of waiting translates into postponed market entry. Due to the higher adjusted return rate in the stochastic case relative to the deterministic case ($\mu_u > r$), fixed costs incurred further away in the future are in prospective terms also smaller. Hence, uncertain productivity growth reduces the relative fixed cost disadvantage of the FDI mode over time. As the relevance of relative fixed costs decreases over time, it is the relative variable costs which become decisive for the optimal entry mode choice (see (28)).

We summarize our results in the following proposition.

Proposition 3:
Assume $I_E < I_F$, $w_F < w_E\tau_E^{\frac{1}{\theta}}$, $\theta = 1, \nu \neq 1$, $\mu_u > \alpha_u$. For $\alpha > 0$ and $\sigma > 0$,

- *if $\left(\frac{w_F}{w_E\tau_E^{\frac{1}{\theta}}}\right)^{\kappa\theta} \leq \frac{I_E}{I_F}\left(\frac{I_E}{I_F}\right)^{-\frac{\kappa}{\beta_u}}, \left.\frac{F_{Fu}(\phi)}{V_{Eu}(\phi)}\right|_{\phi>\phi^*_{Eu}} \geq 1$, FDI is the firm's dominant strategy. $T_F = 0 \Leftrightarrow \phi \geq \phi^*_{F_u}$, $T_F > 0 \Leftrightarrow \phi < \phi^*_{F_u}$;*
- *if $\left(\frac{w_F}{w_E\tau_E^{\frac{1}{\theta}}}\right)^{\kappa\theta} > \frac{I_E}{I_F}\left(\frac{I_E}{I_F}\right)^{-\frac{\kappa}{\beta_u}}$, both mode-choices are possible. $T_i = 0 \Leftrightarrow \phi \geq \phi^*_{i_u}$, $T_i > 0 \Leftrightarrow \phi < \phi^*_{i_u}$;*
- *if $\phi^*_E < \phi^*_F$, $T_E < T_F$.*

Proof: *Follows the same steps as in the proof of proposition 2.*

While we are able to explicitly derive the optimal entry time T_i for the Export and FDI strategy in the deterministic case, this is no longer possible in the uncertain case. Due to the stochastic evolution of productivity, the entry time is also a stochastic variable with probability density function $f(T_i)$ for $\phi \leq \phi^*_{i_u}$. Therefore, we refer in our analysis to the following expected entry time (see appendix E for its derivation from $f(T_i)$):[23]

[23] A detailed derivation is offered by Karatzas and Shreve (1991, p.196) or by Karlin and Taylor (1975, p.363).

$$\mathbb{E}(T_i(\phi^*_{i_u})) = \begin{cases} \frac{1}{\alpha - \frac{1}{2}\sigma^2} \ln\left(\frac{\phi^*_{i_u}}{\phi}\right) & \text{if} \quad \alpha > \frac{1}{2}\sigma^2 \\ \\ \infty & \text{if} \quad \alpha \leq \frac{1}{2}\sigma^2 \end{cases} \qquad \text{with} \quad i \in \{E, F\}. \tag{29}$$

Accordingly, for a sufficiently productive firm ($\phi > \phi^*_{i_u}$) market entry is realized immediately and the optimal entry mode is chosen based on proposition 3. However, for productivity levels $\phi < \phi^*_{i_u}$ market entry is postponed, on average, to $\mathbb{E}(T_i)$. Furthermore, only if the extent of uncertainty σ lies within the range $\{0, \sqrt{2\alpha}\}$ market entry is realized, otherwise the foreign market is never served. This is because a too high uncertainty ($\sigma > \sqrt{2\alpha}$) increases the likeliness of a productivity drop, which in turn reduces the probability of reaching the cut-off levels $\phi^*_{i_u}$ for a given α. As a result $\mathbb{E}(T_i)$ diverges.

The expected entry time depends on α and σ directly and indirectly through the productivity cut-off level. We characterize how it responds to these parameters: differentiating equation (29) with respect to α and σ yields

$$\begin{aligned} \frac{\partial \mathbb{E}(T_i)}{\partial \sigma} &= \frac{\sigma}{\left(\alpha - \frac{1}{2}\sigma^2\right)^2} \ln\left(\frac{\phi^*_{i_u}}{\phi}\right) + \frac{1}{\alpha - \frac{1}{2}\sigma^2} \frac{\partial \phi^*_{i_u}}{\partial \sigma} \frac{1}{\phi^*_{i_u}} \\ \frac{\partial \mathbb{E}(T_i)}{\partial \alpha} &= -\frac{1}{(\alpha - \frac{1}{2}\sigma^2)^2} \ln\left(\frac{\phi^*_i}{\phi}\right) + \frac{1}{\alpha - \frac{1}{2}\sigma^2} \frac{1}{\phi^*_i} \frac{\partial \phi^*_{i_u}}{\partial \alpha}, \end{aligned}$$

leading to the following proposition.

Proposition 4:

Given (5), $\beta_u > \kappa > 0$, $\phi < \phi^*_{i_u}$, $\alpha > 0$ *and* $\sigma > 0$,

- $\frac{\partial \mathbb{E}(T_i)}{\partial \alpha} < 0$,
- $\frac{\partial \mathbb{E}(T_i)}{\partial \sigma} > 0$.

Proof: *See appendix F.*

Overall, an increase in both productivity growth and uncertainty amplifies the dominance of the FDI strategy (see figure 5), but their effect on the expected entry time $\mathbb{E}(T_i)$ differs.

More precisely, at a given point in time a boost in productivity growth accompanied by a constant extent of volatility (or modest increase) leads to an anticipation of foreign market entry through FDI.

In contrast, an increase in volatility along a constant growth rate in productivity (or modest increase) leads to a postponement of the foreign market entry through FDI. Therefore, market entry that is observed in a specific period is shaped by the dynamic cumulative effect of these two forces. For instance, it comprises those firms that are anticipating entry because of a higher productivity growth as well as those firms that, because of an increased uncertainty, had initially retarded entry and for which entry timing has become mature.

According to empirical data, average firm productivity has experienced a strong boost in particular during the second half of the 1980s due to an accelerating IT revolution, accompanied by a modest volatility (Jorgenson, 2001). This temporary positive productivity shock came along with a first surge of FDI flows which lasted until the early 1990s (UNCTAD, 2009). Furthermore, Comin and Philippon (2005) provide evidence for a strong volatility increase on the firm level in the first half of the 1990s, escorted by a drop in FDI flows. Only a subsequent second positive productivity boost at the end of the 1990s and early 2000 (Jorgenson, 2001) gave rise to a second surge of FDI.

It is worth to record here that these waves of FDI flows during the last decades emerged along strong changes in productivity growth and volatility. Accounting for the effects of an interplay between α and σ alterations in our model, we can contribute to the explanation of these cyclical FDI flows in the long run and their surge in the short term.

Finally, it is interesting to note that the likeliness of market entry via FDI is enhanced along an increase of competition in the destination market. This follows from equations (27) and (28) as

$$\frac{\partial F_{F_u}(\phi)}{\partial \nu} > \frac{\partial F_{E_u}(\phi)}{\partial \nu} > 0 \quad \text{and} \quad \frac{\partial \phi^*_{Fu}}{\partial \nu} < \frac{\partial \phi^*_{Eu}}{\partial \nu} < 0.$$

Obviously, an increasing competitive environment in the destination market (i.e. $\nu \to 1$) favors the FDI mode as the optimal first time market entry strategy, likewise.

This result is also consistent with empirical observations on export and FDI patterns. Over the last four decades a broad range of countries experienced a significant increase in deregulation and market-friendly policies, in particular emerging economies (OECD, 2008). Such policies reduced barriers to market entry and triggered sooner or later strong foreign direct investments. As Comin and Philippon (2005) illustrate, deregulation policies have strong impacts on firm level volatility, which again is decisive for the observed timing of market entry.

3 Conclusion

We revise the *proximity-concentration* trade-off argument in an uncertain setting. We rely on the real option methodology, a typical approach in the investment literature, to introduce a type of uncertainty that is idiosyncratic to the firm. To keep our framework analytically tractable, we build the dynamics of our model on a partial equilibrium version of the monopolistic competitive model, while preserving the possibility of including other forms of uncertainty. In spite of the necessary simplifications, the main benefit of this approach is, differently to what is commonly assumed in the FDI literature, that firm's entry into the market is postponable. Therefore, both the timing and the mode of entry are determined. We find that uncertainty amplifies the effects that we derive for the deterministic case and biases the firm's mode choice toward the FDI strategy. We also show that the cyclical pattern of FDI flows in the past can be explained by changes in productivity growth and uncertainty.

In prospective terms, the IT revolution, which started in the mid 1980s, has pushed the productivity of firms, but it has initially required some learning, implying productivity patterns as modeled in our framework. Indeed, productivity decays while the firm is on the learning curve, and it boosts when the firm can exploit the full potential of the new technology. The prediction derived in our model indicates that we should then expect, relative to the past, entry via FDI more commonly than via export. Similarly, the deregulation in many industries starting from the late 1980s as well as the greater emphasis on competition policies, may also have contributed to the surge of FDI from the early 1990s.

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Appendix

A Proof for the Optimal Entry Mode in Area C

For all productivity levels with $\phi > \phi_E^*$, area C in figure 4 unambiguously leads to FDI.

Proof:

For any relative cost structures in area C in figure 4

$$\phi_E^* < \phi_F^* \tag{A.1}$$

and any investment strategy i is postponed for $\phi < \phi_E^*$.

Recall that at ϕ_E^* the value matching (17) and smooth pasting (18) condition hold. The two option value functions $F_E(\phi)$ and $F_F(\phi)$ are strictly convex in ϕ, whereas the net present value functions are convex (strictly convex for $\kappa > 1$).

For any productivity level with $\phi > \phi_E*$

$$\frac{F_F(\phi_E^*)}{V_E(\phi_E^*) - I_E} > 1 \tag{A.2}$$

if

$$\frac{\frac{\partial F_F(\phi_E^*)}{\partial \phi}}{\frac{\partial V_E(\phi_E^*)}{\partial \phi}} > 1 \quad \wedge \quad \frac{\frac{\partial^2 F_F(\phi_E^*)}{\partial \phi^2}}{\frac{\partial^2 V_E(\phi_E^*)}{\partial \phi^2}} > 1. \tag{A.3}$$

Due to the convexity of all four value functions, the two inequalities in (A.3) hold for

$$\left(\frac{w_F}{w_E \tau^{\frac{1}{\theta}}}\right)^{\kappa\theta} < \frac{I_E}{I_F}\left(\frac{I_E}{I_F}\right)^{(-\frac{\kappa}{\beta})}. \tag{A.4}$$

Relative cost patterns in area C always fulfill inequality (A.4). For cost constellations in area A, $\phi_{F_u} < \phi_{E_u}$ and hence FDI is the only entry choice.

B Equivalent Risk-Adjusted Return

Given the Geometric Brownian motion in equation (5) from Ito's lemma we have:

$$\mathbb{E}\left(\frac{d\phi^\kappa}{\phi^\kappa}\right) = \left(\kappa\phi^{\kappa-1}d\phi + \frac{1}{2}\kappa(\kappa-1)\phi^{\kappa-2}\sigma^2\phi^2 dt\right)/\phi^\kappa.$$

Substituting for $d\phi$ leads to

$$\mathbb{E}\left(\frac{d\phi^\kappa}{\phi^\kappa}\right) = \left(\alpha\kappa + \frac{1}{2}\kappa\sigma^2(\kappa-1)\right)dt + \kappa\sigma dz_t. \tag{B.1}$$

From the quadratic equation in (D.1) which is valid for ϕ^κ, with $\Psi(\kappa) = 0$ it follows that

$$\frac{1}{2}\kappa\sigma^2(\kappa-1) = r - (r - (\mu - \alpha))\kappa$$

Hence, the equivalent risk-adjusted rate of return for an exponential variable results as:

$$\mu_u = r - \kappa(\mu - r). \tag{B.2}$$

C Option Values under Uncertainty

Given (5) we derive $F_i(\phi)$ by applying the contingent claims approach and by spanning an asset that leads to the no arbitrage condition (see e.g. Dixit and Pindyck, (1994)),

$$dF_i(\phi) - \frac{\partial F_i(\phi)}{\partial\phi}d\phi - (\mu-\alpha)\frac{\partial F_i(\phi)}{\partial\phi}\phi dt = r\left(F_i(\phi) - \frac{\partial F_i(\phi)}{\partial\phi}\right)dt. \tag{C.1}$$

By using Ito's lemma we receive

$$\frac{1}{2}\sigma^2\phi^2\frac{\partial^2 F_i(\phi)}{\partial\phi^2} + (r - (\mu-\alpha))\phi\frac{\partial F_i(\phi)}{\partial\phi} - rF_i(\phi) = 0. \tag{C.2}$$

The solutions of these homogeneous differential functions are a linear combination of any two linearly independent guess solutions,

$$F_i(\phi) = A_{i1}\phi^{\beta_{u1}} + A_{i2}\phi^{\beta_{u2}}, \tag{C.3}$$

with the two exponents resulting from the fundamental quadratic equation:

$$\beta_{u1} = \frac{1}{2} - \frac{r-(\mu-\alpha)}{\sigma^2} + \sqrt{\left[\frac{r-(\mu-\alpha)}{\sigma^2} - \frac{1}{2}\right]^2 + \frac{2r}{\sigma^2}} > 1 \tag{C.4}$$

$$\beta_{u2} = \frac{1}{2} - \frac{r-(\mu-\alpha)}{\sigma^2} - \sqrt{\left[\frac{r-(\mu-\alpha)}{\sigma^2} - \frac{1}{2}\right]^2 + \frac{2r}{\sigma^2}} < 0. \tag{C.5}$$

The unknown parameters A_{i1}, A_{i2} are derived based on the value-matching (17) and smooth pasting condition (26). Additionally, we introduce a limiting boundary condition $F_i(0) = 0$. This limiting condition implies that $A_{i2} = 0$. Based on these three boundary conditions the two option values result as

$$\begin{aligned} F_i(\phi) &= A_{i_u}\phi^{\beta_u} \qquad (C.6)\\ &= M_i^{\frac{\beta_u}{\kappa}} I_i^{1-\frac{\beta_u}{\kappa}} \Lambda\ \phi^{\beta_u}\\ &\text{with}\quad \Lambda = \frac{1}{(\mu_u-\alpha_u)}\left(\frac{\beta_u(\mu_u-\alpha_u)}{\beta_u-\kappa}\right)^{1-\frac{\beta_u}{\kappa}} - (\mu_u-\alpha_u)\left(\frac{\beta_u(\mu_u-\alpha_u)}{\beta_u-\kappa}\right)^{-\frac{\beta_u}{\kappa}}\\ &\beta_{u1} = \beta_u \quad \text{and} \quad A_{i1} = A_{i_u}, \qquad i \in \{E, F\}. \end{aligned}$$

D The Fundamental Quadratic Equation

Substituting the guess solution (C.6) into the linear differential equation (C.2), we receive the fundamental quadratic equation

$$\Psi = \frac{1}{2}\sigma^2\beta_u(\beta_u - 1) + (r - (\mu - \alpha))\beta_u - r = 0. \tag{D.1}$$

Consider the total differential

$$\frac{\partial \Psi}{\partial \beta_u}\frac{\partial \beta_u}{\partial \sigma} + \frac{\partial \Psi}{\partial \sigma} = 0, \tag{D.2}$$

which can be evaluated at $\beta_u = \beta_{1_u}$. The quadratic equation Ψ increases in β_u with $\partial\Psi/\partial\beta_u > 0$. The derivative of Ψ with respect to σ results as

$$\frac{\partial \Psi}{\partial \sigma} = \sigma\beta_u(\beta_u - 1) > 0, \tag{D.3}$$

because of C.5 . From D.2 necessarily we have

$$\frac{\partial \beta_u}{\partial \sigma} < 0. \tag{D.4}$$

Furthermore, the discount rate of periodical profits in equation (24) turns out to be the negative expression of Ψ evaluated at κ. Note that

$$\mu_u - \alpha_u = r - (r - (\mu - \alpha))\kappa - \frac{1}{2}\kappa(\kappa - 1)\sigma^2. \tag{D.5}$$

For bounded results, this discount rate needs to be strictly positive and, hence, κ must lie between the two roots, specifically: $\beta_u > \kappa > 0$. As a consequence

$$\frac{\partial\left(\frac{\beta_u}{\beta_u - \kappa}\right)}{\partial \sigma} > 0. \tag{D.6}$$

For $\sigma = 0$, we have $\mu = r$ and from equation (D.1) it follows that

$$\beta_u = \frac{r}{\alpha} = \beta_g. \tag{D.7}$$

E Expected Entry Time

By using the Girsanov theorem[24] it is possible to derive the probability density function of the waiting time T_i as

$$f(T_i, \phi, \phi^*_{i_u}) = \frac{\ln\left(\frac{\phi^*_{i_u}}{\phi}\right)}{\sqrt{2\pi\sigma^2 T_i^3}}\; e^{-\frac{\left(\ln\left(\frac{\phi^*_{i_u}}{\phi}\right) - (\alpha - \frac{1}{2}\sigma^2)T_i\right)^2}{2\sigma^2 T_i}} \tag{E.1}$$

$$\text{with} \quad \phi^*_{i_u} > \phi.$$

The Laplace transform of T_i is then given by (see Ross, 1996; Proposition 8.4.1)

$$\mathbb{E}\left(e^{-\lambda T_i^*}\right) = \int_0^\infty e^{-\lambda T_i} f(T_i) dT_i = e^{-\left(\sqrt{(\alpha-\frac{1}{2}\sigma^2)^2 + 2\sigma^2\lambda} - (\alpha - \frac{1}{2}\sigma^2)\right)\frac{\ln\left(\frac{\phi^*_{i_u}}{\phi}\right)}{\sigma^2}} \tag{E.2}$$

and can be used to determine the expected waiting time as

$$\mathbb{E}(T_i) = \int_0^\infty T_i f(T_i) dT_i = -\lim_{\lambda\to 0} \frac{\partial \mathbb{E}(e^{-\lambda T_i})}{\partial \lambda} = \frac{\ln\left(\frac{\phi^*_{i_u}}{\phi}\right)}{\alpha - \frac{1}{2}\sigma^2}. \tag{E.3}$$

More precisely

$$\mathbb{E}(T_i(\phi^*_{i_u})) = \begin{cases} \frac{1}{\alpha - \frac{1}{2}\sigma^2} \ln\left(\frac{\phi^*_{i_u}}{\phi}\right) & \text{if} \quad \alpha > \frac{1}{2}\sigma^2 \\ \\ \infty & \text{if} \quad \alpha \le \frac{1}{2}\sigma^2 \end{cases} \tag{E.4}$$

$$\text{with} \quad \phi^*_{i_u} > \phi \quad \text{and} \quad i \in \{E, F\}.$$

[24] A detailed derivation is offered by Karatzas and Shreve (1991, p.196) or by Karlin and Taylor (1975, p.363).

F Expected Entry Time and Comparative Statics

Exploiting the monotonicity of $V^*_{i_u}$ in $\phi^*_{i_u}$ we prove that $\frac{\partial \phi^*_{i_u}}{\partial \sigma} > 0$ and $\frac{\partial \phi^*_{i_u}}{\partial \alpha} < 0$ by proving that $\frac{\partial V^*_{i_u}}{\partial \sigma} > 0$ and $\frac{\partial V^*_{i_u}}{\partial \alpha} < 0$, respectively.

Rearranging (27) we obtain

$$\frac{M_i \phi^{*\kappa}_{i_u}}{\mu_u - \alpha_u} = V^*_{i_u}(\phi^*_{i_u}) = \frac{\beta_u}{\beta_u - \kappa} I_i. \tag{F.1}$$

The derivative of $V^*_{i_u}(\phi^*_{i_u})$ with respect to σ results as

$$\frac{\partial V^*_{i_u}}{\partial \sigma} = \frac{\partial \beta_u}{\partial \sigma} I_i \left(\frac{-\kappa}{(\beta_u - \kappa)^2} \right). \tag{F.2}$$

From D.1 we can derive

$$\frac{\partial \beta_u}{\partial \sigma} = -\frac{\beta_u \sigma (\beta_u - 1)}{\sigma^2(\beta_u - \frac{1}{2}) + r - (\mu - \alpha)}. \tag{F.3}$$

Substituting into F.2 results in

$$\frac{\partial V^*_{i_u}}{\partial \sigma} = \frac{V^*_{i_u} \sigma (\beta_u - 1) \kappa}{(\sigma^2(\beta_u - \frac{1}{2}) + r - (\mu - \alpha))(\beta_u - \kappa)}. \tag{F.4}$$

For $\beta_u > 1$ and $\kappa \geq 1$ the numerator is always positive. We can prove that the denominator is also always positive. To do so we rewrite (C.5) as

$$(\beta_u - \frac{1}{2})\sigma^2 + r - (\mu - \alpha) = \sigma^2 \sqrt{\left(\frac{r - (\mu - \alpha)}{\sigma^2} - \frac{1}{2} \right)^2 + \frac{2r}{\sigma^2}} > 0. \tag{F.5}$$

The right hand side of this equation is always positive for $\beta_u > 1$ and hence $\frac{\partial V^*_{i_u}}{\partial \sigma} > 0$. Furthermore,

$$\frac{\partial V^*_{i_u}}{\partial \alpha} = \frac{\partial \beta_u}{\partial \alpha} I_i \left(\frac{-\kappa}{(\beta_u - \kappa)^2} \right). \tag{F.6}$$

From F.2 we receive

$$\frac{\partial \beta_u}{\partial \alpha} = \frac{-\beta_u}{(\beta_u - \frac{1}{2})\sigma^2 + r - (\mu - \alpha)} < 0. \tag{F.7}$$

Hence we have $\frac{\partial V^*_{i_u}}{\partial \alpha} > 0$. Since $V^*_{i_u}$ behaves as $\phi^*_{i_u}$ we can state

$$\frac{\partial \phi^*_{i_u}}{\partial \alpha} < 0 \quad \wedge \quad \frac{\partial \phi^*_{i_u}}{\partial \sigma} > 0. \tag{F.8}$$

With these results we can consider the following partial derivatives of E.4:

$$\frac{\partial \mathbb{E}(T_i)}{\partial \sigma} = \frac{\sigma}{\left(\alpha - \frac{1}{2}\sigma^2\right)^2} \ln\left(\frac{\phi^*_{i_u}}{\phi}\right) + \frac{1}{\alpha - \frac{1}{2}\sigma^2} \frac{\partial \phi^*_{i_u}}{\partial \sigma} \frac{1}{\phi^*_{i_u}} > 0 \tag{F.9}$$

$$\frac{\partial \mathbb{E}(T_i)}{\partial \alpha} = -\frac{1}{(\alpha - \frac{1}{2}\sigma^2)^2} \ln\left(\frac{\phi^*_i}{\phi}\right) + \frac{1}{\alpha - \frac{1}{2}\sigma^2} \frac{1}{\phi^*_{i_u}} \frac{\partial \phi^*_i}{\partial \alpha} < 0. \tag{F.10}$$

In both modes expected entry time increases in σ and decreases in α.