Globalization, Endogenous Growth and Unemployment: How to Remove Scale Effects? – Preliminary and Incomplete –

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April 11, 2011

JEL-Classification: E24, F16, F43
Keywords: Schumpeterian Growth, Globalisation, Scale Effects, Rent Protection, Unemployment, Wage Bargaining

Abstract

Eliminating scale effects from a Schumpeterian endogenous growth model by the rent protection approach (Dinopoulos & Syropoulos, 2007) leads to ambiguous results concerning the comparative statics of steady state variables with respect to globalisation variables, if labor markets are not competitive. For one case, I show the existence of a steady state numerically. By contrast, using a simpler approach as in Dinopoulos & Segerstrom (1999) leads to unambiguous results, but opposite to those with the rent protection approach. I also show the existence of a steady state numerically. If labor markets are competitive, the results do not differ.

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1 Introduction

It is widely accepted that to build Schumpeterian growth models, these should not exhibit scale effects. There are different ways to remove them. But which approach you use affects heavily the features of steady state variables, such as the innovation rate, with respect to globalisation variables, if labor markets are *not* competitive, as I will show.

I consider two similar, but different ways of getting rid of the scale effect. Both fall under the category of permanent-effects-on-growth (PEG) approaches[1]. One way assumes that the discovery of new products or the improvement of quality becomes exogenously more difficult with an increasing population size. It was first used by Dinopoulos & Thompson (1996) and is a simplification of a model where the number of industries rises with an increasing population such that the number of researchers per product line remains constant (Dinopoulos & Thompson, 1998; Young, 1998). I call this approach the product line (PL) approach. A second way assumes that firms have to pay for measures which slow down imitating or understanding and overcoming their technology. That's the rent protection (RP) approach, introduced by Dinopoulos & Syropoulos (2007). Means for rent protection capture means against industrial espionage as well as paying lawyers who enforce patents, trademarks, designs and copyrights and also paying the personnel which detects violations against intellectual property rights. A common mean against industrial espionage is improving IT security, which is a steady process since hackers find new

[1] The notion of "permanent effects on growth" stems from Dinopoulos & Thompson (1996) and refers to the insight that changes in policy variables have permanent effects on growth.

ways of entering IT systems. Rent protection activities can be modeled both as flow and as stock variables. To give an example for a stock variable, think of a lobbyist who succeeds in convincing the parliament to pass a law that forbids selling counterfeits. Once this law is established, it "pays" for innovating firms.

The contribution of this paper is that I clarify the question whether the decision between the PL and RP approach matters. In a North-South trade model where the Southern imitation rate is taken as given, Grieben & Şener (2009) use the RP approach and find, inter alia, that a higher relative Southern population level increases the Northern innovation rate, and a higher Southern imitation rate decreases the Northern innovation rate. However, these findings are not robust to the choice between the PL and the RP approach, as I will show below. If labor markets are competitive, there is no difference in the results between the two approaches.

The model is similar to Grieben & Şener (2009), but simpler as I reduced the model to the main aspects which are necessary for the results. The model below is also close to Petsas (2010), who considers semi-endogenous Schumpeterian growth in two countries, but where the location of the quality leader is determined by comparative advantage in research. He makes similar assumptions about the relative wages of the countries. In a working paper version (Petsas, 2008), he compared the PL approach with the TEG (temporary effects on growth) approach and found that the results are not robust concerning the comparative advantage.

The second contribution of my paper is that I derive the transitional dynamics along the balanced-growth path for this kind of models. Most other models only consider the steady state equilibrium without proving its existence and its stability.

Arnold (2002) analyses the effects of globalisation on the growth rate by looking at a North-South trade model. Globalisation is modelled as an increase in the Southern imitation rate. His model shows scale effects, so he claims that his model can at best be interpreted at a medium-run perspective. It is unclear whether his model is robust to a version without the scale effect property.

2 The Model

There are two countries, North (N) and South (S). There is a continuum of industries, and in all industries in the North, firms seek to develop better quality levels of the existing products. The imitation activity in the South is exogenous, while it is endogenous in the North. A new quality level improves the consumer's satisfaction by λ relative to the previous quality level. So quality improves stepwise, not continuously. Once a firm develops a new quality level, it owns a patent for the corresponding technology and hence has a monopoly position, while the technology of the previous quality level immediately becomes common knowledge and is hence produced and sold competitively. If a Southern firm successfully obtains the knowledge about how to produce the goods, this knowledge also becomes common knowledge, implying perfect competition.

In the North, the labor market is unionized and unions bargain with firms over wages. For simplicity, the Southern consumption level is exogenous.

2.1 The Model's Assumptions

2.1.1 Industries and Price Setting

There is a continuum of industries which I index by ω, and ω is between 0 and 1. In all industries, firms face Bertrand price competition.

2.1.2 Consumer's Like Higher Quality Products

Households in the North have an initial size of L_{N0} and grow at rate g_L. They maximize the standard lifetime utility function

$$U = \int_0^\infty L_N(t)e^{-\rho t}u(t)dt, \tag{1}$$

where I assume that $\rho > g_L$, subject to the intertemporal budget constraint

$$\dot{A}(t) = r(t)A(t) + W(t) - c_N(t)L_N(t). \tag{2}$$

where $A(t)$ are the household's assets, $r(t)$ is the interest rate at time t, $W(t)$ is the expected household income and $c_N(t)$ is the per capita consumption expenditure. The instantaneous utility function $u(t)$ is

$$u(t) = \int_0^1 \log \sum_{j=0}^\infty \lambda^j x_N(j,\omega,t)d\omega, \tag{3}$$

where λ is the quality parameter, j is the quality level and x_N is the quantity that is bought of the good from industry ω. Households maximize instantaneous utility subject to

$$c_N(t) = \int_0^1 \sum_{j=0}^\infty p(j,\omega,t)x_N(j,\omega,t)d\omega. \tag{4}$$

For the South, I consider per capita expenditures c_S to be exogenous. Nevertheless, households make the same consmption decisions as in the North. Households in the South also have an initial size L_{S0} and grow at the same rate g_L.

2.1.3 Labor

I only consider the Northern labor market. There is only one type of labor which can work either in the production of goods or in research and development.

2.1.4 Constant Returns To Scale in Production

Firms need one unit of labor to produce one unit of their consumption good.

2.1.5 Constant Returns To Scale in Innovation

There is free entry in innovation, and there are no fixed costs for starting innovation. The innovation process is linear, such that the probability of an innovation in firm m in industry ω increases proportionally with the research activities R invested in innovation – think of research activities as representing lab experiments or developing prototypes – such that

$$\iota_m(\omega) = \frac{R_m(\omega)}{D(\omega)}, \tag{5}$$

where D measures the difficulty of innovation. The innovation probabilities are independent across firms and industries and follow a Poisson distribution. Therefore, the innovation probability in industry ω is

$$\iota(\omega) = \sum_{m=0}^{\infty} \iota_m(\omega) = \sum_{m=0}^{\infty} \frac{R_m(\omega)}{D(\omega)}. \tag{6}$$

Since all industries are structurally identical, we can henceforth omit the industry index ω.

2.1.6 Larger Populations Make Innovation More Difficult

Following Dinopoulos & Segerstrom (1999), R&D difficulty D increases proportionally with the population size, so $D = k \cdot (L_N + L_S)$. This is the PL approach to R&D difficulty. It implicitly captures the idea that a larger population goes in line with more product lines and that therefore the R&D personnel per product line remains constant over time or across countries of different size.[2]

2.1.7 Decentralized Wage Bargaining

After a successful innovation, a firm has to bargain with the labor union about wages. R&D firms must by law pay the same wage to R&D workers. There's no bargaining about the employment level. The union's objective function W_{LU} is simple the wage per worker

$$W_{LU} = w_N \tag{7}$$

As wage bargaining is decentralized, we can neglect other aspects such as the unemployment rate, since bargaining with a single firm does not the aggregate unemployment rate.

2.2 Equilibrium

2.2.1 Households ...

... look at quality-adjusted prices Households maximize their utility in two steps: They first maximize instantaneous utility and always buy the good with the lowest quality-adjusted price $\frac{p(j,\omega,t)}{\lambda^j}$.

[2]Dinopoulos & Thompson (1998) provide microfoundations.

... follow the Keynes-Ramsey rule They then maximize lifetime utility, and I get the standard result:

$$\frac{\dot{c}_N(t)}{c_N(t)} = r(t) - \rho \tag{8}$$

2.2.2 Prices and Profits

I conjecture that the wage in the North is $w_N > 1$ and the Southern wage is the numéraire, $w_S \equiv 1$. The previous quality level is produced competitively and therefore sold at marginal costs. So the price charged by Southern imitators in the North is $p_N^S(j = k-1) = 1$, which is lower than the price which the Northern followers would charge, $p_N^N(j = k-1) = w_N$. So quality leaders compete against the Southern followers in the North. The price charged in the South by Southern followers is $p_S^S(j = k-1) = 1$, and here the Southern followers price out the Northern followers as well.

Top quality producers can charge a quality markup of λ against producers of the previous quality level. To maximize profits, top quality producers charge $p_N^H(j = k) = \lambda - \varepsilon$ in the North and $p_S^H(j = k) = \lambda - \varepsilon$ in the South, where $\varepsilon \to 0$ in order to price out quality followers and catch the whole market. So consumers only buy the top quality product and the profit of the Northern producers is

$$\pi_N = \frac{c_N L_N}{\lambda} (\lambda - w_N) + \frac{c_S L_S}{\lambda} (\lambda - w_N) \tag{9}$$

2.2.3 Firm Value

The firm obtains instantaneous profits π_N for the time period dt. With probability $1 - (\iota + \mu)dt$, it is not replaced after time dt by a different firm and keeps its value v_N plus an increase in the firm value $\dot{v}_N$ per time period dt. With probability $(\iota + \mu)dt$, a better quality level is developed and the firm loses its value. The future value of the firm has to be discounted by the interest rate. Hence, the expected firm value is given by

$$v_N = \frac{1}{1 + rdt} (\pi_N dt + (1 - (\iota + \mu)dt)(v_N + \dot{v}_N dt) + (\iota + \mu)dt0) \tag{10}$$

Rearranging gives

$$rv_N dt = \pi_N dt + (1 - (\iota + \mu)dt)\dot{v}_N dt - (\iota + \mu)dtv_N \tag{11}$$

and collecting terms, dividing by dt and letting $dt \to 0$ gives the firm value

$$v_N = \frac{\pi_N}{r + \iota + \mu - \frac{\dot{v}_N}{v_N}}. \tag{12}$$

2.2.4 Balanced Trade

There is no international debt. The firm's profits are given to households via dividends. So for the North, household expenditures equal firm revenue

$$c_N L_N = n_N \left(c_N L_N + c_S L_S \right) \tag{13}$$

where n_N is the share of the Northern industries[3]. Equation (13) can be rearranged to

$$c_N L_N = c_S L_S \frac{n_N}{1 - n_N}. \tag{BT}$$

2.2.5 Innovation

The innovating firm maximizes the expected firm value v_N minus R&D costs with respect to R&D activity, that is

$$\max_{R_m} v_N \iota_m dt - w_N a_R R_m dt. \tag{14}$$

where a_R is the unit labor requirement of R_m. To have a finite equilibrium value of R_m, we need that the firm value equals R&D costs per unit of innovation.

$$v_N = w_N a_R D. \tag{15}$$

If the firm value were below the *RHS*, no firm would innovate; hence this cannot be an equilibrium with positive innovation. If the firm value were above the *RHS*, all firms would try to put infinitely many resources into R&D and none into production, which cannot be an equilibrium either.

2.2.6 Industry flows

During the time interval dt, the outflow of industries from the North to the South is $\mu dt n_N$, and the inflow is $\iota dt(1 - n_N)$, so that the change in the share of the Northern industries is

$$\dot{n}_N dt = \iota dt(1 - n_N) - \mu dt n_N \tag{16}$$

and dividing by dt gives

$$\dot{n}_N = \iota(1 - n_N) - \mu n_N \tag{17}$$

2.2.7 Labor markets

A firm that wants to produce and sell its newly developed top-quality product immediately finds workers, but before it can start to produce, it has to bargain with a labor union over the wage for production workers. R&D firms must by law pay the same wage to their workers.[4]

[3] It can also be interpreted as the average share of time in which the quality leader is located in the North.

[4] Otherwise, the wage in the R&D sector would be competitive and there would be no unemployment.

Wage Bargaining The Nash bargaining problem is

$$\max_{w_N} \Omega = (W_{LU})^{\alpha} (v_N)^{1-\alpha}. \tag{18}$$

The labor union bargains with each firm separately. Since there is a continuum of industries, it does not take into account the effect of the wage on the employment rate.

The bargained wage of production workers is hence

$$w_N = \alpha \frac{c_N L_N + c_S L_S}{\frac{c_N L_N}{\lambda} + \frac{c_S L_S}{\lambda}} = \alpha\lambda.^5 \tag{19}$$

Labor supply and demand As I allow for unemployment, the labor market equation in the North is

$$n_N \left(\frac{c_N L_N}{\lambda} + \frac{c_S L_S}{\lambda} \right) + a_R R + u_N L_N = L_N. \tag{20}$$

The first term on the left hand side is the demand for production workers: n_N is the share of industries whose quality leader is located in the North, and the term in parentheses is the global demand for consumption goods. The second term is the demand for R&D workers. The third term comprises the unemployed workers. On the right hand side is the supply of workers.

Since I assume that the Southern consumption level c_S is exogenous, I do not consider the Southern labor market.

2.3 Steady State

I solve the steady state for a constant per capita consumption level in the North, which implies $r(t) = \rho$ by (8).

2.3.1 Constant share of industries

In the steady state, the share of industries in each country is constant, that is $\dot{n}_N = 0$. So the steady state share of the North industries is

$$n_N = \frac{\iota}{\iota + \mu}. \tag{21}$$

2.3.2 Consumption

I use (BT) and (21) to derive the steady state consumption in the North,

$$c_N = \eta_S \frac{\iota}{\mu} \tag{22}$$

where $\eta_S \equiv \frac{L_S}{L_N}$.

[5]With $\beta \neq 1$, the result would be $w_N = \frac{\alpha\beta}{1-\alpha+\alpha\beta} \frac{c_N L_N + c_S L_S}{\frac{c_N L_N}{\lambda} + \frac{c_S L_S}{\lambda}} = \frac{\alpha\beta}{1-\alpha+\alpha\beta}\lambda$.

2.3.3 Unemployment

I use the steady state consumption in the North, (22), in the labor market equation (20), to obtain the unemployment rate

$$u_N = 1 - \iota\left(\frac{c_S\eta_S}{\lambda\mu} + A\right), \tag{23}$$

where $A \equiv a_R k(1+\eta_S)$.

2.3.4 Innovation Rate

I use the steady state consumption (22) and the wage (19) in the free entry in innovation condition (15) to obtain

$$c_S\eta_S\frac{\iota+\mu}{\mu}\frac{(1-\alpha)/\alpha}{\rho+\iota+\mu-g_L} = A. \tag{24}$$

The left hand side is the expected benefit of innovation per capita divided by w_N, and the right hand side is the resource requirement of R&D.

2.3.5 Comparative Statics

Proposition 1. *A higher relative Southern population level decreases the innovation rate ι. A higher imitation level μ increases the innovation rate ι.*

For a given innovation rate ι, the expected benefit increases with a higher relative Southern population level η_S. The RHS of (24) also increases with a higher η_S, but it increases less than the LHS.[6] Since the LHS is increasing with ι, the innovation rate must therefore decrease in equilibrium with a higher η_S.

The expected benefit increases in ι, since the percentage change in per capita profits $\left(\frac{c_S\eta_S}{\lambda}\frac{\iota+\mu}{\mu}\frac{1-\alpha}{\alpha}\right)$ with a higher Northern innovation rate is larger than the percentage change in the discount rate $(\rho+\iota+\mu-g_L)$ with a higher innovation rate. Per capita profits increase with a higher ι because it increases the Northern industry share and the per capita profit from sales. The discount rate increases with a higher ι, because it reduces the expected incumbency period of the firm.

If the imitation rate increases, per capita profits decrease and the discount rate increases, leading to an unambiguous decrease of expected benefits. To reestablish the equality, the innovation rate must hence increase, as it increases the expected benefit.

[6] Note that the LHS divided by η_S is equal to $a_R k\left(\frac{1}{\eta_S}+1\right)$, which is hence equal to the partial derivative of the LHS wrt η_S, but the derivative of the RHS wrt η_S is $a_R k$, which is smaller than $a_R k\left(\frac{1}{\eta_S}+1\right)$.

2.4 Transitional dynamics – Very preliminary

I solve the model for the share of Nortern industries, n_N (see Appendix C.1 for the calculations). As the stability properties of the resulting differential equation can not be analyzed algebraically, I investigate the stability issue numerically.[7]

Following Steger (2003) and Basu (1996), λ is between 1.1 and 1.4. I start with $\lambda = 1.4$. To ensure $w_N > 1$ and since $w_N = \alpha\lambda$, I set $\alpha = 0.72$ which yields $w_N = 1.008 > 1$. For the population growth rate, I follow Grieben & Şener (2009) and set it to $g_L = 0.12$, which is the annual population growth rate of middle income and high income countries. The discount rate $\rho = 0.07$ equals the real interest rate to the U.S. stock market (Mehra & Prescott, 1985).

The other parameter values have been chosen in an ad-hoc manner in a first step with the objective to get a reasonable Northern unemployment rate $u_N < 20\%$ and a Northern per-capita consumption $c_N > c_S$. I set the imitation rate to $\mu = 0.145$, Southern per-capita consumption to $c_S = 1$, the relative Southern population level to $\eta_S = 1$, unit labor requirement of R&D to $a_R = 1$, R&D dificulty parameter to $k = 0.5$, and the initial share of Northern industries to $n_{N0} = 0.3$.

The development of the Northern industry n_N share is determined by equation (68). Northern per-capita consumption is given by the balanced trade condition (BT). The unemployment rate is given by (71) and the innovation rate by (65). Figure 1 shows the development of these variables over time. As can be seen, the economy converges to a stable steady state.

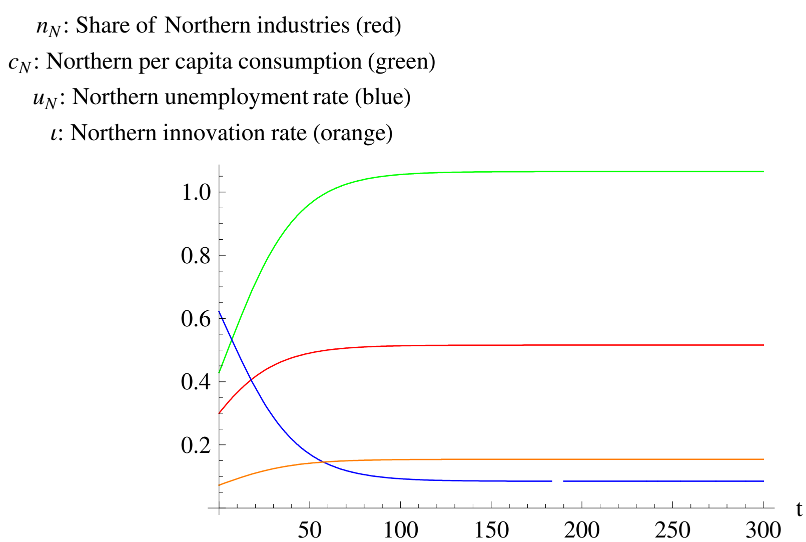

Figure 1: Development of the Northern economy along the balanced growth path

[7] Mathematica file available upon request.

3 The Model With the RP Approach

3.1 Changing Assumptions

I can keep most assumptions: Sections 2.2.1, 2.2.2 and 2.2.4 - 2.2.6 remain valid. In the North, I distinguish now between general purpose (GP) workers who work in R&D and in production, and between specialized workers who work only in rent protection (RP) activities. GP and RP workers are exogenous fractions of the labor force. Only GP workers bargain with the firm, and R&D firms must by law pay the same wage to R&D workers. So general purpose workers earn the same wage in the steady state, and only general purpose workers are unemployed.

3.1.1 R&D difficulty

I change the measure of R&D difficulty D as not being exogenously present as in the PL approach, but firms have to invest for it, e.g., they pay lawyers or lobbyists. The R&D difficulty is a stock variable. It has an initial value of $D(0) = D_0$ and it increases by

$$\dot{D}(t) = n_N \delta X(t), \tag{25}$$

where $X(t)$ is the level of rent protection activities at time t and δ is a rent protection efficiency parameter.

3.1.2 Labor markets

The labor market is, following Dinopoulos & Syropoulos (2007), divided into a labor force of share $1-s$ working in production and in research and development, and into a labor force of share s working in rent protection activities. I refer to the first share of workers as general purpose workers and to the second share as specialized workers.

Only general purpose workers are unionized. The market for specialized workers is perfectly competitive.

3.2 Equilibrium

3.2.1 Profits

The profits of the quality leader are now reduced by rent protection activities:

$$\pi_N = \underbrace{\frac{c_N L_N}{\lambda}(\lambda - w_N^{GP}) + \frac{c_S L_S}{\lambda}(\lambda - w_N^{GP})}_{\pi_N^P} - w_N^{RP} a_X X \tag{26}$$

where π_N^P are the profits from production, w_N^{GP} is the wage for general purpose workers, w_N^{RP} is the wage for rent protection workers, X is the level of rent protection activities and a_X is the unit labor requirement for X.

3.2.2 Labor Markets

As mentioned before, the Northern labor market is divided into a fraction $(1-s)$ of general purpose workers, working in production and in R&D:

$$n_N\left(\frac{c_N L_N}{\lambda}+\frac{c_S L_S}{\lambda}\right)+a_R R=(1-s-u_N)L_N, \tag{20a}$$

and a fraction s working only in rent protection,

$$n_N a_X X = sL_N \tag{20b}$$

where a_X is the unit labor requirement of rent protection activities.

Since only general purpose workers are unionized, only these workers can be unemployed.

3.2.3 Level of R&D difficulty

As I seek to analyze the steady state first, I can use the fact that, for a constant innovation rate, the level of R&D difficulty has to grow at the same rate as the resources invested into R&D, and both grow at rate g_L[8]. Combining then $\frac{\dot{D}}{D}=g_L$ and (25) yields $D=n_N\delta X/g_L$. [9]

Using (20b) in $D(t)$, we get $D(t)=\frac{\delta s}{a_X g_L}L_N(t)$, where the term $\frac{\delta s}{a_X g_L}$ can be interpreted as the parameter k in section 2.1.6. The rent protection approach is hence a PEG approach, but the firms bear costs for the existence of the R&D difficulty.

3.2.4 Optimal rent protection

The firm's optimal rent protection activity is given by[10]

$$v_N\frac{\iota}{X}=w_N^{RP}a_X \tag{27}$$

where the LHS is the marginal reduction of the expected loss with respect to rent protection and the RHS is the marginal cost of rent protection. I can solve for the optimal level of rent protection and I obtain

$$X^*=\frac{v_N\iota}{w_N^{RP}a_X}. \tag{28}$$

[8] Differentiating (20a) with respect to time yields this result.

[9] For the balanced growth path, the derivation is different. I combine (25) and (20b) to obtain $\dot{D}(t)=\frac{\delta s}{a_X}L_N$. Since L_N grows at rate g_L, integrating and using $D(0)=D_0$ gives $D(t)=\frac{\delta s}{a_X g_L}\left(L_N(t)-L_{N0}\right)+D_0$. Taking limits for $t\to\infty$ and using L'Hôpitale's rule, D grows in the steady state at rate g_L.

[10] See Şener (2006) for a derivation.

3.2.5 Firm Value

When I plug this solution into the expected value of the firm, I obtain

$$\begin{aligned} v_N =& \frac{1}{1+rdt}\left[\left(\pi_N^P - v_N\iota\right)dt + (1-(\iota+\mu)dt)(v_N+\dot{v}_N dt)\right] \\ =& \frac{1}{1+rdt}\left[\pi_N^P dt + (1-(2\iota+\mu)dt)v_N + (1-(\iota+\mu)dt)\dot{v}_N dt\right]. \end{aligned} \tag{29}$$

So the cost of rent protection equals ιv_N in equilibrium and hence the innovation rate counts twice.

As with the PL approach, after rearranging, dividing by dt and letting $dt \to 0$, I obtain for the firm value

$$v_N = \frac{\pi_N^P}{r + 2\iota + \mu - \frac{\dot{v}_N}{v_N}} \tag{30}$$

3.2.6 Wages

The result of the bargaining process is still (19), but it holds only for general purpose workers, so $w_N^{GP} = \alpha\lambda$.

Using (15) and (20b) in (28), the wage of rent protection workers is

$$w_N^{RP} = w_N^{GP}\frac{\hat{A}}{s}\iota_N n_N \tag{31}$$

where $\hat{A} \equiv \frac{a_R \delta s}{a_X g_L}$.

3.3 Steady State

3.3.1 Innovation Rate

I derive the steady state equation as before; it is

$$\frac{c_S\eta_S}{\lambda}\frac{\iota+\mu}{\mu}\frac{(1-\alpha)/\alpha}{\rho+2\iota+\mu-g_L} = \hat{A}. \tag{32}$$

The left hand side is, as with the PL approach, the expected benefit of innovation per capita divided by the wage w_N^{GP}, $\frac{v_N}{L_N w_N^{GP}}$, and the right hand side is the effective resource requirement of innovation per capita.

3.3.2 Unemployment Rate

I derive the unemployment rate as before using the steady state consumption value. It is

$$u_N = 1 - s - \iota\left(\frac{c_S\eta_S}{\lambda\mu} + \tilde{A}\right). \tag{33}$$

The unemployment rate is decreasing with a higher s, since a higher share of specialized workers, who work in rent protection activities, means that ceteris paribus, the demand for production and R&D workers remains constant, but since the available share of general purpose workers is decreasing, a larger share of them is employed, hence a lower unemployment rate.

The unemployment rate is decreasing with the innovation rate ι, since a higher ι increases the share of the North industries and it increases the demand for R&D workers.

3.3.3 Comparative statics

Proposition 2. *Iff $\rho - g_L < \mu$, a higher relative Southern population level η_S increases the innovation rate, and a higher imitation rate μ decreases the innovation rate ι.*

A higher relative Southern population level increases ceteris paribus the expected per capita benefit of R&D. Whether the expected per capita benefit increases or decreases for a higher innovation rate depends on the relative size of $\rho - g_L$ and μ.

With the RP approach, the relative size of the percentage changes of the profits and the discount rate is not clear and depends on the relative size of $\rho - g_L$ and μ. So using the RP approach in case of imperfect labor markets leads to ambiguous comparative statics of the multiple variables in the steady state. So it matters in which way you remove scale effects.

In order to prove the existence and the stability of the respective steady state equilibrium, we have to analyze the behavior on the balanced growth path.

3.4 Transitional Dynamics – Very preliminary

Again, I solve for the share of Northern industries, n_N. The parameters λ, α, ρ, g_L and n_0 have the same value as for the PL approach. Again, to get a reasonable value for the Northern unemployment rate and to ensure $c_N > c_S$, I set the imitation rate to $\mu = 0.245$, the share of specialized workers to $s = 0.101$, the unit labor requirement of rent protection to $a_X = 12$, the rent protection effectivity parameter to $\delta = 1$, the initial value of R&D difficulty to $D_0 = 1.1$ and the relative size of the Southern popuation to $\eta_S = 1.01$. These values produce an unemployment rate of $u_N < 20\%$.

Figure 2 shows the development of the model's variables over time. The share of Northern industries is now determined by the differential equation (92) and the unemployment rate by (96), while Northern per-capita consumption and the Northern innovation rate are determined as before.

A Competitive Labor Market

A.1 The PL approach

This model has a perfectly competitive labor market, and I first eliminate scale effects by the PL approach. Sections 2.2.1 – 2.2.6 remain valid, but the assumptions and equilibrium equations for the labor market change.

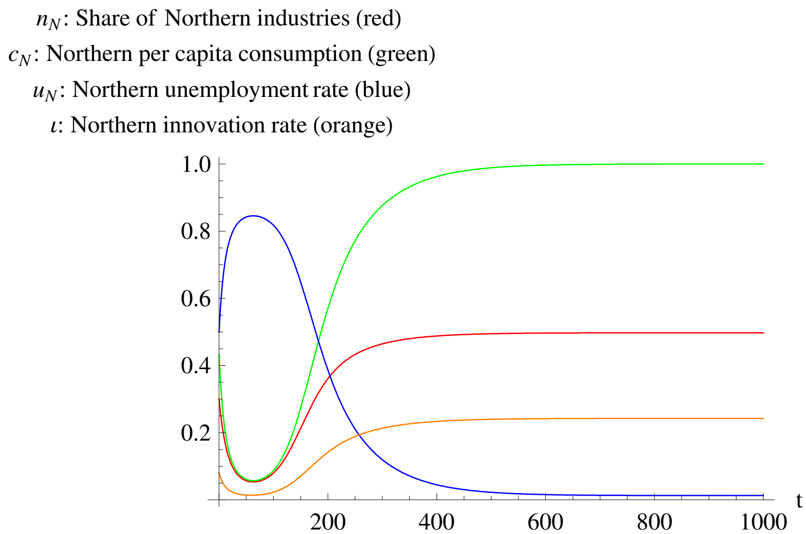

Figure 2: Development of the Northern economy along the balanced growth path

A.1.1 The Labor Market

The Northern labor market is perfectly competitive, so the labor market clears. It is characterized by

$$n_N \left(\frac{c_N L_N}{\lambda} + \frac{c_S L_S}{\lambda} \right) + a_R R = L_N \tag{34}$$

where the left hand side is demand for production workers plus demand for R&D workers, and the right hand side is the supply. The fraction of the Northern industries is n_N.

I determine the wage using the FEIN condition

$$v_N = w_N a_R k (L_N + L_S) \tag{35}$$

and obtain

$$w_N = \frac{c_N L_N + c_S L_S}{a_R k (L_N + L_S) \left(r + \iota + \mu - \frac{\dot{v}_N}{v_N} \right) + \frac{c_N L_N}{\lambda} + \frac{c_S L_S}{\lambda}} \tag{36}$$

A.1.2 Steady State

Wage I substitute the steady state consumption into the free entry in innovation condition (15). The wage is

$$w_N = \frac{\frac{\iota+\mu}{\mu} c_S \eta_S}{A(\rho + \mu + \iota - g_L) + \frac{\iota+\mu}{\mu} \frac{c_S \eta_S}{\lambda}} \tag{37}$$

The Innovation Rate I substitute the Northern industry share (21), steady state consumption (22) into the labor market clearing condition (34). I obtain

$$\frac{c_S \eta_S}{\lambda} \frac{\iota}{\mu} + \iota A = 1 \tag{38}$$

where the left hand side is the sum of the per capita production and the per capita R&D. Note that the innovation rate does not depend on import tariffs.

Comparative Statics

Proposition 3. *In case of competitive labor markets and removing scale effects by the simple PEG approach, a higher relative the South population decreases the the North innovation rate, and a higher the South innovation rate increases the the North innovation rate.*

Why does a higher relative Southern population level decrease the innovation rate? A higher relative Southern population level increases the per capita level of R&D difficulty, $a_R k(1+\eta_S)$. Note that only the North does R&D, so "per capita" means "per Northern inhabitant". Second, it increases the per capita production in the North, $\frac{c_S \eta_S}{\lambda} \frac{\iota}{\mu}$. So for a given innovation level ι, it increases both the labor demand in production and in R&D. Given limited resources, the equilibrium innovation level must therefore decrease.

What about an increasing imitation rate? It decreases the per capita labor demand in production $\frac{c_S \eta_S}{\lambda} \frac{\iota}{\mu}$ for a given innovation level. The per capita labor demand in R&D remains constant. Given limited resources, the equilibrium innovation rate ι must therefore decrease.

A.2 Rent Protection

The labor force is exogenously divided into a fraction s working only for rent protection and a fraction $1-s$ working in R&D and production. So the labor market equations in the North become

$$n_N \left(\frac{c_N L_N}{\lambda} + \frac{c_S L_S}{\lambda} \right) + a_R R = (1-s) L_N \tag{39}$$

and

$$n_N a_X X = s L_N \tag{40}$$

The balanced trade condition and the per capita consumption are still given by (BT) and (22), respectively.

The wage of general purpose workers is

$$w_N^{GP} = \frac{\frac{\iota+\mu}{\mu} c_S \eta_S}{\tilde{A}(\rho + 2\iota + \mu - g_L) + \frac{c_S \eta_S}{\lambda} \frac{\iota+\mu}{\mu}} \tag{41}$$

and the wage of specialized workers is

$$w_N^{RP} = \frac{\hat{A}}{s} \iota n_N w_N^{GP}, \tag{42}$$

where $\hat{A} \equiv \frac{a_R \delta s}{g_L a_X}$.

I use the free entry in innovation condition to obtain the steady state innovation rate,

$$\frac{c_S \eta_S}{\lambda} \frac{\iota}{\mu} + \iota \cdot \tilde{A} = 1 - s \tag{43}$$

Proposition 4. *In case of competitive labor markets, removing the scale effect by the rent protection approach leads to the same result as in case of the simple PEG approach: The steady state innovation rate decreases with a higher relative the South population level and it increases with a higher the South innovation rate.*

B Proofs

B.1 Proof of Proposition 1

Proof. Solving (24) for ι and using the definition of A, I obtain

$$\iota = \mu \frac{a_R k \left(1 + \eta_S\right) \left(\rho - g_L + \mu\right) - \frac{c_S \eta_S}{\lambda} \frac{1-\alpha}{\alpha}}{\frac{c_S \eta_S}{\lambda} \frac{1-\alpha}{\alpha} - a_R k \left(1 + \eta_S\right) \mu}. \tag{44}$$

To have a positive innovation rate ι, the denominator and the numerator must both be positive. If both were negative, then $a_R k \left(1 + \eta_S\right) \left(\rho - g_L + \mu\right) < a_R k \left(1 + \eta_S\right) \mu$, or $\rho < g_L$, which contradicts the assumption that $\rho > g_L$. So both terms must be positive, which I assume henceforth.

The derivative of (44) with respect to η_S is

$$\frac{d\iota}{d\eta_S} = -\mu \frac{\frac{c_S}{\lambda} \frac{1-\alpha}{\alpha} a_R k (\rho - g_L)}{\left(\frac{c_S \eta_S}{\lambda} \frac{1-\alpha}{\alpha} - a_R k (1 + \eta_S) \mu\right)^2} < 0 \tag{45}$$

since $\rho - g_L > 0$ and the derivative with respect to ι_S is

$$\frac{d\iota}{d\mu} = \frac{\left(\frac{c_S \eta_S}{\lambda} \frac{1-\alpha}{\alpha}\right) \left(A(\rho - g_L + \mu) - \frac{c_S \eta_S}{\lambda} \frac{1-\alpha}{\alpha}\right) + A \left(\frac{c_S \eta_S}{\lambda} \frac{1-\alpha}{\alpha} - A\mu\right)}{\left(\frac{c_S \eta_S}{\lambda} \frac{1-\alpha}{\alpha} - A\mu\right)^2} > 0. \tag{46}$$

The sign follows from the positive numerator and denominator of (44). □

B.2 Proof of Proposition 2

Proof. I solve (32) for the innovation rate ι and obtain

$$\iota = \mu \frac{\tilde{A}(\rho - g_L + \mu) - \frac{c_S \eta_S}{\lambda} \frac{1-\alpha}{\alpha}}{\frac{c_S \eta_S}{\lambda} \frac{1-\alpha}{\alpha} - 2\tilde{A}\mu}. \tag{47}$$

A positive the North innovation rate requires the numerator and the denominator of the fraction on the right hand side to be either both positive or both negative. In contrast to section 2.3.4, both cases are possible: For both being positive, a necessary condition is $\rho - g_L > \mu$, and for both being negative, a necessary condition is $\rho - g_L < \mu$.

The derivative of (47) with respect to the relative Southern population level is

$$\frac{d\iota}{d\eta_S} = \mu \frac{\frac{c_S}{\lambda}\frac{1-\alpha}{\alpha}\tilde{A}(\mu - (\rho - g_L))}{\left(\frac{c_S\eta_S}{\lambda}\frac{1-\alpha}{\alpha} - 2\tilde{A}\mu\right)^2} < 0 \text{ iff } \rho - g_L > \mu \tag{48}$$

and the derivative of (47) with respect to the imitation rate is

$$\frac{d\iota}{d\mu} = \frac{\frac{c_S\eta_S}{\lambda}\frac{1-\alpha}{\alpha}\left(\tilde{A}(\rho - g_L + \mu) - \frac{c_S\eta_S}{\lambda}\frac{1-\alpha}{\alpha}\right) + \tilde{A}\mu\left(\frac{c_S\eta_S}{\lambda}\frac{1-\alpha}{\alpha} - 2\tilde{A}\mu\right)}{\left(\frac{c_S\eta_S}{\lambda}\frac{1-\alpha}{\alpha} - 2\tilde{A}\mu\right)^2} > 0$$

$$\text{iff } \rho - g_L > \mu \text{ and } \iota > 0 \tag{49}$$

□

B.3 Proof of Proposition 3

Proof. I solve (38) for the steady state innovation rate,

$$\iota = \frac{1}{\frac{c_S\eta_S}{\lambda\mu} + A}, \tag{50}$$

and deriving (50) gives

$$\frac{\partial\iota}{\partial\eta_S} = \frac{-\left(\frac{c_S}{\lambda\mu} + a_R k\right)}{\left(\frac{c_S\eta_S}{\lambda\mu} + A\right)^2} < 0 \tag{51}$$

and

$$\frac{\partial\iota}{\partial\mu} = \frac{\frac{c_S\eta_S}{\lambda\mu^2}}{\left(\frac{c_S\eta_S}{\lambda\mu} + A\right)^2} > 0. \tag{52}$$

□

B.4 Proof of Proposition 4

Proof. I solve (43) for ι, which gives

$$\iota = \frac{(1-s)}{\frac{c_S\eta_S}{\lambda\mu} + \hat{A}}. \tag{53}$$

and deriving (53) gives

$$\frac{\partial\iota}{\partial\eta_S} = \frac{-(1-s)\frac{c_S}{\lambda\mu}}{\left(\frac{c_S\eta_S}{\lambda\mu} + \hat{A}\right)^2} < 0 \tag{54}$$

and

$$\frac{\partial \iota}{\partial \mu} = \frac{(1-s)\frac{c_S \eta_S}{\lambda \mu^2}}{\left(\frac{c_S \eta_S}{\lambda \mu} + \hat{A}\right)^2} > 0. \tag{55}$$

□

C Derivation of Balanced Growth Paths

C.1 Product Line Approach with Wage Bargaining

The firm value is

$$\begin{aligned} v_N(t) &= \frac{\frac{c_N(t)L_N(t)}{\lambda}(\lambda - \alpha\lambda) + \frac{c_S L_S(t)}{\lambda}(\lambda - \alpha\lambda)}{r(t) + \iota(t) + \mu - \frac{\dot{v}_N(t)}{v_N(t)}} \\ &= \frac{c_N(t)L_N(t)(1-\alpha) + c_S L_S(t)(1-\alpha)}{r(t) + \iota(t) + \mu - \frac{\dot{v}_N(t)}{v_N(t)}} \end{aligned}$$

and using the balanced trade condition (13)

$$\begin{aligned} v_N(t) &= \frac{(1-\alpha)(c_S L_S(t)\frac{n_N(t)}{1-n_N(t)} + c_S L_S(t))}{r(t) + \iota(t) + \mu - \frac{\dot{v}_N(t)}{v_N(t)}} \\ &= \frac{(1-\alpha)c_S L_S(t)\frac{1}{1-n_N(t)}}{r(t) + \iota(t) + \mu - \frac{\dot{v}_N(t)}{v_N(t)}}. \end{aligned} \tag{56}$$

Using the FEIN condition (15), we can write

$$w_N a_R D(t) = \frac{(1-\alpha)c_S L_S(t)\frac{1}{1-n_N(t)}}{r(t) + \iota(t) + \mu - \frac{\dot{v}_N(t)}{v_N(t)}} \tag{57}$$

Solving for $r(t)$ gives

$$\begin{aligned} r(t) &= \frac{(1-\alpha)c_S L_S(t)\frac{1}{1-n_N(t)}}{w_N a_R D(t)} - \iota(t) - \mu + \frac{\dot{v}_N(t)}{v_N(t)} \\ &= \frac{(1-\alpha)c_S L_S(t)\frac{1}{1-n_N(t)}}{w_N a_R k(L_N(t) + L_S(t))} - \iota(t) - \mu + \frac{\dot{v}_N(t)}{v_N(t)} \\ &= \frac{(1-\alpha)c_S \eta_S \frac{1}{1-n_N(t)}}{w_N a_R k(1+\eta_S)} - \iota(t) - \mu + \frac{\dot{v}_N(t)}{v_N(t)} \end{aligned} \tag{58}$$

Differentiating the FEIN condition with respect to time after taking logs yields

$$\frac{\dot{v}_N(t)}{v_N(t)} = \frac{\dot{D}(t)}{D(t)} \tag{59}$$

and using the definition of $D(t)$,

$$\frac{\dot{v}_N(t)}{v_N(t)} = \frac{\dot{D}(t)}{D(t)} = \frac{\dot{L}_N(t)}{L_N(t)} = g_L \tag{60}$$

Using this result in (58) and plugging this into the Keynes-Ramsey rule (8), we have

$$\frac{\dot{c}_N}{c_N} = \frac{(1-\alpha)c_S\eta_S\frac{1}{1-n_N(t)}}{w_N a_R k(1+\eta_S)} - \iota(t) - \mu + g_L - \rho$$

and using $w_N = \alpha\lambda$ from (19), we have

$$\frac{\dot{c}_N}{c_N} = \frac{(1-\alpha)c_S\eta_S\frac{1}{1-n_N(t)}}{\alpha\lambda a_R k(1+\eta_S)} - \iota(t) - \mu + g_L - \rho \tag{61}$$

Taking logs in the balanced trade condition (13) and differentiating with respect to time yields

$$\frac{\dot{c}_N}{c_N} + \frac{\dot{L}_N}{L_N} = \frac{\dot{L}_S}{L_S} + \frac{\dot{n}_N}{n_N}\frac{1}{1-n_N} \tag{62}$$

and since L_N and L_S both grow at rate g_L, the equation reduces to

$$\frac{\dot{c}_N}{c_N} = \frac{\dot{n}_N}{n_N}\frac{1}{1-n_N} \tag{63}$$

which I use to replace the left hand side of (61), yielding

$$\frac{\dot{n}_N}{n_N}\frac{1}{1-n_N} = \frac{(1-\alpha)c_S\eta_S\frac{1}{1-n_N(t)}}{\alpha\lambda a_R k(1+\eta_S)} - \iota(t) - \mu + g_L - \rho \tag{64}$$

Remember that the differential equation for the share of the Northern industries is

$$\dot{n}_N = \iota(1-n_N) - \mu n_N \tag{Eq:IF}$$

and solving for ι yields

$$\iota = \frac{\dot{n}_N}{1-n_N} + \mu\frac{n_N}{1-n_N}. \tag{65}$$

Equation (64) then becomes

$$\frac{\dot{n}_N}{n_N}\frac{1}{1-n_N} = \frac{(1-\alpha)c_S\eta_S\frac{1}{1-n_N(t)}}{\alpha\lambda a_R k(1+\eta_S)} - \frac{\dot{n}_N}{1-n_N} - \mu\frac{n_N}{1-n_N} - \mu + g_L - \rho \tag{66}$$

and after multiplying by $n_N(1-n_N)$, the resulting differential equation for the share of Northern industries is

$$\dot{n}_N = \frac{(1-\alpha)c_S\eta_S}{\alpha\lambda a_R k(1+\eta_S)}n_N - \dot{n}_N n_N - \mu n_N^2 + (g_L - \rho - \mu)n_N \tag{67}$$

or

$$\dot{n}_N(1+n_N) = \underbrace{\left[\frac{(1-\alpha)c_S\eta_S}{\alpha\lambda a_R k(1+\eta_S)} + (g_L-\rho-\mu)\right]}_{\equiv\beta} n_N - \mu n_N^2 \tag{68}$$

This is an autonomous nonlinear differential equation and can be solved by separating it into terms depending on n_N and terms depending on t and integrating both parts separately:

$$\int \frac{1+n_N}{\beta n_N - \mu n_N^2} dn_N = \int dt \tag{69}$$

The general solution is

$$\frac{\log n}{\beta} + \frac{(\beta+\mu)\log(\beta-\mu n_N)}{\beta\mu} = t + c \tag{70}$$

where c has to be determined given the initial value n_{N0}.

The unemployment rate is

$$u_N = 1 - \frac{n_N}{1-n_N}\frac{c_S\eta_S}{\lambda} - \iota A. \tag{71}$$

C.2 Rent Protection Approach with Wage Bargaining

The level of R&D difficulty grows as stated in (25), and using (20b), I determine $D(t)$ as

$$\begin{aligned} D(t) =& D_0 + \int_0^t \dot{D}(\tau)d\tau = D_0 + \frac{s\delta}{a_X} L_{N0}\int_0^t e^{g_L\tau}d\tau \\ =& D_0 + \frac{s\delta}{a_X}\left[\frac{1}{g_L}L_{N0}e^{g_L t} - \frac{1}{g_L}L_{N0}\right] \\ =& D_0 + \frac{s\delta}{a_X g_L}\left[L_N(t) - L_{N0}\right] \end{aligned} \tag{72}$$

which can be rewritten using (20b) as

$$D(t) = D_0 + \frac{\delta}{g_L}\left[n_N(t)X(t) - n_{N0}X(0)\right]. \tag{73}$$

Note that along the balanced growth path, the growth rate of R&D difficulty is not equal to g_L:

$$\frac{\dot{D}(t)}{D(t)} = \frac{\delta n_N(t)X(t)}{D(t)} = \frac{\frac{s\delta}{a_X}L_N(t)}{D(t)} \neq g_L \tag{74}$$

To determine the optimal firm behavior, I follow Dinopoulos & Syropoulos (2007) to determine the Jacobi-Bellman equations for the incumbent as

$$-\dot{J}(1,t) = \max_{p,X}\left\{e^{-r_c(t)}\pi_N(p,X,t) + \left(\mu + \frac{R(t)}{D_0 + \frac{\delta}{g_L}\left[n_N(t)X(t) - n_{N0}X_0\right]}\right)\left[J(0,t) - J(1,t)\right]\right\} \tag{75}$$

where $r_c(t)$ is the cumulative interest rate for time t. Maximising with respect to the price yields limit pricing as before, such that $p = \lambda$. Maximising (75) with respect to X yields

$$-e^{-r_c(t)} w_N^{RP}(t) a_X - \frac{R(t)\frac{\delta}{g_L} n_N(t)}{\left[D_0 + \frac{\delta}{g_L}\left[n_N(t)X(t) - n_{N0}X(0)\right]\right]^2} \left[J(0,t) - J(1,t)\right] \overset{!}{=} 0. \tag{76}$$

Setting $J(0,t) = 0$, using the definition of ι and using $e^{-r_c(t)} v_N(t) = J(1,t)$ yields

$$\frac{v_N(t)}{D(t)} = w_N^{RP}(t) \frac{a_X g_L}{\iota(t) n_N(t) \delta} \tag{77}$$

Since $v_N(t) = \frac{\pi_N^P(t) - w_N^{RP}(t) a_X X(t)}{r(t) + \iota(t) + \mu - \frac{\dot{v}_N(t)}{v_N(t)}}$ and using the definition of $D(t)$, I have

$$\begin{aligned} \pi_N^P(t) - w_N^{RP}(t) a_X X(t) =& w_N^{RP}(t) \frac{a_X g_L}{\iota(t) n_N(t) \delta} \left(r(t) + \iota(t) + \mu - \frac{\dot{v}_N(t)}{v_N(t)} \right) D(t) \\ =& w_N^{RP}(t) \frac{a_X g_L}{\iota(t) n_N(t) \delta} \left(r(t) + \iota(t) + \mu - \frac{\dot{v}_N(t)}{v_N(t)} \right) \left[D_0 + \frac{\delta}{g_L} \left[n_N(t) X(t) - n_{N0} X(0) \right] \right] \end{aligned} \tag{78}$$

which, by taking all terms connected to $X(t)$ to the right hand side, can be rearranged to

$$\begin{aligned} \pi_N^P(t) - w_N^{RP}(t) \frac{a_X g_L}{\iota(t) n_N(t) \delta} \left[D_0 - \frac{\delta}{g_L} n_{N0} X(0) \right] \left(r(t) + \iota(t) + \mu - \frac{\dot{v}_N(t)}{v_N(t)} \right) = \\ w_N^{RP}(t) \frac{a_X}{\iota(t)} X(t) \left(r(t) + \iota(t) + \mu - \frac{\dot{v}_N(t)}{v_N(t)} \right) + w_N^{RP}(t) a_X X(t) \end{aligned} \tag{79}$$

and again after multiplying by $\iota(t)$ and collecting terms on the right hand side

$$\begin{aligned} \pi_N^P(t)\iota(t) - w_N^{RP}(t) \frac{a_X g_L}{n_N(t) \delta} \left[D_0 - \frac{\delta}{g_L} n_{N0} X(0) \right] \left(r(t) + \iota(t) + \mu - \frac{\dot{v}_N(t)}{v_N(t)} \right) = \\ w_N^{RP}(t) a_X X(t) \left(r(t) + 2\iota(t) + \mu - \frac{\dot{v}_N(t)}{v_N(t)} \right) \end{aligned} \tag{80}$$

Solving for $X(t)$, the optimal level of rent protection is hence

$$X^*(t) = \frac{\pi_N^P(t)}{r(t) + 2\iota(t) + \mu - \frac{\dot{v}_N(t)}{v_N(t)}} \frac{\iota(t)}{w_N^{RP}(t) a_X} - \frac{\psi \left(r(t) + \iota(t) + \mu - \frac{\dot{v}_N(t)}{v_N(t)} \right)}{n_N(t) \left(r(t) + 2\iota(t) + \mu - \frac{\dot{v}_N(t)}{v_N(t)} \right)} \tag{81}$$

where $\psi \equiv \frac{D_0 g_L}{\delta} - \frac{s L_{N0}}{a_X}$ is constant.

The Jacobi-Bellman equation for the challengers is

$$-\dot{J}(0,t) = \max_{R_j}\left\{-e^{-r_c(t)}w_N^{GP}a_R R_j(t) + \frac{R_j(t)}{D(t)}[J(1,t)-J(0,t)]\right\} \tag{82}$$

and maximising (82) with respect to R_j yields

$$-e^{-r_c(t)}w_N^{GP}a_R + \frac{1}{D(t)}[J(1,t)-J(0,t)] \stackrel{!}{=} 0. \tag{83}$$

Proceeding similarly to above, the FEIN condition is

$$\frac{v_N(t)}{D(t)} = a_R w_N^{GP} \tag{84}$$

Using FEIN, the growth rate of the firm value is equal to the growth rate of R&D difficulty:

$$\frac{\dot{v}_N(t)}{v_N(t)} = \frac{\dot{D}(t)}{D(t)} \tag{85}$$

since a_R and $w_N^{GP} = \alpha\lambda$ are constant.

Combining (77) and (84), the wage of rent-protection workers is (as in the steady state)

$$w_N^{RP}(t) = w_N^{GP}\frac{a_R\delta}{a_X g_L}\iota(t)n_N(t). \tag{86}$$

I plug $X^*(t)$ from above into v_N and obtain the firm value along the balanced growth path as

$$\begin{aligned}
v_N(t) =& \frac{\pi_N^P(t) - w_N^{RP}(t)a_X X(t)}{r(t)+\iota(t)+\mu-\frac{\dot{v}_N(t)}{v_N(t)}} \\
=& \frac{\pi_N^P(t) - w_N^{RP}(t)a_X\left(\frac{\pi_N^P(t)}{r(t)+2\iota(t)+\mu-\frac{\dot{v}_N(t)}{v_N(t)}}\frac{\iota(t)}{w_N^{RP}(t)a_X} - \frac{\psi\left(r(t)+\iota(t)+\mu-\frac{\dot{v}_N(t)}{v_N(t)}\right)}{n_N(t)\left(r(t)+2\iota(t)+\mu-\frac{\dot{v}_N(t)}{v_N(t)}\right)}\right)}{r(t)+\iota(t)+\mu-\frac{\dot{v}_N(t)}{v_N(t)}} \\
=& \frac{\pi_N^P(t) - \frac{\pi_N^P(t)}{r(t)+2\iota(t)+\mu-\frac{\dot{v}_N(t)}{v_N(t)}}\iota(t) + w_N^{RP}(t)a_X\frac{\psi\left(r(t)+\iota(t)+\mu-\frac{\dot{v}_N(t)}{v_N(t)}\right)}{n_N(t)\left(r(t)+2\iota(t)+\mu-\frac{\dot{v}_N(t)}{v_N(t)}\right)}}{r(t)+\iota(t)+\mu-\frac{\dot{v}_N(t)}{v_N(t)}} \\
=& \frac{\pi_N^P(t)\frac{r(t)+\iota(t)+\mu-\frac{\dot{v}_N(t)}{v_N(t)}}{r(t)+2\iota(t)+\mu-\frac{\dot{v}_N(t)}{v_N(t)}} + w_N^{RP}(t)a_X\frac{\psi\left(r(t)+\iota(t)+\mu-\frac{\dot{v}_N(t)}{v_N(t)}\right)}{n_N(t)\left(r(t)+2\iota(t)+\mu-\frac{\dot{v}_N(t)}{v_N(t)}\right)}}{r(t)+\iota(t)+\mu-\frac{\dot{v}_N(t)}{v_N(t)}} \\
=& \frac{\pi_N^P(t) + w_N^{RP}(t)\frac{a_X\psi}{n_N(t)}}{r(t)+2\iota(t)+\mu-\frac{\dot{v}_N(t)}{v_N(t)}}
\end{aligned} \tag{87}$$

I use FEIN to replace v_N:

$$w_N^{GP}a_R D(t) = \frac{\pi_N^P(t) + w_N^{RP}(t)\frac{a_X\psi}{n_N(t)}}{r(t)+2\iota(t)+\mu-\frac{\dot{v}_N(t)}{v_N(t)}} \tag{88}$$

and solving for $r(t)$, I obtain

$$r(t) = \frac{\pi_N^P(t) + w_N^{RP}(t)\frac{a_X\psi}{n_N(t)}}{w_N^{GP} a_R D(t)} - 2\iota(t) - \mu + \frac{\dot{v}_N(t)}{v_N(t)} \tag{89}$$

which I use in the Keynes-Ramsey rule for Northern per capita consumption:

$$\frac{\dot{c}_N}{c_N} = \underbrace{\frac{\pi_N^P + w_N^{RP}\frac{a_X\psi}{n_N}}{w_N^{GP} a_R D(t)} - 2\iota - \mu + \frac{\dot{v}_N}{v_N}}_{r(t)} - \rho \tag{90}$$

$$\begin{aligned}
&= \frac{\pi_N^P}{w_N^{GP} a_R D(t)} + \frac{w_N^{RP}\frac{a_X\psi}{n_N}}{w_N^{GP} a_R D(t)} - 2\iota - \mu + \frac{\dot{v}_N}{v_N} - \rho \\
&= \frac{\pi_N^P}{\alpha\lambda a_R D(t)} + \frac{w_N^{GP}\frac{a_R\delta}{a_X g_L}\iota n_N \frac{a_X\psi}{n_N}}{w_N^{GP} a_R D(t)} - 2\iota - \mu + \frac{\dot{v}_N}{v_N} - \rho \\
&= \frac{\pi_N^P}{\alpha\lambda a_R D(t)} + \frac{\frac{\delta}{g_L}\psi}{D(t)}\iota - 2\iota - \mu + \frac{\dot{v}_N}{v_N} - \rho \\
&= \frac{\pi_N^P}{\alpha\lambda a_R D(t)} + \frac{\frac{\delta}{g_L}\psi}{D(t)}\iota - \frac{D(t)}{D(t)}\iota - \iota - \mu + \frac{\dot{v}_N}{v_N} - \rho \\
&= \frac{\pi_N^P}{\alpha\lambda a_R D(t)} + \frac{D_0 - \frac{s\delta}{a_X g_L} L_{N0}}{D(t)}\iota - \frac{D(t)}{D(t)}\iota - \iota - \mu + \frac{\dot{v}_N}{v_N} - \rho \\
&= \frac{\pi_N^P}{\alpha\lambda a_R D(t)} + \frac{D_0 - \frac{s\delta}{a_X g_L} L_{N0} - D(t)}{D(t)}\iota - \iota - \mu + \frac{\dot{v}_N}{v_N} - \rho \\
&= \frac{\pi_N^P}{\alpha\lambda a_R D(t)} + \frac{D_0 - \frac{s\delta}{a_X g_L} L_{N0} - \left(D_0 + \frac{s\delta}{a_X g_L}[L_N(t) - L_{N0}]\right)}{D(t)}\iota - \iota - \mu + \frac{\dot{v}_N}{v_N} - \rho \\
&= \frac{\pi_N^P}{\alpha\lambda a_R D(t)} - \frac{\frac{s\delta}{a_X g_L} L_N(t)}{D(t)}\iota - \iota - \mu + \frac{\dot{v}_N}{v_N} - \rho \\
&= \frac{\frac{c_N L_N}{\lambda}(\lambda - \alpha\lambda) + \frac{c_S L_S}{\lambda}(\lambda - \alpha\lambda)}{\alpha\lambda a_R D(t)} - \frac{\frac{s\delta}{a_X g_L} L_N(t)}{D(t)}\iota - \iota - \mu + \frac{\dot{v}_N}{v_N} - \rho \\
&= \frac{(1-\alpha)(c_N L_N + c_S L_S)}{\alpha\lambda a_R D(t)} - \frac{\frac{s\delta}{a_X g_L} L_N(t)}{D(t)}\iota - \iota - \mu + \frac{\dot{v}_N}{v_N} - \rho \\
&= \frac{(1-\alpha)\left(c_S L_S \frac{n_N}{1-n_N} + c_S L_S\right)}{\alpha\lambda a_R D(t)} - \frac{\frac{s\delta}{a_X g_L} L_N(t)}{D(t)}\iota - \iota - \mu + \frac{\dot{v}_N}{v_N} - \rho \\
&= \frac{(1-\alpha) c_S L_S \frac{1}{1-n_N}}{\alpha\lambda a_R D(t)} - \frac{\frac{s\delta}{a_X g_L} L_N(t)}{D(t)}\iota - \iota - \mu + \frac{\dot{v}_N}{v_N} - \rho \\
&= \frac{(1-\alpha) c_S L_S \frac{1}{1-n_N}}{\alpha\lambda a_R D(t)} - \frac{1}{g_L}\frac{\dot{v}_N}{v_N}\iota - \iota - \mu + \frac{\dot{v}_N}{v_N} - \rho
\end{aligned}$$

where $D(t)$ is defined as in (72). Using again (63), we have

$$\frac{\dot{n}_N}{n_N}\frac{1}{1-n_N} = \frac{(1-\alpha)c_S L_S \frac{1}{1-n_N}}{\alpha\lambda a_R D(t)} - \frac{1}{g_L}\frac{\dot{v}_N}{v_N}\iota - \iota - \mu + \frac{\dot{v}_N}{v_N} - \rho \tag{91}$$

Using (65) in (90), I finally obtain a differential equation for n_N:

$$\frac{\dot{n}_N}{n_N}\frac{1}{1-n_N} = \frac{(1-\alpha)c_S L_S \frac{1}{1-n_N}}{a_R\alpha\lambda D(t)} - \left(\frac{1}{g_L}\frac{\dot{v}_N}{v_N}+1\right)\left(\frac{\dot{n}_N}{1-n_N} + \mu\frac{n_N}{1-n_N}\right) - \mu + \frac{\dot{v}_N}{v_N} - \rho \tag{92}$$

The following is just some rearrangement. After multiplying by $1-n_N$, I have

$$\frac{\dot{n}_N}{n_N} = \frac{(1-\alpha)c_S L_S}{a_R\alpha\lambda D(t)} - \left(\frac{1}{g_L}\frac{\dot{v}_N}{v_N}+1\right)(\dot{n}_N + \mu n_N) + \left(\frac{\dot{v}_N}{v_N} - \mu - \rho\right)(1-n_N), \tag{93}$$

then I take all terms with $\dot{n}_N$ to the left hand side

$$\begin{aligned}\dot{n}_N + &\left(\frac{1}{g_L}\frac{\dot{v}_N}{v_N}+1\right)\dot{n}_N n_N = \\ &\frac{(1-\alpha)c_S L_S}{a_R\alpha\lambda D(t)} n_N - \left(\frac{1}{g_L}\frac{\dot{v}_N}{v_N}+1\right)\mu n_N^2 + \left(\frac{\dot{v}_N}{v_N} - \mu - \rho\right)(n_N - n_N^2)\end{aligned} \tag{94}$$

and after collecting and rearranging terms on the right hand side

$$\begin{aligned}\dot{n}_N + &\left(\frac{1}{g_L}\frac{\dot{v}_N}{v_N}+1\right)\dot{n}_N n_N \\ =&\frac{(1-\alpha)c_S L_S}{a_R\alpha\lambda D(t)} n_N - \left(\frac{\mu}{g_L}\frac{\dot{v}_N}{v_N} + \frac{\dot{v}_N}{v_N} - \rho\right) n_N^2 + \left(\frac{\dot{v}_N}{v_N} - \mu - \rho\right) n_N \\ =&\frac{(1-\alpha)c_S L_S}{a_R\alpha\lambda D(t)} n_N + \left(\frac{\dot{v}_N}{v_N} - \mu - \rho\right) n_N - \left(\frac{\mu}{g_L}\frac{\dot{v}_N}{v_N} + \frac{\dot{v}_N}{v_N} - \rho\right) n_N^2\end{aligned}$$

and on the left hand side

$$\begin{aligned}\dot{n}_N &\left(1 + \left(\frac{1}{g_L}\frac{\dot{v}_N}{v_N}+1\right) n_N\right) \\ =&\frac{(1-\alpha)c_S L_S}{a_R\alpha\lambda D(t)} n_N + \left(\frac{\dot{v}_N}{v_N} - \mu - \rho\right) n_N - \left(\frac{\mu}{g_L}\frac{\dot{v}_N}{v_N} + \frac{\dot{v}_N}{v_N} - \rho\right) n_N^2\end{aligned} \tag{95}$$

where $\frac{\dot{v}_N}{v_N}$ is deterministically given by (85) and (74). I now have a nonlinear, non-autonomous differential equation of first order and first degree.

The unemployment rate is

$$u_N = 1 - s - \frac{n_N}{1-n_N}\frac{c_S\eta_S}{\lambda} - \iota\frac{a_R D(t)}{L_N(t)}. \tag{96}$$

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