Abstract

This paper analyzes how globalization affects matching efficiency in the labor market. While previous studies have focused on the mismatch between heterogeneous workers and firms within an industry, we look inside these firms and investigate how trade alters the matching of worker-specific abilities and task-specific requirements. The outcome of this matching depends on how firms organize the internal labor market and this in turn is an endogenous decision. In our model only the most productive firms start exporting in the open economy, and these firms find it attractive to increase the investment into the internal allocation process, thereby improving the matching outcome. Things are different for non-exporters, whose market share shrinks in the open economy, lowering these firms’ incentive to invest into firm-internal labor allocation process. As a result, trade not only increases matching efficiency and thus the average productivity of workers due to adjustments in the firm-internal organization of labor, but also raises the dispersion of labor productivity between heterogeneous producers. Changes in the firm-internal organization of labor also point to a so far unexplored channel through which gains from trade can materialize.

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1 Introduction

In any industrialized economy, labor markets have to solve the complex problem of matching firm-specific skill requirements and worker-specific abilities. In the presence of market imperfections, the outcome of this matching process is typically not efficient. This is not only because some workers do not find a job at all. Rather, a significant share of workers cannot exploit full productivity because they are not matched with the best occupation (see Legros and Newman, 2002; Eckhout and Kircher, 2011). In recent years, this source of inefficiency has also sparked considerable attention in the trade literature. With an increasing general interest in the consequences of trade for underemployment, several authors have highlighted improvements in matching quality as a key aspect of gains from trade in terms of both welfare and employment (Amiti and Pissarides, 2005; Davidson, Matusz, and Shevchenko, 2008; Larch and Lechthaler, 2011). Thereby, the typical approach is to associate the quality of the matching process with its ability to match heterogeneous workers with heterogeneous firms in an efficient way, assuming implicitly that the production process covers just a single task with a specific skill requirement. However, this ignores the sophisticated structure of modern production processes and thus misses an important role of firms in reducing the requirement-ability mismatch by improving the assignment of workers to specific tasks within the boundaries of a single production entity.

Shedding light on the role of firms for matching workers with tasks and discussing how access to trade affects this firm-internal allocation process is the main purpose of this paper. Starting point of our analysis is a Melitz (2003) model, in which firms are heterogeneous due to differences in their productivity levels. As in Acemoglu and Autor (2011), we assume that production consists of a continuum of tasks that differ in their skill requirements. For performing these tasks, firms hire heterogeneous workers in a perfectly competitive labor market. Heterogeneity is horizontal in the sense that workers differ in their ability to perform specific tasks, while being equally productive over the whole range of activities. This implies that all workers have the same value to firms, and lacking information about abilities of individual workers, firms randomly draw their employees from the labor supply pool. This lack of information generates a source mismatch between task-specific skill requirements and worker-specific abilities within the boundaries of a production unit. To

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1 Usually the claim is that more able workers are employed by more productive firms, which is referred to by the term positive assortative matching in the literature. Compared to the large number of theoretical studies on positive assortative matching, there is surprisingly little empirical support for it (see Lentz and Mortensen, 2010).

2 The costs associated with a suboptimal assignment of heterogeneous workers to heterogeneous tasks have recently been highlighted in a study by Burgess, Propper, Ratto, von Hinke Kessler Scholder, and Tominey (2010). They show that productivity losses from a mismatch of workers and tasks in teams can be significant and that one important channel through which incentive payments to managers can improve the outcome of production units is the better assignment of workers to tasks.

3 This is similar to the labor supply assumption in Amiti and Pissarides (2005). However, in their model production consists of a single task, so that each firm has an ideal worker. The view that worker ability is task-specific is well in line with recent empirical evidence on occupation-specific human capital (see, for instance, Kambourov and Manovskii, 2009; Sullivan, 2010).

4 The question of how workers are paired with tasks is also addressed in a large literature on worker assignment. However, assignment models usually presume perfect information of workers and firms and thus produce an efficient allocation of labor (see Dupuy, 2008, for a literature review). Assignment models with imperfect information include Sattinger (1995) and Shimer (2005). However, these studies focus on search frictions and analyze the assignment of
reduce this mismatch, firms can invest into a screening technology for gathering some (imperfect) information about the abilities of workers, whom they have hired in the recruitment process. We model the screening investment as a fixed cost, which does not depend on the size of the firm. A higher investment provides better knowledge about the abilities of workers, therefore allowing a better match of these workers with the tasks required to produce output. The incentives to screen are more pronounced in larger firms, and hence there is an additional source of heterogeneity in our model, which is endogenous and reinforces heterogeneity of firms due to exogenous differences in firm productivity.

We use this model to shed new light on the consequences of trade for labor market outcome, thereby focusing on adjustments in the firm-internal labor market. To be more specific, we are interested in how trade affects underemployment arising from a mismatch between worker-specific abilities and task-specific skill requirements. To keep the analysis simple, we focus on trade between symmetric countries, and we furthermore assume that only the most productive firms start exporting in the open economy. There is strong empirical evidence for this self-selection by productivity (see, for instance, Bernard and Jensen, 1995, 1999) and, as in Melitz (2003), we get this result by considering a fixed cost of foreign market entry. By getting access to the export market, high-productivity firms find it more attractive to screen their workforce, because this improves the matching quality and thus lowers production costs. Low-productivity non-exporters, on the other hand, lose market share and thus lower their investment into the screening technology, which raises these firms' production costs. By changing the cost structure of producers, this asymmetric response to trade liberalization exerts a feedback effect on the entry/exit decision of firms in both the domestic and the export market, which is not present in a Melitz (2003) model with exogenous firm productivity. In addition, it alters the productivity distribution of active firms by driving a wedge between matching efficiency of exporters and non-exporters. This provides an alternative to the ‘learning-by-exporting’ hypothesis for explaining the empirical finding that firms become more productive when starting to export (see Fryges and Wagner, 2008).

Furthermore, adjustments in the firm-internal labor allocation process lower the aggregate mismatch between worker-specific abilities and task-specific skill requirements, thereby generating a productivity stimulus that reinforces the

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5 We model the screening technology in a rudimentary way, and it can hence be associated with any monitoring activity that helps improving the quality of labor allocation within the boundaries of this firm. A measure that has gained particular attention in this respect is job rotation. The literature distinguishes between three types of job rotation: employee learning (job rotation as a training device); employee motivation (job rotation makes work more interesting) and employer learning (job rotation as a way to discover in which jobs different employees are best at). Our model is in line with the employer learning view which was introduced by Ortega (2001) and is empirically supported by Eriksson and Ortega (2006).

6 According to Doeringer and Piore (1971) an internal labor market is “an administrative unit, such as a manufacturing plant, within which the pricing and allocation of labor is governed by a set of administrative rules and procedures. [...] This market is to be distinguished from the external labor market of conventional economic theory where pricing, allocating and training decisions are controlled directly by economic variables” (pp. 1f).

7 Greenaway and Kneller (2007) and Wagner (2007) summarize existing empirical evidence regarding the feedback effects of exporting on firm productivity. Our reading of the literature is that there is some support for such a positive feedback effect, but not all existing studies can identify a significant impact.
gains from trade in an otherwise identical Melitz (2003) model.

The firm-level adjustments to trade liberalization are not so different, in principle, from the adjustments in Helpman, Itskhoki, and Redding (2010). In their model, firms can invest into a screening technology in order to learn about the quality of applicants in the recruitment process. While screening provides an imprecise signal about worker types, it allows the firm to detect (and reject) applicants whose ability is below a certain threshold. The higher the investment, the better is the screening technology and the higher is the average ability of workers employed by the firm. The screening investment is endogenous and responds to trade in a similar way as the screening investment does in our model. It increases in exporting firms and shrinks in non-exporting ones. Aside from these similarities, there is a crucial difference between the two approaches. While in our setting all workers are equally valuable to firms – and only differ in their ability to perform specific tasks – workers in Helpman, Itskhoki, and Redding (2010) differ in their productivity they can elicit in a firm of a specific type. Hence, there is an efficiency loss in the Helpman, Itskhoki, and Redding (2010) model, because firms are not matched with the ideal worker, while there is a mismatch in our setting because workers do not perfectly fit the skill requirements of tasks they are performing within the boundaries of a firm.

By opening up the black box of production and modeling explicitly the firm-internal labor allocation process, our model not only identifies a new channel through which positive trade effects can materialize, but also contributes to a growing literature on the role of globalization for firm organization. Early contributions to this literature have studied the role of openness for the boundaries of firms (see Grossman and Helpman, 2002; Antrás, 2003; Antrás and Helpman, 2004). In contrast to these studies, we focus on the question how trade changes the organization of labor within these boundaries. This renders our analysis akin to Marin and Verdier (2008, 2012) who investigate how trade affects the hierarchy in firms and the empowerment of human capital. In contrast to them, our focus is not on changes in the hierarchy structure but on matching quality, so that our findings are complementary to their results. Finally, the key mechanism discussed in the paper also differs from a pure division of labor effect, which arises if there is a change in the number of tasks performed by a single worker (see Becker and Murphy, 1992). In our setting it is not the number of tasks performed by a single worker but rather the matching of workers with these tasks that matters.

The remainder of the paper is organized as follows. In Section 2 we describe the model structure and characterize the equilibrium in a closed economy. In Section 3, we consider trade between two symmetric countries. We characterize the equilibrium and investigate how a movement from autarky to trade affects allocation of labor ‘inside’ the firm as well as per capita income. In Section 4, we study the consequences of marginal trade liberalization, i.e. a gradual reduction in trade costs. Furthermore, we conduct a calibration exercise to get an idea about the quantitative size of the effects that trade liberalization exerts on welfare and underemployment in our setting. Finally, we briefly discuss in this section, how our model can be extended to capture other forms of underemployment. Section 5 concludes with a brief summary of the most important results.
2 The closed economy

2.1 Model structure

We consider an economy that is populated by an exogenous mass of workers $L$, who supply one unit of labor in an imperfectly competitive labor market. There are two sectors of production: a perfectly competitive final goods industry that produces a homogeneous output good by assembling differentiated intermediate goods; and a monopolistically competitive intermediate goods industry that hires labor for its production of differentiated goods. Similar to Egger and Kreickemeier (2009), we represent the final goods technology by a constant-elasticity-of-substitution (CES) production function without external scale economies. To be more specific, we assume that the technology for producing final output $Y$ is given by

$$Y = \left[ M^{-\frac{1}{\sigma}} \int_{\omega \in \Omega} x(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right]^{\frac{\sigma}{\sigma-1}},$$

(1)

where $x(\omega)$ denotes the quantity of intermediate good $\omega$ used in the final goods production, $M$ is the Lebesgue measure of set $\Omega$ and represents the mass of available intermediate goods, and $\sigma > 1$ denotes the (constant) elasticity of substitution between different product varieties. $Y$ serves as numéraire in our analysis, implying that the price index corresponding to the production function in Eq. (1) is equal to one, by assumption. Denoting by $p(\omega)$ the price of intermediate good $\omega$, we can write total costs of producing output $Y$ as follows: $\int_{\omega \in \Omega} p(\omega)x(\omega)d\omega$. Maximizing final goods profits with respect to $x(\omega)$, then gives intermediate goods demand

$$x(\omega) = \frac{Y}{M} p(\omega)^{-\sigma}.$$

(2)

At the intermediate goods level, there is a continuum of firms, each of them supplying a unique variety under monopolistic competition. Following Acemoglu and Autor (2011), we assume that intermediate goods production is a composite of different tasks. To be more specific, there is a continuum of tasks that is represented by the unit interval. The production technology is of the Cobb-Douglas type and given by

$$x(\omega) = \phi(\omega) \exp \left[ \int_{0}^{1} \ln x(\omega, i)di \right],$$

(3)

where $x(\omega, i)$ is the production level of task $i$ in firm $\omega$ and $\phi(\omega)$ is this firm’s baseline productivity. Task $x(\omega, i)$ is performed (produced) by workers who are employed in a linear-homogeneous production technology, which is the same for all tasks. To keep things simple, we assume that task-level output is equal to the effective labor input: the mass of workers performing the task multiplied by these workers’ average productivity. The productivity of workers in performing a specific task differs, because workers differ in their abilities, while tasks differ in their skill requirements. As in Amiti and Pissarides (2005), we assume that abilities of workers are uniformly distributed along a
circle with length 1, implying that each point on the circle is populated by a mass of \( L \) workers. The skill requirements to perform a task are also uniformly distributed along the ability circle, with each location on this circle therefore representing the skill requirements of a specific task. Hence, all workers are equally useful in the production process, and within a firm there exists an optimal occupation for each worker. We assume that firms are uninformed about the abilities of individual workers prior to the recruitment, and therefore randomly select workers from the labor supply pool, paying them the going, market-clearing wage rate \( w \).

Once workers are recruited, firms have to assign them to tasks. Since tasks enter production function (3) symmetrically and firms pay the same wage to all of their workers, firms maximize their profits by choosing the same employment level, \( l(\omega) \), for all tasks. However, this does not solve the problem of matching specific workers with specific tasks. The quality of this matching depends on how much the firm knows about the ability of workers. As long as the information about the ability of workers is not perfect, matching does not establish a perfect fit between worker-specific abilities and task-specific skill requirements, and there is an efficiency loss involved in this mismatch. We capture this efficiency loss by assuming that effective labor input of a worker when performing a task decreases monotonically in the distance between the task’s and the worker’s location on the circle, which we denote \( j \). Firms can reduce the efficiency loss by screening the workforce and thereby improving the fit between worker ability and skill requirement. Similar to Helpman, Itskhoki, and Redding (2010), we assume that screening involves a fixed investment \( f_\mu = \mu(\omega)^\gamma \) and provides an imprecise signal about worker ability, with the quality of the signal increasing in screening effort \( \mu(\omega) \). Screening with strength \( \mu(\omega) \) allows the firm to divide the ability circle into \( \mu(\omega) \) segments of equal length, so that workers can be matched with tasks whose skill requirements lie within the same segment on the ability circle. This raises the workers’ productivity in performing tasks. We relate the average productivity of workers, \( \kappa(\omega) \), to the ability-skill requirement distance \( j \) by the following function:

\[
\kappa(\omega) = (1 + \mu(\omega)) \exp \left[ \int_0^{1+\mu(\omega)} \frac{1}{1+j} dj \right] - 2, \tag{4}
\]

which can be simplified to \( \kappa(\omega) = \mu(\omega) \). While the specific functional relationship is not essential for our analysis, (4) is chosen in the interest of analytical tractability. Most importantly, it ensures that all active firms make at least a small investment into screening, so that we can avoid corner solutions. However, the main effects of our analysis would be the same for other specifications, as long as they generate a positive link between \( \kappa(\omega) \) and \( \mu(\omega) \). Due to symmetry, productivity \( \kappa(\omega) \) is the same for all tasks, and hence we have suppressed task index \( i \) in Eq. (4). Effective labor input at the task level is therefore given by \( \kappa(\omega)l(\omega) \) – and this equals \( x(\omega, i) \) for any \( i \). In view of (3), total output of firm \( \omega \) is therefore given by

\[
x(\omega) = \phi(\omega)\kappa(\omega)l(\omega). \tag{5}
\]
In contrast to Melitz (2003), firm productivity consists of two parts in our model: a baseline productivity $\phi$, which captures the efficiency of coordinating the bundle of different tasks within the boundaries of the firm, and labor productivity $\kappa(\omega) = \mu(\omega)$, which captures how effectively the heterogeneous abilities of workers are used for performing the different tasks in the production process. Crucially, the latter part of firm productivity is an endogenous variable in our setting and, as outlined above, firms can increase their productivity by investing into a screening technology which improves the matching quality in the firm-internal labor allocation process.\footnote{This mechanism is not too different, in principle, from an R&D investment that lowers variable production costs (see, for instance, Eckel, 2009).}

The baseline productivity is exogenous and drawn by firms in a lottery from the common Pareto distribution, $G(\phi) = 1 - \phi^{-\nu}$. To participate in this lottery, firms have to pay a fee $f_e$ in units of final output $Y$. This investment allows just a single draw and is immediately sunk. After productivity levels are revealed, producers decide upon setting up a plant and starting production. This involves an additional fixed cost $f$ (in units of final output) for setting up a local distribution network. Only firms with a sufficiently high baseline productivity will pay this additional fixed cost and start production, while firms with a low $\phi$ will stay out of the market. This two-stage entry mechanism is similar to Melitz (2003) – with the mere difference that in contrast to Melitz’ dynamic framework, we use the static model variant proposed by Helpman and Itskhoki (2010) and Helpman, Itskhoki, and Redding (2010) to reduce complexity. However, in our setting firms must also install a screening technology for improving the task-worker match. This screening technology comes with an additional fixed cost $f_\mu = \mu_\gamma$. All firms will make an investment into screening because otherwise their productivity falls to zero. However, the extent of screening can differ across firms, and this generates an additional heterogeneity of producers in the closed economy.

### 2.2 Equilibrium in the closed economy

After the lottery, the baseline productivity is revealed, and the firm either stays out of the market or it decides to produce, sets its price $p(\omega)$ and chooses its screening effort $\mu(\omega)$ to maximize profits

$$\pi(\omega) = p(\omega)x(\omega) - w(\omega)l(\omega) - \mu(\omega)\gamma - f$$

subject to (2), (5), and a set of common non-negativity constraints. The solution to this maximization problem establishes two first-order conditions. The first of these conditions determines the optimal pricing behavior, which is represented by a constant markup rule:

$$p(\omega) = \frac{\sigma}{\sigma - 1} \frac{w}{\phi(\omega)\mu(\omega)}.$$  \hspace{1cm} (7)

The second first-order condition determines the profit-maximizing screening effort $\mu(\omega)$, and this condition can be reformulated to

$$r(\omega) = \frac{\sigma\gamma}{\sigma - 1} \mu(\omega)\gamma,$$  \hspace{1cm} (8)
where \( r(\omega) = p(\omega)x(\omega) \) denotes revenues of firm \( \omega \). Eq. (8) establishes a positive relationship between firm-level revenues and screening effort (expenditures). Combining (2), (7) and (8), we get two different expressions for the revenue ratio of two firms, indexed 1 and 2:

\[
\frac{r(\omega_1)}{r(\omega_2)} = \left( \frac{\mu(\omega_1)}{\mu(\omega_2)} \right)^\gamma, \quad \frac{r(\omega_1)}{r(\omega_2)} = \left( \frac{\phi(\omega_2)\mu(\omega_2)}{\phi(\omega_1)\mu(\omega_1)} \right)^{1-\sigma}.
\]

These two expressions jointly determine the relative screening effort of firms 1 and 2 as a function of the of these firms’ baseline productivity ratio. This implies that heterogeneity of the two firms is fully characterized by their baseline productivity differential, and we can therefore use productivity \( \phi \) to index firms from now on. Hence, we can rewrite (9) in the following way:

\[
\frac{\mu(\phi_1)}{\mu(\phi_2)} = \left( \frac{\phi_1}{\phi_2} \right)^{\frac{\sigma-1}{\gamma+\sigma}}, \quad \frac{r(\phi_1)}{r(\phi_2)} = \left( \frac{\phi_1}{\phi_2} \right)^{\frac{\gamma(\sigma-1)}{\gamma+\sigma}}.
\]

Provided that \( \gamma > \sigma - 1 \) (which is required to ensure positive revenues – see below), we can conclude that firms with higher \( \phi \)-levels make higher revenues and choose a higher screening effort. This is well in line with evidence on job rotation, which shows a clear size pattern (see Eriksson and Ortega, 2006). Furthermore, the model is also consistent with the finding that workers are more productive in larger firms (see Idson and Oi, 1999), pointing to the role of better matching quality for explaining this size differential.

To separate active from inactive firms we can characterize a marginal producer, who is indifferent between starting production and remaining inactive. We denote the productivity of this firm by \( \phi^* \), which we refer to by the term \textit{cutoff productivity level}. The zero-cutoff profit condition, which characterizes this firm, is given by \( r(\phi^*)/\sigma = f + \mu(\phi^*)^{\gamma} \). We can combine this indifference condition with (8) to explicitly solve for screening effort and revenues of the marginal producer:

\[
\mu(\phi^*) = \left( \frac{f(\sigma - 1)}{\gamma - \sigma + 1} \right)^{\frac{1}{\gamma}}, \quad r(\phi^*) = \frac{\sigma \gamma f}{\gamma - \sigma + 1}.
\]

In view of (10) and (11), we can calculate average profits of active producers, \( \bar{\pi} \). Defining \( \xi = \gamma(\sigma - 1)/(\gamma - \sigma + 1) \), we obtain

\[
\bar{\pi} = \frac{f \xi}{\nu - \xi},
\]

where \( \nu > \xi \) is assumed to ensure a finite positive level of \( \bar{\pi} \). Noting that \( d \xi/d \gamma < 0 \), we can rewrite condition \( \nu > \xi \) in the following way: \( \gamma > \nu(\sigma - 1)/(\nu - \sigma + 1) \equiv \gamma \). Hence, \( \bar{\pi} \) has a finite positive value if \( \nu > \sigma - 1 \) and \( \gamma > \gamma \). Furthermore, free entry into the productivity lottery requires that, in equilibrium, the expected return to entry \( (1 - G(\phi^*))\bar{\pi} \) equals the participation fee \( f_e \). Therefore, the free entry condition in our static model reads

\[
\bar{\pi} = f_e(\phi^*)^{\nu}.
\]
Together, Eqs. (12) and (13) determine $\bar{\pi}$ and $\phi^*$. This completes the characterization of firm-level variables in the closed economy, and we can now turn to studying the main economy-wide variables of interest: welfare and underemployment.

With just a single consumption good, utilitarian welfare equals per-capita income. Since aggregate profits equal total expenditures for the lottery participation fee and the price of final output equals one, according to our choice of numéraire, per-capita income equals wage rate $w$ in our setting. To solve for the wage rate, we can combine $r(\phi^*) = p(\phi^*)x(\phi^*)$ with $Y = Mr(\phi^*)\nu/(\nu - \xi)$. Substituting (2) and (7) and accounting for (11)-(13), we can calculate

$$w = \sigma - \frac{1}{\sigma} \left( \frac{\nu}{\nu - \xi} \right)^{\frac{1}{\mu(\phi^*)}} = \sigma - \frac{1}{\sigma} \left( \frac{\nu}{\nu - \xi} \right)^{\frac{1}{\gamma}} \left( \frac{f\xi}{\gamma} \right)^{\frac{1}{\gamma}} \left( \frac{f\xi}{f_e(\nu - \xi)} \right)^{\frac{1}{\gamma}}. \quad (14)$$

To obtain an aggregate measure of underemployment, we compute the average distance between task-specific skill requirements and worker-specific abilities. As formally shown in the appendix, this aggregate measure of mismatch is given by

$$u = \frac{1}{\mu(\phi^*)} \frac{\gamma(\nu - \xi)}{\gamma(\nu - \xi) + \xi} = \left( \frac{\gamma - \sigma + 1}{f(\sigma - 1)} \right)^{\frac{1}{\gamma}} \frac{\gamma(\nu - \xi)}{\gamma(\nu - \xi) + \xi}. \quad (15)$$

where the second equality follows from (11).\(^9\) The existence of underemployment due to a mismatch of abilities and skill-requirements is the main difference between our setting and an otherwise identical Melitz (2003) framework with homogeneous workers and a single-task production technology. The source of underemployment also differs from other models that introduce search frictions into a Melitz framework (see, for instance, Helpman and Itskhoki, 2010; Helpman, Itskhoki, and Redding, 2010; Felbermayr, Prat, and Schmerer, 2011). In our setting, it is not the recruitment cost that generates underemployment but rather the mismatch of worker-specific abilities and task-specific skill requirements within the boundaries of firms that generates an inefficient equilibrium labor allocation. This completes the analysis of the closed economy.

3 The open economy

3.1 Basic structure and preliminary insights

In this section, we consider trade between two fully symmetric countries, whose economies are as characterized in the previous section. There are no impediments to the international transaction of

\(^9\)Combining the labor market clearing condition with the constant markup rule, gives $wL\sigma/(\sigma - 1) = Mr(\phi^*)\nu/(\nu - \xi)$, which in view of (11) and (14), can be solved for $M$:

$$M = \sigma - \frac{1}{\sigma} \left( \frac{\nu}{\nu - \xi} \right)^{\frac{1}{\mu(\phi^*)}} \left( \frac{f\xi}{\gamma} \right)^{\frac{1}{\gamma}} \left( \frac{f\xi}{f_e(\nu - \xi)} \right)^{\frac{1}{\gamma}} \frac{L}{\xi f}. \quad (14)$$

However, since production technology (1) excludes external scale effects, changes in the mass of producers do not exert a direct effect on welfare and underemployment. For that reason, we do not analyze the mass of producers in further detail.
final goods, whereas exporting of intermediates involves two types of costs: On the one hand, there are fixed costs \( f_x > 0 \) (in units of final output) for setting up a foreign distribution network and, on the other hand, there are iceberg transport costs, which imply that \( \tau > 1 \) units of intermediate goods must be shipped in order for one unit to arrive in the foreign economy. Both of these costs are also present in the Melitz (2003) framework and – in combination with the heterogeneity of firms in their baseline productivity levels – they generate self-selection of only the best (most productive) producers into exporting, provided that these costs are sufficiently high. The decision to start exporting is more sophisticated, because it influences a firm’s optimal choice of screening effort and thus exerts a feedback effect on profits attainable in the domestic market. Hence, there is an interdependence between the decision to export and a firm’s performance in its domestic market, which does not exist in Melitz (2003). Due to this interdependence, we have to distinguish between variables referring to exporters (denoted by superscript \( e \)) and non-exporters (denoted by superscript \( n \)). Furthermore, we use subscript \( x \) to refer to variables associated with foreign market sales of an exporter, while domestic variables are index free.

Holding economy-wide variables constant, access to exporting does not affect a non-exporter’s profit-maximizing choice of \( p(\phi) \) and \( \mu(\phi) \) as characterized by (7) and (8). Things are different for an exporter, who realizes revenues \( r^e(\phi) \) and \( r^e_x(\phi) = \tau^{1-\sigma} r(\phi) \) in the domestic and foreign market, respectively, implying that in the open economy this firm’s profit-maximizing choice of \( \mu(\phi) \) is given by

\[
(1 + \tau^{1-\sigma}) r^e(\phi) = \frac{\sigma \gamma}{\sigma - 1} \mu^e(\phi) \gamma
\]

instead of (8). However, since condition (16) is structurally the same for all exporters, we can conclude that the ratio of screening effort and the ratio of total revenues in (10) remain unaffected in the open economy, when comparing two firms of the same export status (\( n \) or \( e \)) but differing productivity levels. In contrast, when comparing two firms with the same baseline productivity but differing export status, we obtain

\[
\frac{\mu^e(\phi)}{\mu^n(\phi)} = (1 + \tau^{1-\sigma}) \gamma^{\frac{1}{\gamma - \sigma + 1}} \quad \frac{r^e(\phi)}{r^n(\phi)} = (1 + \tau^{1-\sigma}) \gamma^{\frac{\sigma - 1}{\gamma - \sigma + 1}}.
\]

From the analysis of the closed economy we know that a firm’s screening effort increases with its revenues. Since, all other things equal, exporting generates additional revenues from sales to foreign consumers, it renders screening more attractive, resulting in \( \mu^e(\phi) > \mu^n(\phi) \). On the other hand, the higher screening effort under exporting improves the match between task-specific skill requirements and worker-specific abilities and thus lowers unit production costs. This stimulates sales in both the domestic and the foreign market, implying \( r^e(\phi) > r^n(\phi) \) in Eq. (17). Hence, there is a positive feedback effect of exporting on domestic revenues, which raises the incentives of firms to serve foreign consumers.

Despite the additional complexity arising from the feedback effect that a firm’s exporting decision exerts on its domestic profits, our model preserves a key property of the Melitz (2003) model: The
profit gain from exporting increases in baseline productivity $\phi$. To see this, we can look at a firm’s profit differential $\Delta \pi(\phi) \equiv \pi^e(\phi) - \pi^n(\phi)$, which, accounting for (8), (10), (11), and (16), (17), can be written in the following way:

$$\Delta \pi(\phi) = \left(1 + \tau^{1-\sigma} \frac{f_x}{f} - 1\right) \left(\frac{\phi}{\phi^*}\right)^{\frac{\nu}{\xi}} f - f_x. \quad (18)$$

Hence, if the two trade cost parameters, $f_x$ and $\tau$, are sufficiently high, there is self-selection of only the most productive firms into exporting as in other applications of the Melitz model. This is the case we are focussing on in this paper, and we can therefore characterize a firm that is indifferent between exporting and non-exporting: $\Delta \pi(\phi) = 0$. We denote the (cutoff) productivity of this firm by $\phi^*_x$, implying that firms with $\phi > \phi^*_x$ end up being exporters, while firms with $\phi < \phi^*_x$ end up being non-exporters. Solving $\Delta \pi(\phi^*_x) = 0$ for the ratio between the two productivity cutoffs $\phi^*_x$ and $\phi^*$, we obtain

$$\frac{\phi^*_x}{\phi^*} = \left(\frac{f_x/f}{(1 + \tau^{1-\sigma}) \frac{f_x}{f} - 1}\right)^{\frac{n}{\tau}}, \quad (19)$$

and there is partitioning of firms by export status if $\phi^*_x/\phi^* > 1$. Furthermore, we can use the productivity ratio in (19) to calculate the share of exporting firms in the open economy: $\chi \equiv [1 - G(\phi^*_x)]/[1 - G(\phi^*)] = (\phi^*_x/\phi^*)^{-\nu}$, and thus

$$\chi = \left\{\frac{f}{f_x} \left[1 + \tau^{1-\sigma} \frac{f_x}{f} - 1\right]\right\}^{\frac{n}{\tau}}. \quad (20)$$

From (19) and (20), we can conclude that higher trade costs, i.e. a higher fixed exporting cost $f_x$ or a higher iceberg transport costs $\tau$, raises the minimum productivity level that is necessary to render exporting an attractive choice, thereby lowering the share of exporters in the total population of active firms, $\chi$. With this insights at hand, we are now equipped to solve for the open economy equilibrium.

### 3.2 The open economy equilibrium

The equilibrium in the open economy is characterized by a two-stage entry mechanism that is similar to the closed economy, but additionally involves the decision to start exporting or to sell exclusively to the domestic market (at stage 2). Access to the export market raises profits of the most productive producers, and this provides a stimulus for the average profit of active firms, which in the open economy are given by

$$\bar{\pi} = \frac{f \xi}{\nu - \xi} \left(1 + \chi \frac{f_x}{f}\right). \quad (21)$$

---

10 Derivation details are deferred to the appendix.
instead of (12). Combining Eq. (21) with the free entry condition in (13), we can calculate cutoff productivity \( \phi^* \) relative to its closed economy counterpart (subscript \( a \)): \( \phi^*/\phi^*_a = (1 + \chi f_x/f)^{1/\nu} \). Hence, opening up to trade with a symmetric partner country leads to an upward shift in the cutoff productivity level \( \phi^* \). The mechanism behind this effect is well understood from Melitz (2003). Access to exporting generates additional demand for labor, and hence firms at the lower bound of the productivity distribution have to leave the market in order to restore the labor market equilibrium. This points to an important asymmetry of how firms are affected by trade liberalization in a Melitz model. While the most productive firms experience a profit gain due to access to the export market, the least productive producers experience a profit loss due to stronger competition for scarce labor in the open economy. Of course this asymmetry is directly linked to our assumption that only the most productive firms self-select into exporting, while the least productive ones only serve domestic consumers.

To shed further light on the asymmetry in the firm-level response to trade, we can also study how producers adjust their internal labor market in the open economy. We start with a closer look on non-exporting firms. Provided that the marginal firm in the market is not exporting, its screening effort remains to be given by (11). However, the new marginal producer has a higher baseline productivity than the marginal producer in the closed economy, and hence its screening effort is definitely lower than under autarky. Furthermore, since the link between the ratio of screening effort and the ratio of baseline productivities among non-exporting firms remain to be given by (10), it is clear that all non-exporting firms respond to the trade shock with a reduction in their screening effort. This is intuitive, as the sales level of non-exporting firms declines in the open economy, so that these firms are not willing to keep the (relatively) expensive screening technology they have installed in the closed economy. Contrasting the screening effort of a non-exporter in the closed and the open economy, we can calculate:

\[
\frac{\mu^a(\phi)}{\mu^a(\phi)} = \left( \frac{1}{1 + \chi f_x/f} \right)^{\frac{1}{\nu}} < 1. \tag{22}
\]

Calculating the screening differential for an exporting firm, we get

\[
\frac{\mu^e(\phi)}{\mu^a(\phi)} = \left( \frac{(1 + \tau^{1-\sigma})^{1-\sigma}}{1 + \chi f_x/f} \right)^{\frac{1}{\nu}} = \left( \frac{(1 + \chi^{1/\nu} f_x/f)^{1/\nu}}{1 + \chi f_x/f} \right)^{\frac{1}{\nu}} \left( \frac{1 + \chi f_x/f}{1 + \chi f_x/f} \right)^{\frac{1}{\nu}}, \tag{23}
\]

where the second equality follows from Eq. (20). Noting that \( \nu > \xi \) holds by assumption, it is straightforward to show that \( \mu^e(\phi) > \mu^a(\phi) \): A firm that starts exporting in the open economy realizes higher revenues and thus raises its screening effort relative to autarky. The differential impact of trade on screening in non-exporting and exporting firms is graphically depicted by Figure 1 and summarized in Proposition 1.

**Proposition 1** A country’s opening up to trade, leads to an asymmetric response in the organiza-
tion of firm-internal labor markets. While exporters expand their screening effort and thereby improve the match between task-specific skill requirements and worker-specific abilities, non-exporters lower their screening effort and therefore accept a higher mismatch between skill requirements and abilities.

**Proof.** See the analysis in the text.

In view of its asymmetric consequences at the firm level, it is clear that access to trade exerts counteracting effects on the general equilibrium variables of interest: wage rate (welfare) $w$ and underemployment $u$. Similar to the autarky scenario, the wage rate in the open economy, can be derived by combining $r(\phi^*) = p(\phi^*) x(\phi^*)$ with the adding up condition $Y = M(1 + \chi) x(\phi^*) r(\phi^*) \nu / (\nu - \xi)$. Substituting (2) and (7) – with $M(1 + \chi)$ presuming the role of $M$ in the open economy – and accounting for (11), (21), and (13), we can calculate

$$w = \left( \frac{1 + \chi x f_x / f}{1 + \chi} \right)^{\frac{1}{1+m}} w_a. \quad (24)$$

There are gains from trade if and only if $f_x / f > 1$, while welfare shrinks in response to trade liberalization if $f_x / f < 1$. To get an intuition for this result, we can first recollect that production function (1) does not comprise external scale effects, and this closes one important channel through which gains from trade can materialize in our setting. Still, countries can benefit from trade liberalization, if changes in the composition of firms stimulate labor demand. This compositional effect depends on the relative strength of two self-selection effects in our model. On the one hand, there is self-selection of the best producers into exporting, which raises labor demand ceteris paribus. On the other hand, there is self-selection of the least productive firms out of the market, which lowers labor demand. The two self-selection effects are interdependent and their relative strength depends
on fixed cost ratio \( f_x/f \). If this fixed costs ratio is sufficiently high, it is the self-selection into exporting that dominates rendering the overall effect of trade on labor demand and thus welfare positive.

As outlined in Proposition 1, there are asymmetric firm-level effects of trade on the mismatch between abilities and skill requirements, which imply that underemployment shrinks in exporting firms, whereas it increases in non-exporting ones. However, there is an additional positive effect because labor is relocated towards exporting firms in the open economy and, due to this change in labor composition, the overall impact of trade on the average match quality is positive. To see this, we can explicitly solve for underemployment in the open economy. As formally shown in the appendix, we get:

\[
u = \frac{1 + a(\tau)\chi^{1+\zeta/(\nu\gamma)}f_x/f}{1 + \chi f_x/f} u^a,\]

with \( a(\tau) \equiv \frac{1 + \tau^{1-\sigma} \left( \frac{\gamma-1}{\nu(1-\sigma)} \right) - 1}{(1 + \tau^{1-\sigma})^{\frac{1}{\sigma}} - 1} \).

(25)

Noting that \( a(\tau) < 1 \), it is immediate that \( u < u^a \), which proves that trade reduces the average mismatch between task-specific skill requirements and worker-specific abilities, thereby lowering the underemployment problem. This completes our analysis on how opening up to trade affects welfare and underemployment, and we summarize the main insights from our analysis as follows.

**Proposition 2** Opening up to trade improves the average matching quality in the firm-internal allocation of labor, thereby reducing a country’s underemployment problem. The impact of trade on welfare is not clear-cut in general. Only if fixed costs of exporting \( f_x \) are sufficiently high relative to domestic fixed costs \( f \), there are gains from trade in our setting.

**Proof.** See the analysis in the text. □

While the welfare implications of trade are similar to those identified in other studies that exclude external scale effects in the production of final output (see, for instance, Egger and Kreickemeier, 2009), the impact on underemployment differs significantly from other studies. Introducing a random matching approach Helpman and Itskohki (2010) and Felbermayr, Prat, and Schmerer (2011) identify a decline of underemployment in response to trade, due to a reduction in the share of workers not finding a job at all, i.e. due to a decline in the unemployment rate. However, these papers do not analyze the mismatch between skill requirements and abilities. Such a mismatch is addressed in Davidson, Matusz, and Shevchenko (2008) and Helpman, Itskohki, and Redding (2010), but in their models output production requires just a single task, so that there is mismatch of workers and firms, but no mismatch of workers and tasks inside the firm, which is in the center of this paper’s interest. In the next section, we briefly discuss, how we can integrate these alternative forms of underemployment into our model.
4 Further discussion

The purpose of this section is threefold: We first study the implications of marginal changes in the iceberg transport cost parameter, $\tau$, to shed light on whether the implications of trade liberalization are monotonic in our setting. In a second subsection, we conduct a simple calibration exercise to get an idea about the quantitative effects of trade on the main variables of interest. Finally, we briefly discuss possible extensions of our analysis that allow us to account for additional forms of underemployment.

4.1 Marginal changes in the iceberg transport cost parameter

So far, we have studied the implications of a country’s movement from autarky to trade with arbitrary transport costs. However, in reality trade liberalization is a one time elimination of all barriers but rather a gradual process, involving stepwise reductions of these barriers. To capture this pattern, we now shed light on the consequences of a marginal decline in transport cost parameter $\tau$. Such a decline increases expected income from exporting, and thus raises the share of firms that start exporting in the open economy, according to (19) and (20), as well as average profit income $\bar{\pi}$, according to (21). On the other hand, there is a stimulus on labor demand, which compels the last productive producers to leave the market, thereby leading to an upward shift in cutoff productivity $\phi^*$. Furthermore, a marginal decline in the iceberg transport cost parameter augments the heterogeneity in screening effort between non-exporting and exporting producers, according to (17).

With respect to adjustments in the wage rate, we can infer from (24) that $dw/d\tau >, =, < 0$ if $1 >, =, < f_x/f_x$. Hence, a gradual reduction in the iceberg transport cost parameter exerts a monotonic (positive or negative) impact on welfare. If a movement from autarky to trade raises welfare, the gains from trade are magnified when the process of trade liberalization continues. Finally, from the analysis in Section 3, we know that a country’s movement from autarky to trade with an arbitrary transport cost level unambiguously reduces underemployment. We can therefore safely conclude that a marginal decline in $\tau$ must lower $u$ if $\tau$ has been large initially. In the appendix we show that this effect extends to the case where $\tau$ has already been low prior to the fall in the iceberg transport cost parameter, so that a gradual decline in $\tau$ reduces underemployment $u$ monotonically.

4.2 A calibration exercise

In this subsection, we aim at quantifying the effects of trade liberalization in our setting. For this purpose, we calibrate our model, relying on parameter estimates from Egger, Egger, and Kreickemeier (2011). Their paper sets up a modified Melitz (2003) framework with underemployment and structurally estimates the main parameters of this framework, using firm-level data from five European countries – Bosnia and Herzegovina, Croatia, France, Serbia, and Slovenia – for the period 2000 to 2008. While the parameter estimates in Egger, Egger, and Kreickemeier (2011) are
particularly suitable for our purpose, we have to be careful when employing them in the calibration exercise, because the estimated coefficients sometimes have a different interpretation in our model. For instance, Egger, Egger, and Kreickemeier (2011) apply the indifference condition of the marginal producer to estimate the elasticity of substitution. In our task-based approach to production $\xi \sigma / (\sigma - 1)$ presumes the role that $\sigma$ has in their study, and hence we have to associate the reported coefficient of 6.7 with an estimate for $\xi \sigma / (\sigma - 1)$. Furthermore, Egger, Egger, and Kreickemeier (2011) use the structural relationship between revenues of exporting firms to estimate the coefficient $\xi / \nu$ and report an estimate of 0.86 for the average country and year in their dataset. This structural relationship is the same in our setting, but the definition of $\xi$ differs from Egger, Egger, and Kreickemeier (2011). Unfortunately, there are no direct estimates available for $\gamma$, and we are therefore not able calculate parameter values for $\gamma$, $\sigma$, and $\nu$ separately, when setting $\xi \sigma / (\sigma - 1) = 6.7$ and $\xi / \nu = 0.86$. We tackle this problem of indeterminacy by considering three different values of $\sigma$, namely $\sigma = 2$, $\sigma = 4$, and $\sigma = 6$, in the calibration exercise and calculate the corresponding values of $\gamma$ and $\nu$ for these three scenarios.\footnote{For instance, Davis and Harrigan (2011) consider a relatively low level of $\sigma = 2$ in their calibration of a Melitz model with unemployment due to efficiency wages. Based on empirical findings, other studies use considerably higher values of $\sigma$ when calibrating the Melitz model. For instance, Arkolakis (2010) sets $\sigma = 6$, which falls into the range of parameter estimates reported by Broda and Weinstein (2006).} The computed values are listed in Table 1. There, we also report the average share of exporters in the country sample, $\chi$, as well as the average fixed cost ratio $f_x/f$, which has been estimated by Egger, Egger, and Kreickemeier (2011), but is not reported in the paper.

<table>
<thead>
<tr>
<th>Parameter values</th>
<th>$\Delta w$</th>
<th>$\Delta u$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>$\nu$</td>
<td>$\gamma$</td>
</tr>
<tr>
<td>2</td>
<td>3.90</td>
<td>1.43</td>
</tr>
<tr>
<td>4</td>
<td>5.84</td>
<td>7.44</td>
</tr>
<tr>
<td>6</td>
<td>6.49</td>
<td>47.86</td>
</tr>
</tbody>
</table>

Table 1: Quantifying the impact of trade on welfare and underemployment

Notes: An exporter share of $\chi = 0.46$ and a fixed cost ratio of $f_x/f = 1.45$ have been considered for computing the figures in this table.

Table 1 contains two additional columns, which present the computed values for welfare gains $\Delta w = 100 * (w/w^a - 1)$ and the reduction in underemployment, $\Delta u = 100 * (u/u^a - 1)$ triggered by the (average) country’s movement from autarky to trade. We see that both gains from trade and improvements in the matching quality can be huge if $\sigma$ is small, while the respective effects are significantly less pronounced for higher values of $\sigma$. We can also use our parameter estimates to shed light on the consequences of further trade liberalization on the variables of interest. For instance, setting $\sigma = 2$, a 20% decline in the iceberg transport cost parameter would increase welfare.
by 2.84% and reduce underemployment by 8.34%. In an alternative scenario with $\gamma \to \infty$ and thus an exogenous screening effort that is the same for all producers ($\mu(\phi) = 1$), the gains from a 20% decline of $\tau$ would trigger a welfare gain of $\Delta w = 2.30\%$, when keeping all other parameters at their levels reported in Table 1. Since $\Delta u = 0$ holds by assumption if $\gamma \to \infty$, this points to an additional welfare stimulus from trade liberalization of about 0.5 percentage points when producers can endogenously adjust the labor allocation process inside the firm.\footnote{Setting $\sigma = 6$, the positive employment and welfare effects are less pronounced and the additional welfare stimulus from endogenous adjustments in the firm-internal organization of labor fall to 0.08 percentage points.}

### 4.3 Different forms of underemployment

Studying the problem of allocating heterogeneous workers to heterogeneous tasks inside the boundaries of a firm, we shed light on a specific form of underemployment that has received surprisingly little attention in the existing academic literature. Typically, the focus in this literature is on the allocation of workers to firms and the capability of labor market institutions to generate efficient matches in this respect. If the labor allocation process outside the firm is not efficient, there is underemployment which may arise in two different forms. Either some workers do not find a job at all, while at the same time vacancies remain unfilled, or there is not a perfect match between the requirements of firms and the abilities of workers.

The former source of underemployment and the role of trade for improving the labor market outcome have been addressed in a recent contribution by Felbermayr, Prat, and Schmerer (2011). These authors introduce a model of random matching and Nash bargaining between firms and workers upon wages into an otherwise standard Melitz (2003) model. Since workers are homogeneous from the perspective of firms, there is no incentive to discriminate between applicants and firms end up paying the same wage to all workers similar to our analysis, due to over-hiring (see Felbermayr and Prat, 2011, for a detailed discussion of this effect). There is involuntary unemployment in this framework, because firms have recruitment costs which give workers an insider power in the bargain and implies that wages do not fall to their market clearing level. With recruitment costs being linked to the price of final output, trade lowers the search costs and thus unemployment if the selection effect in the export market is sufficiently pronounced.\footnote{Helpman and Itskhoki (2010) propose a similar framework. However, they consider more than a single final good and asymmetric countries, and hence it is less obvious to contrast the mechanisms in their study with those in our setting.} It is straightforward to extend our model to one with random matching along the lines of Felbermayr, Prat, and Schmerer (2011). This would give a framework that features two sources of underemployment: unemployment of workers who do not find a job and a mismatch between worker-specific abilities and task-specific skill requirements inside the firm. However, such an extension would neither provide novel insights on the unemployment effects of trade highlighted by Felbermayr, Prat, and Schmerer (2011), nor would it change the insights upon the role of trade for firm internal matching highlighted in this study, and we therefore refrain from studying this more sophisticated model variant in further detail.

The mismatch between workers and firms has been highlighted by Davidson, Matusz, and...
Shevchenko (2008) and Helpman, Itskhoki, and Redding (2010). Davidson, Matusz, and Shevchenko (2008) consider a model of positive assortative matching, in which trade improves the fit between firms and workers, thereby generating additional welfare gains. Davidson, Heyman, Matusz, Sjöholm, and Zhu (2010) provide empirical support for this mechanism. Helpman, Itskhoki, and Redding (2010) study the mismatch between firms and workers in a setting, in which heterogeneous firms employ heterogeneous workers in a Melitz-type framework. In their setting neither workers nor firms have information about the technology-specific abilities of an applicant. However, firms can install a screening technology to improve the matching quality. In contrast to our setting, production is represented by a single task and firms screen workers prior to their hiring. With screening being associated with a fixed cost, there is a differential impact of trade on exporting and non-exporting firms, similar to our setting. In addition, while, similar to Felbermayr, Prat, and Schmerer (2011), Helpman, Itskhoki, and Redding (2010) assume that workers and firms are randomly matched, there is wage differentiation across producers, because recruitment costs are now firm-specific. Since trade increases the recruitment cost differential, it raises wage inequality in this setting.

In principle, it is possible to extend our model to one, in which there is a firm-specific aspect in the ability of workers. For instance, if task-specific skill requirements are represented by just a segment of the ability circle (with equal length for all producers), the segments are uniformly distributed over the circle, and workers are assigned to firms by random matching, our model features similar properties as Helpman, Itskhoki, and Redding (2010). In particular, allowing producers to screen the pool of applicants prior to their recruitment, firms with a good draw in the productivity lottery will end up with higher screening expenditures and thus a better pool of workers than competitors with a bad draw, and this heterogeneity in screening effort would be more pronounced in the open than the closed economy. However, it is not clear ex ante how the ability to screen workers prior to their recruitment interacts with the incentives to screen workers for allocating them to specific tasks in the production process. Shedding light on the interaction of these two screening opportunities in determining underemployment is clearly beyond the scope of this paper, but a promising task for future research.

5 Concluding remarks

This paper sets up a model of heterogeneous firms along the lines Melitz (2003). In contrast to existing work that uses this framework, we associate production with a continuum of tasks that differ in their skill requirements. Furthermore, we assume that workers differ in their abilities to perform these tasks, and firms therefore face the complex problem of matching heterogeneous workers with heterogeneous tasks. To solve this allocation problem, firms must have some information about worker ability and they can get this information by screening their workforce. We associate screening with a firm-internal process in which the employer learns about the abilities of his/her workforce. Screening involves fixed costs and provides an imprecise signal about the ability of workers. The
higher the investment into screening, the better is the signal and the better is therefore the match between abilities of workers and skill requirements of tasks. Firms that have a higher *ex ante* productivity install a better screening technology, so that heterogeneity of firms is reinforced by the endogenous organization of the firm-internal labor market.

We use this framework to study the consequences of trade for welfare and underemployment, arising from a mismatch between workers and tasks. Assuming that only the best (most productive) firms self-select into exporting, trade exerts an asymmetric effect on the screening incentives of high- and low-productivity firms. High-productivity firms expand production due to exporting, and therefore find it attractive to install a better (more expensive) screening technology than in the closed economy. In contrast, low-productivity firms do not start exporting and lose market share at home, and in response they lower their screening expenditures. Despite this asymmetry in firm-level effects, we show that the average mismatch between worker-specific abilities and task-specific skill requirements unambiguously shrinks in the open economy. This points to a so far unexplored channel through which trade can improve the labor market outcome and stimulate welfare. In a calibration exercise, we show that this effect can indeed be sizable.

Focussing on the allocation of labor inside the firm and the productivity loss associated with a mismatch of worker-specific abilities and task-specific skill requirements, we highlight one important source of underemployment that has so far received surprisingly little attention in the literature. At the same time, to keep the analysis tractable we have abstracted from other forms of underemployment, which are of course also relevant: the unemployment of workers who do not find a job and the mismatch between workers and firms (instead of workers and tasks inside the firm). Eventually, all aspects of underemployment must be taken into account to get a comprehensive picture of the different channels through which trade affects the labor market outcome and of the interdependence of different forms of underemployment in an open economy. Adding an additional facet to the discussion on underemployment, we hope that we can contribute to achieve this goal. However, more research is needed before one can arrive at a definite conclusion about how globalization affects domestic labor markets.

**References**


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Appendix

Derivation of Equation (12)

Aggregate revenues of all intermediate goods producers equal

\[ R = M \int_{\phi^*}^{\infty} r(\phi) \frac{dG(\phi)}{1 - G(\phi^*)} = M \frac{f\gamma\sigma}{\gamma - \sigma + 1} \frac{\nu}{\nu - \xi}. \quad (26) \]

Dividing \( R \) by \( \sigma \), accounting for (11), and subtracting fixed costs for operating the local distribution network, \( Mf \), and for installing the screening technology,

\[ M \int_{\phi^*}^{\infty} \mu(\phi) \frac{dG(\phi)}{1 - G(\phi^*)} = M \frac{f}{\gamma - \sigma + 1} \frac{\nu}{\nu - \xi}, \quad (27) \]

gives aggregate profits \( \Pi = M\xi f / (\nu - \xi) \). Dividing \( \Pi \) by \( M \), we finally obtain (12). \( QED \)

Derivation of Equation (15)

Total distance of worker-specific abilities and task-specific skill requirements can be calculated by multiplying the average distance of a firm by this firm’s employment level and aggregating the resulting expression over all firms. This gives total underemployment:

\[ U = M \int_{\phi^*}^{\infty} l(\phi) \frac{dG(\phi)}{\mu(\phi)} \frac{1}{1 - G(\phi^*)} = M \frac{l(\phi^*)}{\mu(\phi^*)} \int_{\phi^*}^{\infty} \left( \frac{\phi}{\phi^*} \right)^{\left(\gamma - 1\right)\xi} \frac{dG(\phi)}{1 - G(\phi^*)} = M \frac{l(\phi^*)}{\mu(\phi^*)} \frac{\gamma\nu}{\gamma (\nu - \xi) + \xi}. \quad (28) \]

Dividing \( U \) by economy-wide employment

\[ L = M \int_{\phi^*}^{\infty} l(\phi) \frac{dG(\phi)}{1 - G(\phi^*)} = Ml(\phi^*) \int_{\phi^*}^{\infty} \left( \frac{\phi}{\phi^*} \right)^{\xi} \frac{dG(\phi)}{1 - G(\phi^*)} = Ml(\phi^*) \frac{\nu}{\nu - \xi}, \quad (29) \]

then gives average underemployment \( u \) in (15). \( QED \)

Derivation of Equation (21)

Total revenues in the open economy are given by

\[ R = M \int_{\phi^*}^{\phi^z} r(\phi) \frac{dG(\phi)}{1 - G(\phi^*)} + M(1 + \tau^{1-\sigma}) \int_{\phi^*}^{\infty} r(\phi) \frac{dG(\phi)}{1 - G(\phi^*)}. \quad (30) \]

Substituting (17) and accounting for (11), we can calculate

\[ R = M \frac{f\gamma\sigma}{\gamma - \sigma + 1} \left( 1 + \frac{f}{\nu} \right) \frac{\nu}{\nu - \xi}. \quad (31) \]
Dividing $R$ by $\sigma$ and subtracting fixed costs $M_f, M_\chi f_x$, and

$$M \int_{\phi^*_x}^{\phi^*_2} \mu^n(\phi) \frac{dG(\phi)}{1-G(\phi^*)} + M \int_{\phi^*_2}^{\infty} \mu^c(\phi) \frac{dG(\phi)}{1-G(\phi^*)} = M \frac{f(\sigma - 1)}{\gamma - \sigma + 1} \left(1 + \frac{f_x}{f}\right) \frac{\nu}{\nu - \xi}, \quad (32)$$

we get aggregate profits $\Pi = M \xi f (1 + \chi f_x / f) / (\nu - \xi)$. Dividing $\Pi$ by $M$, finally gives (21). QED

**Derivation of Equation (25)**

Total underemployment in the open economy is given by

$$U = M \int_{\phi^*_x}^{\phi^*_2} \ln(\phi) \frac{dG(\phi)}{\mu^a(\phi) 1 - G(\phi^*)} + M \int_{\phi^*_2}^{\infty} \ln(\phi) \frac{dG(\phi)}{\mu^a(\phi) 1 - G(\phi^*)}. \quad (33)$$

Noting that

$$M \int_{\phi^*_x}^{\phi^*_2} \ln(\phi) \frac{dG(\phi)}{\mu^a(\phi) 1 - G(\phi^*)} = M \frac{\ln(\phi^*)}{\mu^a(\phi^*)} \frac{\gamma \nu}{\gamma (\nu - \xi) + \xi} \left[1 - \chi \left(\frac{\phi^*_x}{\phi^*_2}\right)^{\frac{(\gamma - 1)\xi}{\gamma}}\right], \quad (34)$$

while

$$M \int_{\phi^*_2}^{\infty} \ln(\phi) \frac{dG(\phi)}{\mu^a(\phi) 1 - G(\phi^*)} = M \frac{\ln(\phi^*)}{\mu^a(\phi^*)} \frac{\gamma \nu}{\gamma (\nu - \xi) + \xi} \chi \left(\frac{\phi^*_x}{\phi^*_2}\right)^{\frac{(\gamma - 1)\xi}{\gamma}} (1 + \tau^{1 - \sigma}) \frac{(\gamma - 1)\xi}{\gamma (\sigma - 1)}, \quad (35)$$

we can further calculate

$$U = M \frac{\ln(\phi^*)}{\mu^a(\phi^*)} \frac{\gamma \nu}{\gamma (\nu - \xi) + \xi} \left\{1 + \chi \left[1 + (1 + \tau^{1 - \sigma}) \left(\frac{\phi^*_x}{\phi^*_2}\right)^{\frac{(\gamma - 1)\xi}{\gamma}} - 1\right] \left(\frac{\phi^*_x}{\phi^*_2}\right)^{\frac{(\gamma - 1)\xi}{\gamma}}\right\}, \quad (36)$$

Substituting (19) and (20), and accounting for the definition of $a(\tau)$ in (25), then gives

$$U = M \frac{\ln(\phi^*)}{\mu^a(\phi^*)} \frac{\gamma \nu}{\gamma (\nu - \xi) + \xi} \left[1 + a(\tau) \chi \frac{1 + \frac{f_x}{f}}{1 + \frac{f}{f}}\right]. \quad (37)$$

Dividing $U$ by economy-wide employment

$$L = M \int_{\phi^*_x}^{\phi^*_2} \ln(\phi) \frac{dG(\phi)}{1 - G(\phi^*)} + M (1 + \tau^{1 - \sigma}) \int_{\phi^*_2}^{\infty} \ln(\phi) \frac{dG(\phi)}{1 - G(\phi^*)}$$

$$= ML^a(\phi^*) \frac{\nu}{\nu - \xi} \left(1 + \frac{f_x}{f}\right) \quad (38)$$

and noting that $\mu^a(\phi^*_u) = \mu^a(\phi^*)$, we obtain $u$ in (25). QED
The impact of marginal trade liberalization on underemployment $u$

Let us first define $\rho(\tau) \equiv (1 + \tau^{1-\sigma})^{\frac{\xi}{\sigma-1}}$, with $\rho'(\tau) < 0$. In view of (19), (20) and (25), we can then rewrite $\chi$ and $a(\tau)$ in the following way:

$$
\chi = \left(\frac{f}{f_x}(\rho(\tau) - 1)\right)^{\xi} \quad a(\tau) = \frac{\rho(\tau) - 1}{\rho(\tau) - 1}.
$$

(39)

Totally differentiating $u$ with respect to $\tau$, therefore gives

$$
\frac{du}{d\tau} = \left\{ \frac{\chi f_x/f}{1 + \chi f_x/f} \frac{\xi}{\gamma} f_x \frac{da(\cdot)}{d\rho} \right\} \frac{\rho(\tau)}{(1 + \chi f_x/f)^2} \left[ \frac{\xi}{\gamma \nu} \chi^{\frac{\xi}{\gamma - 1}} a(\tau) \left(1 + \chi \frac{f_x}{f}\right) + \chi^{\frac{\xi}{\gamma - 1}} a(\tau) - 1 \right] \frac{d\chi}{d\rho} \frac{d\rho(\tau)}{d\tau},
$$

(40)

according to (25). Substituting

$$
\frac{da(\cdot)}{d\rho} = -\frac{1}{\rho(\tau) - 1} \left(\frac{a(\tau)}{\gamma} - \frac{\gamma - 1}{\gamma} \frac{1 - \rho(\tau)^{-\frac{1}{\xi}}}{}\right), \quad \frac{d\chi}{d\rho} = \nu \frac{\chi}{\xi \rho - 1},
$$

(41)

we can calculate

$$
\frac{du}{d\tau} = \frac{\Omega}{\rho(\tau) - 1} \frac{\chi f_x/f}{(1 + \chi f_x/f)^2} \frac{d\rho(\tau)}{d\tau},
$$

(42)

with

$$
\Omega \equiv \chi^{\frac{\xi}{\gamma - 1}} \left(1 + \chi \frac{f_x}{f}\right) \frac{\gamma - 1}{\gamma} \frac{1 - \rho(\tau)^{-\frac{1}{\xi}}}{} + \nu \left(\chi^{\frac{\xi}{\gamma - 1}} a(\cdot) - 1\right).
$$

(43)

Noting that $1 + \chi f_x/f = 1 + \chi^{1-\xi/\nu} (\rho(\tau) - 1)$ holds, according to (39), it is easily confirmed that $1 + \chi f_x/f < \rho(\tau)$ for any $\chi < 1$. This implies

$$
\Omega < \chi^{\frac{\xi}{\gamma - 1}} \frac{\gamma - 1}{\gamma} \frac{\rho(\tau) - \rho(\tau)^{-\frac{1}{\xi}}}{} + \nu \left(\chi^{\frac{\xi}{\gamma - 1}} a(\cdot) - 1\right) = -\left(\nu \frac{\gamma - 1}{\gamma}\right) \chi^{\frac{\xi}{\gamma - 1}} [1 - a(\cdot)] - \nu \left(1 - \chi^{\frac{\xi}{\gamma - 1}}\right).
$$

Since the right-hand side of this inequality is negative, we can conclude that $\Omega < 0$ and, in view of $d\rho(\tau)d\tau < 0$, $du/d\tau > 0$ must hold. This confirms that a marginal decline in $\tau$ unambiguously lowers underemployment $u$ in our setting and thus completes the proof. QED