Heterogeneous Fixed Export Cost: Explaining the Existence of Export-only Firms*

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Abstract

The present paper introduces a heterogeneous firm-level fixed cost for serving foreign market and a homogeneous fixed cost for serving domestic market across firms in addition to heterogeneous firm-level productivity. We explain the existence of export-only firms along with for which fixed export cost and productivity are relatively low. We also find that exposure to trade forces firms with a combination of low productivity and high fixed export cost to exit the market and induces firms with a combination of even lower productivity and lower fixed export cost to become export-only firms. The resource reallocation affects industrial productivity not unambiguously due to firms distribution of fixed export cost and productivity. Furthermore, we explore the effects of trade liberalization and innovation on equilibrium. In particular, we explore how shifts in the distributions of fixed export cost and productivity influence the behaviour of firms.

Keywords: Export-only firms, Heterogeneous fixed export cost, Liberalization, Innovation, Shift of distribution

JEL: F12, F13, L1

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1 Introduction

Export-only firms exist, especially in developing countries. Taking China as an example, the average share of export-only firms out of all firms over the period 1999 to 2008 is 6.81%; while the average share of export-only firms out of all exporting firms accounts for 27.56% (see appendix table A1). Export-only firms rarely have been considered in the research on international trade. In the present paper we consider a generalization of Melitz (2003) in order to allow for export-only firms as well as firms both exporting and serving their domestic market and firms serving their domestic markets only. In the model, there are two dimensions of firm-level heterogeneity: fixed export cost and productivity. Akin to fixed export cost, firms have identical fixed costs for serving the domestic market. Firms decide to export only, export and serve its domestic market or serve domestic market only based on their costs. We show that export-only firms are characterized by a combination of low productivities and low fixed export costs, firms serving domestic market are characterized by a combination of relatively low productivities and high fixed export costs and firms exporting and serving domestic market are characterized by high productivities in every level of fixed export cost.

Due to export-only firms, we give quite different arguments about productivity performance between exporters (including export-only firms) and domestic-market-only firms (non-exporters). Export-only firms are less productive than domestic-market-only firms. Therefore the overall productivity of exporting firms can be lower than domestic-market-only firms. We show a different channel through which trade affects industrial productivity. Exposure to trade forces the least productive firms with relatively high fixed export cost to exit the market, but inducing even less productive firms (compared to the market leavers) with relatively low fixed export cost into the market as export-only firms. The resource reallocation affects industrial productivity not unambiguously. Whether the overall industrial productivity is higher or lower after trade is uncertain, and depends on the firms distributions of fixed export cost and productivity, which determines the portfolio of firms exit and enter.

Other than transition from autarky, we also explore the effects of liberalization in terms of decrease in export cost. A decrease of fixed export cost shifts fixed export cost distribution to the left. Given only one peak in the fixed export cost distribution, the decrease raises the productivity cut-off for both markets, and so forces the least productive firms out of the market for every level of fixed export cost. The least productive firms serving both markets cannot get profit anymore and switch to the more profitable single market. A decrease in variable trade cost raises the productivity cut-off for the domestic market and decreases the

productivity cut-off for the foreign market. For firms that can only get profit from both markets, the mixed two countervailing effects affect the productivity cut-off ambiguously. Regarding innovation, we shift the distribution of productivity to the right, and show that innovation affects the productivity cut-off through active firms. In particular, if the innovation increases (decreases) the density of active firms productivity, innovation will raise (decrease) the productivity cut-off.

This paper is an extension of Melitz (2003) model by adding the heterogeneity of fixed export cost. Melitz (2003) introduced heterogeneous firm-level productivity into theoretical research in international trade. He showed that, subject to fixed export cost, exposure to trade induces only high productive firms to export while forcing the least productive firms to exit, and this resource reallocation from less productive firms to higher productive firms contributes to a higher industrial productivity. Melitz's theory is a response to previous empirical firm-level studies on the participation behaviour into trade (e.g. Clerides et al., 1998; Bernard and Jensen, 1999; Aw et al., 2000; Isgut, 2001; Delgado et al., 2002) and is well supported by later empirical studies (e.g. Bernard and Jensen, 2004; Girma et al., 2004; Arnold and Hussinger, 2005; Bernard et al., 2006; Lileeva and Trefler, 2010). Importantly, Melitz's model has become a widely-used framework leading to numerous theoretical contributions in trade (eg. Helpman et al., 2004, 2010; Yeaple, 2005; Melitz and Ottaviano, 2008; Chaney, 2008; Feenstra and Kee, 2008).

Most research is based on developed countries, however when applying Melitz's model to developing countries, it faces three big challenges. The first challenge is the relevance of export-only firms in developing countries that cannot be incorporated into Melitz's model, where firms either serve the domestic market only or serve both domestic and foreign markets. The second challenge is that the productivity of exporters is not always higher than non-exporters (e.g. Li and Yin, 2009; Li, 2010; Li et al., 2010; Liu and Jin, 2011; Tang and Liu, 2011; Tang et al., 2011). The third challenge is that export may not contribute to higher industrial productivity due to the relevance of export-only firms (see the discussion about processing trade, which is strongly related to export-only firms, as in Li and Yin, 2009; Li, 2010; Li and Zhao, 2010; Li et al., 2010; Zhang and Sun, 2010; Tang and Liu, 2011; Tang et al., 2011). The paper addresses these challenges well. The current model is more applicable to developing countries by incorporating export-only firms, and also includes Melitz's model as a special case.

This paper contributes to the literature also by considering simultaneously the heterogeneity of productivity and the heterogeneity of fixed export cost. Fixed export cost plays a key role in firms' participation behaviour into trade and is widely incorporated into theoretical research in international trade since Melitz (2003). However most studies model a homogenous fixed export cost (e.g. Help-

man et al., 2004; Chaney 2005; Ghironi and Melitz, 2005; Yeaple, 2005; Helpman, 2006; Alessandria and Choi, 2007; Bernard et al., 2007; Melitz and Ottaviano 2008; Constantini and Melitz, 2008; Demidova, 2008; Egger and Kreickemeier, 2009; Cherkashin et.al 2010; Helpman et al., 2010; Baldwin and Forslid, 2010; Atkeson and Burstein, 2010; Amiti and Davis, 2011; Baldwin and Harrigan, 2011; Felbermayr et al., 2011). A homogeneous fixed export cost simplifies the model by assuming that if a firm exports, it exports to all destinations. However, exporters do not export to all available destinations, and different exporters (variety) export to a different set of destinations (extensive margin of trade). A variable fixed export cost across destinations is introduced by Chaney (2008) and Helpman et al. (2008) to interpret the extensive margin, by Arkolakis et al. (2008) to interpret the relationship between variety and market size and by Johnson (2012) to interpret the variation in the number of exporting firms across destinations and in their average quality-adjusted prices.

In both cases of homogeneous fixed export cost and destination-specific variable export cost, all firms share the same cost structure. However, different firms may have different ability or policy advantage to serve foreign markets. For example some firms may have a lower fixed export cost due to better negotiation ability, more experience in marketing search, being more familiar with the foreign market regulatory environment, or operating in a more weakly regulated industry. Some firms, especially in developing countries, are involved in global production of multinational firms. They can benefit from this with a low fixed export cost. Das et al. (2007) using plant-level panel data of three manufacturing industries in Columbia find convincing evidence that fixed export cost varies between firms, e.g. large producers have lower average fixed cost than small producers and fixed export cost of knitting mills is different from leather firms among small producers, as well from basic chemical firms among large producers. Kasahara and Lapham (2012) using plant-level data for a set of Chilean manufacturing industries find that fixed export cost (and export sunk cost) varies among apparel, plastics, food, textiles, wood and metal products.

There exist some studies related to heterogeneous firm-specific fixed export cost (Schmitt and Yu, 2001; Jørgensen and Schröder, 2006, 2008; Das et al., 2007; Arkolakis, 2010, 2011; Eaton et al., 2011; Arkolakis et al., 2012; and Krautheim, 2012;) but their focus is quite different from the current paper. Schmitt and Yu (2001) assume that all firms share an identical productivity. Though Jørgensen and Schröder (2006) simultaneously consider heterogeneous firm-level productivity and heterogeneous fixed export cost, they get the equilibrium based on firms earning zero profit in the domestic market, which is problematic (as subsequently recognised by Jørgensen and Schröder, 2008) as profit from export still induces

firms into the market. Jørgensen and Schröder (2008) fix this problem using a free entry and exit dynamics á la Melitz, but they get the equilibrium assuming a weak firm-level productivity heterogeneity with only two types of productivity (high-level or low-level productivity), and in their model there cannot exist exportonly firms. Das et al. (2007) design a model for empirical analysis by decomposing fixed export cost into two parts: a homogeneous per-period sunk cost across firms and a once-time firm-specific start-up cost. Firm-specific fixed export cost is exogenous in these studies. Arkolakis (2010, 2011) and Eaton et al. (2011) introduce an endogenous firm-specific fixed export cost (market penetration cost) by assuming that the fixed export cost is determined by the destination market size and the reached fraction of consumers in the destination market (market penetration), where the latter is determined by productivity. Krautheim (2012) assumes a fixed export cost varying with the number of exporters into each destination, which in turn is determined by the productivity cut-off for each export destination. Arkolakis et al. (2012) develop a general model by assuming that the fixed export cost is a function of firm-specific characteristics and endogenous destination-specific characteristics, and illustrate this general model using examples of Ricardian model and the Melitz (2003) model. All these studies assume that all firms serve the domestic market and focus on how to describe the characteristics of exporters, while this paper focuses on why export-only firms exist. The introduced heterogeneous fixed export cost explains why export-only firms, domestic-market-only firms and firms export and serving domestic market co-exist in the economy.

Last, but not least, a novelty in this paper is brought by assuming a fixed cost for serving the domestic market. Since there is a fixed export cost for serving the foreign market, serving the domestic market is costly as well (Del Gatto et al., 2006) so firms incur a fixed cost for serving the domestic market. Studies ignoring export-only firms assume a fixed export cost for serving the foreign market and a domestic fixed cost for production, which works well as the fixed cost for serving the domestic market can be incorporated into the domestic fixed cost for production. Even the existing literature which recognises the fixed cost for serving domestic market is still assuming this as the only domestic fixed cost (Felbermayr et al. (2011) point out that producers face a fixed market access cost in both domestic and foreign markets, however assume this fixed cost as the only domestic fixed cost, assuming away the production cost). Noticeably, when accounting for export-only firms, there will be challenges because if no fixed cost for serving the domestic market is assumed, any incumbent firm can serve the domestic market costlessly so export-only firms do not exist. The current paper assumes a fixed cost for production and a fixed cost for serving the domestic market, so when serving a market, the firm can choose which market to serve based on the costs. Reasonably, a low productive firm with low fixed export cost only exports, instead

of also serving the domestic market.

The rest of this paper is organized as follows. Section 2 is the set up of the model. Section 3 describes the dynamics of firm entry and exit. Section 4 gives the equilibrium of the model. Section 5 explores the effect of trade liberalization. Section 6 discusses the impact of innovation. Section 7 concludes the paper.

2 Set Up

We assume a world with only two symmetric countries. Akin to Melitz (2003), the two countries have the same labour force and wage rate (as Melitz points out that the model can be easily extended to asymmetric countries by an exogenously fixed relative wage between countries). Consumers and firms face domestic market and foreign market. Firms select to serve only domestic market or only foreign market, or both markets. There is a dynamic process of firms free entry and exit. Every period there is a portion of firms exit and the same portion of firms successfully enter. The equilibrium is determined by zero profit condition. Except sunk cost, firms pay fixed cost and marginal cost for production and pay fixed cost for serving either market. Firms that cannot get profit exit the market. Labour is the only input factor of firms.

2.1 Consumers

The preference function by a consumer is given as CES-type utility (e.g. Dixit and Stiglitz, 1997; Melitz, 2003):

$$U = \left[\int_{\omega \in \Omega} q(\omega)^{\rho} d\omega \right]^{\frac{1}{\rho}} \tag{1}$$

where ω is the variety of goods and Ω represents a continuum of goods available both from domestic and foreign markets. Elasticity of substitution between any two goods is denoted by $\sigma = 1/(1-\rho)$, and $\sigma > 1$ since $0 < \rho < 1$. The aggregate quantity index and price index are determined as:

$$Q = \left[\int_{\omega \in \Omega} q(\omega)^{\rho} d\omega \right]^{\frac{1}{\rho}} \qquad P = \left[\int_{\omega \in \Omega} p(\omega)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}}$$
 (2)

To maximise consumer utility with a limited budget, the consumer behaviour is determined by:

$$q(\omega) = Q\left[\frac{p(\omega)}{P}\right]^{-\sigma} \qquad r(\omega) = p(\omega)q(\omega)$$
 (3)

Where R is total expenditure, so $R = PQ = \int_{\omega \in \Omega} r(\omega) d\omega$.

2.2 Firms

Firms pay an entry sunk cost f_s to enter into the market and then draw their parameters, which are not known till the moment firms start to produce and serve markets. Firms that cannot get a positive profit will exit immediately. Successful entrants will pay a fixed cost of production f and a marginal cost in terms of productivity φ . When serving foreign market, the firm will incur a fixed export cost f_{ex} and iceberg transport cost, whereby $\tau \geq 1$ units product is shipped but only 1 unit arrive at the foreign market. When serving domestic market, a fixed domestic cost f_{do} is introduced just as f_{ex} , which means not only in foreign market, but also in domestic market, firms must search the buyers, set up distribution channels, conform to all rules and adapt the products to serve market well and these kinds of cost are not associated with the volume of sales. Every firm is characterized by its cost structure $(f_{s,i}, f_i, f_{do,i}, \tau_i, f_{ex,i}, \varphi_i)$. We assume that fixed export cost and marginal cost are heterogeneous while the other costs are constant across firms. Therefore every firm is described by $\phi_i = (f_{ex,i}, \varphi_i)$. The fixed export cost is distributed on $(0,\infty)$ by a probability distribution with density $\psi(f_x)$ and cumulative distribution $\Psi(f_x)$. The productivity is distributed on $(0,\infty)$ by a probability distribution with density distribution $g(\varphi)$ and cumulative distribution $G(\varphi)$.

Production

There is a set of firms under monopolistic competition, so each firm will produce only one different variety of the good. Labour is the only input factor for firms production. The aggregate labour is fixed as L and the wage rate is normalized to one. The output is determined by:

$$l = F + q/\varphi \tag{4}$$

where $F \in \{f + f_{do}, f + f_{ex}, f + f_{do} + f_{ex}\}$. We assume iceberg transport cost is identical across firms. With the demand function from (3), firms set price for domestic and foreign market as:

$$p_d(\varphi_i) = \frac{1}{\rho \varphi_i} \qquad p_x(\varphi_i) = \frac{\tau}{\rho \varphi_i}$$
 (5)

Domestic and foreign revenue are determined as:

$$r_d(\varphi_i) = \frac{R}{(\rho P)^{1-\sigma}} \varphi_i^{\sigma-1} \qquad r_d(\varphi_i) = \frac{R}{(\rho P)^{1-\sigma}} (\frac{\varphi_i}{\tau})^{\sigma-1} \tag{6}$$

Obviously, the price and revenue are determined by productivity. A firm with higher productivity sets a lower price and gets higher revenue and a higher profit.

Firm Export Behaviour

When serving markets, firms get profit after covering fixed cost and marginal cost. There is a dynamic process of firms exit. For the incumbent firms, there is a probability δ in every period to be forced to exit by a shock. Since the fixed cost is incurred only once, it is reasonable to calculate the per-period profit using amortized portion of fixed cost for simplicity:

$$f_p = \delta f$$
 $f_x = \delta f_{ex}$ $f_d = \delta f_{do}$ (7)

So the profit of firm ϕ_i is:

$$\pi(\phi_i) = \begin{cases} \frac{r_d(\varphi_i)}{\sigma} - f_p - f_d & \text{only serve domestic market} \\ \frac{r_d(\varphi_i)}{\sigma} + \frac{r_x(\varphi_i)}{\sigma} - f_p - f_d - f_{x,i} & \text{serve both markets} \\ \frac{r_x(\varphi_i)}{\sigma} - f_p - f_{x,i} & \text{only serve foreign market} \end{cases}$$
(8)

Which markets to serve depends on whether the firm can get profit. Firms only serving domestic market will get positive profit from domestic market while negative profit from foreign market. This means $\pi_d(\phi_i) = \frac{r_d(\varphi_i)}{\sigma} - f_p - f_d > 0$ and $\pi_x(\phi_i) = \frac{r_x(\varphi_i)}{\sigma} - f_{x,i} < 0$. So the basic condition of fixed export cost for firms only serving domestic market is stated as:

$$f_{x,i} > \tau^{1-\sigma}(f_p + f_d) = f_{x,h}$$
 (9)

For firms with fixed export cost higher than $f_{x,h}$, the productivity cut-off for export is higher than domestic market, which is similar to Melitz(2003). Let φ_d^* means productivity cut-off for domestic market, and φ_x^* means productivity cut-off for export, $\varphi^* = \min \{ \varphi_d^*, \varphi_x^* \}$ means productivity cut-off of firms, and. Incorporating (6), we can get:

$$\begin{cases}
\pi_d(\varphi_d^*, f_{x,i}) = (\varepsilon P)^{\sigma - 1} \varphi_d^{*\sigma - 1} - (f_p + f_d) = 0 \\
\pi_x(\varphi_x^*, f_{x,i}) = (\varepsilon P)^{\sigma - 1} \tau^{1 - \sigma} \varphi_x^{*\sigma - 1} - f_{x,i} = 0 \\
\varphi^* = \varphi_d^* < \varphi_x^*
\end{cases} \tag{10}$$

whre $\varepsilon = (\frac{R}{1-\rho})^{\frac{1}{\sigma-1}}\rho$. For firms only serving export market, the condition must be met as:

$$f_{x,i} < \tau^{1-\sigma} f_d - f_p = f_{x,l} \tag{11}$$

Since they can only get positive profit from foreign market while negative profit from domestic market, which means $\pi_x(\phi_i) = \frac{r_x(\varphi_i)}{\sigma} - f_p - f_{x,i} > 0$ and $\pi_d(\phi_i) =$

 $\frac{r_d(\varphi_i)}{\sigma} - f_d < 0$. Here we assume $f_{x,l} > 0$. For firms with fixed export cost lower than $f_{x,l}$, the productivity cut-off for export is lower than domestic market. This is the main reason the export-only firms exist.

$$\begin{cases}
\pi_d(\varphi_d^*, f_{x,i}) = (\varepsilon P)^{\sigma - 1} \varphi_d^{*\sigma - 1} - f_d = 0 \\
\pi_x(\varphi_x^*, f_{x,i}) = (\varepsilon P)^{\sigma - 1} \tau^{1 - \sigma} \varphi_x^{*\sigma - 1} - (f_p + f_{x,i}) = 0 \\
\varphi^* = \varphi_x^* < \varphi_d^*
\end{cases}$$
(12)

The incumbents will serve both domestic and foreign markets if

$$\tau^{1-\sigma} f_d - f_p \le f_{x,i} \le \tau^{1-\sigma} (f_p + f_d) \tag{13}$$

And

$$\begin{cases}
\pi_d(\varphi^*, f_{x,i}) + \pi_x(\varphi^*, f_{x,i}) = (\varepsilon P)^{\sigma - 1} (1 + \tau^{1 - \sigma}) \varphi^{*\sigma - 1} - (f_p + f_d + f_{x,i}) = 0 \\
\varphi^* = \varphi_d^* = \varphi_x^*
\end{cases}$$
(14)

For firms with fixed export cost between $f_{x,l}$ and $f_{x,h}$, they have to serve both markets to cover the fixed cost. Firms with productivity cut-off will get negative profit from any single market, but they can serve both markets to get zero profit.

3 Firm Entry and Exit

There is a pool of potential entrants, but only a portion of firms enter and a less portion of firms successfully serve the markets. After paying an entry sunk cost, firm enters into market and randomly draw its parameter $f_{x,i}$ under distribution $\psi(f_x)$. We assume that for every fixed export cost there is a set of firms denoted as $\Gamma(f_{x,i})$ with productivity under distribution of $g(\varphi)$. Any entrant draws productivity from $g(\varphi)$, and if the productivity turns out to be less than productivity cut-off φ^* , the firm exits. The entry and exit process do not affect the distribution of fixed export cost and productivity. So productivity distribution of incumbents with same fixed export cost $\Gamma(f_{x,i})$ is denoted as:

$$\mu(\varphi)|_{\Gamma(f_{x,i})} = \begin{cases} \frac{g(\varphi)}{1 - G(\varphi^*|_{\Gamma(f_{x,i})})} & \text{if } \varphi > \varphi^*|_{\Gamma(f_{x,i})} \\ 0 & \text{otherwise} \end{cases}$$
 (15)

Though productivity is heterogeneous, there can be a representative productivity of all firms serving certain market (Melitz, 2003). Given fixed export cost, all firms with heterogeneous productivity serving domestic market will be simplified as

sharing identical productivity $\tilde{\varphi}_d$ and also all firms serving foreign market sharing identical productivity $\tilde{\varphi}_x$. And they can be expressed as:

$$\tilde{\varphi}_d(\varphi_d^*|_{\Gamma(f_{x,i})}) = \left[\frac{1}{1 - G(\varphi_d^*|_{\Gamma(f_{x,i})})} \int_{\varphi_d^*|_{\Gamma(f_{x,i})}}^{\infty} \varphi^{\sigma - 1} g(\varphi) d\varphi\right]^{\frac{1}{\sigma - 1}}$$
(16)

$$\tilde{\varphi}_x(\varphi_x^*|_{\Gamma(f_{x,i})}) = \left[\frac{1}{1 - G(\varphi_x^*|_{\Gamma(f_{x,i})})} \int_{\varphi_x^*|_{\Gamma(f_{x,i})}}^{\infty} \varphi^{\sigma-1} g(\varphi) d\varphi\right]^{\frac{1}{\sigma-1}}$$
(17)

Obviously $\tilde{\varphi}_d$ and $\tilde{\varphi}_x$ are functions of productivity cut-off and independent with the numbers of firms. Since $\tilde{\varphi}$ is representative productivity of all firms serving certain market, let $\pi_d(\tilde{\varphi}_d)$ and $\pi_x(\tilde{\varphi}_x)$ denote the average profit from domestic market and export respectively. According to Melitz(2003), if the fixed export cost is higher than $f_{x,h}$, the average profit of firms $\Gamma(f_{x,i})$ is determined as:

$$\overline{\pi}|_{\Gamma(f_{x,i}|f_{x,i}>f_{x,h})} = \pi_d(\tilde{\varphi}_d) + p_x \pi_x(\tilde{\varphi}_x) = (f_d + f_p)k(\varphi_d^*|_{\Gamma(f_{x,i})}) + p_x|_{\Gamma(f_{x,i})} f_x k(\varphi_x^*|_{\Gamma(f_{x,i})})$$
(18)

where $k(\varphi) = \left[\frac{\tilde{\varphi}(\varphi)}{\varphi}\right]^{\sigma-1} - 1$ and $p_x|_{\Gamma(f_{x,i})} = \frac{1 - G(\varphi_x^*|_{\Gamma(f_{x,i})})}{1 - G(\varphi_d^*|_{\Gamma(f_{x,i})})}$ is ex-post fraction of firms that export (See Appendix 3.1 for proof). If the fixed export cost is lower than $f_{x,l}$, the average profit of firms is determined as(See Appendix 3.2 for proof):

$$\overline{\pi}|_{\Gamma(f_{x,i}|f_{x,i} < f_{x,l})} = \pi_x(\tilde{\varphi}_x) + \pi_d(\tilde{\varphi}_d)/p_x = (f_x + f_p)k(\varphi_x^*|_{\Gamma(f_{x,i})}) + f_dk(\varphi_d^*|_{\Gamma(f_{x,i})})/p_x|_{\Gamma(f_{x,i})}$$
(19)

If the fixed export cost is between $f_{x,l}$ and $f_{x,h}$, the average profit of firms is determined as(See Appendix 3.3 for proof):

$$\overline{\pi}|_{\Gamma(f_{x,i}|f_{x,i}\in[f_{x,l},f_{x,h}])} = \pi_x(\tilde{\varphi}_x) + \pi_d(\tilde{\varphi}_d) = (f_p + f_d + f_x)k(\varphi^*|_{\Gamma(f_{x,i})})$$
(20)

Firm enters into market because it expects to have a probability of earning a profit per period until it is shocked out the market. Let $v|_{\Gamma(f_{x,i})}$ denote the value of a firm with fixed export cost $f_{x,i}$, so

$$v|_{\Gamma(f_{x,i})} = \frac{1 - G(\varphi^*|_{\Gamma(f_{x,i})})}{\delta} \overline{\pi}|_{\Gamma(f_{x,i})}$$
(21)

The value of entrant is described as:

$$V = \int_{0}^{f_{x,l}} v|_{\Gamma(f_{x,i}|f_{x,i} < f_{x,l})} \psi(f_{x}) df_{x} + \int_{f_{x,l}}^{f_{x,h}} v|_{\Gamma(f_{x,i}|f_{x,i} \in [f_{x,l}, f_{x,h}])} \psi(f_{x}) df_{x}$$

$$+ \int_{f_{x,h}}^{\infty} v|_{\Gamma(f_{x,i}|f_{x,i} > f_{x,h})} \psi(f_{x}) df_{x}$$
(22)

To incorporate (18), (19), (20) and (21) into (22), let $\varphi_{xh}^* = \varphi_x^*|_{\Gamma(f_{x,i}|f_{x,i}>f_{x,h})}$, $\varphi_{xl}^* = \varphi_x^*|_{\Gamma(f_{x,i}|f_{x,i}< f_{x,l})}$, $\varphi_{dh}^* = \varphi_d^*|_{\Gamma(f_{x,i}|f_{x,i}>f_{x,h})}$, $\varphi_{dl}^* = \varphi_d^*|_{\Gamma(f_{x,i}|f_{x,i}< f_{x,l})}$, and $\varphi_m^* = \varphi_{xm}^* = \varphi_{dm}^* = \varphi^*|_{\Gamma(f_{x,i}|f_{x,i}\in [f_{x,l},f_{x,h}])}$, we can get:

$$V = \frac{1}{\delta} \left[\int_{0}^{f_{x,l}} \left[(f_x + f_p) j(\varphi_{xl}^*) + f_d j(\varphi_{dl}^*) \right] \psi(f_x) df_x + \int_{f_{x,l}}^{f_{x,h}} \left[(f_p + f_d + f_x) j(\varphi_m^*) \right] \right]$$

$$\psi(f_x) df_x + \int_{f_{x,h}}^{\infty} \left[(f_d + f_p) j(\varphi_{dh}^*) + f_x j(\varphi_{xh}^*) \right] \psi(f_x) df_x$$
(23)

Where $j(\varphi) = k(\varphi)(1 - G(\varphi))$. V is a weighted average of value of firm with every level of fixed export cost $v|_{\Gamma(f_{x,i})}$, where weight is the density of fixed export cost. Free entry means that value of entrant equaling the sunk cost makes no more entry. In particular,

$$V = f_s \tag{24}$$

4 Equilibrium

In equilibrium, the productivity cut-off, number of export-only firms and domesticonly firms, the price index and quantity index are determined. Among these economic variables, productivity cut-off for domestic market is the great starting point. If the productivity cut-off for domestic market is determined, all the remaining economic variables are determined.

4.1 Determination of Equilibrium

Theorem 4.1. There exists a unique equilibrium.

According to (10), (12) and (14), formula (23) denotes V as a function of productivity cut-off for domestic market (φ_{dl}^* or φ_{dm}^*). So formula (24) determines the equilibrium by getting a unique productivity cut-off for domestic market (see appendix 4.1 for proof).

With productivity cut-off for domestic market determined, the productivity cut-off of every fixed export cost is determined according to formula (10),(12) and (14). So the $p_x|_{\Gamma(f_{x,i})}$ is determined and according to (20) and (21) into (22), the average

profit $(\overline{\pi})$ is determined as:

$$\overline{\pi} = \int_0^{f_{x,l}} \overline{\pi}|_{\Gamma(f_{x,i}|f_{x,i} < f_{x,l})} \psi(f_x) \mathrm{d}f_x + \int_{f_{x,l}}^{f_{x,h}} \overline{\pi}|_{\Gamma(f_{x,i}|f_{x,i} \in [f_{x,l},f_{x,h}])} \psi(f_x) \mathrm{d}f_x$$

$$+ \int_{f_{x,h}}^{\infty} \overline{\pi}|_{\Gamma(f_{x,i}|f_{x,i} > f_{x,h})} \psi(f_x) \mathrm{d}f_x$$
(25)

So the average revenue (\bar{r}) is determined as:

$$\overline{r} = \overline{\pi} + \int_{0}^{f_{x,l}} [f_d/p_x|_{\Gamma(f_{x,i}|f_{x,i} < f_{x,l})} + (f_p + f_x)]\psi(f_x)df_x + \int_{f_{x,l}}^{f_{x,h}} (f_p + f_d + f_x)$$

$$\psi(f_x)df_x + \int_{f_{x,h}}^{\infty} [f_d + f_p + f_x p_x|_{\Gamma(f_{x,i}|f_{x,i} > f_{x,h})}]\psi(f_x)df_x$$
(26)

So number of incumbents is determined as $M = R/\overline{r} = L/\overline{r}$. The number of domestic market-only firms is determined by:

$$M_D = \int_{f_{x,h}}^{\infty} M(1 - p_x|_{\Gamma(f_{x,i}|f_{x,i} > f_{x,h})}) \psi(f_x) df_x$$
 (27)

The number of export only firms is determined by:

$$M_E = \int_0^{f_{x,l}} M(1 - \frac{1}{p_x|_{\Gamma(f_{x,i}|f_{x,i} < f_{x,l})}}) \psi(f_x) df_x$$
 (28)

Price index and quantity index are determined, so all economic variables are determined in this equilibrium.

4.2 Properties of Equilibrium

With all productivity cut-off determined, Fig 1 is drawn to explain the relationship between productivity cut-off and fixed export cost in equilibrium (Hereafter we draw Figures assuming $1 < \sigma < 2$. If $\sigma \ge 2$, everything is identical except the shape of figures. See Appendix 4.2 for other figures).

Theorem 4.2. There co-exist the export-only firms, domestic market-only firms and firms serving both domestic and foreign markets.

1) If the fixed export cost is higher than $f_{x,h}$, active firms with low productivity serve the domestic market only and firms with high productivity serve both markets. These domestic-market-only firms are described as $\phi_i(\varphi_{dh}^* < \varphi_i < \varphi_{xh}^* | f_{x,i} > f_{x,h})$ and shown as A area in Fig 1.

- 2) If the fixed export cost is lower than $f_{x,l}$, active firms with low productivity serve foreign market only and firms with high productivity serve both markets. These export-only firms are described as $\phi_i(\varphi_{xl}^* < \varphi_i < \varphi_{dl}^* | f_{x,i} < f_{x,l})$ and shown as B area in Fig 1.
- 3) If the fixed export cost is between $f_{x,l}$ and $f_{x,h}$, all active firms serve both markets. Any firm serving single market cannot get positive profit. C area in Fig 1 denotes the firms serving both markets.

Theorem 4.3. Productivity of exporters is not always higher than non-exporters (domestic-market-only firms).

It is obvious from Fig 1 that the firms $\phi_i(\varphi_m^* < \varphi < \varphi_{dh}^*|f_{x,l} < f_{x,i} < f_{x,l})$ and firms $\phi_i(\varphi_{xl}^* < \varphi_i < \varphi_{dh}^*|f_{x,i} < f_{x,l})$ serve foreign market, and their productivity is lower than productivity of non-exporters. A firm with low productivity may serve foreign market due to low fixed export cost. Whether the average productivity of exporters is higher than non-exporters depends on the distribution of fixed export cost and productivity.

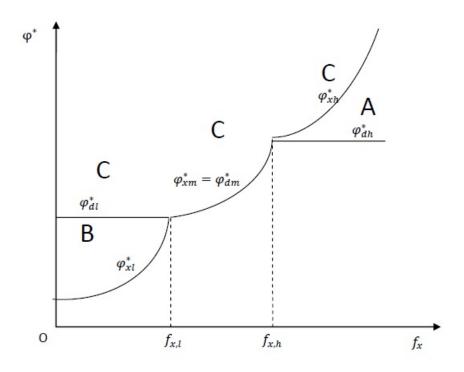


Fig 1: Productivity cut off in equilibrium

5 Trade Liberalization

5.1 Trade Liberalization

In autarky, firms only serve domestic market, so the average profit of firms $\Gamma(f_{x,i})$ is determined as:

$$\overline{\pi}|_{\Gamma(f_{x,i})} = \pi_d(\tilde{\varphi}_d) = (f_d + f_p)k(\varphi_a^*|_{\Gamma(f_{x,i})})$$
(29)

In autarky, fixed export cost plays no role to firms profit and productivity cut-off, so for every level of fixed export cost, the average profit and productivity cut-off are identical. So the value of entrant is denoted as:

$$V_{a} = \frac{1}{\delta} \int_{0}^{\infty} (f_{d} + f_{p}) j(\varphi_{a}^{*}|_{\Gamma(f_{x,i})}) \psi(f_{x}) df_{x} = \frac{1}{\delta} (f_{d} + f_{p}) j(\varphi_{a}^{*}) = f_{s}$$
 (30)

Autarky is same with situation that all firms share an infinite fixed export cost, where the value of entrant is identical to formula (29) since $p_x|_{\Gamma(f_{x,i})} = 0$ in formula (18). Formula (30) determines the unique productivity cut-off φ_a^* since $j(\varphi)$ is monotonically decreasing from infinity to zero.

Theorem 5.1. Let $\varphi_{xl}^*|_{f_x=0}$ denote the productivity cut-off for export of firms with zero fixed export cost, $\varphi_{xl}^*|_{f_x=0} < \varphi_a^* < \varphi_{dh}^*$ (see appendix 5.1 for proof).

However whether the productivity cut-off for domestic market for firms with lower fixed export cost than $f_{x,l}$ is higher than φ_a^* is uncertain, which depends on the fixed export cost distribution. The value of firm decreases as fixed export cost increases (shown in appendix 5.1). If the enough firms tend to have high fixed export cost, the productivity cut-off will be low. At some extent, productivity cut-off for domestic market for firms with low fixed export cost φ_{dl}^* may be lower than φ_a^* as shown in Fig 2a, otherwise φ_{dl}^* is higher than φ_a^* as shown in Fig 2b.

Theorem 5.2. Exposure to trade affects industrial productivity not unambiguously.

Exposure to trade not only forces least productive firms with fixed export cost higher than \overline{f}_x out of market, but also induces even less productive firms with lower fixed export cost than \overline{f}_x into market. In open economy, the opportunity for some firms to get extra profit makes the competition for labour higher than in autarky to bid up the real wage. The lowest productive firms with fixed export cost higher than \overline{f}_x cannot afford the new wage, so they are forced out (D area in Fig 2). For firms with fixed export cost lower than \overline{f}_x even the real wage is higher, they can benefit more from the foreign market. So they can stay in the

market by serving both markets (Fig 2a) or switch from domestic market to foreign market (Fig 2b F area). At the same time, less productive firms are induced into the market. In Fig 2a, these new entrants with low productivity only serve foreign market (E1) and entrants with high productivity can even serve both markets (E2). In Fig 2b, these new entrants can only serve foreign market. Exposure to trade affects industrial productivity not unambiguously. The overall outcome is subject to firms distributions of fixed export cost and productivity, which determine the portfolio of firms exit and enter.

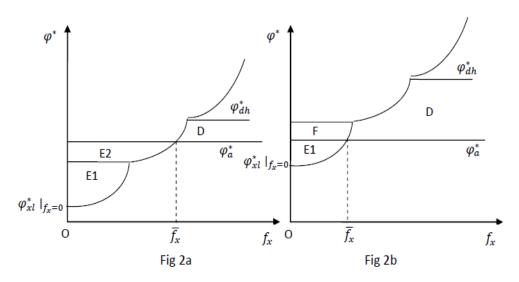


Fig 2: Productivity cut off from autarky to trade

5.2 Decrease in Fixed Export Cost

If single firm change the fixed export cost, it is assumed not affect distribution of fixed export cost. Here decrease in fixed export cost means that distribution of fixed export cost moves to the left. Fig 3a shows an example by assuming that the distribution only has one peak.

Theorem 5.3. Given that fixed export cost distribution has one peak, shifting fixed export cost distribution to left will raise all productivity cut-off.

V determined by formula (23) is a weighted average of value of firm with every level fixed export cost $(v|_{\Gamma(f_{x,i})})$ in formula (21), where weight is the density of fixed export cost. The shift of distribution of fixed export cost will give value of low-fixed-export-cost firm higher weight and value of high-fixed-export-cost firm

lower weight. The value of firm increases as fixed export cost decreases (shown in appendix 5.1). So the shift of fixed export cost leads to a higher V which raises all productivity cut-off. Noticeably, it is vital to assume only one peak of distribution, while where the peak is makes no difference. If the distribution has more than one peak, V in (23) may rise or decrease, leading to change the new productivity cut-off unambiguously. Where the peak is makes no differences because always is that value of low-fixed-export-cost firms has higher weight and value of high-fixed-export-cost firms has lower weight. The new curve of productivity cut-off shifts above as shown in Fig 3b. Shifting fixed export cost distribution to left raises average profit of firms, which makes completion for labour higher to bid up the real wage. So the least productive firms of every fixed export cost are forced out of the market. The least productive firms that serve both markets switch to single market. Those with fixed export cost lower than $f_{x,l}$ (higher than $f_{x,h}$) switch from both markets to foreign (domestic) market only, because they cannot get profit from the domestic (foreign) market for a higher wage.

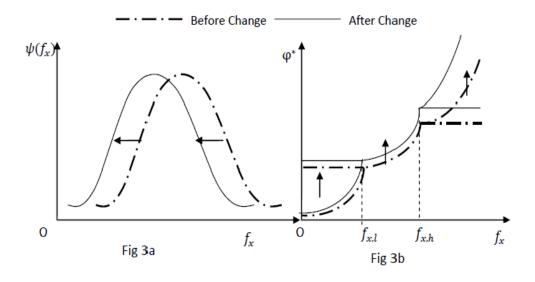


Fig 3: Shift of fixed export cost distribution

5.3 Decrease in Variable Trade Cost

Theorem 5.4. Decrease in variable trade cost will raise the productivity cut-off for domestic market (φ_{dh}^* and φ_{dl}^*) and decrease the productivity cut-off for foreign market (φ_{xl}^* and φ_{xh}^*). For firms that can only get profit from both markets, the

mixed two countervailing effects affect the productivity cut-off ambiguously (see appendix 5.2 for proof).

Decrease in variable trade cost will make firms that serve foreign market get more profit thereby increasing the demand for labour. The real wage is higher. So the productivity cut-off for domestic market becomes higher. For firms with fixed export cost higher than $f_{x,h}$, least productive firms that only serve domestic market are forced out of market. For firms with fixed export cost lower than $f_{x,l}$, the least productive firms serving both markets switch to foreign market only. However, even the real wage is higher, the firms serving foreign market still can benefit from a lower variable trade cost. The productivity cut-off for foreign markets becomes lower to induce more firms into foreign market. Fig 4 is drawn to depict the change of equilibrium.

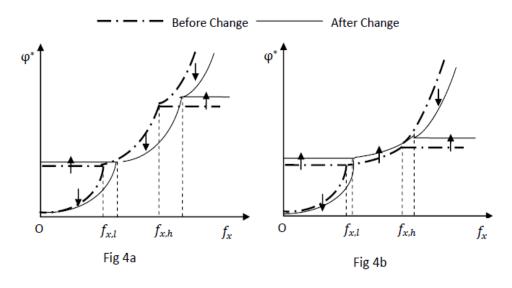


Fig 4: Decrease in variable trade cost

However, the productivity cut-off for firms with medium fixed export cost is affected uncertainly. These firms can only get profit from both markets. The higher wage rate makes profit less. On the other hand, lower variable export cost makes profit from foreign market more. So the mixed two countervailing effects affect the productivity cut-off uncertainly. Whether the new productivity cut-off is higher or not depends on which effect is stronger. If later effect dominates this process, productivity cut-off becomes lower (shown in Fig 4a); otherwise productivity cut-off becomes higher (shown in Fig 4b). Actually which effect is strong depends on the fixed export cost distribution, if more firms have low fixed export cost, the elasticity of real wage rate to variable trade cost is higher to make wage effect

dominates, so the productivity cut-off increases; otherwise the productivity cut-off decreases.

6 Innovation

Single firm adopts innovation to have a higher productivity, which does not affect the equilibrium. Here we assume an innovation across all firms, for example a new computer technology. The distribution of productivity shifts to right to make a new equilibrium.

Theorem 6.1. If $g'(\varepsilon) < 0$ for $\varepsilon \epsilon [\varphi_{xl}^*|_{f_x=0}, \infty)$, innovation will raise all the productivity cut-off, shifting the curve of productivity cut-off up; while if $g'(\varepsilon) > 0$ for $\varepsilon \epsilon [\varphi_{xl}^*|_{f_x=0}, \infty)$, innovation will shift the curve of productivity cut-off down (see appendix 6.1 for proof).

Innovation affects equilibrium through the active firms which have a higher productivity than the lowest productivity cut-off $\varphi_{xl}^*|_{f_x=0}$. If $g'(\varepsilon) < 0$ for $\varepsilon \epsilon [\varphi_{xl}^*|_{f_x=0}, \infty)$, innovation will increase the density of active firms productivity for every level of fixed export cost as shown in Fig 5a. This will increase average productivity of active firms and profit, thereby increasing the competition for labor, further raising the productivity cut-off as shown in Fig 5b; otherwise if $g'(\varepsilon) > 0$ for $\varepsilon \epsilon [\varphi_{xl}^*|_{f_x=0}, \infty)$, innovation will decrease the productivity cut-off as shown in Fig 5d. The distribution of non-active firms will not affect the new equilibrium.

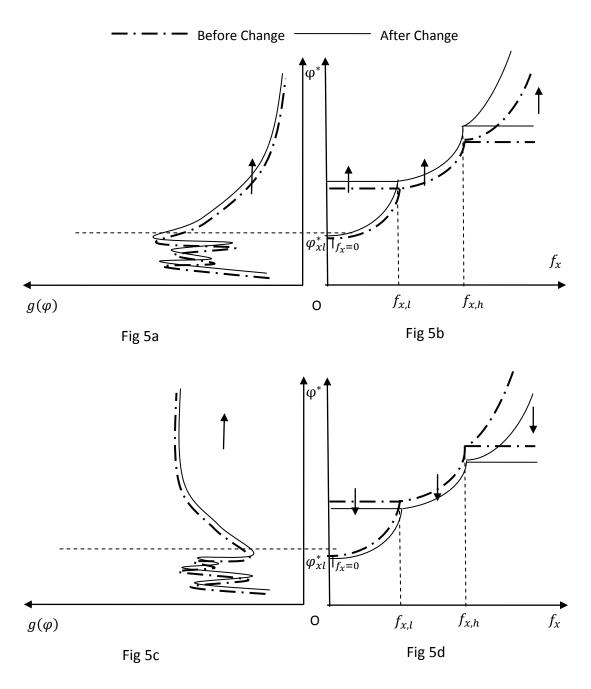


Fig 5 Shift of distribution of productivity

7 Conclusion

This paper has analyzed a new realm of firms export behaviour: serving foreign markets only. These export-only firms are especially relevant in developing countries due to their trade regimes and patterns of participating globalization. Export-only firms, domestic-market-only firms and firms serving both markets co-exist, which can only be theoretically explained with additional firm-level heterogeneity of fixed export cost other than productivity. The existence of fixed cost for serving domestic market also plays a key role here but is rarely given a deserved attention in the previous literatures. The paper shows firms with a combination of low productivity and low fixed export cost serve the foreign market only and firms with a combination of low productivity and high fixed export cost serve domestic market only. The rest firms serve both markets. The productivity of export-only firms is relatively lower than domestic-market-only firms.

The paper shows that heterogeneity of fixed export cost exceedingly affects the impact of exposure to trade on firms export behaviour. Trade not only forces the least productive firms with high fixed export cost out of the market, but induces even less productive firms with low fixed export cost into the market as export-only firms. This model captures export-only firms and also includes Melitzs model as a special case. Trade-induced reallocation is more complicated and affects aggregate industrial productivity not unambiguously. Whether overall productivity is higher depends on the firms distributions of fixed export cost and productivity, which tells the portfolio of firms exit and enter. This paper also sheds light on impact of trade liberalization and innovation on firms behaviour. In particular, the paper explores how shift of fixed export cost distribution and productivity distribution affects the productivity cut-off, forces firms out and switches firms between markets. The thought-provoking point is the effect of decrease in variable export cost. The productivity cut-off for firms with medium export cost is based serving both markets, and decrease in variable export cost makes these firms get less profit from both market and more profit from foreign market. So the mix effect from two markets affects productivity cut-off uncertainly.

Although this paper shows aggregate industrial productivity is affected uncertainly by trade, the welfare from trade in terms of decreased price level and increased product variety is still channelled as other literatures. The model of this paper gives a new empirical implication to explore the relationship between export behaviour and productivity. This paper introduces heterogeneous fixed export cost and highlights the export-only firms with low fixed cost, however ignores that the firms with high enough fixed export cost may select to invest in foreign markets directly. The heterogeneity of fixed export cost provides a potential unified

framework to include FDI, which is an interesting area in future study.

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Appendix

Table A 1: Firm Export Behaviour from 1999 to 2008 in China

year	all firms	D-market	F-market	both	Ratio(1)	Ratio(2)	Ratio(3)
		only	only	\max	%	%	%
1999	155,534	120,836	9,205	25,493	22.31	5.92	26.53
2000	$156,\!852$	$119,\!652$	$10,\!407$	26,793	23.72	6.63	27.98
2001	$166,\!560$	125,757	11,689	29,114	24.50	7.02	28.65
2002	177,283	131,979	12,713	$32,\!591$	25.55	7.17	28.06
2003	193,415	$142,\!510$	14,608	36,297	26.32	7.55	28.70
2005	269,607	193,989	$20,\!484$	55,134	28.05	7.60	27.09
2006	299,605	$220,\!290$	$21,\!559$	57,756	26.47	7.20	27.18
2007	$335,\!460$	$256,\!357$	$22,\!581$	$56,\!522$	23.58	6.73	28.55
2008	410,837	322,610	22,309	65,918	21.47	5.43	25.29

Notes: Source form China Industrial Firm-level database which covers all manufacturing firms with sales more than 5 million RMB, accounting for more than 90% of Chinese industrial output. Some data of year 2004 is missed in our database. We only draw 'gross sales value' and 'export delivery value' from raw database and drop all observations with gross sales value not higher than zero and less than 'export delivery value'. On average, only 1.8% of firms are deleted. Column from 2 to 5 means number of all firms, firm serving domestic market only, firms serving foreign market only and firms serving both market. Ratio (1) means share of exporters (serving both markets plus serving foreign market only) by all firms. Ratio (2) means share of export-only firms by all firms. Ratio (3) means share of export-only firms by all exporters.

Appendix 3.1. Average profit of firms $\Gamma(f_{x,i}|f_{x,i}>f_{x,h})$

If fixed export cost is higher than $f_{x,h}$, $\pi_d(\varphi_d^*) = \frac{r_d(\varphi_d^*)}{\sigma} - f_p - f_d = 0$ and $\pi_x(\varphi_x^*) = \frac{r_x(\varphi_x^*)}{\sigma} - f_x = 0$. According to (3) and (5), we can get $\frac{r_d(\tilde{\varphi}_d)}{r_d(\varphi_d^*)} = (\frac{\tilde{\varphi}_d}{\varphi_d^*})^{\sigma-1}$ and $\frac{r_x(\tilde{\varphi}_x)}{r_x(\varphi_x^*)} = (\frac{\tilde{\varphi}_x}{\varphi_x^*})^{\sigma-1}$. So $\pi_d(\tilde{\varphi}_d) = \frac{r_d(\tilde{\varphi}_d)}{\sigma} - f_p - f_d = \frac{r_d(\varphi_d^*)}{\sigma} (\frac{\tilde{\varphi}_d}{\varphi_d^*})^{\sigma-1} - f_p - f_d = (f_p + f_d)k(\varphi_d^*)$ and $\pi_x(\tilde{\varphi}_x) = \frac{r_x(\tilde{\varphi}_x)}{\sigma} - f_x - \frac{r_x(\varphi_x^*)}{\sigma} (\frac{\tilde{\varphi}_x}{\varphi_x^*})^{\sigma-1} - f_x = f_x k(\varphi_x^*)$. So $\overline{\pi}|_{\Gamma(f_{x,i}|f_{x,i}>f_{x,h})} = \pi_d(\tilde{\varphi}_d) + p_x \pi_x(\tilde{\varphi}_x) = (f_d + f_p)k(\varphi_d^*|_{\Gamma(f_{x,i})}) + p_x|_{\Gamma(f_{x,i})}f_x k(\varphi_x^*|_{\Gamma(f_{x,i})})$

Appendix 3.2. Average profit of firms $\Gamma(f_{x,i}|f_{x,i} < f_{x,l})$

If fixed export cost is lower than $f_{x,l}$, $\pi_x(\varphi_x^*) = \frac{r_x(\varphi_x^*)}{\sigma} - f_p - f_x = 0$ and $\pi_d(\varphi_d^*) = \frac{r_d(\varphi_d^*)}{\sigma} - f_d = 0$. So $\pi_x(\tilde{\varphi}_x) = \frac{r_x(\tilde{\varphi}_x)}{\sigma} - f_p - f_x = \frac{r_x(\varphi_x^*)}{\sigma} (\frac{\tilde{\varphi}_x}{\varphi_x^*})^{\sigma - 1} - f_p - f_x = (f_p + f_x)k(\varphi_x^*)$ and $\pi_d(\tilde{\varphi}_d) = \frac{r_d(\tilde{\varphi}_d)}{\sigma} - f_d = \frac{r_d(\varphi_d^*)}{\sigma} (\frac{\tilde{\varphi}_d}{\varphi_d^*})^{\sigma - 1} - f_d = f_dk(\varphi_d^*)$. And we can get:

$$\overline{\pi}|_{\Gamma(f_{x,i}|f_{x,i} < f_{x,l})} = \pi_x(\tilde{\varphi}_x) + \pi_d(\tilde{\varphi}_d)/p_x = (f_x + f_p)k(\varphi_x^*|_{\Gamma(f_{x,i})}) + f_dk(\varphi_d^*|_{\Gamma(f_{x,i})})/p_x|_{\Gamma(f_{x,i})}.$$

Appendix 3.3. Average profit of firms $\Gamma(f_{x,i}|f_{x,i}\epsilon[f_{x,l},f_{x,h}])$

If fixed export cost is between $f_{x,l}$ and $f_{x,h}$, $\frac{r_d(\varphi^*)}{\sigma} + \frac{r_x(\varphi^*)}{\sigma} - f_p - f_d - f_x = 0$, so $(1 + \tau^{1-\sigma})r_d(\varphi_d^*) = \sigma(f_p + f_d + f_x)$. And we can get $\overline{\pi}|_{\Gamma(f_{x,i}|f_{x,i}\in[f_{x,l},f_{x,h}])} = \frac{r_d(\tilde{\varphi})}{\sigma} + \frac{r_x(\tilde{\varphi})}{\sigma} - f_p - f_d - f_x = \frac{(1+\tau^{1-\sigma})r_d(\tilde{\varphi})}{\sigma} - f_p - f_d - f_x = \frac{(1+\tau^{1-\sigma})r_d(\varphi^*)}{\sigma} (\frac{\tilde{\varphi}}{\varphi^*})^{\sigma-1} - f_p - f_d - f_x = (f_p + f_d + f_x)k(\varphi^*|_{\Gamma(f_{x,i})})$

Appendix 4.1. Unique existence of equilibrium

We take productivity cut-off for domestic market (φ_{dl}^*) as an example. Substitute all other productivity cut-off in formula (23) using φ_{dl}^* from (10), (12) and (14), the value of entrant is a function of φ_{dl}^* , $V(\varphi_{dl}^*)$. Equilibrium is determined by $f_s = V(\varphi_{dl}^*)$. Hereby we prove $V(\varphi_{dl}^*)$ is monotonically increasing from zero to infinity with φ_{dl}^* over $(0, \infty)$.

$$V\prime(\varphi_{dl}^*) = \frac{1}{\delta} \left[\int_0^{f_{x,l}} [(f_x + f_p)j\prime(\varphi_{xl}^*) \frac{\partial \varphi_{xl}^*}{\partial \varphi_{dl}^*} + f_d j\prime(\varphi_{dl}^*)] \psi(f_x) \mathrm{d}f_x + \int_{f_{x,l}}^{f_{x,h}} [(f_p + f_d + f_x) + f_d j\prime(\varphi_{dl}^*)] \psi(f_x) \mathrm{d}f_x + \int_{f_{x,l}}^{f_{x,h}} [(f_p + f_d + f_x) + f_d j\prime(\varphi_{dl}^*)] \psi(f_x) \mathrm{d}f_x + \int_{f_{x,l}}^{f_{x,h}} [(f_p + f_d + f_x) + f_d j\prime(\varphi_{dl}^*)] \psi(f_x) \mathrm{d}f_x + \int_{f_{x,l}}^{f_{x,h}} [(f_p + f_d + f_x) + f_d j\prime(\varphi_{dl}^*)] \psi(f_x) \mathrm{d}f_x + \int_{f_{x,l}}^{f_{x,h}} [(f_p + f_d + f_x) + f_d j\prime(\varphi_{dl}^*)] \psi(f_x) \mathrm{d}f_x + \int_{f_{x,l}}^{f_{x,h}} [(f_p + f_d + f_x) + f_d j\prime(\varphi_{dl}^*)] \psi(f_x) \mathrm{d}f_x + \int_{f_{x,l}}^{f_{x,h}} [(f_p + f_d + f_x) + f_d j\prime(\varphi_{dl}^*)] \psi(f_x) \mathrm{d}f_x + \int_{f_{x,l}}^{f_{x,h}} [(f_p + f_d + f_x) + f_d j\prime(\varphi_{dl}^*)] \psi(f_x) \mathrm{d}f_x + \int_{f_{x,l}}^{f_{x,h}} [(f_p + f_d + f_x) + f_d j\prime(\varphi_{dl}^*)] \psi(f_x) \mathrm{d}f_x + \int_{f_{x,l}}^{f_{x,h}} [(f_p + f_d + f_x) + f_d j\prime(\varphi_{dl}^*)] \psi(f_x) \mathrm{d}f_x + \int_{f_{x,l}}^{f_{x,h}} [(f_p + f_d + f_x) + f_d j\prime(\varphi_{dl}^*)] \psi(f_x) \mathrm{d}f_x + \int_{f_{x,l}}^{f_{x,h}} [(f_p + f_d + f_x) + f_d j\prime(\varphi_{dl}^*)] \psi(f_x) \mathrm{d}f_x + \int_{f_{x,l}}^{f_{x,h}} [(f_p + f_d + f_x) + f_d j\prime(\varphi_{dl}^*)] \psi(f_x) \mathrm{d}f_x + \int_{f_{x,l}}^{f_{x,h}} [(f_p + f_d + f_x) + f_d j\prime(\varphi_{dl}^*)] \psi(f_x) \mathrm{d}f_x + \int_{f_{x,l}}^{f_{x,h}} [(f_p + f_d + f_x) + f_d j\prime(\varphi_{dl}^*)] \psi(f_x) \mathrm{d}f_x + \int_{f_{x,l}}^{f_{x,h}} [(f_p + f_d + f_x) + f_d j\prime(\varphi_{dl}^*)] \psi(f_x) \mathrm{d}f_x + \int_{f_{x,l}}^{f_{x,h}} [(f_p + f_d + f_x) + f_d j\prime(\varphi_{dl}^*)] \psi(f_x) \mathrm{d}f_x + \int_{f_{x,l}}^{f_{x,h}} [(f_p + f_d + f_x) + f_d j\prime(\varphi_{dl}^*)] \psi(f_x) \mathrm{d}f_x + \int_{f_{x,l}}^{f_{x,h}} [(f_p + f_d + f_x) + f_d j\prime(\varphi_{dl}^*)] \psi(f_x) \mathrm{d}f_x + \int_{f_{x,l}}^{f_{x,h}} [(f_p + f_d + f_x) + f_d j\prime(\varphi_{dl}^*)] \psi(f_x) \mathrm{d}f_x + \int_{f_{x,l}}^{f_{x,h}} [(f_p + f_d + f_x) + f_d j\prime(\varphi_{dl}^*)] \psi(f_x) \mathrm{d}f_x + \int_{f_{x,l}}^{f_{x,h}} [(f_p + f_d + f_x) + f_d j\prime(\varphi_{dl}^*)] \psi(f_x) \mathrm{d}f_x + \int_{f_{x,l}}^{f_{x,h}} [(f_p + f_d + f_x) + f_d j\prime(\varphi_{dl}^*)] \psi(f_x) \mathrm{d}f_x + \int_{f_{x,l}}^{f_{x,h}} [(f_p + f_d + f_x) + f_d j\prime(\varphi_{dl}^*)] \psi(f_x) \mathrm{d}f_x + \int_{f_{x,l}}^{f_{x,h}} [(f_p + f_d + f_x) + f$$

 $j(\varphi)=k(\varphi)(1-G(\varphi)),\ j\prime(\varphi)=\frac{(1-\sigma)(1-G(\varphi))(1+k(\varphi))}{\varphi}<0,\ j(\varphi)\ \text{is monotonically decreasing from infinity to zero on }(0,\infty)\ (\text{refer to Melitz}(2003)\ \text{Appendix}).\ \text{From formula }(10),\ (12)\ \text{and }(14),\ \varphi_{xl}^*=\tau\varphi_{dl}^*(\frac{f_x+f_p}{f_d})^{\frac{1}{\sigma-1}},\ \varphi_m^*=\varphi_{dl}^*(\frac{f_x+f_p+f_d}{f_d(1+\tau^{1-\sigma})})^{\frac{1}{\sigma-1}},\ \varphi_{xh}^*=\tau\varphi_{dl}^*(\frac{f_x}{f_d})^{\frac{1}{\sigma-1}},\ \text{and}\ \varphi_{dh}^*=\varphi_{dl}^*(\frac{f_d+f_p}{f_d})^{\frac{1}{\sigma-1}},\ \frac{\partial\varphi_{xl}^*}{\partial\varphi_{dl}^*},\ \frac{\partial\varphi_{xh}^*}{\partial\varphi_{dl}^*},\ \frac{\partial\varphi_{xh}^*}{\partial\varphi_{dl}^*}\ \text{and}\ \frac{\partial\varphi_{dh}^*}{\partial\varphi_{dl}^*}\ \text{are all positive, leading to}\ V\prime(\varphi_{dl}^*)<0.\ \text{When}\ \varphi_{xh}^*\ \text{approaches to infinity (zero)},\ \varphi_{xl}^*\ \text{approaches to infinity (zero)}\ \text{and so do}\ \varphi_m^*,\ \varphi_{xh}^*\ \text{and}\ \varphi_{dh}^*,\ \text{making}\ j(\varphi_{xl}^*),\ j(\varphi_{dl}^*),\ j(\varphi_m^*),\ j(\varphi_{xh}^*)\ \text{and}\ j(\varphi_{dh}^*)\ \text{all approach to zero (infinity), furthermore making}\ V(\varphi_{dl}^*)\ \text{approach to zero (infinity)}.\ \text{So}\ V(\varphi_{dl}^*)\ \text{is monotonically decreasing from infinity to zero with with}\ \varphi_{dl}^*\ \text{over}(0,\infty)\ \text{and there exists a unique equilibrium, where}\ \varphi_{xl}^*,\ \varphi_{dl}^*,\ \varphi_m^*,\ \varphi_{xh}^*\ \text{and}\ \varphi_{dh}^*\ \text{are all determined with a unique value.}$

Appendix 4.2. Figures with $\sigma \geq 2$.

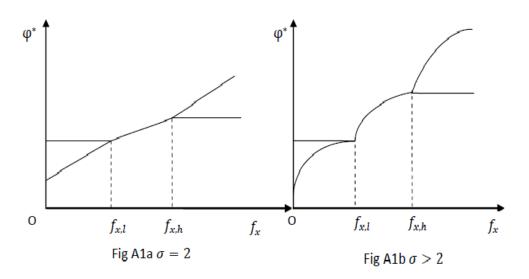


Fig A 1: Figures with $\sigma \geq 2$

Appendix 5.1. $\varphi_{xl}^*|_{f_x=0} < \varphi_a^* < \varphi_{dh}^*$

According to formula (21), $v|_{\Gamma(f_{x,i})} = \frac{1 - G(\varphi^*|_{\Gamma(f_{x,i})})}{\delta} \overline{\pi}|_{\Gamma(f_{x,i})}$, we first prove $\frac{\partial v|_{\Gamma(f_{x,i})}}{\partial f_x} < 0$.

- 1) If the fixed export cost is higher than $f_{x,h}$, $v|_{\Gamma(f_{x,i})} = \frac{1}{\delta}[(f_d + f_p)j(\varphi_{dh}^*) + f_x j(\varphi_{xh}^*)]$. So $\frac{\partial v|_{\Gamma(f_{x,i})}}{\partial f_x} = \frac{1}{\delta}[j(\varphi_{xh}^*) + f_x j \prime(\varphi_{xh}^*) \frac{\partial \varphi_{xh}^*}{\partial f_x}] = \frac{1}{\delta}[G(\varphi_{xh}^*) 1] < 0$.
- 2) If the fixed export cost is lower than $f_{x,l}$, $v|_{\Gamma(f_{x,i})} = \frac{1}{\delta}[(f_x + f_p)j(\varphi_{xl}^*) + f_d j(\varphi_{dl}^*)]$. So $\frac{\partial v|_{\Gamma(f_{x,i})}}{\partial f_x} = \frac{1}{\delta}[j(\varphi_{xl}^*) + (f_x + f_p)j\prime(\varphi_{xl}^*)\frac{\partial \varphi_{xl}^*}{\partial f_x}] = \frac{1}{\delta}[G(\varphi_{xl}^*) - 1] < 0$.
- 3) If the fixed export cost is between $f_{x,l}$ and $f_{x,h}$, $v|_{\Gamma(f_{x,i})} = \frac{1}{\delta}[(f_d + f_p + f_x)j(\varphi_m^*)]$. So $\frac{\partial v|_{\Gamma(f_{x,i})}}{\partial f_x} = \frac{1}{\delta}[j(\varphi_m^*) + (f_x + f_p + f_d)j\prime(\varphi_m^*)\frac{\partial \varphi_m^*}{\partial f_x}] = \frac{1}{\delta}[G(\varphi_m^*) - 1] < 0$.

So $\frac{\partial v|_{\Gamma(f_{x,i})}}{\partial f_x} < 0$. If we assume all firms with infinite fixed export cos, V becomes lower. So in equilibrium, $V\delta = \delta f_s = (f_d + f_p)j(\varphi_a^*) > (f_d + f_p)j(\varphi_{dh}^*)$, and leads to $\varphi_a^* < \varphi_{dh}^*$. And if we assumes all firms with zero fixed export cost, V becomes higher. So in equilibrium, $V\delta = \delta f_s = (f_d + f_p)j(\varphi_a^*) < f_pj(\varphi_{xl}^*|_{f_x=0}) + f_dj(\varphi_{dl}^*|_{f_x=0}) < (f_d + f_p)j(\varphi_{xl}^*|_{f_x=0})$ since $\varphi_{xl}^*|_{f_x=0} < \varphi_{dl}^*$, and leads to $\varphi_{xl}^*|_{f_x=0} < \varphi_{dh}^*$.

Appendix 5.2. $\frac{\partial \varphi_{dl}^*}{\partial \tau} < 0$, $\frac{\partial \varphi_{dh}^*}{\partial \tau} < 0$, $\frac{\partial \varphi_{xl}^*}{\partial \tau} > 0$, and $\frac{\partial \varphi_{xh}^*}{\partial \tau} > 0$.

1) If we take
$$f_s = V(\varphi_{dl}^*, \tau)$$
, so $\frac{\partial \varphi_{dl}^*}{\partial \tau} = -\frac{\partial V/\partial \tau}{\partial V/\partial \varphi_{dl}^*}$.

$$\frac{\partial V}{\partial \tau} = \frac{1}{\delta} \left[\int_{0}^{f_{x,l}} \left[(f_x + f_p) j'(\varphi_{xl}^*) \frac{\partial \varphi_{xl}^*}{\partial \tau} \right] \psi(f_x) \mathrm{d}f_x + \left[(f_{x,l} + f_p) j(\varphi_{xl}^*|_{f_x = f_{x,l}}) \right] + f_x j(\varphi_{dl}^*|_{f_x = f_{x,l}}) \right] \psi(f_{x,l}) \frac{\partial f_{x,l}}{\partial \tau} + \int_{f_{x,l}}^{f_{x,h}} \left[(f_p + f_d + f_x) j'(\varphi_m^*) \frac{\partial \varphi_m^*}{\partial \tau} \right] \psi(f_x) \mathrm{d}f_x + \left[(f_p + f_d + f_{x,h}) j(\varphi_m^*|_{f_x = f_{x,h}}) \right] \psi(f_{x,h}) \frac{\partial f_{x,h}}{\partial \tau} - \left[(f_p + f_d + f_{x,l}) j(\varphi_m^*|_{f_x = f_{x,l}}) \right] \psi(f_{x,l}) \frac{\partial f_{x,l}}{\partial \tau} + \int_{f_{x,h}}^{\infty} f_x j'(\varphi_{xh}^*) \frac{\partial \varphi_{xh}^*}{\partial \tau} \psi(f_x) \mathrm{d}f_x - \left[(f_d + f_p) j(\varphi_{dh}^*|_{f_x = f_{x,h}}) + f_{x,h} j(\varphi_{xh}^*|_{f_x = f_{x,h}}) \right] \psi(f_{x,h}) \frac{\partial f_{x,h}}{\partial \tau} \right]$$

Because $\varphi_{xl}^*|_{f_x=f_{x,l}} = \varphi_{dl}^*|_{f_x=f_{x,l}} = \varphi_m^*|_{f_x=f_{x,l}}$ and $\varphi_{xh}^*|_{f_x=f_{x,h}} = \varphi_{dh}^*|_{f_x=f_{x,h}} = \varphi_m^*|_{f_x=f_{x,h}}$, so $[(f_{x,l}+f_p)j(\varphi_{xl}^*|_{f_x=f_{x,l}})$ $+ f_x j(\varphi_{dl}^*|_{f_x=f_{x,l}})]\psi(f_{x,l})\frac{\partial f_{x,l}}{\partial \tau} - [(f_p+f_d+f_{x,l})j(\varphi_{m}^*|_{f_x=f_{x,l}})]\psi(f_{x,l})\frac{\partial f_{x,l}}{\partial \tau} = 0$, and $[(f_p+f_d+f_{x,h})j(\varphi_m^*|_{f_x=f_{x,h}})]\psi(f_{x,h})\frac{\partial f_{x,h}}{\partial \tau} - [(f_d+f_p)j(\varphi_{dh}^*|_{f_x=f_{x,h}}) + f_{x,h}j(\varphi_{xh}^*|_{f_x=f_{x,h}})]$ $\psi(f_{x,h})\frac{\partial f_{x,h}}{\partial \tau}] = 0$. According to appendix 4.1, partial derivative $\frac{\partial \varphi_{xl}^*}{\partial \tau}$, $\frac{\partial \varphi_m^*}{\partial \tau}$, and $\frac{\partial \varphi_{xh}^*}{\partial \tau}$ are all positive, so $\frac{\partial V}{\partial \tau} < 0$. Since partial derivative $\frac{\partial V}{\partial \varphi_{dl}^*} < 0$, we can get $\frac{\partial \varphi_{dl}^*}{\partial \tau} < 0$, and so $\frac{\partial \varphi_{dh}^*}{\partial \tau} < 0$.

If we take $f_s = V(\varphi_{xl}^*, \tau)$, so $\frac{\partial \varphi_{xl}^*}{\partial \tau} = -\frac{\partial V/\partial \tau}{\partial V/\partial \varphi_{xl}^*}$.

$$\frac{\partial V}{\partial \tau} = \frac{1}{\delta} \left[\int_{0}^{f_{x,l}} f_{d}j'(\varphi_{dl}^{*}) \frac{\partial \varphi_{dl}^{*}}{\partial \tau} \psi(f_{x}) df_{x} + \int_{f_{x,l}}^{f_{x,h}} \left[(f_{p} + f_{d} + f_{x})j'(\varphi_{m}^{*}) \frac{\partial \varphi_{m}^{*}}{\partial \tau} \right] \psi(f_{x}) df_{x} \right]
+ \int_{f_{x,h}}^{\infty} (f_{p} + f_{d})j'(\varphi_{dh}^{*}) \frac{\partial \varphi_{dh}^{*}}{\partial \tau} \psi(f_{x}) df_{x} \right]$$

The partial derivative $\frac{\partial \varphi_{dl}^*}{\partial \tau}$, $\frac{\partial \varphi_{m}^*}{\partial \tau}$, and $\frac{\partial \varphi_{dh}^*}{\partial \tau}$ are all negative, so $\frac{\partial V}{\partial \tau} > 0$. Since partial derivative $\frac{\partial V}{\partial \varphi_{xl}^*} < 0$, we can get $\frac{\partial \varphi_{xl}^*}{\partial \tau} > 0$, and so $\frac{\partial \varphi_{xh}^*}{\partial \tau} > 0$.

 $\frac{\partial \varphi_m^*}{\partial \tau} = \frac{\varphi_m^*}{\varphi_{xl}^*} \frac{\partial \varphi_{xl}^*}{\partial \tau} - \frac{\varphi_m^* \tau^{\sigma-2}}{1 + \tau^{\sigma-1}} = \frac{\varphi_m^*}{\tau} \left(\frac{\tau}{\varphi_{xl}^*} \frac{\partial \varphi_{xl}^*}{\partial \tau} - \frac{\tau^{\sigma-1}}{1 + \tau^{\sigma-}} \right), \text{ which depends on elasticity of } \varphi_{xl}^*$ on τ , which is further related to the fixed export cost distribution.

Appendix 6.1. Effects of Innovation

$$j(\varphi) = k(\varphi)(1 - G(\varphi)) = \left[\left(\frac{\tilde{\varphi}(\varphi)}{\varphi} \right)^{\sigma - 1} - 1 \right] (1 - G(\varphi)) = \int_{\varphi}^{\infty} g(\varepsilon) \left[\left(\frac{\varepsilon}{\varphi} \right)^{\sigma - 1} - 1 \right] d\varepsilon.$$

If $g'(\varepsilon) < 0$ for $\varepsilon \epsilon [\varphi_{xl}^*|_{f_x=0}, \infty)$, innovation will shift the productivity to right, making a higher $j(\varphi)$. So for every fixed export cost, $j(\varphi^*)$ is higher, which means

 $j(\varphi_{dl}^*)$, $j(\varphi_{xl}^*)$, $j(\varphi_{xh}^*)$, $j(\varphi_{xh}^*)$ and $j(\varphi_{dh}^*)$ are all higher, leading to a higher $V(\varphi_{dl}^*)$ according to appendix 4.1. So in new equilibrium, φ_{dl}^* becomes higher, so do all the other productivity cut-off.

If $g'(\varepsilon) > 0$ for $\varepsilon \epsilon [\varphi_{xl}^*|_{f_x=0}, \infty)$, for every fixed export cost, $j(\varphi^*)$ is lower, leading to a lower $V(\varphi_{dl}^*)$. So φ_{dl}^* becomes lower and so do all the other productivity cut-off.