The short run welfare implications of openness to FDI and monetary policy*

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Very Preliminary

Abstract

We study the welfare effect of firms' cross-border production location decisions in an open-economy macroeconomic model, and discuss its implication on monetay policy conduct. We show that short run FDI fluctuations exacerbate utility loss over business cycles in an environment with nominal shocks, but has little impact on welfare over business cycles caused by productivity shocks. The first best outcome occurs when the economy retains long run FDI, but restricts short run changes in the production location of firms. We show that the presence of cross-border FDI flow has interesting implications on the conduct of monetary policy. Contrary to conventional wisdom, output gap targeting is welfare-improving in an environment when the natural level of output is unobservable.

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1 Introduction

From a macroeconomic perspective, foreign direct investment (FDI) is usually seen as a win-win strategy. It promotes economic growth for developing countries, and reduces production cost for developed countries. Economists have gone a long way in identifying and quantifying the various benefits of FDI. Previous studies mainly focus on the long run equilibrium gain of openness to FDI. This paper adds to the literature by examining the welfare implications of openness to FDI from a short run perspective. It is shown that over business cycles, relocation of multinational enterprises (MNE) hurts world welfare under nominal shocks, but has little effect on welfare under productivity shocks. Overall, fluctuations of FDI over the business cycle reduces welfare, and the first best outcome is achieved when the long-term FDI is retained and short run fluctuations is prohibited. Given the adverse welfare effects of short run FDI fluctuations, we further argue that monetary policy can target firm relocations of the short run nature. Contrary to conventional wisdom of monetary policy, we show that when the potential level of output is unknown to the policymaker, a Taylor type interest rate rule that targets output deviation from its steady state value improves welfare when FDI is present.

We incorporate firms' production location decisions in a two-country model based on Devereux and Engel (2001). In our model, firms move to produce in a foreign country to take advantage of lower production cost abroad, rather than to gain access to consumers of the local market. FDI is modeled as firms' fixed sunk investment cost when they decide to move production overseas. Location decisions need to be made one period ahead of actual production. Firms in each country are heterogeneous in their productivity levels. The two countries are asymmetric in terms of the average productivity of firms. In the model, households set wage in a Calvo manner, inducing sluggish adjustment of the real wage. The uncertainty in the model comes from both monetary shocks and productivity shocks.

In the model, a firm's relative profitability of producing abroad depends positively on its productivity, and on the expected gap in real wages between domestic and foreign workers. A firm decides its production location by comparing the net gain from switching to a low production cost location with the sunk cost the switch imposes. Given a lower expected real wage abroad, there exists a cutoff firm who will just break even in expectation by switching its production location. Every firm with a higher productivity than the cutoff firm must earn a strictly positive profit abroad, and thus becomes an MNE in the following period.

We show that the countries' average productivities determine the long run direction and volume of cross-border FDI flow. Specifically, the country with higher average productivity becomes the source country of FDI, and the country with lower productivity the recipient country. This is because without cross-border relocation of production, the country with lower productivity has lower real wage, making it profitable for the more productive firms in the source country to relocate.

The steady state of the model is Pareto optimal. Consequently, deviations from steady state following a nominal shock represent inefficiencies caused by nominal rigidity. In this process, firm relocation tends to futher aggrevate the harm. Following a foreign depreciaton, the influx of foreign firms increases labor demand. However, without FDI, foreign households are already working too much compared with the flexible-price long run equilibrium level. Yet when cross-border relocation is allowed, the increased labor demand from the new MNEs causes foreign households to work even more. Likewise, domestic households will work even less. Thus when the economy is hit by monetary shocks, firm relocation pushes aggregate employments further away from efficient levels.

To assess how FDI affects welfare, we compute the expected world utility by numerically simulating the model with second order approximation. We compare expected world utility in the benchmark model with utilities in two alternative scenarios - when all firms produce domestically, and when only long-term relocations are allowed. Our key findings are three-fold. First, firm relocations in the steady state are welfare-improving. Second, with nominal shocks, short run FDI fluctuations further deteriorate welfare loss caused by nominal frictions through the aforementioned employment effects. With real shocks, short run FDI fluctuations has little welfare effect as a result of several counteracting forces. Finally, the first best outcome can be achieved in a setting where long term firm relocation is retained, but short run, temporary relocations are eliminated. In the benchmark simulation, world utility is highest at every volatility level of real and nominal shocks when the distribution of MNEs is kept constant at the steady state level, but moving further back-and-forth is prohibited in the short run.

We then carry out a simple examination of monetary policy implications of the aforementioned adverse welfare effect of short run FDI fluctuations. Policymaker faces a realistic environment in which the potential level of GDP is unobservable. We argue that a traditionaly Taylor rule that targets output deviations from its steady state trend improves welfare, contrary to conventional wisdom that monetary policy should target inflation only. The close correlation of GDP and the employment is the key reason behind the result. The direct channel through which MNE relocations create welfare loss is their distortion of aggregate employments under nominal shocks. As output and employment move closely together, targeting the deviation of output automatically puts the deviation of the employment under control. We show that the benefit of output-gap targeting depends on the relative importance of nominal shocks versus real shocks, but in all our simulations, zero output-gap targeting strength is never optimal.

This paper contributes to the literature on gains from openness to FDI. Previous works have mainly focuses on long run gains from productivity improvement. McGrattan and Prescott (2009) examine host country's gains from FDI in a growth model in which a country can exploit another country's technological capital by opening to FDI. In the Burstein and Monge-Naranjo (2009) model, foreign management know-how is incorporated as an additional factor of production into a neoclassical model. The authors estimate that the host country has large output and welfare gains from opening to FDI. Ramondo and Rodriguez-Clare (2013) and Ramondo (2014) employ models based on new trade theory to measure gains from FDI. Ramondo (2014) focuses on horizontal FDI, in which affiliates of MNE sell output only in the host country. Ramondo and Rodriguez-Clare (2013), on the other hand, incorporates various types of FDI into a trade model, and uses it to examine how trade interacts with FDI. These papers look at long run gains from openness to FDI, derived from increased productivity or availability of goods. Our work contributes to the literature by providing a welfare analysis from a short run perspective.

Our study is also related to a recent and booming literature that analyzes dynamics of international macroeconomic models with multinational production. Burstein, Kurz, and Tesar (2008) and Arkolakis and Ramanarayanan (2009) look at how vertical integration of countries affect their business cycle synchronization. Motivated by observations of US multinational activity in Mexico, Bergin, Feenstra, and Hanson (2009, 2011) argue that vertical FDI increases employment volatility of the recipient country, relative to the source country. Cavallari (2010, 2013) incorporate firm entry into an international macro model, and show that the models' predictions better match international business cycle moments in terms of consumption and investment spillovers, as well as output comovement.

The rest of the paper is organized as follows. We set up the household's and the firm's problems and describe environment of the model in section 2. We examine the long run equilibrium of the economy by looking at the non-stochastic steady state in section 3. Section 4 studies the first order dynamics of the model. We first solve the model analytically under a specific assumption to gain insights into the model dynamics, and then we solve the model using numerical simulation to further study the details in the general case. Section 5 examines welfare effects of firm relocation. Section 6 discusses monetary policy conduct. Section 7 concludes.

2 The Model

There are two countries, North and South, each inhabited by a continuum of households whose total mass is normalized to 1. That is, the world population is 2. In addition, there is a continuum of firms whose total mass is also 2. We assume that half of these firms are owned by Northern households and the other half by Southern households. The only inherent difference between North and South is the average productivity of firms. Each firm is a monopolistic supplier of a differentiated good. The nature of these firms will be specified below.

2.1 The household's problem

A Northern household h tries to maximize a time-separable utility function

$$U_t = E_t \sum_{j=0}^{\infty} \beta^j u_{t+j} \left(C_{t+j}(h), N_{t+j}(h) \right).$$

The period utility function depends on consumption of a composite good $C_t(h)$ and labor supply $N_t(h)$:

$$u_t(C_t(h), N_t(h)) = \frac{1}{1-\sigma} C_t(h)^{1-\sigma} - \frac{1}{1+\phi} N_t(h)^{1+\phi},$$

where the parameters satisfy $\sigma > 0$ and $\phi > 0$. The composite good is a Dixit-Stiglitz CES aggregate of differentiated individual consumption goods defined as

$$C_t(h) = \left[\int_0^2 C_t(h,f)^{\frac{\mu-1}{\mu}} df \right]^{\frac{\mu}{\mu-1}},$$

where $C_t(h,f)$ is household h's consumption of the differentiated good produced by firm f. We assume that the elasticity of substitution μ is greater than 1. Each Northern household h chooses its consumption basket taking the set of individual goods' prices $P_t(f)$ as given.

We follow Erceg, Henderson, and Levin (2000)'s approach to incorporate a Calvo type wage-setting mechanism. We assume that households are monopolistic suppliers of labor. Each household supplies a differentiated type of labor to the firms. Firms produce final products using the "composite labor", a CES aggregate of differentiated types of labor defined as

$$L_t = \left[\int_0^1 N_t(h)^{\frac{1}{1+\eta}} dh \right]^{1+\eta}.$$

Labor is assumed to be immobile across countries, and thus Northern households supply labor only to firms who produce in the North, which include domestically-producing Northern firms, and possibly Southern firms who have relocated to produce in the North as multinationals.

This implies the Northern wage index, defined as the minimum cost of hiring one unit of the composite labor, is given by

$$W_t = \left[\int_0^1 W_t(h)^{-\frac{1}{\eta}} dh \right]^{-\eta}, \quad (1)$$

where $W_t(h)$ is the wage rate set by household h for its differentiated type of labor. Like any individual firm, each household h has zero weight in the continuum of household, and therefore its particular wage rate has no effect on the aggregate wage. Thus any individual household sets a wage taking the aggregate wage rate as given. Like the monopolistically competitive firms, each household faces a downward-sloping labor demand curve for its particular type of labor given by

$$N_t(h) = \left(\frac{W_t(h)}{W_t} \right)^{-\frac{1+\eta}{\eta}} L_t, \tag{2}$$

where L_t is the aggregate employment in period t.

Households engage in Calvo type wage setting. In each period t, with probability $(1-\theta)$, household h is able to update the wage rate it offers. Otherwise, its wage rate will be $\Pi W_{t-1}(h)$, where Π is the unconditional long-run gross rate of inflation of the economy, and $W_{t-1}(h)$ is the wage rate household h charged last period. Thus the wage rate is indexed to the long run level of aggregate inflation.

There is an integrated world financial market where a complete set of state-contingent nominal bonds are traded. These bonds are (arbitrarily) denominated in the Northern currency. Thus, household h faces the recursive period budget constraint

$$\begin{aligned} & P_t C_t(h) + \sum_{\nabla^{t+1} \in \Omega^{t+1}} Z\left(\nabla^{t+1} | \nabla^t\right) D\left(h, \nabla^{t+1}\right) \\ \leq \quad & (1+\tau_w) W_t(h) N_t(h) + \Gamma_t(h) + T_t(h) + D\left(h, \nabla^t\right), \end{aligned} \tag{3}$$

where $D\left(h, \nabla^{t+1}\right)$ is the units of nominal bond household h acquires. Each of these bonds pays one unit of Northern currency in period $t+1$ in state ∇^{t+1}. The period t price of such a bond is denoted $Z\left(\nabla^{t+1} | \nabla^t\right)$. The set of all possible states in period $t+1$ is denoted Ω^{t+1}. $\Gamma_t(h)$ is the profit earned by Northern firms that is distributed to household h lump-sum, $T_t(h)$ is government transfer, and $\tau_w = \eta$ is the rate of government subsidy to labor.

As Ricardian equivalence holds, we assume that the government has a balanced budget every period without loss of generality. Government in this model subsidizes domestic workers and firms at rates τ_w and τ_p, respectively, in order to offset monopolistic distortion. Thus the total government transfer (or negative of tax) to domestic households equals to negative of the expenditure on labor and firm subsidies:

$$T_t = -\tau_w \int_0^1 W_t(h) N_t(h) dh - \tau_p \int_0^1 P_t(f) Y_t(f) df.$$

We assume that the monetary authority sets nominal interest rate i_t according to an inflation-targeting rule

$$i_t = \bar{i} + \rho(\pi_t - \bar{\pi}) + \nu_t,\ \rho > 1,\ \nu_t \sim N\left(0, \sigma_m^2\right), \tag{4}$$

where $\bar{i}$ is the steady state long run interest rate, π_t and $\bar{\pi}$ denote the period t and steady state inflation rates respectively, and ν_t is the exogenous monetary disturbance. The monetary shock ν_t can be seen as a policy error. The error may be there because the natural interest rate is unobservable, or because effective money supply is difficult to measure.

Intra-temporal utility maximization gives us the familiar consumption price index P_t, defined as the minimum cost of acquiring one unit of the composite consumption good:

$$P_t = \left[\int_0^2 P_t(f)^{1-\mu} df\right]^{\frac{1}{1-\mu}},$$

while the demand for an individual product f is given by

$$C_t(f) = \left(\frac{P_t\,(f)}{P_t}\right)^{-\mu} C_t, \tag{5}$$

where C_t is the aggregate Northern consumption.

Household h's first order conditions from the utility-maximization problem include

$$Z\left(\nabla^{t+1}|\nabla^t\right) = prob\left(\nabla^{t+1}|\nabla^t\right)\beta\frac{\left[C_t(h)\right]^{\sigma} P_t}{\left[C_{t+1}(\nabla^{t+1}, h)\right]^{\sigma} P_{t+1}}, \tag{6}$$

which, by summing across all states, gives us the Euler equation

$$Z_t = \beta E_t\left[\frac{C_t^{\sigma} P_t}{C_{t+1}^{\sigma} P_{t+1}}\right].$$

In the previous equation, Z_t is the price of the riskless nominal portfolio that pays one unit of Northern currency in every state at time $t+1$, i.e. the inverse of the nominal interest rate. Note that the existence of complete market allows us to drop the household index in consumption.

We assume that the government chooses the subsidy rate $\tau_w = \eta$ to counteract the monopolistic distortion caused by the market power of wage-setters. The first order condition regarding the optimal wage-setting is then

$$E_t \sum_{j=0}^{\infty} (\beta\theta)^j \left[V_{N(h),t+j} - \frac{W_t(h)\Pi^j}{P_{t+j}} U_{c,t+j}\right] N_{t+j}(h) = 0. \tag{7}$$

where $V_{N(h),t+j} = N(h)_{t+j}^{\phi}$ is the marginal disutility of labor for household h in period $t+j$, and $U_{c,t+j} = C_{t+j}^{-\sigma}$ is the marginal utility of consumption. The conditional expectation is taken over states in which the household is unable to update its wage. When a household gets the chance to update its wage in period t, it takes into account the fact that with probability θ every period, its current reset wage will remain effective into the future. Thus it optimally sets a wage by weighing the discounted sum of future marginal disutility of working against the marginal utility of consumption made possible by the extra income from working. As the individual labor demand function (2) makes clear, household h's reset wage at period t depends only on the current and expected future aggregate variables. Thus all wage-resetters in period t must set the same wage rate $\tilde{W}_t$. Given that a constant fraction $(1-\theta)$ of household reset wage in every period, the aggregate wage rate evolves according to

$$W_t = \left[(1-\theta)\, \tilde{W}_t^{-\frac{1}{\eta}} + \theta \left(\Pi W_{t-1} \right)^{-\frac{1}{\eta}} \right]^{-\eta}. \tag{8}$$

Southern households are symmetric. Southern variables are denoted by an asterisk over their Northern counterpart.

2.2 Firms' Problem

Each firm produces a differentiated product using a technology that is linear in the input of the composite labor. A Northern firm who produces domestically has the production function

$$Y_t(\varphi) = \varphi A_t L_t(\varphi), \tag{9}$$

where φ is the time-invariant, firm-specific productivity, and A_t represents the aggregate productivity shock to Northern *firms*. $Y_t(\varphi)$ and $L_t(\varphi)$ denote the output and labor hired by firm φ, respectively. As firms in a given country only differ in terms of their productivity levels, we index Northern firms by their productivities φ from now on.

Assume each country gets an aggregate, country-specific productivity shock. The shock affects all firms owned by residents of this country in completely symmetric ways. An increase in A_t could capture such technology innovations as the use of internet and/or managerial improvements in a country.

We abstract from idiosyncratic firm productivity shocks. Although it is an interesting aspect to explore for some purposes, the current study focuses on macroeconomic outcomes of MNE production. Idiosyncratic shocks change firms' relative positions in the productivity distribution, but as we will show below, the volume and behavior of cross-border FDI flow is determined by the relative aggregate productivity of North and South.

Note that aggregate productivity affects firms headquartered in the country, rather than labors who reside in the country. In models without firm relocation, these are equivalent. But when firms are allowed to move across border, it becomes an important distinction. We think in our context, firm-augmenting technology is more realistic and reasonable because, when considering cross-border FDI flows, firms are typically the patent holders. This is also consistent with the literature on welfare gains from multinational production, which emphasizes how MNEs take production technology and managerial know-how to their foreign subsidiaries.[1]

Therefore, when a Northern firm produces as a multinational in the South, its production function is

$$Y_{Nt}^{*}(\varphi) = \varphi A_t L_{Nt}^{*}(\varphi),$$

where the subscript N is a reminder of the origin of the firm. Thus the firm takes its firm-specific technology and the home country productivity with it to the South. The only component foreign in its production is the employment of Southern workers.

We assume the productivity shocks follow the processes

$$\log A_t = \zeta \log A_{t-1} + \epsilon_t,$$

$$\log A_t^{*} = \zeta \log A_{t-1}^{*} + \epsilon_t^{*},$$

in which $0 < \zeta < 1$, and ϵ_t and ϵ_t^{*} are normally distributed.

Firm heterogeneity in productivity is introduced to capture the empirical regularity that multinationals are found to be the more productive firms in their industry. They earn higher revenue, hire more workers, and make larger profit. For example, Helpman, Melitz, and Yeaple (2004) examines the characteristics of multinational firms compared with exporters, as well as firms who only serve the domestic market. Their paper concludes that among these three sets of firms, multinationals have the highest average productivity, followed by exporters and then domestic firms.

Following Helpman, Melitz, and Yeaple (2004) and Chaney (2008), productivities are assumed to be drawn from a Pareto distribution with shape parameter α. Then Northern firm productivity cumulative distribution function is given by

$$\Pr(\varphi' < \varphi) \equiv G(\varphi) = 1 - \left(\frac{\varphi_m}{\varphi}\right)^{\alpha}, \varphi \geq \varphi_m,$$

where φ_m is the lower bound of Northern productivity distribution. The shape parameter α is an inverse measure of the heterogeneity of firms. A lower value of α indicates a fatter upper tail of the productivity distribution. We assume that $\alpha > \mu - 1$ to ensure that the

[1]Examples include, but are not limited to, Ramondo and Rodriguez-Clare (2013), McGrattan and Prescott (2009), and Burstein and Monge-Naronjo (2009).

$(\mu-1)^{\text{th}}$ moment of the distribution is bounded.[2] We normalize the Northern minimum productivity φ_m to one, but assume that South has a minimum productivity $\varphi_m^* < 1$. Thus Northern firms have, on average, a higher productivity.[3]

Firms' production location decisions determine cross-border FDI flows. To produce in period t, a firm has to first decide, in period $t-1$, where to produce next period. The firm can choose to produce either in its domestic country or in the foreign country.[4] If it decides to move its production facility overseas, it has to pre-commit a fixed real investment cost F in terms of the aggregate consumption good. Producing at home requires no additional cost. We assume that the location decision is irreversible by time t, when production takes place. We also assume the location decision-making is repeated every period.[5] The volume of FDI flow in this model is defined as the total fixed investment taken by all multinational firms. That is, FDI inflow into the South in period t is $m_t F$, where m_t is the mass of Northern firms who will produce as MNE in the South in period $t+1$.

Firms are optimally subsidized at the rate $\tau_p = 1/(\mu-1)$, such that for every unit of final product the firm sells to consumers, the firm receives $(1+\tau_p)p_t(\varphi)$. We assume that each country's government subsidizes its own firms, even if the firm is producing abroad.

We also assume that goods are freely traded with no cost. Given this assumption, the law of one price holds for each individual good. The firm thus receives the same unit price on its sales in both countries, when expressed in a common currency. The firm's total units sold worldwide is denoted $Y_t(\varphi)$.

Firms flexibly set prices of their outputs. Combined with the production function (9) and demand function (5), the firm's profit maximization problem at time t when producing domestically can be written

$$\max_{P_t(\varphi)} (1+\tau_p) P_t(\varphi) Y_t(\varphi) - W_t \frac{Y_t(\varphi)}{\varphi}$$

[2] This is necessary because otherwise, the total revenue made by MNEs will be unbounded when the cutoff productivity goes to infinity.

[3] An alternative assumption is to assume that the distribution of Northern firms has a different shape parameter α than that of South firms. All qualitative results we present below only require that Northern firms are more productive on average and do not depend on how the difference in average productivity is introduced.

[4] A firm can, in principle, produce in both countries. However, given the setup of the model, it does not make sense to have more than one production facility. A firm always produce in the low cost location.

[5] The assumption of a periodically repeated location decision is made mainly for tractability, but it should not change the qualitative results of the model. As will become clear later, the mechanism that induces short-term relocation of firms is the real wage gap caused by the sticky wage. In a Calvo setting, the wage rate adjusts sluggishly. If we assume that the firms' location decision is relevant for multiple periods, the decision rule will involve comparing a string of discounted future benefits with the sunk cost of relocation. While being more complicated, this alternative formulation does not change the qualitative nature of the decision.

subject to the demand function

$$Y_t(\varphi) = \left(\frac{P_t(\varphi)}{P_t}\right)^{-\mu} Y_t^w, \tag{10}$$

where $Y_t^w \equiv C_t + C_t^* + m_t F$ is the total world demand for the composite good. Note that as the fixed cost is assumed to be in units of the composite consumption good, period t world demand is increased by the amount $m_t F$ when cross-border relocations are present. We denote the total mass of MNEs producing in period t by m_{t-1}. The $t-1$ subscript reflects the one-period-ahead decision making on production location. Since consumption utility functions are identical across countries, purchasing power parity (PPP) holds for the aggregate prices: $P_t = S_t P_t^*$. Therefore a firm's relative prices in the two countries must be equal. This fact enables us to abbreviate the individual demand into (10).

As the result of the optimal subsidy, firms produce the optimal output, at which prices equal to their respective marginal costs. A Northern firm producing at home will optimally set price at

$$p_t(\varphi) = \frac{W_t}{\varphi A_t}, \tag{11}$$

and for Northern firms producing in South, the optimal price reflects its Southern labor cost and Northern productivity:

$$P_{Nt}^*(\varphi) = \frac{W_t^*}{\varphi A_t}. \tag{12}$$

Now we turn to a typical firm's location choice problem. If a firm decides to produce in the foreign country, it has to pre-commit a sunk cost F. When a Northern firm with productivity φ produces domestically, it earns the profit

$$\Pi_t^H(\varphi) = \left(\frac{1}{\mu-1}\right) W_t^{1-\mu} (\varphi A_t)^{\mu-1} P_t^\mu Y_t^w, \tag{13}$$

and if producing in the South, it earns the profit

$$\Pi_t^F(\varphi) = \left(\frac{1}{\mu-1}\right) (S_t W_t^*)^{1-\mu} (\varphi A_t)^{\mu-1} P_t^\mu Y_t^w, \tag{14}$$

expressed in Northern currency. The firm will choose to produce as a multinational if and only if the discounted expected gain from moving abroad is large enough to compensate for the current fixed cost F, i.e.

$$E_t\left[\delta_{t+1}(\Pi_{t+1}^F(\varphi) - \Pi_{t+1}^H(\varphi))\right] \geq P_t F,$$

where $\delta_{t+1} \equiv \beta P_t C_t^\sigma / P_{t+1} C_{t+1}^\sigma$ is the stochastic discount factor. Substitute the profit functions (13) and (14) into the expression above, use the optimal prices (11) and (12), and rearrange, we can reduce the inequality to

$$\left(\frac{\beta}{\mu-1}\right)\varphi^{\mu-1}E_t\left\{\frac{C_t^{\sigma}}{C_{t+1}^{\sigma}}\left(A_{t+1}\right)^{\mu-1}Y_{t+1}^{w}\left[\left(\omega_{t+1}^{*}\right)^{1-\mu}-\left(\omega_{t+1}\right)^{1-\mu}\right]\right\}\geq F, \tag{15}$$

where $\omega_t \equiv W_t/P_t$ denotes the Northern real wage and $\omega_t^* \equiv W_t^*/P_t^*$ denotes the Southern real wage. Inequality (15) outlines the break-even condition for Northern firm φ. The left-hand-side is the real expected discounted gain from switching production abroad, and the right-hand-side is the real cost.

The break-even condition (15) suggests three points. First, a lower real wage in the South, and therefore a lower marginal cost of production, is the source of potential gain for a Northern firm to become multinational. On the other hand, if Southern real wage is lower, no Southern firm will find it profitable to move to produce in the North. Thus there can only be one-way FDI flow in the model. Second, the scope of the gain is a monotonically increasing function of the firm's productivity level, implying that there exists a cutoff productivity $\tilde{\varphi}_t$ such that in period t, a Northern firm with productivity $\varphi = \tilde{\varphi}_t$ will just break even in expectation by switching its production location to the South in period $t+1$, and every Northern firm with productivity level $\varphi > \tilde{\varphi}_t$ must expect to make a positive net discounted profit by relocating and thus must decide to produce as a multinational in the next period. Finally, a higher expected aggregate productivity in the North makes Northern firms more likely to be a multinational, as it enhances the marginal gain from accessing lower cost labor in the same manner as the firm-specific productivity.

Southern firms have symmetric production functions and face the same choices. The only asymmetry between North and South is the average productivity levels of firms.

2.3 Aggregation

In this section, we aggregate across firms producing in each country, so that each economy can be described as having a "representative firm" whose productivity depends on the MNE distribution. We write down the equations for the case when some Northern firms are producing in the South as multinationals. The case of Southern FDI flow into the North is symmetric to the current analysis.[6]

Northern firms with productivity $\varphi < \tilde{\varphi}_{t-1}$ produce in the domestic country in period t. Using the production function (9), the demand for individual goods (5), and the optimal price-setting (11), we can write the aggregate labor demand as a function that depends only

[6] We show later that given the assumption that Northern firms have higher average productivity, FDI flow from North to South is the relevant case to examine.

on aggregate variables and a measure of aggregate productivity in the North:

$$L_t = \int_1^{\tilde{\varphi}_{t-1}} \left(\frac{Y_t(\varphi)}{\varphi}\right) g(\varphi)\, d\varphi = \omega_t^{-\mu} Y_t^w A_t^{\mu-1} \bar{\varphi}_{Nt-1}^{\mu-1}, \tag{16}$$

where

$$\bar{\varphi}_{Nt-1} \equiv \left[\int_1^{\tilde{\varphi}_{t-1}} \varphi^{\mu-1} g(\varphi)\, d\varphi\right]^{\frac{1}{\mu-1}} \tag{17}$$

is a measure of the aggregate productivity of firms who produce in the North. Likewise, the labor demand in the South is given by

$$\begin{aligned} L_t^* &= \int_{\tilde{\varphi}_{t-1}}^{\infty} \left(\frac{Y_{Nt}^*(\varphi)}{\varphi}\right) g(\varphi)\, d\varphi + \int_{\varphi_m^*}^{\infty} \left(\frac{Y_t^*(\varphi^*)}{\varphi^*}\right) g(\varphi^*)\, d\varphi^* \\ &= (\omega_t^*)^{-\mu} Y_t^w \left[A_t^{\mu-1} \bar{\varphi}_{Nt-1}^{*\mu-1} + A_t^* \bar{\varphi}^{*\mu-1}\right], \end{aligned} \tag{18}$$

where

$$\bar{\varphi}_{Nt-1}^* \equiv \left[\int_{\tilde{\varphi}_{t-1}}^{\infty} \varphi^{\mu-1} g(\varphi)\, d\varphi\right]^{\frac{1}{\mu-1}} \tag{19}$$

is the aggregate productivity of Northern firms who produce in the South in period t, i.e., the multinationals, and

$$\bar{\varphi}^* \equiv \left[\int_{\varphi_m^*}^{\infty} \varphi^{*\mu-1} g(\varphi^*)\, d\varphi^*\right]^{\frac{1}{\mu-1}}$$

is the mean productivity of Southern firms.

Meanwhile, the total profit earned by Northern firms, denoted by Γ_t, includes profits of Northern firms who produce domestically, and Northern firms who produce in the South (in case multinational firms are present). These profits are reduced by the amount of the fixed sunk investment cost

$$\Gamma_t = \Gamma_{Nt} + \Gamma_{Nt}^* - (1 - G(\tilde{\varphi}_t))\, P_t F.$$

Using the optimal price-setting rule (11), for a Northern firm φ who produces domestically, its profit is

$$\begin{aligned} \Gamma_{Nt}(\varphi) &= P_t(\varphi) Y_t(\varphi) - W_t L_t(\varphi) \\ &= \left(\frac{1}{\mu-1}\right) W_t L_t(\varphi). \end{aligned}$$

Integrating over domestically-producing Northern firms using a similar procedure as in (16), we find that the total profit made by these firms is given by, in real terms,

$$\frac{\Gamma_{Nt}}{P_t} = \left(\frac{1}{\mu-1}\right) (\omega_t)^{1-\mu} Y_t^w A_t^{\mu-1} \bar{\varphi}_{Nt-1}^{\mu-1}. \tag{20}$$

Likewise, the total real profit made by the set of Northern MNEs is

$$\frac{\Gamma^*_{Nt}}{P_t} = \left(\frac{1}{\mu - 1}\right) (\omega^*_t)^{1-\mu} Y^w_t A^{\mu-1}_t \bar{\varphi}^{*\mu-1}_{Nt-1}. \tag{21}$$

In the next section, we will use these aggregate profits to derive steady state consumption.

3 Steady State FDI

We will proceed by linearizing the model around a non-stochastic steady state, assuming zero initial wealth. The steady state describes the long run equilibrium toward which the economy tends to move absent shocks. In the steady state, all real variables are constant and all nominal variables grow at the constant rate $\bar{\pi}$. We denote the value of a variable at steady state by an overbar, except for the aggregate productivities (17) and (19), whose steady state values are simply denoted by the variables without the time subscript.

As there can't be two-way FDI flows, the direction of FDI flow in the steady state could fall in one of three cases: Northern firms relocate to produce in the South, Southern firms relocate to produce in the North, or there could be no FDI flow. We write down the relevant equations for the first case here. The reversed case is symmetric, and the zero FDI case is a special case of the former two. We prove that given the assumption of higher productivity in the North, steady state FDI flows from the higher productivity country to the lower productivity one.

In the non-stochastic steady state, every household sets the same wage rate and supply the same amount of labor, implying $\bar{L} = \bar{N}(h)$ and $\bar{W} = \bar{W}(h)$ across households. The wage distribution degenerates into a single value. The optimal wage-setting rule (7) reduces to the simple familiar labor supply rule that equates the marginal rate of substitution to the real wage

$$\bar{\omega} = \bar{L}^{\phi} \bar{C}^{\sigma}, \tag{22}$$

$$\bar{\omega}^* = \bar{L}^{*\phi} \bar{C}^{*\sigma}. \tag{23}$$

Dividing the Northern labor supply by its Southern counterpart, we get

$$\frac{\bar{\omega}}{\bar{\omega}^*} = \left(\frac{\bar{L}}{\bar{L}^*}\right)^{\phi} \left(\frac{\bar{C}}{\bar{C}^*}\right)^{\sigma}. \tag{24}$$

Incorporating steady state aggregate labor demands (16) and (18) into (24), the labor market equilibrium in the steady state thus requires

$$\left(\frac{\bar{\omega}}{\bar{\omega}^*}\right)^{\frac{1}{\phi}+\mu} = \left(\frac{\bar{C}}{\bar{C}^*}\right)^{\frac{\sigma}{\phi}} \left(\frac{\bar{\varphi}_N^{\mu-1}}{\bar{\varphi}_N^{*\mu-1} + \bar{\varphi}^{*\mu-1}}\right), \tag{25}$$

where $\bar{\varphi}_N$ and $\bar{\varphi}_N^*$ are the steady state values of the aggregate productivities defined in (17) and (19).

The budget constraint states that in the steady state, each country's consumption is the sum of its residents' labor income and the net profits earned by domestic firms, minus the government subsidy to firms. Thus for the North,

$$\bar{C} = \bar{\omega}\bar{L} + \bar{\Gamma}_t/\bar{P}_t - \tau_t \int_0^1 P_t(f)Y_t(f)df.$$

And $\bar{\Gamma}_t$, the profits made by Northern firms, is the sum of profits made by domestic producers and that made by multinationals. Taking the steady state value of (16), (20), and (21) and substituting into the above expression, we find that the steady state consumption is given by

$$\bar{C} = \bar{\omega}^{1-\mu}\bar{Y}^w\bar{\varphi}_N^{\mu-1} - \left(1 - G\left(\bar{\tilde{\varphi}}\right)\right)F. \tag{26}$$

Similarly, the Southern consumption is given by

$$\bar{C}^* = \bar{\omega}^{*1-\mu}\bar{Y}^w\left(\bar{\varphi}_N^{*\mu-1} + \bar{\varphi}^{*\mu-1}\right). \tag{27}$$

We prove that in the steady state, there are positive FDI flows and its direction depends on the average productivities of the countries.

Proposition: When the countries have different average productivities and are otherwise identical, in the steady state, multinational firms originating from the high-productivity country relocate to produce in the low-productivity country.

Proof: see appendix

As FDI flows from the North to the South in the steady state, in later approximations around the steady state, we assume that the shocks are never big enough to reverse the direction of FDI flow. This should be reasonable, as our focus is economic fluctuations over business cycles of normal magnitude.

4 First order dynamics of the FDI flow

In this section, we examine the dynamics of the model in a first order approximation. In the first subsection, we analytically solve the first-order dynamics of firms' location decisions under a special assumption to gain insights into the model. In the second subsection, we present results from numerical analysis of the general model.

4.1 First order dynamics at zero sunk cost

An analytically tractable solution to the cross-country distribution of firms is possible when we approximate the model around a steady state where the fixed investment cost is zero. As we will see, despite being a special case, the analytical study in this section unveils the basic mechanism behind the first-order dynamics of the location distribution of firms.

As PPP holds for prices of consumption baskets in this model, we take the relative labor cost ω_t/ω_t^* as the relevant real exchange rate. We first show how are the relative real wage and FDI flows are related. We also look at how nominal exchange rate and FDI flows are related.

Following Melitz(2003) and subsequent literature, we take the fixed cost F as the embodiment of barrier to cross-border FDI flows. When F goes to infinity, it becomes excessively expensive to relocate oversea and cross-border direct investment shuts down, leaving the economy in a domestic production equilibrium. In such an equilibrium, labors in each country work for domestic firms, and consumption equals to the total revenue of the firms. When F shrinks to zero, there is no barrier to direct investment, and the break-even condition (15) states that FDI flows will only stop when real wages in the two countries are equalized.

As the break-even condition makes clear, the fixed cost also serves as a filter that selects the most productive firms into oversea production. When a finite fixed cost exists, a firm has to be productive enough to earn a profit at least as large as the sunk cost to move to a location overseas. Thus the set of multinational firms are the upper truncation of the productivity distribution. When there is no fixed cost at all, we don't exactly know who among the Northern firms become multinationals. But here we consider the special case $F = 0$ to be the limit of the series of equilibria when F shrinks toward zero. Thus we impose the requirement that the most productive firms are the first to become MNEs.

Denote the log deviation from steady state of a variable x by $\hat{x} \equiv \log(x/\bar{x})$. The Euler equation can be linearized to

$$\hat{Z}_t = \sigma \hat{C}_t + \hat{P}_t - \sigma E_t \hat{C}_{t+1} - E_t \hat{P}_{t+1}. \tag{28}$$

As is well known, the combination of complete market and PPP implies that consumption ratio C_t/C_t^* is constant at an initial level. Substracting the linearized Southern Euler equation from its Northern counterpart, and using $\hat{C}_t = \hat{C}_t^*$ and PPP, we get the uncovered interest parity condition

$$E_t \hat{S}_{t+1} - \hat{S}_t = i_t - i_t^*.$$

Following Gali (2007), we define the nominal interest rate as $i_t \equiv -\log Z_t$. Using the interest rate rule (4), we solve the exchange rate depreciation as a function of the monetary shocks

$$\Delta \hat{S}_t = -\frac{1}{\rho}\left(\nu_t - \nu_t^*\right), \tag{29}$$

where $\Delta x_t \equiv x_t - x_{t-1}$ denotes the change of variable x from period $t-1$ to t. The nominal exchange rate follows a random walk, and a relative Northern monetary tightening causes a Northern appreciation.

Using the optimal wage-setting rule (7), together with the individual labor demand (2) and aggregate wage evolution (8), we get an equation governing the evolution of the aggregate real wage[7]:

$$(\hat{\omega}_t - \hat{\omega}_{t-1}) = \kappa(mrs_t - \hat{\omega}_t) + \beta E_t(\hat{\omega}_{t+1} - \hat{\omega}_t) + \beta E_t \pi_{t+1} - \pi_t \tag{30}$$

where $mrs_t \equiv \phi \hat{L}_t + \sigma \hat{C}_t$ is the marginal rate of substitution (MRS) between consumption and leisure, and

$$\kappa = \frac{(1-\beta\theta)(1-\theta)}{\theta\left(1+\phi\left(\frac{1+\eta}{\eta}\right)\right)}$$

captures the responsiveness of wage-setting to the current period deviation of MRS from the real wage. Subtracting the Southern counterpart of the wage-setting equation (30) from the Northern one, and rearranging, we get

$$(1+\beta+\kappa)\hat{\omega}_t^d = \kappa\phi\hat{L}_t^d + \hat{\omega}_{t-1}^d + \beta E_t \hat{\omega}_{t+1}^d + \beta E_t \pi_{t+1}^d - \pi_t^d, \tag{31}$$

where $\hat{x}_t^d \equiv \hat{x}_t - \hat{x}_t^*$ denotes the North-South difference in variable x.

From linearizing the aggregate labor demand equations (16) and (18), we get

$$\hat{L}_t^d = -\mu\hat{\omega}_t^d + \hat{\bar{\varphi}}_t^r,$$

where

$$\bar{\varphi}_t^r \equiv \frac{\bar{\varphi}_{N,t-1}^{\mu-1} A_t^{\mu-1}}{\bar{\varphi}_{N,t-1}^{*\mu-1} A_t^{\mu-1} + \bar{\varphi}^{*\mu-1} A_t^{*\mu-1}}$$

denotes the relative aggregate productivity of firms producing in the two countries. Its first order expansion is

$$\hat{\bar{\varphi}}_t^r = (\mu-1)\left[\hat{\bar{\varphi}}_{N,t-1} - \left(\frac{\bar{\varphi}_N^{*\mu-1}}{\bar{\varphi}_N^{*\mu-1} + \bar{\varphi}^{*\mu-1}}\right)\hat{\bar{\varphi}}_{N,t-1}^*\right] + \frac{(\mu-1)\bar{\varphi}^{*\mu-1}}{\bar{\varphi}_N^{*\mu-1} + \bar{\varphi}^{*\mu-1}}\hat{A}_t^d.$$

[7] The derivation follows closely that of Erceg, Henderson, and Levin (2000). See the appendix of that paper for the procedure.

Given the assumption that the productivity distributions are Pareto, the aggregate productivities (17) and (19) can be expressed as functions of the cutoff productivity $\tilde{\varphi}_{t-1}$. We can write the relative labor demand as a function of the real wage gap, the cutoff productivity, and the relative aggregate productivity:

$$\hat{L}_t^d = -\mu\hat{\omega}_t^d + \frac{z}{\kappa\phi}\hat{\tilde{\varphi}}_{t-1} + \frac{(\mu-1)\bar{\varphi}^{*\mu-1}}{\bar{\varphi}_N^{*\mu-1} + \bar{\varphi}^{*\mu-1}}\hat{A}_t^d, \tag{32}$$

where

$$z = \kappa\phi\alpha\bar{\tilde{\varphi}}^{\mu-1-\alpha}\left(\frac{1}{\bar{\varphi}_N^{\mu-1}} + \frac{1}{\bar{\varphi}_N^{*\mu-1} + \bar{\varphi}^{*(\mu-1)}}\right).$$

The relative labor demand depends positively on the cutoff productivity. A higher $\hat{\tilde{\varphi}}_{t-1}$ means less Northern firms are producing as MNEs in the South, increasing the labor demand in the North relative to South.

When the fixed cost is set to zero, the break-even condition (15) can be linearized to a simple relation

$$E_t(\hat{\omega}_{t+1}^d) = 0. \tag{33}$$

This states that when there's no barrier to FDI investment, firms will relocate until the real wage rates in the two countries are equalized in expectation.

PPP implies that the difference in inflation rates must be reflected in the nominal exchange rate depreciation

$$\pi_t^d = \Delta\hat{S}_t.$$

We can use the solution to the nominal exchange rate depreciation (29) to substitute out π_t^d and π_{t+1}^d in (31). Substituting also the relative labor demand (32) and the break-even conditon (33) into (31) , we obtain a solution to the real exchange rate

$$\hat{\omega}_t^d = \alpha_1\hat{\tilde{\varphi}}_{t-1} + \alpha_2\hat{\omega}_{t-1}^d + \alpha_3\nu_t^d + \alpha_4\hat{A}_t^d, \tag{34}$$

where

$$\alpha_1 = \frac{z}{(1+\kappa+\beta+\kappa\phi\mu)} > 0,$$

$$\alpha_2 = \frac{1}{(1+\kappa+\beta+\kappa\phi\mu)} \in (0,1),$$

$$\alpha_3 = \frac{1}{\rho(1+\kappa+\beta+\kappa\phi\mu)} \in (0,1),$$

and

$$\alpha_4 = \frac{\kappa\phi(\mu-1)}{(1+\kappa+\beta+\kappa\phi\mu)}\left(\frac{\bar{\varphi}^{*(\mu-1)}}{\bar{\varphi}_N^{*\mu-1} + \bar{\varphi}^{*(\mu-1)}}\right) > 0.$$

These parameters are all positive. The equilibrium relative real wage depends positvely on the lagged cutoff productivity, because higher cutoff productivity means more Northern firms are producing at home, increasing the relative labor demand in the North. The sluggish wage adjustment means the relative real wage also depends positively on its own lagged value and the monetary shocks. A relative monetary tightening in the North causes real wage rate in the North to rise relative to South. Finally, a Northern productivity gain causes the real wage to rise in the North through increased labor demand.

We substitute (34) into the linearized break-even condition (33) to get a solution to the cutoff productivity

$$\hat{\tilde{\varphi}}_t = -\alpha_2 \hat{\tilde{\varphi}}_{t-1} - \frac{\alpha_2^2}{\alpha_1} \hat{\omega}_{t-1}^d - \frac{\alpha_2 \alpha_3}{\alpha_1} \nu_t^d - \left(\frac{\alpha_2 \alpha_4}{\alpha_1} + \zeta \frac{\alpha_4}{\alpha_1} \right) \hat{A}_t^d. \tag{35}$$

The fraction of Northern firms who will be producing as multinationals in period $t+1$ is $1 - G(\tilde{\varphi}_t)$. Thus the cutoff productivity is negatively related to the size of FDI. Thus a Northern monetary contraction or a positive productivity shock in the North increases the Northern FDI flow to the South.

The break-even condition for the marginal multinational firm is the key to understand the response of FDI flow to the exogenous shocks. Essentially, it states that firm relocation will continue until the expected North-South real wage gap shrinks to a level at which the marginal firm cannot expect to make a profit anymore. (In the special case $F = 0$, the real wages have to equalize in expectation.) A relative Northern monetary tightening increases the relative Northern real wage, as under the sluggish wage-setting, the nominal wages are not responsive enough to offset the appreciation of the nominal exchange rate. Also as a consequence of the sticky wage, the relative real wage is history-dependent. Thus the current wage gap is expected to persist into the next period. This induces more Northern firms to commit to producing abroad in the next period, causing an increased FDI flow into the South.

A relative Northern productivity gain drives up the relative real wage, as the average firm producing in the North is more productive. Firm relocation increases for two reasons. First, productivity processes are persistent. The higher Northern productivity today is expected to be inherited next period. Thus the real wage gap is expected to continue existing, causing more firms to relocate. Second, the sticky wage-setting still implies that the current wage gap is going to continue into the next period, again causing more MNE relocation. The relocation caused by the persistence of productivity will happen with or without sticky wage, and is captured in equation (35) by the last term $(\zeta \alpha_4 / \alpha_1) \hat{A}_t^d$. It is easy to verify that the last term represents the efficient firm relocation under flexible wages. Note that the existence of sticky wage tends to "strengthen" the FDI flow that would happen under flexible wages, so that firm relocation overshoots the efficient level in the period when productivity shock hits.

4.2 Numerical simulation

In this subsection, we numerically simulate the model for the general case when the fixed cost is positive, and highlight several key features.

The parameterization mainly follows Erceg, Henderson, and Levin (2000). We set a (quarterly) discount rate $\beta = 0.99$, and utility parameters $\sigma = \phi = 1.5$. We choose an elasticity of substitution $\mu = 4$, and wage mark-up rate $\eta = 1/3$. The probability of wage stickiness is set to $\theta = 0.75$. This corresponds to an average wage contract duration of one year. Following Russ (2007), we set $\alpha = \mu + 0.1$ to make the dispersion of firm productivity close to one, as the cross-industry estimation by Helpman, Melitz, and Yeaple (2004) suggests.[8] The Southern minimum productivity is set to 1/2. Next, we set the fixed cost to 0.01. In the steady state, about 7% Northern firms produce as multinationals. Regarding the interest rate rule, the inflation response coefficient is set to $\rho = 1.5$. The persistence coefficient of the aggregate productivity is set to $\zeta = 0.9$. Finally, the monetary and productivity shocks have a standard deviation of 0.02.[9]

Figure 1 plots the impulse response functions of a set of variables of interest to a Southern monetary expansion (that is, a one standard deviation drop in ν_t^*). The monetary expansion causes an inflation and drives down real wage in the South. As Northern currency appreciates against the South, imported goods price depreciates and drags down aggregate price level, causing an increase in the Northern real wage. The enlarged real wage gap induces more Northern firms to relocate production to the South, causing the mass of MNE to increase.

[8] Helpman, Melitz, and Yeaple (2004) estimated productivity dispersion, measured by the value of the coefficient $\alpha - (\mu - 1)$, using industry level data for U.S. and European firms. The estimations of these industry-specific dispersion parameters range from 0.52 to 2.97. The qualitative simulation results in this section are robust to these alternative dispersion parameters. See Helpman, Melitz, and Yeaple (2003), NBER working paper#9439, Table A.1.

[9] We try different parameter values to make sure the qualitative results are robust to alternative parameterizations.

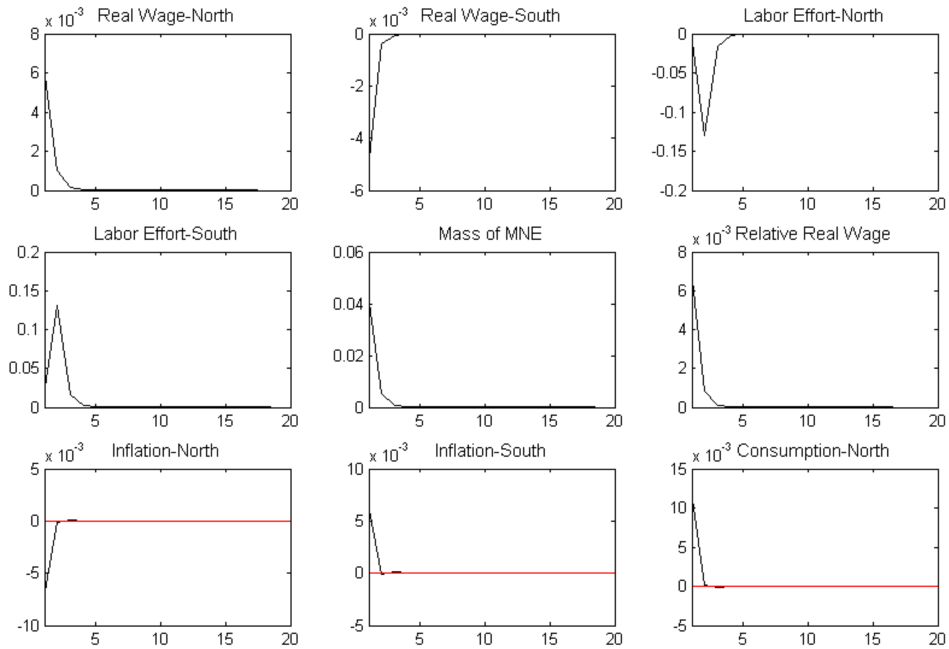


Figure 1: IRF to a Southern monetary expansion

Figure 2, on the other hand, plots IRFs to an increase in Northern productivity (a one standard deviation shock to ε_t). Higher productivity lowers marginal cost, putting downward pressure on prices of goods produced by Northern firms. Deflation drives up real wages in both countries. Meanwhile, at any given real wage gap, Northern firms find it more profitable to relocate to the South, as their productivity is higher, increasing the expected gain from relocation. The IRFs show that the mass of MNE moves positively with the relative real wage period-by-period. Specifically, in the second period after the shock, the relocation of MNEs to the South puts so much upward pressure on the labor demand in the South, that it drives the relative real wage down to a level slightly below the steady state. This happens because the productivity gain by Northern firms is large enough, so that relocation is profitable even if the real wage gap is smaller. When the relative real wage recovers again in the following period, so does the mass of MNE.

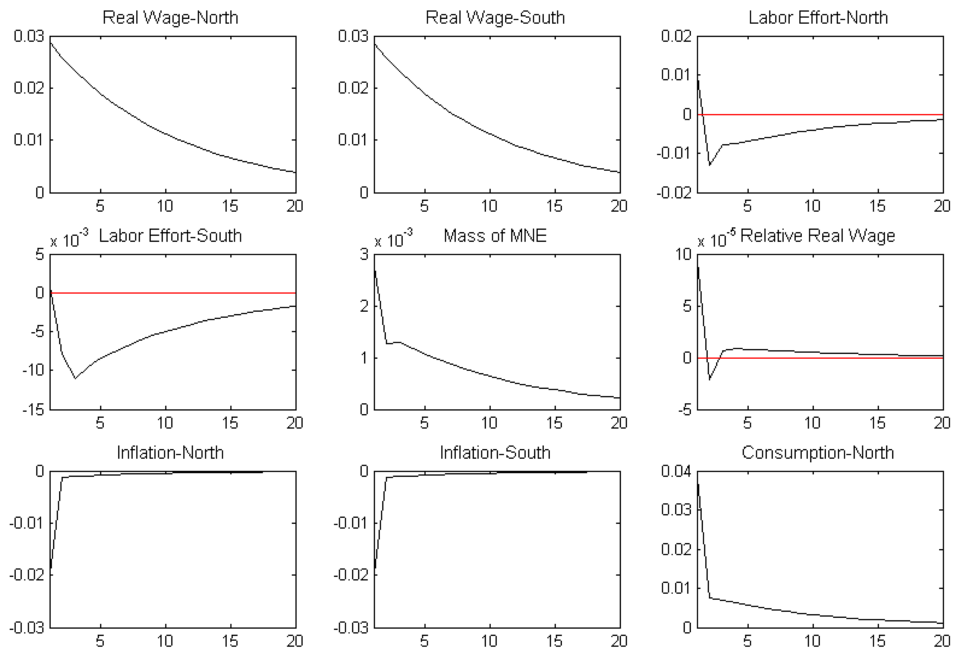


Figure 2: IRF to a Northern productivity increase

The prediction of the model seems to be consistent with the view that a country can attract more FDI inflow by engineering a depreciation, but it also predicts that the outcome is undesirable from the depreciating country's point of view.

Going back to Figure 1, where the economy experiences an unexpected Southern monetary expansion, the sharp drop (rise) of the Northern (Southern) aggregate employment one period after the shock is caused by the relocation of the new MNEs. The more productive Northern firms are attracted by the lower real wage abroad after the shock, and their relocation means the demand for Southern labor is further increased. Note that without these new MNEs, Southern (Northern) workers would already be working too much (little) compared to the flexible-price long run equilibrium level, as the Southern monetary expansion lowers (increases) the Southern (Northern) real wage. The entry of new MNEs thus drives the aggregate employments in both countries further away from their long run equilibrium level. Figure 3 shows the IRFs of outputs and period utilities after the Southern monetary expansion. Southern output increases sharply as MNEs move production facilities to the South, as we use GDP to measure output, and products of MNEs are counted into host country GDP.

Note that as complete market distributes consumptions consistently over time, the change in relative utility of the two countries is mainly determined by the change in employment levels. This suggests that the welfare effects of these cross-border relocations after a monetary shock is asymmetric across the two countries. As the Southern real wage depreciation attracts more FDI inflows, households are worse off as they end up working more, while households in the North enjoy more leisure. Thus the model suggests that an unanticipated depreciation attracts more FDI inflows, but leaves the domestic agents worse off as they work more under sticky wage contracts. It is not beneficial to the domestic household to manipulate the exchange rate to attract FDI inflow. Rather, the reversed operation of an artificial appreciation that drives firms abroad is welfare-improving. Of course, the gain is only to the appreciating country, the world as a whole loses utility from such nominal rigidity-caused deviations.

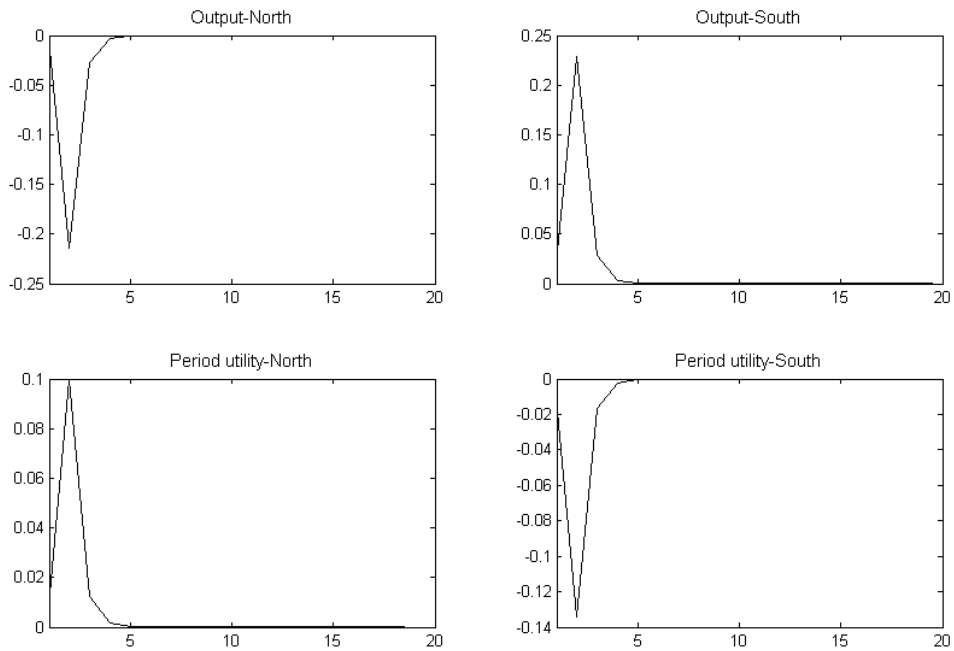


Figure 3: Output and period utility after a Southern monetary expansion

The fact that firm relocation exacerbates employment deviations under nominal shocks is consistent with the empirical finding of Bergin, Feenstra, and Hanson (2009).[10] In Figure 3 we examine how *period* utility changes after the shock. The interesting question is, if relocation of MNE exacerbates the business cycle fluctuations around the efficient steady state level, what is the welfare implication of multinational production as an institution? We turn to this question in the next section.

[10]Bergin, Feenstra, and Hanson (2009) emphasize that Mexican *maquiladora* industry has higher employment volatility than their US counterpart. In this model though, we stress that the effect is bilateral. Of course, if the model is revised so that the South is much smaller than the North, we would expect to see the same firm relocations to create larger impact on the South.

5 Welfare Effects of FDI

To formally analyze how the option of cross-border relocation affects expected utility, in this section we numerically simulate the model in second order approximation to examine how multinational production affects world welfare. We answer the question by examining the model in environments with nominal shocks and real shocks separately. Our model with firm relocation (henceforth referred to as the "benchmark model") is compared with two alternative models. The domestic production model is the benchmark model without cross-border firm relocation. It can be thought of as the limit of the benchmark model as the fixed cost F goes to infinity. The steady state MNE model (henceforth "ssMNE model") , on the other hand, is the benchmark model with the invariant steady state MNE distribution. That is to say, all Northern firms who are MNEs in the steady state is required to stay producing in the South, and no more moving back-and-forth is allowed over the business cycle. In this way, we can hypothetically separate the steady state relocations of the long term nature from the marginal relocations that only happen over business cycle. We show that, qualitatively, the welfare effect of multinational production differ under nominal shocks and real shocks. Under nominal shocks, it decreases welfare by magnifying business cycle fluctuations. Under real shocks, the marginal MNE relocation has ambiguous welfare effect. In terms of magnitude, we show that the welfare impact of FDI under nominal shocks is much larger than that under real shocks. We conclude that in an environment with mixed nominal and real shocks, the model that allows only long term firm relocation produces highest level of welfare.

5.1 Welfare implication of firm relocation under monetary shocks

In this sub-section, we shut down real shocks, and examine the welfare effects of FDI under nominal shocks only. The world welfare is defined as the unconditional expectation of the unweighted sum of Northern and Southern household average period utilities. That is,

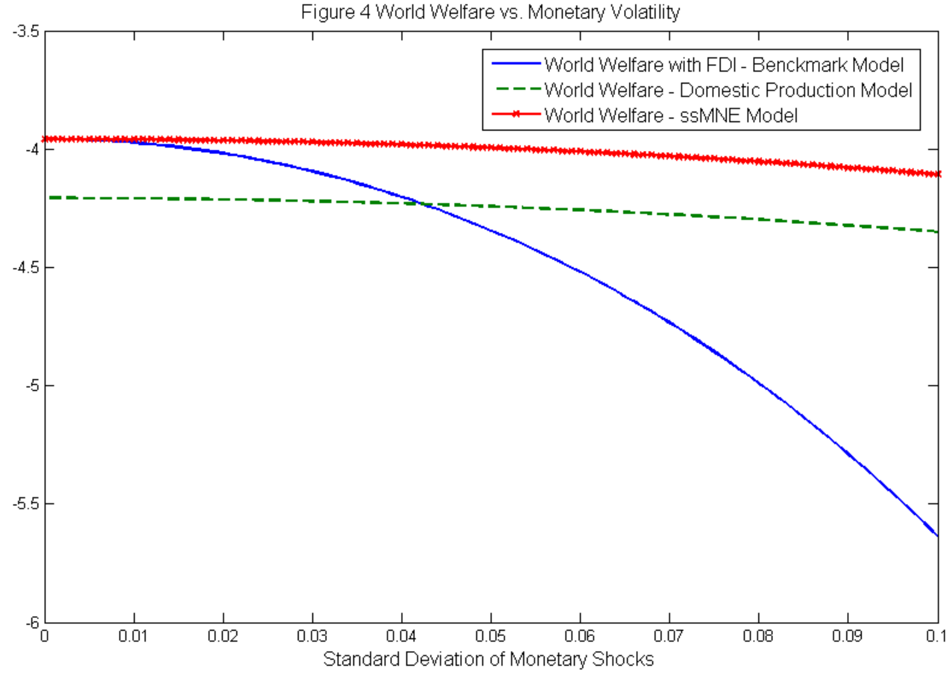
$$W_t = U(C_t) + \int_0^1 V(N_t(h))dh + U(C_t^*) + \int_0^1 V(N_t^*(h^*))dh^*.$$

We compare the expected world welfare in the benchmark model with that in the domestic production model and the ssMNE model. We show that while the steady state world welfare is higher when FDI is allowed, the welfare decreases at a faster rate as the volatility of monetary shocks increases, compared to the domestic production model. As a result, when the volatility of monetary shocks gets large enough, the world will be better off if cross-border FDI flows are completely shut down. Yet given that the long run relocation of firms in the steady state is welfare-improving, the scenario when MNE distribution is kept at its steady state level induces the highest level of utility.

To compute the expected world utility, we simulate the model for two thousands periods, and drop the first two hundred. We repeat this for various values of monetary volatilities, measured by the standard deviations of the shocks to interest rate rule ν and ν^*. We compute the expected utility under these different values of standard deviations, ranging from 0 to 0.1, at increment of 0.001.

Figure 4 reports the results of these simulations. The solid blue line depicts the world utility computed from the benchmark model, when all firms can freely relocate cross-border subject to the sunk cost F. The dotted green line is the world utility under domestic production, i.e., when all firms are restricted to produce in their domestic country. Clearly, the long run FDI flows that takes place in the steady state are welfare-improving. This steady state welfare improvement is consistent with insights from previous literature on the long run gain of FDI. North gains from access to Southern labor that lowers cost of production. South gains by accessing superior technology from the North.

Figure 4 World Welfare vs. Monetary Volatility



However, while welfare under both the FDI and domestic production scenarios decrease with the volatility of monetary shocks, the rate of decrement is higher with FDI flows. An inspection of terms that affect welfare show that FDI's impact on aggregate employment is the culprit. When the volatility of monetary shocks gets higher, on average, the relative real wage is driven further away from its flexible-price equilibrium level. This then induces more temporary relocation of firms, causing aggregate employments to deviate even more, as we have shown in the last section. Thus the option to relocate in the short run amplifies the inefficiency caused by the nominal rigidity in wage-setting. As a result, if the monetary volatility is sufficiently high, it becomes welfare-improving to shut down cross-border FDI flows altogether. In our baseline calibration, the utility with FDI drops below the utility under domestic production at a standard deviation of 4.3%.

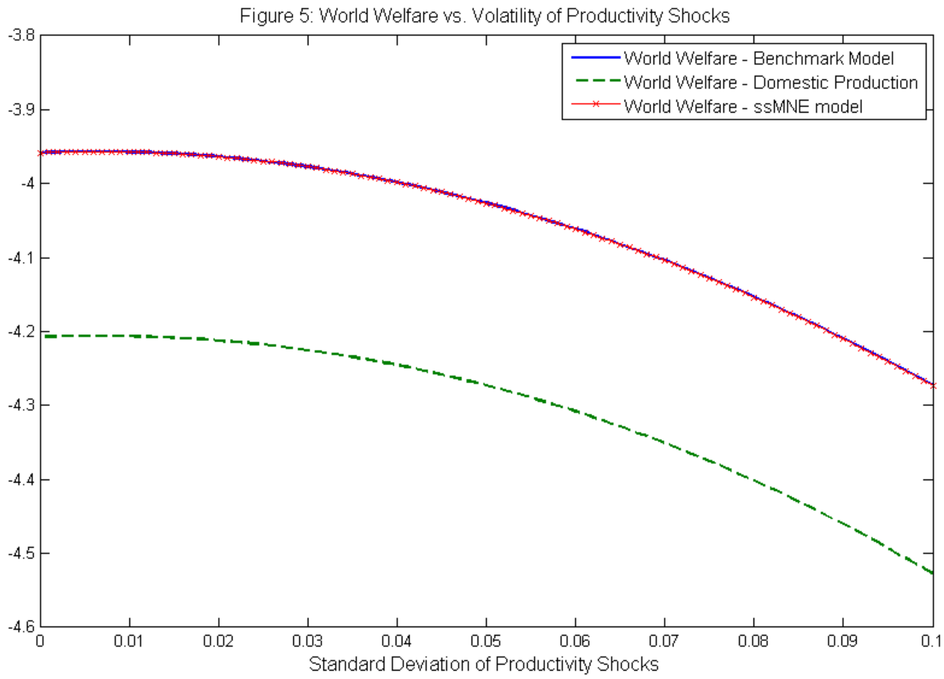
These welfare results under nominal shocks can be thought of as an application of the theory of second best. When a friction causes a market failure, allowing market participants to freely exploit the opportunities caused by the friction could deteriorate the overall welfare loss. In our specific case, the same FDI flow motivated by lower production cost abroad could cause good or harm depending on the underlying cause of that lower real wage in the first place. It is welfare-improving when the real wage gap is caused by the fundamental difference in productivity, but exacerbates the welfare loss when the wage gap is caused by short run nominal friction.

Therefore, there should be no surprise that the first best outcome occurs under the scenario that keeps the MNE relocations of the long-run nature. The top red line with cross markers in Figure 4 depicts the world welfare when we simply keep the steady state distribution of firms, but disallow further moving back-and-forth. World utility is highest under this scenario at every monetary volatility level, as the economy is able to reap the gain from long run firm relocation, but avoid the harm caused by short run relocations aimed to take advantage of nominal rigidities.

5.2 Welfare implication of firm relocation under productivity shocks

This subsection examines the welfare effects of FDI in an environment with productivity shocks. We again compare world welfare under the benchmark model, the domestic production model, and the ssMNE model. In the relevant range of productivity shock volatility, the domestic production model produces lowest level of welfare. The benchmark model and the ssMNE model perform remarkably similarly.

Figure 5 depicts the welfare in the three models when the standard deviation of productivity shocks increases from zero to 0.1. The solid blue curve is welfare under the benchmark model, the dotted green curve is welfare under domestic production model, and the thin red curve with cross marks the ssMNE model.



Clearly, both benchmark and ssMNE model dominate the domestic production model. In the relevant range of productivity shock volatility, the loss from absence of FDI mainly comes from the failure to capture the long run gain of FDI. The welfare loss compared to the benchmark model further enlarges only when the volatility of productivity shocks gets unreasonably large.

A comparison of the welfare under the benchmark model and the ssMNE model reveals that with real shocks only, the welfare effect of the marginal MNE relocation over the business cycle is negligible. Graphically, the two lines representing welfare under the benchmark model and the ssMNE model almost overlap. Numerically, the welfare in the benchmark model at a standard deviation of 10% only exceeds the welfare in the ssMNE model by a tiny margin.

A more careful examination indicates that the proximity of the welfare under the two models is a result of counteracting forces. The marginal MNE relocation benefits the economy in two ways. It increases mean consumptions in both countries, as the relocation motivated by productivity changes allows MNEs to reduce production cost. MNE relocations also serve as a "employment risk-sharing" mechanism between the two countries. Consider a positive productivity shock in the North. Absent firm relocations, the Northern productivity gain makes domestic workers work harder. Meanwhile, Southern consumers enjoy more leisure because Southern consumption increases under complete asset market, driving down the marginal utility gain of working. Relocations of MNEs "redistribute" labor efforts in an equalizing way. With Northern productivity gain, more Northern firms find it profitable to move abroad, increasing labor demand in the South, while reducing it in the North. As utility function is concave in leisure, this redistribution improves world welfare, in the same way as complete markets redistributes consumption.

With these benefits, why wouldn't the marginal MNE relocations be welfare-improving? The answer lies in nominal rigidity. With sticky wages, MNE distribution deviates from the efficient flexible wage equilibrium. When a positive productivity shock hits the North, for example, Northern firms "over-relocate" upon impact. We have analytically shown this mechanism in section 4.1, after deriving the cutoff productivity (35). The over-relocation drives labor efforts away from efficient level and causes loss of welfare.

The resulting welfare implication then reflects a tradeoff between the gain from increased consumption and labor efforts sharing and the loss from the distortion. The gain presents under flexible or sticky wages, but the loss is only a consequence of nominal rigidity. This suggests that the net welfare gain from the relocation of marginal MNEs increases with the degree of nominal wage flexibility. Indeed, when we compute world welfare under flexible wages (can be considered as the limiting case of the benchmark model when the probability of reseting wage equals to one), the benchmark model yields higher utility at every level of real shock volatility.

Note that though, under productivity shocks, the difference in the magnitude of welfare produced by the benchmark model and the ssMNE model is extremely small. Even when wage is completely flexible, so that the loss from nominal distortions is eliminated, and the standard deviation of the productivity shocks reaches 0.1, the gain in world welfare from having marginal MNE relocations is merely 0.0022. Equivalently, that means agents in the model are willing to give up 0.14% of steady state consumption to go from the ssMNE model to the benchmark model. In contrast, in the benchmark calibration, when the standard deviation of nominal shocks is 0.02, agents are willing to give up 3.24% of steady state consumption to shut down the marginal relocations.

The difference in magnitude of welfare implications thus suggest that, at the business cycle frequency, the welfare effect of FDI flows mainly comes from its magnification of fluctuations caused by nominal shocks. In an environment when the source of economic fluctuations is unobservable, the ssMNE model in which the marginal MNE relocation is disabled produces the first best outcome for any reasonable mixture of monetary and productivity shocks.

6 Welfare Effects of output gap response

6.1 Interest Rate Rule and Output Gap Response

Assume that monetary authorities in both countries set nominal interest rates according to a Taylor type rule that target both inflation and output gap. The Northern central bank set nominal interest rate according to

$$i_t = \rho + \rho_\pi (\pi_t - \bar{\pi}) + \rho_y \hat{Y}_t + \nu_t, \tag{36}$$

where ρ denotes the steady state net nominal interest rate, which equals to $\frac{1}{1-\beta} - 1$ in the zero-inflation steady state, and ν_t is still the shock to the nominal interest rate. The Southern nominal interest is set in a symmetric manner. Assume that the shocks to the nominal interest rates are normally distributed.

In equation (36), $\hat{Y}_t$ denotes the output gap, defined as the percentage deviation of current output from the *steady state* output. Theoretically, it is optimal for monetary policymakers to target instead the deviation of output from its natural level - the output that would result in a frictionless environment. But in practice, it is very difficult to disentangle observed economic fluctuations into nominal shocks versus real shocks-induced parts, and thus the natural level of output is almost always unknown. As a result, it is more practical to instead respond to the output deviation from the steady state (or balanced growth path) value.

Likewise, if shocks to the economy were observable, monetary authority should optimally adjust the nominal interest rate to its natural level, implying the constant term ρ in equation (36) should be the time-varying natural rate. Thus, the nominal shock ν_t can be seen as a policy error due to the fact that the natural rate is unobservable.

Also, it is noteworthy that the output measure we use is the gross domestic product (GDP), which measures total product in a given period within a nation's geographical border. When multinationals are present, a country's total GDP counts products produced by foreign-owned firms. Thus when a Northern firm moves to South to produce, its output is no longer counted in the Northern GDP, but is included in the Southern GDP.

Following the convention, Northern GDP is defined as the aggregate product of all firms that produce in the North. The aggregation takes the same form as consumption:

$$Y_t = \left[\int_1^{\tilde{\varphi}_{t-1}} (Y_{Nt}(\varphi))^{\frac{\mu-1}{\mu}} g(\varphi)\, d\varphi \right]^{\frac{\mu}{\mu-1}}. \tag{37}$$

Likewise, for the South, GDP includes the products of those Northern firms that produce in the South as multinationals:

$$Y_t^* = \left[\int_{\varphi_m^*}^{\infty} \left(Y_t^*(\varphi)\right)^{\frac{\mu-1}{\mu}} g(\varphi)\, d\varphi + \int_{\tilde{\varphi}_{t-1}}^{\infty} \left(Y_{Nt}^*(\varphi)\right)^{\frac{\mu-1}{\mu}} g(\varphi)\, d\varphi \right]^{\frac{\mu}{\mu-1}}. \tag{38}$$

Incorporating output-targeting interest rate rules does not change the basic properties of the model. We will show in the next section that aggregate output measured by GDP is closely associated with the aggregate employment in each country. Given our earlier finding that short-run FDI fluctuations magnify the welfare losses by adversely affecting aggregate employments, the output gap response is a way to respond to the "employment gap". Thus we expect output gap targeting to be an effective tool to control the short run fluctuations in FDI flows.

Nominal shocks do not shift the natural level of output - the output level that would result with flexible prices - over business cycle. Therefore, under nominal shocks, the deviation of output from its steady state level is equivalent to its deviation from the natural level. On the other hand, when productivity shocks hit, the natural level of output does deviate from its steady state level, and there is an important cost responding to output gap. The cost rises from the fact that in practice, the natural level of output is unobservable. As a result, although monetary policy should only be used to correct for the deviation of output from its natural level, in reality it is more practical to target instead the deviation from the steady state or balanced growth path. Under this type of constraints, when a real shock causes a deviation of the natural level of output, the output gap response will be inefficient as it acts as another disturbance. To see this, note that our definition of the output gap can be decomposed into two parts: a first component that represents the deviation of output from its natural level, and a second component that represents the deviation of the natural level of output from the steady state level, i.e.

$$\hat{Y}_t \equiv \log\left(Y_t/\bar{Y}\right) = \log\left(\frac{Y_t}{Y_t^n} \times \frac{Y_t^n}{\bar{Y}}\right) = \log\left(\frac{Y_t}{Y_t^n}\right) + \log\left(\frac{Y_t^n}{\bar{Y}}\right),$$

where Y_t^n denotes the natural level of output. Note that nominal rigidities cause the first term to deviate from zero, which should be the concern of monetary policy. A real shock causes the natural level of output to optimally deviate from steady state, in which case the second term will be non-zero. Yet when monetary authorities can't separate the second term from the first, the interest rate rule (36) can be written as

$$i_t = \rho + \rho_\pi \left(\pi_t - \bar{\pi}\right) + \rho_y \log\left(\frac{Y_t}{Y_t^n}\right) + \rho_y \log\left(\frac{Y_t^n}{\bar{Y}}\right) + \nu_t. \tag{39}$$

In this case, the "overcompensation" represented by the term $\rho_y \log\left(Y_t^n/\bar{Y}\right)$ looks like an augmentation to the nominal interest shock ν_t, and it increases the economic fluctuations as such. It is easy to infer from this expression that the inefficiency caused by mistakenly responding to the natural level deviation is increasing in both the magnitudes of the real shocks and the response coefficient ρ_y.

Based on this argument, conventional wisdom has in general favored inflation targeting over output targeting. (See, e.g. Gali (2008)) However, when multinationals present, there is an additional benefit of output gap response. As we have shown, the short run changes in FDI flows cause further distortions in the labor market, which ultimately cause the welfare loss we highlighted. When a firm changes its production location in response to a nominal shock, it causes a corresponding change in labor demand and output of its host country. In that sense, when monetary policy takes into account the output gap, it automatically corrects for the short run changes in employment, and thus discourages those relocation of temporary nature. This gain, however, should be weighed against the cost the conventional wisdom has emphasized.

Substituting demand and optimal prices into the definition of country outputs (37) and (38), we can conveniently write each country's output as a function of its current period's aggregate employment and a measure of the aggregate productivity of firms who produce in that country:

$$Y_t = \bar{\varphi}_{Nt-1} A_t L_t,$$

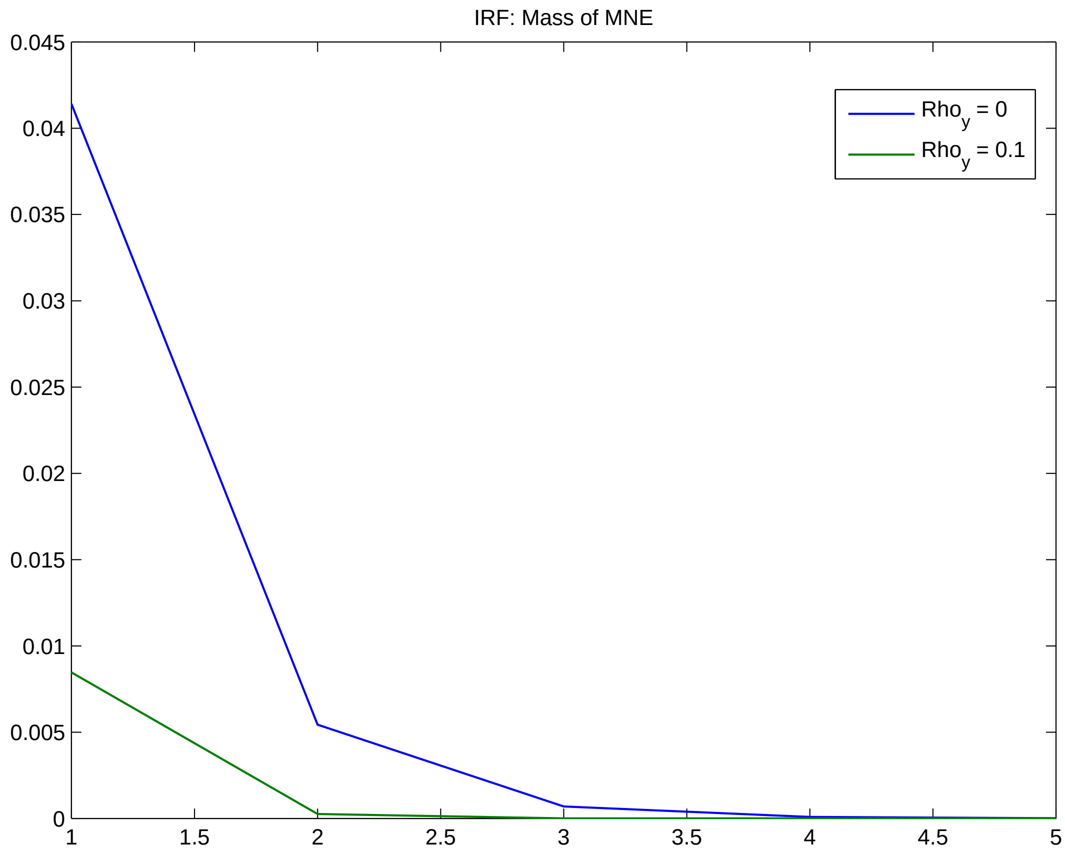
and

$$Y_t^* = \left[(\bar{\varphi}_{Nt-1}^* A_t)^{\mu-1} + (\bar{\varphi}^* A_t^*)^{\mu-1} \right]^{\frac{1}{\mu-1}} L_t^*.$$

These expressions resemble a country-level aggregate productivity function in models with representative firms. The difference, however, is that the aggregate productivity of the recipient country is positively affected by the source country's aggregate productivity shock. Given a certain level of labor supply L_t^*, a positive Northern productivity shock increases the Southern GDP.

6.2 Simulation results

This section is dedicated to the discussion of monetary policy in the model environment. We re-examine the desirability of responding to output gap when cross-border relocation is allowed. As we show in the previous section, output gap targeting when real shocks present induces a cost that resembles an enlarged volatility for nominal shocks. However, intuition suggests that when cross-border firm relocation is allowed, targeting output gap is similar to targeting the employment gap, which is the direct culprit of welfare-loss under nominal shocks. The figure below presents a piece of evidence that supports this intuition. The blue line depicts the IRF of mass of MNEs in the benchmark model, when $\rho_y = 0$, under a one-standard-deviation shock to the Northern interest rate. The green line, on the other hand, depicts the same IRF when the output response coefficient is set to a moderate 0.1. In the baseline simulation, more than 4% of Northern firms become new multinationals following the shock, whereas setting ρ_y to 0.1 decreases the new multinationals to less than 1%.

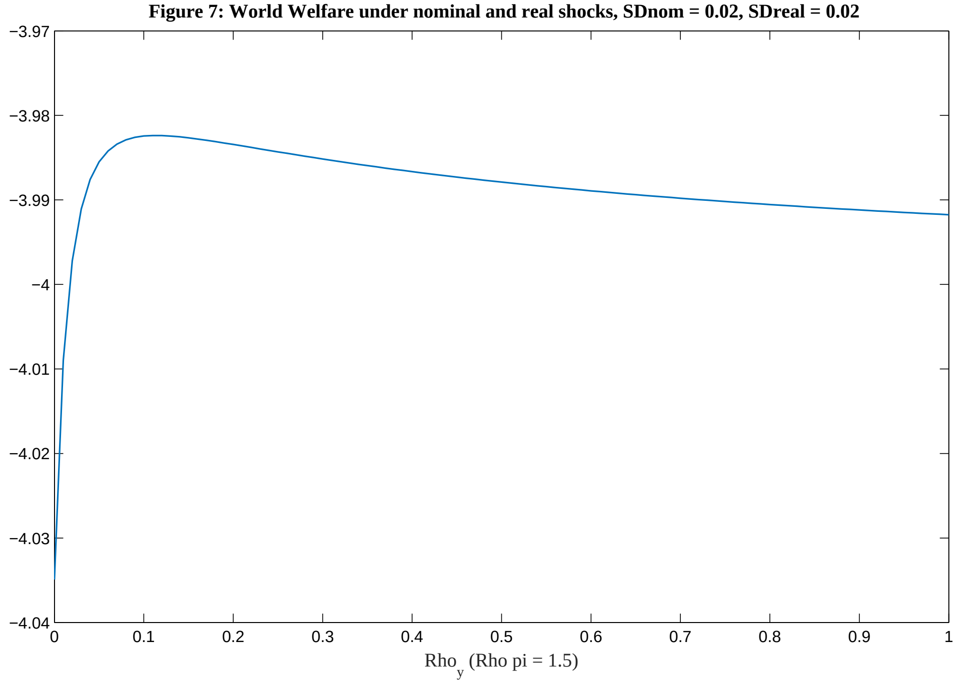


The analysis above suggests that output gap targeting is an effective way to tame the short run fluctuations in FDI flows under nominal shocks. The more important question is then, how is the expected welfare affected by output targeting? The answer to this question obviously depends on how large is the gain compared to the cost of mistakenly responding to the deviation of the natural level of output from steady state. Equation (39) points out that the cost is larger when that deviation is larger. Thus a reasonable guess is that the relative benefit of output gap targeting is increasing with the relative importance of nominal shocks compared with real shocks, as reducing the volatility of FDI flows caused by nominal shocks is welfare-improving, yet correcting for those natural level deviations caused by real shocks is counter-productive. Next we formally verify this conjecture by computing the expected world utility corresponding to different values of output response coefficient in a second order approximation.

The expected world utility is computed in the same way as in chapter one. We simulate the model in second order approximation for 2000 periods, drop the first 200 periods, and compute the average period utility. We repeat this process to make sure that we get stable results.

Figure 7, 8, and 9 depict how the expected period world utility changes as the output response coefficient ρ_y is increased from the baseline value of zero to one. The upper black line represents the steady state level of world welfare and the lower green line represents the expected world utility at different values of ρ_y. In the baseline case in figure 7, when both nominal and real shocks have the same standard deviation of 0.02, increasing the output response coefficient improves utility at first, and decreases it when we increase ρ_y further. Clearly there is an optimal value of output gap response at which the expected world welfare is maximized.

Figure 7: World Welfare under nominal and real shocks, SDnom = 0.02, SDreal = 0.02



That pattern is more apparent when the importance of the productivity shocks are increased. To contrast with the last case, we set the standard deviation of the productivity shocks to 0.1, while keeping the volatility of nominal shocks the same as before. The world utility in this environment is depicted in figure 8. As ρ_y increases from zero, the expected world utility first increases, and then drops. This is because ρ_y serves as a multiplier to the deviations of the natural level of output in (39). When ρ_y is small, the cost of output gap response is negligible, but it does a beneficial job of discouraging the FDI fluctuations caused by nominal shocks. As we continue increasing the output response coefficient though, its benefit is exhausted and the importance of the cost takes over. Fixing the magnitude of the real shocks, increasing ρ_y directly contributes to the volatility of the system. The lesson from this case is then that, when real shocks are volatile compared with nominal shocks, the optimal level of output response strength is significantly smaller.

Figure 8: World Welfare: SDnom = 0.02, SDreal = 0.1

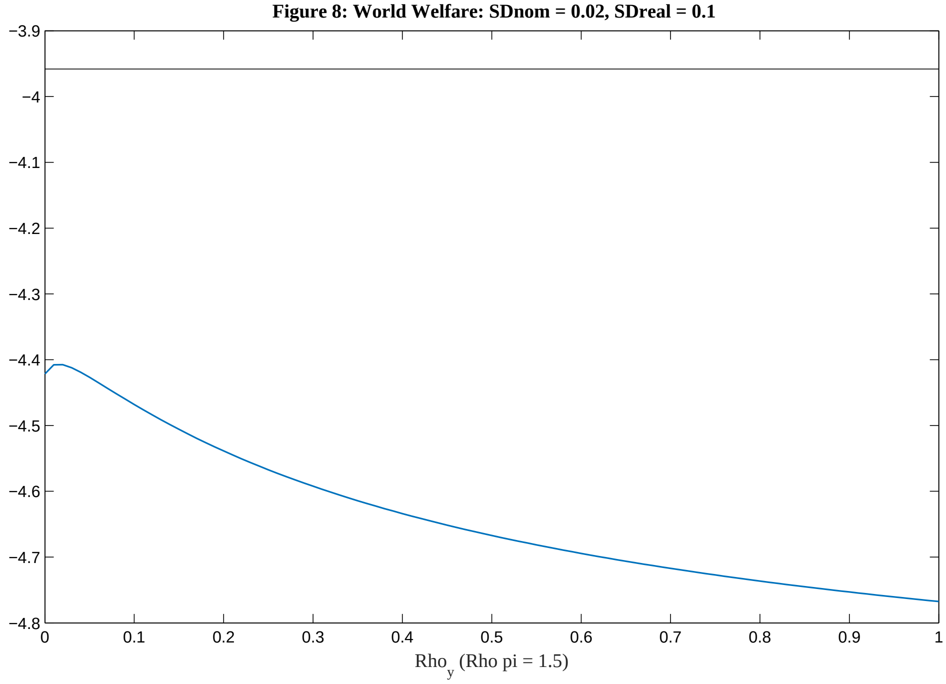
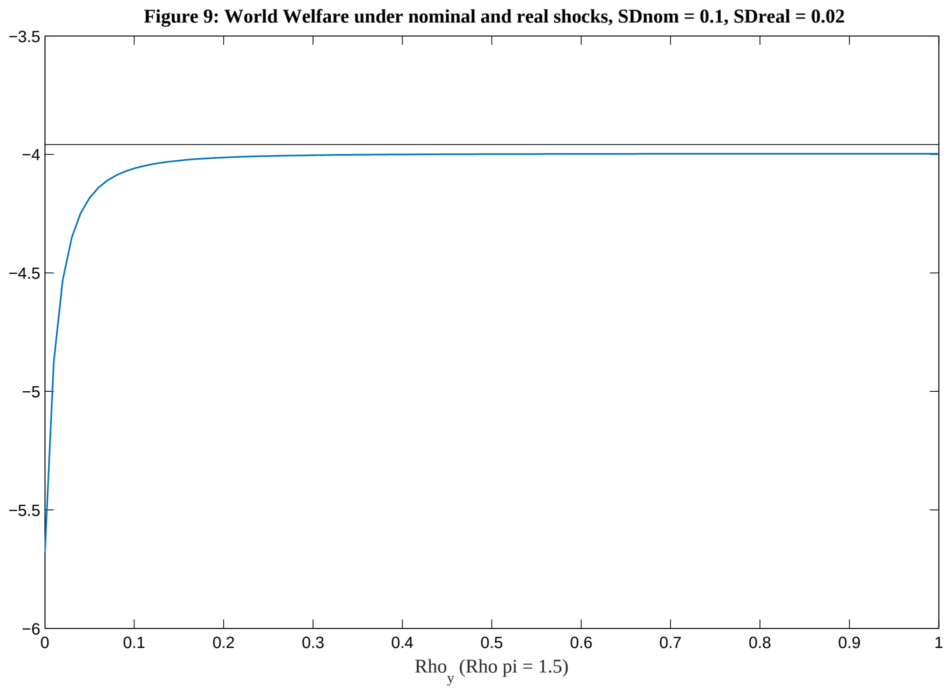


Figure 9 is the case when the nominal shocks are relatively more volatile than the real shocks. The distortion of output gap targeting appears to be substantially tamed, and increasing ρ_y pushes the expected utility very close to its steady state level.



To summarize, when we increase the output gap response coefficient, the expected world utility in second order approximation exhibit different patterns under different assumptions regarding the relative volatility of real and nominal shocks. Theory suggests that output response causes more distortion when the real shocks are volatile, as the natural level of output experiences more fluctuations. The simulation results in this section conform with this intuition. Although this is a simple analysis, one implication seems to be apparent: When multinational firms present, setting output response coefficient to zero is never optimal.

7 Conclusions

This paper examines the short run welfare effects of FDI flows in an open-economy macro model, and discuss the implication on monetary policy conduct. Welfare analysis shows that short run FDI fluctuations exacerbate utility loss in an environment with nominal shocks, but has little impact on welfare over business cycles caused by productivity shocks. Thus the first best outcome can be achieved if long term FDI is retained, but short run changes in production location is switched off.

Furthermore, as each country's aggregate employment is directly connected to its aggregate output, adopting a monetary policy that takes into account the output gap is an effective way to tame the fluctuations of FDI flows over the business cycle. As a result, output gap targeting helps reducing the welfare loss associated with these cross-border firm relocation found in chapter one.

The net benefit of output gap response is increasing in the relative volatility of nominal shocks compared to the real ones. There exists an optimal output response coefficient at which the expected world utility is maximized. Moreover, we have shown that the distortion caused by mistakenly responding to the deviation of the natural level of output is proportional to the magnitude of the response coefficient. Thus, when the coefficient is small, the loss is negligible, but its marginal benefit is largest when the coefficient is small. As a result, in all our simulations, it is always optimal to respond to output gap to some extent.

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A Proof: FDI flows from the country with high average productivity to the country with low average productivity

We prove by contradiction.

Case 1: Assume there is no cross-border investment in the steady state.

Given that the productivities are distributed on $[0, \infty)$, the breakeven condition (15) implies that the real wages must equalize. Thus (25) becomes

$$\frac{\bar{C}}{\bar{C}^*} = \left(\frac{\bar{\varphi}^{\mu-1}}{\bar{\varphi}^{*\mu-1}} \right)^{-\frac{\phi}{\sigma}} \tag{40}$$

where

$$\bar{\varphi} \equiv \left[\int_1^\infty \varphi^{\mu-1} g(\varphi)\, d\varphi \right]^{\frac{1}{\mu-1}}$$

is the uncondition average productivity of North firms. By assumption, $\bar{\varphi} > \bar{\varphi}^*$. Therefore (40) implies $\bar{C}/\bar{C}^* < 1$. On the other hand, without foreign production, each country gets their domestic production. Setting $\bar{\varphi}_H^{*\mu-1} = 0, \overline{\bar{\varphi}}_H = \bar{\varphi}$ and $G(\overline{\tilde{\varphi}}) = 1$ in the two budget constraint (26) and (27) and then divide, we get

$$\frac{\bar{C}}{\bar{C}^*} = \frac{\bar{\varphi}^{\mu-1}}{\bar{\varphi}^{*\mu-1}} > 1,$$

which leads to a contradiction with the prediction from the labor market equilibrium condition.

Case 2: Assume that in steady state, FDI flows from South to North.

If this is true, the breakeven condition (15) implies that the real wage must be higher in South, i.e. $\bar{\omega}^* > \bar{\omega}$. We have the counterpart of (25)

$$\left(\frac{\bar{C}}{\bar{C}^*} \right)^{\frac{\sigma}{\phi}} = \left(\frac{\bar{\omega}}{\bar{\omega}^*} \right)^{\frac{1}{\phi}+\mu} \left(\frac{\overline{\bar{\varphi}}_F^{*\mu-1}}{\overline{\bar{\varphi}}_S^{\mu-1} + \bar{\varphi}^{\mu-1}} \right), \tag{41}$$

where

$$\overline{\bar{\varphi}}_S^* \equiv \left[\int_{\varphi_m^*}^{\overline{\tilde{\varphi}}^*} \varphi^{\mu-1} g(\varphi)\, d\varphi \right]^{\frac{1}{\mu-1}},$$

and

$$\overline{\bar{\varphi}}_S^* \equiv \left[\int_{\overline{\tilde{\varphi}}^*}^{\infty} \varphi^{\mu-1} g(\varphi)\, d\varphi \right]^{\frac{1}{\mu-1}}$$

are the aggregate productivities of Southern firms who produce domestically and abroad, respectively, and $\overline{\tilde{\varphi}}^*$ is the cutoff productivity. Since $\bar{\varphi}^{\mu-1} > \bar{\varphi}^{*\mu-1} > \overline{\bar{\varphi}}_S^{*\mu-1}$, and $\bar{\omega}^* > \bar{\omega}$,

(41) implies that $\bar{C}/\bar{C}^* < 1$.

By symmetry, the two budget constraints (26) and (27) imply

$$\bar{C} = \bar{\omega}^{1-\mu}\bar{Y}^w\left(\overline{\bar{\varphi}}_S^{\mu-1} + \bar{\varphi}^{\mu-1}\right),$$

$$\bar{C}^* = \bar{\omega}^{*1-\mu}\bar{Y}^w\overline{\bar{\varphi}}_S^{*\mu-1} - \left(1 - G^*\left(\overline{\bar{\varphi}}^*\right)\right)F.$$

The labor market equilibrium states that $\bar{C}/\bar{C}^* < 1$, and the last term is a non-negative number. Thus we must have

$$\bar{\omega}^{1-\mu}\bar{Y}^w\left(\overline{\bar{\varphi}}_S^{\mu-1} + \bar{\varphi}^{\mu-1}\right) \quad < \quad \bar{\omega}^{*1-\mu}\bar{Y}^w\overline{\bar{\varphi}}_S^{*\mu-1}$$

This implies

$$\left(\frac{\bar{\omega}}{\bar{\omega}^*}\right)^{1-\mu} < \frac{\overline{\bar{\varphi}}_S^{*\mu-1}}{\overline{\bar{\varphi}}_S^{\mu-1} + \bar{\varphi}^{\mu-1}} < 1, \qquad (42)$$

as North has higher average productivity. But this implies $\bar{\omega}^* < \bar{\omega}$, which leads to a contradiction, as the break-even condition states that when there is positive FDI flow from South to the North, Southern real wage must be higher than the Northern. Therefore Case#2 can't be true. The only possible equilibrium is for the country with high productivity to relocate to the country with low productivity.