

Selection Effects with Heterogeneous Firms

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Nottingham Lectures in International Economics 2012

March 13, 2012

Background

- Firms and Trade: Selection Effects

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- Melitz (2003): More efficient firms export

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- Melitz (2003): More efficient firms export
- More efficient firms more likely to do everything:
 - FDI rather than export: Helpman-Melitz-Yeaple (2004)
 - Internalise rather than outsource: Antràs-Helpman (2004)
 - Pay higher wages: Egger-Kreickemeier (2009), Helpman-Itzhak-Horowitz-Redding (2010)
 - Adopt more skill-intensive technology: Bustos (2011)

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 - Adopt more skill-intensive technology: Bustos (2011)
- Increasingly applied to other fields:
 - International macroeconomics: Ghironi-Melitz (2005)
 - International tax competition: Davies-Eckel (2010)
 - Environmental economics: Forslid-Okubo-Ulltveit-Moe (2011)

Motivation

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 - Typical of modern manufacturing? (Milgrom-Roberts 1990)
 - “Supermodularity” important: substantively? technically?

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 - Distribution of firm productivities: Pareto
 - Symmetric countries

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 - If successful firms are large in every dimension, is monopolistic competition the right market structure?
 - A universal tendency? Or are there interesting counter-examples?

Our Contribution

- Distinguish “first-order” and “second-order” selection effects
 - “First-order” much more robust
- General result on second-order selection effects
 - Proved first for a monopoly firm choosing between exports and FDI
 - We then show that it extends to a wide variety of market structures:
 - monopolistic competition, oligopoly;
 - ... and to a wide variety of firm choices:
 - choice of technique, in-house vs outsourcing, multi-market vs export-platform FDI

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- Technical: Application of *supermodularity*
 - Arises very naturally in this context
 - e.g., FDI: Both production costs and market access costs differ by finite amounts
 - Supermodularity imposes a natural restriction on the finite “difference-in-differences” of firm profits

Related Literature

• Firm Selection

- Exports: Melitz (*Em* 2003)
- FDI: Helpman-Melitz-Yeaple (*AER* 2004)
- Export-Platform FDI: Helpman-Melitz-Yeaple (WP 2003, Appendix), Mrázová-Neary (2010)
- Outsourcing: Antràs-Helpman (*JPE* 2004)
- Heterogeneous workers: Helpman-Itskhoki-Redding (*Em* 2010)
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• Supermodularity

- General: Milgrom-Roberts (*AER* 1990), Milgrom-Shannon (*Em* 1994), Athey (*QJE* 2002)
- Trade:
 - Matching: Grossman-Maggi (*AER* 2000), Costinot (*Em* 2009), Costinot-Vogel (*JPE* 2010)
 - Trade and Environmental Agreements: Limao (*JIE* 2005)
 - Firm Selection: Costinot (WP 2007)

Outline of the Talk

- 1 First-Order Selection Effects
- 2 Supermodularity
- 3 Selection into FDI versus Exporting
- 4 Selection Effects in Monopolistic Competition
- 5 Selection Effects in Oligopoly
- 6 Alternative Firm Choices
- 7 Summary and Conclusion
- 8 Supplementary Material

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Operating Profits

- Consider a single firm located in one country
- It wishes to serve consumers in a foreign country
- $\pi(t, c)$: Maximum operating profits it can earn; weakly decreasing in:
 - t : Access cost (tariffs and transport costs) it faces
 - c : Exogenous cost parameter (inversely related to productivity)
 - Often, though not always, equal to marginal production cost
 - In some applications, an inverse indicator of quality
 - Other determinants optimally chosen or exogenous; examples later

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 - Total profits: $\Pi^X = \pi(t, c) - f_X < 0$; fixed cost f_X independent of c
 - With $c_1 > c_2$, we have $\pi(t, c_1) < \pi(t, c_2)$
 - So, firm 2 must be the exporter
 - $\Pi_2^X < 0 \Rightarrow \Pi_1^X \ll 0$; $\Pi_1^X > 0 \Rightarrow \Pi_2^X \gg 0$

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- Robust? Very; extends to:
 - Continuum of firms
 - Arbitrary distribution of variable costs
 - Asymmetric countries
 - Arbitrary assumptions about demand and technology
- All that is needed is π decreasing in c : a very mild assumption

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Supermodularity

- Δ_c : The finite difference between the values of a function evaluated at two different values of c :

$$\Delta_c \pi(t, c) \equiv \pi(t, c_1) - \pi(t, c_2) \quad \text{when} \quad c_1 \geq c_2$$

- Non-positive: measures the profit disadvantage of a higher-cost firm
- π differentiable in c : $\frac{\Delta_c \pi(t, c)}{c_1 - c_2} \rightarrow \frac{\partial \pi}{\partial c}$ as $c_1 \rightarrow c_2$

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Definition

The function $\pi(t, c)$ is supermodular in t and c if and only if:

$$\Delta_c \pi(t_1, c) \geq \Delta_c \pi(t_2, c) \quad \text{when} \quad t_1 \geq t_2.$$

► Implications

Supermodularity: Some Intuition

$$\Delta_c \pi(t_1, c) \geq \Delta_c \pi(t_2, c)$$

$$\Rightarrow 0 \geq \pi(t_1, c_1) - \pi(t_1, c_2) \geq \pi(t_2, c_1) - \pi(t_2, c_2)$$

- A higher tariff reduces in absolute value the profit disadvantage of a higher-cost firm.

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$$\Rightarrow \pi(t_2, c_2) - \pi(t_1, c_2) \geq \pi(t_2, c_1) - \pi(t_1, c_1) \geq 0$$

- The “Matthew Effect”: “To those who have, more shall be given”:
 - A lower tariff is of more benefit to a firm with more sales
 - A lower-cost (more productive) firm usually has higher sales
 - ... though not always ...

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- Analogous to Hicksian complementarity in consumer theory or strategic complementarity in game theory

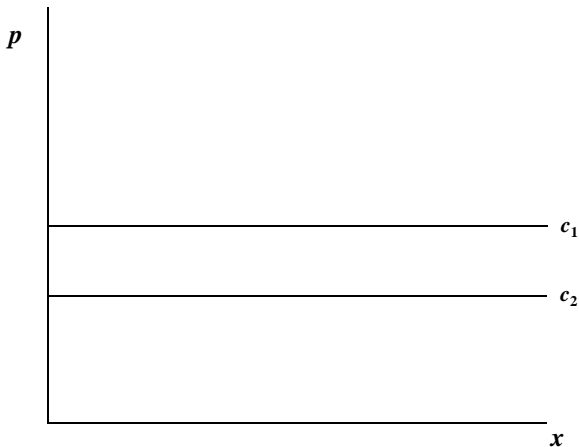
Example 1: Marginal Cost Independent of Output

- Constant marginal cost c
- Inverse demand function: $p(x)$
- So, firm's operating profits equal:

$$\pi(t, c) \equiv \underset{x}{Max} [\{p(x) - c - t\} x]$$

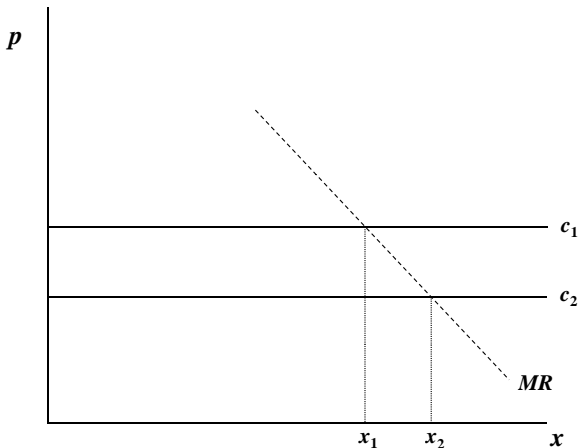
[► Maths](#)

Example of Supermodularity



- Less productive firm has higher marginal cost
- And therefore ...

Example of Supermodularity



- Less productive firm has higher marginal cost
- And therefore lower output, so it benefits less from a tariff reduction.

Example 2: Marginal Cost Varies with Output

- Key features of Example 1: π continuous in t and c and depends only on their sum
 - Given these, supermodularity \Leftrightarrow *convexity* of π in both t and c

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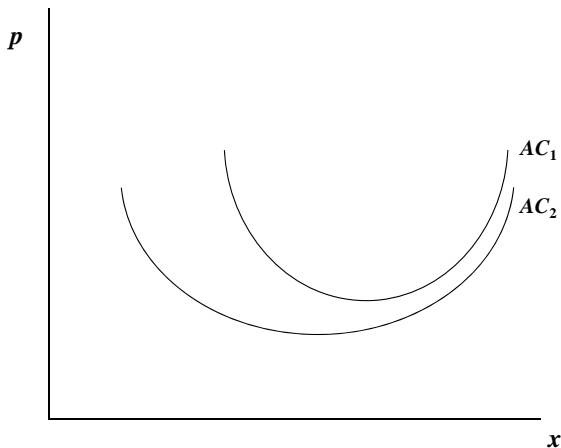
- Key features of Example 1: π continuous in t and c and depends only on their sum
 - Given these, supermodularity \Leftrightarrow *convexity* of π in both t and c
- Suppose instead that marginal cost varies with output:

$$\pi(t, c) \equiv \underset{x}{Max} [\{p(x) - t\} x - C(c, x)]$$

- $C(c, x)$: Total variable costs; $C_c > 0$, $C_x > 0$

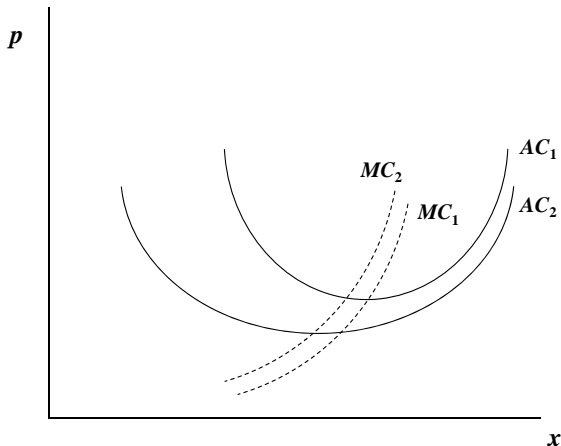
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Example of Submodularity



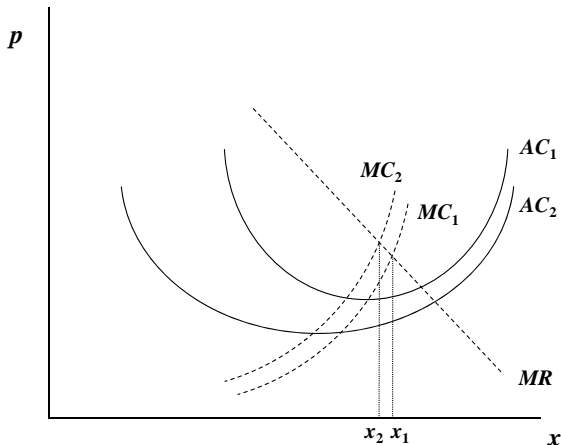
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- Less productive firm is *relatively* more productive at higher output
- So much so that it has *lower* marginal cost and *higher* output

Example 2: Conclusion

- Example 2 provides an exception to supermodularity because inter-firm differences in efficiency work in opposite directions *on average* and *at the margin*.
- Supermodularity holds as long as they work in the same direction.

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- Firm has two options:
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- Define the *tariff-jumping gain* γ :

$$\gamma(t, c, f) \equiv \pi(0, c) - \pi(t, c) - f = \Pi^F - \Pi^X$$

- $f \equiv f_F - f_X > 0$ is the excess fixed cost of FDI relative to exporting

Firm Selection into Tariff-Jumping

$$\gamma(t, c, f) \equiv \pi(0, c) - \pi(t, c) - f$$

- Apply Δ_c to the tariff-jumping gain:

$$\Delta_c \gamma(t, c, f) = \Delta_c \pi(0, c) - \Delta_c \pi(t, c)$$

- From the definition of supermodularity, $\Delta_c \gamma(t, c, f)$ is negative if and only if π is supermodular in t and c

► Recall definition

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- Since γ measures the incentive to engage in FDI relative to exporting:

Proposition

If and only if the profit function π is supermodular in t and c , higher-cost firms will serve the foreign market by exports, while lower-cost firms will serve it via FDI, for all admissible f .

Technical Details of Proof

- Sufficiency is immediate:
 - SM [Supermodularity of π in t and c] $\Rightarrow \gamma$ weakly decreasing in c
 - \Rightarrow Selection, if it occurs, must follow the CS pattern
 - CS ["Conventional Sorting"]: High-cost firms export, low-cost in FDI

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 - ▶ Figure

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- However, CS for *all* admissible f *does* imply SM
 - So, SM is also necessary
 - “Admissible”: Range of f for which selection occurs:
 - $f \in (0, \bar{f})$, where $\bar{f} \equiv \max_c \gamma(t, c, 0)$ for given $t > 0$
 - Quasi-linearity of Π is crucial for this
 - [▶ Figure](#)

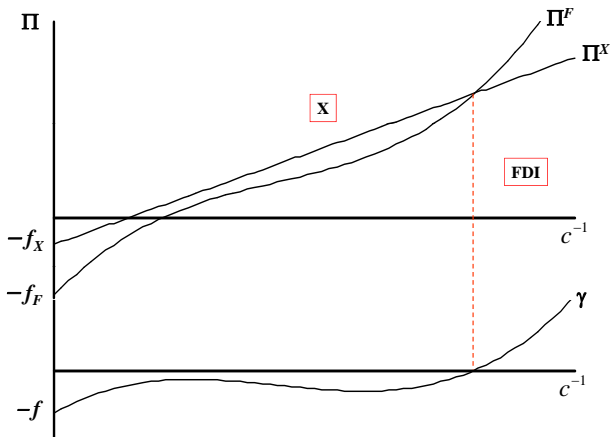
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- None of this matters in many practical applications
 - Mostly we assume π is differentiable
 - So sufficiency is sufficient: we need only check if π_{tc} is non-negative
 - Since: $\pi_{tc} \geq 0 \Rightarrow \text{SM} \Rightarrow \text{SCP} \Rightarrow \text{CS}$
 - [▶ Next](#)

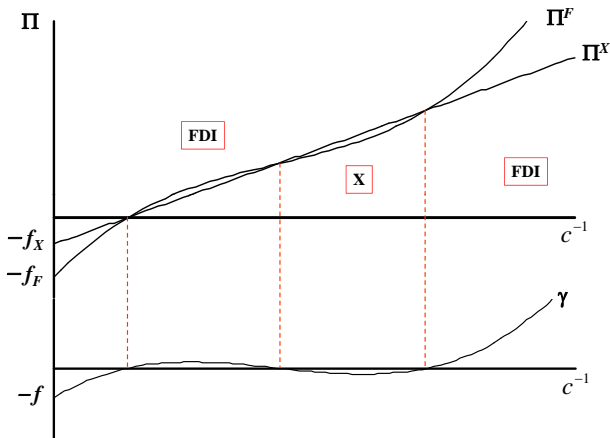
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SCP But Not SM



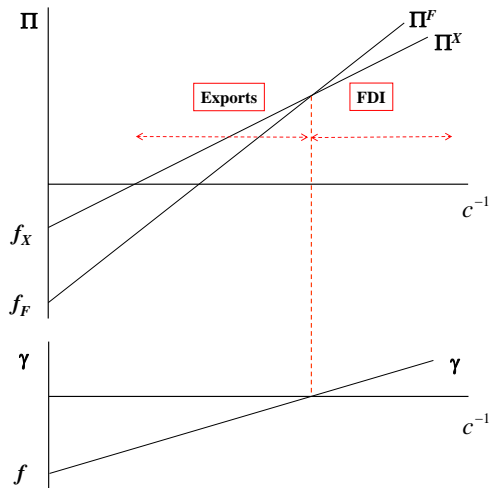
- SCP holds but not SM, and so CS holds for *some* f

Not SM \Rightarrow Not SCP for some f



- SM does not hold, so we can always find an f such that SCP and so CS does not hold

[Back](#)

SM \Rightarrow SCP

- SM holds, so SCP and CS hold for *all* admissible f

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 - Things are different if they depend on t and c : see below
- Proposition 1 generalises the result of Helpman-Melitz-Yeaple (2004)
- ... or does it?

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 - General Preferences
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Selection Effects with CES Preferences

- As in Helpman-Melitz-Yeaple: CES preferences $x = Ap^{-\sigma}$
- Profits: $\pi(t, c) = (\tau c)^{1-\sigma} B$
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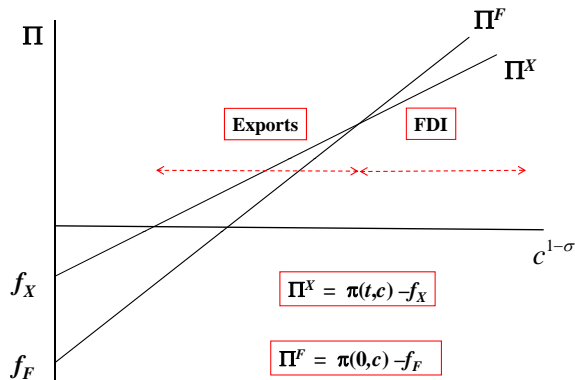
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- No: Cross-section comparison between two different firms only, both of measure zero

Inferring Selection Effects the Hard Way



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- Details:

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- SM: $\Delta_c \pi(t, c) > \Delta_c \pi(0, c)$

$$\Leftrightarrow \pi(t, c_1) - \pi(t, c_2) > \pi(0, c_1) - \pi(0, c_2)$$

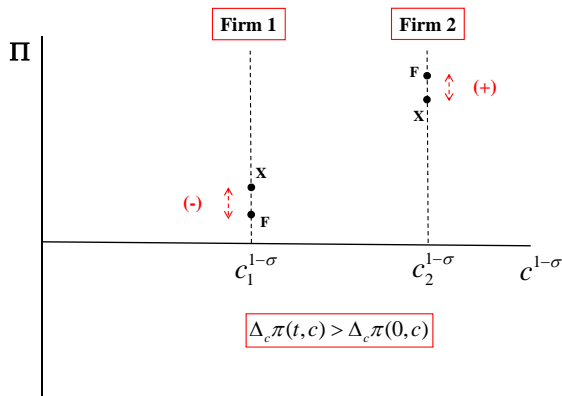
$$\Leftrightarrow \pi(0, c_2) - \pi(t, c_2) > \pi(0, c_1) - \pi(t, c_1)$$

- Now subtract $f = f_F - f_X$ from both sides:

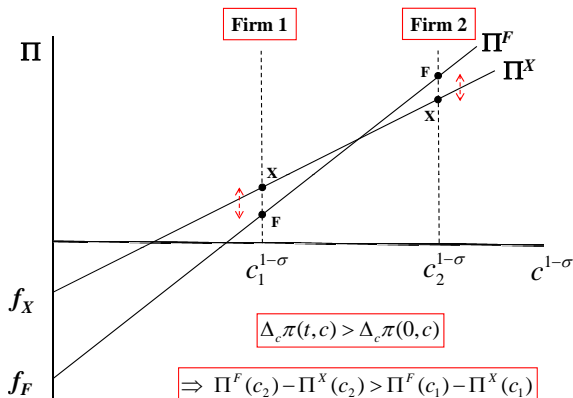
$$\Leftrightarrow \Pi^F(c_2) - \Pi^X(c_2) > \Pi^F(c_1) - \Pi^X(c_1)$$

- Repeat for every pair of firms ...

Selection Effects: Two Tiny Firms at a Time



Inferring Selection Effects from Supermodularity



Selection Effects with General Preferences

- In general, the result is ambiguous:

[▶ Skip to next](#)

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- Operating profits: $\tilde{\pi}(x; \tau, c) = [p(x) - \tau c] x$
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Selection Effects with General Preferences II

- Write demand elasticity as a function of x : $\varepsilon(x) \equiv -\frac{\partial x}{\partial p} \frac{p}{x} = -\frac{p}{xp'}$
 - Krugman (1979): $\varepsilon_x < 0$ (e.g., quadratic, Stone-Geary, CARA, logistic)
 - CES case: $p = x^{-1/\sigma} \Rightarrow \varepsilon_x = 0, \varepsilon = \sigma$
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 - e.g., $p = (x - \beta)^{-1/\sigma}, \beta > 0$

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Proposition

With iceberg transport costs, the profit function is supermodular in t and c for all levels of output if the demand function is superconvex, $\varepsilon_x \geq 0$.

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- By contrast, if $\varepsilon_x < 0$, submodularity may hold for sufficiently high x .

Selection Effects with General Preferences III

- Submodularity possible whenever $\varepsilon_x < 0$:

$$\varepsilon_x = -\frac{1}{x} (1 + \varepsilon - \varepsilon\rho) \quad \rho \equiv -\frac{xp''}{p'}$$

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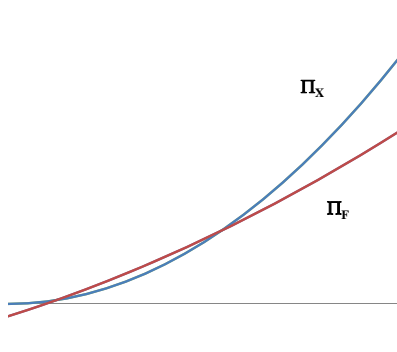
Proposition

With general demands and iceberg transport costs, the profit function is supermodular in t and c if and only if: $\varepsilon + \rho - 3 \geq 0$.

$$\pi_{tc} = -x - \tau c \tilde{\pi}_{xx}^{-1} = -x - \frac{p + xp'}{2p' + xp''} = \frac{\varepsilon + \rho - 3}{2 - \rho} x$$

- So: Submodularity more likely for less elastic and more concave demand

Quadratic Preferences; Iceberg Transport Costs



Quadratic preferences (Melitz-Ottaviano (2008), Nefussi (2006))

► Details

► Compare Ad Valorem Case

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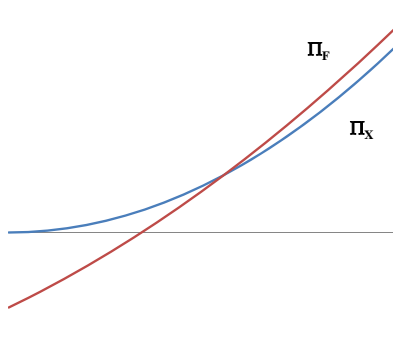
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- BUT: Separability of π does not hold if c measures quality

Quadratic Preferences; Ad Valorem Transport Costs



Summary: Monopolistic Competition

- Is the profit function always supermodular?

Preferences:	$\varepsilon_x \geq 0$	$\varepsilon_x < 0$
Iceberg transport costs:	Yes	No
Proportional transport costs:	Yes	Yes

Outline of the Talk

- 1 First-Order Selection Effects
- 2 Supermodularity
- 3 Selection into FDI versus Exporting
- 4 Selection Effects in Monopolistic Competition
- 5 Selection Effects in Oligopoly**
- 6 Alternative Firm Choices
- 7 Summary and Conclusion
- 8 Supplementary Material

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- Though need to be careful in considering boundary cases
- Examples:
 - Leahy-Montagna (2009): Outsourcing
 - Porter (2011): FDI

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- So far, focus on choice between exports and FDI only
- Analogous results apply to other firm choices:
 - Export-platform versus multi-plant FDI [▶ Details](#)
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 - In each case, supermodularity (between firm's cost parameter and a parameter representing the marginal cost of the choice variable) is necessary and sufficient for the standard selection effect
- Of course, this does not apply to first-order selection effects;
 - e.g., choice between serving a market or not:
 - Melitz (2003): Export decision
 - Depends *only* on $\pi(t, c) - f_X$
 - π decreasing in c ensures conventional sorting.
- Melitz result is very robust.

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- Firm/HQ chooses organizational form to maximize “realized” profits:
 - $\pi(w, \psi, c) \equiv \text{Max}_x (1 - \psi) [p(x) - wc] x$
 - Wages: w_N in North, w_S in South, $w_N > w_S$
 - ψ : profit loss due to incomplete contracting between HQ and supplier
 - Structural microfoundations in Antràs-Helpman (2004)
 - No transport costs

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- Composite VFDI case with $\psi_N < \psi_S$ and $w_N > w_S$ ambiguous

► Skip to Conclusion

Heterogeneous Fixed Costs

- Analysis unaffected if fixed costs depend on t only
 - e.g. Kleinert-Toubal (2006, *RIE* 2010):
 - Fixed costs rising with distance rationalize a gravity equation for FDI
 - and avoid counter-factual prediction that falling trade costs lower FDI
 - But selection effects are unchanged
 - Why? Δ_c operator applied to γ eliminates $f_F(t)$

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 - Supermodularity and so conventional sorting are reinforced
 - Example 2: Oldenski (2009): Task-based trade in services
 - Higher-productivity firms in service sectors are more vulnerable to contract risk when located abroad;
 - Now: $\Delta_c f_F(c) = f_F(c_1) - f_F(c_2) \leq 0$, so $\pi(t, c)$ may be submodular
 - Conventional sorting may be reversed: higher-productivity firms may find it more profitable to locate at home.

Endogenous Fixed Costs

[▶ Skip to Conclusion](#)

- Assume firm invests in market-specific process R&D
 - Similar results apply to advertising, marketing, etc. (Arkolakis *JPE* 2010)
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- Supermodularity may not hold if $C_{kc} < 0$
- i.e., if investment lowers the cost disadvantage of a lower productivity firm
- This cannot happen in some commonly-used special cases ...
- ... But: We can find examples exhibiting submodularity ...

Endogenous Fixed Costs: Details

$$\pi(t, c) \equiv \underset{x, k}{Max} [\{p(x) - C(c, k) - t\}x - F(k)]$$

Proposition

$\pi(t, c)$ is supermodular in (t, c) if $C(c, k)$ is supermodular in (c, k) .

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$$C(c, k) = c_0 + c\phi(k), \quad F'' = 0$$

Lemma

$\pi(t, c)$ is supermodular in (t, c) if and only if $\phi(k)$ is log-convex in k .

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Lemma

$\pi(t, c)$ is supermodular in (t, c) if and only if $\phi(k)$ is log-convex in k .

- $\pi(t, c_0, c)$ is always supermodular in (t, c_0)
- Second-order condition requires: $x C_{kk} + F'' > 0 \Rightarrow C_{kk} > 0$ if $F'' = 0$
 - So, when $F'' = 0$, C must be convex in k , but need not be log-convex.

Endogenous Fixed Costs: Proofs

Proof of Proposition:

- Supermodularity again depends on sign of x_c : $\pi_{tc} = -x_c$
- First-order conditions: $p - C - t + xp' = 0$ and $-xC_k - F' = 0$

$$\Rightarrow \begin{bmatrix} 2p' + xp'' & -C_k \\ -C_k & -(xC_{kk} + F'') \end{bmatrix} \begin{bmatrix} dx \\ dk \end{bmatrix} = \begin{bmatrix} C_c dc + dt \\ xC_{kc} dc \end{bmatrix}$$

$$\Rightarrow \pi_{tc} = -x_c = \underbrace{D_+^{-1} [C_c (xC_{kk} + F'')]}_{+} - \underbrace{xC_k C_{kc}}_{-}$$

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- Second-order conditions: $D > 0$ (determinant) and $xC_{kk} + F'' > 0$
- ... both of which work in favour of $x_c < 0$ and so of supermodularity
- But: supermodularity could still fail if $C_{kc} < 0$

Proof of Lemma:

- $C(c, k) - c_0 = c\phi(k)$ log-convex in $k \Leftrightarrow C_k C_{kc} - C_c C_{kk} < 0$.
 - $\Phi(k) \equiv \ln \phi(k)$; $\Phi'' > 0 \Leftrightarrow \phi\phi'' - (\phi')^2 > 0$
 - $C_k C_{kc} - C_c C_{kk} = c [(\phi')^2 - \phi\phi'']$

Endogenous Fixed Costs: Special Cases

- ① d'Aspremont-Jacquemin (*AER* 1988): $C(c, k) = c_0 - c^{-1}k$, $F(k) = \frac{1}{2}\gamma k^2$
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- ② Spence (*Em* 1984): $C(c, k) = c_0 + ce^{-\theta k}$, $F(k) = k$
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 - $C(c, k) - c_0 = ce^{-\theta k}$ is log-linear in k
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 - So: No selection effects in the Spence case
 - All firms produce the same level of output (though more productive firms invest less and make higher profits)
- However, a less convex R&D cost function can be submodular ...
 - ... as well as being economically interesting ...

Example of Submodularity

③ $C(c, k) = c_0 + ce^{-\theta k^a}$:

- $a = 1$: Spence (1984); $a = 2$: Gaussian function

Example of Submodularity

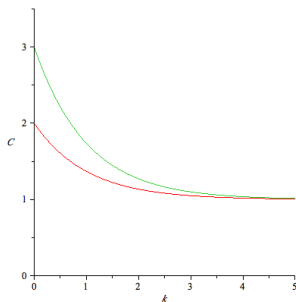
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- \rightarrow firm selection into proximity versus concentration reversed

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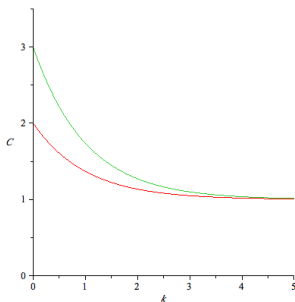


$a = 1$

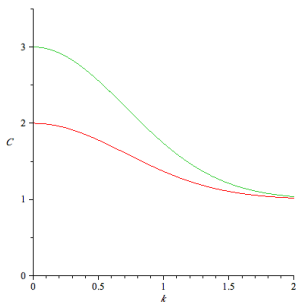
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$a = 2$

Outline of the Talk

- 1 First-Order Selection Effects
- 2 Supermodularity
- 3 Selection into FDI versus Exporting
- 4 Selection Effects in Monopolistic Competition
- 5 Selection Effects in Oligopoly
- 6 Alternative Firm Choices
- 7 Summary and Conclusion**
- 8 Supplementary Material

Summary and Conclusion

- First-order selection effects very robust (e.g. Melitz (2003))
- Second-order selection effects less robust
 - Supermodularity of profits in tariffs and production costs is necessary and sufficient for “conventional sorting”
 - More efficient firms engage in FDI, less efficient in exporting
 - Fixed costs may not predict outcome, except in CES case
- Result holds under a variety of assumptions about market structure and extends to a broad range of models
- Supermodularity pervasive but not universal; counter-examples:
 - FDI without CES: Horizontal with icebergs, or Vertical
 - Task-based trade in services
 - Threshold effects in R&D
- Technique very easy to apply analytically

Thank you for listening. Comments welcome!

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 - Examples 1 and 2

Examples 1 and 2

- Example 1: $\pi(t, c) \equiv \underset{x}{Max} [\{p(x) - c - t\} x]$
 - From the envelope theorem: $\pi_t = -x(t, c)$.
 - Hence the second cross-partial derivative is positive: $\pi_{tc} = -x_c > 0$.
 - Proof: The first-order condition is: $p - c - t + xp' = 0$
 - Differentiate this to get: $dx = -H^{-1}dc$
 - $H \equiv -(2p' + xp'') > 0$ from second-order condition
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- Example 2: $\pi(t, c) \equiv \underset{x}{Max} [\{p(x) - t\} x - C(c, x)]$
 - Envelope theorem still holds: $\pi_t = -x(t, c)$

$$\Rightarrow \pi_{tc} = -x_c = H^{-1}C_{xc}$$

- $H \equiv -[2p' + xp'' - (2C_x + xC_{xx})] > 0$ from second-order condition
- But π is *submodular* if C_{xc} is negative

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Proof of Proposition 1

- Proof of Necessity:

- Let $t > 0$.
- If π is not supermodular in t and c , then there exist some c_1 and c_2 such that $c_1 > c_2$ and $\pi(t, c_1) - \pi(t, c_2) < \pi(0, c_1) - \pi(0, c_2)$.
- Rearranging terms gives: $G_1 > G_2$, $G_1 \equiv \pi(0, c_1) - \pi(t, c_1)$,
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 - Rearranging terms gives: $G_1 > G_2$, $G_1 \equiv \pi(0, c_1) - \pi(t, c_1)$, $G_2 \equiv \pi(0, c_2) - \pi(t, c_2)$
 - Set f such that: $f = \frac{1}{2} (G_1 + G_2)$.
 - For this f , we have: $\gamma(t, c_1, f) = G_1 - f = \frac{1}{2} (G_1 - G_2) > 0$ and $\gamma(t, c_2, f) = G_2 - f = \frac{1}{2} (G_2 - G_1) < 0$.
 - Since γ measures the incentive to engage in FDI relative to exporting, the higher-cost firm will serve the foreign market via FDI while the lower-cost firm will serve it by exports.
- So, if SM does not hold, we can find some f such that CS is reversed.
 - So, SM is necessary for CS

Proof of Proposition 3

We want to express π_{tc} in terms of ε and ε_x :

► Return

$$\varepsilon(x) = -\frac{p(x)}{xp'(x)} \rightarrow \varepsilon_x = -\frac{1}{x} + \frac{p(p' + xp'')}{(xp')^2}$$

$$\rightarrow 2p' + xp'' = -\frac{p}{x\varepsilon^2} (\varepsilon - 1 - x\varepsilon_x) < 0$$

$$p - \tau c + xp' = 0 \rightarrow \tau c = p + xp' = p - \frac{p}{\varepsilon} = \frac{\varepsilon - 1}{\varepsilon} p$$

Finally, substitute these results into π_{tc} :

$$\pi_{tc} = -x - \tau c (2p' + xp'')^{-1} = -x + \frac{\varepsilon - 1}{\varepsilon - 1 - x\varepsilon_x} \varepsilon x$$

Collecting terms gives the desired expression:

$$\pi_{tc} = \frac{(\varepsilon - 1)^2 + x\varepsilon_x}{\varepsilon - 1 - x\varepsilon_x} x$$

Example: Quadratic Preferences

- Quadratic preferences:

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- $p = A - bx \Rightarrow p' = -b, p'' = 0, H = 2b$
- FOC: $p - \tau c - bx = 0 \Rightarrow x = \frac{1}{2b} (A - \tau c)$
- \Rightarrow Maximized operating profits: $\pi(t, c) = bx^2 = \frac{1}{4b} (A - \tau c)^2$
- $\Rightarrow \pi_{tc} = -x + \frac{\tau c}{2b} = -\frac{1}{2b} (A - 2\tau c) \geq 0$

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 - Mid-cost ones engage in FDI
- But: *Sub*-modular for low-cost firms!
 - Low $c \Rightarrow 2\tau c < A \Rightarrow \pi_{tc} < 0$
 - Lowest-cost/most productive firms export

Exports vs FDI with Many Countries

- Assume firm wants to serve n identical foreign countries [e.g., EU]
 - Same inverse demand function $p(x)$ in each
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- Profits from establishing plants in m member countries and exporting from them to the remaining $n - m$ countries:

$$\Pi^{Fm} = m [\pi(0, c) - f_F] + (n - m) [\pi(t_U, c) - f_X]$$

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 $\rightarrow \Pi^{Fm} - \Pi^X = m\gamma(t, c, f) + (n - m)\phi(t, t_U, c)$

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- Linear in m , so optimal m is either one or n
- So, a firm that engages in FDI will establish either a single export-platform plant or n plants, one in each country.
- Assumption of constant marginal cost c is crucial here.

Profitable Modes of Serving Foreign Markets

- Gain from multi-market FDI relative to export-platform FDI:

$$\Pi^{Fn} - \Pi^{F1} = (n - 1) [\gamma(t, c, f) - \phi(t, t_U, c)]$$

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- Like $\Pi^{Fn} - \Pi^X = n\gamma(t, c, f)$, both can be either positive or negative.

Profitable Modes of Serving Foreign Markets

- Gain from multi-market FDI relative to export-platform FDI:

$$\Pi^{Fn} - \Pi^{F1} = (n - 1) [\gamma(t, c, f) - \phi(t, t_U, c)]$$

- Gain from export-platform FDI relative to exporting:

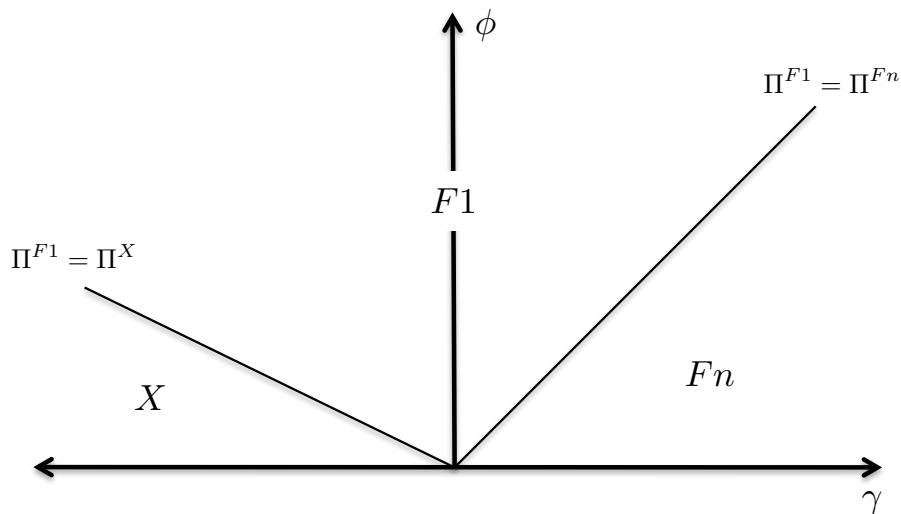
$$\Pi^{F1} - \Pi^X = \gamma(t, c, f) + (n - 1) \phi(t, t_U, c)$$

- Like $\Pi^{Fn} - \Pi^X = n\gamma(t, c, f)$, both can be either positive or negative.
- Summarising the results so far:

Lemma

There are only three profitable modes of serving the n markets: exporting to all, export-platform FDI (with one plant), and multi-market FDI (with n plants).

Figure: Profitable Modes of Serving Foreign Markets



Firm Selection

- Export-platform FDI versus exports:

$$\Delta_c (\Pi^{F1} - \Pi^X) = \Delta_c \gamma(t, c, f) + (n - 1) \Delta_c \phi(t, t_U, c) \quad (1)$$

- If and only if π is supermodular in t and c , both $\Delta_c \gamma(t, c, f)$ and $\Delta_c \phi(t, t_U, c)$ are negative, so (1) is negative

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- Multi-market versus export-platform FDI:

$$\begin{aligned} \Delta_c (\Pi^{Fn} - \Pi^{F1}) &= (n - 1) \Delta_c [\gamma(t, c, f) - \phi(t, t_U, c)] \\ &= (n - 1) \Delta_c \gamma(t_U, c, f) \end{aligned}$$

- This too is negative if and only if profits are supermodular in t and c

Firm Selection

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- Multi-market versus export-platform FDI:

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- This too is negative if and only if profits are supermodular in t and c

Proposition

If and only if π is supermodular in t and c , then the least efficient firms that serve foreign markets will do so via exporting, the next most efficient via export-platform FDI, and the most efficient via multi-market FDI, for all admissible f .

- More effective worker screening: Helpman-Itskhoki-Redding (*Em* 2010)