

# Unraveling Firms: Demand, Productivity and Markups Heterogeneity

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- Econometric Models: Based on Olley and Pakes (1996) seminal contribution with a proxy variable approach to tackle the issue of omitted (to the econometrician) variables.
- Applied contributions: Wide ranging use of estimated firm TFP as a key variable: business cycles (Macro literature), firm size distribution, survival and growth (IO literature), self selection of firms into export status and intensive margin (Trade literature), etc.

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- It is common practice to use **revenue as a measure of output** meaning that (at best) one can measure a composite of productivity and the price: **revenue productivity**
- **Prices** depend upon PRODUCTIVITY, MARKET POWER, QUALITY, ETC. Therefore, revenue productivity measures **conflate all these elements**.



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- What this paper **is not about**: getting productivity “more right” than other frameworks.
- What this paper **is about**: getting a broader spectrum of analysis by means of several dimensions.

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  - ① Show how they are correlated among them as well as with revenue TFP measures.
  - ② Show how and to what extent they allow to say something about two key outcomes: firm response to rising imports from China and import status.

# Road Map

A short summary of the baseline econometric model (we can allow for several extensions)

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Application to Chinese import competition and firm import status



# The Model: Cost Minimization; Flexible and fixed factors

We have 3 production factors: labour ( $L$ ), intermediate inputs ( $M$ ) and capital ( $K$ ). Whereas intermediate inputs are perfectly flexible, labour is semi-flexible (chosen in between  $t-1$  and  $t$ ) while capital is predetermined in  $t$ .

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- Capital is difficult to adjust in the short-run and its level, chosen in  $t-1$  with respect to the long-run plans and performance of the firm, can be considered fixed in the wake of a short-term shock.
- Intermediate inputs are easy to adjust in the short-run and firms do so in the wake of short-term shocks in the “standard” way:

value of marginal product = marginal cost of interm. inputs.

- Labour is in between the two. It can adjust to short-term shocks but only imperfectly so:

value of marginal product  $\neq$  marginal cost of labour

# The Model: Cost Minimization; Flexible and fixed factors

Consequently, firms are dealing in  $t$  with the following short run cost minimization problem (we use here a Cobb-Douglas but could as well use a Translog):

$$\min_M \{M_{it} W_M\} \text{ s.t. } Q_{it} = A_{it} L_{it}^{\alpha_L} M_{it}^{\alpha_M} K_{it}^{\gamma - \alpha_M - \alpha_L}$$

where  $A_{it}$  is firm quantity TFP and we use lower case to indicate logs ( $\log A_{it} = a_{it}$ ).

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where  $A_{it}$  is firm quantity TFP and we use lower case to indicate logs ( $\log A_{it} = a_{it}$ ).

We assume, as standard, that productivity follows a Markov process (could also be non-linear):

$$a_{it} = \phi_a a_{it-1} + \nu_{ait},$$

where the autoregressive component  $\phi_a a_{it-1}$  captures the persistency of TFP (works well empirically) and  $\nu_{ait}$  is an idiosyncratic short-term shock.

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where:

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If we were to regress log quantity on log capital, log labour and log intermediate inputs to get  $a_{it}$  as a residual we would make a mistake. Indeed  $a_{it}$  is endogenous because it is known to the firm who makes inputs choices accordingly.

More specifically, we can expect capital to be correlated with  $\phi_a a_{it-1}$  but not with  $\nu_{ait}$  while labour and intermediate inputs would be correlated with both  $\phi_a a_{it-1}$  and  $\nu_{ait}$ .

# The Model: Preferences

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The best way to think about  $\Lambda_{it}$  is to consider a representative consumer solving the following problem (we also characterize  $\Lambda_{it}$  in the case of discrete/continuous choice models: Nocke and Schutz, 2016):

$$\max_{\tilde{Q}} \left\{ U(\tilde{Q}) \right\} \text{ s.t. } \int_i P_{it} Q_{it} di - B_t = 0$$

where  $\tilde{Q}$  is a vector of elements  $\Lambda_{it} Q_{it}$ . Therefore, while the representative consumer chooses quantities  $Q$ , these quantities enter into the utility function as  $\tilde{Q}$  and  $\Lambda_{it}$  can be interpreted as a measure of product appeal/quality of a particular variety.

# The Model: Preferences

Note that static profit maximization (monopolistic competition as well as oligopoly) implies:

$$\frac{\partial r_{it}}{\partial q_{it}} = \underbrace{\frac{\partial R_{it}}{\partial Q_{it}}}_{\text{marginal revenue}} \frac{Q_{it}}{R_{it}} = \underbrace{\frac{\partial C_{it}}{\partial Q_{it}}}_{\text{marginal cost}} \frac{Q_{it}}{P_{it} Q_{it}} = \frac{\frac{\partial C_{it}}{\partial Q_{it}}}{P_{it}} = \frac{1}{\mu_{it}},$$

where  $\mu_{it}$  is the profit maximizing markup.



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The way we introduce  $\lambda_{it}$  implies (**proof in the paper**)

$$\frac{\partial r_{it}}{\partial \lambda_{it}} = \frac{1}{\mu_{it}}$$

and so we obtain a first-order linear approximation of the revenue function:

$$r_{it} \simeq \frac{1}{\mu_{it}}(q_{it} + \lambda_{it})$$

# The Model: Preferences

We use the revenue equation  $r_{it} \simeq \frac{1}{\mu_{it}}(q_{it} + \lambda_{it})$  for two purposes:

- 1 Back out the value of product appeal:  $\lambda_{it} \simeq \mu_{it}r_{it} - q_{it}$ .
- 2 To improve parameters' identification by combining the quantity equation (production function) and the revenue equation.

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$r_{it} \simeq \frac{1}{\mu_{it}}(q_{it} + \lambda_{it})$  is an exact solution to the case of generalized CES preferences (Spence, 1976):

$$U(\tilde{Q}_t) = \int_{i \in I_t} (\tilde{Q}_{it})^b di = \int_{i \in I_t} \Lambda_{it}^b (Q_{it})^b di$$

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We also work out in the paper some other cases where we get the exact solution for the revenue function (Gaussian demand).

# The Model: Demand and Productivity shocks

- As already said we assume, as standard, that productivity follows a Markov process. We make the same assumption for demand shocks. In the case of a linear (**we can generalize to non-linear as well introduce correlated unobserved heterogeneity**) Markov process this means:

$$a_{it} = \phi_a a_{it-1} + \nu_{ait}$$

$$\lambda_{it} = \phi_\lambda \lambda_{it-1} + \nu_{\lambda it}$$

# The Model: Markups

- As for markups we do not need to make specific assumptions about the process they follow. Given firms minimize the cost of freely adjustable intermediate inputs (Hall, 1986; DLW, 2012):

$$\frac{\partial C_{it}}{\partial Q_{it}} = \frac{\partial C_{it}}{\partial M_{it}} \frac{\partial M_{it}}{\partial Q_{it}} = W_{Mt} \frac{\partial M_{it}}{\partial Q_{it}}.$$

Now define the markup as:

$$\mu_{it} \equiv \frac{P_{it}}{\frac{\partial C_{it}}{\partial Q_{it}}}.$$

We thus have:

$$\frac{P_{it}}{\mu_{it}} = W_{Mt} \frac{\partial M_{it}}{\partial Q_{it}}.$$

# The Model: Markups

- Multiplying by  $Q_{it}$  and dividing by  $M_{it}$  on both sides:

$$\frac{P_{it}Q_{it}}{M_{it}\mu_{it}} = \frac{R_{it}}{M_{it}\mu_{it}} = W_{Mt} \frac{\partial M_{it}}{\partial Q_{it}} \frac{Q_{it}}{M_{it}} = W_{Mt} \frac{\partial m_{it}}{\partial q_{it}}.$$

Re-arranging we finally have:

$$\mu_{it} = \frac{\frac{\partial q_{it}}{\partial m_{it}}}{\frac{W_{Mt}M_{it}}{R_{it}}} = \frac{\frac{\partial q_{it}}{\partial m_{it}}}{s_{Mit}} = \frac{\alpha_M}{s_{Mit}}.$$

where  $\alpha_M$  is the production function coefficient of intermediate inputs and  $s_{Mit}$  is the share of expenditure on intermediate inputs in revenue.

# The Model: Summing up

## Key equations

$$q_{it} = \alpha_L l_{it} + \alpha_M m_{it} + (\gamma - \alpha_L - \alpha_M) k_{it} + a_{it}. \quad (1)$$

$$\mu_{it} = \frac{\alpha_M}{s_{Mit}}, \quad (2)$$

$$r_{it} \simeq \frac{1}{\mu_{it}} (q_{it} + \lambda_{it}), \quad (3)$$

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- We can then use the intermediate inputs coefficients, as well as the revenue share of intermediate inputs, to recover  $\mu_{it}$  from (2)
- With markups, as well as log revenue and log quantity, we can finally back out  $\lambda_{it}$  from (3)

# The Model: How to estimate production function parameters

$$\begin{aligned}a_{it} &= \phi_a a_{it-1} + \nu_{ait} \\ \lambda_{it} &= \phi_\lambda \lambda_{it-1} + \nu_{\lambda it}\end{aligned}\tag{4}$$

There are two complementary ways (**we use both**). Both rely on the assumptions that capital is predetermined in  $t$  while both  $\nu_{ait}$  and  $\nu_{\lambda it}$  are unanticipated shocks in  $t-1$  so that, for example,  $E\{\nu_{ait}k_{it}\} = E\{\nu_{ait}l_{it-1}\} = E\{\nu_{ait}m_{it-1}\} = E\{\nu_{ait}k_{it-1}\} = 0$ .

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- Use both the revenue and quantity equations and substitute equilibrium condition to get a system of two equations to be estimated (linear in key parameters) [▶ details](#)
- Focus on the quantity equation and, based on the existence and invertibility of the cond. inputs demand funct.  $m(k_{it}, l_{it}, a_{it}, \lambda_{it}, \mu_{it})$ , use  $p_{it}$  and  $r_{it}$  as proxies for  $\mu_{it}$  and  $\lambda_{it}$  (De Loecker et al, 2016)

## Data: Production

We use firm-level production data for Belgian manufacturing firms.

**Prodcom** is a monthly survey of industrial production. Eurostat established the survey in order to improve the comparability of production statistics across the EU by the use of a common product nomenclature called Prodcom (8-digit codes based on NACE 4-digits).

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Examples of 8-digit products (roughly 5,000 codes):

- 1 “Fresh or chilled cuts of geese; ducks and guinea fowls” (Prodcom code 15121157)
- 2 “Envelopes of paper or paperboard” (Prodcom code 21231230)
- 3 “Band saws for working wood, cork, bone and hard rubber, hard plastics or similar hard materials” (Prodcom code 29404233)

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Around **7,000 firms a year** over the period 1995-2009. Data is organised by product-year-month-firm. We borrow information on quantity (unit of measurement depends on product) and value (euros) of production sold.

We aggregate the data at the firm-year-product level.

# Data: Balance sheet and Trade

- Annual accounts from National Bank of Belgium. For this study, we selected those companies that filed a full-format or abbreviated balance sheet between 1996 and 2007 and with at least one full-time equivalent employee.
  - ▶ The resulting dataset has been shown to be [representative of the Belgian economy](#).
  - ▶ We take information on FTE employment, material costs, capital stock and turnover. More than **15,000 firms per year in manufacturing** with complete information.

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  - ▶ We take information on FTE employment, material costs, capital stock and turnover. More than **15,000 firms per year in manufacturing** with complete information.
- Standard EU-type micro trade data at the product-country-firm-month level over the period 1995-2008 with different rules for EU and non-EU trade.
  - ▶ We borrow information on firm import status.

# Estimates of production function parameters

Industry	Description	Labour	Materials	Capital	$\gamma$
1	Food products, beverages and tobacco	0.397 <sup>a</sup> (0.029)	0.728 <sup>a</sup> (0.040)	0.045 <sup>a</sup> (0.014)	1.169 <sup>a</sup> (0.061)
2	Textiles and leather	0.325 <sup>a</sup> (0.020)	0.636 <sup>a</sup> (0.019)	0.020 <sup>c</sup> (0.012)	0.981 <sup>a</sup> (0.014)
3	Wood except furniture	0.340 <sup>a</sup> (0.050)	0.632 <sup>a</sup> (0.049)	0.026 (0.021)	0.998 <sup>a</sup> (0.058)
4	Pulp, paper, publishing and printing	0.427 <sup>a</sup> (0.065)	0.629 <sup>a</sup> (0.092)	-0.070 <sup>a</sup> (0.017)	0.986 <sup>a</sup> (0.141)
5	Chemicals and rubber	0.328 <sup>a</sup> (0.040)	0.648 <sup>a</sup> (0.052)	0.034 <sup>c</sup> (0.019)	1.010 <sup>a</sup> (0.071)
6	Other non-metallic mineral products	0.316 <sup>a</sup> (0.039)	0.622 <sup>a</sup> (0.051)	0.047 <sup>a</sup> (0.015)	0.985 <sup>a</sup> (0.078)
7	Basic metals and fabric. metal prod.	0.338 <sup>a</sup> (0.015)	0.629 <sup>a</sup> (0.012)	0.024 <sup>a</sup> (0.008)	0.991 <sup>a</sup> (0.005)
8	Machinery, electric. and optical equip.	0.347 <sup>a</sup> (0.033)	0.630 <sup>a</sup> (0.023)	0.026 <sup>b</sup> (0.011)	1.004 <sup>a</sup> (0.008)
9	Transport equipment and n.e.c.	0.313 <sup>a</sup> (0.032)	0.636 <sup>a</sup> (0.031)	0.025 (0.016)	0.974 <sup>a</sup> (0.039)

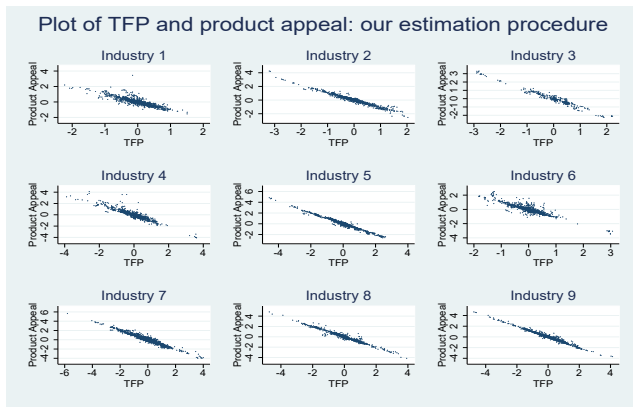
Notes:  $\gamma$  denotes returns to scale. Bootstrapped standard errors in parenthesis (200 replications). <sup>a</sup>  $p < 0.01$ , <sup>b</sup>  $p < 0.05$ , <sup>c</sup>  $p < 0.1$ .

# Standard deviation of TFP, product appeal and markups by industry

Industry	Description	TFP	product appeal	markups
1	Food products, beverages and tobacco	0.416	0.477	0.154
2	Textiles and leather	0.604	0.671	0.130
3	Wood except furniture	0.843	0.914	0.180
4	Pulp, paper, publishing and printing	0.775	0.843	0.152
5	Chemicals and rubber	0.952	0.970	0.079
6	Other non-metallic mineral products	0.520	0.607	0.123
7	Basic metals and fabric. metal prod.	0.860	0.896	0.169
8	Machinery, electric. and optical equip.	0.917	0.925	0.139
9	Transport equipment and n.e.c.	1.021	1.020	0.151

Demand heterogeneity at least as sizeable as TFP heterogeneity. Less variation in markups

# Within 8-digit products correlation between TFP and product appeal by industry



Productivity shocks  $a$  are very strongly and **negatively correlated** with demand shocks  $\lambda$ : **Nissan vs. Mercedes**

## Which Plant is better?



Mercedes plant Rastatt  
Cars/Employee in 2000: 53



Nissan plant Sunderland  
Cars/Employee in 2000: 100



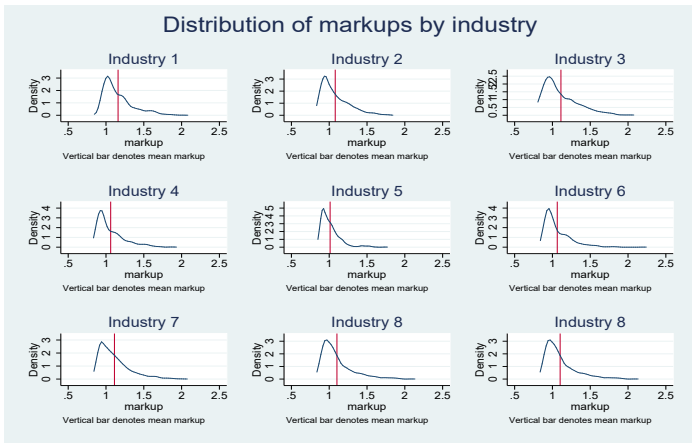
<http://www.prnewswire.co.uk/news-releases/nissans-sunderland-car-plant-sets-new-european-productivity-standards-154794285.html>

Both plants are profitable and perhaps generate a **very similar revenue productivity**.

Yet, their business model is quite different: they **are differentiated in the quality-cost space**

# Distribution of Markups

Figure: Distribution of markups by industry





# Reality check regressions of markups and prices

Estimation method	OLS		First differences	
	Markups	Prices	Markups	Prices
TFP	0.3424 <sup>a</sup> (0.0193)	-0.9093 <sup>a</sup> (0.0080)	0.3724 <sup>a</sup> (0.0246)	-0.8866 <sup>a</sup> (0.009)
$\lambda$	0.3570 <sup>a</sup> (0.0185)	0.0735 <sup>a</sup> (0.0076)	0.3707 <sup>a</sup> (0.0246)	0.1048 <sup>a</sup> (0.0089)
capital	-0.0252 <sup>a</sup> (0.0030)	-0.0101 <sup>a</sup> (0.0012)	0.0037 (0.0029)	-0.0046 <sup>c</sup> (0.0019)
Year dummies	Yes	Yes	Yes	Yes
Prod dummies	Yes	No	Yes	No
N Obs	11,100	11,100	7,768	7,768
$R^2$	0.6338	0.9878	0.3971	0.9925

Markups  $\mu$  are **reasonably correlated** with demand and productivity shocks. Yet,  $R^2$  is not one so this is an **important additional dimension** of heterogeneity.

# Correlations across time

Industry	1	2	3	4	5	6	7	8	9
TFP									
lag TFP	0.9743 <sup>a</sup> (0.0155)	0.9718 <sup>a</sup> (0.0126)	0.9865 <sup>a</sup> (0.0157)	0.9577 <sup>a</sup> (0.0231)	0.8715 <sup>a</sup> (0.0264)	0.9711 <sup>a</sup> (0.0114)	0.8665 <sup>a</sup> (0.0245)	0.7482 <sup>a</sup> (0.0579)	0.8332 <sup>a</sup> (0.0355)
N Obs	901	843	232	710	702	867	2,000	785	738
R <sup>2</sup>	0.8742	0.8785	0.962	0.8986	0.7867	0.9371	0.7035	0.5986	0.73
$\lambda$									
lag $\lambda$	0.9654 <sup>a</sup> (0.0136)	0.9688 <sup>a</sup> (0.0136)	0.9886 <sup>a</sup> (0.012)	0.9532 <sup>a</sup> (0.0323)	0.8737 <sup>a</sup> (0.0265)	0.9503 <sup>a</sup> (0.0153)	0.8769 <sup>a</sup> (0.0247)	0.7465 <sup>a</sup> (0.059)	0.8292 <sup>a</sup> (0.0413)
N Obs	901	843	232	710	702	867	2,000	785	738
R <sup>2</sup>	0.8763	0.8825	0.9688	0.871	0.7813	0.8942	0.7216	0.6001	0.7239
markup									
lag markup	0.9476 <sup>a</sup> (0.0139)	0.9464 <sup>a</sup> (0.0162)	0.9242 <sup>a</sup> (0.0372)	0.9504 <sup>a</sup> (0.0178)	0.9327 <sup>a</sup> (0.0261)	0.9344 <sup>a</sup> (0.0193)	0.9006 <sup>a</sup> (0.0143)	0.8717 <sup>a</sup> (0.02)	0.9742 <sup>a</sup> (0.0336)
N Obs	901	843	232	710	702	867	2,000	785	738
R <sup>2</sup>	0.9112	0.8724	0.8637	0.8965	0.8762	0.8878	0.8018	0.7837	0.8133

# Decomposing revenue TFP

Revenue TFP could be defined in our framework as  $TFP_{it}^R \equiv r_{it} - \bar{q}_{it}$   
where  $\bar{q}_{it} = \alpha_L (l_{it} - k_{it}) + \alpha_M (m_{it} - k_{it}) + \gamma k_{it}$ .

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By substituting and simplifying we get:

$$TFP_{it}^R = \frac{1}{\mu_{it}} (a_{it} + \lambda_{it}) + \frac{1 - \mu_{it}}{\mu_{it}} \bar{q}_{it}$$

So revenue productivity is a non-linear function of  $a$ ,  $\lambda$ ,  $\mu$  and production scale.

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So revenue productivity is a non-linear function of  $a$ ,  $\lambda$ ,  $\mu$  and production scale.

It can also be made linear by considering So markups-adjusted quantity TFP, product appeal and scale:  $\tilde{a}_{it} = \frac{a_{it}}{\mu_{it}}$ ,  $\tilde{\lambda}_{it} = \frac{\lambda_{it}}{\mu_{it}}$ ,  $\tilde{q}_{it} = \frac{(1 - \mu_{it})\bar{q}_{it}}{\mu_{it}}$ .

$$TFP_{it}^R = \tilde{a}_{it} + \tilde{\lambda}_{it} + \tilde{q}_{it}$$

# Regression of OLS revenue TFP and De Loecker and Warzynski revenue TFP on $\tilde{a}$ , $\tilde{\lambda}$ and $\tilde{q}$ by industry

Industry	1	2	3	4	5	6	7	8	9
OLS revenue TFP									
$\tilde{a}$	0.1529 <sup>a</sup> (0.0134)	0.6517 <sup>a</sup> (0.0164)	0.7786 <sup>a</sup> (0.0222)	0.3242 <sup>a</sup> (0.0218)	0.2783 <sup>a</sup> (0.0135)	0.6023 <sup>a</sup> (0.0128)	0.8696 <sup>a</sup> (0.0057)	0.8806 <sup>a</sup> (0.0178)	0.7055 <sup>a</sup> (0.0158)
$\tilde{\lambda}$	0.1666 <sup>a</sup> (0.0136)	0.6639 <sup>a</sup> (0.0166)	0.7869 <sup>a</sup> (0.0223)	0.3428 <sup>a</sup> (0.0222)	0.2950 <sup>a</sup> (0.0139)	0.6201 <sup>a</sup> (0.0132)	0.8776 <sup>a</sup> (0.0057)	0.8878 <sup>a</sup> (0.0179)	0.7223 <sup>a</sup> (0.0163)
$\tilde{q}$	0.0761 <sup>a</sup> (0.0125)	0.7170 <sup>a</sup> (0.0148)	0.8476 <sup>a</sup> (0.0223)	0.3166 <sup>a</sup> (0.0248)	0.3308 <sup>a</sup> (0.0191)	0.6055 <sup>a</sup> (0.0159)	0.8894 <sup>a</sup> (0.0050)	0.8814 <sup>a</sup> (0.0167)	0.7586 <sup>a</sup> (0.0187)
N Obs	1,317	1,225	348	975	1,055	1,215	2,814	1,108	1,055
R <sup>2</sup>	0.7611	0.9567	0.9688	0.8122	0.9233	0.9515	0.9766	0.9743	0.8955
De Loecker and Warzynski revenue TFP									
$\tilde{a}$	0.0246 (0.0201)	0.6946 <sup>a</sup> (0.0278)	0.0128 (0.3747)	0.4004 <sup>a</sup> (0.0263)	0.2730 <sup>a</sup> (0.0137)	0.6934 <sup>a</sup> (0.0122)	0.9099 <sup>a</sup> (0.0053)	0.8530 <sup>a</sup> (0.0448)	0.7230 <sup>a</sup> (0.0141)
$\tilde{\lambda}$	0.0269 (0.0204)	0.6945 <sup>a</sup> (0.0282)	-0.0931 (0.3730)	0.4084 <sup>a</sup> (0.0268)	0.2926 <sup>a</sup> (0.0142)	0.7048 <sup>a</sup> (0.0124)	0.9148 <sup>a</sup> (0.0054)	0.8522 <sup>a</sup> (0.0448)	0.7388 <sup>a</sup> (0.0146)
$\tilde{q}$	-0.0969 <sup>a</sup> (0.0165)	0.7490 <sup>a</sup> (0.0280)	0.7327 <sup>b</sup> (0.3474)	0.3928 <sup>a</sup> (0.0377)	0.2359 <sup>a</sup> (0.0197)	0.6652 <sup>a</sup> (0.0157)	0.9236 <sup>a</sup> (0.0048)	0.8604 <sup>a</sup> (0.0422)	0.7704 <sup>a</sup> (0.0176)
N Obs	1,317	1,225	348	975	1,055	1,215	2,814	1,108	1,055
R <sup>2</sup>	0.7071	0.9168	0.7932	0.6465	0.9201	0.9091	0.9836	0.9222	0.9052

# Import competition from China: what we are used to

Numerous studies have explored the many, besides the well-documented negative effects on employment (David et al., 2013), impacts of the spectacular rise of Chinese trade.

Bloom et al. (2016) provide evidence supporting the claim that import competition from China caused an increase in technical change, as well as an increase in revenue TFP, for European firms selling products most affected by rising imports from China.

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**Our framework** allows to go **further**. Consider decomposing changes in revenue TFP in terms of quantity TFP, demand and scale.



# Import competition from China: what we ARE NOT are used to

Table: Quota analysis on the “Textile and Apparel” industry

Outcome measure	Labour	Rev.TFP	$\tilde{a}$	$\tilde{\lambda}$	$\tilde{q}$
$Quota_{CPA6}$	-0.0360 <sup>c</sup> (0.0207)	0.0074 <sup>b</sup> (0.0033)	0.1095 <sup>c</sup> (0.0593)	-0.1163 <sup>b</sup> (0.0571)	0.0143 <sup>b</sup> (0.0064)
Observations	700	700	700	700	700
R-squared	0.0052	0.0055	0.0033	0.0039	0.0079

Table: Chinese import penetration analysis

Outcome measure	Labour	Rev.TFP	$a$	$\lambda$	$\bar{q}$	$\mu$
$IPC_{CPA6,t}^{EU15}$	-0.7393 <sup>a</sup> (0.2412)	0.1847 <sup>a</sup> (0.0423)	1.3205 <sup>a</sup> (0.4028)	-1.0728 <sup>a</sup> (0.4140)	-0.8483 <sup>a</sup> (0.2574)	0.0493 (0.0650)
Observations	10,161	10,161	10,161	10,161	10,161	10,161
R-squared	0.1741	0.0292	0.0174	0.0111	0.2141	0.0369
Firm FE and year dummies	Yes	Yes	Yes	Yes	Yes	Yes
Kleibergen-Paap rk LM statistic under-id	146.71	146.71	146.71	146.71	146.71	146.71
P-value	0	0	0	0	0	0
Kleibergen-Paap rk Wald F statistic weak id.	226.34	226.34	226.34	226.34	226.34	226.34

## Import Status: what we are used to

It is a stylized fact that **importers are**, on average, **more productive** than non-importers. Evidence comes from many datasets and countries and is based on standard revenue-based measures of productivity.

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**Our data is no exception** to such stylized fact. Consider for example import status of a firm regressed on its DLW revenue-based TFP

# Import Status: what we are used to

**Table:** The revenue productivity advantage of importers by industry: De Loecker and Warzynski (DLW) revenue TFP

Industry	1	2	3	4	5	6	7	8	9	All
DLW TFPR	2.3207 <sup>a</sup> (0.1541)	1.1475 <sup>a</sup> (0.2095)	0.5283 <sup>a</sup> (0.0381)	0.9602 <sup>a</sup> (0.1632)	0.1967 (0.2972)	2.3030 <sup>a</sup> (0.3459)	0.6613 <sup>a</sup> (0.1394)	1.2348 <sup>a</sup> (0.2419)	0.4739 <sup>b</sup> (0.2244)	0.7752 <sup>a</sup> (0.0450)
Observ.	1,317	1,225	348	975	1,055	1,215	2,814	1,108	1,055	11,112
R-squared	0.4685	0.3137	0.5749	0.6471	0.3465	0.2996	0.3212	0.4636	0.3015	0.4245

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Our framework allows to go further. Consider now import status of a firm regressed on  $a$ ,  $\lambda$  and  $\bar{q}$

# Import Status: what we ARE NOT are used to

Ind	1	2	3	4	5	6	7	8	9	All
$a$	0.0870 <sup>b</sup> (0.0413)	0.0914 (0.0781)	-0.0601 (0.1645)	0.0324 (0.0379)	0.6182 <sup>a</sup> (0.1057)	0.2420 <sup>a</sup> (0.0384)	0.0094 (0.0335)	0.1802 <sup>a</sup> (0.0581)	0.3554 <sup>a</sup> (0.0654)	0.1069 <sup>a</sup> (0.0165)
$\lambda$	-0.0429 (0.0354)	0.0876 (0.0706)	-0.0652 (0.1564)	0.0581 <sup>c</sup> (0.0315)	0.6141 <sup>a</sup> (0.1029)	0.1554 <sup>a</sup> (0.0275)	-0.0063 (0.0321)	0.1493 <sup>b</sup> (0.0580)	0.3309 <sup>a</sup> (0.0643)	0.0877 <sup>a</sup> (0.0154)
$\bar{q}$	0.2421 <sup>a</sup> (0.0083)	0.1779 <sup>a</sup> (0.0161)	0.2286 <sup>a</sup> (0.0247)	0.1067 <sup>a</sup> (0.0104)	0.1870 <sup>a</sup> (0.0202)	0.2578 <sup>a</sup> (0.0104)	0.2970 <sup>a</sup> (0.0081)	0.2468 <sup>a</sup> (0.0185)	0.2616 <sup>a</sup> (0.0114)	0.2238 <sup>a</sup> (0.0040)
Obs	1,317	1,225	348	975	1,055	1,215	2,814	1,108	1,055	11,112
$R^2$	0.6090	0.3886	0.5895	0.6728	0.4298	0.4400	0.5042	0.5514	0.5324	0.5419

## Import Status: what we ARE NOT are used to

Ind	1	2	3	4	5	6	7	8	9	All
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$R^2$	0.6090	0.3886	0.5895	0.6728	0.4298	0.4400	0.5042	0.5514	0.5324	0.5419

This shows importing firms are **large firms** that are **more productive** and/or sell **more appealing** products.

Quantity TFP and product appeal seems to be **equally important** in drawing the line between importing and non-importing firms

# Conclusions

- We provide a framework allowing to simultaneously retrieve heterogeneity in productivity, markups and demand across firms while leaving the correlation among the three unrestricted.
- We use production data on Belgian firms to quantify our model.
- We are able to unravel standard measures of revenue productivity into different components. This is important at different levels:
  - ① **At the micro level:** it makes a huge difference to know that some firms or industries lack in competitiveness because of poor physical TFP (due for example to low expenditure in process R&D) or poor product quality (due for example to low expenditure in product R&D).
  - ② **At the macro level:** It allows looking at aggregate revenue productivity cycles, like for example the severe downturn of aggregate revenue productivity since the financial crisis, not only in terms of changes in some underlying production capacity of the economy but also as changes in markups and/or demand.



# How Important is Demand Heterogeneity? Plot of Log Price and Log Quantity



▶ back

# The Model: Estimation

Being sufficiently explicit about demand/market structure allows us to use **both** the **revenue** and **quantity** equations to estimate parameters. The proxy variable approach only exploits the quantity equation while building on invertibility.

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The **revenue equation**  $r_{it} \approx \frac{1}{\mu_{it}} (q_{it} + \lambda_{it})$  can be rewritten as:

$$LHS_{it} = \frac{\gamma}{\alpha_M} k_{it} + \frac{\alpha_L}{\alpha_M} (l_{it} - k_{it}) + \phi_a LHS_{it-1} - \phi_a \frac{\gamma}{\alpha_M} k_{it-1}$$

$$- \phi_a \frac{\alpha_L}{\alpha_M} (l_{it-1} - k_{it-1}) + (\phi_\lambda - \phi_a) \left( \frac{r_{it-1}}{s_{Mit-1}} - \frac{q_{it-1}}{\alpha_M} \right) + \frac{1}{\alpha_M} (\nu_{ait} + \nu_{\lambda it})$$

where  $LHS_{it} \equiv \frac{r_{it} - s_{Mit}(m_{it} - k_{it})}{s_{Mit}}$  is a function of observables.

# The Model: Estimation

We can rewrite this equation in a linear form as:

$$LHS_{it} = b_1 z_{1it} + b_2 z_{2it} + b_3 z_{3it} + b_4 z_{4it} + b_5 z_{5it} + b_6 z_{6it} + b_7 z_{7it} + u_{it},$$

where  $z_{1it} = k_{it}$ ,  $z_{2it} = (l_{it} - k_{it})$ ,  $z_{3it} = LHS_{it-1}$ ,  $z_{4it} = k_{it-1}$ ,  
 $z_{5it} = (l_{it-1} - k_{it-1})$ ,  $z_{6it} = \frac{r_{it-1}}{s_{Mit-1}}$ ,  $z_{7it} = q_{it-1}$ ,  $u_{it} = \frac{1}{\alpha_M} (\nu_{ait} + \nu_{\lambda it})$

as well as  $b_1 = \frac{\gamma}{\alpha_M}$ ,  $b_2 = \frac{\alpha_L}{\alpha_M}$ ,  $b_3 = \phi_a$ ,  $b_4 = -\phi_a \frac{\gamma}{\alpha_M}$ ,  $b_5 = -\phi_a \frac{\alpha_L}{\alpha_M}$ ,  
 $b_6 = (\phi_\lambda - \phi_a)$ ,  $b_7 = -(\phi_\lambda - \phi_a) \frac{1}{\alpha_M}$ .

# The Model: Estimation

Given our assumptions, the error term  $u_{it}$  is uncorrelated with current capital as well as with inputs use, quantity and revenue in  $t - 1$ .

Therefore,  $z_{1it}$  as well as  $z_{3it}$  to  $z_{7it}$  are uncorrelated to  $u_{it}$ .

As for  $z_{2it}$  we can use the lagged value  $(l_{it-2} - k_{it-2})$  as an instrument.

The equation can thus be estimated via linear IV and after doing this we set  $\frac{\widehat{\gamma}}{\widehat{\alpha}_M} = \widehat{b}_1$ ,  $\frac{\widehat{\alpha}_L}{\widehat{\alpha}_M} = \widehat{b}_2$  and  $\widehat{\phi}_a = \widehat{b}_3$  and do not exploit parameters' constraints in the estimation.

# The Model: Estimation

We then use these 3 parameters in a 2nd stage equation. The quantity equation can be manipulated to get:

$$q_{it} = \frac{\gamma}{\hat{b}_1} (m_{it} - k_{it}) + \frac{\gamma \hat{b}_2}{\hat{b}_1} (l_{it} - k_{it}) + \gamma k_{it} + \gamma \frac{\hat{\phi}_a}{\hat{b}_1} LHS_{it-1} \\ - \frac{\gamma \hat{b}_2 \hat{\phi}_a}{\hat{b}_1} (l_{it-1} - k_{it-1}) - \gamma \hat{\phi}_a k_{it-1} - \hat{\phi}_a \left( r_{it-1} \frac{\gamma}{\hat{b}_1 s_{Mit-1}} - q_{it-1} \right) + \nu_{ait}$$

Note that the only unobservable here is the idiosyncratic productivity shock  $\nu_{ait}$  while the only parameter left to identify is  $\gamma$ .

# The Model: Estimation

We can write the above as the following linear regression:

$$\overline{LHS}_{it} = b_8 z_{8it} + \nu_{ait}$$

$$\begin{aligned}\overline{LHS}_{it} &= q_{it} - \hat{\phi}_a q_{it-1} \\ z_{8it} &= \frac{1}{\hat{b}_1} (m_{it} - k_{it}) + \frac{\hat{b}_2}{\hat{b}_1} (l_{it} - k_{it}) + k_{it} + \frac{\hat{\phi}_a}{\hat{b}_1} LHS_{it-1} \\ &\quad - \frac{\hat{b}_2 \hat{\phi}_a}{\hat{b}_1} (l_{it-1} - k_{it-1}) - \hat{\phi}_a k_{it-1} - \frac{\hat{\phi}_a r_{it-1}}{\hat{b}_1 S_{Mit-1}}\end{aligned}$$

as well as  $b_8 = \gamma$ .

Concerning  $z_{8it}$  we can use several moment conditions for identification:

$$\begin{aligned}E\{\nu_{ait} k_{it}\} &= E\{\nu_{ait} l_{it-1}\} = E\{\nu_{ait} m_{it-1}\} = E\{\nu_{ait} k_{it-1}\} = \\ E\{\nu_{ait} q_{it-1}\} &= E\{\nu_{ait} r_{it-1}\} = 0.\end{aligned}$$

## The Model: Estimation

The above procedure allows recovering the coefficients of the production function. With these we can then compute markups (as seen above) from:

$$\mu_{it} = \frac{\alpha_M}{s_{Mit}}$$

and finally product appeal (as seen above):

$$r_{it} \simeq \frac{1}{\mu_{it}} (q_{it} + \lambda_{it})$$



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The above procedure allows recovering the coefficients of the production function. With these we can then compute markups (as seen above) from:

$$\mu_{it} = \frac{\alpha_M}{s_{Mit}}$$

and finally product appeal (as seen above):

$$r_{it} \simeq \frac{1}{\mu_{it}} (q_{it} + \lambda_{it})$$

Alternatively one can compute production function coefficients using the procedure developed in De Loecker et al. (2016) (requires existence and invertibility of a suitable conditional input demand for intermediate inputs) and use the two above equations to get markups and product appeal. We have tried this and found extremely similar results.