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**Factor Price Frontiers with International  
Fragmentation of Multistage Production**

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# **Factor price frontiers with international fragmentation of multistage production**

**by**

**Wilhelm Kohler**

## **Abstract**

I develop a generalized factor price frontier which incorporates endogenous adjustment of international fragmentation in multistage production, allowing for a continuum of stages. This allows us to address fragmentation, not only as an exogenous event, but also as an integral part of endogenous adjustment to a variety of changes not directly related to fragmentation. A two-dimensional general equilibrium analysis explores how the margin of fragmentation, as well as factor prices, respond to a change in the final output price, and to an improvement in the “technology of fragmentation”. A key distinction arises between the “average” and “marginal” labour intensity, respectively, of domestic production in the multistage industry. The paper identifies conditions under which outsourcing to a low-wage country is a “friend” or an “enemy” to domestic labour, as well as conditions under which the Jonesian magnification effects underlying the Stolper-Samuelson theorem are reinforced, or mitigated, by endogenous changes in the margin of fragmentation. Protection may result in a broader or a narrower range of stages produced domestically.

JEL classification: D33, F11, F15, F23

Keywords: factor price frontiers, multistage production, fragmentation

## **Outline**

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- 3. A generalized factor price frontier*
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- 5. Protection under endogenous fragmentation*
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## **Non-Technical Summary**

International fragmentation means that firms do not carry out entire production processes in the home country, but locate certain stages in foreign countries which offer lower cost conditions. Such outsourcing of value-added is often perceived as a threat to domestic employment and wages.

Outsourcing has two direct effects. First, it implies cost-savings to domestic firms. *Ceteris paribus*, this should allow them to pay higher factor rewards, and under conditions of perfect competition the cost-savings will indeed be passed on to some factor(s). Secondly, the industry's domestic factor demand is altered. If a very labour-intensive stage is moved abroad, domestic production will become more capital-intensive. It is unclear, *a priori*, whether the outcome of these two effects is higher wages or higher returns to other factors, possibly even at the expense of labour. It depends on how "outsourcing-industries" are related to other sectors of the domestic economy. The wage-effect of international fragmentation can only be determined by a full general equilibrium analysis.

Economists often use factor price frontiers to explore how factor prices are determined in general equilibrium. Such frontiers depict alternative combinations of factor prices that an economy can "afford", given its technology and output prices. They are usually constructed assuming that there is no international fragmentation of production, or that the margin of fragmentation remains constant. However, if outsourcing is determined by foreign cost-advantages for certain stages or production, then alternative domestic factor price combinations must imply different degrees of outsourcing. Hence, the notion of a factor price frontier for a given margin of fragmentation is questionable.

This paper develops a generalized factor price frontier which incorporates an endogenous adjustment of fragmentation when moving from one point on the frontier to another. It assumes two factors (capital and labour) and a foreign economy where labour is relatively cheap. Moreover, it assumes that foreign factor prices as well as final output prices, determined on world markets, are unaffected by domestic outsourcing as such.

Using the generalized frontier, it is shown that outsourcing may be a "friend" or "enemy" to domestic labour, depending on how the aggregate and marginal capital intensity, respectively, of the "outsourcing-industry" compare to the rest of the economy. Aggregate

intensity refers to the overall domestic part of production. Marginal intensity refers to the single stage where firms are indifferent between domestic and foreign production. The same comparison also determines whether a multistage industry responds to a rise in the final goods price by broadening the range of stages produced domestically, or by specializing on an ever narrower range of stages. Endogenous fragmentation generally makes capital and labour closer substitutes. The real income redistribution attendant upon a change in the final goods price (Stolper-Samuelson theorem) is aggravated by endogenous fragmentation if that price increases, while it is mitigated for a price cut. The generalized frontier is a convenient tool to describe the technology of outsourcing. It shows that the factor price effect of “easier” fragmentation similarly depends on the above mentioned capital intensity comparison.

# 1 Introduction

The 1990s have witnessed a new form of economic globalization, often called fragmentation or outsourcing, whereby firms no longer carry out all stages of production in their home country, but locate some stages in foreign countries where economic conditions are more advantageous. The phenomenon has drawn considerable attention in connection with the widespread concern about wage inequality. There is now a sizable body of literature documenting the empirical importance of international fragmentation, which has, in turn, spurred considerable theoretical research on its causes and effects, particularly its effects on factor prices and income distribution.<sup>1</sup>

Despite significant progress, the current state of theoretical analysis of international fragmentation has certain shortcomings. It is prone to a casuistic approach, leading to a variety of different results that sometimes seem contradictory and difficult to reconcile. In Kohler (2003), I have made an attempt to identify general principles behind seemingly contradictory results. A further shortcoming is that the analysis often lacks an explicit modeling of the multistage nature of production which, by necessity, underlies international fragmentation. In this paper, I propose a modeling framework, where the multistage nature of industrial production is made explicit, and where the engineering sequence of stages is juxtaposed with economic incentives for international fragmentation. The model draws on Dixit & Grossman (1982) in assuming that there is a continuum of fragments which makes the margin of outsourcing (or fragmentation) a continuous variable. This should, in turn, facilitate an easier use of tools commonly relied upon in general equilibrium trade theory. More specifically, when economists explore determinants of factor prices in general equilibrium they often use factor price frontiers. While there are treatments of outsourcing assuming a continuum of stages [e.g., Feenstra & Hanson (1996, 1997)], these do not develop the underlying price frontier.

The factor price frontier is a representation of alternative factor price combinations that are supported, in a competitive equilibrium, by an economy's technology and by given prices for final outputs. In the empirical literature, such frontiers have been used, in more or less explicit ways, to justify so-called "mandated wage regressions" [see for instance Leamer (1998), and Feenstra & Hanson (1999)]. In theoretical models of fragmentation, however, they have

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<sup>1</sup>Throughout this paper, outsourcing and fragmentation are used synonymously, and we always refer to *international* outsourcing. For empirical literature see, for instance, Irwin (1996), Feenstra (1998), Hummels, Rapoport, Ishii and Yi (1998), Hummels, Ishii & Yi (2001), and several papers in Arndt & Kierzkowski (2001). On the driving force of technological advances in communication and transport, see Jones & Kierzkowski (1990), Harris (1995 and 2001), and Jones & Kierzkowski (2001a). For theoretical treatments, see Feenstra & Hanson (1996 and 1997), Arndt (1997 and 1999), Venables (1999), Jones (2000), Deardorff (2001a and 2001b), Jones & Kierzkowski (2001a and 2001b), and Helpman & Grossman (2002).

so far not been used extensively. There is a general argument to the effect that international outsourcing acts like a *technological change* which effectively shifts the domestic economy's factor price frontier [Feenstra & Hanson (1999)]. This essentially assumes that outsourcing is an *exogenous* event. But under very general conditions outsourcing is importantly influenced by domestic factor prices. Hence, it should be seen as an *endogenous* phenomenon which is driven by factor price changes traced out by the factor price frontier. In terms of the wage regressions in the empirical literature, this implies that including an outsourcing regressor-variable poses a simultaneity problem. If we have a model of how domestic factor prices changes affect outsourcing, and how outsourcing in turn affects the competitive equilibrium, then it should be possible to develop what may be called an "*endogenous fragmentation factor price frontier*", i.e., a *generalized* factor price frontier which represents alternative factor price combinations supported by the specific conditions or outsourcing, in addition to the fundamental technological knowledge and final output prices.

Such a generalized factor price frontier opens up a more general view on international fragmentation than is usually taken in the literature. First, it is a very convenient *dual* representation of what may be called the "technology" of fragmentation, which allows us to address a variety of factors that may be responsible for the increasing empirical importance of international outsourcing, and which should serve as a useful basis for empirical modeling. As an example, I shall demonstrate how the generalized factor price frontier may be used to explore specific improvements in the "technology" of international fragmentation. And secondly, such a frontier allows us to consider outsourcing as an integral (endogenous) part of adjustment to certain shocks not directly related to outsourcing, but where adjustment under endogenous fragmentation may be different in important respects from adjustment under exogenous fragmentation, or adjustment without fragmentation. As an example, I shall consider the protective effect of an increase in the final output price of a multistage industry, whether policy induced or otherwise.

The paper is structured as follows. In the next section, I first develop a dual representation of technology for a single sector featuring multistage production under fragmentation. In doing so, I rely on the notion of a continuum of stages, as in Dixit & Grossman (1982), assuming two factors (capital and labour) with given prices abroad. Section 3 derives a generalized factor price frontier which incorporates an endogenous adjustment of the margin of international fragmentation to domestic factor price changes, and it explores the properties of this frontier. Section 4 moves to a general equilibrium perspective by adding a single-stage sector and a domestic endowment constraint. Section 5 presents a comparative static analysis of protection for the multistage industry in the form of a higher final output price. Does the sector respond by an increased level of outsourcing, and if so, does this benefit or harm domestic labour? Does an endogenous adjustment of international fragmentation to changes in the final output price entail a reinforcement, or a mitigation, of the magnification

effects underlying the Stolper-Samuelson theorem? Does higher protection imply that a broader or narrower range of stages is produced domestically? Section 6 shows how the dual representation of multistage production with outsourcing may be used to explore possible improvements in the “technology” of fragmentation. I address a very simple scenario of such an improvement, focusing on the effects on real factor rewards. Section 7 concludes the paper by way of a brief summary.

## 2 Multistage production and fragmentation

Industrial processes are often characterized by a well-defined engineering sequence of stages, whereby at stage  $i$  primary factors combine in a specific way in order to add to the value generated by stages up to  $i$ , until at some final stage the final good becomes available. We assume that sector 1 is featured by such multistage production, while sector 2 involves a single stage according to a linearly homogeneous technology, using capital and labour which are perfectly mobile across sectors. Suppose that  $c_2(w, r)$  is the minimum unit-cost function for sector 2, where  $w$  and  $r$  denote the wage rate and capital rental, respectively, expressed in units of the numéraire good 2. For sector 1,  $f(w, r, i)$  denotes the minimum cost associated with a unit of stage- $i$  production, and a unit of the final good requires  $a(i)$  units of stage- $i$  production. Assuming linear homogeneity of production at all stages,  $f(w, r, i)$  is independent on the level at which stage- $i$  is operated.<sup>2</sup> Following Dixit & Grossman (1982), I assume that  $i$  is a continuous variable in the closed interval  $[0, 1]$ . Without loss of generality, we assume that as  $i$  moves from 0 to 1, production moves “downstream”, so that the final good becomes available as  $i = 1$ . This the aforementioned *engineering sequence* of stages.

We assume that the economy is integrated in world commodity markets, but for some reason factor prices are not equalized internationally. Firms in sector 1 may exploit international factor price differences by outsourcing some stages to foreign factor markets where factor prices are  $\bar{w}$  and  $\bar{r}$ , in which case production becomes fragmented.

We can now envisage an *economic sequence* of stages by identifying the cost-advantage of outsourcing. There are two different forces at work here. One is the cost of international fragmentation, the other is the factor price difference. For simplicity, we assume that the cost of extra transport and communication is of the familiar iceberg-type. I.e.,  $\tau(i) > 1$  units of foreign stage- $i$  production need to be carried out for 1 unit of this stage to become available domestically towards further processing in subsequent stages of production. This is a crude but convenient way to introduce the “distance” factor into the analysis of international frag-

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<sup>2</sup>Strictly speaking, stage  $i$  of production should be denoted by  $i+di$ , and its unit cost by  $f(w, r, i)di$ . For ease of notation and wording, stage  $i+di$  will henceforth simply be called stage  $i$ .



mentation.<sup>3</sup> In addition, there may be Ricardian efficiency gaps between the two economies. If one unit of *domestic* labour is required to secure a certain amount of stage- $i$  output, securing that same amount via outsourcing requires  $\rho(i) > 0$  units of *foreign* labour. And the same applies to capital, i.e., the efficiency gaps are assumed to be Hicks-neutral. If  $\rho(i) > 1$ , then the foreign economy has a technological disadvantage, and vice versa if  $\rho(i) < 1$ .

The cost-advantage from factor price differences is, of course, the core of the analysis, and it is less straightforward, depending on how the factor intensity of production changes with  $i$ . For simplicity, I assume that more “downstream” stages are always more capital intensive than more “upstream” stages at all relevant factor price ratios. I.e., the capital intensity of stages,  $f_r(w, r, i)/f_w(w, r, i)$ , *increases monotonically* and continuously with  $i$ . What this implies for the cost-advantage of outsourcing depends on how domestic factor prices  $w$  and  $r$  deviate from foreign factor prices  $\bar{w}$  and  $\bar{r}$ . Assuming that foreign factor prices are unaffected by domestic outsourcing,<sup>4</sup> we define

$$\gamma(w, r, i) \equiv \frac{f(w, r, i)}{f(\bar{w}, \bar{r}, i)} \quad (1)$$

as the relative cost-advantage of outsourcing stemming from factor price differences. Given the assumption on  $f_r(w, r, i)/f_w(w, r, i)$ , this measure of foreign cost-advantage falls with  $i$  if  $w/r > \bar{w}/\bar{r}$ , and it increases with  $i$  if  $w/r < \bar{w}/\bar{r}$ . I will deal with the case where  $w/r > \bar{w}/\bar{r}$ , whence  $\gamma(w, r, i)$  monotonically falls in  $i$ , meaning that there is an incentive to outsource early (labour intensive) stages of production. It will become apparent below that the approach is not restricted to this case in any fundamental way, but can be applied – *mutatis mutandis* – to other cases as well.<sup>5</sup> Assuming  $w/r > \bar{w}/\bar{r}$  at the outset when domestic factor prices are endogenous may seem questionable. From a theoretical point of view, as long as we restrict ourselves to local comparative statics, this simply amounts to an unspecified assumption

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<sup>3</sup>In many crucial aspects of this paper, artificial barriers – tariff and non-tariff – act pretty much like these iceberg-costs, but consistent modeling would imply further complications in that tariff revenue or quota rents need to be modeled explicitly.

<sup>4</sup>This implies that the foreign economy can accommodate any additional factor demand that may arise from international fragmentation by means of Rybczynski-type internal reallocation at constant factor prices. The attendant output effects are, in turn, accommodated by world commodity markets at unchanged final output prices. In this sense, the assumption of constant foreign factor prices can be interpreted as the domestic and the foreign country being two small economies. An equivalent interpretation is to treat the foreign economy as the rest of the world where any factor demand that may arise from domestic outsourcing is of a negligible magnitude.

<sup>5</sup>There is, of course, the possibility of factor intensity reversals. In the present setup, this would mean that the monotonicity of the capital intensity with respect to  $i$  itself depends on factor prices. The implication of assuming monotonicity of  $\gamma(w, r, i)$  in  $i$  is that factor intensity reversals are not ruled out in principle, but they do not occur in the relevant range of wage-rental ratios  $[w/r, \bar{w}/\bar{r}]$ .

about the two countries' relative factor endowments. From an empirical point of view, such an assumption will often be quite reasonable, but it is clear that for some scenarios one might want to look at the twin assumptions of constant foreign foreign factor prices and  $w/r > \bar{w}/\bar{r}$  may not be reasonable.

Combining factor price considerations with the technology of fragmentation and Ricardian productivity gaps, we now assume that the overall cost-advantage

$$\Gamma(w, r, i) \equiv \gamma(w, r, i)/[\tau(i)\rho(i)] \quad (2)$$

preserves the monotonicity of  $\gamma(w, r, i)$ . A sufficient condition for this is that  $[\tau(i)\rho(i)]$  is non-decreasing in  $i$ . Figure 1 depicts  $\log \gamma(w, r, i)$  as a function of  $i$ , and then derives  $\log \Gamma(w, r, i)$  by combining it with  $\log \tau(i)$  and  $\log \rho(i)$ , assuming linearity for easier drawing. Figure 2 depicts domestic factor price contours satisfying  $\Gamma(w, r, i) = 1$  for alternative levels of  $i \in [0, 1]$ , including the two extreme cases where  $i = 0$ , and where  $i = 1$ . Several points are worth pointing out about this set of contours.

1. Points above the contour for any  $i = i'$  indicate domestic factor prices where domestic production of stage  $i'$  (and a fortiori of stages  $i < i'$ ) is not competitive, relative to outsourcing. The opposite holds true for points below such a contour. Points above the contour for  $i'$  are thus consistent with equilibrium only if *some* stages higher than  $i'$  are located abroad, and vice versa for points below.
2. For an arbitrary domestic factor price ratio  $(w/r)' > \bar{w}/\bar{r}$ , the slopes of these contours are equal in absolute value to the capital intensity of stage  $i$ ,  $f_r(w, r, i)/f_w(w, r, i)$ . The slope is larger for higher  $i$  (more “downstream” stages).
3. Since  $\Gamma(w, r, i)$  is monotonically increasing in  $i$ , contours for higher  $i$  lie farther “north-east”.
4. The radial distance between the two extreme contours along a ray  $(w/r)'$  is determined by the gap between  $(w/r)'$  and  $\bar{w}/\bar{r}$ , and by the extent to which the capital intensity increases as production moves further “downstream”.
5. For wage-rental ratios equal to  $\bar{w}/\bar{r}$  the contours for all  $i \in [0, 1]$  have equal slopes, their distance from the origin differing only due to  $\tau(i)\rho(i)$ .
6. The whole set of contours is defined independently of the price of the final good  $p_1$ . This will be a crucial for the analysis to follow.

For future reference, we call the case depicted in figures 1 and 2 *case I-a*. The analysis will largely stick to this case, but it is worth considering alternative assumptions, in order to see that it is actually quite general. Suppose the capital intensity falls as we move “downstream”

the value-added process and labour is relatively cheap in the domestic economy,  $w/r < \bar{w}/\bar{r}$ . We may label this *case II-a* where  $\gamma(w, r, i)$  is monotonically increasing in  $i$ . It is straightforward that this case is covered by a diagram like figure 2, with a reverse labeling of axes. This assumes, of course, that monotonicity of  $\gamma(w, r, i)$  is preserved by  $\Gamma(w, r, i)$ . International fragmentation again implies outsourcing of early stages, but these are now more capital intensive than later stages. The case where the capital intensity increases with  $i$  and where  $w/r < \bar{w}/\bar{r}$  implies that  $\gamma(w, r, i)$  is monotonically falling in  $i$ , as in case I-a. For obvious reasons, we label this *case I-b*. As in figure 2, on any ray through the origin the slope of contours for  $\Gamma(w, r, i) = 1$  rises with  $i$ , but unlike figure 2 contours for higher  $i$  are now closer to the origin, since the domestic economy has relatively cheap labour. Therefore, domestic advantage lies with early stages of production, and international fragmentation starts with outsourcing the most “downstream” stages. The remaining *case II-b* emerges if the capital intensity falls with  $i$ , and if domestic labour is relatively expensive (as in the benchmark case I-a). In this case  $\gamma(w, r, i)$  is monotonically rising in  $i$ , and it is clear that it is formally equivalent to case I-b with reversed axes, again assuming that the monotonicity of  $\gamma(w, r, i)$  is preserved by  $\Gamma(w, r, i)$ . This discussion reveals that the approach pursued below can easily be applied – mutatis mutandis – to cases where the factor intensity ordering is different.

The set of  $\Gamma$ -contours is a convenient device to characterize the “technology” of international fragmentation.<sup>6</sup> To proceed towards an “endogenous fragmentation factor price frontier” we must look at overall minimum unit cost involving all stages of production. We use  $i^*$  to denote the *cost minimizing margin of international fragmentation*, separating the stages produced at home from those outsourced to foreign factor markets.<sup>7</sup> Then, the *minimum* unit-cost of good 1 is

$$c_1(w, r, i^*) = \int_0^{i^*} a(i)\tau(i)\rho(i)f(\bar{w}, \bar{r}, i) di + \int_{i^*}^1 a(i)f(w, r, i) di, \quad (3)$$

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<sup>6</sup>The term “technology” may seem somewhat unusual here, since it incorporates foreign factor *prices*, while technology is usually viewed as a more fundamental concept not related to prices. But exchanging domestic production for imports *is*, in essence, an alternative technology to obtain the imported goods, the technology being described by the terms of trade. In this sense, foreign factor prices constitute the terms of trade for outsourcing.

<sup>7</sup>See Kohler (2003) for a related definition of the margin of international fragmentation with discrete stages.

where  $i^*$  satisfies the first order condition<sup>8</sup>

$$\Gamma(w, r, i^*) = 1 \text{ if } 0 < i^* < 1, \quad (4a)$$

$$\Gamma(w, r, i^*) \leq 1 \text{ if } i^* = 0, \quad (4b)$$

$$\text{and } \Gamma(w, r, i^*) \geq 1 \text{ if } i^* = 1. \quad (4c)$$

The second order condition on the margin  $i^*$  is satisfied from the monotonicity assumption relating to 2. To simplify notation, we shall henceforth use  $\bar{v}_1(\bar{w}, \bar{r}, i^*)$  and  $v_1(w, r, i^*)$  to denote the factor cost of foreign and domestic value-added, respectively, per unit of the final good:

$$\bar{v}_1(\bar{w}, \bar{r}, i^*) \equiv \int_0^{i^*} a(i)\tau(i)\rho(i)f(\bar{w}, \bar{r}, i) di \quad \text{and} \quad v_1(w, r, i^*) \equiv \int_{i^*}^1 a(i)f(w, r, i) di, \quad (5)$$

where  $i^*$  is again determined by 4 above.

### 3 A generalized factor price frontier

The multistage technology of production, together with the technology of international fragmentation determine the set of factor prices that are consistent with a zero profit equilibrium, given a certain final output price. This set can be described by means of a factor price frontier. I use  $p_1$  to denote the price of good 1, expressed in units of the numéraire good 2, and given exogenously at  $\bar{p}_1$  from world markets, taking the domestic economy to be small. Assuming perfect competition and free entry, a production equilibrium in sector 1 requires zero profits, i.e.,

$$c_1(w, r, i^*) = \bar{p}_1, \quad (6)$$

where  $c_1(w, r, i^*)$  is taken from 3. Notice that the cost minimizing margin of fragmentation  $i^*$  varies endogenously with factor prices – in line with 4 – as we move along this frontier. Moreover, it also depends on the final goods price, i.e., we have  $i^* = i^*(w, r, \bar{p}_1)$ . We shall see below how  $i^*$  responds to changes in the final goods price. Equations 6 and 4 together are a representation of the *domestic endogenous fragmentation factor price frontier* (henceforth labeled ef-fpf) for the multistage industry 1.

To see more clearly how the ef-fpf differs from the conventional zero profit line, it is useful to look at the factor price contour  $c_1(w, r, i') = \bar{p}_1$  for some arbitrary margin of fragmentation  $i = i'$ , ignoring for a moment the optimality condition 4. In view of 3 and 5, the conventional fpf satisfies

$$v_1(w, r, i') = \bar{p}_1 - \bar{v}_1(\bar{w}, \bar{r}, i'). \quad (7)$$

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<sup>8</sup>See also Dixit & Grossman (1982) where the issue is comparative advantage and trade as such, rather than international fragmentation.

The right-hand side of 7 may be called the *effective* price of the domestic value-added chain per unit of the final good, given the final output price and the cost of outsourcing stages up to  $i'$ .<sup>9</sup> The position of the  $v_1(w, r, i')$ -contour in factor-price-space depends on  $\bar{p}_1$  as well as  $i'$ . For a constant final output price there is, thus, a whole set of  $v_1(w, r, i)$ -contours for alternative levels of outsourcing  $i$ . All of these are downward sloping and convex, and the familiar envelope property implies that at any point the slope reflects the aggregate capital intensity of all domestic stages ( $i > i'$ ) of sector-1 production. Differentiating 7, and observing 5, we obtain

$$\left. \frac{dw}{dr} \right|_{v_1(w, r, i') = \bar{p}_1 - \bar{v}_1(\bar{w}, \bar{r}, i')} = \frac{\int_{i'}^1 a(i) f_r(w, r, i) di}{\int_{i'}^1 a(i) f_w(w, r, i) di}. \quad (8)$$

Confronting the  $v_1(w, r, i')$ -contour with the set of contours  $\Gamma(w, r, i)$  which describes the technology of international fragmentation, it becomes clear that if  $0 < i' < 1$  there is only one point of the  $v_1(w, r, i')$ -contour which also belongs to the “endogenous fragmentation factor price frontier”. This point is where the  $v_1(w, r, i')$ -line – the dashed line in figure 2 – and the line for  $\Gamma(w, r, i') = 1$  intersect. Since by assumption all “interior” domestic stages of production are more capital intensive than the marginal stage  $i'$ , at that intersection – point b in figure 2 – the  $\Gamma(w, r, i')$ -line is flatter than the  $v_1(w, r, i')$ -contour. All points on the  $v_1(w, r, i')$ -contour to the left of the intersection point, with a wage-rental ratio higher than  $(w/r)^i$ , are not part of the ef-fpf, since they violate the optimality condition 4 on the *equilibrium* margin of fragmentation. Moving to the left on the  $v_1(w, r, i')$ -contour leads to points above the line for  $\Gamma(w, r, i') = 1$ , where  $\Gamma(w, r, i') > 1$  implies that domestic production of stage  $i'$  is inefficient. Firms would forego the possibility of reducing unit-costs for the final output by outsourcing further stages of production to the foreign economy. Similar reasoning applies to points on the  $v_1(w, r, i')$ -contour to the right of point b, where a cost reduction can be achieved by reducing the margin of fragmentation below  $i'$ .

The ef-fpf is determined by equations 6 and 4. Using the implicit solution for the equilibrium margin of fragmentation  $i^*(w, r, \bar{p}_1)$ , the formal expression for the ef-fpf may be written as

$$c_1[w, r, i^*(w, r, \bar{p}_1)] = \bar{p}_1, \quad (9)$$

as opposed to  $c_1(w, r, i') = \bar{p}_1$  (equivalent to 7) for the traditional frontier which takes  $i = i'$  as exogenously given. The argument above implies that for  $i' = i^*$  the contour defined by 7 (treating  $i$  fixed at  $i'$ ) is tangent from below to the ef-fpf as defined in 9. This is a reflection of the envelope property with respect to the equilibrium margin of fragmentation  $i^*$ . Formally,

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<sup>9</sup>See Kohler (2003) for a more detailed elaboration on the relationship between the concept of effective prices, as used in the theory of effective protection, and international fragmentation.

based on 3, the ef-fpf satisfies  $dc_1(w, r, i^*) = 0$ , i.e.,

$$\int_{i^*}^1 a(i) f_w(w, r, i^*) dw di + \int_{i^*}^1 a(i) f_r(w, r, i^*) dr di + a(i^*) \tau(i^*) \rho(i^*) f(\bar{w}, \bar{r}, i^*) - a(i^*) f(w, r, i^*) = 0.$$

For interior  $i^*$ , the first order condition 4a guarantees that the two terms in the second line cancel, hence

$$\left| \frac{dw}{dr} \right|_{c_1[w, r, i^*(w, r, \bar{p}_1)] = \bar{p}_1} = \frac{\int_{i^*}^1 a(i) f_r(w, r, i) di}{\int_{i^*}^1 a(i) f_w(w, r, i) di}. \quad (10)$$

Comparing with 8 for  $i' = i^*$ , we have the aforementioned tangency condition. Moreover, given our assumptions, the ef-fpf is convex and continuous.

As  $i^*$  adjusts in line with 4, the ef-fpf crosses successive  $\Gamma$ -contours, until  $i^*$  reaches its upper or lower limit at  $i^* = 1$  or  $i^* = 0$ , respectively. For  $i^* = 1$ , with a wage rental ratio equal to  $(w/r)_1$ , the ef-fpf smoothly pastes with the  $\Gamma(w, r, 1)$ -line at point c, while for  $i^* = 0$  it pastes with the traditional fpf for  $c_1(w, r, 0) = \bar{p}_1$  (full domestic production of all stages) at point a, with a wage rental ratio equal to  $(w/r)_0$ . It is important to realize that, unlike the set of  $\Gamma$ -contours, these limiting wage-rental ratios depend on the final goods price; see below. As the wage-rental ratio moves from  $(w/r)_0$  to  $(w/r)_1$ , the  $\log \Gamma(w, r, i)$ -line in figure 1 shifts to the right with successively higher intersection points with the horizontal zero-line. It is worth pointing out once more that the whole set of  $\Gamma$ -contours changes as the foreign wage-rental ratio changes. The position of these contours is governed by the condition that along the ray  $\bar{w}/\bar{r}$  the  $\Gamma$ -contours have the same slope for all  $i \in [0, 1]$ , with their distance to the origin determined by  $\tau(i)\rho(i)$ .

Perhaps the most crucial point to be emphasized is that the ef-fpf, while featuring the same slope as the conventional fpf (interpreted as the average capital intensity of domestic stages), features a higher elasticity of substitution between capital and labour. Any increase in the wage-rental ratio, in addition to causing a substitution of capital for labour for all domestic stages of production, also causes an endogenous adjustment of the margin of fragmentation whereby the least labour intensive stages are relocated abroad, which reinforces the traditional substitution effect. In other words, endogenous international fragmentation makes capital and labour closer substitutes than would be the case for a constant level of outsourcing. It has often been conjectured that globalization has increased the elasticity of labour demand. That elasticity is equal to the capital share times the elasticity of substitution, hence this result suggests a rigorous theoretical rationale for the conjecture, expressed for instance in Fabbri, Haskel & Slaughter (2002). One must, however, be cautious in drawing conclusions. The ef-fpf is a partial equilibrium device in that it looks only at the multistage sector 1. Taking into account general equilibrium interrelationships, the overall elasticity of labour demand depends on possibilities of factor reallocation between the multistage sector and other sectors of the economy; see below. But the result points to the general importance of accounting

for fragmentation when empirically estimating (or using) elasticities of labour demand in the context of globalization. We may summarize this section by means of the following proposition.

**Proposition 1** *For given foreign factor prices and a given final output price, the set of factor prices supported by a c.r.s. technology featuring a continuum of stages with varying capital intensities and cost-minimizing international fragmentation can be described by an “endogenous fragmentation factor price frontier” (ef-fpf) which has the following properties: a) It is continuous and convex, b) its slope measures the average capital intensity of the domestic stages of production, but differs from the capital intensity at the margin of international fragmentation, and c) endogenous fragmentation makes capital and labour closer substitutes in the multistage industry than appears from the conventional fpf which takes the margin of fragmentation as exogenously given.*

## 4 General equilibrium

It is obvious that the ef-fpf alone is not enough to determine factor prices, or changes in these prices brought about, for instance, by certain scenarios of globalization. This can only be achieved by means of a general equilibrium analysis which also highlights conditions of factor reallocation between sectors. This section therefore extends the analysis by adding a single stage (numéraire) sector.<sup>10</sup> I present the general equilibrium conditions for a small economy and then turn to comparative statics, largely relying on diagrammatic analysis using the ef-fpf for sector 1 derived above.

In addition to the zero profit conditions for sector 1, given by equations 6 and 4, general equilibrium requires zero profits in sector 2, as well as full employment of labour and capital, which are assumed to be in given supply  $L^0$  and  $K^0$ , respectively. Using  $c_2(w, r)$  to denote the minimum unit-cost function for sector 2, zero profits imply

$$c_2(w, r) = 1. \quad (11)$$

Full employment requires

$$v_{1w}(w, r, i^*)q_1 + c_{2w}(w, r)q_2 = L^0 \quad (12a)$$

$$\text{and } v_{1r}(w, r, i^*)q_1 + c_{2r}(w, r)q_2 = K^0, \quad (12b)$$

where  $q_j$  denotes final output in sector  $j$ . Equations 12 make use of Shephard’s Lemma, whereby subscripts  $w$  and  $r$  denote partial derivatives, whence  $v_{1w}(w, r, i^*)$  indicates labour

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<sup>10</sup>In the sequel sector 2 will be alternatively referred to as the numéraire sector and the single-stage sector.

demand per unit of final good 1, based on definitions 5. Analogous interpretations hold for  $v_{1r}(w, r, i^*)$ , as well as for  $c_{2w}(w, r)$  and  $c_{2r}(w, r)$ . Note that domestic factor demands in sector 1 also depend on the margin of international fragmentation  $i^*$  which is in turn a function of  $w$  and  $r$ , as well as  $p_1$ , determined by equations 6 and 4, i.e.,  $i^* = i^*(w, r, \bar{p}_1)$ .

Equations 4, 6, 11 and 12 form a simultaneous system of 5 equations determining equilibrium values for 5 endogenous variables:  $w, r, i^*, q_1$  and  $q_2$ . Both  $c_2(w, r)$  and  $v_1(w, r, i^*)$  are homogeneous of degree 1 in  $w$  and  $r$ . Hence the zero profit and full employment conditions imply the usual equality between the value of output and aggregate income:

$$\bar{p}_1 q_1 + q_2 = wL^0 + rK^0 + \bar{v}_1(\bar{w}, \bar{r}, i^*)q_1 \quad (13a)$$

$$\text{or, equivalently, } \bar{\pi}_1(\bar{p}_1, \bar{w}, \bar{r}, i^*)q_1 + q_2 = wL^0 + rK^0, \quad (13b)$$

where  $\bar{\pi}_1(\bar{p}_1, \bar{w}, \bar{r}, i^*) = \bar{p}_1 - \bar{v}_1(\bar{w}, \bar{r}, i^*)$  is the effective price of domestic sector-1 value-added per unit of the final good. Introducing Marshallian demand functions  $d_1(wL^0 + rK^0, \bar{p}_1)$  and  $d_2(wL^0 + rK^0, \bar{p}_1) = wL^0 + rK^0 - d_1(wL^0 + rK^0, \bar{p}_1)$ , we have the trade balance equation

$$d_2 - q_2 + \bar{v}_1(\bar{w}, \bar{r}, i^*)q_1 = \bar{p}_1(q_1 - d_1), \quad (14)$$

stating that imports of good 2 plus the value of outsourcing in sector 1 are equal to the value of final good 1 exports. Note that equations 13 and 14 are no independent equilibrium conditions, but implied by the aforementioned set of five equilibrium conditions.

The full employment conditions may be rewritten as

$$\frac{v_{1w}(w, r, i^*)}{v_{1r}(w, r, i^*)}\kappa_1(w, r, i^*) + \frac{c_{2w}(w, r)}{c_{2r}(w, r)}\kappa_2(w, r) = \frac{L^0}{K^0} \quad (15a)$$

$$\text{and } \frac{v_{1r}(w, r, i^*)}{v_{1w}(w, r, i^*)}\lambda_1(w, r, i^*) + \frac{c_{2r}(w, r)}{c_{2w}(w, r)}\lambda_2(w, r) = \frac{K^0}{L^0}, \quad (15b)$$

where  $\kappa_j$  and  $\lambda_j$  are the allocation shares of capital and labour, respectively, employed in industry  $j$ . Naturally, we have  $\kappa_1 + \kappa_2 = 1$  and  $\lambda_1 + \lambda_2 = 1$ . I.e., in equilibrium, the given overall capital-labour endowment ratio must be equal to a weighted average of the sectoral capital intensities, with weights equal to the respective labour allocation shares (equation 15b). Analogously for the labour-capital endowment ratio and the capital allocation shares (equation 15a). An implication of this familiar property to which we turn below is that whenever the capital intensities of both activities rises, full employment requires an expansion – in terms of an increase in  $q_i$  – of the less capital intensive activity, and vice versa.

Figure 3 depicts general equilibrium for some initial price  $\bar{p}_1^0$  by combining the corresponding factor price frontier  $\text{ef-fpf}_1^0$  with a frontier representing 11, labeled  $\text{fpf}_2$ . With good 2 being our numéraire, the position of  $\text{fpf}_2$  is independent of goods prices; the superscripts A,B and C will be explained shortly. Assuming that both sectors are viable domestically, equilibrium factor prices  $w^{*0}$  and  $r^{*0}$  are found at the intersection point  $E^0$ , which also determines



the equilibrium margin of international fragmentation  $i^{*0}$ . We assume that the intersection point lies on the segment  $ac$  of the multistage factor price frontier, whence  $0 < i^{*0} < 1$ . The aggregate capital intensity of all domestic stages of industry 1 is equal to the slope of  $ef\text{-}fpf_1^0$ , which exceeds the marginal capital intensity at stage  $i^{*0}$ , equal to the slope of the line  $\Gamma(w, r, i^{*0}) = 1$ . In the sequel,  $k_1^v$  and  $k_1^m$  denote the equilibrium aggregate and marginal capital intensities, respectively, of domestic stages of production, while  $k_2$  denotes the equilibrium capital intensity of sector 2. Since we are looking at *case I-a* (see above), we have  $k_1^v > k_1^m$ . Depending on the technology in sector 2, we can now envisage three alternative types of equilibria.

**Case A:** The factor price frontier for sector 2 (labeled  $fpf_2^A$ ) is steeper at the intersection point than the  $v_1(w, r, i^{*0})$ -line, i.e., the numéraire sector 2 is more capital intensive than domestic value-added in sector 1. By necessity, its capital intensity is then also higher than the marginal capital intensity of sector 1:  $k_2 > k_1^v > k_1^m$ .

**Case B:** The factor price frontier for sector 2 (labeled  $fpf_2^B$ ) is flatter than the  $\Gamma_1(w, r, i^{*0})$ -line. The capital intensity of sector 2 is lower than the marginal capital intensity of sector 1. By necessity, sector 2 is also less capital intensive than domestic sector 1 value-added:  $k_2 < k_1^m < k_1^v$ .

**Case C:** The factor price frontier for sector 2 (labeled  $fpf_2^C$ ) is flatter than the  $v_1(w, r, i^{*0})$ -line, but steeper than the  $\Gamma_1(w, r, i^{*0})$ -line. I.e., the capital intensity of sector 2 is larger than sector 1's marginal capital intensity, but lower than observed capital intensity of value-added in sector 1:  $k_1^m < k_2 < k_1^v$ .

Before turning to comparative statics, we may note two interesting general equilibrium implications of endogenous fragmentation. As compared to a case where the margin of fragmentation is considered constant, endogenous fragmentation, by making capital and labour closer substitutes in the multistage process, also increases the elasticity of substitution along the economy's production possibilities frontier. The other is that it also increases the "likelihood" of factor intensity reversals between sectors.<sup>11</sup>

## 5 Protection under endogenous fragmentation

Turning to comparative statics under endogenous fragmentation, we first look at the effect of a rise in the domestic final goods price of the multistage sector on domestic factor prices, on the margin of fragmentation, and on outputs. As far as these effects are concerned, we may

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<sup>11</sup> As opposed to reversals between stages within sector 1, which was dealt with above.

also interpret this as a trade policy scenario: an import tariff if good 1 is imported, or an export subsidy if it is exported.<sup>12</sup> I shall rely on the diagrammatic representation, focusing on the three different cases identified in figure 3, assuming a small change from an interior equilibrium where  $0 < i^{*0} < 1$ .

### 5.1 Factor rewards and real income distribution

Suppose the relative change is  $\hat{p}_1 > 0$ . A first point to note is that the whole set of contours for  $\Gamma(w, r, i) = 1$  remains unaffected by such a change. To see how the ef-fpf<sub>1</sub> contour shifts, it is again convenient to first look at the traditional factor price frontier for a fixed  $i = i^{*0}$ , given by 7 above. Differentiating 7, it is easily seen that this frontier shifts out proportionally by a factor equal to  $(1 + \hat{p}_1/\theta_{1v}^0)$ , where  $\theta_{1v}^0 \equiv v_1(w^{*0}, r^{*0}, i^{*0})/c_1(w^{*0}, r^{*0}, i^{*0})$  is the share of domestic to overall value-added in sector 1 at the initial margin of international fragmentation. In other words, the effective price of domestic value added increases by  $100 \times \hat{p}_1/\theta_{1v}^0$  percent. A low  $\theta_{1v}^0$  thus acts as a leverage on the effective price increase.

In line with the theory of effective protection, a higher effective price mandates higher domestic factor prices. If we write  $\pi_1^0$  for the initial effective price, then the new position of the initial  $v_1(w, r, i^{*0})$ -contour is now determined by  $v_1(w, r, i^{*0}) = \bar{\pi}_1^0(1 + \hat{p}_1/\theta_{1v}^0)$ . Due to homothetic technology, the slope of this line is the same as the initial line for  $v_1(w, r, i^{*0}) = \bar{\pi}_1^0$  at a common wage-rental ratio  $w^{*0}/r^{*0}$ . To avoid clutter, these  $v_1(w, r, i^{*0})$ -contours have not been drawn in figure 3. While passing on the effective price increase proportionally to both factors would satisfy 6, it would violate 4a. The reason is that this moves the economy above the  $\Gamma(w, r, i^{*0})$ -line, which remains unchanged by  $\hat{p}_1$ . Hence, a point where both factor prices increase by a factor  $(1 + \hat{p}_1/\theta_{1v}^0)$ , while belonging to the traditional fpf with an *exogenous* margin of fragmentation, does not belong to the new ef-fpf<sub>1</sub>. Formally, such a point is fully comparable to a point on the  $v(w, r, i')$ -line in figure 1 to the left of point b. Applying the logic pertaining to that figure, we conclude that, for a wage-rental ratio equal to  $w^{*0}/r^{*0}$ , the new ef-fpf<sub>1</sub> passes through a point above the line  $v_1(w, r, i^{*0}) = \bar{\pi}_1^0(1 + \hat{p}_1/\theta_{1v}^0)$ , and it is steeper there than at the initial equilibrium. Intuitively, as domestic factor owners proportionally reap the benefit of a higher effective price, firms lose competitiveness at the initial margin  $i^{*0}$ . At an unchanged wage-rental ratio, the new ef-fpf<sub>1</sub> therefore features an equilibrium margin of fragmentation higher than  $i^{*0}$ . As a result, domestic value-added in sector 1 becomes a more capital intensive process, and the cost-savings effect of further outsourcing mandates factor price increases beyond a factor equal to  $(1 + \hat{p}_1/\theta_{1v}^0)$ .

If we apply the above reasoning to the three cases considered in the previous section, we

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<sup>12</sup>The welfare effect is, of course, different for an exogenous world price change and a policy induced change in the domestic price, at a given world price.

arrive at the following proposition.

**Proposition 2** *If there is a multistage industry where the capital intensity of value-added stages is monotonically higher for more “downstream” stages, if there is a viable numéraire sector which is not amenable to international fragmentation, if factors are completely mobile between sectors, and if the domestic economy has relatively expensive domestic labour, then an increase in the given world market price for the final good of this industry affects the equilibrium margin of international fragmentation  $i^*$ , the domestic wage rate  $w$ , and domestic capital rental  $r$  as follows:*

**Case A:** *If the numéraire sector is more capital intensive than aggregate domestic value-added in the multistage sector, then  $i^*$  rises (more outsourcing), while  $r$  falls and  $w$  is increased.*

**Case B:** *If the capital intensity of the numéraire sector is lower than the marginal capital intensity of the multistage sector, then  $i^*$  increases, with a higher  $r$  and a lower  $w$ .*

**Case C:** *If the numéraire sector exhibits a capital intensity lower than the aggregate of all domestic stages, but higher than that of the marginal stage, then the result is a lower  $i^*$  (less outsourcing), while  $w$  falls and  $r$  rises.*

In discussing this result, several things are worth pointing out. First, outsourcing may be a “friend” or an “enemy” to domestic labour. The crucial point is not only whether the numéraire sector is less capital intensive than domestic value-added (in which case a rise in  $\bar{p}_1$  always works against labour), but also whether its capital intensity is lower than the marginal capital intensity of domestic value-added in the multistage industry. If this latter condition is violated, then a rise in  $\bar{p}_1$ , while still hurting labour, lowers outsourcing. In this sense it can then be said that outsourcing and labour are “friends”. Secondly, a rise in  $\bar{p}_1$  will never be a Pareto improvement. This is ruled out by the presence of a *viable* single-stage sector which is not amenable to international fragmentation. This result is a reflection of the Stolper-Samuelson theorem, which in this sense is upheld under endogenous fragmentation. A further point worth mentioning is that in Case A of the proposition the final goods price change drives international factor prices further apart.<sup>13</sup>

An interesting question to ask is whether endogenous fragmentation reinforces or mitigates the Jonesian magnification effects underlying the Stolper-Samuelson theorem [see Jones (1965)]. We assume that the economy is diversified with  $0 < i^* < 1$  in both the initial and the new equilibrium reached after the final goods price change. For a notionally unchanged margin of fragmentation, the effective price for domestic value added changes by a larger

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<sup>13</sup>This possibility has been pointed out by Deardorff (2001b). Here, it is a corollary of a general proposition relating to a change in the final goods price.

absolute amount than the final goods price:  $|\hat{p}_1/\theta_{1v}^0| > |\hat{p}_1|$ . Clearly, the effect on mandated factor prices is driven by  $|\hat{p}_1/\theta_{1v}^0|$ .

A first point to note is that – in all cases considered – the economy moves along a downward sloping  $\text{fpf}_2$ -line, hence some factor always suffers a real income loss if  $\hat{p}_1 > 0$ , as mentioned above. Moreover, some factor will necessarily gain in real terms if  $\hat{p}_1 < 0$ . The cost-savings effect from an endogenous adjustment of outsourcing cannot, therefore, avoid the magnification effects as such. To see if it reinforces or mitigates these effects, we may use the dual logic introduced by Mussa (1979). Again, we first look at the  $v_1(w, r, i^{*0})$ -schedule and how it shifts upon  $\hat{p}_1$ , identifying the traditional magnification effect under a constant  $i = i^{*0}$ . In our model, however, such a situation will involve a fragmentation *disequilibrium*, and if we can identify how the endogenous margin of fragmentation changes, then we should also be able to see if the magnification effect is strengthened, or mitigated.

We have seen above that  $\hat{p}_1 > 0$  causes the effective price of domestic value added to increase by a factor of  $(1 + \hat{p}_1/\theta_{1v}^0)$ . The  $\text{fpf}_1$ -line for an unchanged margin of fragmentation  $i = i^{*0}$  thus shifts out proportionally to a position defined by  $v_1(w, r, i^{*0}) = \pi_1^0(1 + \hat{p}_1/\theta_{1v}^0)$ .<sup>14</sup> To avoid clutter, no  $\text{fpf}_1$ -line has been drawn in figure 1, but it is easy to imagine its initial position as being tangent from below to the  $\text{ef-fpf}_1^0$ -line at  $E^0$ . If the  $\text{fpf}_1$ -line shifts out, its intersection point with the  $\text{fpf}_2$  line determines the new factor prices, featuring a magnification effect in all possible cases considered. For instance, in case A the new equilibrium point will be found at an intersection point with  $\text{fpf}_2^A$ -line where  $r^{*1}$  is lower than  $r^{*0}$  and  $w^{*1} > w^{*0}(1 + \hat{p}_1/\theta_{1v}^0)$ . Similar logic applies to cases B and C.<sup>15</sup> The fragmentation *disequilibrium* is now easily identified by recognizing that *none* of these intersection points lies on the  $\Gamma(w, r, i^{*0})$ -contour. Cases A and B lead to a point above that contour, while case C leads to a point below. In this latter case, the domestic wage rate has fallen by so much that foreign production of stage  $i^{*0}$  has become inefficient, and firms in sector 1 will shift the margin of fragmentation below  $i^{*0}$ , thereby further reducing unit cost of final output and allowing domestic factor prices to rise beyond the frontier  $v_1(w, r, i^{*0}) = \pi_1^0(1 + \hat{p}_1/\theta_{1v}^0)$ . Allowing for endogenous adjustment of fragmentation, we thus arrive at an intersection point between the  $\text{ef-fpf}_1^f$ -line and the  $\text{fpf}_1^C$ -line which involves a *further* reduction of  $w$  and a *further* increase in  $r$ . The magnification effect is thus reinforced. It is relatively easy to see that the same logic leads to a reinforcement of the magnification effect also in cases A and B where the fragmentation disequilibrium associated with the conventional magnification effect implies that  $i^*$  rises above  $i^{*0}$ , meaning that reinforcement happens via further outsourcing. Moreover, it is relatively easy to verify by the same logic that the magnification effects will

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<sup>14</sup>We assume small changes in which case we may argue with local approximations.

<sup>15</sup>This dual logic to identify the Stolper-Samuelson theorem has been introduced by Mussa (1979).

be mitigated for a fall in the final goods price,  $\hat{p}_1 < 0$ . We may thus state the following proposition.

**Proposition 3** *If there is a single stage sector alongside a multistage industry, then the real income effects from the Stolper-Samuelson theorem are upheld under endogenous fragmentation, irrespective of the cost-savings effect from outsourcing. Compared with the conventional case without fragmentation, or a case where the margin of fragmentation is exogenous and constant, an endogenous adjustment of the margin of fragmentation strengthens the magnification effects underlying this theorem for both factors, if the final goods price rises. If the final goods price falls, then the attendant magnification effect is mitigated for both factors.*

Notice that the proposition makes no reference to whether endogenous adjustment of fragmentation implies a higher or lower level of outsourcing. This is a reflection of the above mentioned ambiguity as regards the “enemy”-relationship of outsourcing to labour. On the other hand, in case A labour ends up unambiguously worse off if any change in the final goods price, whether a rise or a fall, is accompanied by an endogenous adjustment of fragmentation, than if the margin of fragmentation is assumed constant. The reverse is true for cases B and C. Case B is quite relevant against the background of some of the concerns about economic globalization. If a country hosts the relatively capital intensive stages of a multistage process which is relatively capital intensive domestically, then the possibility of international outsourcing to a low-wage country aggravates the adverse wage rate effect (beneficial effect on capital rental) which stems from an increase in final output price of this industry. Notice, however, that this may well entail not an increase, but a lowering of the level of international outsourcing (Case B above).

## 5.2 Production and endowments

The presence of a second sector is also crucial for whether endowment changes have any effect on international fragmentation. It is relatively easy to see that such an effect necessarily arises if the multistage sector is the only domestic activity. In this case, equilibrium requires that the domestic endowment ratio  $K^0/L^0$  be tangent to the ef-fpf<sub>1</sub>-line. Then, any change in the domestic capital-labour endowment ratio would move the economy along the ef-fpf<sub>1</sub> line to a different  $\Gamma$ -contour, representing a lower (higher) level of outsourcing if the economy becomes more (less) labour abundant. Conversely, if the economy is diversified, then a small change in the endowment ratio will not affect outsourcing. This is a notable difference to the case considered by Feenstra & Hanson (1996, 1997), where the multistage sector is the only activity in the economy and where international capital movement causes further outsourcing. In a multisector economy, provided the economy remains in the cone of diversification, outsourcing is driven only by prices and is insensitive towards endowment changes.

In line with the theory of effective protection, one is inclined to interpret any increase in the final goods price at unchanged prices of imported intermediate inputs as having a protective effect on that sector. This is unambiguously true in the present model if we measure the protective effect in terms of sectoral outputs  $q_i$ . This follows from the above mentioned general equilibrium property that a rise in the capital intensity of both sectors requires a reallocation of both factors towards the less capital intensive activity, and vice versa. However, introducing as an alternative measure of the protective effect the stages of value added produced domestically, we can state the following proposition.

**Proposition 4** *Only in case C where the capital intensity of the numéraire sector lies in between the marginal and average capital intensity, respectively, of domestic value added in the multistage sector does an increase in the final output price of the multistage product (or an increase in the effective price) protect domestic value added in terms of both, the overall output and the number of stages produced. In cases A and B an increase in  $p_1$ , while causing a reallocation of both factors into the multistage industry, disprotects the marginal stage of domestic value added, whence expansion of the industry is paralleled by outsourcing further stages to the foreign economy.*

The explanation of the *disprotective* effect in cases A and B is that any attempt to reap the benefits of a higher effective price through higher domestic factor rewards makes the domestic economy uncompetitive at the marginal stage. Referring to figure 3, the resulting movements along  $\text{fpf}_2^A$  or  $\text{fpf}_2^B$ , attendant upon the outward shift of the  $\text{ef-fpf}_1$ -line to lead the economy to the northeast of the  $\Gamma(w, r, i^{*0})$ -line.<sup>16</sup> It is interesting to envisage consecutive price increases for the multistage product. In case A the process of adjustment is characterized by a reallocation of capital and labour towards sector 1, but with an ever *narrower* specialization in downstream stages of the production process until the economy becomes completely specialized in the multistage industry, but producing only a narrow range of stages at the downstream end. Figure 3 indicates such a situation at point d, where the endowment ratio coincides with the slope of the new  $\text{ef-fpf}_1^1$ -line. A similar scenario arises for case B. Which of the two is relevant depends on the endowment ratio of the economy, scenario B arising for a more labour abundant country.<sup>17</sup> In case C, on the other hand, increasing

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<sup>16</sup> It is worth pointing out that such a *disprotective* effect on marginal stages is absent in Dixit and Grossman (1982) who employ a specific-factors model. They note a different counter-intuitive possibility: Gaining additional stages of the production process may make the multistage activity less intensive in the mobile factor to such an extent that the output increase due to protection does not suffice to absorb the entire labor set free in the unprotected sector. The result is an *unambiguous* real income loss to labor, which is not possible in the simpler specific-factors model.

<sup>17</sup> For case A to arise, the domestic endowment ratio must lie between the slopes of  $\text{ef-fpf}_1^0$  and  $\text{fpf}_2^A$ , while

specialization in the multistage industry takes the form of an ever *broader* range of stages produced domestically.

## 6 Improved “technology” of international fragmentation

In the present model, all aspects of the “technology” of international fragmentation are conveniently summarized in dual form by the set of contours  $\Gamma(w, r, i) = 1$  which are also determined by foreign factor prices  $\bar{w}$  and  $\bar{r}$ , as emphasized above. In all of the above scenarios, the whole set of  $\Gamma$ -contours was assumed to remain constant. In other words, any outsourcing effect considered took place under a given technology of international fragmentation.

Much of the concern about economic globalization, however, evolves around technological changes which directly bear on outsourcing and international fragmentation.<sup>18</sup> Our set of  $\Gamma$ -contours should be a convenient tool to explore such changes. In this paper, I shall restrict myself to simple changes in the “iceberg”-cost of fragmentation captured by the function  $\tau(i)$ , but it is obvious that the approach can be applied to changes in foreign factor prices, as well as extended to more complex aspects of the fragmentation technology. This is, however, beyond the scope of the present paper.

Suppose that  $\tau(i)$  can be decomposed into a general term applicable to all stages of production, and a stages-specific term:  $\tau(i) = \tau_0\zeta(i)$ . I shall look at the simplest case where the technological improvement only affects the general term. I.e., for all  $i \in [0, 1]$ , we have  $d\tau(i) = d\tau_0\zeta(i) < 0$ . Given homotheticity of  $\gamma(w, r, i)$ , it is obvious that this leads to a proportional inward shift of each  $\Gamma(w, r, i)$ -contour in factor price space. An alternative way to capture this change is to say that the initial contours for now represent a different margin of international fragmentation  $i$ , the change being determined by

$$di = -\frac{\zeta(i)\rho(i)}{\gamma_i(w, r, i)} d\tau_0, \quad (16)$$

where in line with our assumptions  $\gamma(w, r, i)$  is falling in  $i$ , i.e.,  $\gamma_i < 0$ ; see above.

The immediate effect is that the initial equilibrium  $E^0$  in figure 3 is now a fragmentation *disequilibrium*, because the contour formerly labeled  $\Gamma(w, r, i^{*0})$  now represents

$$\Gamma(w, r, i^{*0} - di) = 1, \quad (17)$$

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for case B it must lie between  $ef\text{-}fp_1^0$  and  $fp_2^B$ . By necessity, therefore, case B features a more labor abundant domestic economy than case A.

<sup>18</sup>See, for instance Jones & Kierzkowski (1990), as well as Harris (1995, 2001) and Jones and Kierzkowski (2001a), where it is argued that technological advances in transport and communication make fragmentation less costly.

where  $di$  is taken from 16. Given that  $di$  in 16 is negative, this is an alternative way of saying that  $E^0$  lies above the new  $\Gamma$ -contour for  $i^{*0}$ . Unit-costs may be lowered by extending the margin of fragmentation beyond  $i^{*0}$ . But there is an additional effect stemming from infra-marginal stages  $i < i^{*0}$ . These are now obtained cheaper than before, the cost-effect being

$$\int_{i=0}^{i^{*0}} \zeta(i) \rho(i) f(\bar{w}, \bar{r}, i) d\tau_0 di < 0.$$

This acts just like an increase in the effective price, and the implication of this is as considered above. There is an inward shift in the conventional (constant margin  $i^{*0}$ ) factor price frontier determined by 7 for  $i' = i^{*0}$ . It becomes clear that the direct effect on the  $\Gamma$ -contours and this latter effect on the  $v_1(w, r, i^{*0})$ -contour reinforce each other towards an increase in the margin of fragmentation. Indeed, applying the logic used above to determine the effects of an increase in the final goods price, we realize that an improvement in the technology of fragmentation similarly shifts the ef-fpf<sub>1</sub>-line outward, while at the same time rotating it in a clockwise fashion. However, the pattern of effects is not quite the same, as summarized in the following proposition.

**Proposition 5** *An equal reduction of the costs of international fragmentation accross all stages unambiguously raises the equilibrium margin of international fragmentation  $i^*$ . It leads to a lower real wage rate and a higher real capital rental if factor intensities are as in cases B and , while opposite factor price effects arise in case A.*

The crucial difference to the protection scenario considered above is that  $i^*$  *always* increases. The reason is that, in addition to the rotated shift in the ef-fpf<sub>1</sub>-line, we now also observe a change in the set of  $\Gamma$ -contours. A number of points are worth mentioning. First, a Pareto improvement is ruled out, the crucial point being the presence of a viable single-stage sector, as noted above. Absent such a non-fragmentation activity, a Pareto improvement is possible, since the scenario features a savings in real resource use. Indeed, Feenstra & Hanson (1996, p.101, and 1997, p.378) emphasize such a possibility for a scenario which looks different, but in a fundamental sense is quite similar to the one considered here.<sup>19</sup> The crucial point here is that their setup rules out any non-fragmentation activity. The impossibility of a Pareto improvement is quite independent on the driving force behind the change in  $i^*$ .<sup>20</sup>

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<sup>19</sup>In the Feenstra-Hansen case the two factor considered are high- and low-skilled labor, which are nested in a production function with capital. In their scenario, an increase in  $i^*$  is brought about by international capital movement from the country where low-skilled labor is relatively expensive to the other country where it is relatively cheap – the foreign country in our setup. Obviously, this must increase productivity of both types of labor in the foreign country, which in our setup acts exactly like a downward-shift in  $\tau(i)$ .

<sup>20</sup>See Kohler (2003) for a general statement.



Secondly, in contrast to Feenstra & Hanson (1996 and 1997), the distributional change attendant upon a shift in the margin of international fragmentation is ambiguous. Again, it is the presence of a viable single-stage sector which makes the difference. Thus, suppose the single-stage sector were non-viable. Then the rotated shift in the ef-fpf<sub>1</sub>-line caused by  $d\tau_0 < 0$  would unambiguously lower the domestic wage-rental ratio. This follows from the fact that full employment now requires that the slope of the ef-fpf<sub>1</sub>-line must be equal to the given domestic capital-labour endowment ratio.<sup>21</sup>

## 7 Conclusion

In this paper I have studied international fragmentation of production using a framework with a continuum of fragments. With a continuum of stages, outsourcing is a continuous process. I have developed a generalized version of the familiar factor price frontier, where the margin of international fragmentation *endogenously* adjusts *along* the frontier. I have embedded this frontier in a simple general equilibrium model of a small economy hosting a single-stage industry in addition to the multistage process that is subject to fragmentation.

I have shown that such a generalized frontier proves useful in two distinct ways. First, it is a convenient dual representation of the “technology” of international fragmentation, which can be used to analyze the factor price effects of a variety of changes directly related to outsourcing. As an example, I have considered a uniform change in the cost of international fragmentation across all stages. Secondly, I have shown that it is a useful tool to address changes not directly related to outsourcing, but where the possibility of international fragmentation may be an important integral part of endogenous adjustment. As an example for this, I have looked at changes in the final output price of a multistage industry which may outsource a variable range of its stages to a low-wage foreign country.

In all of the scenarios considered, a key distinction arises between the *aggregate* capital intensity of all *domestic* stages of value-added, and the capital intensity of the process at the *margin* of international fragmentation. While the generalized factor price frontier reflects the average capital intensity, it deviates in a systematic way from the marginal capital intensity. An exogenous change in the final goods price leads to a “rotated” shift of the generalized frontier. Taking into account the factor price frontier for the single-stage sector, one can identify cases where international outsourcing is a “friend” or an “enemy” to domestic labour. The relevant conditions relate to the capital intensity of the single-stage sector, relative to the average and marginal capital intensity, respectively, of the multistage industry. The presence

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<sup>21</sup>Kohler (2003) presents a general result on the distributional consequences of international fragmentation which encompasses as special cases the results obtained by Feenstra & Hanson (1996 and 1997), as well as Arndt (1997 and 1999).

of a viable single-stage sector alongside the multistage industry proves crucial for whether or not a shift in the margin of fragmentation caused by an improvement in the technology of fragmentation is a Pareto improvement. Moreover, the endogenous adjustment of the margin of international fragmentation in certain cases reinforces the magnification effects of the Stolper-Samuelson theorem, while mitigating them in others. A multistage sector will always respond to the increase of its final output price by an increase in domestic value added, but this may be achieved by a specializing on an ever narrower range of stages, or by increasing the range of stages produced domestically.

The generalized factor price frontier developed in this paper also sheds light on “mandated factor price regressions” which have become popular in the empirical literature on wages and globalization. It has often been acknowledged that introducing an outsourcing regressor-variable involves a simultaneity problem which is difficult to resolve. While the present paper has taken a short-cut in that the generalized factor price frontier is a reduced form relationship, the steps undertaken in the analysis, particularly those relating to the “technology” of international fragmentation, should prove helpful when attempting to solve the simultaneity problem by specifying a full simultaneous equations system incorporating a separate “outsourcing equation”.

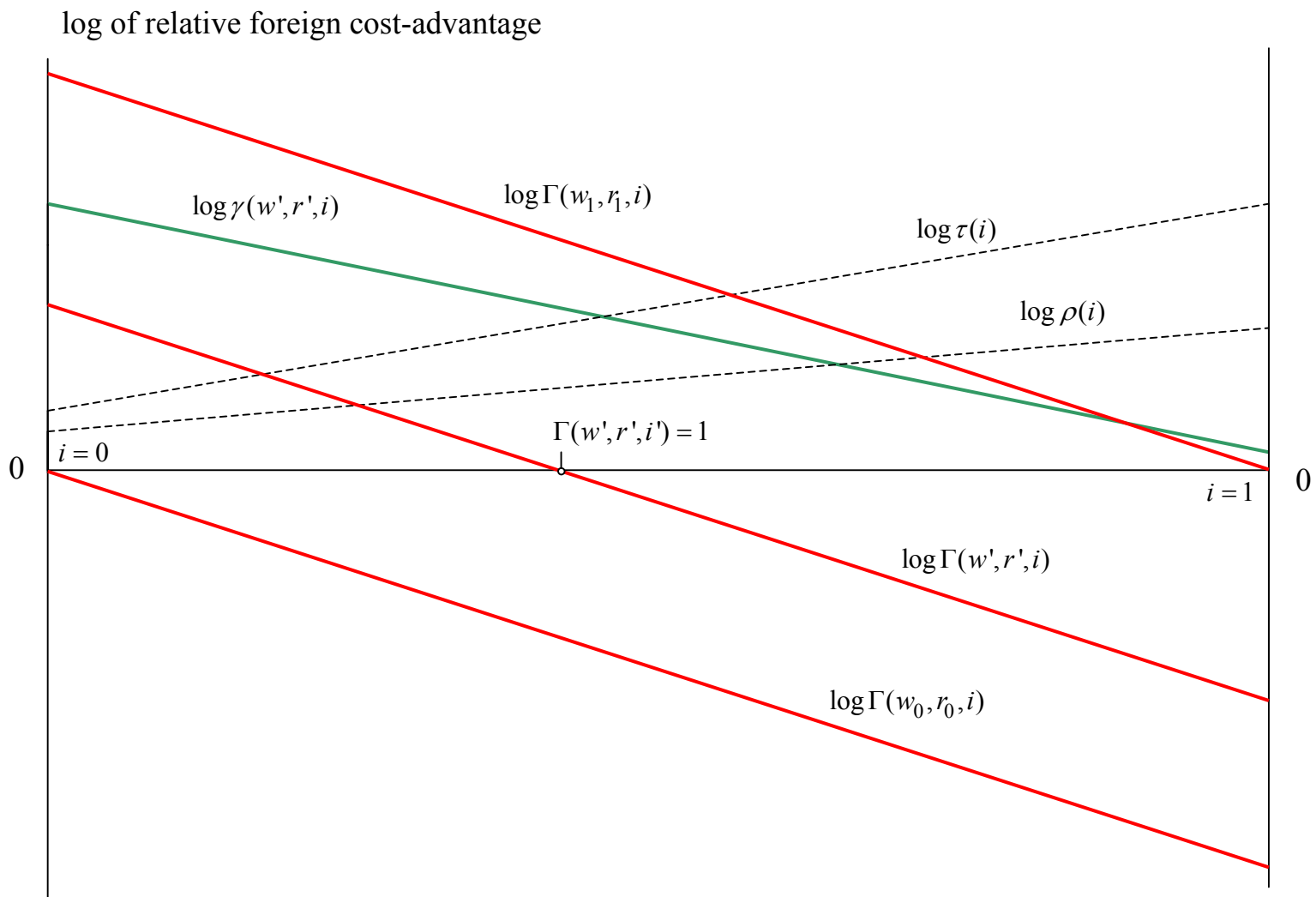
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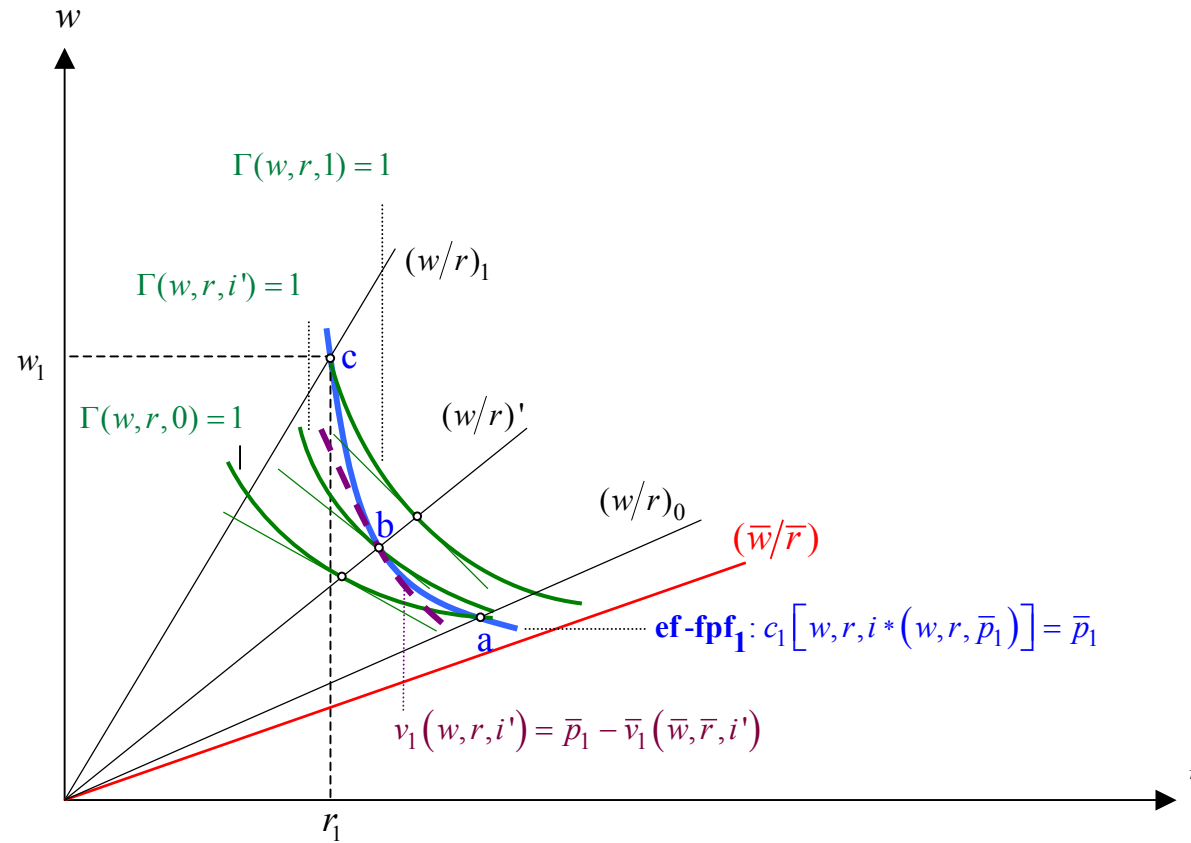
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**Figure 1:**  
*The relative foreign cost advantage of outsourcing*



**Figure 2:**  
*The factor price frontier for a multistage industry with international outsourcing*



**Figure 3:**  
*Factor price frontier for a multistage industry in general equilibrium*

