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Lindahl vs. Cournot Nash: The Role of the Income Distribution

by

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Abstract

It is known that a Lindahl equilibrium is not necessarily Pareto-superior to the non-cooperative Cournot-Nash outcome. This paper derives conditions under which the Lindahl Pareto-dominates the Cournot-Nash solution. We show that all are better off in the Lindahl equilibrium as compared to the Cournot-Nash equilibrium when the exogenously given income distribution is not skewed too much or the number of countries is high. The underlying effects are related to the famous exploitation of the rich by the poor countries occurring in Cournot-Nash equilibrium (which follows from Warr neutrality) and the fact that underprovision of the public good in Cournot-Nash equilibrium is particularly serious in large economies. Finally, our results are applied to infer some favourable conditions for successful international cooperation aiming at the provision of global public goods.

JEL classification: D3, H4.

Keywords: Public Goods, Cournot-Nash Equilibria, Lindahl Equilibria, International Environmental Agreements.

Contents

Non-Technical Summary

Provision of international public goods, such as protection of the ozone layer and global climate, are examples of important environmental issues that draw on public good theory. The noncooperative equilibrium of a pure public good model is typically Pareto-inefficient. International agreements, such as the Montreal Protocol and the Kyoto protocol, are essentially attempts to mitigate this inefficiency. By contrast with Cournot-Nash, a Lindahl equilibrium is Pareto-efficient. For this and other reasons, the Lindahl equilibrium is often appealed to as a natural cooperative allocation. Yet a move from the Cournot-Nash to Lindahl equilibrium will not command the support of all players unless each participant is made no worse off by the move. It is known that this is not always the case – poorer players, who in a precise sense exploit the richer players in a Cournot-Nash equilibrium, may be disadvantaged by such a move. This paper identifies the mechanism that makes some worse off at the Lindahl equilibrium than at the Cournot-Nash equilibrium. Its message is reassuring – in situations in which there are many players, it is likely that the move from Cournot-Nash to Lindahl represents a Pareto improvement. However, extreme inequalities in initial endowments can generate losers.

1. Introduction

During the last decade provision of international public goods, like protection of the ozone layer and the global climate, has become an important issue in international politics (see e.g. Kaul et al., 2003), stimulating interest in the theory of public goods (e.g. Sandler, 2004). In particular the theory of private or non-cooperative provision of a public good, which was not fully developed until the 1980s (see Cornes and Sandler, 1986/1996, and Bergstrom, Blume and Varian, 1986), provides a useful tool for describing the behavior of independent states in the essentially anarchic world system. Even though the emergence of a "world government" cannot seriously be expected, the countries nonetheless try to overcome the inefficiency of the non-cooperative outcomes by co-ordinating their activities through international agreements. Important examples of such attempts are the Montreal protocol in the case of ozone layer protection and the Kyoto protocol in the case of global warming. Therefore, considering co-operative solutions becomes important in the theory of international public goods. The treatment of Pareto-efficient outcomes, however, is deeply rooted in the theory of public goods in which, beginning with Lindahl (1919), properties of such allocations as the possible outcomes of cooperative arrangements have been sought.[1]

The economic literature on international cooperation, particularly in the field of global environmental protection, focuses on the question whether or not a country will voluntarily comply with an international agreement (see Finus, 2001, for an overview on the relevant literature). Prior to the stability of an international environmental agreement (IEA), however, is the approval of the conditions laid down in an IEA. A country will only be ready to accept the contractual terms if it attains higher welfare under the cooperative arrangement than under the non-cooperative outcome. Only then does initiating cooperation become individually rational for this country. The objective of this paper is to deal with the problem of "individual rationality" of cooperation leading to a Pareto-efficient allocation for a public-good economy in a specific but salient case.

In the theory of public goods, the most prominent outcome within the set of Pareto-efficient allocations is the solution already proposed by Lindahl (1919). In a *Lindahl equilibrium* all agents - as "cost-share takers" for appropriately chosen cost shares - unanimously choose the same provision level of the public good. Lindahl equilibria are analogous to competitive market equilibria in a private goods economy which gives the commonly used, but

[1] The fundamental optimality condition for a public-good economy is due to Samuelson (1954). See Sandmo (2004) for a recent discussion of efficient public-good allocations in the case of international environmental problems.

rather impractical, justification for the Lindahl concept.[2] But cost shares or individualised public-good prices in a Lindahl equilibrium coincide with individual marginal willingness to pay, so that Lindahl solutions fulfil the famous equivalence principle "each according to his benefit" and can thus be seen as fair burden-sharing arrangements. Above all it is this normatively appealing property which makes Lindahl equilibria interesting for the theory of international cooperation and international environmental economics (see, e.g., Eyckmans, 1997, or Sandler, 2002).[3]

It is well known that the Lindahl solution does not necessarily Pareto-dominate the Cournot-Nash outcome, as e.g. Cornes and Sandler (1985, p. 110-111) have noticed using a geometric example in the two-agent case. Therefore, it cannot be taken for granted that the Lindahl solution will find unanimous approval of all countries involved.[4] Using the standard public-good framework[5] in this paper we further analyse the welfare consequences of a move from the Cournot-Nash solution to the Lindahl equilibrium in a more general setting. As a general condition for the individual rationality of such a move we show that a country will always be made better off by this transition if in the Lindahl equilibrium it bears a lower cost share for financing the public good than in the Cournot-Nash solution. Applying this criterion to the case when preferences are identical shows that rich countries prefer the Lindahl over the Cournot-Nash solution as the move to the Lindahl solution helps in avoiding the "exploitation of the rich by the poor" which occurs in the Cournot-Nash equilibrium. The same reasoning suggests, however, that poor countries may be made worse off in the Lindahl equilibrium. But implementing the Pareto-efficient Lindahl solution also means that the underprovision of the public good, which is a main characteristic for the non-cooperative Cournot-Nash solution, is overcome.[6] This effect, which by itself benefits all countries, becomes dominant for large economies. Hence, in such a setting, there is a strong presumption that Lindahl equilibria Pareto-dominate their Cournot-Nash counterparts.

[2] See Myles (1995) or the short discussion of the Lindahl concept in Silvestre (2003).

[3] For other applications of the Lindahl solution or the generalised concept of cost-share equilibria to the problem of international cooperation see Weber and Wiesmeth (1991) and Sandler and Hartley (2001). Referring to global environmental protection, Eyckmans (1997) describes a "matching-mechanism" (see also, e.g., Althammer and Buchholz (1993) or Varian (1994)) to implement Lindahl-like solutions in order to solve international environmental problems.

[4] In the context of international environmental problems the problem of the individual rationality of proportional cost-sharing solutions is emphasized by Eyckmans (1997, p.329).

[5] For a comparison of Cournot-Nash and Lindahl equilibria in a rather uncommon and less transparent framework with a continuum of consumers see also Shitovitz and Spiegel (1998).

[6] For a general proof that the provision level of the public good in the Lindahl equilibrium lies above that in the Cournot-Nash outcome see Buchholz and Peters (2001).

2. The Framework

There are n countries $i = 1,...,n$. For simplicity we treat each country as a single agent. Country i's utility function is $u_i(x_i, G)$, where x_i is private consumption of agent i and G is the provision level of a pure public good equally consumed in all n countries. Utility of each country is strictly quasi-concave and twice continuously differentiable and both goods are strictly normal to all agents. By $mrs_i(x_i, G)$ we denote country i's marginal rate of substitution between the public and the private good at (x_i, G). The marginal rate of transformation mrt between the private and the public good is equal to one for every country. By y_i we denote the initial private good endowment ("income") of country i. Then $Y = \sum_{i=1}^{n} y_i$ is aggregate income available in all countries. In the following, we want to compare two different types of equilibria that may arise in this standard public-good economy: the Cournot-Nash equilibrium N and the Lindahl equilibrium L.

In a *Cournot-Nash equilibrium* N, given the contributions of all other countries, each country independently chooses its contribution to the public good to maximise its own utility. The Cournot-Nash solution N is generally not Pareto-efficient. Let G^N denote public-good supply and x_i^N private good consumption of country i at the Cournot-Nash equilibrium. Formally, country i's contribution to the public good $g_i^N := y_i - x_i^N \geq 0$ at N maximises its utility

(1) $$u_i(y_i - g_i, g_i + G_{-i}^N)$$

among all feasible public good contributions $g_i \in [0, y_i]$ where $G_{-i}^N = \sum_{j \neq i} g_j^N$ is the total contribution to the public good of all other countries except country i at N. By $p_i^N = g_i^N / G^N$ we denote the cost share of public good supply at N that falls on country i. If $p_i^N = 0$ for some country *i* this country is a complete free rider at N, making no contribution to the public good. It is a standard result in the theory of public goods that - under the general assumptions made here - existence of a unique Cournot-Nash equilibrium N is always ensured.

The *Lindahl equilibrium* L provides a cooperative solution that leads to a particular Pareto-efficient solution. Taking its cost share as given, every country would choose that level of the public good which is provided in the Lindahl equilibrium. Price-taking behaviour, well known from perfect competition for private goods, is assumed to prevail at the Lindahl equilibrium L for the public good. Let G^L be public-good demand, x_i^L private consumption and $g_i^L = y_i - x_i^L$ country i's public good contribution at L. Then for each country i the public good level G^L maximises

(2) $$u_i(y_i - p_i^L G, G)$$

for all $G \in \left[0, y_i / p_i^L\right]$, where $p_i^L = g_i^L / G^L$ gives country i's individual Lindahlian cost share. Under the standard assumptions on preferences a unique Lindahl equilibrium exists.

In contrast to the Cournot-Nash equilibrium, a Lindahl equilibrium is Pareto-efficient. Therefore, the utility of at least one country must increase when the cooperative Lindahl solution replaces the non-cooperative Cournot-Nash outcome. Let W denote the group of countries that are "winners" which are better off in *L* as compared to *N*. In the next section we derive a sufficient criterion that ensures that a country belongs to the group W.

3. A General Criterion

In order to provide a condition that implies higher utility for country i at L than at N, we compare country i's cost shares p_i^N and p_i^L in both solutions. We restrict attention to the only relevant case in which the total income is not concentrated on a single country. A basic criterion is now given that applies to active contributors to the public good in N, i.e. for which $p_i^N > 0$ holds.

Proposition 1: If $p_i^L \le p_i^N$ holds for a country *i* then $i \in W$.

Proof: We have

(3) $$u_i(y_i - p_i^L G^L, G^L) \ge u_i(y_i - p_i^L G^N, G^N) \ge u_i(y_i - p_i^N G^N, G^N)$$

where the first inequality follows from the price adjustment of country i in Lindahl equilibrium and the second inequality is a direct consequence of $p_i^L \le p_i^N$. If $p_i^L < p_i^N$, the second inequality is strict. Now assume $p_i^L = p_i^N < 1$. Confronted with the cost share p_i^L country i as a price taker would demand more of the good than G^N, since $mrs_i(x_i^N, G^N) = 1$ holds for any contributing country i. Then, the first inequality in (3) is strict. The remaining case $p_i^L = p_i^N = 1$ would imply that country i holds the whole aggregate income, i.e. $y_i = Y$, which is excluded by assumption. **QED.**

This result can be interpreted as follows: Compared to the Lindahl equilibrium the public good is provided at an inefficiently low level in the Cournot-Nash solution. Switching to Lindahl clearly removes this underprovision. By itself, this benefits all countries. But, simultaneously, the cost shares in financing the public good will generally also change in the transition from N to L. This second effect clearly benefits those countries whose cost shares fall. These agents enjoy a double advantage. For countries whose cost shares rise in the move from N to L, these two effects are opposed, so that it is *a priori* not clear whether the transition from N to L would pay for these countries.

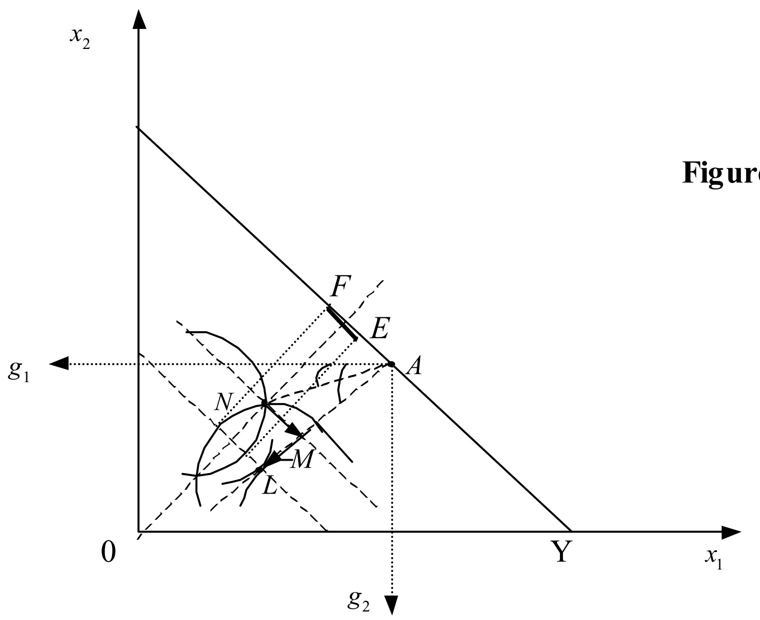

Figure 1

If cost shares do not change by moving from N to L, i.e. $p_i^L = p_i^N$ holds for all countries $i = 1,...,n$, only the first effect is present: All countries are therefore winners, and the move from the Cournot-Nash to the Lindahl solution implies a Pareto improvement.

The interplay of these two welfare effects is described diagrammatically in Figure 1. The horizontal axis measures private consumption of country 1 and the vertical axis that of country 2. The distribution (y_1, y_2) of initial endowments from which the countries start is described by some point A on the straight line with slope -1 which intersects both axes at Y. Furthermore, we choose an initial income distribution (y_1, y_2) that leads to the interior Cournot-Nash equilibrium, i.e. both countries actively contribute to the public good. If we additionally assume that both countries have the same preferences, Warr neutrality implies that the interior Cournot-Nash equilibrium lies on the 45º-line (see Warr, 1983, and Cornes and Sandler, 1996).

In Figure 1 the move from N to L may be decomposed into two steps:

Step 1: The pure change in the cost shares keeping public-good supply at the Cournot-Nash level $G = G^N$. In Figure 1 this is the move from N to M. M and N are on the same straight line with slope -1. This move helps country 1 whose cost share falls and hurts country 2 whose cost share rises.

Step 2: The relaxation of the rationing constraint that initially insists on $G = G^N$. In Figure 1 the adaptation of the two countries to their Lindahl cost shares is described by the move from M to L. By virtue of the construction of the Lindahl allocation this component cannot hurt anyone but will benefit both countries.

Figure 1 depicts the situation in which the effect described by Step 1 is stronger for country 2 than the effect described by Step 2 so that country 2 loses by moving from N to L. The welfare effect for country 1 whose cost share falls, however, is definitely positive.

Having presented the basic mechanism at work, we will now characterise the group of "winners" more precisely by applying the criterion in Proposition 1.

4. Characterising Winners: The Role of the Income Distribution

In Figure 1 of the previous section it is country 1 that definitely benefits in the move from N to L. In the diagram this follows as M lies below the 45º-line, which, in turn, is a consequence of the unequal income distribution in A. In the two-country case with homogeneous preferences it is always the country with the higher income level that gains by moving from the Cournot-Nash to the Lindahl allocation whereas the poorer country might lose. The more skewed the income distribution is, the greater the distance between N and M becomes, and the more likely it is that country 2 will be worse off at L relative to N.

These welfare effects can be illustrated by a numerical example in which both countries have the same symmetric Cobb-Douglas utility function $u_i(x_i, G) = x_i G$ and aggregate initial income is Y. An interior Cournot-Nash equilibrium results when the incomes of both countries lie between $\frac{1}{3}Y$ and $\frac{2}{3}Y$. At each interior Cournot-Nash equilibrium countries have an identical utility level $\left(\frac{Y}{3}\right)^2$ which is independent of the initial income distribution. Utility at the Lindahl equilibrium, however, depends on the distribution of income. If country 1's income is y_1 its utility at L is $\frac{y_1 Y}{4}$. Comparing the utilities both countries have in N and in L (for all $y_1 \in (\frac{1}{3}Y, \frac{2}{3}Y)$) we find that L is Pareto-superior to N if and only if the income distribution (y_1, y_2) is such that the income levels of both countries lie in the interval $(\frac{4}{9}Y, \frac{5}{9}Y)$. For a more skewed income distribution the poorer country will lose in the move from N to L. Only for a subset of all income distributions leading to an interior Cournot-Nash solution N will the Lindahl allocation L be preferred by all countries. This result may be shown by additional calculations to remain valid even if we include corner solutions in which only one country contributes to the public good in N.

We now show that countries with a high relative income are winners when moving from the Cournot-Nash to the Lindahl allocation also in the general case for more than two countries. For the general analysis let $G_i(p_i, y_i)$ be country i's demand function for the public good, where p_i denotes i's individual cost share. Formally $G_i(p_i, y_i)$ maximises country i's utility $u_i(y_i - p_i G, G)$ among all $G \geq 0$. For all countries $i = 1,...,n$ we evaluate these demand functions for the Cournot-Nash cost shares p_i^N. Without loss of generality, we can assume that the countries are ranked according to their public-good demand given p_i^N, i.e.

(4) $$G_1(p_1^N, y_1) \leq ... \leq G_n(p_n^N, y_n).$$

Among the countries for which $p_i^L \leq p_i^N$ holds, let m be the country with the highest number according to (4). Then by normality $G^L \geq G_m(p_m^N, y_m)$ and, as $G_i(p_i^N, y_i) \leq G_m(p_m^N, y_m)$ for any $i \leq m$, $p_i^L \leq p_i^N$ is implied. (Otherwise country i as price-taker would not demand G^L). Therefore, given the ranking (4), it follows from Proposition 1 that there is some $m \leq n$ such that, for every country $i \leq m$, we have $i \in W$. This means that those countries will gain by the move from N to L that - as cost-share takers - only would demand a low level of the public good when confronted with their Cournot-Nash cost shares. This is a specific expression of the fact that these countries are overburdened in N.

In the special case when all countries have identical preferences, this general insight provides a characterisation for the group of winners.

Proposition 2: If all countries have the same preferences, then there exists an $m \leq n$ such that $i \in W$ for all countries i with an income y_i exceeding the threshold y_m: $y_i \geq y_m$.

Proof: First, note that $G_i(p_i^N, y_i) \geq G^N$ holds for all countries i being contributors in N (As $mrs_i(x_i^N, G^N) = 1$ for every contributor otherwise $p_i^N > 1$ would follow.). The index m denotes the contributor with the smallest income among all countries i for which $p_i^L \leq p_i^N$. Normality implies that $G_i(p_i^N, y_i) \leq G_m(p_m^N, y_m)$ for every country i with $y_i \geq y_m$. This is depicted in Figure 2 where it is used that in the case of identical preferences each contributor has the same private good consumption level x^N in *N*.

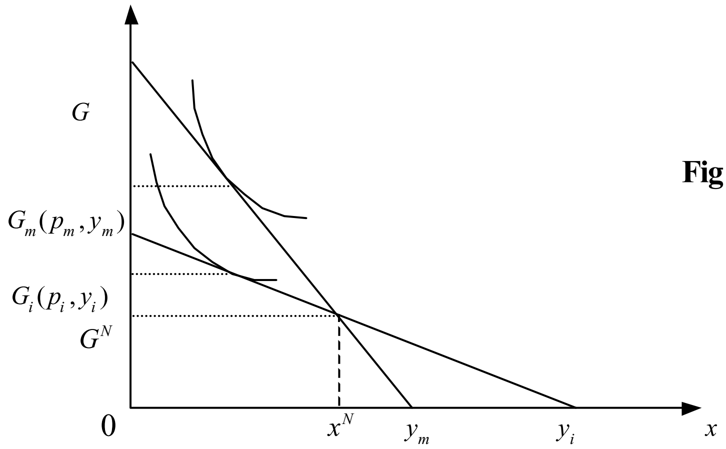

Figure 2

The ranking of countries according to (4) then gives the assertion of the Proposition. **QED**.

If preferences are identical for all countries and, moreover, have the particular form $u_i(x_i, G) = f(G)x_i + h(G)$, public-good supply is the same in every Pareto-efficient solution. (see Bergstrom and Cornes, 1983). In this special case it then follows from Proposition 3 that country i is in W if its income lies above average income, i.e. $y_i \geq Y/n$ holds.

These results concerning the income distribution can be interpreted as follows: It has been observed (see Sandler, 1992, and Sandler and Hartley, 2001) that in the case of homogeneous preferences there is an "exploitation of the rich by the poor" in Cournot-Nash equilibrium, i.e. that contributors with a high income do not have higher utility in *N* than agents with low income. Moving to the Lindahl equilibrium then - by imposing a higher cost share on the "poor" agents - removes this redistribution implicitly given by Cournot-Nash allocation. Therefore, it can be expected that in particular rich countries will gain from this effect whereas poor countries have to forego the benefits accruing to them by the indirect redistribution in the Cournot-Nash equilibrium. If, however, the distribution of income is fairly equal, "exploitation of the rich by the poor" at *N* naturally can only be of limited extent. Consequently, countries cannot suffer very much when this redistribution through voluntary provision no longer exists. Then a Pareto-improvement becomes likely when L instead of N is implemented.

When preferences are heterogeneous across countries analogous results hold. In this general case the Lindahl solution *L* Pareto-dominates the Cournot-Nash equilibrium N when the initial income distribution is balanced in a specific sense. Having such a "balanced" income distribution does not necessarily mean that the income levels of the countries have to be similar as the different preferences must also be taken into account.

To make this precise it can be shown that, if the heterogeneous preferences have the property that $\lim_{x_i \to 0} mrs_i(x_i, G) = 0$ for any $G > 0$, there exists a certain distribution $(\hat{y}_1, ... \hat{y}_n)$ of total income Y such that $p_i^L = p_i^N$ holds for the cost shares of all countries $i = 1,...,n$ in *N* and *L*. (For a proof see the Appendix A1.) Then the condition of Proposition (1) is fulfilled, and everyone prefers *L* over *N*.

It follows from a continuity argument that, not only for $(\hat{y}_1,...,\hat{y}_n)$, but also for any income distribution close to $(\hat{y}_1,...,\hat{y}_n)$, the Cournot-Nash equilibrium is Pareto-dominated by its Lindahl counterpart. The following Proposition summarises these findings:

Proposition 3: For any combination of individual preferences there exists an open set of income distributions such that the Lindahl solution is Pareto-superior to the Cournot-Nash equilibrium.

In a diagram analogous to Figure 1 this set of income distributions is a sub-interval on the endowment line which contains those income distributions for which the Lindahl equilibrium Pareto-dominates the Cournot-Nash equilibrium N. (For the case of identical preferences this is the interval EF in Figure 1.) All these Lindahl solutions lie within the lens which is delimited by the indifference curves of the both countries passing through *N*.

As a general rule, it is to be expected that the balanced income level $\hat{y}_i$ of a country i is the higher the weaker its preferences for the public good are. Look, e.g., at the case with two countries where $u_1(x_1, G) = x_1 G$ and $u_2(x_2, G) = x_2^{1/2} G$. Country 2 has a stronger preference for the public good than country 1. From the condition $p_1^L = p_1^N$ it is easy to calculate

$\hat{y}_2 = \sqrt{\frac{3}{8}}\hat{y}_1 < \hat{y}_1$ for every amount of total income Y (See Appendix A2). This phenomenon can be attributed to the fact that countries with a strong preference for the public good will rather become "overburdened" in the Cournot-Nash equilibrium even when their income is moderate (See also Shrestha and Feehan, 2003). According to our main argument such a country then will gain by moving from N to L.

Referring to the comparison of Cournot-Nash and Lindahl equilibria, it has been noted by Cornes and Sandler (1985, p. 111) that "lump-sum transfers may be important instruments in the search for acceptable reforms of public good provision". Now we have described in more detail the nature of the income redistribution if we require that countries unanimously prefer the Lindahl solution over the Cournot-Nash equilibrium. This becomes more likely if the income distribution is more balanced in the sense described above. In the special case of homogeneous preferences, this means that the income distribution is more equal. In the general case with different preferences a country with a weaker preference for the public good is more likely to require a transfer in order to make the Lindahl allocation attractive for it.

The sensitivity of the size of the winner set W with respect to the income distribution is also highlighted by a particular converse of Proposition 3. This result only depends on the same weak assumption on preferences as Proposition 3, i. e. on having $\lim_{x_i \to 0} mrs_i(x_i, G) = 0$ for each type of country $i = 1,...,n$ and any $G > 0$.

Proposition 4: There always exists an income distribution such that W only consists of a single country.

For a sketch of the proof we consider income distributions that are so skewed that only one country, say country 1, makes a positive contribution to the public good at the Cournot-Nash equilibrium N. Moving to the Lindahl solution then increases public-good supply and reduces country 1's cost share in the public good. Even when this induces country 1 to spend more on the public good the increment in expenditure per additional unit of the public good is limited under the assumption on preferences being made here. Therefore, there remains a certain positive share of additional public-good supply that has to be financed by the other countries. When, however, the income for these originally free-riding countries becomes small their willingness to pay for more of the public good goes to zero. This means that they cannot gain from participating in the Lindahl cost-sharing scheme.

5. Increasing the Size of the Economy

If some countries lose when moving from the Cournot-Nash to the Lindahl solution then the negative cost-shifting effect (see Step 1) is stronger for these countries than the positive public-good expansion effect (see Step 2). The number of countries involved considerably affects the relative strength of the two effects. When the number of countries increases the second effect will, in many important cases, dominate the first. Then the Lindahl solution Pareto-dominates the Cournot-Nash outcome. This result in particular is true when the original economy consisting of n possibly heterogeneous countries ("types") is replicated. This may be confirmed by a short proof of a conjecture first stated by Danziger (1976).

For an arbitrary replication factor k, let $G^N(k)$ denote public-good supply in the Cournot-Nash equilibrium N of the replication economy. Then it is known from Andreoni (1988) that there exists an upper limit for the provision level of the public good $\overline{G}^N = \lim_{k\to\infty} G^N(k)$. As public-good supply in N is increased when k rises, utility at N is bounded from above by $u_i(y_i, \overline{G}^N)$ in any replication economy for each type $i = 1,...,n$.

Now consider the Lindahl equilibrium L for different values of k and let $p_i^L(k)$ denote type i's Lindahl price in this replication economy. From the Samuelson condition $k\sum_{i=1}^{n} p_i^L(k) = 1$ it follows that $\lim_{k\to\infty} p_i^L(k) = 0$ for every type $i = 1,...,n$. Then for any indifference curve of agent i there exists a replication factor k_i^* such that the Lindahl budget line (defined by $x_i + p_i^L(k_i^*)G = y_i$) intersects this indifference curve. This, on the one hand, implies that $\lim_{k\to\infty} G^L(k) = \infty$ holds, i.e. that public-good supply in the Lindahl equilibrium is not bounded from above when the replication goes to infinity.

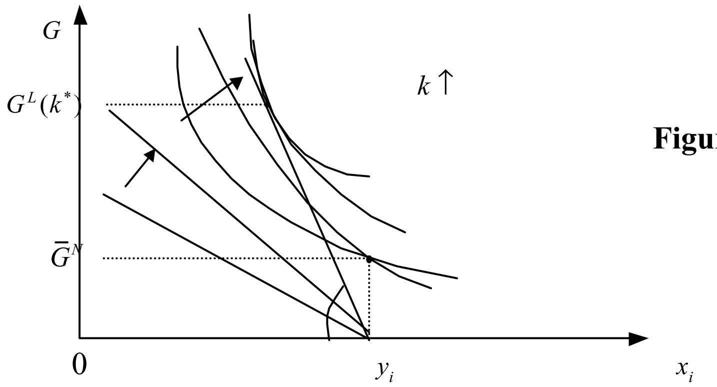

Figure 3

If, on the other hand, we consider country i's indifference curve passing through $(y_i, \bar{G}^N)$ (see Figure 3), we obtain the result that a country of type i prefers L over N as soon as $k \geq k_i^*$. This gives the following result:

Proposition 5: The Lindahl equilibrium will always become Pareto-superior to the Cournot-Nash equilibrium when the initially given group of countries is replicated sufficiently often.

Increasing the number of countries by replication ensures that the income distribution does not become too dispersed. In fact, the Lorenz curve of the income distribution is the same for any number k of replications. This naturally limits the exploitation effect existing in the Cournot-Nash equilibrium. At the same time, as $G^L(k)$ is unbounded but $G^N(k)$ is bounded when k goes to infinity, i.e. $\lim_{k\to\infty} G^L(k)/G^N(k) = \infty$. Consequently, the underprovision problem gets more serious when the number of countries is increased in this way. Comparing our two basic effects it therefore is understandable why Lindahl equilibria can always become Pareto-superior over Cournot-Nash solutions by replication.

The number of replications that is sufficient to induce Pareto-superiority of L over N may be quite small. Look, as in Section 3, at the case in which all countries have identical Cobb-Douglas preferences represented by $u(x_i, G) = x_i G$. Then for any given size n of the original economy and each income distribution within this economy the Lindahl equilibrium will Pareto-dominate the Cournot-Nash allocation for any replication, when $k \geq 3$ (See Appendix A3). This example, therefore, suggests that Pareto superiority of L over N is not only a remote theoretical possibility but may already occur in rather small groups.

However, when countries are very different the Lindahl solution L need not be Pareto-superior to the Cournot-Nash equilibrium N. This conclusion holds irrespective of the total number of countries and is clearly implied by Proposition 4.

6. Conclusion

In a public-good economy the distribution of income plays a significant role in determining whether the Lindahl equilibrium is Pareto-superior to the Cournot-Nash equilibrium. When the income distribution is unbalanced, relatively poor countries benefit greatly from implicitly "exploiting" the rich in the Cournot-Nash outcome - so greatly indeed that they may lose by a move to the Lindahl solution. When the Lindahl scheme is applied as the cost-sharing rule of a cooperative solution, then our argument can explain why international cooperation becomes more likely when countries are homogeneous. Moreover, our theoretical insights might give some further justification why transfers between countries may help to promote international cooperation. In the case of greenhouse-gas abatement the Kyoto protocol already includes some indirect redistribution mechanisms (through Joint Implementation, the Clean Development Mechanism and the possibility for world-wide emission trading), and in many proposals aiming at an improvement of global climate policy international transfers are assigned an important role (see, e.g., Panayotou, Sachs and Zwane, 2001, and Schelling, 2002).

The redistribution effect which hurts poor countries when the Lindahl instead of the Cournot-Nash allocation is implemented, however, is mitigated when the number of countries is high. Then by the move to the Lindahl solution the amount of the public good is increased substantially, which is in the interest of all countries. This observation gives another theoretical explanation why a high degree of participation in a cooperative arrangement like an international climate convention is required to make cooperation successful.[7] Then the adverse welfare effects that are caused for some countries under a specific cost-sharing rule like the Lindahl-mechanism are washed out when the removal of the underprovision of the public good becomes dominant in a large group of cooperating countries.

This paper has described basic effects that determine the individual rationality of cooperation situation in which burden-sharing among nations follows the Lindahl scheme. It might be an interesting objective for future research to extend this analysis to other cooperation mechanisms beyond the most prominent Lindahlian cost-sharing method.

Appendix

A1: We suppose that $\lim_{x_i \to 0} mrs_i(x_i, G) = 0$ holds for each country i for any $G > 0$. This condition means that - as in the Cobb-Douglas case - the indifference curves approach the G-axis but do not intersect it. The desired income distribution $(\hat{y}_1, ... \hat{y}_n)$ then is constructed in the following way: For given preferences and total income Y let $\hat{G}^N$ be public-good supply and $\hat{x}_i^N$ private consumption of country i in an *interior* Cournot-Nash equilibrium in which all agents are contributors. If $e_i(G)$ is agent i's income expansion path along which

[7] For another explanation focussing on the reactions of outsiders see Buchholz, Haslbeck and Sandler (1998) and Buchholz and Peters (2005).

$mrs_i(x_i, G) = 1$ then $\hat{G}^N$ is given by the budget constraint $\hat{G}^N + \sum_{i=1}^{n} e_i(\hat{G}^N) = Y$ and $\hat{x}_i^N = e_i(\hat{G}^N)$ for $i = 1,...,n$.

For each $i = 1,...,n$ we now construct a function $\tilde{x}_i(G)$ on $G \geq \hat{G}^N$ which is implicitly defined by

$$\text{(5)} \qquad mrs_i(\tilde{x}_i, G) = \frac{\hat{x}_i^N - \tilde{x}_i}{G - \hat{G}^N}.$$

In a x_i - G - diagram condition 5 means that the straight line that connects the points $(\hat{x}_i^N, \hat{G}^N)$ and $(\tilde{x}_i, G)$ has to be tangential to the indifference curve passing through $(\tilde{x}_i, G)$.

Under the assumptions we have made on preferences a private consumption level $\tilde{x}_i(G)$ that fulfils eq. (5) always exists and is uniquely determined for any $G \geq \hat{G}^N$. For country i now define the function $\pi_i(G) := mrs_i(\tilde{x}_i(G), G)$ for all $G \geq \hat{G}^N$. Then - as $\tilde{x}_i(\hat{G}^N) = \hat{x}^N$ - we have $\pi_i(\hat{G}^N) = 1$ and $\lim_{G \to \infty} \pi_i(G) = 0$. Therefore from continuity there is a public good level $\hat{G}^*$ such that $\sum_{i=1}^{n} \pi_i(\hat{G}^*) = 1$. By normality $\hat{G}^*$ is uniquely determined. Then the income level $\hat{y}_i$ is characterised by

$$\text{(6)} \qquad \hat{y}_i = \hat{x}_i^N + \pi_i(\hat{G}^*)\hat{G}^N$$

for every $i = 1,...,n$. Obviously, $\sum_{i=1}^{n} \hat{y}_i = \sum_{i=1}^{n} \hat{x}_i + (\sum_{i=1}^{n} \pi_i(\hat{G}^*))\hat{G}^N = Y$ such that $(\hat{y}_1,...,\hat{y}_n)$ indeed is a distribution of Y. As $\hat{x}_i^N < \hat{y}_i$ for every $i = 1,...,n$ the Cournot-Nash equilibrium that results given the income distribution $(\hat{y}_1,...,\hat{y}_n)$ is interior and thus corresponds to $(\hat{x}_i^N,...,\hat{x}_n^N, \hat{G}^N)$. Each country i endowed with $\hat{y}_i$ would choose the public-good level $\hat{G}^*$ when confronted with individual public-good price $\pi_i(\hat{G}^*)$. Therefore $\hat{G}^*$ is public-good supply in the Lindahl allocation L when the income distribution is $(\hat{y}_1,...,\hat{y}_n)$. The cost share of agent i in N then is $\hat{p}_i^N = (\hat{y}_i - \hat{x}_i^N)/\hat{G}^N = \pi_i(\hat{G}^*)$, such that Pareto superiority of L over N is a straightforward implication of Proposition 1.

A2: Let (y_1, y_2) be some distribution of total income Y. Given the Cobb-Douglas preferences as specified in Section 4 an interior Cournot-Nash equilibrium N on the one hand must have $G^N = x_1^N = \frac{2Y}{5}$ and thus $g_1^N = y_1 - x_1^N = \frac{5y_1 - 2Y}{5}$. Then

(7) $$p_1^N = \frac{g_1^N}{G^N} = \frac{5y_1 - 2Y}{2Y}.$$

On the other hand we calculate for some given income distribution the respective terms for the Lindahl equilibrium L which gives $G^L = \frac{4Y - y_1}{6}$ and $g_1^L = \frac{y_1}{2}$. Then

(8) $$p_1^L = \frac{g_1^L}{G^L} = \frac{3y_1}{4Y - y_1}$$

To determine the balanced income distribution $(\hat{y}_1, \hat{y}_2)$ we have to equate $p_1^N = p_1^L$. This gives a quadratic equation which has the solution

(9) $$(\hat{y}_1, \hat{y}_2) = (\frac{8 - 2\sqrt{6}}{5}, \frac{2\sqrt{6} - 3}{5})Y.$$

It is then a small algebraic exercise to show that $\frac{\hat{y}_2}{\hat{y}_1} = \sqrt{\frac{3}{8}}$.

A3: Let Y be aggregate income in the original economy. First consider the k-replication when the income distribution in the original economy is extremely skewed, i.e. is of the form $(Y,0\ldots,0)$ Then, public good supply in the Cournot-Nash equilibrium N is $\breve{G}^N(k) = \frac{kY}{k+1}$. (This follows from the aggregate budget constraint $\breve{G}^N(k) + k\breve{x}^N(k) = kY$ as for the given Cobb-Douglas preferences private consumption $\breve{x}^N(k)$ in N has to be equal to $\breve{G}^N(k)$). Now consider an arbitrary distribution $(y_1, \ldots y_n)$ of Y in the original economy. Then it follows, e.g., from Bergstrom, Blume and Varian (1986) that public-good supply $G^N(k)$ in the Cournot-Nash equilibrium of the k-replication is bounded from above by $\breve{G}^N(k)$. Given identical preferences the voluntary public-good supply is the higher the more we concentrate total income within the economy. Trivially, private consumption of each country i is bounded from above by country i's income y_i. Thus Cournot-Nash utility in the k-replication is at most

(10) $$u_i^N(k) \le \frac{kYy_i}{k+1}$$

for country i with income y_i. In the Lindahl equilibrium L of the k-replication country i has utility

(11) $$u_i^L(k) = \frac{kYy_i}{4}.$$

Obviously, the expression in (11) is not smaller than that in (10) as soon as $k \ge 3$.

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