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## *Extending the Melitz Model to Asymmetric Countries*

**by**

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# Extending the Melitz Model to Asymmetric Countries

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**Abstract**

In this paper we extend the Melitz (2003) heterogeneous firm trade model to include differences in country sizes and production technologies. We begin by characterising a "superior" technology, in terms of both survival cutoffs and representative firm productivity, and then examine a (costly) trading equilibrium between a leading and laggard country. We find that the intra-industry market share reallocations towards more efficient firms are stronger in the leading country. The numbers of firms depend on both country size and technology differences. Other things equal, the leading or the larger country tends to run a trade surplus in differentiated products, and, if technology or size differences are sufficiently large, the smaller or laggard country will cease production of differentiated products. A fall in unit trade costs will always benefit the leading country, but may hurt the laggard if technology differences are large enough *and* the laggard has the larger market.

**JEL classification:** F12

**Keywords:** heterogeneous firms, technology differences, trade liberalisation

**Outline**

Non-Technical Summary

The growing evidence of persistent and significant differences in productivity among firms within industries has begun to be reflected in trade modelling. Standard homogeneous firm, monopolistic competition trade models have been extended to allow firms to differ in their marginal costs. In a typical framework entering firms incur a fixed (entry) cost and then “draw” their variable cost from a known distribution. Those whose draw is below an endogenously determined cutoff find production profitable, those whose draw is above the cutoff exit. Opening such industries to (costly) international trade gives two equilibrium cost cutoffs – an export cutoff, below which firms both export and produce for the domestic market, and a survival cutoff above which firms exit. Firms whose costs lie between these two cutoffs produce for the domestic market only. Trade improves industry efficiency by reallocating resources to the more efficient (exporting) firms, and by inducing the least productive to exit.

In this paper we extend this model to allow for differences in industry cost distributions across countries. We begin by characterising a “superior” technology in terms of both a lower survival cutoff and a more efficient representative firm in the “leading” country in autarky. We find that the latter requires a stronger condition (reverse hazard rate stochastic dominance) than the former (first order stochastic dominance). We also show that neither of these is sufficient to determine whether the leading or the laggard country has the larger representative firm in autarky. This depends on the probability that an entrant survives, and the country with the higher survival probability has the smaller representative firm.

In the trading equilibrium, the leading country will have a lower domestic cutoff, but a higher export cutoff than the laggard. A higher proportion of surviving firms export in the leader. Both countries gain from trade as their survival cutoffs are lower than in autarky. The number of firms in each country will depend on country size, through the “home market effect”, and technology differences. The larger or the leading country will run a trade surplus in differentiated products, other things equal, but if market size and technology differences pull in opposite directions it is possible that a large laggard or a small leader could lose its industry. Finally a reduction in unit trade costs will lead to welfare gains in both countries, as long as technology differences are not too great. Otherwise, the leader always gains but the laggard may lose, although only if it has the larger market.

# 1. Introduction

The notion that the pattern of trade among countries can be based on differences in technologies available to their producers has a long and venerable tradition. The concept of comparative advantage was built on this, and later extended to other sources of cost differences, relative factor abundance in particular. But as long as trade models assumed constant returns to scale, individual firms were not well defined, and technology remained an industry rather than firm characteristic. This potentially changed with the development of new trade theory, which placed the firm centre stage and emphasised product variety and economies of scale at the firm level. Yet even here a representative firm framework tended to be employed, where a country's firms were assumed to be homogeneous, at least within an industry.

More recently this approach has changed, partly because empirical research has confirmed the presence of persistent, significant differences in productivities among firms within an industry[1]. It is then natural to ask whether a representative firm framework adequately captures interactions between firm productivities and trade. What role, if any, do within industry technology differences have in influencing trade patterns or the gains from trade? Furthermore, where firms within an industry differ, one might expect import competition and export opportunities to have different impacts on firms of different efficiency. What role does opening up the economy play in shaping the range of productivities observed among firms?

The development of heterogeneous-firm trade models has begun to investigate these effects. Melitz (2003) extends the Krugman single factor, homogeneous firm model to include firm heterogeneity through differences in marginal production costs. Bernard, Redding and Schott (2004) develop a similar model, but allow for a net trade pattern based on factor endowment differences. In both (costly) trade results in the exit of high cost firms and a division of the surviving firms into low cost (which sell on the domestic market and export) and higher cost (that produce for the domestic market only). Opening up to trade then increases industry productivity through a combination of exit by low productivity

[1] See, for example, Tybout (2003).

firms and expansion of the highest productivity firms into export markets[2]. An additional source of gains from trade is revealed.

This literature generally assumes that firms in different countries have “access to” the same technologies. Countries are either identical (except perhaps for size) or comparative advantage is based on endowment differences. Thus in Melitz (2003) entering firms undertake sunk entry costs, then draw their production productivity from a distribution that is the same in all countries. However, there is no particular reason to believe that firms in different countries will have access to identical productivity distributions[3], which raises the question of how differences across countries will affect trading outcomes. It also returns us to the earliest explanation of comparative advantage – Ricardian productivity differences. Differences in the productivity distribution of its firms lie at the heart of characterising an industry as having a superior technology which conveys a “comparative advantage” in that sector. Here we investigate the effects of industry technology differences by extending the Melitz model in two dimensions[4]. First, we allow firm productivities in different countries to be drawn from different distributions. Second, we follow Helpman, Melitz and Yeaple (2004) in adding a standard homogeneous good sector with constant returns to scale. The latter allows the size of the differentiated product sector to vary with technology differences, which has important implications for possible outcomes as we show[5].

From the perspective of technology differences the Melitz model has two key features. First, each equilibrium is characterised by industry productivity cutoffs - a single domestic cutoff for each country in the autarky equilibrium, and a domestic survival and an export cutoff for each country in the trading equilibrium. Firms with productivities below the relevant cutoffs are unable to sell in the relevant market. These cutoffs are functions of the

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[2] The theoretical literature on exporting and productivity is reviewed in Falvey and Yu (2005). Baldwin (2005) highlights the properties of the Melitz model and discusses some possible extensions. For a related discussion of issues of firm organisation in a broader context (i.e. allowing exports and FDI etc.), see Helpman (2005).

[3] See Bart and Monnihof (1996) for evidence on differences in firm size distributions across 5 OECD countries.

[4] We have investigated several of these issues in an earlier paper which relied on the Pareto Distribution for explicit solutions. See Falvey, Greenaway and Yu (2004). This work has been extended and updated in Falvey, Greenaway and Yu (2005).

[5] Bernard, Redding and Schott (2004) allow for more than one sector with heterogeneous firms producing differentiated products, where productivity distributions are identical within sectors across countries, but sectors differ in their intensity of factor use. Opening an economy to costly trade shifts resources to more efficient firms within each sector, and also shifts resources to the sector intensive in the use of the relatively abundant factor. As they note, the latter effect induces endogenous Ricardian productivity differences at the sector level, positively correlated with factor endowment based comparative advantage.

technologies only and the domestic cutoff turns out to be the crucial factor in determining welfare effects. Our analysis therefore begins by considering the effects of technology differences on the autarky and trading cutoffs. Ours is not the only investigation of this feature. Demidova (2005) also adapts the Melitz model to consider differences in firm productivity distributions between two trading partners. Her approach is to assume that the leading country's productivity distribution hazard rate stochastically dominates (HSD) that of the laggard country, which is shown to be a sufficient condition for the leading country to have a lower domestic survival cutoff (in both trade and autarky). She also shows that a country loses from a technology improvement in its trading partner. Our analysis of cutoffs essentially parallels that of Demidova, except that we work with firm unit costs rather than productivities[6], and we are able to establish that the leading country will have a lower survival cutoff (again in both trade and autarky) under the weaker sufficient condition of first order stochastic dominance (FSD)[7].

The second key feature of the Melitz analysis is that outcomes can be formulated in terms of a representative firm, where the unit cost attached to the firm that represents the industry is endogenous, depending on the equilibrium cutoffs. To this point a superior technology has been interpreted as one that gives a lower equilibrium survival cutoff. But one might alternatively view a better technology as one that generates a more efficient representative firm in equilibrium. We therefore also investigate the effects of technology differences on the efficiency of the representative firm, and are able to establish that if the leading country's cost distribution reverse hazard rate stochastically dominates (RHSD) that of the laggard country then it will have the more efficient representative firm in autarky. This is a stronger condition than FSD[8]. Interestingly even this is not sufficient to establish that the leading country will have the larger representative firm, however. This turns out to depend on survival probabilities.

Representative firm size is particularly relevant when we consider the number of firms in each country. It is important to do this because firm numbers are also affected by technology differences, and a concentration on equilibrium cutoffs can be misleading once the technology difference is sufficiently large that the industry in the laggard country closes

[6] Since one is simply the inverse of the other this is a trivial reconfiguration done for convenience only.
[7] HSD implies FSD, but not vice versa.
[8] RHSD implies FSD, but not vice versa.

down. Such specialization is firmly in the Ricardian tradition and we are able to partially paramaterise when it can occur. Of course firm numbers also depend on country sizes[9]. Since this framework involves fixed and per unit trade costs, it exhibits standard new economic geography features[10]. The larger country tends to have a disproportionate share of the output of the differentiated product industry, unless its market size advantage is offset by a technology disadvantage. Similarly, the leading country tends to have a disproportionate share (according to its size) of this output, unless its technological advantage is offset by a market size disadvantage. Indeed, when market size and technological advantages pull in opposite directions, it is possible for either the leading or larger country to have no differentiated product industry at all.

Finally, we consider the effects of a fall in (unit) trade costs on the trading equilibrium. Melitz showed that, when countries are identical, this raises the domestic productivity cutoff and reduces the export productivity cutoff. As a result aggregate productivity increases and all countries gain. This outcome continues to hold when countries differ in their technologies, as long as the technology gap is not too large. If it is, then, as Demidova also shows, the outcome can be reversed for the laggard country, whose domestic productivity cutoff would fall, signalling a reduction in its welfare. This can only happen in specific circumstances, however, as we show. Its occurrence relies on fixed trade costs being sufficiently large and the laggard country having the larger market size. Unless the leading country is smaller than the laggard, the latter's industry has shut down before the technology gap becomes large enough for immiserising trade liberalisation to occur[11].

In outline the remainder of the paper is as follows. The next section sets up the model and analyses the autarky equilibria and Section 3 considers the trading equilibrium. The effects of trade are discussed in section 4, and section 5 presents conclusions.

## 2. The Closed Economy

[9] Melitz and Ottaviano (2005) modify preferences so that market size affects cutoffs directly.
[10] Baldwin and Okubo (2005) consider the economic geography implications of firm heterogeneity in more detail.
[11] As we show below, if both countries are the same size, then the laggard industry shuts down at exactly the point where a reduction in unit trade costs would become immiserising.

We consider a model in which a single factor (labour) is used to produce output in two sectors. Sector H produces a homogenous good and sector D differentiated products. There are two countries, which can differ in both the size of their labour forces and their technologies in the D sector. Where technologies differ, we will label the home country as the technological leader and the foreign country (whose variables are indicated by a ~) as the laggard.

## 2.1 Demand and production

The preferences of a representative consumer are Cobb-Douglas across the outputs of the two sectors, with $\beta$ being the fraction of expenditure on differentiated products. Production in sector H exhibits constant returns to scale, and we choose the homogeneous good as the numeraire. Selecting units so that one unit of labour is required to produce one unit of the homogeneous good in each country, implies the wage rates are also unity. Full employment is maintained through adjustment in the size of the H sector. In sector D, market structure is assumed to be Dixit-Stiglitz monopolistic competition, and preferences across varieties are of a standard CES love of variety form, with elasticity of substitution $\sigma = 1/[1-\rho] > 1$, where $0 < \rho < 1$. This yields a constant elasticity of demand function for each variety produced by a corresponding unique firm *i*:

$$q_i = Ap_i^{-\sigma} \qquad [1]$$

where $A = \beta L P^{\sigma-1}$ and $P = \left(\int_{v \in V} p(v)^{1-\sigma} dv\right)^{\frac{1}{1-\sigma}}$ denotes the aggregate price index in the home country[12].

Firms incur two types of costs for production: a constant marginal cost $a$ which is the units of labour input required to produce one unit of output and is assumed to differ across firms, plus a fixed production cost *f* which is identical across firms. Taking $A$ as given, the price, sales, revenue and operating profits of a firm with marginal cost *a* are

[12] Total utility is $U = H^{1-\beta} Q^{\beta}$ with sub-utility function $Q = (\int_{v \in V} q^{\rho}(v) dv)^{1/\rho}$, $q(v)$ and *V* denote the consumption of variety *v* and the available variety set, respectively. Consumption of differentiated goods can be treated as consuming an aggregate *Q* with price index *P*.

$$p(a)=\frac{a}{\rho};\ q(a)=A\left(\frac{a}{\rho}\right)^{-\sigma};\ r(a)=A\left(\frac{a}{\rho}\right)^{1-\sigma};\ \text{and}\ \pi(a)=Ba^{1-\sigma}-f \qquad [2]$$

where $B=(1-\rho)\rho^{\sigma-1}A$. Firm revenue and operating profit are monotonically decreasing in a firm's own cost level $a$, but increasing in *A* and *B*, where the latter reflect market size and the extent of competition, and will be referred to as the "business environment" below.

### 2.2 Firm entry, exit and heterogeneity

There exists a continuum of potential entrants in sector D. To enter, each firm has to make an irreversible investment of $f_E$. Following Melitz (2003) we assume that, after entry, firms draw a marginal unit cost from a common *ex ante* cumulative distribution ($G(a)$) which is assumed to be exogenous and determined by the country's technology conditions. Once an entrant's cost is revealed, it will decide whether to stay or exit depending on whether its operating profit is positive or not. If we let $a_D$ denote the "survival ceiling" (i.e. the maximum cost level at which a firm can avoid operating losses), then

$$\pi(a_D)=Ba_D^{1-\sigma}-f=0 \qquad [3]$$

Since profit is decreasing in a firm's marginal cost, entrants whose marginal costs are higher than $a_D$ will find it unprofitable to produce and exit immediately. Entrants with marginal costs lower than the ceiling will find it profitable to operate, pay the fixed production cost $f$ and serve the market.[13] Hence entry and exit follow a self-selection process: low cost (lucky) firms survive and higher cost (unlucky) firms fail.

### 2.3 Equilibrium

Free entry of firms will drive the expected profit, net of entry costs, to zero, i.e. $E(\pi)-f_E=0$. Letting $\alpha_D$ denote the autarky equilibrium survival cutoff, from [3] we can write $B=f\alpha_D^{\sigma-1}$, so that [2] and [3] imply

$$E(\pi)=\int_0^{\alpha_D}\pi(a)dG(a)+\int_{\alpha_D}^{\infty}0dG(a)=f\int_0^{\alpha_D}\left\{\left[\frac{\alpha_D}{a}\right]^{\sigma-1}-1\right\}dG(a)=fQ(\alpha_D)=f_E \qquad [4]$$

[13] Melitz (2003) considers a multiperiod world with an exogenous probability that each surviving firm will exit in any period, leading to a stationary equilibrium with constant entry and exit flows. Here we follow Helpman, Melitz and Yeaple (2004) and assume that each surviving firm only operates for one period. This simplifies the analysis and does not significantly affect the results.

where $Q(x) \equiv \int_0^x \left\{ \left[ \frac{x}{a} \right]^{\sigma-1} - 1 \right\} dG(a)$. It is clear that $E(\pi)$ is increasing in $f$ and $\alpha_D$, so that a higher entry cost ( $f_E$ ) or a lower production fixed cost ( $f$ ) implies a higher equilibrium cost cutoff ( $\alpha_D$ )[14]. Since only entrants with marginal cost lower than $\alpha_D$ can survive, we can derive the cumulative distribution of surviving firms' costs (denoted $S(a)$ ) from the cumulative distribution of entrants' costs, as

$$S(a) = \frac{G(a)}{G(\alpha_D)}, \qquad 0 < a \leq \alpha_D$$

The probability of successful entry can be defined as $S_D \equiv G(\alpha_D)$.

We are now in a position to compare autarky equilibrium cutoffs across countries. Since our main interest is in how differences in industry cost distributions affect these, we adopt the simplifying assumption that our two countries are identical in terms of all fixed costs and demand parameters. Equation [4] then makes it clear that differences in cost distributions only affect the autarky equilibrium cutoffs if they impact on the expected profit function. Further it is only differences in cost distributions below the survival cutoff that are relevant, since all marginal costs above this receive an equal (zero) weight in the expected profit calculation. If the countries' cost distributions yield the same expected profit at $\alpha_D$, then $\alpha_D$ will be the autarky equilibrium survival cutoff for both, regardless of how their cost distributions differ for costs beyond this. The expected profit conditions therefore provide the necessary condition for technology differences to affect the autarky equilibrium cutoffs. While this might suggest that the most direct approach to defining technology differences is through the Q(.) function, the fact that expected profits reflect demand parameters as well as costs distributions, makes interpretation of differences in terms of the latter problematic. We therefore follow tradition and base our specification of technology differences in terms of the cost distributions alone. This makes their interpretation more straightforward, but restricts our analysis to sufficient conditions.

A natural definition of a technology advantage is where the home country has a *superior technology* in the differentiated goods sector over the unit cost range [0,x] if

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[14] The equilibrium cost ceiling will also be lower if the product differentiation is lower (i.e. a higher $\sigma$ ). Note that equi-proportional changes in $f_E$ and $f$ will have no impact on the cost ceiling.

$$G(a) \geq \tilde{G}(a) \text{ for all } a \in [0,x] \text{ and } G(a) > \tilde{G}(a) \text{ for some } a \in [0,x] \qquad [5]$$

That is, the foreign cost distribution (first order) stochastically dominates (FSD) that of the home country over this range. The home technology is thereby "superior" in the sense that the proportion of entrants drawing a unit cost less than or equal to any point in this range is no lower in the home country than in the foreign, and is strictly higher at some point in the range. Equivalently, the probability of a home entrant drawing a unit cost less than or equal to any point in this range is no lower than for a foreign entrant, and is in fact higher at some point in the range[15]. We can define an analogous *technological improvement* in the differentiated product sector (over the range [0,x]) in the home country if $G_1(a) \geq G_0(a)$ for all $a \in [0,x]$ and $G_1(a) > G_0(a)$ for some $a \in [0,x]$, where subscripts 0 and 1 denote the status before and after the technology change, respectively.

Since one can show (Appendix 1(a)) that Q(.) is increasing in G(.), [5] implies that if the home country has a superior technology over the range [0,x] home entrants also have higher expected profits– i.e. $Q(x) > \tilde{Q}(x)$. Suppose the home country has a superior technology over the range $[0, \tilde{\alpha}_D]$, where $\tilde{\alpha}_D$ is the equilibrium cutoff in the foreign country. With identical fixed costs in the two countries, the zero expected profit condition ([4]) can only be satisfied in both if $Q(\alpha_D) = \tilde{Q}(\tilde{\alpha}_D)$ which requires $\alpha_D < \tilde{\alpha}_D$. That is

> *Proposition 1. If the home country has a superior technology (in the sense of first order stochastic dominance) over the range of costs observed in the foreign autarky equilibrium, then the home country will have the lower survival ceiling in autarky.*

When all entrants have a better chance to draw a low cost, competition is more intense and the maximum survival cost level is driven down.

### 2.4 The representative firm

While the equilibrium cost cutoff is a natural indicator of industry efficiency and welfare in this model, one might also be interested in comparing the characteristics of countries' representative firms. For example one might view the country whose representative firm is more efficient in autarky as having the technological advantage. To consider this we follow

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[15] This definition also has the consistency property that if the home country has a superior technology over some range, the foreign country cannot have a superior technology over any of its subranges.

Melitz in identifying a "representative" surviving firm[16], whose (weighted average) unit cost ( $\bar{\alpha}$ ) is such that (for the home country)

$$P = M^{\frac{1}{1-\sigma}} p(\bar{\alpha}); \qquad \Pi = M\pi(\bar{\alpha}); \quad \text{and} \qquad R = Mr(\bar{\alpha}) \qquad [6]$$

where $\Pi$ and $R$ are sector total profits and revenue respectively. We can solve for $\bar{\alpha}$ from [6] using $p(\bar{\alpha}) = \bar{\alpha}/\rho$ from [2] and $P = \left[ M\int_0^{\alpha_D} p(a)^{1-\sigma} dS(a) \right]^{\frac{1}{1-\sigma}}$, yielding

$$\bar{\alpha} = \left[ \int_0^{\alpha_D} a^{1-\sigma} dS(a) \right]^{\frac{1}{1-\sigma}} = \left[ \frac{\int_0^{\alpha_D} a^{1-\sigma} dG(a)}{G(\alpha_D)} \right]^{\frac{1}{1-\sigma}} \qquad [7]$$

A direct comparison of the unit costs of representative firms is complicated in that both the distributions of unit costs and, consequently, the autarky equilibrium cutoffs differ across countries. But it is clear that a country will tend to have a larger representative firm, if its firms are relatively more likely to draw lower costs. This can be made more precise if we define the home country as having a *superior cost distribution* over the range [0, x] if the foreign cost distribution reverse hazard rate stochastically dominates the home cost distribution – i.e. $\tilde{G}(a) \succ_{RHSD} G(a)$ if $\frac{\tilde{g}(a)}{\tilde{G}(a)} > \frac{g(a)}{G(a)}$ for all $a \in [0, x]$.[17] Assuming the home country has a superior cost distribution we can show (Appendix 1(b)) that $\bar{\alpha}(x) < \tilde{\bar{\alpha}}(x)$. Further, one can show that a superior cost distribution over the range [0,x] implies a superior technology over the same range[18]. Hence if the home country has a superior cost distribution over the range of foreign autarky costs, it will have the lower cost cutoff, and since $d\bar{\alpha}(x)/dx > 0$, the more efficient representative firm. These results are summarised in

> *Proposition 2: If the leading country has a superior cost distribution (in the sense of reverse hazard rate stochastic dominance) it will have a lower autarky cost cutoff and its representative firm will be more efficient than that in the laggard country.*

Does a more efficient representative firm imply a larger firm? For this and subsequent derivations, it is useful to write the unit cost of the representative firm as

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[16] Note that the unit cost of the representative firm is endogenous and will differ across countries and equilibria.

[17] Again an advantage of this definition is that if the home country has a superior cost distribution over a range, the foreign country cannot have a superior cost distribution over any subrange.

[18] If $\tilde{G}(a) \succ_{RHSD} G(a)$ then $\tilde{G}(a) \succ_{FSD} G(a)$, i.e. $G(a) > \tilde{G}(a)$.

$$\bar{\alpha} = \left[ \frac{Z(\alpha_D)}{G(\alpha_D)} \right]^{\frac{1}{1-\sigma}} \quad [8]$$

where $Z(x) \equiv \int_0^x a^{1-\sigma} dG(a)$. After some manipulation (Appendix 1(c)), we can then write the revenue of the representative firm as

$$r(\bar{\alpha}) = A\left[\frac{\bar{\alpha}}{\rho}\right]^{1-\sigma} = \frac{f}{1-\rho}\frac{H(\alpha_D)}{G(\alpha_D)} \quad [9]$$

where $H(x) = Q(x) + G(x) = x^{\sigma-1}\int_0^x a^{1-\sigma} dG(a) = x^{\sigma-1}Z(x)$. This allows cross-country comparisons of the revenues of their respective representative firms[19]. Given that fixed costs and demand parameters are identical, we have

$$r(\bar{\alpha}) \geq (\leq) \tilde{r}(\tilde{\bar{\alpha}}) \qquad \text{as} \qquad \frac{H(\alpha_D)}{G(\alpha_D)} \geq (\leq) \frac{\tilde{H}(\tilde{\alpha}_D)}{\tilde{G}(\tilde{\alpha}_D)}$$

Using the definitions of $H(.)$ (and $\tilde{H}(.)$), noting that the autarky survival cutoffs are determined such that $Q(\alpha_D) = \tilde{Q}(\tilde{\alpha}_D)$, and recalling that $G(\alpha_D) = S_D$ and $\tilde{G}(\tilde{\alpha}_D) = \tilde{S}_D$ are the survival probabilities, we have

$$r(\bar{\alpha}) \geq (\leq) \tilde{r}(\tilde{\bar{\alpha}}) \qquad \text{as} \qquad \tilde{S}_D \geq (\leq) S_D$$

The country with the lower survival probability has the larger representative firm. Under our assumptions this could still be either the leading or the laggard country[20].

### 2.5 Numbers of firms.

To determine the numbers of varieties produced, we need to solve for the equilibrium mass of operating firms (*M*). This can be derived by equating industry total cost (TC) with total revenue The total operating cost of the representative surviving firm is $C(\bar{\alpha}) = f + q(\bar{\alpha})\bar{\alpha} = f + \rho r(\bar{\alpha})$. The expected cost of an entrant is then the entry cost ($f_E$) plus the expected operating cost if the firm survives ($C(\bar{\alpha})$) multiplied by the probability of

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[19] The comparison is less direct than it might appear because the business environments ($A$, $\tilde{A}$) will differ in the two markets. From [2] and [3] we have $A = \left\{[1-\rho]\rho^{\rho-1}\right\}^{-1} f \alpha_D^{\sigma-1}$, and so $\alpha_D < \tilde{\alpha}_D$ implies $A < \tilde{A}$.

[20] If $G(a)$ is a Pareto distribution, $G(a) = \left[a/\hat{a}\right]^k$, then the probability of survival is the same in both countries if the home technological superiority arises from a lower cost upper bound ($\hat{a}$), and lower in the leading country when its superiority arises through a higher shape parameter (*k*). See Falvey, Greenaway and Yu (2005).

survival ($S_D$). If we then multiply this by the number of entrants ($M_E$) we have industry total cost. Using [2], [4] and [9], this can be written as

$$TC = M_E \sigma [ f_E + f S_D ] = M_E \sigma F = M \sigma \frac{F}{S_D} \qquad [10]$$

where $F = f_E + S_D f$ is entrant (expected) fixed cost. Average firm cost can be written as a "markup" on *F*. Industry total cost can be written as a constant markup ($\sigma$) on *F* multiplied by the number of entrants, or an equilibrium specific markup ($\sigma / S_D$) multiplied by the number of survivors. Equating this with total revenue ($\beta L$) allows us to solve for the autarky equilibrium numbers of entrants and survivors as

$$M = S_D M_E = S_D \frac{\beta L}{\sigma F} = \frac{\beta L}{r(\bar{\alpha})} \quad \text{and} \quad \tilde{M} = \tilde{S}_D \tilde{M}_E = \tilde{S}_D \frac{\beta \tilde{L}}{\sigma \tilde{F}} = \frac{\beta \tilde{L}}{\tilde{r}(\bar{\tilde{\alpha}})} \qquad [11]$$

The number of varieties in a closed economy is proportional to its market size and decreasing in the size of its representative firm. These results can be summarised in

> *Proposition 3. In autarky: (a) if the two countries are of equal size, the country with the lower probability of firm survival will have a smaller mass of operating firms and a representative operating firm of larger size; (b) if the two countries have identical technologies, the larger country will have the larger mass of operating firms, and the representative firms will be of equal size.*

## 2.6 Welfare.

To evaluate welfare differences, we solve for the price level in equilibrium, which is proportionally increasing in the survival ceiling, but decreasing in market size (Appendix 1(c)):

$$P = \alpha_D \left[ \frac{\beta L}{f} \right]^{\frac{1}{1-\sigma}} \eta \qquad [12]$$

where $\eta = \left\{ (1-\rho)^{\sigma - 1} \rho \right\}^{-1} > 0$. Therefore, other things equal, the price level is lower in the leading country or the larger market. In the latter a larger country attracts a greater number of producers which drives down the aggregate price. Welfare per capita
($u$) is then given by

$$u = \frac{U}{L} = \frac{([1-\beta]L)^{1-\beta} (\beta L P^{-1})^{\beta}}{L} = \psi P^{-\beta} \qquad [13]$$

where $\psi = (1-\beta)^{1-\beta} \beta^{\beta}$ is a constant. Welfare per capita is determined by and negatively related to the aggregate price index only. Hence

> *Proposition 4: Other things equal, consumers in the larger country or the leading country will be better off in autarky.*

This reflects the standard home market effect and the leading country's absolute advantage in the differentiated product sector.

## 3. The Open Economy

We now allow trade, adopting the standard simplifying assumption that both countries produce the homogenous good, which is costlessly tradable[21]. Hence wage rates are equalised across countries. In the absence of trade costs on differentiated products, all firms will sell in both markets, implying entrants in either country will face identical fixed costs and business environments, which equalises their survival ceilings. If the home country has a superior technology, its entrants will always face a higher expected profit. Hence all potential entrants will prefer to enter the home country and export. The trade pattern is then straightforward: production of the differentiated good will be concentrated in the leading country, which produces in both sectors and exports (imports) differentiated goods (the homogeneous good), while the laggard country specialises in the production of the homogenous good. The same outcome will occur if trade costs are so low that all operating firms in both countries find it profitable to export. Then entrants in different countries still face identical business environments, as well as the same survival ceiling. Hence the leading country will specialise in the differentiated good sector and the trade pattern is just as described. Thus trade with low transport costs leads to a production pattern consistent with specialisation according to comparative advantage.

### 3.1 Trade costs

However, if trade costs are sufficiently high that only a proportion of domestic firms find it profitable to export, then entrants in different countries face different business

[21] This requires that preferences are not too strongly biased towards the differentiated good (i.e. $\beta$ is sufficiently small). Otherwise, the labour demand in the differentiated good sector may exceed the total labour endowment in one country.

environments[22]. Consequently, this will generate different survival ceilings across countries, which attract positive numbers of entrants and leads to positive production in the differentiated good sector in both countries. As a result each country exports differentiated goods leading to intra-industry trade, which is our focus here. We adopt the standard assumptions in this literature that the home and foreign markets are segmented, and that there are two types of trade costs: melting-iceberg per-unit transport costs $t \geq 1$[23], and an additional fixed cost $f_X$ associated with exporting, which is independent of entrants' potential export sales[24].

Domestic market entry conditions remain as above. Firms face uncertainty about their productivity before entry; and once this has been resolved post-entry they must decide whether to stay or exit. Sales in their domestic market incur a fixed cost and for survival in its domestic market a firm's marginal cost must fall below a survival ceiling ($a_D$ for the home market) determined as before:

$$a_D^{1-\sigma} B = f \qquad \text{and} \qquad \tilde{a}_D^{1-\sigma} \tilde{B} = f \qquad [14]$$

Such firms are now potential exporters, and their profits from entry into the relevant export market are given by[25]:

$$\pi_X(a) = [at]^{1-\sigma} \tilde{B} - f_X \qquad \text{and} \qquad \tilde{\pi}_X(a) = [at]^{1-\sigma} B - f_X$$

Which yields implicit export ceilings $a_X$, $\tilde{a}_X$ that equate the export profits with zero:

$$a_X^{1-\sigma} \tilde{B} = f_X t^{\sigma-1} \qquad \text{and} \qquad \tilde{a}_X^{1-\sigma} B = f_X t^{\sigma-1} \qquad [15]$$

If the implied export ceiling exceeds the domestic cost cutoff then all successful entrants would become exporters. As we show below, this will be the outcome for firms in the leading country once its technology is sufficiently superior. We assume, however, that when technologies are similar the equilibrium features the empirically more relevant case where $a_X < a_D$ and $\tilde{a}_X < \tilde{a}_D$ which indicates the co-existence of exporters and non-

[22] Under our assumptions, production of differentiated goods must lead to exports, because there always exist a proportion of entrants with very low costs, who will always find it profitable to export.
[23] The melting iceberg trade cost assumption implies that differences in firm level "efficiency" apply to both the production and transportation of goods.
[24] In our one period model, $f_X$ includes both sunk export costs and fixed export costs. See Roberts and Tybout (1997) and Clerides, Lach and Tybout (1998) for a discussion of the nature and source of the fixed export costs.
[25] A firm that wished only to export would effectively face a fixed cost of $f + f_X$.

exporters in both countries.[26] From [14] and [15] we can solve for export cutoffs as a function of trade costs and the domestic survival cutoffs in the export market – i.e.

$$\frac{a_D}{\tilde{a}_X} = \frac{\tilde{a}_D}{a_X} = \left[\frac{f_X}{f}\right]^{\frac{1}{\sigma-1}} t \equiv \phi \qquad [16]$$

The existence of non-exporters in both countries in the symmetric case (where they have identical technologies and $a_X = \tilde{a}_X$ and $a_D = \tilde{a}_D$) then requires that $\phi > 1$. Condition [16] also implies that the least efficient importer is more efficient than the least efficient domestic producer in each market (i.e. $a_X < \tilde{a}_D$ and $\tilde{a}_X < a_D$).

### 3.2 Equilibrium cutoffs[27]

In the open economy, expected operating profit for potential home entrants is the sum of expected profit from domestic sales (if $a < a_D$) and from exporting (if $a < a_X$). Where non-exporters exist this can be written as (Appendix 2(a))

$$E(\pi) = \int_0^{a_D} \pi_D(a) dG(a) + \int_0^{a_X} \pi_X(a) dG(a) = fQ(a_D) + f_X Q(a_X) \qquad [17]$$

When all firms export, the corresponding expression is

$$E'(\pi) = fQ(a_D) + f_X Q'(a_D, \phi^{-1}\tilde{a}_D) \qquad [17']$$

where $Q'(x,y) = \int_0^x \left\{ \left[\frac{y}{a}\right]^{\sigma-1} - 1 \right\} dG(a)$. In each case the first term on the right hand side is expected domestic profit and the second, expected profit from exporting. Equating expected operating profit with entry costs and using [16] to write $a_X = \phi^{-1}\tilde{a}_D$, then [17] and [$17'$] give combinations of $a_D$ and $\tilde{a}_D$ yielding zero net expected profits for home entrants:

$$J(a_D, \tilde{a}_D) = \begin{cases} fQ(a_D) + f_X Q(\phi^{-1}\tilde{a}_D) - f_E = 0 & a_D \geq \phi^{-1}\tilde{a}_D \quad [18A] \\ fQ(a_D) + f_X Q'(a_D, \phi^{-1}\tilde{a}_D) - f_E = 0 & a_D < \phi^{-1}\tilde{a}_D \quad [18B] \end{cases}$$

This zero net expected profit schedule is illustrated in Figure 1[28]. If there were no exporting opportunities for home firms (i.e. $\tilde{a}_D = 0$), the solution is identical to the autarky equilibrium cutoff ($\alpha_D$). From that point, the larger the foreign cutoff, the greater the

[26] Country-specific empirical studies typically show that exporters are in the minority, the proportion of exporting firms range from 21% (US) to around 80% (Sweden). See Greenaway, Gullstrand and Kneller (2003) for a review.

[27] As noted in the Introduction, our discussion in this subsection closely parallels Demidova (2005), who provides a careful analysis of the effects of technology differences on productivity cutoffs and welfare in the open economy.

[28] Demidova (2005) similarly plots expected profits as functions of productivity cutoffs.

expected export profits for home firms and hence the lower the home cutoff needed for zero expected profits for entering firms. Note that a higher foreign cutoff increases home entrants' expected profits through two channels – by increasing the range of profitable exporters (i.e. increasing $a_X = \phi^{-1}\tilde{a}_D$) and by improving the foreign business environment ($\tilde{B}$) for existing exporters. However, once the foreign cutoff is sufficiently high (and domestic cutoff sufficiently low), that all home firms export, then [18B] becomes the relevant expected profit equation. Now the range of home exporters depends on the home cutoff, and an increase in the foreign cutoff only increases expected export profits of home entrants by improving the business environment in the foreign market. Lower home cutoffs imply a lower range of home exporters, and higher foreign cutoffs are required to provide the increasing export profits necessary to offset falling domestic profits. Thus the $J$ schedule is asymptotic to the $\tilde{a}_D$ axis. The foreign schedule (designated by $\tilde{J}$) is constructed similarly from

$$\tilde{J}(\tilde{a}_D, a_D) = \begin{cases} f\tilde{Q}(\tilde{a}_D) + f_X\tilde{Q}(\phi^{-1}a_D) - f_E = 0 & \tilde{a}_D > \phi^{-1}a_D \quad [19A] \\ f\tilde{Q}(\tilde{a}_D) + f_X\tilde{Q}'(\tilde{a}_D, \phi^{-1}a_D) - f_E = 0 & \tilde{a}_D < \phi^{-1}a_D \quad [19B] \end{cases}$$

[Figure 1 about here]

The intersection of these schedules determines the equilibrium domestic and export cost ceilings. We restrict attention to the case where there is a unique equilibrium. Both zero expected profit schedules are downward sloping (Appendix 2(b)). When non-exporters are present[29]

$$\left.\frac{\partial a_D}{\partial \tilde{a}_D}\right|_{J=0} = -\left[\frac{\tilde{a}_D}{a_D}\right]^{\sigma-2} \frac{Z(\phi^{-1}\tilde{a}_D)}{Z(a_D)} \frac{1}{t^{\sigma-1}} \quad \text{and} \quad \left.\frac{\partial a_D}{\partial \tilde{a}_D}\right|_{\tilde{J}=0} = -\left[\frac{\tilde{a}_D}{a_D}\right]^{\sigma-2} \left[\frac{\tilde{Z}(\phi^{-1}a_D)}{\tilde{Z}(\tilde{a}_D)} \frac{1}{t^{\sigma-1}}\right]^{-1} \quad [20]$$

The foreign schedule is steeper than the home schedule at the equilibrium as

$$\Delta \equiv 1 - \frac{Z(\phi^{-1}\tilde{a}_D)}{Z(a_D)} \frac{\tilde{Z}(\phi^{-1}a_D)}{\tilde{Z}(\tilde{a}_D)} \frac{1}{[t^{\sigma-1}]^2} > 0 \quad [21]$$

since $\phi^{-1}\tilde{a}_D < a_D; \phi^{-1}a_D < \tilde{a}_D$, and $Z, \tilde{Z}$ are increasing in their arguments, and $t \geq 1$.

The equilibrium labelled $E^M$ in Figure 1 corresponds to the case where both countries have non-exporting firms and identical technologies (the "Melitz" equilibrium). In this case the

[29] The schedules will be concave functions as shown, as long as $\sigma > 2$. See Appendix 2(b).

countries have the same domestic ($a_D^M$) and export ($a_X^M$) cost cutoffs. Note that the opportunity for exporting means that the equilibrium survival ceilings are below the autarky ceilings in both countries - the domestic market selection effect is stronger (i.e. $a_D < \alpha_D$) and there is an additional export market selection effect (i.e. $a_X < a_D$). Both effects reallocate market shares towards more efficient firms and contribute to an aggregate productivity gain. But, as Melitz notes, in this model it is the opening of export markets, not the entry of imports that induces exit of the least productive. Import competition reduces the number of surviving firms, but does not change their distribution.

We can now consider cases where the home country has a superior technology. While this is a comparative statics exercise, it is convenient to describe it in terms of successive improvements in home technology, from a position where both technologies are initially identical. For an improvement in home technology to affect equilibrium cutoffs, it must have an impact on the expected profits of home entrants at the initial equilibrium cutoffs. Our definition of a superior technology in [5], implies that if the home technology improves over the range $[0, a_D]$, expected profits from export sales do not fall (since $G_1(a) \geq G_0(a)$ over the range $[0, a_X]$), and expected profits from domestic sales increase (since $G_1(a) \geq G_0(a)$ over the range $[0, a_D]$ and $G_1(a) > G_0(a)$ for some $a$ in $[0, a_D]$). For any given foreign cutoff, a lower home cost cutoff is required for zero expected profits for domestic firms. The home expected profit schedule therefore shifts down, at least near the initial equilibrium point. Improvements in the home technology therefore move the equilibrium point down the unchanged foreign schedule. The decline in the home survival cutoff pulls down the foreign export cutoff (since $\tilde{a}_X = \phi^{-1} a_D$), reducing expected profits for entrants in the foreign market. The foreign survival cutoff therefore increases to compensate, pulling up the home export cutoff in turn (since $a_X = \phi^{-1} \tilde{a}_D$). For small technology differences the net outcome is an equilibrium such as $E^N$, with the following ordering of cost cutoffs

$$\tilde{a}_D^N > a_D^M > a_D^N > a_X^N > a_X^M > \tilde{a}_X^N \qquad [22]$$

*Proposition 5: In the trading equilibrium, when non-exporters exist in both countries, the leading country has a lower survival ceiling but a higher export ceiling than the* *laggard country. Compared with the symmetric equilibrium, the domestic cutoff is higher (lower) and the export cutoff lower (higher) for the laggard (leading) country.*

For firms in the technologically leading country, self-selection into the domestic market is stronger but into the export market is weaker than for firms in the laggard country. Equation [21] also indicates that the *ex ante* probability of exporting is greater in the leading country[30] - i.e. $G(a_X^N) > \tilde{G}(\tilde{a}_X^N)$. This means entrants in the leading country are more likely to become exporters.

For larger technology differences, the curves intersect below X (where $a_D = \phi^{-1}\tilde{a}_D$) on the foreign schedule in Figure 1 and all home firms export. Since this is the empirically less relevant case we do not investigate it in detail[31], and simply note that the equilibrium is a point such as $E^A$ in Figure 1. The equilibrium cost cutoffs satisfy

$$\tilde{a}_X^A < a_D^A < \tilde{a}_D^A$$

This leads us to[32]:

*Proposition 5A: When all firms export in the leading country, its single cutoff is lower than the survival ceiling and higher than the export ceiling in the laggard country.*

As we shall see, the crucial variables for the comparative statics results below are the elasticities of substitution between the cost cutoffs along the expected profit schedules. Returning to the case where not all home firms export, if we let ^ denote a proportional change and write $\hat{a}_D = -\varepsilon\hat{\tilde{a}}_D$ (or equivalently $\hat{a}_D = -\varepsilon\hat{a}_X$) on the home schedule and $\hat{\tilde{a}}_D = -\tilde{\varepsilon}\hat{a}_D$ (or equivalently $\hat{\tilde{a}}_D = -\tilde{\varepsilon}\hat{\tilde{a}}_X$) on the foreign schedule, then from [20] we have

[30] Recall that $G(a) \geq \tilde{G}(a)$, therefore $G(a_X^N) \geq \tilde{G}(a_X^N) > \tilde{G}(\tilde{a}_X^N)$.

[31] Some details are provided in the Appendices. Note that the foreign expected profit schedule is still steeper than the home schedule at equilibria in this range as shown in Appendix 2(b).

[32] Comparing the equilibrium cutoffs when all home firms export with those when some home firms are non-exporters is complicated by the switch in regime when non-exporters are driven from the home market. Improvements in the home technology raise the home export cutoff as long as it is tied to the foreign survival cost cutoff (i.e. until X), but once that link is broken, further improvements in the home technology mean that the range of home exporters declines in step with the home survival cutoff. From Figure 1 we can infer that $\tilde{a}_D^A > \tilde{a}_D^N > a_D^M > a_D^N > a_D^A > \tilde{a}_X^A$.

$$\varepsilon = \frac{f_X}{f}\frac{H(\phi^{-1}\tilde{a}_D)}{H(a_D)} = \frac{1}{t^{\sigma-1}}\left[\frac{\tilde{a}_D}{a_D}\right]^{\sigma-1}\frac{Z(\phi^{-1}\tilde{a}_D)}{Z(a_D)} \qquad [23A]$$

and

$$\tilde{\varepsilon} = \frac{f_X}{f}\frac{\tilde{H}(\phi^{-1}a_D)}{\tilde{H}(\tilde{a}_D)} = \frac{1}{t^{\sigma-1}}\left[\frac{a_D}{\tilde{a}_D}\right]^{\sigma-1}\frac{\tilde{Z}(\phi^{-1}a_D)}{\tilde{Z}(\tilde{a}_D)} \qquad [23B]$$

The sizes of these elasticities indicate the magnitude of the response of each domestic market cutoff required by a change in the export cutoff to maintain zero expected profits. As such they reflect the relative importance of exports to (expected) profits at the margin, and in Section 4.3 below we show that the profit substitution elasticity of each country gives the ratio of export revenue to domestic revenue[33]. For given survival cutoffs this in turn depends on the size of trade costs and the distribution of unit costs. This is seen most clearly in the right hand terms in [23A] and [23B], where a low value of $\varepsilon$ or $\tilde{\varepsilon}$ occurs if trade costs are sufficiently high (i.e. $t$ is large and $\phi^{-1}$ is small, implying $a_X, \tilde{a}_X$ are small for given $a_D, \tilde{a}_D$ ) or the distribution of unit costs allows for relatively few exporters (i.e. $Z(a_X)/Z(a_D)$, $\tilde{Z}(\tilde{a}_X)/\tilde{Z}(\tilde{a}_D)$ are low for given survival cutoffs).

What is known of the properties of these profit substitution elasticities at the trading equilibrium? First, [21] implies that $\Delta = 1 - \varepsilon\tilde{\varepsilon} > 0$, so that in equilibrium the product of these elasticities is always less than unity. Second, [23A] and [23B] show that each elasticity individually is always less than unity if $f_X < f$. Third, at equilibria where the home country has a superior technology, $\tilde{\varepsilon} < 1$ since $t \geq 1$, $a_D \leq \tilde{a}_D$, $\phi^{-1}a_D < \tilde{a}_D$ and $\tilde{Z}(.)$ is an increasing function. Further as the home technology improves, the equilibrium $\tilde{\varepsilon}$ declines, since the equilibrium values of $a_D$ falls and $\tilde{a}_D$ rises. Fourth, beyond some point, the equilibrium value of $\varepsilon$ must increase with improvements in the home technology. Figure 1 shows that the equilibrium value of the ratio $\tilde{a}_D/a_D$ increases as the home technology improves, and, while technological improvements could affect the shape of the $Z(.)$ function in various ways, we know that the equilibrium values of $\phi^{-1}\tilde{a}_D$ and $a_D$ converge as the home technology improves, implying that $Z(\phi^{-1}\tilde{a}_D)/Z(a_D)$ converges to unity from below. Hence there is a strong presumption (Appendix 2(c)) that the equilibrium

[33] Bernard, Redding and Schott also highlight the impotance of the "wedge between revenue in the export and domestic markets …in determining how trade liberalisation increases the expected value of entry into an industry" (2004; p18).

value of $\varepsilon$ increases as the home technology improves. If $f_X > f$ then the equilibrium $\varepsilon$ will eventually exceed unity, since [23A] shows that $\varepsilon = f_X / f$ at the point where the last non-exporter is driven from the home market (i.e. when $\phi^{-1}\tilde{a}_D = a_D$). Finally, in the symmetric equilibrium where both countries have identical technologies $\varepsilon = \tilde{\varepsilon} < 1$. As the home technology improves, $\tilde{\varepsilon}$ falls and our presumption is that $\varepsilon$ increases. Hence we conclude that $\varepsilon - \tilde{\varepsilon} \geq 0$, and that an improvement in the home technology widens this gap. For ease of reference these properties are summarised in:

*Proposition 6: In equilibria where not all firms export (a) the product of the profit substitution elasticities is less than unity* ($\Delta = 1 - \varepsilon\tilde{\varepsilon} > 0$)*; (b) the laggard's elasticity is less than unity* ($\tilde{\varepsilon} < 1$) *and falls as the leader's technology improves; (c) if* $f_X < f$ *then the leader's elasticity is always less than unity; (d) the leader's elasticity increases as its technology improves and eventually exceeds unity if* $f_X > f$ *; (e) the leader's elasticity is never less than the laggard's elasticity* ($\varepsilon - \tilde{\varepsilon} \geq 0$) *and the difference between them increases as the leader's technology improves.*

### 3.3 Numbers of firms

The equilibrium cutoffs derived above are only relevant where market sizes are such that firms can exist in both countries. To derive the corresponding masses of firms, we note that expenditure on the differentiated good in either country is now split between domestic output and imports – i.e.

$$\beta L = R_D + \tilde{R}_X \qquad \text{and} \qquad \beta\tilde{L} = \tilde{R}_D + R_X \qquad [24]$$

We can write $R_D$ as the domestic revenue of the representative home firm ($r(\bar{a}_D)$) multiplied by the mass of home firms ($M$), and $\tilde{R}_X$ as the export revenue of the representative foreign exporter ($\tilde{r}(\tilde{\bar{a}}_X)$) multiplied by the mass of foreign exporters, which in turn is the mass of foreign firms ($\tilde{M}$) multiplied by the probability of a foreign firm exporting ($\tilde{G}(\tilde{a}_X)/\tilde{G}(\tilde{a}_D)$). This yields

$$\beta L = Mr(\bar{a}_D) + \tilde{M}\frac{\tilde{G}(\tilde{a}_X)}{\tilde{G}(\tilde{a}_D)}\tilde{r}(\tilde{\bar{a}}_X)$$

where $r(\bar{a}_D) = \frac{f}{[1-\rho]} \frac{H(a_D)}{G(a_D)}$ and $\tilde{r}(\bar{\tilde{a}}_X) = \frac{f_X}{[1-\rho]} \frac{\tilde{H}(\tilde{a}_X)}{\tilde{G}(\tilde{a}_X)}$ (Appendix 2(d)). Using the definition of $\tilde{\varepsilon}$ from [23B], we can write $\frac{\tilde{G}(\tilde{a}_X)}{\tilde{G}(\tilde{a}_D)} \tilde{r}(\bar{\tilde{a}}_X) = \frac{f}{[1-\rho]} \frac{\tilde{H}(\tilde{a}_D)}{\tilde{G}(\tilde{a}_D)} \tilde{\varepsilon} = \tilde{r}(\bar{\tilde{a}}_D)\tilde{\varepsilon}$. When home non-exporters exist we can apply the same procedure to the foreign market, and [24] becomes

$$r(\bar{a}_D)M + \tilde{\varepsilon}\tilde{r}(\bar{\tilde{a}}_D)\tilde{M} = \beta L \qquad [25A]$$

$$\varepsilon r(\bar{a}_D)M + \tilde{r}(\bar{\tilde{a}}_D)\tilde{M} = \beta \tilde{L} \qquad [25B]$$

This system can be solved for the equilibrium mass of firms in each country

$$M = \frac{\beta L}{r(\bar{a}_D)\Delta}\left[1 - \frac{\tilde{L}}{L}\tilde{\varepsilon}\right] \text{ and } \tilde{M} = \frac{\beta \tilde{L}}{\tilde{r}(\bar{\tilde{a}}_D)\Delta}\left[1 - \varepsilon \frac{L}{\tilde{L}}\right] \qquad [26]$$

The conditions for a positive mass of firms in each individually and in both simultaneously (in which case we have intra-industry trade in differentiated products) are given by

$$1 > \frac{\tilde{L}}{L}\tilde{\varepsilon} \text{ for } M > 0 \qquad [27A]$$

$$1 > \frac{L}{\tilde{L}}\varepsilon \text{ for } \tilde{M} > 0 \qquad [27B]$$

and for a positive mass in both countries

$$\tilde{\varepsilon}^{-1} > \frac{\tilde{L}}{L} > \varepsilon \qquad [27C].$$

Equation [27C] puts bounds on the range of country size differences that permit intra-industry trade depending on trade costs and technology differences. There are no firms in the home market if the left inequality fails, and no firms in the foreign market if the right inequality fails. Despite its more efficient technology, the leading country may have no firms if it is sufficiently small. If both countries have identical technologies, then $\varepsilon = \tilde{\varepsilon} < 1$, and there is a range of relative country sizes which satisfy [27C]. What happens to this range as the home technology improves? In Proposition 6 we concluded that $\tilde{\varepsilon}^{-1}$ becomes larger, for given trade costs. The leading country can be relatively smaller and still produce differentiated products. But our presumption is that $\varepsilon$ increases as the home technology improves, implying that the laggard is less likely to produce differentiated products the larger the technology difference. If both countries are the same size, the case examined by Melitz and Demidova, the laggard only produces differentiated products as long as $\varepsilon < 1$.

*Proposition 7: In the trading equilibrium (a) the industry in the smaller country will shut down if the difference in country size is sufficiently large; (b) a superior technology will allow the industry in the smaller country to survive for larger differences in country size (e.g. if $\tilde{L} > L$ and $\tilde{\varepsilon}^{-1} > \tilde{L}/L$); and (c) the industry in the larger country will shut down if its technology is sufficiently backward (e.g. if $\tilde{L} > L$ and $\varepsilon > \tilde{L}/L$).*

Once the technology difference is so large that the foreign industry shuts down we are in the specialisation case where all entry occurs in the leading country, as discussed in Section 3.1.

## 4. The Effects of Trade

What we have shown so far is how technological differences across asymmetric countries can lead to differences in firm level self-selection into the domestic and export markets as well as the number of domestic firms. We now make comparisons between the autarky and trade equilibria for each country, across trading equilibria and across countries. We also consider the effects of a reduction in unit trade costs.

### 4.1 Gains from trade

The expressions for the aggregate price index and welfare per capita are the same as in the closed economy (i.e. [12] and [13]: $P = a_D \left[ \frac{\beta L}{f} \right]^{\frac{1}{1-\sigma}} \eta$ and $u = \psi P^{-\beta}$

As shown above, for both countries opening up to trade reduces survival cost ceilings and hence raises productivity, which is consistent with Melitz (2003). As a consequence, aggregate prices also fall, which improves welfare in each country[34]. It is worth noting that the gains from trade are independent of changes in the total number of varieties available to consumers, as described by Krugman (1980) in a homogeneous firm framework. As Melitz notes, and we show below, in this setting the total number of varieties available may fall or rise as a result of trade.

[34] But as Demidova (2005) shows in her Proposition 2 a country loses if its trading partner becomes more efficient. This is also the implication of our Proposition 5.

### 4.2 Industry costs

In the trading equilibrium, the total operating cost on domestic sales of the representative firm is $f + \rho r(\bar{a}_D)$, and the corresponding cost for exports is $f_X + \rho r(\bar{a}_X)$. As shown above, industry total cost can be written in terms of "markups" on entrant (expected) fixed cost ($F = f_E + fS_D + f_X S_X$) which now includes those associated with exporting. For the home industry this gives

$$TC = M_E \sigma F = M\sigma \frac{F}{S_D} = M_X \sigma \frac{F}{S_X}$$

where $M_E$ denotes total entrants, $M(= S_D M_E)$ surviving domestic producers, $M_X(= S_X M_E)$ exporters, $S_D = G(a_D)$ the probability that a domestic entrant survives and $S_X = G(a_X)$ the probability that a domestic entrant exports. Making the corresponding comparisons with autarky (whose values are denoted with superscript A), from [10], we find

$$\frac{F}{S_D} - \frac{F^A}{S_D^A} = \frac{f_E[G(\alpha_D) - G(a_D)] + f_X G(a_X)]}{G(a_D)G(\alpha_D)[f_E + fS_D^A]} > 0$$

since $G(a_D) < G(\alpha_D)$. An equivalent expression applies for the foreign country. Hence

> *Proposition 8: Industry total cost per surviving firm is higher in the trading equilibrium than in autarky, in both countries.*

In this sense firms are larger on average in the trading equilibrium than in autarky in both countries.

A comparison between two trading equilibria (0,1) where the home country has an improved technology in 1, yields

$$\frac{F^1}{S_D^1} - \frac{F^0}{S_D^0} = f_E \left[ \frac{G(a_D^0) - G(a_D^1)}{G(a_D^0)G(a_D^1)} \right] + f \left[ \frac{G(a_X^1)}{G(a_D^1)} - \frac{G(a_X^0)}{G(a_D^0)} \right] > 0$$

A technology improvement reduces the home equilibrium survival probability ($G(a_D^1) < G(a_D^0)$) and increases the home equilibrium export probability ($G(a_X^1) > G(a_X^0)$). The corresponding expression for the foreign country is negative, since $\tilde{G}(\tilde{a}_D^1) < \tilde{G}(\tilde{a}_D^0)$ and $\tilde{G}(\tilde{a}_X^1) < \tilde{G}(\tilde{a}_X^0)$. To summarise

*Proposition 9: An improvement in the technology in the leading country leads to a new trading equilibrium in which industry total cost per surviving firm is higher in the leading country and lower in the laggard country.*

The greater the technological superiority of the leading country, the larger its firms on average and the smaller the firms in the laggard on average.

**4.3 Balance of trade in differentiated products.**

Using [25A] and [25B], we calculate the total revenues of firms located in the two countries

$$R = R_D + R_X = [1+\varepsilon]r(\bar{a}_D)M \quad \text{and} \qquad \tilde{R} = \tilde{R}_D + \tilde{R}_X = [1+\tilde{\varepsilon}]\tilde{r}(\tilde{\bar{a}}_D)\tilde{M}$$

This shows that the profit substitution elasticities capture the ratio of export revenue to domestic revenue in each country. Dividing these revenues by country expenditures on differentiated products and using [26] gives us "expenditure ratios"

$$\frac{R}{\beta L} = \frac{[1+\varepsilon]}{\Delta}\left[1-\tilde{\varepsilon}\frac{\tilde{L}}{L}\right] \text{ and } \qquad \frac{\tilde{R}}{\beta \tilde{L}} = \frac{[1+\tilde{\varepsilon}]}{\Delta}\left[1-\varepsilon\frac{L}{\tilde{L}}\right]$$

The influence of country size and technology differences on the trade balance in differentiated products is then apparent. If both countries have identical technologies (i.e. $\varepsilon = \tilde{\varepsilon}$, and $\Delta = 1-\varepsilon^2 = [1-\varepsilon][1+\varepsilon]$), then

$$L \geq \tilde{L} \text{ implies that } \frac{R}{\beta L} \geq 1 \geq \frac{\tilde{R}}{\beta \tilde{L}}.$$

The industry in the larger country has a greater share of total expenditure on differentiated products in the trading equilibrium than in autarky.

If both countries are of identical size (i.e. $L = \tilde{L}$), then

$$\frac{R}{\beta L} = 1 + \frac{\varepsilon - \tilde{\varepsilon}}{\Delta} \qquad \text{and} \qquad \frac{\tilde{R}}{\beta \tilde{L}} = 1 - \frac{\varepsilon - \tilde{\varepsilon}}{\Delta}$$

In Proposition 6 we concluded that if the leading country has a superior technology then $\varepsilon - \tilde{\varepsilon} \geq 0$, and further improvements in the leading country's technology, relative to a constant laggard technology, lead to equilibria in which $\varepsilon - \tilde{\varepsilon}$ is larger[35]. We now see that this reflects a switch in expenditure towards firms in the leading country. Hence

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[35] Sign $d\left[[\varepsilon-\tilde{\varepsilon}]/\Delta\right]$ = sign$\left\{[1-\tilde{\varepsilon}^2]d\varepsilon - [1-\varepsilon^2]d\tilde{\varepsilon}\right\}$. This is positive as long as both elasticities are less than unity, since Proposition 6 implies $d\varepsilon > 0$ and $d\tilde{\varepsilon} < 0$ when the home technology improves. If the home profit elasticity exceeds unity, then the existence of the foreign industry is in doubt anyway.

*Proposition 10: In the trading equilibrium (a) if both countries have identical technologies the larger country has a trade surplus in differentiated products; (b) if both countries are of identical size, then the leading country has a trade surplus in differentiated products; and (c)* an *improvement in the technology of the leading country tends to raise (reduce) its trade surplus (deficit) with the laggard country..*

### 4.4 Numbers of firms.

We are now in a position to compare the numbers of firms located in and selling in the two markets in the trading equilibrium and in autarky. Equating industry total costs and total revenue, the masses of firms are[36]

$$M = \frac{S_D}{\sigma F}\beta L \frac{[1+\varepsilon]}{\Delta}\left[1-\tilde{\varepsilon}\frac{\tilde{L}}{L}\right] \text{ and } \quad \tilde{M} = \frac{\tilde{S}_D}{\sigma \tilde{F}}\beta \tilde{L}\frac{[1+\tilde{\varepsilon}]}{\Delta}\left[1-\varepsilon\frac{L}{\tilde{L}}\right]$$

$$M_X = \frac{S_X}{S_D}M \; ; \; \tilde{M}_X = \frac{\tilde{S}_X}{\tilde{S}_D}\tilde{M} \; ; \qquad M_T = M + \tilde{M}_X \text{; and } \quad \tilde{M}_T = \tilde{M} + M_X$$

where $M_T, \tilde{M}_T$ denote the numbers of firms selling in each market in the trading equilibrium. Comparing these to those in autarky gives

$$\frac{M}{M^A} = \frac{F_A/S_D^A}{F/S_D}.\frac{[1+\varepsilon]}{\Delta}\left[1-\tilde{\varepsilon}\frac{\tilde{L}}{L}\right] \qquad \text{and} \qquad \frac{\tilde{M}}{\tilde{M}^A} = \frac{\tilde{F}_A/\tilde{S}_D^A}{\tilde{F}/\tilde{S}_D}.\frac{[1+\tilde{\varepsilon}]}{\Delta}\left[1-\varepsilon\frac{L}{\tilde{L}}\right]$$

Each of these expressions is composed of a cost ratio multiplied by an expenditure ratio. Proposition 8 indicates that the cost ratios are both less than unity. Proposition 10 indicates that the expenditure ratios are both equal to unity if the countries are identical in size and technologies, otherwise the larger (smaller) country or the leading (laggard) country has an expenditure ratio greater (less) than unity. This leads to

*Proposition 11. In the trading equilibrium (a) identical countries will each have fewer surviving firms than in autarky; (b) if countries have identical technologies, the smaller country will have fewer surviving firms than in autarky, but the larger country may have more than in autarky if the size difference is sufficiently large; and (c) if countries are the same size, the laggard country will have fewer firms than in autarky, but the leading country may have more or less firms than in autarky.*

[36] We also considered the numbers of entrants, but all comparisons of entrant numbers across countries and equilibria gave ambiguous results and are not reported here.

Firms are larger on average in the trading equilibrium in both countries. Aggregate world expenditure on differentiated products is unchanged. Hence if countries are identical, the number of firms in each falls. If they have identical technologies, their average firms are the same size, but Proposition 10 indicates that firms in the larger country receive a disproportionately larger share of aggregate expenditure. The number of firms in the smaller country therefore unambiguously declines due to the increase in average firm size and shift in expenditure. But there are opposing forces on the number of firms in the larger country, which tends to fall due to the increase in average firm size, and to rise due to the increase in expenditure.[37] If the countries are identical in size, then Proposition 10 indicates that firms in the leading country receive a disproportionately larger share of total expenditure. Again firms in both countries are larger than in autarky so that the number of firms in the laggard country unambiguously falls on both counts. The outcome for the leading country depends on the balance of the average firm size and expenditure switching effects.[38]

Now compare the numbers of varieties available to consumers. This is where the gains from trade arise in the homogeneous firm variants of this model, although not here, as we saw above. As one might expect, the full expressions are quite cumbersome, so we consider special cases where the countries differ in at most one dimension. If both countries are identical in terms of size and technologies we find

$$\frac{M_T}{M^A} = \frac{F^A / S_D^A}{F / [S_D + S_X]}$$

Whether the number of varieties available for consumption is greater or less than in autarky is ambiguous even in this case, depending on relative fixed costs and the probabilities of survival and exporting. We can show that

$$M_T > (<)\, M^A \text{ as } f_E S_X > (<) f_E [S_D^A - S_D] + S_D^A S_X [f_X - f]$$

Since the probability of survival is lower than in autarky ($S_D^A > S_D$), a sufficient condition for the number of varieties to fall is that $f_X > f + [f_E / S_D^A]$ (which itself requires that $f_X > f$). Hence we conclude;

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[37] If we consider the extreme case where $\tilde{M} = 0$, then $R = \beta[L + \tilde{L}]$ and $\frac{M}{M^A} = \frac{[F^A / S_D^A]}{[F/S]} \frac{L + \tilde{L}}{L}$.

[38] Again, if we consider the extreme case where $\tilde{M} = 0$, then $R = 2\beta L$ and $\frac{M}{M^A} = 2 \frac{[F^A / S_D^A]}{[F/S]}$.

*Proposition 12: Two identical countries will consume fewer varieties in the trading equilibrium than in autarky if fixed costs of exporting are sufficiently high.*

If both countries have identical technologies, but differ in size, it is important to consider how market size impacts on the number of varieties available to consumers in both markets. For the home country we have

$$M_T = M + \tilde{M}_X = \frac{\beta}{\sigma F[1-\varepsilon]}\left\{S_D\left[L - \varepsilon\tilde{L}\right] + S_X\left[\tilde{L} - \varepsilon L\right]\right\}$$

An increase in the size of the home market will increase the number of domestically produced varieties, but reduce the number of foreign produced varieties and hence the number of varieties imported by home consumers. Since[39] $S_D > \varepsilon S_X$ the former effect dominates and the number of varieties consumed in an expanding market increases. An increase in the size of the foreign market, other things equal, will reduce the number of varieties produced in the home market, but increase the number produced in the foreign market itself, and hence the number available to home consumers through imports. In this case the total effect is ambiguous, with the latter effect dominating if $S_X > \varepsilon S_D$. Thus it is possible that an expansion of either market leads to an increase in the number of varieties consumed in both, if the proportion of exporters is sufficiently high (i.e. $\varepsilon < S_X / S_D$ ).

If the home country has the larger market, it will both produce more varieties in autarky (as $M/\tilde{M} = [L - \tilde{L}\varepsilon]/[\tilde{L} - L\varepsilon]$) and, since the same proportion of varieties produced is exported from each country, it will consume more varieties in the trading equilibrium than the foreign country. But what then happens to the relative strength of this home market effect in moving from autarky to the trading equilibrium? For this we consider

$$\text{sign}\left\{\frac{M_T}{\tilde{M}_T} - \frac{M^A}{\tilde{M}^A}\right\} = \text{sign}\left\{[L - \tilde{L}][\varepsilon S_D - S_X]\right\}$$

Whether the home market effect is strengthened or weakened in the trading equilibrium, depends on whether an expansion in either market increases or reduces the number of varieties available in the other.

If both countries are the same size, but have different technologies, then

[39] Recall $\varepsilon < 1$ in this case.

$$\frac{M}{\tilde{M}} = \frac{\tilde{F}/\tilde{S}_D}{F/S_D} \cdot \frac{1+\varepsilon}{1+\tilde{\varepsilon}} \frac{[1-\tilde{\varepsilon}]}{[1-\varepsilon]}$$

this expression is composed of two ratios, the second of which exceeds unity when $\varepsilon > \tilde{\varepsilon}$. But the first is less than unity,[40] which leaves the overall outcome ambiguous. So we do not know which country produces the larger number of varieties. Not surprisingly, this ambiguity carries over to the numbers of varieties available to consumers, where

$$M_T > (<) \tilde{M}_T \quad \text{as} \qquad M\left[1 - \frac{S_X}{S_D}\right] > (<) \tilde{M}\left[1 - \frac{\tilde{S}_X}{\tilde{S}_D}\right]$$

Since $S_X / S_D$ is the proportion of surviving firms that export, [1-$S_X / S_D$ ] is the proportion of domestic products sold only on the domestic market. Thus the country with the larger number of non-exporters has the larger number of varieties available to its consumers.

### 4.4 Changes in unit trade costs

The effects of a proportional change in per unit trade costs ($\hat{t}$) on equilibrium cutoffs, when not all home firms export can be solved by differentiating the expected profit conditions [18A] and [19A] (see Appendix 2(e)) obtaining

$$\begin{bmatrix} 1 & \varepsilon \\ \tilde{\varepsilon} & 1 \end{bmatrix} \begin{bmatrix} \hat{a}_D \\ \hat{\tilde{a}}_D \end{bmatrix} = \begin{bmatrix} \varepsilon \\ \tilde{\varepsilon} \end{bmatrix} \hat{t}$$

which yields solutions

$$\hat{a}_D = \frac{\varepsilon}{\Delta}[1-\tilde{\varepsilon}]\hat{t} \qquad \text{and} \qquad \hat{a}_X = \frac{1}{\Delta}[\tilde{\varepsilon}-1]\hat{t} \qquad [28A]$$

$$\hat{\tilde{a}}_D = \frac{\tilde{\varepsilon}}{\Delta}[1-\varepsilon]\hat{t} \qquad \text{and} \qquad \hat{\tilde{a}}_X = \frac{1}{\Delta}[\varepsilon-1]\hat{t} \qquad [28B]$$

$$\hat{a}_D - \hat{\tilde{a}}_D = \frac{1}{\Delta}[\varepsilon-\tilde{\varepsilon}]\hat{t} \quad \text{and} \qquad \hat{a}_X - \hat{\tilde{a}}_X = \frac{1}{\Delta}[\tilde{\varepsilon}-\varepsilon]\hat{t} \qquad [29]$$

Given our conclusion in Proposition 6 that $\varepsilon - \tilde{\varepsilon} \geq 0$ and that this gap rises as the home technology improves, equation [29] implies that the leading country has the greater proportionate reduction (increase) in its survival (export) ceiling.

[40] To see this start from the trading equilibrium where both countries have identical technologies. Then consider a trading equilibrium where home firms have a superior technology. By Proposition 9, industry total cost per surviving firm has risen in the leading country and fallen in the laggard country in the latter equilibrium, and hence is higher in the leading than in the laggard country.

If $f_X < f$ or the technology difference is small (so that we are close to the Melitz equilibrium and both $\varepsilon$ and $\tilde{\varepsilon}$ are less than unity), we can write

$$\hat{t} > \hat{a}_D > \hat{\tilde{a}}_D > 0 > \hat{a}_X > \hat{\tilde{a}}_X > -\hat{t}$$

A fall in trade costs ($\hat{t} < 0$) then leads to an unambiguous fall in both domestic cutoffs and an unambiguous increase in both export cutoffs. However, if $f_X > f$ and the technology gap is large enough, then $\varepsilon > 1$ in the initial equilibrium. In this case the foreign country has an increase in its survival ceiling and a fall in its export cutoff with a fall in trade costs. But note that this outcome is only relevant if the laggard country has a positive mass of firms in the initial equilibrium – i.e. from [27B] if $\tilde{L}/L > \varepsilon$. Falling per unit trade costs can only raise the survival cutoff in the laggard if it is the larger country.

> *Proposition 13: When non-exporters exist in both countries, a reduction in unit trade costs always leads to a fall in the survival cutoff in the leading country, thus increasing its welfare. However, the laggard country may face an increase in its survival cutoff and therefore a reduction in its welfare if it is the larger country and the technology gap is sufficiently large.*

This striking result that the positive productivity effect of trade liberalization for similar countries might be reversed for the laggard when technology differences are sufficiently large, can be explained as follows. Recall that they key channel for productivity gains in the Melitz model is the export opportunities induced by trade. When countries are symmetric, a reduction in trade costs must lead to a rise in the export cutoff and the probability of exporting in all countries. Market shares are reallocated towards more productive firms and the domestic survival cutoff falls. Aggregate productivity increases and welfare is improved. When the technology gap is large (i.e. $\varepsilon > 1$), however, falling trade costs may actually *reduce* the export cutoff in the laggard. Since $\tilde{a}_X = a_D / \phi$, and both $\phi$ and $a_D$ fall with the decline in trade costs, the net outcome depends on their relative changes. From [28A] we see that a larger technology gap (smaller $\tilde{\varepsilon}$) leads to a larger reduction in $a_D$, whose impact in reducing the competitiveness of laggard exporters may overwhelm the advantage they receive from the fall in trade costs.

## 5. Conclusions

Our aim in this paper was to investigate the effects of technology differences in the Melitz model. Differences in technology across countries were characterized by differences in the distributions of unit costs that entrants drew from before making their production decision. Because only firms with unit costs below some (endogenous) threshold survive, it is differences in the low-cost tail of the distributions that are relevant. We therefore defined the home country as having a superior technology over a range of unit costs if its cost distribution implied that home firms were no less likely to draw a cost below any point in this range and were more likely to draw a unit cost below some point in this range. If the home country had a superior technology over the range up to the foreign survival cutoff, then the expected profits of home entrants would be positive at that cutoff. The home cutoff would therefore have to be smaller. Ensuring that the representative firm in the leading country was more efficient than its counterpart in the laggard, required a stronger condition. But even this was insufficient to determine whether the leading country's representative firm would be larger in autarky.

In a trading equilibrium where the home country has a superior technology, the home survival cutoff is lower and the home export cutoff is higher than the foreign. This implies that the most efficient group of firms, is the laggard's exporters, followed by the leader's exporters, the leader's domestic firms and the laggard's domestic firms. Both countries gain from trade, but the foreign survival cutoff is closer to its autarky value than it would be if the trading partners had identical technologies. Unless it is significantly smaller, the country with the superior technology will have a higher level of welfare.

We were somewhat limited in what we could say about the size and hence numbers of the representative firms in the two countries. Firm size depends on fixed costs and the probability of firm survival (and firm exporting in the trading equilibrium). While the probability of an entrant exporting is higher in the leading than in the laggard country, it is possible that the probability of an entrant surviving could be higher in either country. But surviving firms are larger on average in the trading equilibrium than in autarky in both countries, implying fewer firms survive in both if they are sufficiently similar in size and technology, and in the smaller or laggard country otherwise.

Whether the technological superiority of the leading country can be such as to force the closure of the differentiated product sector in the laggard, depends on relative country size and the relative size of the fixed costs involved in exporting. Standard economic geography effects are in operation, so that the differentiated product industry will tend to agglomerate in the larger country. Suppose the two countries are approximately equal in size. Then if the (additional) fixed costs of exporting are less than the fixed costs of production, the industry in the laggard is able to survive. But if the fixed costs of exporting are higher there is some technology difference that will force the industry in the laggard to close down.

Finally, a fall in (unit) trade costs will benefit both trading partners, as long as the technology difference is not too large. Both gain from a fall in their survival cutoffs. The leading country has the larger proportionate reduction in its cutoff. But if the technology difference is large, the fixed costs of exporting are relatively high and the laggard country is the larger, then it may lose from a reduction in unit trade costs.

## Appendix 1. Closed economy equilibrium

### (a) Survival costs ceilings

Integration by parts gives us

$$Q(x) \equiv \int_0^x \left[ \left( \frac{x}{a} \right)^{\sigma-1} - 1 \right] dG(a) \ = \left[ \left( \frac{x}{a} \right)^{\sigma-1} - 1 \right] G(a) \Bigg|_{a=0}^{a=x} - \int_0^x G(a) d \left( \frac{x}{a} \right)^{\sigma-1}$$

$$= (\sigma - 1) x^{\sigma-1} \int_0^x \frac{G(a)}{a^\sigma} da \qquad \text{[A1]}$$

Then $Q(x)$ is increasing in $x$, and if we assume $\lim_{a \to 0} \frac{G(a)}{a^\sigma} = 0$, then $Q(x) \in [0, \infty)$.

If the home country has a superior technology over the range [0,x], then $G(a) \geq \tilde{G}(a)$ over this range and $G(a) > \tilde{G}(a)$ for some $a$ in this range. This implies that $\frac{G(a)}{a^\sigma} \geq \frac{\tilde{G}(a)}{a^\sigma}$ for all $a$ in the range, and $\frac{G(a)}{a^\sigma} > \frac{\tilde{G}(a)}{a^\sigma}$ for some $a$ in this range, so that

$\int_0^x \frac{G(a)}{a^\sigma} da > \int_0^x \frac{\tilde{G}(a)}{a^\sigma} da$. From [A1] we then have $\qquad Q(x) > \tilde{Q}(x) \qquad$ [A2]

Suppose that the home country has a superior technology over the range $[0, \tilde{\alpha}_D]$. In equilibrium $E(\pi) = fQ(\alpha_D) = f_E = f\tilde{Q}(\tilde{\alpha}_D) = E(\tilde{\pi})$

Therefore [A2] implies that $\alpha_D \leq \tilde{\alpha}_D$.

### (b) Size of representative firm

To isolate the influence of differences in cost distributions, we compare these representative unit costs at a common cutoff (say $x$) obtaining from [7] after some rearrangement,

$$\left[\bar{\alpha}(x)\right]^{1-\sigma} - \left[\tilde{\bar{\alpha}}(x)\right]^{1-\sigma} = \int_0^x a^{1-\sigma} \left[ \frac{g(a)}{G(x)} - \frac{\tilde{g}(a)}{\tilde{G}(x)} \right] da \qquad \text{[A3]}$$

Since $\sigma > 1$, $\left[\bar{\alpha}(x)\right]^{1-\sigma}$ is a decreasing function which implies $\bar{\alpha}(x) < \tilde{\bar{\alpha}}(x)$ if [A3] is positive. We observe that $\int_0^x \frac{g(a)}{G(a)} da = \int_0^x \frac{\tilde{g}(a)}{\tilde{G}(a)} da = 1$, so that the integral on the right consists of a decreasing function of unit costs ($a^{1-\sigma}$) weighted by differences in terms that have the same sum. This indicates that the home representative firm will be more efficient at any common cutoff, if the weights are positive at small unit costs and negative at high unit costs – i.e. the home cost distribution is relatively biased towards lower cost firms.

We wish to show that $\int_0^x h(a)\frac{g(a)}{G(x)}da > \int_0^x h(a)\frac{\tilde{g}(a)}{\tilde{G}(x)}da$ where $h'(a) < 0$ and

$\tilde{G}(a) \succ_{RHSD} G(a)$ which means that $\frac{\tilde{g}(a)}{\tilde{G}(a)} > \frac{g(a)}{G(a)}$ for all $a \in [0, x]$. Note that the latter

implies that $\frac{d\frac{\tilde{G}(a)}{G(a)}}{da} = \frac{\tilde{g}(a)G(a) - g(a)\tilde{G}(a)}{\left[G(a)\right]^2} > 0$ so that $\frac{\tilde{G}(a)}{G(a)}$ is increasing in *a*.

We can rewrite $\int_0^x h(a)\frac{dG(a)}{G(x)} = h(a)\frac{G(a)}{G(x)}\bigg|_0^x - \int_0^x \frac{G(a)}{G(x)}dh(a) = h(x) - \int_0^x \frac{G(a)}{G(x)}h'(a)da$ ,

which gives $\int_0^x h(a)\frac{g(a)}{G(x)}da - \int_0^x h(a)\frac{\tilde{g}(a)}{\tilde{G}(x)}da = \int_0^x h'(a)\left[\frac{\tilde{G}(a)}{\tilde{G}(x)} - \frac{G(a)}{G(x)}\right]da$ [A4]

Since $\frac{\tilde{G}(a)}{G(a)}$ is increasing in *a*, we have $\frac{\tilde{G}(x)}{G(x)} > \frac{\tilde{G}(a)}{G(a)}$ and thus $\frac{G(a)}{G(x)} > \frac{\tilde{G}(a)}{\tilde{G}(x)}$ for *x* > *a*. Each term in the integral [A4] is the product of two negative terms and is therefore positive.

**(c) Number of firms**

From [2] the revenue of the representative firm is

$$r(\bar{\alpha}) = A\left[\frac{\bar{\alpha}}{\rho}\right]^{1-\sigma}$$

From the definitions of A and B, and [4] we have $[1-\rho]\rho^{\sigma-1}A = B = \alpha_D^{\sigma-1}f$ which gives

$$r(\bar{\alpha}) = \frac{B}{[1-\rho]\rho^{\sigma-1}}\left[\frac{\bar{\alpha}}{\rho}\right]^{1-\sigma} = \frac{f}{[1-\rho]}\alpha_D^{\sigma-1}\bar{\alpha}^{1-\sigma} \qquad \text{[A5]}$$

Using [7] to substitute for $\bar{\alpha}$ , we obtain

$$r(\bar{\alpha}) = \frac{f}{[1-\rho]}\alpha_D^{\sigma-1}\frac{Z(\alpha_D)}{G(\alpha_D)} = \frac{f}{[1-\rho]}\frac{H(\alpha_D)}{G(\alpha_D)}$$

Then $M = \frac{\beta L}{r(\bar{\alpha})} = \frac{[1-\rho]\beta L}{f}\frac{G(\alpha_D)}{H(\alpha_D)}$

which, since $Q(a_D) = f_E/f$ from [5] and $S_D = G(a_D)$ , can be rewritten as

$$M = \frac{[1-\rho]\beta L S_D}{f_E + f S_D}$$

**(c) Price Level**

From [6] and [11] the aggregate price index can be written as:

$$P = M^{\frac{1}{1-\sigma}} p(\bar{\alpha}) = \left[\frac{\beta L}{r(\bar{\alpha})}\right]^{\frac{1}{1-\sigma}} p(\bar{\alpha})$$

Since $p(\bar{\alpha}) = \dfrac{\bar{\alpha}}{\rho}$, and [9] implies that $\left[r(\bar{\alpha})\right]^{\frac{1}{1-\sigma}} = A^{\frac{1}{1-\sigma}}\left[\dfrac{\bar{\alpha}}{\rho}\right]$ we have $P = \left[\dfrac{\beta L}{A}\right]^{\frac{1}{1-\sigma}}$.

From the definitions of $A$ and $B$ and [A4] we have $A = \dfrac{B}{[1-\rho]\rho^{\sigma-1}} = \dfrac{\alpha_D^{\sigma-1} f}{[1-\rho]\rho^{\sigma-1}}$,

which gives $P = \left[\dfrac{\beta L[1-\rho]\rho^{\sigma-1}}{\alpha_D^{\sigma-1} f}\right]^{\frac{1}{1-\sigma}} = \alpha_D \left[\dfrac{\beta L}{f}\right]^{\frac{1}{1-\sigma}} \eta$ [A6]

where $\eta = \{[1-\rho]^{\sigma-1}\rho\}^{-1} > 0$.

## Appendix 2 . Equilibrium in the Open economy

### (a) Expected profit schedules

When some home firms are non-exporters

$E(\pi) = \int_0^{a_D} \pi_D(a)dG(a) + \int_0^{a_X} \pi_X(a)dG(a)$.

From [14] and [15], $\pi_D(a) = Ba^{1-\sigma} - f$ , and $\pi_X(a) = \tilde{B}\left[ta\right]^{1-\sigma} - f_X$, so in equilibrium $B = fa_D^{\sigma-1}$ and $\tilde{B} = f_X\left[ta_X\right]^{\sigma-1}$. Thus

$$E(\pi) = f\int_0^{a_D}\left[\left(\frac{a_D}{a}\right)^{\sigma-1} - 1\right]dG(a) + f_X\int_0^{a_X}\left[\left(\frac{a_X}{a}\right)^{\sigma-1} - 1\right]dG(a) = fQ(a_D) + f_XQ(a_X)$$

When all home firms export we have $\pi_X(a) = \tilde{B}\left[ta\right]^{1-\sigma} - f_X$ as before, but our solution for $\tilde{B}$ must now come from $\tilde{B} = f\tilde{a}_D^{\sigma-1}$, yielding

$\pi_X(a) = \tilde{B}\left[ta\right]^{1-\sigma} - f_X = f_X\left[\left(\frac{f}{f_X}t^{1-\sigma}\right)\left(\frac{\tilde{a}_D}{a}\right)^{\sigma-1} - 1\right]$. Using $\phi = \left[\frac{f_X}{f}\right]^{\frac{1}{\sigma-1}}t$, this becomes

$\pi_X(a) = f_X\left[\left(\frac{\phi^{-1}\tilde{a}_D}{a}\right)^{\sigma-1} - 1\right]$. In which case $E(\pi) = fQ(a_D) + f_XQ'(a_D, \phi^{-1}\tilde{a}_D)$, where

$Q'(a_D, \phi^{-1}\tilde{a}_D) = \int_0^{a_D}\left[\left(\frac{\phi^{-1}\tilde{a}_D}{a}\right)^{\sigma-1} - 1\right]dG(a)$.

### (b) Slopes of the expected profit schedules.

When non-exporters exist in the home country we have $E(\pi) = fQ(a_D) + f_XQ(\phi^{-1}\tilde{a}_D) = f_E$

So $$\left.\frac{\partial a_D}{\partial \tilde{a}_D}\right|_{J=0} = -\frac{f_X}{f}\frac{\frac{\partial Q(\phi^{-1}\tilde{a}_D)}{\partial \tilde{a}_D}}{\frac{\partial Q(a_D)}{\partial a_D}}$$

From [A1] $$\frac{\partial Q(a_D)}{\partial a_D} = [\sigma-1]\left\{\left[\int_0^{a_D}\frac{G(a)}{a^{\sigma}}da\right][\sigma-1]a_D^{\sigma-1} + a_D^{\sigma-1}\frac{G(a_D)}{a_D^{\sigma}}\right\}$$

Which can be written as $\frac{\partial Q(a_D)}{\partial a_D} = \frac{[\sigma-1]}{a_D} H(a_D) = [\sigma-1] a_D^{\sigma-2} X(a_D)$ [A7]

Similarly $\frac{\partial Q(\phi^{-1}\tilde{a}_D)}{\partial \tilde{a}_D} = \frac{[\sigma-1]}{\tilde{a}_D} H(\phi^{-1}\tilde{a}_D) = [\sigma-1] \frac{\tilde{a}_D^{\sigma-2}}{\phi^{\sigma-1}} X(\phi^{-1}\tilde{a}_D)$

And $\left.\frac{\partial a_D}{\partial \tilde{a}_D}\right|_{J=0} = -\frac{a_D}{\tilde{a}_D}\frac{f_X}{f}\frac{H(\phi^{-1}\tilde{a}_D)}{H(a_D)} = -\frac{f_X}{f}\frac{1}{\phi^{\sigma-1}}\left[\frac{\tilde{a}_D}{a_D}\right]^{\sigma-2}\frac{X(\phi^{-1}\tilde{a}_D)}{X(a_D)}$ [A8]

Substituting for $\phi$ from [16] we have

$$\left.\frac{\partial a_D}{\partial \tilde{a}_D}\right|_{E(\pi)=f_E} = -\frac{1}{t^{\sigma-1}}\left[\frac{\tilde{a}_D}{a_D}\right]^{\sigma-2}\frac{X(\phi^{-1}\tilde{a}_D)}{X(a_D)}$$

The foreign entrant expected profit schedule correspondingly yields

$$\left.\frac{\partial a_D}{\partial \tilde{a}_D}\right|_{\tilde{J}=0} = -\frac{a_D}{\tilde{a}_D}\left[\frac{f_X}{f}\frac{\tilde{H}(\phi^{-1}a_D)}{\tilde{H}(\tilde{a}_D)}\right]^{-1} = -\left[\frac{\tilde{a}_D}{a_D}\right]^{\sigma-2}\left[\frac{1}{t^{\sigma-1}}\frac{\tilde{X}(\phi^{-1}a_D)}{\tilde{X}(\tilde{a}_D)}\right]^{-1} \quad \text{[A9]}$$

When all home firms export the home expected profit schedule becomes

$E(\pi) = fQ(a_D) + f_X Q'(a_D, \phi^{-1}\tilde{a}_D) = f_E$

So $\left.\frac{\partial a_D}{\partial \tilde{a}_D}\right|_{J'=0} = -\frac{f_X \frac{\partial Q'(a_D,\phi^{-1}\tilde{a}_D)}{\partial \tilde{a}_D}}{\left[f\frac{\partial Q(a_D)}{\partial a_D} + f_X\frac{\partial Q'(a_D,\phi^{-1}\tilde{a}_D)}{\partial a_D}\right]}$ [A10]

Now $Q'(a_D, \phi^{-1}\tilde{a}_D) \equiv \int_0^{a_D}\left\{\left[\frac{\phi^{-1}\tilde{a}_D}{a}\right]^{\sigma-1} - 1\right\} dG(a) = [\phi^{-1}\tilde{a}_D]^{\sigma-1}\int_0^{a_D} a^{1-\sigma} dG(a) - G(a_D)$

Which, using [A10] can be written as

$Q'(a_D, \phi^{-1}\tilde{a}_D) = \gamma[Q(a_D) + G(a_D)] - G(a_D)$

where $\gamma = \left[\frac{\phi^{-1}\tilde{a}_D}{a_D}\right]^{\sigma-1}$, so that $\gamma \geq 1$ over the relevant range.

It then follows that $\frac{\partial Q'(a_D,\phi^{-1}\tilde{a}_D)}{\partial \tilde{a}_D} = H(a_D)\frac{\partial \gamma}{\partial \tilde{a}_D} = \frac{[\sigma-1]}{\tilde{a}_D}\gamma H(a_D)$

While $\frac{\partial Q'(a_D,\phi^{-1}\tilde{a}_D)}{\partial a_D} = \gamma\left[\frac{\partial Q(a_D)}{\partial a_D} + g(a_D)\right] - g(a_D) + H(a_D)\frac{\partial \gamma}{\partial a_D}$

$$= \gamma\frac{[\sigma-1]}{a_D}H(a_D) + [\gamma-1]g(a_D) - H(a_D)\gamma\frac{[\sigma-1]}{a_D} = [\gamma-1]g(a_D)$$

where $g(a) = dG(a)$. Substituting these solutions in [A9] we obtain

$$\left.\frac{\partial a_D}{\partial \tilde{a}_D}\right|_{J'=0} = -\frac{a_D}{\tilde{a}_D}\frac{f_X[\sigma-1]H(a_D)\gamma}{\left\{f[\sigma-1]H(a_D)+f_X a_D g(a_D)[\gamma-1]\right\}} = -\left[\frac{\tilde{a}_D}{a_D}\right]^{\sigma-2}\frac{1}{t^{\sigma-1}}\frac{1}{[1+K]} \qquad \text{[A11]}$$

where $K = \frac{f_X}{f}\frac{a_D g(a_D)}{[\sigma-1]}[\gamma-1] \geq 0$ as $\gamma \geq 1$.

The slope of the foreign schedule is as given in [A9], and the foreign schedule is still steeper at the equilibrium since

$$\Delta' = 1 - \frac{1}{1+K}\frac{\tilde{Z}(\phi^{-1}a_D)}{\tilde{Z}(\tilde{a}_D)}\frac{1}{[t^{\sigma-1}]^2} > 0.$$

Note that at the point of regime change [ $a_D = \phi^{-1}\tilde{a}_D$ ] the slopes of both elements of the home expected profit schedule are equal since $\left.\frac{\partial a_D}{\partial \tilde{a}_D}\right|_{J=0} = -\left[\frac{f_X}{f}\right]^{\frac{\sigma-2}{\sigma-1}} = \left.\frac{\partial a_D}{\partial \tilde{a}_D}\right|_{J'=0}$

When all home firms export, the elasticity of substitution on the home expected profit schedule (i.e. $\hat{a}_D = -\varepsilon'\hat{\tilde{a}}_D$) becomes

$$\varepsilon' = \frac{f_X}{f}\gamma\frac{1}{1+K} = \frac{1}{t^{\sigma-1}}\left[\frac{\tilde{a}_D}{a_D}\right]^{\sigma-1}\frac{1}{1+K} = \left[\frac{\tilde{a}_D}{t a_D}\right]^{\sigma-1}\frac{1}{1+K}$$

**Second Derivatives**

When home non-exporters are present we can show that

$$\left.\frac{\partial^2 a_D}{\partial \tilde{a}_D^2}\right|_{J=0} = -\frac{a_D}{\tilde{a}_D^2}\varepsilon\left\{[1+\varepsilon][\sigma-2]+\varepsilon\zeta(a_D)+\zeta(\phi^{-1}\tilde{a}_D)\right\}$$

where $\zeta(x) = \frac{xg(x)}{H(x)} > 0$. A sufficient condition for this to be negative is that $\sigma > 2$.

When all home firms export, the equivalent expression is

$$\left.\frac{\partial^2 a_D}{\partial \tilde{a}_D^2}\right|_{J'=0} = -\frac{a_D}{\tilde{a}_D^2}\varepsilon'\left\{[1+\varepsilon'][\sigma-2]+\frac{\delta' K\xi}{[1+K]}-\frac{f_X}{f}\zeta(a_D)\frac{\left(\gamma+\varepsilon'[2\gamma-1]\right)}{[1+K]}\right\}$$

where $\xi \equiv \frac{a_D}{g(a_D)}\frac{\partial g(a_D)}{\partial a_D}$, about which we know little.

At the point of regime change we have

$$\left.\frac{\partial^2 a_D}{\partial \tilde{a}_D^2}\right|_{J=0} = -\frac{f_X}{f}\frac{[1+\varepsilon]}{\phi \tilde{a}_D}\left\{[\sigma-2]+\zeta(a_D)\right\} < -\frac{f_X}{f}\frac{[1+\varepsilon']}{\phi \tilde{a}_D}\left\{[\sigma-2]-\frac{f_X}{f}\zeta(a_D)\right\} = \left.\frac{\partial^2 a_D}{\partial \tilde{a}_D^2}\right|_{J'=0}$$

since $\varepsilon = \varepsilon' = \frac{f_X}{f}$ when $a_D = \phi^{-1}\tilde{a}_D$.

Note that when not all home firms export

$$\left.\frac{\partial \varepsilon}{\partial \tilde{a}_D}\right|_{J=0} = \varepsilon\left\{[1+\varepsilon][\sigma-1]+\varepsilon\zeta(a_D)+\zeta(\phi^{-1}\tilde{a}_D)\right\} > 0$$

while

$$\left.\frac{\partial \varepsilon'}{\partial \tilde{a}_D}\right|_{J'=0} = -\frac{\varepsilon'}{\tilde{a}_D}\left\{[1+\varepsilon'][\sigma-1]+\frac{\varepsilon' K\xi}{[1+K]}-\frac{f_X}{f}\zeta(a_D)\frac{\left(\gamma+\varepsilon'[2\gamma-1]\right)}{[1+K]}\right\}$$

**(c) The effects of technology change on profit substitution elasticities**

From the definition of the substitution elasticities, we have

$$\varepsilon = \frac{f_X H(a_X)}{fH(a_D)} = \frac{f_X[G(a_X)+Q(a_X)]}{f[G(a_D)+Q(a_D)]} \quad \text{and} \quad \tilde{\varepsilon} = \frac{f_X[\tilde{G}(\tilde{a}_X)+\tilde{Q}(\tilde{a}_X)]}{f[\tilde{G}(\tilde{a}_D)+\tilde{Q}(\tilde{a}_D)]}$$

where $a_X = \phi^{-1}\tilde{a}_D$, and $\tilde{a}_X = \phi^{-1}a_D$. In equilibrium entrant expected profits must equal entry costs in each market – i.e. $fQ(a_D) + f_X Q(a_X) = f_e = f\tilde{Q}(\tilde{a}_D) + f_X\tilde{Q}(\tilde{a}_X)$.

Now suppose that, from an initial equilibrium ($a_D^0$ etc.), there is an improvement in the home technology from $G^0(a)$ to $G^1(a)$, where $G^1(a) \geq G^0(a)$ for all $a \in [0, a_D]$ and $G^1(a) > G^0(a)$ for some $a \in [0, a_D]$. Since $Q(a)$ is increasing in $G(a)$, this implies that $fQ^1(a_D^0) + f_X Q(a_X^0) > f_e$, leading to a change in equilibrium cutoffs as described above – i.e. $a_D^1 < a_D^0, \tilde{a}_X^1 < \tilde{a}_X^0, \tilde{a}_D^1 > \tilde{a}_D^0$ and $a_X^1 > a_X^0$. The result is that:

(a) $\tilde{Q}(\tilde{a}_D^1) > \tilde{Q}(\tilde{a}_D^0)$ and $\tilde{Q}(\tilde{a}_X^1) < \tilde{Q}(\tilde{a}_X^0)$, since $\tilde{Q}(.)$ is an increasing function, and the foreign technology has not changed;

(b) $Q^1(a_D^0) \geq Q^0(a_D^0) > Q^1(a_D^1)$ and $Q^1(a_X^1) > Q^1(a_X^0) \geq Q^0(a_X^0)$, since home entrant expected profits are equal in the two equilibria (i.e. $fQ^1(a_D^1) + f_X Q^1(a_X^1) = f_e = fQ^0(a_D^0) + f_X Q^0(a_X^0)$) and expected profits from exporting can have increased both from the technology improvement and from the increase in the home export cutoff;

(c) $\tilde{G}(\tilde{a}_D^1) > \tilde{G}(\tilde{a}_D^0)$ and $\tilde{G}(\tilde{a}_X^1) < \tilde{G}(\tilde{a}_X^0)$, since the foreign technology has not changed; and

(d) $G^1(a_X^1) > G^0(a_X^0)$ since the home technology has improved and the export cutoff has risen. But $G^1(a_D^1) >$ or $< G^0(a_D^0)$, since, although the home survival cutoff has fallen, the home technology has improved.

These outcomes allow us to conclude that $\tilde{H}(\tilde{a}_D^1) > \tilde{H}(\tilde{a}_D^0)$ and $\tilde{H}(\tilde{a}_X^1) < \tilde{H}(\tilde{a}_X^0)$, hence $\tilde{\varepsilon}^1 < \tilde{\varepsilon}^0$. Similarly $H^1(a_X^1) > H^0(a_X^0)$. Furthermore and $H^1(a_D^1) < H^0(a_D^0)$ *unless* $G^1(a_D^1) - G^0(a_D^0) > Q^0(a_D^0) - Q^1(a_D^1) > 0$. There is a very strong presumption therefore that $\varepsilon^1 > \varepsilon^0$, and an even stronger presumption that $\varepsilon^1 - \tilde{\varepsilon}^1 > \varepsilon^0 - \tilde{\varepsilon}^0 \geq 0$, i.e. that the difference between the two elasticities increases as the home technology improves.

**(d) Numbers of Firms**

When non-exporters exist in the home country, the domestic revenue of the average home firm is $r(\bar{a}_D) = A\left[\frac{\bar{a}_D}{\rho}\right]^{1-\sigma}$ where

$$\bar{a}_D = \left[\int_0^{a_D} a^{1-\sigma} dS(a)\right]^{\frac{1}{1-\sigma}} = \left[\frac{\int_0^{a_D} a^{1-\sigma} dG(a)}{G(a_D)}\right]^{\frac{1}{1-\sigma}} = \left[\frac{X(a_D)}{G(a_D)}\right]^{\frac{1}{1-\sigma}}$$

Similarly export revenue of the average foreign exporter is $r(\tilde{\bar{a}}_X) = A\left[\frac{t\tilde{\bar{a}}_X}{\rho}\right]^{1-\sigma}$

where $\tilde{\bar{a}}_X = \left[\int_0^{\tilde{a}_X} a^{1-\sigma} d\tilde{S}_X(a)\right]^{\frac{1}{1-\sigma}} = \left[\frac{\int_0^{\tilde{a}_X} a^{1-\sigma} d\tilde{G}(a)}{\tilde{G}(\tilde{a}_X)}\right]^{\frac{1}{1-\sigma}} = \left[\frac{\tilde{X}(\tilde{a}_X)}{\tilde{G}(\tilde{a}_X)}\right]^{\frac{1}{1-\sigma}}$.

Then using $A = \frac{B}{[1-\rho]\rho^{\sigma-1}} = \frac{f a_D^{\sigma-1}}{[1-\rho]\rho^{\sigma-1}}$ and $A = \frac{f_X\left[t\tilde{a}_X\right]^{\sigma-1}}{[1-\rho]\rho^{\sigma-1}}$ gives

$$r(\bar{a}_D) = \frac{f}{[1-\rho]}\frac{a_D^{\sigma-1}X(a_D)}{G(a_D)} = \frac{f}{[1-\rho]}\frac{H(a_D)}{G(a_D)} \text{ and } r(\tilde{\bar{a}}_X) = \frac{f_X}{[1-\rho]}\frac{\tilde{a}_X^{\sigma-1}\tilde{X}(\tilde{a}_X)}{\tilde{G}(\tilde{a}_X)} = \frac{f_X}{[1-\rho]}\frac{\tilde{H}(\tilde{a}_X)}{\tilde{G}(\tilde{a}_X)}$$

The expenditure equation for the home market is therefore

$$\beta L = R_D + R_I = Mr(\bar{a}_D) + \tilde{M}\frac{\tilde{G}(\tilde{a}_X)}{\tilde{G}(\tilde{a}_D)}r(\tilde{\bar{a}}_X) = M\frac{f}{[1-\rho]}\frac{H(a_D)}{G(a_D)} + \tilde{M}\frac{f_X}{[1-\rho]}\frac{\tilde{H}(\tilde{a}_X)}{\tilde{G}(\tilde{a}_D)}$$

Using the definition of $\tilde{\varepsilon}$ from [23B] we have $f_X\tilde{H}(\tilde{a}_X) = \tilde{\varepsilon} f\tilde{H}(\tilde{a}_D)$, and since

$r(\tilde{\bar{a}}_D) = \frac{f}{[1-\rho]}\frac{\tilde{H}(\tilde{a}_D)}{\tilde{G}(\tilde{a}_D)}$, we have $\beta L = Mr(\bar{a}_D) + \tilde{\varepsilon}\tilde{M}r(\tilde{\bar{a}}_D)$

The same procedure applied to the foreign expenditure equation yields $\beta\tilde{L} = \varepsilon M r(\bar{a}_D) + \tilde{M} r(\tilde{\bar{a}}_D)$. These can then be solved straightforwardly for the equilibrium numbers of firms in the two countries as in the text.

When all home firms export, the home expenditure equation remains as above, but the foreign expenditure equation requires some modification. We have $r(\tilde{\bar{a}}_D) = \tilde{A}\left[\frac{\tilde{\bar{a}}_D}{\rho}\right]^{1-\sigma}$,

where $\tilde{\bar{a}}_D = \left[\frac{\tilde{X}(\tilde{a}_D)}{\tilde{G}(\tilde{a}_D)}\right]^{\frac{1}{1-\sigma}}$, and $r(\bar{a}_X) = A\left[\frac{t\bar{a}_D}{\rho}\right]^{1-\sigma}$, where $\bar{a}_D = \left[\frac{X(a_D)}{G(a_D)}\right]^{\frac{1}{1-\sigma}}$. Proceeding as above $r(\tilde{\bar{a}}_D) = \frac{f}{[1-\rho]}\frac{\tilde{H}(\tilde{a}_D)}{\tilde{G}(\tilde{a}_D)}$. To solve for representative home firm export sales, we use $\tilde{B} = f\tilde{a}_D^{\sigma-1}$, obtaining $r(\bar{a}_X) = \frac{f\tilde{a}_D^{\sigma-1}}{[1-\rho]}\frac{t^{1-\sigma}}{a_D^{\sigma-1}}\frac{a_D^{\sigma-1}X(a_D)}{G(a_D)} = \frac{ft^{1-\sigma}}{[1-\rho]}\left[\frac{\tilde{a}_D}{a_D}\right]^{\sigma-1}\frac{H(a_D)}{G(a_D)}$.

From [16] $ft^{1-\sigma} = f_X\phi^{1-\sigma}$, so that $r(\bar{a}_X) = \frac{f_X}{[1-\rho]}\left[\frac{\phi^{-1}\tilde{a}_D}{a_D}\right]^{\sigma-1}\frac{H(a_D)}{G(a_D)}$. Then

$$\beta\tilde{L} = \tilde{R}_D + \tilde{R}_I = Mr(\bar{a}_X) + \tilde{M}r(\tilde{\bar{a}}_D) = M\frac{f_X}{[1-\rho]}\gamma\frac{H(a_D)}{G(a_D)} + \tilde{M}\frac{f}{[1-\rho]}\frac{\tilde{H}(\tilde{a}_D)}{\tilde{G}(\tilde{a}_D)}.$$

Since $r(\bar{a}_D) = \frac{f}{[1-\rho]}\frac{H(a_D)}{G(a_D)}$, and $f_X\gamma = \varepsilon'[1+K]f$, we can write

$$\beta\tilde{L} = [1+K]\varepsilon' r(\bar{a}_D)M + r(\tilde{\bar{a}}_D)\tilde{M}.$$

Again this combined with the home equation can be solved straightforwardly for the equilibrium numbers of firms – i.e.

$$r(\bar{a}_D)M + \tilde{\varepsilon}\tilde{r}(\tilde{\bar{a}}_D)\tilde{M} = \beta L$$

$$[1+K]\varepsilon' r(\bar{a}_D)M + \tilde{r}(\tilde{\bar{a}}_D)\tilde{M} = \beta\tilde{L}$$

and yields solutions

$$M = \frac{\beta L}{r(\bar{a}_D)\Delta'}[1-\frac{\tilde{L}}{L}\tilde{\varepsilon}] \text{ and } \tilde{M} = \frac{\beta\tilde{L}}{\tilde{r}(\tilde{\bar{a}}_D)\Delta'}\left\{1-\frac{\tilde{L}}{L}[1+K]\varepsilon'\right\}$$

If all home firms export, the condition for the existence of a home industry remains as in [24A]. At this point it is useful to define $\mu = [1+K]\varepsilon' = \frac{f_X}{f}\gamma = \left[\frac{\tilde{a}_D}{ta_D}\right]^{\sigma-1}$. The condition for the foreign industry to survive ($\tilde{M} > 0$) is now

$$1 > \frac{L}{\tilde{L}}\mu$$

and for intra-industry trade is

$$\tilde{\varepsilon}^{-1} > \frac{\tilde{L}}{L} > \mu$$

As before there are no firms in the home country if the left inequality fails and no firms in the foreign country if the right inequality fails. If $f_X > f$ then $\mu > 1$ and the industry in the laggard country only survives if that country is sufficiently larger than the leading country. Even if $f_X < f$ it will still disappear once $\gamma$ becomes large enough[41].

**(e) Effects of changes in trade costs**

To determine the effects of changes in unit trade costs on equilibrium cutoffs, we differentiate the zero expected profit conditions [18A] and [19A], using [A7] etc. Converting to proportional changes we have

$$\begin{bmatrix} fH(a_D) & f_X H(\phi^{-1}\tilde{a}_D) \\ f_X \tilde{H}(\phi^{-1}a_D) & f\tilde{H}(\tilde{a}_D) \end{bmatrix} \begin{bmatrix} \hat{a}_D \\ \hat{\tilde{a}}_D \end{bmatrix} = \begin{bmatrix} f_X H(\phi^{-1}\tilde{a}_D) \\ f_X \tilde{H}(\phi^{-1}a_D) \end{bmatrix} \hat{\phi}$$

Now the definitions of the profit substitution elasticities from [23A] and [23B] imply that $f_X H(\phi^{-1}\tilde{a}_D) = \varepsilon\, fH(a_D)$ and $f_X \tilde{H}(\phi^{-1}a_D) = \tilde{\varepsilon}\, f\tilde{H}(\tilde{a}_D)$. Substituting these expressions and canceling the common terms gives us $\begin{bmatrix} 1 & \varepsilon \\ \tilde{\varepsilon} & 1 \end{bmatrix} \begin{bmatrix} \hat{a}_D \\ \hat{\tilde{a}}_D \end{bmatrix} = \begin{bmatrix} \varepsilon \\ \tilde{\varepsilon} \end{bmatrix} \hat{\phi}$ as in the text.

If all home firms export the system becomes

$$\begin{bmatrix} 1 & \varepsilon' \\ \tilde{\varepsilon} & 1 \end{bmatrix} \begin{bmatrix} \hat{a}_D \\ \hat{\tilde{a}}_D \end{bmatrix} = \begin{bmatrix} \varepsilon' \\ \tilde{\varepsilon} \end{bmatrix} \hat{t}$$

with solutions $\hat{a}_D = \frac{\varepsilon'}{\Delta'}[1-\tilde{\varepsilon}]\hat{t} = \hat{a}_X$

$$\hat{\tilde{a}}_D = \frac{\tilde{\varepsilon}}{\Delta'}[1-\varepsilon']\hat{t} \qquad \text{and} \qquad \hat{\tilde{a}}_X = \frac{1}{\Delta'}[\varepsilon'-1]\hat{t}$$

$$\hat{\tilde{a}}_X = \frac{1}{\Delta'}[\varepsilon'-\tilde{\varepsilon}]\hat{t}$$

[41] While $\tilde{a}_D$ is capped at $\tilde{\alpha}_D$, $a_D$ tends to 0 as the technology difference becomes large.

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**Figure 1**: **Equilibrium survival cost cut offs in the open economy**